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No. 24.

REPORT

UPON

THE PRIMARY TRIANGULATION

OF THE

UNITED STATES LAKE SURVEY.

BY

LIEUT.-COL. C. B. COMSTOCK, CORPS OF ENGINEERS,

BREVET BRIGADIER-GENERAL, U. S. A.,

AIDED BY THE

ASSISTANTS ON THE SURVEY.

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LETTER OF TRANSMITTAL.

LAKE-SURVEY OFFICE,
Detroit, Mich., August 12, 1882.

General H. G. WRIGHT,
Chief of Engineers, U. S. A.:

GENERAL: I have the honor to submit herewith the final report of the Lake Survey.

The original observations have not been given in detail from the fear of making the report too bulky. But the note-books containing the original observations, as well as the reductions, will be filed in your office.

It had been hoped to express all distances in the report in terms of the international metre to be adopted by the *Bureau International des Poids et Mesures*, but the delay in the preparation of that metre has made it impracticable, and accordingly all distances are given in English feet. The value of the Lake-Survey metre, *R*1876, is probably known in terms of what the international metre will be within one or two thousandths of a millimeter, and this value is given in an appendix to the report. *R*1876 has been sent to the *Bureau International des Poids et Mesures* for an accurate determination of its length. When this is known, the report contains the data for expressing all its distances in terms of the metre.

The results for the difference of longitude of Detroit and Cambridge were not available in time for insertion in the body of the report. They are given in an appendix, and show that no modification of the longitude of Detroit with reference to Washington, as given in the report, is necessary.

In the preparation of the report, while all the assistants have been engaged in the work, I have been especially indebted to Mr. R. S. Woodward, who has aided in preparing the manuscript and has attended to the proof-reading. The following chapters of the report were prepared under my supervision entirely or mainly by the persons named:

Chapter I by First Lieutenant P. M. Price, U. S. Engineers.

Chapter VIII by Assistant Engineer E. S. Wheeler.

Chapters XVI to XX, and Chapters XXIV and XXV by Assistant Engineer R. S. Woodward.

Chapter XXII by Assistant Engineer L. L. Wheeler.

I wish to call your attention to the valuable services of the following assistants, who have remained upon the work till its close.

E. S. Wheeler, employed in measurement of bases and comparisons of standards.

A. R. Flint, O. B. Wheeler, R. S. Woodward, and J. H. Darling, employed in primary triangulation.

T. Russell and T. W. Wright, employed in computation.

To these gentlemen, and to those assistants, whether civilians or officers of Engineers, who have previously been employed on the Survey while it has been under my charge, I wish to tender my thanks for their efficient aid and for their cordial co-operation in all attempts to make the work precise and reliable.

Very respectfully, your obedient servant,

C. B. COMSTOCK,
Lieutenant-Colonel of Engineers,
Brevet Brigadier-General, U. S. A.

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ERRATA.

Page 1, last line, read *improvements* for *improvments.*
Page 16, 10th line, read *local times* for *local time.*
Page 42, Chart 15, 5th column, read *J. H. Forster, C. P. Rabaut* for *J. H. Foster, P. C. Rabaut.*
Page 52, quantities in last column of tables should have minus signs.
Page 54, 15th line from bottom, 2d column, read 0.100114 for 0.109114.
Page 82, 20th line, read $0^{in}.0000003333$ for $0^{in}.0000006666$.
Page 89, under tube 2, Keweenaw Base, 4th line, 4th column, read l'_1 for l_1.
under tube 2, Sandy Creek Base, 6th line, 2d column, read 84.8 for 84.2.
Page 96, 9th line, read *squares of* for *square of.*
4th line from bottom, read § 14 for § 11.
Page 98, 6th line of § 6, read § 14 for § 11.
Page 99, 20th line of § 7, read § 14 for § 11.
Page 101, 1st line, read § 14 for § 11.
Page 103, 17th and 33d lines, read 1½ *feet* for 6 *feet.*
Page 105, last line, read § 32 for § 31.
Page 109, heading of 2d column of table, read Time of maximum *or minimum* length of bar.
Page 121, unit of quantities in last three columns of table is the inch.
Page 122, ditto.
Page 128, heading of 5th column of table, read Product of mean l by number of tubes.
Page 130, end of 10th line from bottom, read $[ax]$ for ax.
last line, read $\frac{\pm 0.690}{6} = \pm 0.115$ for $\frac{\pm 0.686}{6} = \pm 0.114$.
Page 131, 3d line of § 6, read $(0.115)^2$ for $(0.114)^2$.
Page 151, 16th line from bottom, read (2) for (1).
Page 153, 5th date in table, read 9:21 *a. m.* for 9:21 *p. m.*
last line, last column, read —0.4.
Page 158, 6th date, 2d column, read 60.2 for 62.2.
Page 161, 3d date, 5th column, read —0.3 for —0.
Page 173, 1st line of § 9, read 1000th for 100th.
Page 178, 6th line from bottom, 3d column, read +91.73 for +91.93.
Page 179, 6th line of table, 4th column, read —0.7 for 0.7.
Page 196, 7th line, read $-42^{\mu}.2$ for $-4^{\mu}.2$.
19th line, read $\frac{3^{\mu}.1}{25^{\mu}} \times 1^{\circ}$.
Page 206, 8th line of Aug. 26, 3d column, read 2541 for 3541.
Page 220, 4th line of § 41, insert *these* after *when* and commas after *bars* and *normal lengths.*
Page 221, 15th line, read *naturally* for *natually.*
Page 222, 23d line, read $0^{\mu}.5$ for 0^{μ}.
Page 223, 21st line from bottom, read $S_2 - B$ for $S_1 - B$.
Page 232, 24th line from bottom, read *inclination of the ends* for *inclinations of the end.*
Page 233, 20th line from bottom, read *are* for *is.*
Page 236, 13th line from bottom, read $2^{\mu}.4$ for $2^{\mu}4$.
Page 237, 13th line, insert *is referred* after *which.*
16th line, read $E_B - E_{S_2}$ for $E_B - E_S$.
Page 240, 1st line of § 60, read §§ 15–18 for § 18.
Page 251, 16th line, read $100b_1 \sin \frac{f}{2}$ for $100a_2 \sin \frac{f}{2}$.
17th line, read $100a_2 \sin f$ for $100b_1 \sin f$.
Page 253, 2d table, 4th column, 4th line, read 100.9 for 101.9.
Page 256, 19th line from bottom, read 2488.4 for 2488.0.
18th line from bottom, read 2986.2 for 2986.0.

Page 261, end of 5th paragraph, insert *See Plate XIII.*
Page 262, equation (3), read E_{S_1} for E_S.
Page 263, 10th line, read $\frac{2\beta}{\alpha'-\alpha}$ for $\frac{2\beta}{\alpha-\alpha}$.
Page 269, heading of 7th column of table, read $Z_1-S_1=m$ for $Z_1-S_1=n$.
Page 271, 7th line from bottom, read $\frac{E_{S_1}}{E_{Z_1}-E_{S_1}}$ for $\frac{E_{S_1}}{E_Z-E_{S_1}}$.
Page 280, 13th line from bottom, read $0^{mm}.01$ for $0^{mm}.1$.
Page 282, at head of 1st column, read 1877 for 1878.
under Three Cut-off Plates, Sandusky Base, First Measurement, insert in 1st column, 1878.
Page 283, under Three Cut-off Plates, Olney Base, Second Measurement, 1st column, read 1879 for 1877.
Page 287, 19th line from bottom, read $+570^{\mu}.4$ for $+570^{\mu}$.
Page 288, 8th line of § 33, read $\pm 7^{mm}.14$ for $\pm 7^{mm}.4$.
11th line from bottom, read $(Z_1-S_1)^2$ for $Z_1-S_1)$.
Page 292, heading of 7th column of table, read $Z_1-S_1=m$ for $Z_1-S_1=n$.
Page 293, 5th column of table, 1st quantity, read -108151.8 for $+108151.8$.
Page 295, 3d line, read § 34 for § 33.
Page 301, 8th line from bottom, read $\frac{E_{S_1}}{E_{Z_1}-E_{S_1}}$ for $\frac{E_{S_9}}{E_{Z_9}-E_{S_8}}$.
Page 303, 11th line from bottom, insert *at* $60^{\circ}.292$ F. after R 1876 in parenthesis.
8th line from bottom, read $(60.292-59)$ for $(t-59)$.
Page 304, 4th line of § 7, read § 34 for § 33.
Page 311, 3d line of equations (5″), read $(m-n')$ for $(n-n')$.
Page 318, 12th line from bottom, 7th word, read *were* for *was*.
Page 321, 9th and 10th lines from bottom, read 242° for 240°.
Page 333, last equation under Crebassa — 26, read $+26.88$ for $+26.28$.
Page 334, absolute term of equation IV, read 34.197 for 34.179.
first term of equation IX, read 34.3099 for 24.3099.
sixth term of equation IX, read $+71.6531$ for -71.6531.
Page 335, fifth term of equation XX, read $+[19_1]$ for $-[19_1]$.
last term of $[28_3]$, read $+0.02667$ for $+0.02267$.
Page 336, 16th line from bottom, read $[15_{6+1}]$ for $[12_{6+1}]$.
Page 337, 16th term of equation 5, read -0.02990 for $+0.02990$.
Page 338, 5th term of equation 29, read XXIX for XIX.
Page 342, 5th line from bottom, read § 32 for § 31.
Page 343, 5th triangle, 4th column, read 5.0952226 for 5.0952326.
7th triangle, last column, read 5.2967696 for 5.2967796.
10th triangle, 4th column, read 5.6066564 for 5.6065564.
Page 344, 2d triangle, 4th column, read 4.9149578 for 5.9149578.
6th triangle, last column, read 5.0057294 for 5.0017294, and read 4.6912740 for 5.6912740.
7th triangle, last column, read 4.9666298 for 4.9666398.
9th triangle, 4th column, read 4.8480655 for 4.8480755.
Page 347, 2d line from bottom, read *south* 55° 50′ *east*, 36.2 *feet distant* for *south* 55° 60′ *east* 35.2 *feet distant*.
Page 361, 2d term of equation XII, read $-[3_1]$ for $+[3_1]$.
absolute term of equation XXIII, read -2.067.
Page 362, value of $[15_1]$, read *XXIII* for *XXII*.
Page 372, under Oshkosh — 39, 1st line, 7th column, read $-0''.767$ for $-0''.769$.
Page 375, 1st term of $[21_{4+5}]$, read *XXVIII* for *XXVII*.
Page 379, 5th term of equation 49, read *XLVIII* for *XLIX*.
Page 385, 2d term of $[43_{6+7}]$, read *LXXIII* for *LXIII*.
4th term of $[44_1]$, read -0.24390.
Page 394, 24th line, read 46.32 *metres* for 4632 *metres*.
Page 417, 3d term of $[18_3]$, read -1.02714 for $+1.02714$.
Page 419, 6th term of $[23_3]$, read -0.17593 for -0.17583.
9th term of $[23_7]$, read $+0.61779$ for -0.61779.
Page 420, 1st term of $[26_3]$, read $+0.66667$ for $+0.66677$.
Page 431, 14th line from bottom, read *north* 86° 56′ *east* for *north* 85° 56′ *east*.
Page 450, under Pittsford — 52, 3d line, 6th column, read $+0.277$ for -0.277.
under Pittsford — 52, 3d line, 7th column, read -0.028 for $+0.028$.
Page 451, under Dundee — 57, 4th line, 6th column, read -0.135 for -0.155.
Page 454, last term of $[51_1]$, read *LXXXVI* for *LXXVI*.
3d term of $[52_1]$, read *LXXIX* for *LXIXX*.
2d term of $[53_4]$, read -0.36364 for $+0.36364$.
5th term of $[56_1]$, read $+0.13033$ for -0.13033.

Page 465, absolute term of equation XV, read $+32.277$ for -32.277.
Page 467, 1st term of $[63_2]$, read $+0.85714$ for $+1.85714$.
Page 469, 1st term of $[68_{4+5}]$, read -0.28571 for. $+0.28571$.
Page 476, value of correlate XXXII, read $+0.4752$ for $+0.4572$.
Page 482, 4th triangle, 4th column, read 4.6981910 for 4.6981919.
Page 483, 3d line from bottom, read 84° 38′ *east* for 83° 38′ *east*.
Page 486, 4th line from bottom, read 8.2 *metres* for 8.02 *metres*.
Page 490, under Houghton — 48, 4th line, 7th column, read $+0.227$ for $+0.277$.
Pages 496–501, tops of pages, read *Section XII* for *Section XIII*.
Page 529, under Batavia — 27, 1st line, 7th column, read -0.072 for $+0.072$.
under Batavia — 27, 2d line, 7th column, read -0.747 for $+0.747$.
Page 535, 4th term of $[22_{4b}]$, read -0.03106 for -0.3106.
1st term of $[29_2]$, read $+0.72727$ for $+0.27727$.
Page 536, 4th term of equation 18, read -0.06258 for $+0.06258$.
5th term of equation 18, read -0.09558 for $+0.09558$.
Page 554, under Manteno — 7, absolute terms of first two normal equations, read -0.574 for $+0.574$
Page 555, under Watseka — 10, absolute terms of normal equations, read -0.440 for $+0.440$.
Page 559, under Palermo — 20, 5th line, 6th column, read $+0.164$ for -0.164.
Page 566, value of general correction $[14_2]$, read $+0.031$ for -0.031.
Page 569, under Claremont — 30, 4th normal equation, 6th term, read (30_6) for (36_6).
Page 575, 5th term of $[30_{3+4}]$, read 0.87264 for 087264.
Page 579, 3d term of equation 51, read 0.16909 for 0.06909.
Page 581, 2d term of equation 64, read $+0.20336$ for -0.20336.
19th term of equation 68, read $+2.42875$ for -2.42875.
Page 585, last line, 4th column, read 4.8466356 for 5.8466356.
Page 589, 1st triangle, 1st line, 6th column, read 4.9190836 for 4.9190835.
Page 590, 2d triangle from bottom, 1st line, 6th column, read 4.9758813 for 4.97 8813.
Page 603, (29 and 29*a*) — (28 and 28*a*), 4th column, read -0.2195 for $+0.2195$.
Page 604, last line, 3d column, read 2.78 for 2.72.
Page 624, 17th line from bottom of table, read *Gr.* 3680 for 3680.
Page 638, foot of table, probable error of weighted mean, read $\pm 0''.085$ for $\pm 0''.850$.
Page 642, 5th line below table, read $1''.20$ for $1''.09$.
Page 644, last line of first table, read 191° for 119°.
Page 645, 4th line, read *measurement* for *measument*.
Page 650, for Polaris, July 11, read $\delta = 88° \ 38' \ 03''.88$ for $\delta = 88° \ 38' \ 30''.88$.
Page 654, for δ Urs. Min., last column, read $05''.74$ for $054''.74$.
Page 666, for λ Urs. Min., 3d column, 8th line, read $09''.01$ for $19''.01$.
Page 668, for λ Urs. Min., 3d column, 9th line, read $58''.37$ for $58''.57$.
Page 677, for 51 Ceph., Nov. 24, 8th column, 4th line, read $30''.47$ for $30''.41$.
Page 682, for Polaris, Nov. 20, read $\delta = 88° \ 40' \ 26''.86$ for $\delta = 88° \ 40' \ 20''.86$.
Page 701, for λ Urs. Min., Aug. 25, 2d column, 1st line, read 352° for 357°.
Page 703, for λ Urs. Min., read *Aug.* 28 for *Aug.* 27.
Page 708, 6th line from bottom, read A_e for A.
Page 710, for δ Urs. Min., Sept. 20, 2d column, 1st line, read $33''.2$ for $32''.2$.
Page 719, 6th column of table, opposite 14 Pegasi, read 52.126 for 51.126.
Page 725, 9th line from bottom, put comma after *trigonometrical station*.
Page 738, 3d column, 3d line of Cedar River, read 44 for 46.
Page 743, insert 1873 in 2d column of Ada 1.
Page 755, 2d column, 3d line of Rochester, read *Aug.* 13 for *Aug.* 31.
Page 760, 5th column, 2d line of Lapeer, read -0.101 for $+0.101$.
Page 765, 2d column of Fort McKavett, read 1876 for 1869.
Page 766, 4th column, opposite Red Wing and Valley Junction, read 759 for 760.
4th column, opposite Deadwood, read 764 for 766.
Page 767, 4th column, opposite Newaygo, Saginaw, and Saint Louis, read 759 for 760.
4th column, opposite Stanton, Lapeer, Flint, Saint Johns, Ionia, and Corunna, read 760 for 761.
4th column, opposite Fort Fetterman and Camp Robinson, read 764 for 766.
4th column, opposite Lansing, Hastings, Pontiac, Howell, and Charlotte, read 761 for 762.
4th column, opposite Allegan and Galena, read 762 for 763.
Page 768, 4th column, opposite Kalamazoo, Marshall, and Jackson, read 762 for 763.
4th column, opposite Fort Laramie, Fort McDermit, and Ogden, read 764 for 766.
4th column, opposite Rock Island and Louisiana, read 763 for 764.
4th column, opposite Fort Leavenworth and Forts Richardson to McKavett, read 765 for 767.
Page 771, 7th line of Douglas, read $24^m.5$ for $25^m.5$.
Page 774, 7th line, read *replaced by* $\frac{1}{2}\,dZ$ *and* $\frac{1}{2}\,dM$ for *replaced by* dZ *and* dM.
Page 786, Longitude of D (St. Ignace), read 42′ for 43′.

Page 789, 1st line, 4th column of table, read 56′ for 51′.
Page 802, latitude of G (2d line of table) read 44° for 41°.
Page 819, 7th line from bottom, read *For triangles having* for *For a triangle having.*
Page 836, 17th line, read $+27^{\mu}.3$ for $+27^{\mu}.2$.
Page 837, 3d line, read $+0^{\mu}.0069\,t^2$ for $+0^{\mu}.0969\,t^2$.
27th line, read $+0^{\mu}.0039\,t^2$ for $+0^{\mu}.0030\,t^2$.
Page 838, 4th line from bottom, read *abschliesst* for *abschliest.*
Page 845, 5th line from bottom of page, 23d column, read $+1.0$ for $+0.1$.
Page 848, 17th line, read $+22.88$ for $+22.28$.
Page 854, table c, 3d line under A_8, read -9.321 for $+9.321$.
Page 856, table a, 2d line under C_{10}, read -40 for -41.
Page 857, 8th centimeter, under D_2, read $+25$.
Page 859, last line, 24th column, read -0.6.
Page 861, 6th line of §3, read $(0^{\mu}.076 \pm 0^{\mu}.029)$ for $(0^{\mu}.076 \pm 0^{\mu}.29)$.
Page 864, opposite Oct. 3, last column, read -3.
Page 873, 7th column, 3d line, read $+0.74$ for -0.74.
6th column, 5th line, read -0.56 for $+0.56$.
under COLLIMATION, read 750 Groombridge for 759 Groombridge.
Page 874, under OBSERVATION-EQUATIONS, 5th equation, read $+32.60a$ for $+32.66a$.
under OBSERVATION-EQUATIONS, 3d and 4th equations from last, read $+1.2\rho$ and $+1.3\rho$ for -1.2ρ and -1.3ρ.
Page 875, under OBSERVATION-EQUATIONS, 5th equation from last, read $+2.9\rho$ for -2.9ρ.
Page 876, under OBSERVATION-EQUATIONS, 6th equation from last, under *Weight*, read 0.112 for 0.012.
under OBSERVATION-EQUATIONS, last equation, read $+0.02$ for -0.02.
Page 877, opposite β Ophiuchi, 9th column, read 17^{h} for 77^{h}.
opposite 110 Herculis, 6th column, read $+0.14$ for -0.14.
under OBSERVATION-EQUATIONS, 1st equation, read $+\Delta\theta$ for $\pm\Delta\theta$.
under OBSERVATION-EQUATIONS, 5th equation from last, read $+\Delta\theta$ for $-\Delta\theta$.
Page 879, under OBSERVATION-EQUATIONS, 10th equation from last, read $r = -0.01$ for $r = +0.01$.
Page 882, under OBSERVATION-EQUATIONS, last equation, read $-4.74a$.
Page 885, 3d line, 7th column, read $+0.20$ for -0.20.
28th line, 1st column, read *η Herculis.*
Page 887, 2d line, 10th column, read $04^{s}.8$ for $05^{s}.8$.
under OBSERVATION-EQUATIONS, 5th equation from last, read $+\Delta\theta$ for $-\Delta\theta$.
Page 888, under OBSERVATION-EQUATIONS, 3d equation from last, read -0.06 for $+0.06$.
under NORMAL EQUATIONS, 2d equation, 3d term, and 3d equation, 2d term, read -2.44 for $+2.44$.
Page 889, heading to 6th column, read $C\,(c + \Delta\,i + ab'n)$.
Page 891, opposite ε Aquilae, 5th column, read $+0.14$ for -0.14.
Page 892, under OBSERVATION-EQUATIONS, second equation from last, under *Weight*, read 0.06 for 0.006.
under RESULTS, read $\Delta\theta = -0^{s}.005$ for $\Delta\theta = +0^{s}.005$.
Page 893, 4th line, 6th column, read $08^{s}.000$ for $68^{s}.000$.
6th line from bottom, 8th column, read $03^{s}.389$ for $05^{s}.389$.
Page 895, 2d line, 7th column, read $41^{s}.000$ for $41^{s}.010$.
Page 898, 15th line of § 4, read *determined with Needle No. 2* for *that are marked with a star.*
Page 911, 1st column, 4th item, read *Saint Louis, Mich.,* for *Saint Louis, Mo.*
Page 916, the following items in last column should be aligned to read as follows:

Place.	Observer.
Kenosha	From a survey under Captain Cuyler.
Benona	G. A. Marr.
Seven miles south of Sheboygan	J. P. Mayer.
Two miles south of Manitowoc	J. R. Mayer.

Page 919, 1st column, 8th line from bottom, read *Port Clinton* for *Point Clinton.*
Page 920, under LAKE ONTARIO, 3d column, 2d line from bottom of table, read 77° 43′ for 78° 43′.

PART I.

HISTORY.

CHAPTER I.

HISTORICAL ACCOUNT OF THE SURVEY OF THE NORTHERN AND NORTH-WESTERN LAKES—MAY, 1841, TO JULY 1, 1881.

INTRODUCTORY.

§ **1.** The Lake Survey was begun in 1841 under an appropriation of $15,000, made in May of that year. At this time the country bordering on the lower lakes was already pretty well settled and works for the improvement or formation of harbors had been commenced at most of the important points on Lakes Erie and Ontario. The upper-lake region was but thinly settled, and there were no good harbors on Lake Huron, and but one, the harbor of Chicago, on Lake Michigan. Settlers were, however, pouring in rapidly, and there was even then a large and constantly increasing commerce between the lake ports, especially from Buffalo to Detroit and Chicago. Communication with Lake Superior could only be had by portage around the Sault Ste. Marie, but the great mineral wealth of the Lake Superior country was attracting attention, and a survey for a ship-canal had been made in 1840 by officers of the Topographical Engineers. The lake commerce was carried on under many difficulties, which caused much loss of life and property each year.

There were no charts of the lakes except the admiralty charts compiled from the surveys of Captain H. W. Bayfield, of the Royal Navy (English), and these were not in general use by the masters of American vessels. These charts were the results of rapid reconnaissances, and although they showed the coast lines with an accuracy which is remarkable considering the rough methods of surveying employed, they were of little value as hydrographical charts of the American coast, because they showed the depths of water in comparatively few places, and but a small number of the many reefs and shoals which are found along the lake shores.

There were few light-houses and beacons to indicate the positions of dangers to navigation, and, in the absence of charts, pilots were obliged to rely upon their own knowledge, which was frequently only acquired by the vessel's grounding on a shoal or striking a hidden rock.

The navigation of the lakes is attended with peculiar dangers, because, while violent gales are frequent and the storms rival those of the ocean itself, a vessel is never more than a few hours' run from the shore, and cannot, as is generally the case at sea, drift before the wind until the storm is over, but in a long-continued gale must be thrown upon the shore, unless a port or harbor of refuge can be entered. In 1841, a vessel leaving Chicago found no harbor or shelter in storms until the Manitou or Beaver Islands were reached, and after passing the Straits of Mackinac, it was again exposed without refuge on Lake Huron, except in the vicinity of Presqu' Isle, until the head of the Saint Clair River was reached. In sailing from Chicago to Buffalo, the greatest difficulties were encountered in the vicinity of the Straits of Mackinac and in the west end of Lake Erie, on account of the many islands, shoals, and reefs found in those localities; and at the mouth of the Saint Clair River, at which no improvments had been made in 1841, and where the channels were not only circuitous and narrow, but so shoal, that vessels in low-water seasons frequently were compelled to have their cargoes taken over the bars in lighters.

It was therefore with the double object of furnishing reliable charts to lake vessels, and of determining from the surveys the works of improvement which were necessary to the prosperity of the lake commerce, that Congress in 1841 directed a survey of the lakes, and that annual appropriations, with the single exception of the year 1847, have since been made for carrying on the Survey. Some idea of the magnitude of the work may be had from the following dimensions:

The American shore-line of the Great Lakes and their connecting rivers, if measured in steps of 25 miles, is about 3,000 miles, but if the indentations of the shore and the outlines of the islands be included, the developed shore-line is about 4,700 miles in length.

Along rivers, and where a lake is narrow, it is necessary for navigation that both shores be mapped. This increases the length of the shore-line to be surveyed between Saint Regis, N. Y., and Duluth, Minn., to about 6,000 miles.*

During the first ten years of the Survey, whilst a general geodetic survey of the entire chain of lakes was contemplated for the future, the actual operations were mainly confined to surveys of special localities where improvements were called for or where the navigation was difficult; and where the surveys were more extended they were little more than reconnaissances. This course was made necessary because the appropriations were inadequate to the purchase of the finer instruments and the support of the larger force necessary for more extensive and more exact surveys, and also because of the pressing need of improvements at particular localities, for which preliminary surveys were essential.

The Survey from the beginning has been conducted under the War Department, at first by the Chief of Topographical Engineers, and, after the consolidation of that corps with the Corps of Engineers in 1863, by the Chief of Engineers. An officer of engineers has always been in immediate charge of the Survey, and during the earlier years the assistants were almost exclusively engineer officers, but as the scope of the Survey was enlarged, and more assistants were required than could be spared from among the officers of engineers, civilian assistants were employed, many of whom have served for a long number of years on the Survey.

In the following account of the progress of the Survey, the narrative will be divided into eight sections, corresponding to the changes in the officers in charge of the Survey.

The following tables are appended:

1. Catalogue of published charts.
2. Statement of the annual appropriations.
3. Officers in charge of the Survey.
4. Officers who have served as assistants on the Survey.
5. Civil assistants employed on the Survey.
6. Showing the annual issue of charts.

THE SURVEY UNDER THE CHARGE OF CAPT. W. G. WILLIAMS.

MAY, 1841 TO 1845.

§ **2.** Colonel J. J. Abert, Chief of Topographical Engineers, in a letter of instructions, dated May 17, 1841, directed Captain Williams to take charge of the Survey of the Northern and Northwestern Lakes. Captain Williams was then general superintendent of harbor improvements on Lake Erie, with an office at Buffalo, N. Y. He still retained the charge of these harbor works, and the office of the Lake Survey was therefore first established at Buffalo.

Captain Williams remained in charge of the Survey until the fall of 1845. His annual reports to the Chief of Topographical Engineers, and the reports of the latter to the Secretary of War, furnish the data for the following account of the Survey under his administration. The reports are very meager as respects the details of the methods of surveying and of the instruments employed. They only furnish a brief statement of the field of operations, and are mainly devoted to showing the capabilities for improvement of the different localities, and the necessity of such improvements in the interests of commerce. There are, however, in the Lake-Survey office, a few of the field note-

* From memoranda respecting the Lake Survey by General C. B. Comstock, published in the Report of the Chief of Engineers for 1875.

books used by Captain Williams and his assistants, and from them it appears that the general method of making the surveys was as follows:

METHODS OF SURVEY.

The *astronomical observations* for time and latitude were made with a sextant, the differences of longitude being determined apparently by the chronometric method. Azimuths were determined by observations on Polaris or the sun.

Base-lines for the triangulation were generally measured with three well-seasoned wooden rods, each about 10 feet long. These were either laid upon stakes previously leveled, or had grooves cut along their under sides so that they could be laid upon a tightly-stretched rope supported upon stakes driven at suitable intervals. When the stretched rope was used, the base was measured in sections of about 500 feet each. The Mackinac base was measured in 1844 with an iron-bar apparatus devised by Lieutenant Gunnison. There were four bars, each 10 feet long, $1\frac{1}{4}$ inches wide, and $\frac{5}{8}$ inch thick, having a thermometer placed at the middle, with its bulb sunk into the bar. During the measurement each of these bars rested on five rollers, which were mounted on a carriage of the best mahogany wood a few inches shorter than the iron bar and stiffened by iron plates. The rollers at the ends of the carriage had adjusting screws, by which a longitudinal or a cross motion could be given to the bar. Each carriage was supported by two tripods, with adjustable heads for leveling. When measuring, the bars were made level by means of a 12-inch striding level. Contact was made by bringing the rear end of the bar so that it just touched a hair suspended from the forward end of the preceding bar, the hair being made vertical by an attached plumb-bob swung in water. Minor bases were measured with the ordinary surveyor's chain.

The *angles of the triangulation* were measured with theodolites reading to 5 or 10 seconds by verniers. Generally, the pointings were made in succession at the stations around the horizon, each angle being read twice. By some observers the telescope was reversed, and the setting of the horizontal limb changed for the second set of readings. The triangulation stations consisted of a single center-post of the requisite height, stiffened by braces from the ground. To support the platform for the observer a pyramidal frame-work, composed of four corner-posts and the necessary braces, was built around the center-post, but the platform and its supports were entirely disconnected from the center-post. The corner-posts of the pyramid were extended until they met above the center-post, and a few feet of the upper portion of the pyramid was boarded up and whitewashed, and thus served as a target. There is no record of these stations having been permanently marked. Heights were determined by barometrical measurements or leveling with a Y-level.

The *shore-line* was surveyed either with a compass and chain or with a theodolite and chain, points of the shore being determined by chained off-sets at right angles to the compass or theodolite courses. The principal features of the topography in the immediate vicinity of the shore were located in the same way, or by intersections from two or more stations, and the details were sketched in. But little topographical work, however, was done, beyond the rough sketching of the general character of the shores.

For the *hydrographical work* the soundings were taken either from a row-boat or a steamer, the boats running on known courses, or between buoys and stations on the shore, and the soundings being taken at regular intervals of time. The soundings with row-boats, or the in-shore hydrography, were usually made by the parties called shore-parties, who surveyed the shore-line in connection with the latter work. The need of a steamer in making soundings over large areas, and for moving and supplying the various parties, was early felt, and in his report for 1842 Captain Williams submitted an estimate for the building of an iron steamer. The steamer was not finished until the spring of 1844, and in the mean time, as the surveys went on, permanent points and stations were located, to be afterwards used in the hydrographic work. The steamer was at first called the Abert, but her name was subsequently changed to the Surveyor.

PROGRESS OF THE FIELD-WORK.

The season for field operations was usually about five months, from May to October, the remaining seven months being spent in the office in making the reductions, computations, and plottings of the previous season's work.

The following officers of the Corps of Topographical Engineers served at various times as assistants to Captain Williams on the Lake Survey, viz: Captain Howard Stansbury, Lieutenants Joseph E. Johnston, J. N. Macomb, J. H. Simpson, W. H. Warner, I. C. Woodruff, J. W. Gunnison, J. D. Webster, J. W. Abert, and W. B. Franklin.

The instructions to Captain Williams required him to establish a point of commencement for the Survey on the north extremity of the southern cape of the entrance to Green Bay, and also to make surveys of the places of difficult navigation in the vicinity of the Straits of Mackinac. The point of commencement was selected not only on account of the importance to navigation of a survey of the entrance to Green Bay, but also because it was regarded as a favorable point, from which to extend a system of triangulation to the Beaver and Manitou Islands and thence to the east shore of Lake Michigan. All this region of country was heavily timbered, and a great deal of labor was required to clear lines for the measurement of bases and for the triangulation. During the summer of 1841 a detailed topographical survey of the island of Mackinac was completed, reconnaissance surveys were made in the northern part of Lake Michigan, and a site for a base-line near the entrance to Green Bay was selected and partly cleared.

A better view of the work of the Survey can perhaps be presented by tracing separately the progress made at the several localities than by giving a detailed narrative of the operations of each successive year.

1. *Green Bay.*—During the seasons of 1842 and 1843 Lieutenant Simpson finished clearing and measured the base-line at the entrance, and located and built triangulation stations on both shores of the bay and on the islands at its mouth. He also read the angles at many of these stations. The survey of the shore-line on the southeast side of the bay was completed from the town of Green Bay to and around the Porte des Morts entrance. In 1845 Captain Williams and Lieutenant Gunnison, on the steamer Abert, were engaged in triangulation and hydrographic work on Green Bay.

2. *Straits of Mackinac.*—For this survey a base-line about 6 miles long was selected on the south shore of the straits, the west end being near McGulpin's Point and the line running back of Old Fort Mackinac. The line having been cleared in previous years, the base was measured by Lieutenant Gunnison in the summer and fall of 1844 with the apparatus previously described. In 1842 and 1843 Lieutenant Woodruff was at work about the straits. He was engaged in both triangulation and shore-line work, and also assisted Captain Williams in astronomical observations. The shore-line was surveyed down as far as the entrance to Grand Traverse Bay, and about thirty miles of the northern shore of the straits was also surveyed.

3. *Lake Michigan.*—In 1842 and 1843 Lieutenants Gunnison and Webster ran a line along the western shore from Chicago northward until they connected with Lieutenant Simpson's survey of the shore-line of Green Bay. In 1843 Lieutenant Gunnison also made a survey of Grand River on the east side of the lake. The shore-line in the vicinity of Saint Joseph was surveyed by Lieutenant Franklin in 1844. These surveys were made with especial reference to harbor improvements, for which plans and estimates were made by the same officers. In 1843 and 1844 Lieutenant Macomb, assisted in the first year by Lieutenant Abert, made surveys in the vicinity of Grand Traverse Bay and the Manitou Islands, connecting the latter by triangulation with the main shore.

4. *Lake Saint Clair.*—In 1842 Lieutenants Macomb and Warner made a survey of the delta of the Saint Clair River with a view to plans for the improvement of the channels.

5. *Lake Erie.*—In 1844 Captain Williams, assisted by Lieutenants Simpson and Woodruff, on the steamer Abert, was engaged in surveying the harbors on this lake. In 1845 the survey of the west end of Lake Erie, embracing the area west of a line from Sandusky to Pointe Pelée, was commenced by Lieutenants Macomb and Woodruff. A base-line for the trigonometric survey of the islands was measured on South Bass Island, and considerable triangulation and shore-line work was effected during the season. In his annual report for 1845, Colonel Abert states that all the lake harbors, except those upon Lake Superior, have been surveyed, and that he is prepared, if authorized by Congress, to compile and publish a portfolio of them.

THE SURVEY UNDER THE CHARGE OF LIEUT.-COL. JAMES KEARNEY.

1845 TO APRIL 9, 1851.

§ **3.** Lieutenant-Colonel Kearney assumed charge of the Survey in the fall of 1845. During his administration the office of the Survey was removed to Detroit, where it has since remained.

The data for an account of the Survey while under his charge are the brief summaries of operations contained in the annual reports of the Chief of Topographical Engineers for 1846, 1848, and 1850, the annual report of Lieutenant-Colonel Kearney for 1849, and the incomplete set of note-books in the office records. There is no report for 1847.

METHODS OF SURVEY.

The methods of making the surveys were the same as those employed under Captain Williams, except that the triangulation seems to have been more carefully executed, and a greater number of readings was taken in the measurement of angles.

In his report for 1849 Colonel Kearney urges very strongly upon the bureau the necessity for the purchase of improved instruments suitable for a geodetic survey, and asks for an appropriation of $10,000 for that purpose. He states that the Survey was at that time, and had from the beginning, been totally destitute of any astronomical and geodetic instruments, and nearly destitute of any topographical or hydrographical instruments that were fit to be taken into the field.

PROGRESS OF THE FIELD-WORK.

On account of the demand for officers of the Topographical Engineers for service with the armies in Mexico, there were but three officers, besides the superintendent, on the Survey in 1846, 1847, and 1848, viz, Lieutenants Macomb, Woodruff, and Gunnison in 1846, and Lieutenants Macomb, Gunnison, and Scammon in 1847 and 1848.

The operations for these years were restricted, with the exception of the examination of a few points on Lake Ontario and a reconnaissance of Lake Champlain made in 1846, to the completion of the survey of the west end of—

1. *Lake Erie.*—In this work the triangulation was mostly done by Lieutenant Macomb, and the topography and hydrography by Lieutenants Woodruff, Gunnison, and Scammon, assisted by Messrs. R. W. Burgess, J. F. Peter, and J. H. Forster. The survey of the west end was finished in 1848, and the drawings were forwarded to the bureau at Washington for compilation and engraving in 1849. A chart of the whole of Lake Erie on a scale of 1 : 400,000, which had been compiled in the Lake-Survey office, was also forwarded to Washington in the same year. The engraving of these two charts, and of one of Kelley's and the Bass Islands on a larger scale, was completed, and their systematic distribution to navigators was begun in 1852.

2. *Straits of Mackinac.*—Colonel Kearney was directed to resume the survey of the straits in 1849. One triangulation party on the steamer, now called the Surveyor, under Lieutenant Macomb, and five topographical and hydrographical parties, under Lieutenant Scammon and Messrs. Houghton, Hearding, Burgess, and Potter, were sent into the field in the spring. Lieutenant Macomb was occupied during the season in reconnoitering for stations for the primary triangulation, clearing lines of sight, building stations, and preparing the base-line formerly used by Captain Williams for remeasurement. He also assisted the shore-parties in surveying shoals and reefs distant from the land. The three shore-parties first mentioned, under the direction of Lieutenant Scammon, made a survey based upon a secondary and tertiary triangulation of the shore-line and the adjacent islands from Point Saint Ignace on the north shore to the Chenaux group of islands. The other two parties, under Mr. Burgess, surveyed Bois Blanc and Round Islands and the adjacent waters.

3. *Miscellaneous.*—In 1850 the appropriation for the Lake Survey was not made until the 28th of September, too late to allow the survey of the Straits of Mackinac to be resumed that season. The only field-work accomplished, therefore, was a survey of the Sandusky River up to Fremont, Ohio, and a survey of the harbor of Port Clinton, Ohio, both of which were made late in the fall, in obedience to orders from the Topographical Bureau.

THE SURVEY UNDER THE CHARGE OF CAPT. J. N. MACOMB.

APRIL 9, 1851, TO SEPTEMBER, 1856.

§ **4.** Captain Macomb assumed charge of the Lake Survey on the 9th of April, 1851, and in this year the Lake Survey proper may be said to have been begun, as nearly all the localities surveyed in previous years have since been resurveyed with greater accuracy than was possible with the means available when the original surveys were made. Captain Macomb's letter-book is among the records of the office. It contains copies of official letters to the Topographical Bureau, the monthly reports of the progress of the work, and the annual reports of Captain Macomb. These furnish the materials for the following account of the survey under his administration.

Larger appropriations being granted by Congress, Captain Macomb was enabled to procure better instruments, to introduce improved methods, and to prosecute the work more systematically than had been possible in the earlier years of the Survey. He was also able to employ a greater number of assistants, who, starting on the Survey in subordinate positions, were, as they acquired experience in its different branches, promoted to more responsible positions, until they finally became chiefs of parties.

DISTRIBUTION OF THE PUBLISHED CHARTS.

The engraving of the three Lake Erie charts prepared under the direction of Lieutenant-Colonel Kearney having been finished in 1852, their systematic distribution to vessels was begun in the same year. The regulations then adopted for the issue of the charts are still in force, and provide that the charts shall be furnished gratuitously to any American or Canadian vessel navigating the lakes, on the presentation of a certificate of a collector of customs stating the names of the owners, place of registration, tonnage, and certifying that the vessel's papers are in full force. The charts were issued at the Detroit office and at an agency established in Buffalo. A record is kept at both places, and duplicate charts are issued only on satisfactory proof that the originals were lost or destroyed unavoidably, or by an accident, for which no blame attaches to the owners.

A table giving a list of the published charts, the scales on which they are drawn, the dates of publication, the officers under whose direction the surveys were made, the draughtsmen, and the engravers, when known, is appended, and therefore no further reference to the publication of the charts will be made. It should, however, be stated that of late years the practice has been to have the charts photolithographed immediately on their completion by the draughtsman, and to issue these photolithographs whilst the plates are being engraved.

METHODS OF FIELD-WORK.

There were two general classes of parties for the field-work; the steamer-party for the primary triangulation and off-shore hydrography, and the shore-parties for the topographical and in-shore hydrographical work.

Captain Macomb took personal charge of the first party, which usually consisted of two assistants and the necessary crew for the Surveyor, and, in addition to the triangulation and off-shore work, made frequent inspections of the shore-parties, furnished them with supplies, and occasionally moved them from camp to camp.

Astronomical work.—In 1852 a field astronomical transit and a Würdemann zenith-telescope were procured for the Survey, and during the remainder of Captain Macomb's superintendence determinations of longitude were made by the method of lunar culminations, and latitude determinations were made by Captain Talcott's method of observing with the zenith-telescope the differences of meridian zenith distances of pairs of stars culminating near the zenith and at nearly equal distances on opposite sides of it. The first precise astronomical work done by the Lake Survey was the determination of the latitude of Detroit in the spring of 1852, the observers being Captain Macomb and Lieutenant Raynolds. Azimuth determinations were made by observations on Polaris or other close circumpolar stars at their elongations. These were made with all the accuracy required for mapping purposes, but not with that precision and attention to the elimination of instrumental errors which is necessary in the most accurate geodetic work; and this remark, in fact, applies to most of the theodolite work done previous to 1870.

Base-lines.—A great advance was made in the accuracy of the triangulation work by the purchase, in 1852, of a Bache-Würdemann compensating base-apparatus. This was constructed for the Survey under the direction of Captain Thomas J. Lee, of the Topographical Engineers, and was in all essential particulars similar to that previously made by Mr. William Würdemann for the Coast Survey, of which a description, with plates, is given in the Coast-Survey Report for 1854. The modifications introduced in the Lake-Survey apparatus for the purpose of making it lighter and smaller consisted mainly in the reduction of the length of the tubes from 20 feet to 15 feet. It is a contact apparatus, the measurement of a base being made with two tubes, each made up of a bar of iron and a bar of brass firmly joined together at one end, and carrying at the other a compensating lever so constructed that the tube shall have nearly the same length at all temperatures. The apparatus was accompanied by a 15-foot standard brass bar, whose length and coefficient of expansion were determined by Würdemann. It was first used on the Mackinac base in 1852, this being the only primary base measured during Captain Macomb's charge of the Survey.

Primary triangulation.—For this work a 10-inch Gambey repeating theodolite, reading by two verniers to 5 seconds, was generally used, although some of the work was done with another 10-inch instrument made by Würdemann, the limb of which was originally the vertical limb of the first-mentioned instrument. Several of the smaller Würdemann and Gambey theodolites were also occasionally used. The 10-inch Gambey was a very good instrument, and remained in constant use on the Survey until it was taken to the Mississippi River in 1876, and has since then been used in the secondary triangulation of that river. All the angles of the main triangulation seem to have been read by the principle of repetition, but there seems to have been no systematic distribution of the readings on different parts of the limb in order to eliminate periodic and accidental errors of graduation. The stations were built as already described, and were placed near the shore in order to be made available in locating the steamer-soundings. The longest lines did not exceed 20 miles, and were usually much shorter.

Off-shore hydrography.—The general method of running the lines of off-shore soundings was to anchor a buoy at each extremity of the line in water sufficiently deep for the steamer to pass round it, and to take the soundings at regular intervals of time as the steamer passed over the line between the buoys, running at a speed of about 4 miles an hour. The position of the steamer at the time of taking a sounding was also frequently determined by reading sextant angles between stations or other objects on shore. The positions of shoals and reefs were indicated by placing upon them tripod stations, which were located by intersections from the stations on shore. These tripods were made of such a height as to stand two-thirds out of water when placed in position, and were secured by piling stones on a platform built between the legs of the tripod above the water. They not only answered the purpose for which they were mainly intended, but also served in the absence of the buoys, which of late years have been placed in such positions by the Light-house Establishment as beacons to warn navigators of the positions of dangerous places, and were much appreciated by the masters of vessels. On several occasions, before proceeding to his field of operations in the Straits of Mackinac, Captain Macomb went to the west end of Lake Erie to place these tripods on shoals already known, and to locate and mark others which had been discovered since the original surveys. As the operations of the Survey were enlarged, it became evident that more than one steamer was required for the vigorous and economical prosecution of the work, and accordingly Captain Macomb inserted an item for the building of an iron steamer in his estimates for 1854. This side-wheel steamer, having a length of 143 feet, and a beam of 21 feet, was completed and turned over to the Survey in July, 1856. She was called the Search, a name appropriate to one of her most important uses, that of seeking out and exposing hidden dangers.

Topography and in-shore hydrography.—Each shore-party consisted of a chief of party, three or four assistants, and the requisite number of chainmen, leadsmen, and boatmen to furnish the necessary assistance to the topographers, and crews for three or four six-oared cutters. They had a complete camp equipage, and established their camps on shore, and after surveying for six or seven miles on either side of a camp, moved to a new location. Two to four such parties took the field each season. They extended a secondary or tertiary triangulation, developed from bases measured with wooden rods or chains, over their field of work. Frequent observations for azimuth and variation of the compass were made, on Polaris usually. The shore-line and the important features of the topography were determined with the theodolite and chain. The shores of the upper lakes being generally either densely wooded or marshy, the topography back from the shore was as a rule simply sketched, detailed surveys being made only where there were settlements or towns. The in-shore hydrography usually covered the area from the shore to the 3- or 4-fathom curve, but included the development of all shoals or dangerous places within several miles of the shore. The lines of soundings were run between buoys anchored at convenient points and sounding stations on the shore, the soundings being taken at regular intervals of time.

PROGRESS OF THE FIELD-WORK.

The portions of the lakes surveyed under the direction of Captain Macomb were the Straits of Mackinac and the approaches thereto for 30 or 40 miles on either side of the island of Mackinac, part of the north end of Lake Michigan, including the Beaver Island group, and the whole of the Saint Mary's River. The survey of Saginaw Bay was commenced the last season he was in charge. Local surveys of a few harbors on Lake Superior were made by a party under Lieutenant Raynolds in 1855.

1. *Straits of Mackinac.*—The base-line for the triangulation of the straits was the one selected by Captain Williams in 1842, and approximately measured by Lieutenant Gunnison in 1844. The eastern section of the original line, which was about 6 miles long, was found, however, to lie through very unfavorable ground, principally swamps, and it was decided to reduce the length of the base to about 4 miles. Captain T. J. Lee came on from Washington with the new apparatus, and made the measurement in September and October, 1852. He was assisted in this work by Captain Macomb and his officers and assistants. Comparisons of the tubes with the standard bar were made before and after the measurement. As a test of the accuracy of the measurement, a length of 1,500 feet was remeasured in a contrary direction. The ends of the base were marked by fine lines drawn upon silver 10-cent pieces, set in the upper surfaces of stone posts sunk 2 feet under the surface of the ground. Reference stones were also set at short distances from the end marks. The detailed report of Captain Lee on the measurement of this base is printed as Appendix G of the Report of the Chief of Topographical Engineers for 1854. The triangulation and off-shore hydrography were carried on by Captain Macomb and his party on the Surveyor during the seasons of 1851, 1852, and 1853, both being completed in the latter year, with the exception of a small amount of hydrographical work which was done in 1854. Many hitherto unknown shoals and reefs were discovered. An astronomical station was established upon the northwest end of Round Island, opposite Mackinac, at which observations for latitude and azimuth were made by Captain Macomb and Lieutenant Raynolds in 1853. The longitude of the station was determined in September, 1854, by a series of observations of lunar culminations made by Lieutenant Raynolds. The meridian of this station was the principal meridian for all the surveys of Captain Macomb in that portion of the lakes. In 1851 Lieutenant Scammon continued his survey of the north shore from the point at which his work terminated in 1849 to Point Detour at the mouth of the Saint Mary's River. His field of operations included the group of islands called the Chenaux, lying in the indentations of the shore, and separated from it and from each other by narrow but generally deep and navigable channels, some of which afford excellent harbors. In 1852 he surveyed that part of the north shore lying between Pointe la Barbe and Pointe Epoufette. The remaining portions of this shore from Pointe Epoufette about 10 miles westward, and from Pointe la Barbe to Point Saint Ignace, were surveyed by Lieutenant Rose in 1853. The south shore and Mackinac and Round Islands were surveyed by Lieutenant Raynolds' party, that part from Old Fort Mackinac to Hammond's Bay being done in 1851, the survey of the islands and of a short distance west of Old Fort Mackinac being made the following year. In 1853, Lieutenant Raynolds, assisted by Lieutenant Rose and Messrs. G. W. Lamson and H. Gillman, extended the survey westward to Waugoshance light-house, and then south nearly to Little Traverse Bay.

2. *Saint Mary's River.*—In view of the near completion of the ship-canal around the Sault Ste. Marie, the act making appropriations for the Survey for 1853 contained a proviso requiring

the immediate examination and survey of obstacles to navigation in the Saint Mary's River. This duty was assigned to Captain Scammon, who, assisted by Lieutenant Mendell and a large party, made surveys in that year of the East Neebish Rapids and of the expansion of the river called Lake George, and submitted plans for the improvement of navigation at both places. Later in the season, Captain Scammon surveyed the entrance to the river by the Detour Passage. Early in 1854, Captain Scammon proceeded to the East Neebish Rapids to mark out the channel by a system of landmarks on the shores, and having accomplished this satisfactorily, resumed the survey of the river, and at the close of the season had completed the data for a continuous chart of the river from Lake Huron to the Sault. This survey depended upon a carefully executed secondary triangulation, with several bases measured with wooden rods. In 1855 the survey of the river was continued from the Sault to Point Iroquois at the east end of Lake Superior by Lieutenant G. W. Rose. The ship-canal was opened to navigation in this year.

3. *Lake Michigan—North end and Beaver Island group.*—Having finished the triangulation and off-shore hydrography necessary for the completion of the chart of the Straits of Mackinac early in 1854, Captain Macomb, assisted by Lieutenant Rose, devoted the remainder of the season of 1854, and the entire season of 1855, in which he was assisted by Mr. J. A. Potter, to the extension of the triangulation to the Beaver Island group and along the north shore of the lake, and to off-shore hydrographical work in the same locality. The shore-party work in the north section of the Beaver Island Group was carried on by Lieutenant Raynolds in 1854, until September, when he was detached to make astronomical observations on Round Island, and his party, having been placed under the charge of Mr. G. W. Lamson, was transferred to the north shore. In 1855, the surveys of this section were completed by two shore parties under the charge of Messrs. W. H. Hearding and G. W. Lamson, the survey of the north shore being extended to a point about 5 miles east of the Monistique River.

4. *Lake Superior.*—On account of the opening of the Sault Canal in 1855, it became important that surveys of the principal harbors of Lake Superior should be made, and accordingly in that year a party under Lieutenant Raynolds proceeded to Ontonagon with orders to survey that harbor and as many of the harbors to the east of it as was possible during the season, the surveys to include the shore for six or seven miles on either side of each harbor. Lieutenant Raynolds completed the surveys of Ontonagon, Eagle River, Eagle Harbor, and Agate Harbor, meeting many difficulties at the three latter places, owing to the abrupt and rocky shores, heavily wooded, which necessitated the placing of the triangulation stations inland and the cutting of lines of sight. He also determined the latitudes of Ontonagon and Eagle River, and the longitude of the latter by lunar culminations.

5. *Miscellaneous surveys.*—In order to publish a chart of the head of Green Bay it was found that further surveys were needed to render those made under Captain Williams available. Therefore, in July and August, 1852, Captain Macomb and Lieutenant Gunnison, with the steamer Surveyor, were engaged in that work. Observations for latitude and azimuth were made on Green Island by Captain Macomb and Lieutenant Gunnison, respectively, after which several triangulaion stations were built, and while Captain Macomb read the angles at these stations, Lieutenant Gunnison made soundings in the bay. In August of the following year, Lieutenant Raynolds was sent to Green Bay to finish some topographical and other work necessary for the chart, which was published that year. In September, 1855, Mr. G. W. Lamson surveyed the boundary lines between the United States lands and the private claims on the island of Mackinac.

In the fall of 1855, Captain Macomb on the Surveyor made a reconnaissance with a view to connecting the triangulation of the Straits of Mackinac with that of the Saint Mary's River, and with the contemplated Lake Superior system. He found at the head of Lake George, on the Saint Mary's River, a height which commanded a view of the heights near Point Iroquois and of those on the north coast of Lake Huron. He caused stations to be built on this height and on one of those at Point Iroquois. During this reconnaissance the Surveyor passed through the new canal at the Sault, being the first government vessel to make the passage. A third point on the north coast of Lake Huron was selected by Mr. G. W. Lamson, and a station was built upon it in the spring of 1856. These stations, however, were never occupied, the connections desired being eventually made by a different system.

6. *Saginaw Bay.*—On account of the rapidly increasing commerce of the Saginaw region, and in compliance with the urgent requests for surveys and charts of that section, it was decided to place the whole force of the Survey there for the season of 1856. On the 3d of May, Captain Macomb received orders assigning him to duty in New Mexico on being relieved from the command of the Lake Survey by Lieutenant-Colonel Kearney, who, however, did not reach Detroit until the latter part of September. Lieutenants Raynolds and Rose having also been transferred to other duty early in the spring, Messrs. W. H. Hearding, G. W. Lamson, and D. F. Henry were assigned to the charge of three shore-parties. The Surveyor was not in commission that season, being laid up for extensive repairs, which her long service had rendered necessary. The new steamer Search was, however, available in July, and the party on her, under command of Captain Meade, who joined the Survey in June, was engaged in making a general reconnaissance for the triangulation of the bay, in the building of stations, and in hydrographical work. Captain Macomb was with this party for a time. Mr. Hearding made a minute survey of the mouth of the Saginaw River and of the bar in front of it, with reference to improving the entrance, and then carried the shore-line survey north to the Opinkawning River. The shore-line, from six miles above the mouth of the Sable River southward to include Tawas Harbor, was surveyed by Mr. Lamson's party.

Lieutenant C. N. Turnbull, having joined the Survey in June, was assigned to the charge of the party, until then under Mr. Henry. Lieutenant Turnbull made observations for latitude on Charity Islands. This party made surveys of the Charity Islands and of the east shore from Oak Point to Sand Point, where they selected and cleared a site for a base-line.

THE SURVEY UNDER CHARGE OF LIEUT.-COL. JAMES KEARNEY.

SEPTEMBER, 1856, TO MAY 20, 1857.

§ **5.** The only field-work done during the short period Colonel Kearney was for the second time in charge of the Survey, was a resurvey of the Saint Clair Flats, made by Captain Meade after his return from Saginaw Bay in the fall of 1856. This resurvey was based on three stations whose relative positions had been determined by Lieutenant J. N. Macomb in 1842. The usual office-work was carried on during the winter.

In the spring, failing health required that Colonel Kearney should relinquish the command of the Survey, and he was relieved by Captain George G. Meade, who, a few years later, was the commander of the Army of the Potomac.

THE SURVEY UNDER THE CHARGE OF CAPT. GEORGE G. MEADE.

MAY, 1857, TO SEPTEMBER 1, 1861.

§ **6.** The principal work accomplished by Captain Meade was the survey of the whole of Lake Huron, including the completion of that of Saginaw Bay, the entire force of the Survey being engaged on this duty during the years 1857, 1858, and 1859. In 1860 the survey of the northeast end of Lake Michigan was extended southward to include the Fox and Manitou Islands and Grand and Little Traverse Bays, and the data were thus obtained for a much-needed chart of a dangerous part of the lake passed over by the vessels sailing between the Straits of Mackinac and Chicago. Local surveys of a few harbors on Lake Superior were made in 1859, and in 1861 the general survey of the lake was begun at its western end.

METHODS OF SURVEY.

The general methods of survey employed by Captain Meade were similar to those followed by Captain Macomb. The nature of the field of operations required a combination of triangulation and astronomical work for the determination of the positions of points on the shores of Lake Huron, and made some change necessary in the method of executing the off-shore hydrography. Larger appropriations permitted a considerable expansion of the scope of the Survey, the introduction of more accurate methods in obtaining longitudes, and the commencement of a series of magnetic, water-level, and meteorological observations at many points on the lakes. The methods and instruments employed in these observations will be noticed under their appropriate headings. The method of executing the

Off-shore hydrography on Lake Huron has since been used on all the other lakes, and consists in running lines of soundings, made from a steamer, from the outer limit of the in-shore hydrography to a distance of eight or ten miles into the lake. The lines are about a mile apart, and are run by the steamer's compass, their direction being perpendicular to the general line of the coast. The position of the steamer at the time a sounding is taken is located by simultaneous theodolite pointings of two observers on shore, and is checked when practicable by sextant angles read on the steamer. The general character of the bottom of a lake is determined by running lines of soundings entirely across the lake at intervals of ten or fifteen miles.

Detroit observatory.—The use of a suitable lot on Washington avenue, near Grand River avenue, having been offered gratuitously to the Survey by John Hull, esq., a wooden building was erected on it in the spring of 1857 and fitted up as a field-observatory for both astronomical and magnetic observations. The astronomical work, under Captain Meade's direction, was mainly done by Lieutenants C. N. Turnbull and O. M. Poe, and Mr. James Carr, and the magnetic work by Lieutenant William Proctor Smith. Early in 1858 a new astronomical transit (No. 15) and a new zenith-telescope (No. 12), both made by Würdemann, the telescope of each having a focal length of 32 inches, and an aperture of 2½ inches, were purchased for the Survey, and at the same time a break-circuit sidereal clock (No. 184), a chronograph with spring governor, and four sidereal chronometers, all made by Bond & Sons, of Boston, were received. In the following spring another break-circuit sidereal clock (No. 256), a second chronograph, and four more sidereal chronometers, all by the same makers, were added to the list of instruments. A favorable opportunity for determining the longitude of the Detroit observatory by connecting through the magnetic telegraph with an observatory whose longitude was well established did not occur until the winter of 1858–'59, when it was decided to connect with the observatory of the Western Reserve College, at Hudson, Ohio, the longitude of which from the Cambridge observatory had been determined in 1849. The uninterrupted use of the wire between Detroit and Hudson after 9 o'clock at night was offered free of charge by Anson Stager, esq., the general superintendent of the Western Union Telegraph Company. The observations were made in January and February, 1859, the Detroit observer being Lieutenant Turnbull, who used a chronograph, and Professor C. A. Young, of the Western Reserve College, observing at Hudson, and using a Morse register for recording his observations. The telegraphic connections were so made that the transits of stars at either meridian were recorded at both stations with their respective local times, and the same stars being used by both observers, each star gave from each record a determination of the longitude, and the mean of these two determinations was free from wave-time. At the conclusion of the observations at Hudson, Professor Young visited Detroit for the determination of the relative personal equation of himself and Lieutenant Turnbull. The latitude of the Detroit observatory was determined by Lieutenant Turnbull by seven nights' observations with the zenith-telescope in April and May, 1859. In May, 1860, the difference of longitude between the Detroit observatory and that of the University of Michigan at Ann Arbor, was telegraphically determined by Lieutenant Poe, at Detroit, and Professor James C. Watson, at Ann Arbor, by the exchange on two nights of arbitrary signals, the observatory at Ann Arbor not being provided with the necessary apparatus for the exchange of star-signals. The results not being entirely satisfactory, a second connection with Ann Arbor was made in April and May, 1861. On this occasion both observatories had chronographs, and the method of operations was similar to that employed between Detroit and Hudson. Professor Brünnow, of the University of Michigan, observed at Ann Arbor, and Lieutenant Poe and Mr. James Carr at Detroit. Subsequently Professor Brünnow connected Ann Arbor with the Hamilton College observatory at Clinton, N. Y., the observer at the latter place being Professor C. H. F. Peters. The difference of longitude between the Hamilton College and Cambridge observatories having been previously determined, a second connection between Detroit and Cambridge was thus made.

Magnetic observations.—Previous to 1858 the magnetic observations had been limited to the determination by the ordinary compass of magnetic declinations at those places where observations for azimuth were made by the shore or triangulation parties. In that year a portable declinometer, with detached theodolite, for the determination of the magnetic declination and horizontal intensity, made by Jones, of London, a Barrow dip-circle, and a Fox dip-circle, were imported, and the making of magnetic observations was assigned to Lieutenant W. P. Smith in addition to his other duties. At various times during the seasons of 1858, '59, '60, Lieutenant Smith made determinations of the magnetic elements at Detroit, Cambridge, Mass. (where he had been sent to compare his instruments with those of the Cambridge observatory, and to be instructed by Professor Bond in the use of them), Toronto, and at twenty-seven points on the lakes, of which three were on Lake Ontario, four on Lake Erie, six on Lake Michigan, three on Lake Superior, ten on Lake Huron, and one on the Straits of Mackinac. Tables giving the results of these observations are published in the Lake-Survey reports for 1859 and 1860.

Water-level and meteorological observations.—Up to the time of Captain Meade's assuming charge of the Survey, readings for water-level were taken on temporary gauges at the localities where surveys were being carried on, and the soundings were reduced to a certain stage of water, which was usually either the mean level during the period of the survey, or the mean level during a particular season. With a view to establishing a uniform plane of reference for the soundings, as well as deciding numerous interesting questions in regard to the fluctuations of the water-level of the lakes, including the question of tides, Captain Meade, in his annual report for 1857, recommended that simultaneous water-level readings, accompanied by complete meteorological observations, should be made over the entire lake region. This recommendation was approved by the bureau, and the instruments, including four self-registering tide-gauges, were ordered, but were not received at Detroit until August, 1858, too late to be distributed to the different stations that season. Early in the spring of 1859, Captain Meade himself distributed and set up these instruments at Sacket's Harbor, Charlotte, and Fort Niagara, on Lake Ontario; Buffalo, Erie, Cleveland, and Monroe piers, on Lake Erie; Forestville, Pointe aux Barques, Tawas, Thunder Bay Island, and Presqu' Isle, on Lake Huron; Grand Haven, Michigan City, and Milwaukee, on Lake Michigan; Head of Saint Mary's River, Marquette, Ontonagon, and Superior City, on Lake Superior. The instruments furnished each station were a water-gauge, barometer, psychrometer, thermometer, rain-gauge, and wind-gauge. Competent observers were employed to make daily or more frequent observations, the records being sent to the office at Detroit at the end of each month. These records were reduced and tabulated in the office, and detailed tables of the results are published in the annual reports for 1860 and 1861. In 1860, Mr. J. M. Bigelow was placed in charge of the meteorological division of the Survey. His reports discussing the results of the observations are published with the above-mentioned tables. The report for 1861 also contains a discussion of the tides and *seiches* (irregular oscillations) on Lakes Michigan and Superior, by Mr. O. N. Chaffee.

PROGRESS OF THE FIELD-WORK.

1. LAKE HURON—*Sand Point base-line.*—The Saginaw Bay triangulation depends upon a base about 4 miles long, measured in October, 1857, upon the sand-spit extending into the bay from its eastern shore. The Bache-Würdemann apparatus was used. Mr. William Würdemann came on from Washington to correct a slight want of compensation in tube No. 2, and at the same time thoroughly cleaned and adjusted the whole apparatus. The measurement was made by Captain Meade, assisted by Lieutenant Turnbull and Messrs. Potter, Carr, and Casgrain. A length of 960 feet was remeasured as a test of the accuracy of the work. The ends of the line and two intermediate points were marked by stone posts. The latitude and longitude of the west end were determined by Lieutenant Turnbull, the former by observations with the zenith-telescope, the latter by lunar culminations.

Triangulation and astronomical work.—In 1857 Captain Meade on the Search finished selecting the sites and building the stations for the Saginaw Bay triangulation. He had also intended to read the angles, but finding that the duties of general supervision would prevent his accomplishing both the triangulation and off-shore hydrography, he decided to devote himself to the latter, and to assign Lieutenant Poe with a party on the Surveyor to the triangulation work. Lieutenant Poe used the 10-inch repeating Gambey theodolite the greater part of the time. He read the angles from the Charity Islands to the head of the bay in 1857. In order to see over the line from Pointe aux Barques to Pointe au Sable, 27 miles long, much higher stations than had as yet been used on the survey were required. In 1858 Lieutenant Poe built a station with a 100-foot center-post on Pointe aux Barques, and one with an 82-foot center-post on Pointe au Sable. Before occupying them he was taken seriously ill, and had to withdraw from field duty for the remainder of the season. Mr. James Carr relieved him and finished the Saginaw Bay triangulation. It was found impracticable to extend the triangulation beyond the line Point aux Barques–Pointe au Sable on account of the character of the shores of Lake Huron. From Pointe aux Barques to the head of Saint Clair River the coast is comparatively straight and flat, and at that time was densely wooded, with a few scattering settlements on the immediate shores of the lake. To have carried a system of primary triangulation along this coast, it would have been necessary to cut out almost every line of sight through the heavy timber, and would have involved an expenditure of time and money entirely unwarranted by the appropriations made for the Survey. It was therefore decided to carry along the shore in connection with the shore-party work such a minor triangulation as the nature of the ground permitted, and to check this by determining the positions of certain points by astronomical observations; or, in other words, to establish a series of astronomical bases, the latitudes of the ends of these bases being determined by observations with the zenith-telescope, and the differences of longitude between them and the principal meridian at Sand Point being obtained by the repeated transfers of chronometers from one station to the other on the Lake-Survey steamers.

In the longitude work, after the receipt of the second clock and chronograph, a clock and chronograph were used at each of the stations between which the chronometers were to be transferred. The errors and rates of the clocks were carefully determined by as frequent observations as possible with an astronomical transit, and the errors of the chronometers on local time were determined by comparisons with the clocks. From four to twelve chronometers were used, and eight to twelve transfers were made between each pair of stations. Astronomical stations were established at Forestville, Sanilac, and Fort Gratiot, and their latitudes and differences of longitude between each other and Sand Point were determined during the seasons of 1858 and 1859 by Lieutenant Turnbull and Mr. James Carr. Early in the latter year the difference of longitude between Fort Gratiot and Detroit was determined in the same manner, and the Saginaw Bay triangulation and longitudes dependent upon it were thus connected with the Detroit meridian as determined by telegraph. A small portion of the south end of Lake Huron was covered by a triangulation extending from the head of the Saint Clair River to the line Sanilac–Cape Ipperwash, the angles being read by Lieutenant J. L. Kirby Smith in 1860.

Similar considerations prevented the extension of the Saginaw Bay triangulation to the northward, and it was therefore connected with the triangulation of the Straits of Mackinac by a system of latitudes and azimuths, the stations being at Sturgeon Point, Thunder Bay Island, Presqu' Isle, and Sand Bluff, a few miles east of Hammond's Bay. From the observed latitudes and azimuths at these stations their longitudes and distances apart were computed. The observations at the two first-mentioned stations were made by Lieutenant Turnbull in 1858. As a check upon the work, the difference of longitude between Thunder Bay Island and Sand Point was determined directly by the exchange of chronometers. The latitude and azimuth observations at Presqu' Isle and Sand Bluff were made in 1859 by Lieutenant Poe.

After completing these observations, Lieutenant Poe, with a party on the schooner Coquette, fixed the positions of points on the north shore of Lake Huron by extending the triangulation of the Straits of Mackinac eastward to the line Presqu' Isle–Great Duck Island. In order to incorporate into the chart of Lake Huron Bayfield's surveys of such portions of the Canada coast as were not included in the work of the Lake Survey, it was desirable to determine the latitudes and longitudes of several points on that coast, and Goderich, which was in telegraphic connection with Detroit, and Cove Island, at the entrance to the Georgian Bay, were selected as being the most suitable points. Lieutenant Poe, assisted by Lieutenant Beckham, and Mr. Carr, assisted by Mr. Austin, were assigned to this duty in 1860. Lieutenant Poe, at Goderich, and Mr. Carr, at Detroit, determined the longitude of Goderich by telegraphic signals. Mr. Carr then observed for latitude at Cape Ipperwash, and afterwards occupied the Cove Island station, and chronometers were transferred between it and Goderich. Lieutenant Poe then moved to Mackinac, and chronometers were transferred between the station on that island and Cove Island. A comparison was thus made between the longitude of the Mackinac station, as determined by the astronomical and triangulation work on the west shore of Lake Huron, and that by the astronomical work on the east shore, and the results were found to agree very well. Lieutenant Poe and Mr. Carr finished the season's work by determining the telegraphic difference of longitude between Detroit and Grand Haven. Mr. Carr, who observed at Grand Haven, also determined its latitude.

Off-shore hydrography.—The off-shore hydrography for a distance of 15 miles between Saginaw Bay and Thunder Bay; and from Thunder Bay around the northwest end of the lake to Cockburn's Island, was done by a party on the Surveyor, under the command of Mr. J. A. Potter, in 1859. That of the rest of the lake, including Saginaw Bay, was done by the party on the Search, under the command of Captain Meade, during the seasons of 1857, '58, '59. In 1860 Lieutenant W. P. Smith on the Search ran the lines of soundings across the lake, and made observations of the temperature of the water at different depths.

Topography and in-shore hydrography.—All of this work was done by parties under the charge of Messrs. Hearding, G. W. Lamson, and H. C. Penny in 1857 and 1858, and of Messrs. Hearding and Penny in 1859, in which year it was completed. Mr. Hearding's party continued their survey of the Saginaw River, made in 1856 eastward to Quannakisse Bayou, surveyed the coast from Oak Point on the east shore of Saginaw Bay to Pointe aux Barques, and from this point south to the head of the Saint Clair River, and made surveys of Cockburn and Drummond Islands, connecting with Captain Scammon's survey of the Saint Mary's River. Mr. G. W. Lamson's party extended the survey of Tawas Harbor, made in 1856, south to the Opinkawning River, and surveyed Thunder Bay, covering it with a triangulation depending upon a base two miles in length, measured with wooden rods. Mr. Penny's party surveyed the east shore of Saginaw Bay from Sand Point to Quannakisse Bayou, the west shore of Lake Huron from near the mouth of the Sable River to the south side of Thunder Bay, and from the north side of Thunder Bay until a connection was made with the survey of the straits at Hammond's Bay, and the Canadian coast from the head of Saint Clair River to Cape Ipperwash. Lieutenant Poe had charge of this party during the early part of 1857, until he was assigned to triangulation duty.

2. LAKE MICHIGAN.—Nearly the entire force of the Survey was sent to the northeast end of Lake Michigan in the spring of 1860.

Primary triangulation.—Lieutenant J. L. Kirby Smith, with a party on the schooner Coquette, located and built the stations required to extend the Mackinac triangulation from the line Hat Island - Pointe aux Chênes over the field of the present survey. He also, with the assistance of Mr. Carr, read the angles at all the stations except those in Grand Traverse Bay. These were occupied the following year by Mr. O. N. Chaffee. It had been intended to connect this triangulation with the meridian of Detroit by transferring chronometers between Pointe aux Becs Scies and Grand Haven, of which the longitude was determined with this object, but the exigencies of the service prevented the carrying out of the project that season.

Off-shore hydrography.—Mr. J. A. Potter, with the steamer Surveyor, did the off-shore hydrography of the main coast, including the Traverse Bays and around the Manitou Islands. In September, Captain Meade brought the Search from Lake Huron and did part of the sounding around the Fox Islands. Mr. Potter did the rest of it with the Surveyor in 1861.

Topography and in-shore hydrography.—Three shore-parties were engaged in this work. That of Mr. Hearding commenced its survey where the work under Captain Macomb ended, near Middle Village, and carried it south to Deep Water Point, on the east arm of Grand Traverse Bay. Mr. Penny's party surveyed from Traverse City to Pointe aux Becs Scies. Mr. Henry's party, after finishing the Manitou and Fox Islands, surveyed the peninsula which separates the two arms of Grand Traverse Bay, from Deep Water Point to Traverse City.

3. LAKE SUPERIOR.—In 1859, a party under Mr. G. W. Lamson was sent to Lake Superior and made surveys of the harbor of Marquette and of Grand Island and its approaches. The off-shore hydrography about Grand Island was done by Mr. Potter on the Surveyor in 1861. A general reconnaissance of Lake Superior was made by Captain Meade on the Search in 1859. The general survey of Lake Superior was commenced in the spring of 1861, the parties being sent to the extreme west end of the lake. Just previous to the season for commencing field operations the breaking out of the rebellion caused the withdrawal from the Survey of all the officers engaged

upon it as assistants, and no officers but the superintendent were employed upon the Survey from that time until the close of the war.

Primary triangulation.—This was assigned to Mr. Henry, who had charge of a party on the schooner Coquette. He located, cleared, and prepared the ground for the preliminary measurement of a base-line about four miles long on Minnesota Point, south of Duluth, and also built and occupied six stations for the development of the triangulation from the base to the eastward. Mr. Hearding and Mr. Casgrain made the preliminary measurement of the base with wooden rods, duplicating the work as a check upon its accuracy.

Off-shore hydrography.—Mr. Carr, on the Search, did the off-shore work on both shores of the head of the lake along a total distance of 63 miles.

Topography and in-shore hydrography.—Mr. Hearding's party surveyed the Saint Louis River from the head of navigation at the town of Fond du Lac to its mouth at the Bay of Saint Louis, and then made surveys of this bay and of Superior and Allouez Bays. These bays are separated from the lake by the narrow sand-spits called Minnesota and Wisconsin Points. Mr. Penny's party, connecting with Mr. Hearding's survey, extended it along both shores of the lake, the total length of shore-line surveyed being 63 miles.

4. MISCELLANEOUS.—Various local surveys were made from year to year by the different assistants, in obedience to orders from the Topographical Bureau. Many of these were for light-house purposes. Surveys were made nearly every year at the Saint Clair Flats and at the Lake George Flats of the Saint Mary's River, for the purpose of determining the effects of the improvements which were being carried on by Captain A. W. Whipple. A detailed survey of Maumee Bay and River, as far up as Toledo (of which a chart was published), was made by Messrs. Hearding, Penny, and Potter in the fall of 1857, after their return from Lake Huron.

Before the parties returned from the field in 1861, Captain Meade was relieved from the charge of the Lake Survey and ordered to duty with the armies in the field. He turned over the command of the Survey to Lieutenant-Colonel J. D. Graham, of the Topographical Engineers, on the 31st of August, 1861.

THE SURVEY UNDER THE CHARGE OF COL. J. D. GRAHAM, CORPS OF ENGINEERS.

SEPTEMBER 1, 1861, TO APRIL 15, 1864.

§ **7.** When detailed as superintendent of the Lake Survey, Colonel Graham was in charge of the harbor improvements on all the lakes from Champlain to Superior, and had his office at Chicago. On removing to Detroit, he still retained the charge of the harbor works, and was in addition assigned to duty as engineer of the tenth and eleventh light-house districts, embracing all the lakes except Champlain. Colonel Graham was in charge of the Survey during three winter and two summer or field seasons.

OFFICE WORK.

During the winter seasons the work of the office was carried on in the usual manner. Much attention was given by Colonel Graham to the reduction, tabulation, and discussion of the water-level and meteorological observations, which were continued at the stations established by Captain Meade, and at others which had been previously established by Colonel Graham in connection with his harbor works. In the annual reports for 1861, 1862, and 1863, the meteorological data are discussed by Mr. Bigelow, and the fluctuations of the water-level by Colonel Graham, who had made these a subject of study for several years. Colonel Graham, before taking charge of the Survey had, in his annual report for 1860, presented a memoir, accompanied by a record of extended observations at Chicago, demonstrating the existence and character of a semi-diurnal lunar tidal wave in Lake Michigan. In the report for 1863, Colonel Graham revises his memoir and announces as the final result of his observations that the height of the mean semi-diurnal lunar tidal wave at Chicago is 0.142 foot.

METHODS OF SURVEY.

Colonel Graham made but two changes in the methods of field-work. He introduced the stadia for horizontal and vertical measurements in the topographical work, to which more care was given than formerly, although the use of the stadia did not become general among the shore-parties until some years later. The second change was the substitution of the method by powder-flashes for the chronometric method in determining differences of longitude where telegraphic facilities did not exist. At each of the stations whose difference of longitude it was desired to obtain, an astronomical party with a field astronomical transit and clock or chronometer determined the local time by observing the meridian transits of stars. The observations for time were made in two series with an interval of an hour or more between them. During the interval, a series of signals one minute apart was made, either at one of the stations or by a third party from a height visible from both astronomical stations, by flashing small charges of gunpowder. These signals were noted by the observer at each station, and the difference of the local time thus noted was the difference of longitude of the two stations. These flashes could be seen on a clear night over distances of 50 to 60 miles with the naked eye, and with the aid of a telescope over distances of 100 miles.

PROGRESS OF THE FIELD-WORK.

The field seasons during which Colonel Graham was in charge were those of 1862 and 1863. He had intended to continue the survey of Lake Superior, commenced in 1861 by Captain Meade, but the appropriation in 1862 became available at so late a date that the parties could not take the field until the latter part of August, when it would have been unadvisable to send them to so distant a field. The survey of Green Bay was therefore commenced in 1862 and continued in 1863. In the latter year a party was also sent to Lake Superior to make local surveys. Before commencing work on Green Bay in 1862, Mr. O. N. Chaffee's party on the Surveyor and the shore-party of Mr. J. R. Mayer were occupied for some weeks, the first in surveying several shoals about the Fox Islands which had not been discovered when the original surveys were made, and the second in making a detailed topographical survey of South Fox Island, using the stadia.

1. GREEN BAY—*Astronomical work.*—In 1862, an astronomical party under Mr. James Carr made latitude observations with the zenith-telescope at Rock Island, Green Island, and Fort Howard. In 1863 two parties, one under Mr. James Carr, and one for a short time under Mr. Austin and then under Mr. O. B. Wheeler, took the field for the purpose of determining latitudes with the zenith-telescope and longitudes by powder-flash signals. In determining longitudes the station at Fort Howard was taken as the primary meridian, with the intention of subsequently connecting it telegraphically with Detroit through Chicago. These two parties determined the longitudes of Rock and Green islands, and the latitudes and longitudes of Oconto on the west coast of the bay, and of stations on Washington Island, on the bluff at the entrance to Big Bay de Noquette, on South Manitou Island, and on the southeast end of Beaver Island.

Triangulation.—In both seasons Mr. Henry was in charge of this branch of the work, having a party on the schooner Coquette. He was mainly engaged in making reconnaissances for a base-line and for the system of triangulation, in building stations and clearing lines of sight. Some of the stations were occupied, and a preliminary measurement of the base-line, which had been established on Chambers' Island, was made with the wooden-rod apparatus in 1863.

Off-shore hydrography.—This was done in both seasons by parties on the Search and Surveyor, in charge respectively of Mr. Hearding and Mr. O. N. Chaffee. They completed the off-shore work of that part of the bay north of the Sister Islands, and in Lake Michigan carried it from Bayley's Harbor, on the east coast of the peninsula which separates Green Bay from the lake, northward to Point Detour, the northern point of the entrance to Green Bay. Much of the time of the steamers was occupied in the building of primary triangulation stations, and in moving and supplying the shore-parties.

Topography and in-shore hydrography.—This work was done in 1862 by two parties under Messrs. Penny and Mayer, and in 1863 by three parties under Messrs. Penny, Gillman, and Mayer. In the latter year, Mr. O. N. Chaffee's steamer-party ran the shore-line of Washington and Rock islands and the northern part of Little Bay de Noquette, including the secondary triangulation of the whole of this bay. In the two seasons, surveys were made of the west coast of Green Bay from a point six miles south of Cedar River to the north to include both shores of Little Bay de Noquette, of the peninsula between Green Bay and Lake Michigan from Egg Harbor on the former to Bayley's Harbor on the lake, of Detroit Island, and of Plum and Pilot islands in the Porte des Morts entrance to Green Bay.

2. LAKE SUPERIOR.—In the spring of 1863, a party under Mr. J. U. Mueller, assisted by Mr. E. S. Wheeler, was sent to survey Portage Entry on Keweenaw Bay. They surveyed the shore of the bay for a distance of four miles on either side of the mouth of Portage River, Portage River from its mouth to its head, including Portage Lake, the lower portions of Sturgeon, Pike, and Pilgrim rivers, and Torch River from its mouth in Portage Lake to its source in Torch Lake. This survey embraced the positions of the cities of Houghton and Hancock, and of the principal copper-mining establishments in their vicinity.

On the 15th of April, 1864, Colonel Graham was relieved of the charge of the Lake Survey, and of his harbor improvements and light-house duties, by Colonel W. F. Raynolds, Additional Aide-de-Camp and Major of Engineers. Colonel Raynolds was relieved of the charge of the harbors on Lakes Michigan and Erie in October, 1864, of other harbor-works in 1866, and of his light-house duties in 1870.

THE SURVEY UNDER THE CHARGE OF BVT. BRIG. GEN. W. F. RAYNOLDS, LIEUTENANT-COLONEL OF ENGINEERS.

APRIL 15, 1864, TO MAY 12, 1870.

§ **8.** The survey of Lake Superior was the main work of the Lake Survey during the six field-seasons that General Raynolds had charge. At the close of the season of 1869 but three islands of the Apostle Group remained to be surveyed in order to complete the topographical work on the American shores of the lake. A little hydrography and the greater part of the primary triangulation were unfinished. Besides the Lake Superior work, the survey of Green Bay was completed, that of Lake Michigan was extended south to Two Rivers on the west shore and to Little Pointe au Sable on the east shore, the whole of Saint Clair River, and a large part of Lake Saint Clair were surveyed, and many special surveys of harbors or of localities at which works of improvement were in progress were made in different seasons under orders from the Engineer Department. In October, 1865, a third steamer, a screw propeller, length 122 feet and beam 18 feet, was purchased from the Navy Department. She was named the Ada. This steamer was built on the Clyde for a blockade runner and was captured by the Navy. Extensive alterations and additions in the way of upper works and cabin accommodations were necessary to make her suitable for use on the Survey. The office work and the publication and issue of charts were continued in the usual manner.

METHODS OF SURVEY.

The operations of the Survey were continued under General Raynolds on the same general plan that had been pursued by his immediate predecessors.

Triangulation and astronomical work.—Where it was practicable, the surveys depended upon a system of primary triangulation with carefully measured bases. Along the shore of Lake Superior, from Grand Island to White Fish Point, and on both shores of Lake Michigan south of the entrance to Green Bay on the west and of Pointe aux Becs Scies on the east shore, where a primary triangulation was impracticable except at a great cost for stations and for clearing lines of sight, the positions of points were determined either by observations for longitude and latitude or by the method of latitudes and azimuths. At most of the primary triangulation stations latitude and longitude observations were made. Latitudes were determined in all cases by several nights' work with the zenith-telescope. Longitudes were determined by several nights' work either by the method of telegraphic star-signals, in which each observer registers the time of passage of a star over his meridian on all the chronographs employed in the work, or, when the telegraph was not available, by the method of powder-flash signals previously described. The personal equation was not in all cases determined or eliminated with precision. The two primary bases, on Keweenaw Point, Lake Superior, and Chambers' Island, Green Bay, were measured with the Bache-Würdemann apparatus. In June, 1865, agates were set in the ends of the 15-foot brass standard bar, changing its length. To determine its new length five brass yards were made for the Survey by Würdemann, who assigned their lengths and marked the same upon them. The yards were received at Detroit in March, 1867. Several series of comparisons of the standard bar with these yards were made at different times by Colonel Farquhar and Mr. Henry for the purpose of determining the length and coefficient of expansion of the standard bar. Comparisons of the tubes of the base-apparatus with the standard bar were made before and after measuring a base. The triangulation of Lake Superior was much retarded by the lack of suitable instruments. In 1864 the 10-inch Gambey theodolite was the only one in the possession of the Survey fit for primary work. In September, 1865, General Raynolds made a requisition for three large theodolites. These were ordered by the Engineer Department from Oertling & Sons, of Berlin, but were not received at Detroit until the spring of 1869. They were used on Lake Superior during that season. They had 20-inch horizontal limbs, read by three microscopes to single seconds. Two of them turned out to be very poor, and the third was not a good instrument, all having large accidental errors of graduation. In 1867 the triangulation work was done with three instruments borrowed from the U. S. Coast Survey. These were a 24-inch theodolite by Troughton, reading to single seconds by three microscopes; a 14-inch Brünner repeating theodolite, reading by two verniers to five seconds, and a 12-inch Gambey repeating theodolite, reading by two verniers to five seconds. In the primary work a large number of measurements, as a general rule, were made on each angle, but there was no uniformity as to this number, and the readings were not taken with the system which is necessary for the elimination of errors. An improvement in the method of building stations was adopted in 1864 by changing the center-post from a single piece to a tripod, the legs of which were firmly braced together and had a sufficient inclination to give the structure great stability. The tripod has the advantage of enabling the target and instrument either to be centered over the geodetic point or their reductions to this point to be determined, and has since been exclusively used for primary stations. The geodetic point was marked by a hole drilled in the top of a stone post sunk below the surface of the ground, or in the natural rock, where that was found. A brass frustum of a cone with a cross cut on its top surface was sometimes leaded into the top of the marking-stone. During the season of 1865 Assistant Engineers O. B. Wheeler and S. W. Robinson introduced on the Survey a method of sending messages by means of flashes of sunlight from a mirror, made short or long to correspond with the Morse telegraphic code. This method of telegraphing has since been frequently employed. It was especially useful to the parties engaged in the primary triangulation of Lake Superior, messages having been successfully sent in that work over lines 50 to 90 miles long.

Topography and hydrography.—In regard to the topographical work, General Raynolds, in his report for 1866, states that—

> The character of the country in which the surveys are being prosecuted forbids that attention to the details of topography which would otherwise be desirable. It is the exception to find anything but a dense forest, in which it is impossible to make an accurate survey without opening every foot of the lines of sight. No sketching can be done that is reliable. Parties within easy hearing distance cannot see each other. And last, though by no means least, during the summer season, which is the only one in which work can be done at all, the forests are so full of venomous insects that it is next to impossible for an instrument to be used. The stadia has been found most available for overcoming these difficulties.

The hydrographic work was done in the manner hitherto in use on the Survey. The steamer-parties, in addition to doing the off-shore hydrography, were generally charged with the building of stations, with the moving and supplying of other parties, and sometimes with the reading of angles.

Water-level and meteorological observations were continued at many of the stations established by Captain Meade. Mr. Bigelow continued in charge of the reduction and discussion of these observations until the 1st of January, 1867, when Mr. Henry was placed in charge of this department. Extended tables and reports, containing information in regard to the meteorology of the lakes, form appendices to the annual reports of General Raynolds.

Out-flow of the lakes.—In 1867 the investigation of the subject of the supply of water in the chain of lakes was taken up. This duty was assigned to Mr. D. F. Henry, who was directed to carefully gauge the rivers forming the connecting links of the chain. Observations were made during the seasons of 1867–'68–'69. In 1867, a party under Mr. A. R. Flint was engaged in this work on the Saint Clair and Saint Lawrence rivers, and another party under Mr. Lewis Foote worked on the Saint Mary's and Niagara Rivers. The method followed during this season was generally that used by Humphreys and Abbot in their work on the Mississippi River. The bases, however, were much longer than those used on the Mississippi River, being from 700 to 1,100 feet in length, instead of 200 feet. The signals between the ends of the base were sent by telegraph. Several other departures from the method of using double floats, recommended in the Report on the Physics and Hydraulics of the Mississippi River, were made. In 1868, the Saint Lawrence, Niagara, and Saint Clair Rivers were gauged by parties under the immediate charge of Messrs. Flint, Foote, and David Wallace, respectively. In this season meters were used for obtaining the velocity of the currents. These were of two or three different kinds. Some were full propeller-wheels of four blades, some of two blades, and one, devised by Mr. Henry, was constructed of a set of Robinson anemometer cups set in a suitable frame, and was used where the current was so sluggish that a propeller-wheel would not move. The record was made by electrical apparatus, the circuit being broken at each revolution of the wheel. An escapement was attached to the armature of a relay coil, and a record of the revolutions of the wheel kept by a decimal train of wheel-work. The coefficient of velocity in these meters is variable for different velocities, and it is difficult to obtain it with accuracy. The friction of the parts in all those, except the anemometer cups, used in this work caused considerable uncertainty as to the accurate determination of this coefficient. In 1869 the work was continued on the Niagara and Saint Clair Rivers, under the immediate charge of Messrs. Foote and Wallace. Both meters and double floats were used for the object of testing and comparing the two methods. The detailed reports of Mr. Henry form appendices to the Lake-Survey Reports for 1868, 1869, and 1870. In these reports Mr. Henry differs from and criticises the methods and conclusions of Generals Humphreys and Abbot, as given in their report on the Mississippi River. A criticism of Mr. Henry's methods of observation and reduction, as well as of his conclusions, by General H. L. Abbot, is published in the Report of the Chief of Engineers for 1870.

PROGRESS OF THE FIELD-WORK.

1. River and Lake Saint Clair.—In 1867, before the navigation was open on the Upper Lakes, three large parties, on the steamers Search, Surveyor, and Ada, commanded respectively by Lieutenants James Mercur and B. D. Greene, and Mr. O. N. Chaffee, made a complete survey of the Saint Clair River from Lake Huron to the head of the delta, and of the south channel into Lake Saint Clair. The survey was completed early in June. In 1868 the parties, having returned from Lake Superior in September, were immediately reorganized for the purpose of making a survey of Lake Saint Clair. Colonel Farquhar and Lieutenant Gregory, in charge of large parties on the Ada and Search, respectively, made surveys of the flats and of the eastern shore, and the two shore parties of Messrs. Mayer and Molitor surveyed the west shore. Lieutenant B. D. Greene relieved Colonel Farquhar of the charge of the Ada soon after the parties took the field. About 40 miles of the east shore and all the main triangulation of the lake were yet to be done when the parties were withdrawn from the field, owing to the lateness of the season.

2. Green Bay and Lake Michigan—*Base-line.*—The base on Chambers' Island for the Green Bay triangulation, which had been measured with wooden rods in 1863, was grubbed and graded during the summer of 1864, and its measurement with the Bache-Würdemann apparatus was commenced by Professor C. A. Young and Mr. Henry in September of the same year. When about one-third of the line had been measured, Professor Young's duties at his college required him to leave, and the measurement was completed by Mr. Henry, assisted by Messrs. Robinson and Le Baron. The base was about 3⅓ miles in length. The ends and four intermediate points were each marked by cuts on a frustum of a brass pyramid leaded into the top of a stone 5 feet long and 6 inches square on top, sunk just below the surface of the ground. One hundred and fifty-one tubes were remeasured as a test of the accuracy of the work. The Chambers' Island light-house, built in 1868, was placed directly over the north end of the base, and its subsequent use as a base of verification for the triangulation extending down from Lake Superior was thus prevented.

Triangulation.—Mr. O. N. Chaffee had charge of the triangulation work on Green Bay during the seasons of 1864 and 1865, his party being on the schooner Coquette in the first season, until she was wrecked in the fall, and on the Surveyor in the second season. In 1864 the stations north of Chambers' Island, including those on Big Bay de Noquette, were occupied, and the stations south of Chambers' Island were occupied in 1865. During the summer of 1864, Mr. Henry, on the Surveyor, was engaged in reconnoitering for the best method of making a connection between the triangulation extending from the Mackinac base to the Beaver, Fox, and Manitou Islands, and the Green Bay system. Having found that stations on Burnt Bluff at the south end of Big Bay de Noquette, and on Rock Island in the entrance to Green Bay, could be seen from the South Fox and North Manitou Islands, stations were built at these four points. The angles at the two latter points were read by Messrs. O. B. Wheeler and G. E. Swinscoe.

Astronomical work.—In 1864 the longitude of Fort Howard, at the head of Green Bay, was determined. General Raynolds and Mr. S. W. Robinson observed at Detroit, and Professor C. A. Young at Fort Howard, having a clock and chronograph at each station. Intermediate observations were made at Chicago by Mr. O. B. Wheeler, who also determined the latitude of his station, and at Ann Arbor by Professor J. C. Watson. The star transits at each of the four stations were recorded on both chronographs. In 1865, the telegraph line having been extended from Fort Howard to Escanaba and Marquette, the differences of longitude between Marquette and Escanaba, and Escanaba and Fort Howard, were determined. The first of these differences was determined by O. B. Wheeler and S. W. Robinson, each station being provided with a clock and chronograph, and the observers exchanging stations to eliminate personal equation. For the difference between Escanaba and Fort Howard, Mr. O. B. Wheeler and Professor C. A. Young were the observers, both having clocks and chronographs, and exchanging stations to eliminate personal equation. Mr. S. W. Robinson occupied an intermediate station at Menomonee, his observations being recorded on the chronographs at the ends of the line. Latitude observations were made at Escanaba by Messrs. Wheeler and Robinson, and at Menomonee by the latter. After completing these observations, Mr. Wheeler determined the latitude of the triangulation station at Fishdam at the head of Big Bay de Noquette, Mr. Robinson of the stations at Death's Door Bluff on the east side of Green Bay and of Cedar River on the west side of the bay, and Professor Young of the station at Boyer's Bluff on Washington Island.

Off-shore hydrography.—The off-shore work of that part of Green Bay south of the Sister Islands was done by parties on the Search in 1864 and on the Surveyor in 1865, under the charge, respectively, of Messrs. A. C. Lamson and O. N. Chaffee, Mr. Lamson's party doing that part about Chambers' Island only.

Topography and in-shore hydrography.—In 1864 the islands in the entrance to Green Bay, between Washington Island and Point Detour and both shores of Big Bay de Noquette, were surveyed by parties under the charge of Messrs. J. R. Mayer and A. F. Chaffee. Mr. A. Molitor's party assisted in grading the base-line, and made a survey of Chambers' Island and of about 15 miles of the west shore of the bay between Cedar and Menomonee Rivers. In 1865 Mr. A. C. Lamson's and Mr. A. F. Chaffee's parties, respectively, surveyed the east and west shores from Egg Harbor and Menomonee to the head of the bay, and Mr. O. N. Chaffee's party made a survey of the head of the bay and the lower part of Fox River. These surveys completed the shore-line work of Green Bay.

3. LAKE MICHIGAN.—In 1864, Mr. W. T. Casgrain's shore-party took up the survey of the north shore of Lake Michigan at the point about 5 miles east of the Monistique River, where the survey under Captain Macomb in 1855 had terminated, and extended it southward until it joined Mr. Mayer's work near Point Detour. The off-shore soundings along this section were made in the same year by the party on the Search, under the charge for a short time of Mr. Hearding, and, after his resignation, of Mr. A. C. Lamson. In 1866, the survey of the east shore of Lake Michigan was extended from Pointe aux Becs Scies to Little Pointe au Sable by Mr. A. F. Chaffee's party, while that of the west shore was carried from Bayley's Harbor to Two Rivers by Mr. Henry Gillman's party. The off-shore work was completed between the same limits on the east shore and was carried to about 5 miles south of Kewaunee on the west shore by a party on the Ada, under command of Mr. O. N. Chaffee, until illness compelled him to leave the field, after which Mr. A. R. Flint had charge of the party. Five lines of soundings across the lake were also run by the steamer. The method of latitudes and azimuths was adopted for fixing the positions of points along the shores of the lake. In the fall of 1866, Lieutenant M. R. Brown and Mr. G. Y. Wisner determined

the latitudes of Kewaunee, Whitefish Point, and Sheboygan, Mr. S. W. Robinson those of Cana Island, Clay Banks, and Rawley's Point on the west shore, and Mr. O. B. Wheeler those of Big and Little Pointes au Sable on the east shore. The azimuths at these stations were not observed until 1871 and 1872.

4. LAKE SUPERIOR—*Base-line.*—After an extended reconnaissance in the summer of 1865, Mr. D. F. Henry selected and marked out a base-line for the primary triangulation of Lake Superior on the west shore of Keweenaw Bay south of Portage Entry. The line was about 5½ miles long, very level, and so located that the triangulation could be developed from it with good angles. It was, however, through a dense forest, and a great deal of labor was required to clear it and remove the stumps and roots from the ground. In the fall of 1866, about 8,000 feet of the line having been prepared, the measurement with the Bache-Würdemann apparatus was commenced by Mr. D. F. Henry, assisted by Messrs. E. S. Wheeler and David Wallace, but only two hundred and eighty-four tubes were measured, the ground being so soft and shaky from the recent grading and the heavy rains which fell during September as to make the measurement entirely unreliable. In August, 1867, the shore-parties of Lieutenant B. D. Greene and Messrs. A. C. Lamson and H. Gillman were detached from other work and sent to the base-line to finish grading and preparing it for measurement, and on September 7 the measurement was commenced at the South Base station and continued to the North Base station, the work being finished on the 25th of October. The ends of the line were marked by the intersection of two fine lines cut on a frustum of a brass pyramid leaded into the top of a stone 5 feet long. Five intermediate points were marked in the same way. A station was built near the center of the line and the measurement was checked by triangulation. Mr. Henry was in charge of the work, and was assisted by Messrs. E. S. Wheeler and D. Wallace.

Triangulation and astronomical work.—After 1865 the triangulation and astronomical work on Lake Superior were done in connection with each other by the same parties. The only astronomical work of 1864 was the determination of the latitudes of Copper Harbor and Portage Entry by Mr. O. B. Wheeler. In 1865 Messrs. O. B. Wheeler and S. W. Robinson determined the latitudes of Mount Houghton, on Keweenaw Point, and Marquette, and their difference of longitude by powder-flashes, and also the differences of longitude between Marquette, Escanaba, and Fort Howard by telegraph. No triangulation work was done in 1864, but in 1865 Mr. Henry, with a party on the Search, commenced the triangulation of Keweenaw Bay, after selecting the base-line. He built stations on both sides of the bay, and also the Wheal Kate station on Keweenaw Point, about 15 miles northwest of the base. The angles of the secondary triangulation of the head of the bay south of the base-line were read. A general reconnaissance of the central portion of the lake was made, which resulted in fixing approximately the points of the main triangulation. From the base the system expands through very favorable triangles until the first large triangle, Wheal Kate–Vulcan–Huron Mountains, is obtained. Station Vulcan is near the northern extremity of Keweenaw Point, the line Vulcan–Wheal Kate being in round numbers 48 miles long. From this line the triangle Vulcan–Wheal Kate–Isle Royale is reached, and then comes Vulcan–Isle Royale–Saint Ignace. From the line Vulcan–Saint Ignace, 93 miles long, the largest triangle of the system, Vulcan–Saint Ignace–Tip Top is obtained; the line Vulcan–Tip Top being 101 miles long, and Saint Ignace–Tip Top 92 miles long. In 1866 the building of stations and moving of parties were done by Messrs. Henry and A. C. Lamson, who were in command of the steamers Search and Surveyor, respectively. Three triangulation and astronomical parties were in the field, under Lieutenant M. R. Brown and Messrs. O. B. Wheeler and S. W. Robinson, Lieutenant Brown having general charge of the work. On account of the lack of triangulation instruments the operations were confined almost exclusively to astronomical observations. Stations Vulcan, Wheal Kate, Huron Mountains, Isle Royale, Saint Ignace, and Northeast were occupied, their latitudes being determined with the zenith telescope, and their differences of longitude by powder-flashes. Station Northeast was built in 1866 on a hill near Tip Top, but on lower ground than the latter, and was abandoned when the Tip Top station was built in 1867. In the latter part of the season of 1866, Lieutenant J. F. Gregory was in command of the Search and read the angles of the triangulation of Keweenaw Bay from South Base as far north as the line Traverse Island–Huron Island. In 1867 Brevet Lieutenant-Colonel F. U. Farquhar, Captain of Engineers, was in command of the Search and had charge of the reconnaissance for triangulation and the building of stations, and also exercised a general supervision over all the work on Lake Superior in the absence of General Raynolds. The triangulation and astronomical work were under the immediate direction of Lieutenant James Mercur until August 15, and of Lieutenant Gregory after that date. With the instruments borrowed from the Coast Survey, the angles of the triangle Vulcan - Tip Top - Saint Ignace, were read by Lieutenant Mercur and Messrs. O. B. Wheeler and G. Y. Wisner, respectively. After Lieutenant Gregory relieved Lieutenant Mercur the three parties were transferred to the south shore and determined latitudes, and differences of longitude by powder-flashing, of points between Marquette and the Sault Ste. Marie. Marquette, Grand Island, Pointe au Sable, Whitefish Point, Tahquamenon Island, Point Iroquois, Sault Ste. Marie, and two points between Pointe au Sable and Whitefish Point were occupied. The triangulation of Keweenaw Bay was completed in this season by Mr. O. N. Chaffee.

The only triangulation-work of 1868, besides the selection of the sites for and the building of stations at Farquhar's Knob, Porcupine Mountains, and East Sawteeth by Colonel Farquhar, was a secondary triangulation of Whitefish Bay made by Mr. O. N. Chaffee. It depended upon a secondary base measured with wooden rods near Waiska Bay. In 1869 Brevet Major J. A. Smith, Captain of Engineers, had general supervision of the work on Lake Superior. Lieutenant Gregory was in command of the Search, and was charged with the duty of reading the angles of the secondary triangulation between Isle Royale and the Canadian shore north of it, and also with moving and supplying the six triangulation and astronomical parties employed in reading the angles of the main triangulation. Lieutenant Gregory was transferred to the command of the Ada on the 1st of October, and Lieutenant W. R. Livermore, having reported for duty on the Survey, was assigned to the Search. These six parties were divided into two sections, one to work west from the line Vulcan - Saint Ignace, and the other to work east of that line. The first section was under the direction of Lieutenant E. H. Ruffner, and consisted of his own party and those of Messrs. G. Y. Wisner and E. S. Wheeler. They occupied, respectively, the stations Vulcan, Isle Royale, and Wheal Kate, and after reading the angles of this triangle Mr. Wisner occupied the secondary station, West Isle Royale. Lieutenant Ruffner then moved to Farquhar's Knob and Mr. Wisner returned to Isle Royale, and the angles of the triangle Wheal Kate - Farquhar's Knob - Isle Royale were measured. Mr. Wisner then occupied Porcupine Mountains, but on account of the lateness of the season the readings for the triangle Farquhar's Knob - Wheal Kate - Porcupine Mountains were not completed. The latitude of Farquhar's Knob was determined. The second section was under the direction of Mr. O. B. Wheeler, and consisted of his own party and those of Messrs. G. A. Marr and A. R. Flint. They occupied respectively the stations Michipicoten, Saint Ignace, and Tip Top. In connection with the observers at Vulcan and Isle Royale they measured the angles of the triangles Vulcan - Isle Royale - Saint Ignace, Vulcan - Saint Ignace - Tip Top, and Vulcan - Tip Top - Michipicoten. Paugon, Gargantua, and Mamainse were afterwards occupied and the readings of the angles at them completed. The latitudes of all these stations except Paugon and Mamainse were determined and their differences of longitude were obtained by powder-flashes. The first section used the three new Oertling theodolites, and the second had two of the Coast-Survey instruments and the 10-inch Gambey repeating theodolite belonging to the Lake Survey.

Off-shore hydrography.—The off-shore hydrography was all done by the steamer-parties, who also moved and supplied the shore and triangulation parties, built stations, and frequently assisted both in the triangulation and topographical work, the chiefs of the steamer-parties usually exercising a general supervision over the other parties.

It will not be attempted to describe the details of their work for each season, nor to note all the temporary changes that occurred in the officers or assistants in command of them, but the account of their operations will be confined to a brief statement of the general field in which they worked and of the officers in charge of them. In 1864 Mr. W. H. Hearding, in charge of the Search, did the off-shore hydrography in the vicinity of Copper Harbor and of Portage Entry, and also marked Stannard's Rock with a tripod station to serve as a beacon for navigators. In 1865 the off-shore hydrography around Keweenaw Point was done by the party on the Search under the command of Mr. D. F. Henry. In 1866 the Search and Surveyor were both engaged on Lake Superior, the former under the command of Mr. D. F. Henry and the latter of Mr. A. C. Lamson. The party on the Search sounded out some detached shoals east of Keweenaw Bay, made a minute hydrographical survey of Stannard's Rock, and ran several lines of soundings across the lake. Mr. Lamson did the off-shore hydrography of Keweenaw Bay and of the coast east of it as far as Laughing Fish Point. In 1867 Colonel Farquhar on the Search and Mr. O. N. Chaffee on the Ada finished the off-shore work from Grand Island to the head of the Saint Mary's River. Lieutenant Gregory on the Surveyor had general supervision of the work on Isle Royale until August 15, when he was relieved by Lieutenant B. D. Greene. The Surveyor party surveyed Passage and Gull Islands east of Isle Royale, and finished about one-half of the off-shore soundings around Isle Royale. In 1868 Colonel Farquhar, in command of the Ada, made the off-shore soundings along the north coast from Isle Royale to the head of the lake, and along the south shore from Eagle River to Montreal River. Lieutenant Gregory on the Search finished the hydrographical surveys around Isle Royale, and Mr. O. N. Chaffee on the Surveyor made a hydrographical survey of Whitefish Bay. At the close of this season the only portion of the off-shore hydrography remaining unfinished was that of the south shore from Montreal River to Brulé River, including the Apostle Islands group. This work was completed in 1869 by the party on the Ada, which also assisted the shore-parties in the secondary triangulation and topography of the Apostle Islands. The Ada was commanded at times by Major J. A. Smith and Lieutenant J. C. Mallery until October 1, when Lieutenant Gregory took charge of her. Previous to that time Lieutenant Gregory was in charge of the Search, and in addition to his duties in connection with the triangulation made general soundings across the lake.

Shore-line work.—In 1864 Mr. Henry Gillman's shore-party made a survey of Copper Harbor, finished the survey of Torch Lake, and surveyed the shore of Keweenaw Bay from Portage Entry to Pequaquawaming Point. In 1865 Mr. Gillman filled up the gap between the surveys of Eagle River and Ontonagon, while the parties of Messrs. J. R. Mayer and A. Molitor filled up that between Copper Harbor and Portage Entry, making a careful topographical survey of a large part of the mineral region of Keweenaw Point. In 1866 the two latter parties surveyed the shore between Keweenaw Bay and Marquette. In 1867 Mr. Gillman filled up the gap between the surveys of Marquette and Grand Island, while Messrs. Mayer and A. Molitor surveyed the shore from Grand Island to the head of Saint Mary's River. In this year, parties under Lieutenant B. D. Greene and Mr. A. C. Lamson were at work on the south and north shores, respectively, of Isle Royale until the 23d of August, when, as has been stated, the parties were transferred to the Keweenaw base. In 1868 two parties, under Lieutenants B. D. Greene and J. C. Mallery, completed the survey of Isle Royale. A party under Lieutenant J. E. Griffith made a survey of the south shore from Ontonagon to about four miles west of the Montreal River, and was then transferred to the north shore, of which it surveyed about nineteen miles. The transfer was made after Lieutenant Griffith had left the field on account of ill health, and Mr. Gillman had taken charge of his party. The rest of the north shore from Pigeon River to the head of the lake was surveyed by parties under Lieutenant W. E. Rogers and Mr. Mayer. In 1869 the only portion of the survey of the American shore remaining unfinished, that between Bad River and Brulé River, including the Apostle Islands, was completed, with the exception of three small islands of the Apostle group, by the parties of Messrs. A. C. Lamson and J. R. Mayer.

5. MISCELLANEOUS.—In 1868 the large theodolites which had been ordered from Berlin not having yet arrived, the three astronomical and triangulation parties which had been organized for work on Lake Superior were assigned to the duty of determining the latitudes and longitudes of points along the shores of Lakes Ontario and Erie, between Ogdensburg and Detroit. Lieutenant E. H. Ruffner had general charge of this work, the observers being himself, Mr. O. B. Wheeler, and Mr. G. Y. Wisner. They commenced work at the three eastern stations, after finishing which the two observers farthest east swung around the one on the west, and so on. The method of star-signals was used in longitude-work, the middle observer recording his observations on the chronographs at the ends of the line. The points occupied were Ogdensburg, Watertown, Oswego, Rochester, Buffalo, Dunkirk, in New York; Erie, in Pennsylvania; Ashtabula, Cleveland, Sandusky, Toledo, in Ohio; Monroe and Detroit, in Michigan.

General Raynolds was relieved of the charge of the Lake Survey by Brevet-Brigadier General C. B. Comstock, Major of Engineers, on the 12th of May, 1870.

THE SURVEY UNDER THE CHARGE OF BVT. BRIG. GEN. C. B. COMSTOCK, MAJOR OF ENGINEERS.

MAY 12, 1870, TO CLOSE OF SURVEY.

AND UNDER THE TEMPORARY CHARGE OF CAPT. H. M. ADAMS, CORPS OF ENGINEERS, DURING THE ABSENCE IN EUROPE OF GENERAL COMSTOCK,

FROM AUGUST 14 TO NOVEMBER 20, 1874, AND FROM MAY 24, 1877, TO JUNE 25, 1878.

§ **9**. Since General Comstock assumed charge, the Survey of the Northern and Northwestern Lakes has been completed, and a continuous chain of triangulation, depending upon eight carefully measured bases, has been extended from Saint Ignace Island, on the north shore of Lake Superior, to Parkersburg in Southern Illinois, a distance of 10°, and from Duluth, Minn., via Chicago, to the east end of Lake Ontario, a distance along its axis of 1,300 miles. This triangulation was incidental to the survey of the lakes, but was measured with the greatest precision, in order that it might be of value in a more accurate determination of the form and dimensions of the earth.

The description of the survey since 1870 will be arranged under the heads of (1) office work, (2) methods of field-work, (3) field-work of the Survey by years. As most of the subjects under these heads, except those relating to topography and hydrography, will be treated of at length in the other chapters of this report, the descriptions here given will be brief and general in their character.

OFFICE WORK.

1. *General work of the office.*—The office work comprises the reduction and plotting of the field-work of the several parties, the drawing of the final charts, the correspondence, money and property accounts, the issuing of the published charts, the examinations of instruments, and the investigations of the various scientific subjects connected with the Survey. Mr. J. H. Southall was the chief clerk in charge of the money accounts and of the general correspondence of the office from July 25, 1870, to July 31, 1878, when he resigned and Mr. N. S. Fisher was appointed in his place. Mr. J. Lohman has been the property clerk since February 1, 1872, and his duties have included the issuing of charts and the keeping of the registers of note-books, charts, and field sketches, reports and computations, instruments, &c.

2. *Computations and plotting of field-work.*—The computations incident to the Survey are made under the direct supervision of General Comstock, and are submitted for his approval before being registered and adopted for use. All computations not self-checking are duplicated. As a general rule, each astronomical observer, on returning to the office, is required to make the first reduction of his own work, and each chief of a triangulation party reduces and tabulates in the "record of triangulation" the angles read by himself. These two classes of observers, in connection with a small force of computers (of whom Messrs. O. B. Wheeler, T. W. Wright, T. Russell, and C. H. Kummell have been longest employed), who usually remain in the office during the entire year, make the computations for the adjustment of the primary and secondary triangulation, and for the geodetic positions of the points of triangulation, and also prepare the data required by the draughtsmen in the projection and drawing of the final charts. Of the shore-parties, the chief and one assistant are usually retained in the office. They compute their triangulation, the co-ordinates of the stations, and of the points located with the chain and theodolite, and plot both the topography and in-shore hydrography on a scale of 1 : 10,000 on sheets called "detail sheets." These sheets are first divided by fine lines into squares of 1,000 meters on a side, and then the principal stations are plotted by their co-ordinates and checked by the lengths of the sides of triangles. The work which has been plotted in the field on the "field sheets" is then plotted upon the detail sheets being adjusted to the positions of the principal stations. The soundings, after being corrected for error of lead-line, and reduced to a plane of reference, are plotted by interpolation between sounding stations and buoys. The off-shore hydrography is plotted on a scale of 1 : 60,000. Each detail sheet contains a list of the note-books and field sheets from which it is plotted.

3. *Final charts.*—The final charts are compiled and drawn by draughtsmen employed especially for that purpose. The data for the projections and the co-ordinates of all points fixed by the triangulations, primary, secondary, and tertiary, are furnished from the office computations. The details of the topography and hydrography are filled in from the detail sheets. The final charts, when completed and verified, are forwarded to the Engineer Department at Washington, where they are at once photolithographed and afterwards engraved. The system of publication adopted since 1870 has been to publish a general chart of each lake on a scale of 1 : 400,000, and to divide the shore-line of each lake into convenient sections, and publish a separate chart of each section on a scale of 1 : 80,000. These are called coast charts. The charts of the rivers and a few special localities are on larger scales. The sailing lines are laid down on all charts. A list of the authorities, a water-table showing the mean level and fluctuations of the water for certain periods, a table of magnetic variations, a table of light-houses, a list of sailing directions, and a statement of the dangers to be avoided are printed on each chart.

4. *Reduction of water-level and meteorological observations.*—The reduction of water-level and meteorological observations was in charge of Mr. O. B. Wheeler from March, 1871, to July 1, 1878, and of Mr. A. R. Flint after that date. Their reports are published in the annual reports. The Signal Service having established meteorological stations at or near many of the places occupied by the Lake Survey, and the Chief Signal Officer having directed that copies of the observations made at those places should be furnished the Lake Survey, the meteorological observations by the Survey were discontinued in January, 1872, at all stations but Port Austin and Monroe, Mich., and Sacket's Harbor, New York. The observations at these stations were also discontinued in 1876. Water-level observations have been made continuously since 1870 at Milwaukee, Cleveland, Erie, and Charlotte; since 1873 at the above places and also at Sault Ste. Marie, Marquette, Escanaba, Port Austin, Detroit, and Sacket's Harbor; and at various times for a year or more at Superior City, Duluth, Monroe, Buffalo, and Fort Niagara.

The method of observation at the several stations is as follows: A convenient fixed point, called the zero of gauge, is chosen. The distance of the surface of water from this zero is measured with a rod graduated to hundredths of a foot. These measurements are taken three times a day, at 7 a. m., 1 p. m., and 7 p. m., local time. The zero of gauge at each station is connected with at least two permanent bench-marks, and levels are run each season to see if any change has taken place in the position of the zero. As an additional precaution, a check-point is established at each station, from which readings are taken twice a month in the same way as from the zero of the gauge. This serves to detect any change in the position of the zero, and, if any change should be found by the leveling, the check-point readings will show when the change occurred. In 1876 a re-reduction of the Lake-Survey observations since 1859 was made, and a series of observations at several other stations having been furnished to the Survey, a set of annual water-level curves, showing the height of water referred to an established plane for each month of the years, from 1859 to 1876, on each of the lakes, except for Lake Superior, for which the dates are 1870–1876, was drawn and published in the annual report for that year. These curves have been continued and published each year since. A full description of the planes of reference and bench-marks is also given in the report for 1876.

5. *Tides and seiches on the lakes.*—In 1871 General Comstock commenced the examination of the subject of the tides and *seiches* on Lakes Michigan and Superior. For Lake Michigan, the records of a self-registering tide-gauge at Milwaukee for several years were available. The heights of the water for solar hours were read off and tabulated, for the entire lunations of which there was a record from 1867 to 1871, inclusive, and for lunar hours for the complete lunations in 1867. The examination of the solar hourly mean heights showed that there was a solar semi-diurnal tide of about *four-hundredths* of a foot, the tide following the sun's upper transit being considerably the larger. This inequality was explained by the known existence of a lake breeze at Milwaukee during the summer months, a comparison of the solar diurnal curve for April and November, when the lake breeze should be weak, and that for July and August, when the lake breeze should be strongest, with that of the whole season, showing that for the former months the inequality nearly disappears, while for the latter it is considerably increased. The examination of the lunar hourly

mean heights showed a lunar semi-diurnal tide of *eight-hundredths* of a foot. The details of the examination and the theory of these tides form Appendix A of the annual report for 1872.

An examination of the *seiches*, or irregular oscillations of the lakes, led General Comstock to conclude that they resulted from atmospheric disturbance, or, more definitely, barometric oscillations and their accompanying winds over some part of the lake. The details of his examination and theory are given in Appendix B of the above report.

In 1872 a tolerably complete record of the water-level at Duluth during three lunations was obtained from a self-registering tide-gauge, and examined for evidences of solar and lunar tides at that point. The result showed a lunar semi-diurnal tide of 0.14 foot. The same value was found for the solar tide following the sun's upper transit, while that following the lower transit was very small, almost disappearing. The record did not extend over a sufficient length of time to show whether this was to be attributed to the effect of a lake breeze, as was the case at Milwaukee. The fact that the same value, 0.14 foot, was found for both solar and lunar tides, while the solar tide, theoretically, should be much smaller, and that the solar tide following the lower transit nearly disappears, suggested some cause whose period is a solar day, acting to increase the solar tide following the upper transit, and to diminish the other. A land and lake breeze would be such a cause, but the data were not sufficient for the discussion of the question.

6. *Standards of length.*—In the winter of 1870 the question of standards of length, their coefficients of expansion, and those of the tubes of the base-apparatus was taken up by General Comstock, and a large amount of work has since been expended at various times in comparisons of the standards with each other and with the base-tubes, and in determining their coefficients of expansion. The basement of the Lake-Survey office was fitted up as a comparing-room and used until the winter of 1876. The Repsold base-apparatus having arrived in November, 1876, and a very elaborate series of comparisons being required in connection with the determination of its constants, a more complete comparing-room was built on the first floor of the office. Comparisons have been made there almost without interruption since the fall of 1877. The greater part of this work has been done under the immediate direction of General Comstock by Mr. E. S. Wheeler, aided at different times by one or more of the other assistants. Previous to 1870, the standards of length used on the Survey were (1) the 15-foot brass bar, having a cross-section of 1.1 inches by 0.33 inch, made by Würdemann to accompany the base-apparatus. Its length and expansion were given by Würdemann. After the measurement of the Chambers Island base, agates were set in its ends, changing its length. Its new length was determined at the Lake-Survey office by comparing it with (2) five brass yards made for the purpose in 1867 by Würdemann. These yards had a cross-section 0.98 inch by 0.37 inch, and their lengths were assigned by Würdemann and marked upon them. On examining these yards in 1870–'71, considerable discrepancies were found in their relative lengths as given above, and their end surfaces were also found to be so irregular that no final and satisfactory value of the 15-foot bar could be obtained from them, and they were discarded. (3) General Comstock then procured five new standard yards. They were constructed in the Office of Weights and Measures in Washington, under the direction of Professor J. E. Hilgard. They are all similar brass bars 1 inch deep by 0.6 inch wide, and 34.7 inches long, having at each end an axial cylinder 0.4 inch in diameter and 0.6 inch long. In the ends of these cylinders agates are held by the brass, which is burnished down on them. Their lengths were determined by comparisons made by General Comstock with the "Transfer yards A and B" in the Office of Weights and Measures. (4) Early in 1875 were received two steel yards made for the Survey by Troughton & Simms, of London, the lengths and expansions of which had been determined by Colonel A. R. Clarke, of the Royal Engineers, by comparisons with the Ordnance-Survey Standard. These yards are called "Clarke yards A and B." They are bars of steel 0.73 inch deep and 0.50 inch thick. The ends are cylinders 0.35 inch in length and 0.25 inch in diameter, with agates set in their ends. Each yard is inclosed in an iron box, so arranged that the yard remains in the box during comparisons. Each box has niches for four thermometers. The lengths of the brass yards and of the 15-foot brass bar were redetermined by comparisons with the Clarke yards, and these values have since been used. (5) For determining the values of the micrometer-screws of the contact-level comparators, used in the comparisons, a standard inch, graduated to tenths, and the last tenth to hundredths, has been used. This standard inch was made by Troughton & Simms, and the values

of each of its tenths and hundredths were determined by Colonel Clarke by comparisons with the Ordnance-Survey standard foot. (6) There are three standards which accompany the Repsold base-apparatus. One is a steel metre called "R. 1876," whose length and expansion were determined at the Imperial Bureau of Weights and Measures, Berlin. The second is a metre made of two bars, zinc and steel, and called the "metallic thermometer metre"; its constants have not yet (May, 1880) been determined. The third is a decimeter whose values were also determined at the above bureau. These three standards arrived in 1878.

7. *Thermometers.*—The lengths of the standard bars and yards and of the tubes of the base-apparatus depend upon their temperatures, and as the corrections to the thermometers then in use were unknown, the determination of these corrections was undertaken in the winter of 1870. Investigations and experiments relating to this subject have been carried on from time to time to the present date with the object of determining with the greatest precision the values of the thermometers used as standards, and of deducing from these the corrections to be applied to the indications of the working thermometers used directly to give the temperatures of the standards and tubes. The working thermometers are compared with the standard by completely immersing them and the standard in a glass vessel containing water, whose temperature is varied through the desired range, and reading the indications of both by means of a micrometer attached to the telescope of a small theodolite. The water in the vessel is kept in motion by small revolving paddles in order to secure a uniformity of temperature in it. The comparisons and tests of the freezing and boiling points and calibration of the standards, made in the Lake-Survey Office, have been made by Mr. Thomas Russell under the immediate direction of General Comstock.

In 1870 a standard Würdemann thermometer was procured and used until the receipt in 1872 of a standard Troughton & Simms thermometer, whose corrections had been determined by a series of comparisons with the standard thermometer at the Kew Observatory in England. In 1875 sixteen thermometers specially adapted for use with the Clarke yards were received. Eight of these thermometers had been compared by Colonel Clarke with the Ordnance-Survey standard. Soon afterwards five excellent thermometers of Casella's make were received. The corrections of these thermometers were determined at Kew Observatory, for both horizontal and vertical positions, in January, 1875. Two Baudin standard thermometers were procured from Paris in 1876, but the bulb of one of them was broken in transportation, and that of the other was cracked while being calibrated. A new bulb was blown on one of them by James Green, of New York, and it was afterwards calibrated and its freezing and boiling points determined at the Lake-Survey Office. As the expansions of glass and mercury depend upon both the first and second powers of the temperature, a mercurial thermometer which is correct at its fixed points, 32° Fahr. and 212° Fahr., may be found to be in error by several tenths of a degree at 100° Fahr. when compared with an air-thermometer. As such errors affect the precise determinations of the lengths and coefficients of expansion of the standards of measure, it was deemed advisable to have one of the standard thermometers compared with an air-thermometer, and Casella 21472 was accordingly sent to Professor H. Ste. Claire Deville, Paris, in 1876, for this purpose. Professor Deville, however, found it impossible to compare it with an air-thermometer, but he made a careful study of it and reported that it had no errors of calibration exceeding 0°.1, and that it differed from a tested standard of great perfection by only 0°.02 and 0°.03 at 61° Fahr. and 91° Fahr. The same thermometer was sent in the autumn of 1879 to Professor H. A. Rowland, of Johns Hopkins University, Baltimore, to be compared with an air-thermometer there. A direct comparison with the air-thermometer was not obtained, but Professor Rowland compared it very carefully on different days, in both horizontal and vertical positions, with two fine Baudin standards, which had been previously compared with an air-thermometer, and furnished a system of corrections by which to reduce its indications to those of a "perfect-gas thermometer." This thermometer is now the standard Lake-Survey thermometer, and a system of corrections depending upon it has been adopted for the other thermometers.

8. *Report on European surveys.*—In 1875 and 1876 numerous manuscripts, books, and maps relating to European surveys were, at the request of General Comstock, procured by the Engineer Department through the Department of State and forwarded to General Comstock for examination and report. They were carefully looked over and abstracts and translations of the more important papers and notes on the various maps were made, mainly by Captain H. M. Adams, Lieutenant P. M.

Price, and Mr. F. W. Lehnartz. These papers, with general remarks upon them and a discussion of the character and cost of similar surveys for the United States, by General Comstock, were published as a supplement to the Lake-Survey report for 1876.

METHODS OF FIELD-WORK.

1. *Astronomical work.* (*a*) *Longitudes.*—The instruments that have been used for time determinations are two Troughton & Simms astronomical transits, one of 43 inches focal length and 3 inches aperture, the second of 29 inches focal length and 2½ inches aperture; two Würdemann transits, Nos. 1 and 15, each having a focal length of 31 inches and an aperture of 2½ inches; a combined transit and zenith-telescope made by Pistor & Martins, Berlin, dated 1849, having a focal length of 24 inches and an aperture of 2½ inches; a combined instrument made for the Lake Survey by the same makers, and received in Detroit in 1874, having a focal length of 24 inches; and a transit made for the Survey by Buff & Berger, of Boston, having a focal length of 39 inches, a magnifying power with the diagonal eye-piece of 87, and an aperture of 3 inches. The last instrument was received in the spring of 1876, and since that time has been used constantly as the observatory instrument at Detroit. The two Würdemann transits have been used more than the others. The break-circuit sidereal clock No. 184, by Bond & Sons, was used in the observatory until the spring of 1875, when it was replaced by No. 256. Two of Bond & Sons' chronographs and several mean and sidereal chronometers by different makers have been used. In 1874, two break-circuit sidereal chronometers were procured for use by the field observers. Some longitude work by powder-flashing was done on Lake Superior in 1870–'71, but with that exception the method of telegraphic signals has been employed exclusively. For the determination of the longitude of primary triangulation points the field observer has used a chronograph, and in 1871 a clock, but since then a break-circuit chronometer. For these determinations, four complete nights' work has been required, and the reductions have been made by the method of least squares. For the determination of the positions of points in aid of State surveys, two full nights' work is required. The field observer uses a chronometer, and makes his observations by the eye-and-ear method, and these are reduced by the method of high and low stars. A clock and chronograph are used for all the observations made at Detroit. Time stars are selected from the American Ephemeris and from the German Catalogue of "539 Sternen," and circumpolar stars, which must be over 75° declination, are taken from one of the above or from the "General Bericht der Europäische Gradmessung" of 1870.

The following is the programme for one night's work, by which a complete time determination is made before and after the exchange of signals:

Level readings.
Circumpolar star (reversed on).
Level readings.
Five or more time stars.
Level readings.
Reversal of transit.
Level readings.
Five or more time stars.
Level readings.
Circumpolar star (reversed on).
Level readings.
Exchange of signals.
A second time determination following the same programme as above.

Level readings are also taken between the time stars when the interval is sufficient. When both observers have chronographs, signals are exchanged by automatic clock or chronometer beats by sending alternately from each station for $1^m\ 20^s$ or for $2^m\ 20^s$ until two sets of signals have been sent from and received at each station. When the field observer has no chronograph, the signals are sent (1) from Detroit by switching in the clock for seven minutes, the field observer noting the coincidences of the armature breaks and the beats of a mean solar chronometer, which is compared before and after the exchange of signals with the sidereal chronometer used in the time observations, these comparisons being also made by coincidences; (2) from the field station by automatic breaks, if the observer has a break-circuit chronometer, otherwise by hand-breaks, coincident with the beats of his chronometer, usually by sending for 15 seconds, then omitting for 15 seconds, then sending, and so on for a period of two minutes. The personal equation of the observers is determined on two nights before going into the field, and on two nights after returning from it, by the field observer setting up his instrument near that of the Detroit observer, and both making complete time determinations entirely independent of each other. The difference of the determined times, corrected for the few feet of distance between the instruments, gives the personal equation, the clock and chronometer being compared by the exchange of signals, in precisely the same manner as when one of the observers is in the field.

(*b*) *Latitudes.*—All latitude determinations are made by Captain Talcott's method of opposite and nearly equal meridian zenith distances. Some of the observations have been made with the combined zenith-telescope and transit of Pistor & Martins, but most of them have been made with the several Würdemann zenith-telescopes belonging to the Survey. The telescopes of the latter instruments have a focal length of 32 inches and an aperture of $2\frac{1}{2}$ inches, except that of No. 19, which has an aperture of 3 inches. For the determination of the latitudes of primary triangulation points, four nights' work is required, and for points in aid of State surveys, two nights. From twenty to thirty pairs of well determined stars must be observed each night. In 1871–'72 a "Catalogue of the Mean Declinations of 981 Stars between 12 hours and 26 hours of Right Ascension and 30° and 60° of North Declination for January 1, 1875," was prepared by Professor T. H. Safford under the direction of General Comstock, and printed at the Government Printing Office, Washington, 1873. Stars for latitude work have been selected mainly from this catalogue since its publication, and all the important latitude work of previous years has been recomputed with the star-places as given by it.

(*c*) *Azimuths.*—Azimuths are observed with the primary triangulation instruments. Previous to 1875, three nights' observations were required for the determination of primary azimuths, the method followed being that given in the Coast-Survey Report for 1866. In 1875 General Comstock issued instructions changing this method, and since then observations for primary azimuths have been made on five nights. The horizontal limb of the instrument is so placed that on each night the reading to the reference mark will exceed that of the preceding night by one-fifth of the distance between the microscopes, this reading remaining constant for the night. To avoid the assumption of perfect stability of instrument from first pointing to mark to last pointing to star, or of a change in azimuth strictly proportional to the time, the following is the programme for one star for one night, the star being near elongation:

Pointing to mark.
Pointing to star.
Level readings.
Pointing to star.
Pointing to mark.

Reverse the telescope, keeping the same pivot in the same wye.

Pointing to mark.
Pointing to star.
Level readings.
Pointing to star.
Pointing to mark.

Pointing to mark.
Pointing to star.
Level readings.
Pointing to star.
Pointing to mark.

Reverse the telescope as above.
Pointing to mark.
Pointing to star.
Level readings.
Pointing to star.
Pointing to mark.

As many closely circumpolar stars are observed on each night as is practicable. For secondary azimuths two such stars are observed on two nights, and the above programme is followed except that on the completion of each set of readings—to mark, star, star, mark—the horizontal limb is advanced through an angle equal to one-fourth the distance between the microscopes or verniers, and that on the second night the first reading to the mark is greater than that on the first night by one-eighth of this distance. The error of the chronometer is obtained by observing the meridian transits of well-determined stars.

2. *Base-lines.*—The bases measured previous to 1877, viz, at Minnesota Point and Keweenaw Point, Lake Superior; Fond du Lac, Wis.; Sandy Creek, Lake Ontario; and Buffalo, N. Y.; were measured with the Bache-Würdemann compensating apparatus made for the Survey in 1852. In the fall of 1876, a new base-apparatus, made for the Survey under the special instructions of General Comstock, by Repsold & Sons, of Hamburg, was received at Detroit, and with it the bases near Chicago and Olney, Ill., and Sandusky, Ohio, have been measured. The accuracy of the measurement of all these lines has been tested both by duplicating the measurement of a part or the whole of the line, and also by dividing it into two or more sections and checking the lengths of these sections by a triangulation. A full description, with plates, of the Repsold apparatus and of the manner of using it, will be found in Chapter VIII of this report. With it a base is measured with one tube, which is a line-measure, metallic thermometer, four metres long, consisting of a bar of zinc and a bar of steel joined at their middle points. This tube measures the distances between microscopes, with attached micrometers, which are mounted on stable iron stands, so constructed that the microscopes can be placed directly above the ends of the tube. In 1871, a contact apparatus consisting of two iron tubes, for the measurement of secondary bases, was constructed for the Survey. It is similar to the apparatus described in Appendix 45 of the Coast-Survey report for 1857. Although accompanied by trestles, it has where possible been used without them on a straight stretch of railroad track. Two measurements with this apparatus of a base about a mile in length should not differ by more than the $\frac{1}{100000}$ part of the length of the line.

3. *Primary and secondary triangulation.* (*a*) *Instruments.*—In 1870 the theodolites available for primary work were the three Oertling theodolites received the previous year, and the 10-inch repeating Gambey theodolite without vertical circle, which had been in use on the Survey since 1851. The Oertling instruments were so badly constructed and graduated that their use was discontinued as soon as they could be replaced by better instruments, but one of them being used in 1871, and none of them after that year. In 1871 three new theodolites were received, viz: (1) A Troughton & Simms non-repeating instrument, having a 14-inch horizontal limb, reading by three microscopes to single seconds. The telescope has a focal length of 24 inches, the diameter of the object-glass being $2\frac{1}{4}$ inches. (2) A Troughton & Simms repeating theodolite, always used, however, as a non-repeater, with a 12-inch limb, reading by two microscopes to single seconds, the telescope having a focal length of 19 inches. This instrument, having been injured in transportation, and requiring repairs, was not used until 1872. (3) A Pistor & Martins non-repeating theodolite, having a 14-inch horizontal limb, reading by two microscopes to 2 seconds. The telescope has a focal length of 25 inches. In May, 1872, (4) a non-repeating theodolite, made by Repsold & Sons, was received. This instrument has a 10-inch limb, reading by two microscopes to 2 seconds. The telescope is prismatic, the eye-piece being at one end of the horizontal axis, and has a focal length of 20 inches. In the spring of 1876 two new non-repeating theodolites (5) (6), made for the Survey by Troughton & Simms, were received. These instruments have a 14-inch limb, reading by three microscopes to single seconds. The telescopes, which transit, have a focal length of 30 inches. These six instruments have vertical circles, which are read either by verniers or micro-

scopes, and are provided with the appliances necessary in astronomical work. They are all good instruments, capable of doing very precise work.

(*b*) *Stations.*—The general plan of the stations is the same as already described, although several modifications, for the purpose of increasing their stability and convenience for use, have been made from time to time. They have generally been built by contract since 1874. Each observer occupying a station is required to thoroughly mark it in the following manner: (1) The geodetic point is the center of a ¼-inch hole drilled in the top of a stone 2 feet by 6 inches by 6 inches, sunk 2½ feet below the surface of the ground. When the occupation of the station is finished, a second stone post, rising 8 inches above the ground, is placed over the first stone. (2) Three stone reference posts, 3 feet long, rising about a foot above the ground, are set within a few hundred feet of the station, where they are least likely to be disturbed. (3) A sketch of the topography within a radius of 400 metres about the station is made, and the distances and azimuths of the reference marks are accurately determined.

(*c*) *The reading of angles.*—The targets are usually boards or frames of such a width as to subtend an angle of about four seconds, nailed to the center-pole, which rises above the pyramid of the station. The targets are covered with black and white cloth, the upper part usually being black and the lower white. On the longer lines the target is a flash from a heliotrope or small mirror. In all cases the co-ordinates of the center of the target, referred to the geodetic point, are determined with the utmost precision by plumbing down with a small theodolite or plumb-line. When measuring angles the center of the instrument is either placed directly over the geodetic point or is referred to this point by the above method. The instrument is protected from the sun and wind by a square tent. Since 1872 angles have been read with a non-repeating instrument by the following method: The instrument having been put in as perfect adjustment as possible, a station is selected as a starting point, and a pointing to it is made, and then the other stations are pointed at in succession, closing on the first station, the telescope being moved in the direction of *positive* graduation. This gives what is called a *positive result* for the value of each angle. A third, pointing to the first station, is immediately made, and the other stations are again pointed at in succession, closing on the first station, the telescope being moved this time in the direction of *negative* graduation. This gives a *negative result* for each angle. The pointings thus made are called one *set*. A mean of these positive and negative results constitutes a *combined result*. The positive and negative results being obtained within a short space of time, during which the twist of the station and of the horizontal limb can be assumed as uniform, the errors arising from these sources are eliminated from the combined result.

When many stations are visible, as a general rule not more than five are pointed at in one set, so that a set may not require more than ten or fifteen minutes. The same initial station is not used for each set, but the stations in succession are taken as starting points. Sixteen combined results are usually required to be obtained for each angle, although in special cases this number is increased to twenty-four. The mean of the combined results for each angle should not have a probable error greater than $0''.3$. In secondary work eight combined results are usually required for each angle. The instrumental errors are eliminated as follows: The eccentricity by reading at each pointing all the verniers or microscopes; the periodic and accidental errors of graduation by reading each angle an equal number of times on every 30° or less around the limb. For example, in obtaining sixteen combined results with a 3-vernier instrument the horizontal limb would be turned 15° after each two combined results; with a 2-vernier instrument it would be turned 22½° after each two combined results. Collimation error and small errors of level and inequality of pivots are eliminated by turning the microscopes or verniers through 180°, and then turning the telescope 180° about its transit axis, lifting it a little from the wyes if it will not revolve in altitude without, this reversal being made either after each combined result or after every set of two or more combined results, care being taken to obtain an equal number of results in each position of the telescope. With a repeating instrument each angle is read separately in sets of five repetitions, the errors being eliminated in the same manner as with a non-repeating instrument. In the primary work the error in closing a triangle should not be greater than 3 seconds, and in secondary work this error should not exceed 6 seconds.

4. *Precise levels.*—The heights above the ocean of the Great Lakes at definite stages of water

having never been determined with precision, this work was begun in the spring of 1875. The Coast Survey having established at Albany, N. Y., a bench-mark whose height above mean tide at New York was accurately determined, it was decided to adopt this bench-mark as a starting point from which to run a line of levels to Oswego, on Lake Ontario, and then to connect the different lakes by similar lines. The mean level of each of the lakes for a period of several months was determined by taking at certain points tri-daily water-gauge readings, the zeros of these gauges being accurately connected with permanent bench-marks. The points at which the water-gauges and bench-marks were established were Oswego and Port Dalhousie, on Lake Ontario; Port Colborne, and Gibraltar near the mouth of the Detroit River, on Lake Erie; Lakeport, on Lake Huron; Escanaba, on Lake Michigan; and Marquette, on Lake Superior. It was assumed that the mean water surface of each lake during the summer months was level from one end of the lake to the other. The water-level observations, which had extended over a period of several months, in order to eliminate the effects of winds and barometric changes, therefore gave the differences of level between the bench-marks at Oswego and Port Dalhousie; between those at Port Colborne and Gibraltar; and between those at Lakeport and Escanaba, the surfaces of Lakes Michigan and Huron having practically the same level. The differences of level between the bench-marks at Albany and Oswego, between those at Port Dalhousie and Port Colborne, between those at Gibraltar and Lakeport, and between those at Escanaba and Marquette were determined by running between those places duplicate lines of precise levels, the work being done independently by two separate parties. As a full report on this subject, giving the methods and results of the leveling and a description of the instruments will be found in Chapter XXII of this report, it is unnecessary to enter into further details in this place.

5. *Magnetic observations.*—The instruments used in magnetic observations have been a Jones declinometer and a Würdemann theodolite magnetometer for determining declination and horizontal intensity, and a Barrow and a Troughton & Simms dip-circle for determining the dip. The instructions have required observers to proceed in the following manner: In making the observations the first requirement is to select a station free from local attraction. This is tested by taking two radii from the station, making an angle of 90° with each other, and from 300 to 600 feet long, and with the magnetometer seeing if the magnetic bearings at the two ends of each radius agree within 5 minutes. If not, another station where this condition is fulfilled must be chosen. At each station two results on different nights for the astronomical azimuth are obtained, it being required that these results shall agree within 2 minutes. Two days of declination work at both morning and afternoon elongations, giving mean results for the two days not differing by more than 10 minutes, are obtained. The horizontal intensity is determined by two complete and independent sets of observations of oscillations and deflections at two distances, the results agreeing within .025. Dip is observed on two days, both needles being used each day, with their poles direct and reversed, the means for the two days agreeing within 10 minutes. If an agreement within the above limits is not obtained additional observations are made.

6. *Topography and hydrography.*—The following account of the methods in topography and hydrography is, with the exception of a few slight changes, a copy of the report on that subject by Captain H. M. Adams, published in the Lake-Survey report for 1876.

For topography and in-shore hydrography, the parties are distributed along the lake shore at a distance of about 12 miles from each other, and each party is required to extend its work 6 miles on each side, to connect with the work of the adjacent parties. Topography is developed inland about three-fourths of a mile, this distance being increased, when necessary, to include towns or important localities of any kind. The in-shore hydrography is extended lakeward one half-mile, or to the 4-fathom curve. From the primary triangulation secondary points are determined for each of the above 12-mile sections, and the work done by a topographical party is connected with one or more of these points. A comparison of the distances between the primary and secondary triangulation points, as given by that triangulation and by the shore-party work of 1877, shows an average discrepancy of 1 in 2,222 in 22 lines, varying in length from 1,016 to 23,938 meters, the average length being 12,942 meters. The greatest discrepancy was 1 in 1,000. In the following description the methods used for making the detailed survey of one of the 12-mile sections will be given.

Triangulation stations (tertiary) are established from 500 to 2,000 metres apart, so as to include the area to be surveyed. These stations generally consist of a post about 8 inches in diameter and 5 or 6 feet long, set into the ground and braced. Over the post may be placed a whitewashed tripod, if the station is to be seen at a long distance. In measuring angles, the instrument is mounted on a trivet placed on the post. A base-line is selected, and from it the net-work of triangles is developed to include the whole area, and this triangulation is verified by closing on a second measured line. Base-lines are measured with a chain 20^m long, made of links $0^m.5$ long. The links are made of steel wire $0^m.005$ in diameter, and are connected in the usual way by rings. When the chain is used for measuring a base-line, its length is determined by comparison with a standard steel bar 1^m by $0^m.008$ by $0^m.008$ before and after measuring the base. Two or more measurements are made of each base, and the following degree of accuracy has been attained. In fifteen cases examined, the mean discrepancy in chaining base-lines was 1 in 17,485; the greatest discrepancy, 1 in 5,729; the least discrepancy, 1 in 45,104.

For the triangulation, an alt-azimuth instrument reading to 10 or 20 seconds is used. When a station is occupied two or more pointings are made to each station, visible from the one occupied, and important angles are determined by repetition. The following degree of accuracy is attained in closing triangles: In twenty-six cases examined, the average error in closing triangles was 14 seconds; nine of the above gave an average error of +8 seconds; seventeen of the above gave an average error of —7 seconds; the maximum error was —42 seconds. In carrying the triangulation from one base to another, the following degree of accuracy was obtained: At Niagara River, from first to second base, twenty-four triangles, discrepancy in connection, 1 in 2,675; at Buffalo, N. Y., from first to second base, eighteen triangles, discrepancy, 1 in 5,421; at East Porter, N. Y., from first to second base, twenty-two triangles, discrepancy, 1 in 5,040. When the country is obstructed by woods or otherwise, so as to make a triangulation impossible, a line is chained along shore, or along a road parallel to the shore.

Azimuth is determined by observation on Polaris at elongation on at least two nights. The following degree of accuracy in this work has obtained: In seventeen cases examined, the mean discrepancy resulting from two nights' observations was 20 seconds; the greatest discrepancy was 1 minute 10 seconds. A comparison of azimuth is made when the work of two parties is joined. After correction for convergence of meridians, the resulting difference is due to error in observation for azimuth and to error in carrying the azimuth. In twenty-six cases examined, the average error in the connection of azimuths of two parties was 1 minute 23 seconds. In the work of 1877 the work of different parties joined fourteen times, and the average error in the connection of azimuths was 51 seconds.

For topography, the stadia is used to delineate the ground covered by triangulation. In filling in the details, all roads, streams, buildings, fences, cultivation, and woods are indicated, and contours are determined for every 10 or 20 feet of elevation. In the instruments used with the stadia rod, the horizontal wires for stadia measurements are fixed, and each rod is divided and marked for the instrument with which it is to be used. The wires are frequently tested, to determine if any change takes place, by reading the rods on a chained line. Four men are required for a topographical party, one assistant in charge of instrument, one man to carry instrument, record, and do such general work as may be required, two men with stadia rods. The work is conducted as follows: The theodolite is set up over a triangulation station, pointed and set on a line whose azimuth is known or assumed for the time, in case true azimuth has not been determined by observation, so that the horizontal limb shall give by reading the azimuth of the line. Readings are then taken to prominent points within range (400^m) that are to be located; the distances are at the same time read from the stadia rods, which are carried from point to point by the stadia men, as directed by the assistant in charge. The vertical angles and distances read from the stadia are arguments with which from a table horizontal distances and elevations are determined. The angles and distances are recorded in the note-book, and the work is plotted to the scale of 1 : 10000 on a protractor-sheet 21 by 16 inches, fastened to a light drawing-board; further details are sketched in. The instrument is next moved to a second point, previously located by stadia from the first point, set up, and pointed to the first station on the back azimuth of the line. The distance back to the first station is read and recorded for a check, and points located from the first station may also be located from the second station. The prominent points in range are next located as before, and readings are taken to triangulation stations in sight, in order to check the azimuth when the same points are reached with the stadia work. In this way the work is continued and closed on a triangulation point, so that the length of the meandered stadia line is checked by the triangulation. By computing co-ordinates for the stadia work for 1875, the average amount of discrepancy in 141 lines, varying in length from 965^m to 6,486^m, mean 2,450^m, when compared with lines determined by triangulation or by chaining, was found to be 1 in 649. This error ought in no case to exceed 1 in 300.

In-shore hydrography.—Sounding stations are established along the shore at intervals of 100 to 500 metres, and a line of buoys 500 to 1,000 metres apart is placed at the outer limit of the space to be sounded, generally in about 4 fathoms of water. When the distance from the shore to the outer limit of sounding exceeds one mile, a second line of buoys is placed midway between the shore and the outer limit. The sounding stations are located from the principal stations in running the shore-line. The buoys are located from the principal shore-line stations by intersections from three points. This serves as a check on the shore-line and topography, and on an obstructed, broken, irregular shore, the buoys may form a part of the system of tertiary triangulation, care being taken to check the work by closing on a measured line. For sounding, a six-oared cutter, one assistant to record, one helmsman, one leadsman, and six oarsmen are employed. The lead-line is compared with the standard measure every day when in use. Lines of soundings taken on time are run from the sounding stations on shore to the buoys, so as to cover the area to be sounded out. The method in most general use is that of running lines of soundings to and from the sounding stations on shore. The direction of the lines of soundings is determined by an assistant on shore with a theodolite, and the outer end of each line is fixed by its intersection with the line of buoys. In sounding, the boat starts from a sounding station, which is occupied by an assistant with a theodolite, who directs the helmsman in the boat by waving a flag to the right or left, so that the line is run by the boat on an azimuth determined for the lines of soundings, generally so as to make the lines perpendicular to the shore. The line is continued in this way till it intersects the line of buoys. The assistant on shore now moves to the next sounding station, and the boat moves in the same direction on the line of buoys, and, again directed by the flag and a range selected by the helmsman or established by the assistant on shore, runs another line of soundings back to the station on shore now occupied by the assistant. In running to and from shore the distance from the sounding station at the inner end of each line is estimated and recorded. Soundings are taken on time at intervals of 15^s, 30^s, or 1^m, depending on the depth. The soundings near the shore or in shoal water, being most important, are taken at every 15^s in running out and in with the boat. The average speed of the cutter in sounding for a whole day is 70 metres per minute. The number of soundings taken in a space 1,000 metres square near shore in ten cases examined varied from 66 to 322. The sounding stations 200 metres apart, and sounding every 15^s, we should have 285 soundings for a space 1,000 metres square, or one sounding for a space of 59 metres square.

Off-shore hydrography.—Off-shore soundings are lines of soundings run by a steamer, perpendicular to the general direction of the coast, and about one mile apart, commencing with the hydrography done by shore-parties and extending out ten miles from land. Observers with theodolites are placed on shore at two stations about six miles apart, readings are taken to two or more prominent stations, and these readings are repeated often enough to detect any movement in the lower limb of the instrument. The steamer, at starting, whistles, drops the balloon, and a sounding is taken at the same time, and in running out and in the balloon is dropped and a sounding is taken every ten minutes. At the instant the balloon drops, the observers on shore take readings to the steamer and note the time. On the steamer the time of dropping the balloon is noted, and a sextant-angle is read, if possible, between two points located on shore by the triangulation. The watches used by the observers on shore are compared with the watch used on the steamer, before and after the day's work. In water less than 20 fathoms deep, soundings are taken every five minutes. The lead-line is compared with a standard measure every day when in use. The notes are plotted every evening, to make sure that the soundings are properly distributed. Lines of soundings are also run entirely across the lakes, 15 miles apart. The steamer in this case is located from the shore, when soundings are taken, as long as it remains in sight. Permanent bench-marks are established, and water-gauges are kept, to which all soundings can be reduced.

FIELD-WORK BY YEARS.

1870.

Lake Superior.—The triangulation and astronomical work were carried from the line Wheal Kate–Farquhar's Knob as far west as the line Detour–Split Rock by Lieutenant Ruffner and Messrs. O. B. Wheeler, G. Y. Wisner, and A. R. Flint, the general direction of the work being in charge of Major J. A. Smith, who was in command of the Search. Lieutenant J. H. Weeden, with a party on the Surveyor, finished the survey of the Apostle Islands. The base-line on Minnesota Point, which had been measured with wooden rods under the direction of Captain Meade in 1861, was remeasured with the Bache-Würdemann apparatus by General Comstock, assisted by Lieutenant Weeden and Messrs. E. S. Wheeler, A. C. Lamson, and E. J. Knight.

Lake Michigan.—The survey of Lake Michigan was resumed where it had closed in 1866, by a party under Lieutenant A. N. Lee on the Ada, and by the shore-parties of Lieutenant W. R. Livermore and Mr. J. R. Mayer, working on the east and west shores, respectively. A number of additional lines of off-shore soundings were run along the north shore. The off-shore work on the west shore was extended to a point about 16 miles south of Sheboygan, and on the east shore some work was done about Little Point Sable. The shore-parties made about the same progress. On account of the late date of the passage of the appropriation bill, these three parties did not take the field until the 1st of August.

Lake Champlain.—On September 9, Lieutenant Livermore's party was withdrawn from Lake Michigan and sent to Lake Champlain, where it made a survey of Burlington Bay, and nearly completed that of Plattsburg Bay.

1871.

Lake Saint Clair.—In April a large force, consisting of two steamer-parties, two shore-parties under Messrs. Lamson and Mayer, and the triangulation parties of Messrs. Wisner and Flint, all under the general direction of Lieutenant Livermore, were sent to Lake Saint Clair to complete the survey of that lake. This was accomplished in about a month, and the parties returned to Detroit to reorganize for the summer's work.

Lake Superior.—Messrs. Wisner, Flint, Marr, and J. C. Jones, with the steamer Search to move and supply them, took up the triangulation where it ended the previous year and completed it to the west end of the lake, connecting with the Minnesota Point base. They also remeasured as much of the triangulation of Keweenaw Bay as was necessary to determine the line Wheal Kate–Vulcan from the Keweenaw base. The angles of the large triangle Vulcan–Tip Top–Saint Ignace were also partially remeasured. The difference of longitude between Saint Paul, Minn., and North Base station, near Duluth, was determined by General Comstock at Duluth and Mr. Flint at Saint Paul; and the difference of longitude between Detroit and North Base was determined by General Comstock and Mr. Wisner at the latter and Mr. O. B. Wheeler at Detroit. Observations for latitude were made at both ends of the Minnesota Point base and at station Aminicon by Mr. Wisner; at stations Brulé, Saint Ignace, and Crebassa by Mr. Marr; and at Saint Paul and stations Buchanan and South Base, Keweenaw Point, by Mr. Flint. Azimuth observations were made at the Minnesota Point base, South Base Keweenaw Point, and station Aminicon, by Mr. Wisner. The differences of longitude between many of the stations occupied were determined by the method of powder-flashes. When not employed in connection with the triangulation parties the steamer Search was engaged in running lines of deep-sea soundings, and advantage was taken of the opportunity thus presented to have dredgings and examinations of the bottom made by Professor Sidney I. Smith, whose report on the deep-water fauna of Lake Superior is published as an appendix to the annual report of the Survey for 1871.

Lake Michigan.—Work was again carried on on both shores of the lake, Lieutenant Lee on the Ada and the shore-party of Mr. Mayer extending that on the west shore to Milwaukee, while Lieutenant Weeden on the Surveyor extended the off-shore soundings on the east shore to Grand River, and the shore-parties of Messrs. Foote and Custer completed their work as far as the south end of the lake. Lieutenant C. F. Powell, on the chartered steamer Powers, was engaged in hydrographical work in the north end of the lake. Mr. E. S. Wheeler made a reconnaissance for and built primary stations from the head of Green Bay to Milwaukee.

Saint Lawrence River.—The survey of this river was commenced, under the direction of Lieutenant Livermore, by the shore-parties of Messrs. Lamson and Towar. The topography and part of the hydrography and triangulation of that part of the river between the boundary line near Saint Regis, N. Y., and Brockville were done during the season.

Lake Champlain.—A party under Lieutenant E. Maguire completed the survey of Burlington Bay and made a survey of 13 miles of the narrows at the south end of Lake Champlain.

Astronomical work at Detroit.—In March, 1871, the observatory which has since been used for all astronomical work at Detroit was built immediately in rear of the office building, on the corner of Grand River avenue and Park Place. This observatory is provided with two stone observing-piers and a stone pier for the astronomical clock. In the following May the difference of longitude between the east pier of the observatory and the dome of the Naval Observatory at Washington was determined, Mr. O. B. Wheeler observing at Detroit and Professor J. R. Eastman at Washington. The latitude of the Detroit observatory was determined in 1872 by General Comstock. During the summer, observations were made for the telegraphic longitude of Austin, Nev., Battle Mountain, Nev., and Fort Leavenworth, Kans. Mr. O. B. Wheeler observed at Detroit. The work with the two first-named places was in connection with a party under the charge of Lieutenant G. M. Wheeler, and that with Fort Leavenworth was in connection with Lieutenant E. H. Ruffner, engineer officer of the Department of the Missouri.

1872.

Lake Superior.—The triangulation was carried eastward from the line Vulcan-Huron Mountains as far as Granite Island and Triloba, and a reconnaissance was made as far as Green Bay. The observers were Messrs. G. Y. Wisner, G. A. Marr, and R. S. Woodward, who had the Surveyor as a supply steamer.

Lake Michigan.—The hydrography and topography were continued by the party on the Search under the command of Captain A. N. Lee and by the shore-parties of Messrs. Mayer and Custer. The off-shore work on the west shore was extended to Racine, and on the east shore to 10 miles south of Saint Joseph, and the lines of soundings across the lake were completed. The topography and in-shore hydrography of both shores were completed, with the exception of the portion between Evanston and Waukegan. Lieutenants D. W. Lockwood and E. Maguire on the east shore and Lieutenant C. F. Powell on the west shore were engaged in carrying the lines of latitudes and azimuths southward from Pointe aux Becs Scies and Portage Canal, respectively. The base-line near Fond du Lac, Wis., about 24,000 feet in length, was measured by Mr. E. S. Wheeler, assisted by Messrs. Olds, Burton, and Pattengill. The angles of the triangulation extending southward from Green Bay were read by Mr. Flint from the head of the bay to station Horicon. Mr. Flint also observed for the azimuth of a line near Fort Howard and read the angles about the Fond du Lac base. He also located stations from Racine to Chicago and from Chicago to Sturgis, Mich.

Saint Lawrence River.—The survey of the river was made continuous from Saint Regis to Wellesley Island, with the exception of a gap in the triangulation near Ogdensburg, by Captain Livermore in command of the Ada, and by the shore-parties of Messrs. Lamson and Towar, and the triangulation parties of Messrs. Foote and Metcalf.

Sandusky Bay.—Before going to the Saint Lawrence River, Mr. Lamson made a detailed survey of Sandusky Bay, in Lake Erie.

1873.

Lake Superior.—The triangulation was carried eastward to Grand Island, and the stations as far south as Green Bay were built by the parties of Messrs. Wisner and Marr, the first named being in command of the Surveyor. Mr. Wisner also redetermined the latitude and azimuth of the Keweenaw base. The Keweenaw base was remeasured by Mr. E. S. Wheeler, assisted by Messrs. Burton, Pratt, and Goldsmith.

Lake Michigan.—The hydrography and topography of Lake Michigan were completed by the party of Lieutenant D. W. Lockwood on the Search and the shore-party of Mr. Custer. The steamer-party also built the stations necessary for the triangulation of Green Bay. Lieutenants Maguire and Powell, on the east and west shores, respectively, completed the lines of latitudes and azimuths, connecting them with the main triangulation near Saint Joseph and Milwaukee. Messrs. Flint and Woodward carried the triangulation south to the line Palatine-Deerfield, and from the head of Green Bay north to Peshtigo Point. Mr. Flint observed for latitude and azimuth at Minnesota Junction.

Saint Lawrence River.—The topography and hydrography of the Saint Lawrence River were completed to Lake Ontario, and the triangulation was carried to Clayton, near the head of the river, by Captain Livermore in command of the Ada, the shore-parties of Messrs. Mayer and Towar, and the triangulation parties of Messrs. Foote and Metcalf. The survey of the river depends upon a carefully executed secondary triangulation, consisting of 140 triangles extending over a distance of 106 miles from Boundary Post, No. 2, near Saint Regis, to Bear Point, near Cape Vincent. The angles as read closed the triangles within 6 seconds, with the exception of 8 triangles near the east end of the work, the greatest error being 12.6 seconds. Five bases averaging about a mile in length were measured by Captain Livermore with the secondary base-apparatus. Special difficulties were encountered in the hydrographical work on account of the swift current in many parts of the river. A description of the methods adopted to meet these difficulties will be found in Captain Livermore's report published in the annual report for 1872.

Detroit River.—During the summer of 1873 and the following winter, a complete survey of the Detroit River, including a survey of the city of Detroit, was made. The hydrography and topography were done by a party under the charge of Mr. A. C. Lamson, and the triangulation depending upon a secondary base near Wyandotte, mainly by Messrs. Marr and Towar.

Magnetic observations.—In 1872 and 1873, Captain A. N. Lee made a series of magnetic observations at sixteen different points in the lake region. The results of these observations, together with those made at four points on Lake Superior by General Comstock in 1870 and 1871, are published in Appendices 6 and D of the Lake-Survey Reports for 1873 and 1874, respectively.

Astronomical work in aid of State surveys, &c.—Application having been made by the authorities of several of the States bordering on the lakes for the determination by the Lake Survey of the positions of points in their interior, to serve as a basis for the State geological and other surveys, the work was commenced in 1873. No officer of the Survey being available, on account of the pressure of other duties, Professor S. W. Robinson, who was assisted by Mr. T. W. Wright, was employed as the field observer. Latitude and longitude determinations were made at Galena, Ill., and Valley Junction, Wis., the longitude work being done in connection with Mr. O. B. Wheeler at Detroit. Mr. Wheeler, in connection with Dr. Kampf, at Ogden, Utah, also determined the longitude of the observatory established at Ogden by Lieutenant G. M. Wheeler, United States Engineers.

1874.

Lake Superior.—The triangulation was completed by Mr. Marr, and the stations necessary to connect the triangulation of Lake Superior with that of Green Bay were occupied by Mr. Wisner. The work of both observers was finished early in July.

Lake Michigan.—The Green Bay triangulation was finished by Messrs. Wisner, Flint, and Woodward. Latitude and azimuth determinations were made at the Ford River station by Mr. Wisner. During the remainder of the season Messrs. Flint and Woodward carried the Lake Michigan triangulation from Deerfield around the south end of the lake to station Bald Tom, Mr. Flint observing for latitude and azimuth at Willow Springs, near Chicago.

Lake Ontario.—The secondary triangulation at the head of the Saint Lawrence River was finished by Lieutenant T. N. Bailey and Messrs. T. Russell and J. H. Darling. The survey of Lake Ontario was commenced by the parties of Lieutenant Lockwood and Lieutenant Bailey on the Ada and Surveyor, respectively, the shore-parties of Messrs. Mayer, Towar, Terry, and Eisenmann, the triangulation parties of Messrs. Wisner, T. Russell, and Darling, and the reconnaissance party of Mr. Marr. The off-shore hydrography of the east end of the lake was completed as far as Oswego by Lieutenant Bailey. The shore-party work at the east end was finished and extended westward to Big Sodus Bay. A reconnaissance for the primary triangulation of the east end of the lake was made by Mr. E. S. Wheeler, who also selected a site for a base between the mouths of Big and Little Sandy Creeks. This base, about three miles long, was measured later in the season by Mr. E. S. Wheeler, assisted by Messrs. Olds and Pratt, and latitude and azimuth observations were made at its north end by Mr. Wisner, who also read the angles about the base. The party on the Ada was engaged during the season in building stations and in moving and supplying the other parties. The angles of the primary triangles were partially read as far west as Oswego, N. Y., and points for stations were selected as far as Buffalo.

Astronomical work in aid of State surveys.—The geographical positions of Corunna, Pontiac, Lapeer, Flint, Saint Johns, Ionia, and Grand Rapids, all county seats in the State of Michigan, were determined. Lieutenant Maguire was the observer at Detroit. Captain H. M. Adams observed at Corunna, and Lieutenant C. F. Powell at the six other stations.

1875.

Lake Ontario.—The triangulation was extended to Buffalo by Messrs. Wisner, Woodward, T. Russell, Darling, and Metcalf. The hydrography and topography of the lake were completed by the parties of Captain H. M. Adams and Lieutenant Powell, on the Ada and Surveyor, respectively, and the shore-parties of Messrs. Mayer, Lamson, Towar, Terry, and Eisenmann. The survey was extended along the Canadian south shore to a point five miles west of Port Dalhousie.

Lake Erie.—After completing the above work and making a survey of the Niagara River, the same parties commenced the survey of Lake Erie, and before the close of the season extended it along the north shore to a point 5 miles west of Port Colborne, and along the east and south shores to Conneaut, Ohio. The peninsula called Long Point on the north shore was also partly surveyed. The Buffalo base, about 4 miles long, near Buffalo, N. Y., was measured by Mr. E. S. Wheeler, assisted by Messrs. Pratt, Olds, and W. S. Russell. The triangulation about the base was done by Mr. Wisner, who also made azimuth observations at station Tonawanda, near the base. The latitude of Tonawanda was determined by Mr. Flint, who, in connection with Lieutenant Lockwood at Detroit, also observed for the longitudes of Tonawanda, and of Mannsville near the Sandy Creek base. Mr. Marr selected points for the triangulation between Buffalo and Cleveland.

Astronomical work in aid of State surveys.—The longitudes and latitudes of eleven county seats in the State of Michigan were determined. Lieutenant Lockwood observed at Detroit; Lieutenant P. M. Price was the observer at Charlotte, Marshall, Kalamazoo, Paw Paw, and Stanton; and Lieutenant T. N. Bailey observed at Howell, Lansing, Jackson, Hastings, Allegan, and Newaygo.

Precise leveling.—Duplicate lines of levels were run between Albany and Oswego, and between Port Dalhousie and Port Colborne, along the line of the Welland Railway, by Messrs. L. L. Wheeler and F. W. Lehnartz.

Miscellaneous.—A reconnaissance for primary triangulation was made by Mr. Flint, who selected sites for stations from Chicago to the Ohio River, and from Michigan City, Ind., to Bronson, Mich. The construction of the harbor of refuge at Sand Beach on the west shore of Lake Huron having made such progress as to be valuable to commerce, a detailed survey of it was made by Mr. A. T. Morrow, in August.

1876.

The parties were sent into the field in June, but as no appropriation for continuing the work had been made by the 25th of June, it became necessary, in order to avoid a violation of the law, to withdraw them and discharge the civil assistants on or before the 30th. The new appropriation having been passed in the mean time, the parties again took the field early in August.

Lake Erie.—The hydrography and topography were completed about Long Point and were extended westward to Vermilion, Ohio, by the party of Lieutenant P. M. Price, who was in command of the Ada, and the shore-parties of Messrs. Lamson, Towar, and Terry. The triangulation was carried westward to the vicinity of Painesville, Ohio, by the parties of Messrs. Wisner, Flint, Woodward, and Darling. Points for stations were selected as far as Sandusky by Mr. Marr.

Precise levels.—A duplicate line of levels was run from Escanaba on Green Bay to Marquette on Lake Superior, by Messrs. L. L. Wheeler and Lehnartz.

Astronomical work.—Captain H. M. Adams made the observations at the Detroit observatory during the season. Lieutenant Lockwood observed for longitude at Mount Forest, Ill., near the Chicago base, and for longitude, latitude, and the magnetic elements at Saginaw and Saint Louis, in Michigan. In the fall he observed for longitude and latitude at Cairo, Ill. At the request of Captain W. S. Stanton, engineer officer of the Department of the Platte, telegraphic signals were exchanged between Detroit and Fort Fetterman, Wyo. At a similar request from Lieutenant W. Hoffman, Eleventh Infantry, acting engineer officer of the Department of Texas, signals were exchanged with Forts Stockton, Concho, McKavett, Richardson, and Griffin, in Texas. For the two latter points the Detroit observations were made by Messrs. O. B. Wheeler and A. R. Flint.

Magnetic observations.—Lieutenant Bailey was engaged in making magnetic observations until he was relieved from the Survey in August, after which time the work was continued by Lieutenant Powell, who observed at four points. The results of all the magnetic work done in 1876 are given in the annual report for 1877.

Mississippi River.—By the act of Congress making appropriations for the Lake Survey for the fiscal year beginning on July 1, 1876, the duty of making a survey of the Mississippi River was added to the other duties of the Lake Survey, and a small appropriation was made for the commencement of the work. Accordingly, in November, 1876, Lieutenants D. W. Lockwood and P. M. Price, and Mr. F. M. Towar, with a party equipped for general work, were sent to Cairo, Ill., to begin the survey. This party returned to Detroit about the middle of the following February.

1877.

Lake Erie.—The work on this lake was finished this season, the parties engaged in it being the steamer party on the Ada, under command of Lieutenant D. W. Lockwood until September 7, and of Lieutenant P. M. Price after that date; the shore-parties of Messrs. Lamson, Towar, and Terry; and the triangulation parties of Messrs. Wisner, Woodward, and Darling. Mr. Wisner determined the azimuth of the Sandusky base.

Triangulation connecting Lakes Michigan and Erie.—The base near Chicago, Ill., about 4½ miles in length, was measured with the new Repsold apparatus by Mr. E. S. Wheeler, assisted by Messrs. Pratt, Lehnartz, Johnson, and Davis. Mr. Flint read the angles about the base, and then continued the work eastward to the line Penn - Milton.

Precise levels.—A duplicate line of levels was run between Gibraltar, near the mouth of the Detroit River, and Lakeport, on Lake Huron, by Messrs. L. L. Wheeler and Lehnartz.

Astronomical work.—Captain H. M. Adams at Detroit and Lieutenant C. F. Powell at Memphis, Tenn., determined the difference of longitude of those places. At the request of Captain W. S. Stanton, United States Engineers, signals were exchanged between Detroit and Fort Laramie, Wyo., Deadwood, Dak., and Camp Robinson, Nebr. The Detroit observations for this work were made by Lieutenant P. M. Price.

Mississippi River.—The Survey of the Mississippi River was taken up at Memphis, Tenn., on October 1 by parties under the charge of Lieutenants D. W. Lockwood and C. F. Powell, and a precise leveling party under Mr. Lehnartz. These parties returned to Detroit in the following February.

1878.

On account of the small appropriation available for the Survey it was necessary to reduce the force employed, and consequently but little field-work was accomplished this season.

Triangulation.—Mr. Woodward was engaged in rereading some of the angles of the Lake Ontario triangulation which had shown discrepancies larger than was desired. Mr. Wisner carried the triangulation across the State of Michigan eastward as far as Hillsdale. The base-line near Sandusky, Ohio, about 4 miles in length was measured by Mr. E. S. Wheeler, assisted by Messrs. T. W. Wright, J. Eisenmann, J. B. Johnson, and B. H. Colby. This base having two angles in it, thus breaking it into three nearly equal parts, a fifth point was selected so as to give well-shaped triangles, and the angles of the triangles formed by these five points were read by Lieutenant P. M. Price.

Astronomical work.—Major F. U. Farquhar, United States Engineers, having requested the determination of certain points in connection with his surveys, Captain H. M. Adams and Lieutenant C. F. Powell were assigned to this duty in September. Captain Adams made the Detroit observations and Lieutenant Powell observed for the longitude, latitude, and magnetic elements at Louisiana, Mo., Rock Island, Ill., and Red Wing, Minn.

Mississippi River.—On the disappearance of the yellow fever at Memphis in November the survey of the river was resumed at Scanlan's Landing, where it had closed the previous spring. Lieutenant D. W. Lockwood was charged with supplying the parties, making the borings, examinations, &c. Lieutenant P. M. Price and Mr. G. Y. Wisner did the triangulation. The topography and hydrography were done by the parties of Messrs. J. A. Ockerson, J. H. Darling, and J. Eisenmann, and the precise leveling by Mr. L. L. Wheeler. The parties returned to Detroit in March, 1879.

1879.

Triangulation.—The unfinished section from Hillsdale, Mich., to Lake Erie of the triangulation connecting Lakes Michigan and Erie was completed about July 1 by Messrs. Wisner, Woodward, and Darling. These observers, together with Mr. Flint, were occupied during the remainder of the season in extending the triangulation south from the Chicago base. The angles were read down as far as Parkersburg, about 10 miles south of Olney, Ill., that being the last station. Latitude and azimuth determinations were made at station Parkersburg by Lieutenant P. M. Price and Mr. A. R. Flint, respectively. The longitude of Olney, Ill. (the nearest telegraph station to Parkersburg), was determined by the observations of Captain Lockwood at Detroit in connection with those of Lieutenant Price at Olney. A base-line about 4 miles long was measured on the prairie 10 miles north of Olney by Mr. E. S. Wheeler assisted by Messrs. Ockerson, Childs, G. L. Fisher, and Stevens.

1880.

Early in the spring of this year Messrs. Childs, Darrow, and Gould were sent out to make topographical sketches about a number of the primary triangulation stations which had not been sketched when occupied. In May, Mr. Wisner determined the latitudes of two stations between Chicago and Parkersburg, making his observations at Fairmount and the west end of the Olney base. These parties all completed their labors about the 30th of June.

1881.

In May and June, 1881, the difference of longitude of Harvard College observatory, Cambridge, Mass., and the Lake-Survey observatory at Detroit was determined by Assistant Engineers A. R. Flint and O. B. Wheeler, and the difference of longitude of Toledo, Ohio, and Detroit was determined by Assistant Engineers O. B. Wheeler and T. Russell.

§ 10. *Catalogue of published charts of the Lake Survey.*

TABLE I.

Index number of chart.	Name of chart.	Scale.	Year of publication.	Surveys made under direction of—	Compilation and reduction by—	Engraved by—
	LAKE SUPERIOR.					
11	Eagle Harbor	1-5, 000	1858	Capt. J. N. Macomb	Lieut. O. M. Poe and H. Gillman	W. H. Dougal
12	Agate Harbor	1-10, 000	1858	......do	C. P. Rabaut	Do.
16	Eagle River	1-10, 000	1859	......do	......do	Do.
17	Ontonagon Harbor	1-16, 000	1859	......do	Joshua Barney	Do.
20	Marquette Harbor	1-50, 000	1860	Capt. George G. Meade	C. P. Rabaut	Do.
24	Grand Island	1-25, 000	1862	......do	Joshua Barney	Do.
25	West end of Lake Superior	1-32, 000	1863	......do	......do	Do.
30	Portage Lake and River	1-30, 000	1865	Col. J. D. Graham and Col. W. F. Raynolds.	J. U. Mueller	Do.

§ **10.** *Catalogue of published charts of the Lake Survey*—Continued.

TABLE I—Continued.

Index number of chart.	Name of chart.	Scale.	Year of publication.	Surveys made under direction of—	Compilation and reduction by—	Engraved by—
	LAKE SUPERIOR—Continued.					
28	Copper Harbor	1-10, 000	1866	Col. W. F. Raynolds	Joshua Barney	
29	L'Anse and Keweenaw Bay	1-30, 000	1866	Col. J. D. Graham and Col. W. F. Raynolds.	J. U. Mueller	
34	Huron Islands	1-30, 000	1869	Col. W. F. Raynolds	O. N. Chaffee	
32	Lake Superior, No. 2	1-400, 000	1870	Lieut. Col. W. F. Raynolds, Capt. J. N. Macomb, Capt. G. G. Meade, and Col. J. D. Graham.	J. U. Mueller	
31	Lake Superior, No. 1	1-400, 000	1872	Col. W. F. Raynolds, Maj. C. B. Comstock, Capt. J. N. Macomb, Capt. G. G. Meade, and Col. J. D. Graham.	......do	
38	Isle Royale	1-120, 000	1872	Lieut. Col. W. F. Raynolds and Maj. C. B. Comstock.	Edward Molitor	W. H. Dougal.
36	Lake Superior, No. 3	1-400, 000	1873	Lieut. Col. W. F. Raynolds, Maj. C. B. Comstock, Capt. J. N. Macomb, Capt. G. G. Meade, and Col. J. D. Graham.	J. U. Mueller	
	RIVER STE. MARIE.					
5	East Neebish Rapids	1-15, 000	1854	Capt. J. N. Macomb	F. Herbst	Do.
13	River Ste. Marie, No. 1	1-40, 000	1858	Capt. J. N. Macomb and Capt. G. G. Meade.	J. U. Mueller	Do.
14	River Ste. Marie, No. 2	1-40, 000	1858	Capt. J. N. Macomb	......do	Do.
	STRAITS OF MACKINAC.					
4	Straits of Mackinac	1-120, 000	1856	Lieut. Col. James Kearney and Capt. J. N. Macomb.	A. Boschke	Selmar Siebert.
	LAKE MICHIGAN.					
3a	Head of Green Bay	1-30, 000	1853	Capt. W. G. Williams and Capt. J. N. Macomb.		
10	Beaver Island Group	1-120, 000	1857	Capt. J. N. Macomb	J. U. Mueller	W. H. Dougal.
26	Grand and Little Traverse Bays	1-120, 000	1863	Capt. G. G. Meade	......do	Do.
27	North end Green Bay	1-120, 000	1864	Col. J. D. Graham and Col. W. F. Raynolds.	Joshua Barney	
35	South end Green Bay	1-120, 000	1864	......do	Edward Molitor	
33	North end Lake Michigan	1-400, 000	1867	Capt. J. N. Macomb, Capt. G. G. Meade, Col. J. D. Graham, and Col. W. F. Raynolds.	J. U. Mueller	Do.
40	City of Chicago	1-20, 000	1874	Maj. C. B. Comstock	A. de Witzleben	
50	South end Lake Michigan	1-400, 000	1876	Lieut. Col. W. F. Raynolds and Maj. C. B. Comstock.	J. U. Mueller	
51	Coast chart No. 5, Lake Michigan.	1-80, 000	1876	Maj. C. B. Comstock	M. von Hippel	
52	Coast chart No. 3, Lake Michigan.	1-80, 000	1876	......do	F. A. Fisher	
54	Coast chart No. 2, Lake Michigan.	1-80, 000	1877	......do	W. A. Wansleben	
55	Coast chart No. 1, Lake Michigan.	1-80, 000	1877	......do	F. A. Fisher	
57	Coast chart No. 6, Lake Michigan.	1-80, 000	1877	......do	Max Franke	
58	Coast chart No. 7, Lake Michigan.	1-80, 000	1877	......do	W. A. Wansleben	
59	Coast chart No. 4, Lake Michigan.	1-80, 000	1877	......do	M. von Hippel	
62	Coast chart No. 9, Lake Michigan.	1-80, 000	1878	......do	Max Franke	
63	Coast chart No. 8, Lake Michigan.	1-80, 000	1878	......do	M. von Hippel	
	LAKE HURON.					
6	Saginaw River	1-10, 000	1857	Capt. J. N. Macomb		Do.
9	Tawas Harbor	1-16, 000	1857	......do		Do

§ 10. *Catalogue of published charts of the Lake Survey*—Continued.

TABLE I—Continued.

Index number of chart.	Name of chart.	Scale.	Year of publication.	Surveys made under direction of—	Compilation and reduction by—	Engraved by—
	LAKE HURON—Continued.					
18	Saginaw Bay	1-120, 000	1860	Capt. G. G. Meade and Capt. J. N. Macomb.	J. U. Mueller	W. H. Dougal.
19	Thunder Bay	1-40, 000	1860	Capt. G. G. Meade	Joshua Barney	Do.
21	Presqu' Isle and Middle Island	1-40, 000	1860	......do	......do	Do.
22	Lake Huron	1-400, 000	1860	......do	J. U. Mueller	Do.
23	South end Lake Huron	1-120, 000	1861	......do	Joshua Barney	Do.
47	Sand Beach Harbor of Refuge	1-8, 000	1876	Maj. C. B. Comstock	A. de Witzleben	
	SAINT CLAIR RIVER.					
37	Saint Clair River	1-40, 000	1872	Lieut. Col. W. F. Raynolds and Maj. C. B. Comstock.	Edward Molitor	Do.
	LAKE SAINT CLAIR.					
7	Saint Clair Flats	1-32, 000	1857	Lieut. Col. James Kearney		Do.
41	Lake Saint Clair	1-50, 000	1874	Lieut. Col. W. F. Raynolds and Maj. C. B. Comstock.	Edward Molitor	
	DETROIT RIVER.					
39	Mouth of Detroit River	1-20, 000	1874	Maj. C. B. Comstock		E. Molitor.
56	Detroit River	1-40, 000	1876	......do	E. Molitor	
	LAKE ERIE.					
1	Lake Erie	1-400, 000	1852		John Lambert	
2	West end Lake Erie	1-120, 000	1852	Lieut. Col. James Kearney	......do	W. Smith.
3	Kelley's and Bass Islands	1-50, 000	1852	......do	......do	W. H. Dougal and J. V. N. Throop
8	Buffalo Harbor	1-30, 000	1857	Capt. W. G. Williams and Capt. I. C. Woodruff.		W. H. Dougal.
15	Maumee Bay	1-30, 000	1858	Capt. G. G. Meade	J. H. Foster, W. H. Hearding, and P. C. Rabaut.	Do.
43	Sandusky Bay	1-20, 000	1874	Maj. C. B. Comstock	J. U. Mueller	
68	Coast chart No. 2, Lake Erie	1-80, 000	1879	......do	Max Franke	
69	Coast chart No. 3, Lake Erie	1-80, 000	1879	......do	A. de Witzleben	
70	Coast chart No. 4, Lake Erie	1-80, 000	1879	......do	Max Franke	
72	Coast chart No. 6, Lake Erie	1-80, 000	1879	......do	E. Molitor	
71	Coast chart No. 5, Lake Erie	1-80, 000	1880	......do	A. de Witzleben	
73	Coast chart No. 7, Lake Erie	1-80, 000	1881	......do	E. Molitor	
74	Coast chart No. 1, Lake Erie	1-80, 000	1880	......do	Max Franke	
75	Lake Erie	1-400, 000	1880	......do	J. U. Mueller	
	NIAGARA RIVER.					
48	Niagara Falls	1-10, 000	1876	......do	C. Donovan	
	LAKE ONTARIO.					
60	Coast chart No. 1, Lake Ontario	1-80, 000	1877	......do	A. de Witzleben	
61	Lake Ontario	1-400, 000	1877	......do	J. U. Mueller	
64	Coast chart No. 2, Lake Ontario	1-80, 000	1878	......do	W. A. Wansleben	
65	Coast chart No. 3, Lake Ontario	1-80, 000	1878	......do	Max Franke	
66	Coast chart No. 4, Lake Ontario	1-80, 000	1878	......do	A. de Witzleben	
67	Coast chart No. 5, Lake Ontario	1-80, 000	1878	......do	W. A. Wansleben	
	SAINT LAWRENCE RIVER.					
42	Saint Lawrence River, No. 1	1-30, 000	1874	......do	Emil Mahlo	
44	Saint Lawrence River, No. 2	1-30, 000	1875	......do	F. A. Fisher	
45	Saint Lawrence River, No. 3	1-30, 000	1875	......do	A. de Witzleben	
46	Saint Lawrence River, No. 4	1-30, 000	1876	......do	F. A. Fisher	
49	Saint Lawrence River, No. 5	1-30, 000	1876	......do	W. A. Wansleben	
53	Saint Lawrence River, No. 6	1-30, 000	1876	......do	Max Franke	

§ 11. *Statement of the appropriations made for the Survey of the Northern and Northwestern Lakes since its commencement.*

TABLE II.

Appropriation made for—	Act of Congress approved—	Amount.	Appropriation made for—	Act of Congress approved—	Amount.
Hydrographical Survey of Northern and Northwestern Lakes.	Mar. 3, 1841	$15,000 00	Surveys of the Northern and Northwestern Lakes.	Feb. 9, 1863	$106,879 00
Do	May 18, 1842	20,000 00	Do	July 2, 1864	100,000 00
Do	Mar. 1, 1843	30,000 00	Do	Feb. 28, 1865	125,000 00
Do	June 17, 1844	20,000 00	Do	June 12, 1866	50,000 00
Do	Mar. 3, 1845	20,000 00	*Do	Mar. 2, 1867	77,500 00
Do	Aug. 8, 1846	25,000 00	*Do	Mar. 2, 1868	77,500 00
Do	Aug. 12, 1848	25,000 00	Do	July 20, 1868	75,000 00
Do	Mar. 3, 1849	10,000 00	Do	Mar. 3, 1869	100,000 00
Do	Sept. 28, 1850	25,000 00	Do	July 15, 1870	100,000 00
Do	Mar. 3, 1851	25,000 00	Do	Mar. 3, 1871	175,000 00
Surveys of the Northern and Northwestern Lakes.	Aug. 30, 1852	25,000 00	Do	June 10, 1872	175,000 00
			Do	Mar. 3, 1873	175,000 00
Do	Mar. 3, 1853	50,000 00	Do	June 23, 1874	175,000 00
Do	Aug. 5, 1854	50,000 00	Do	Mar. 3, 1875	150,000 00
Do	Mar. 3, 1855	50,000 00	†Do	July 31, 1876	100,000 00
Do	Aug. 30, 1856	50,000 00	‡Do	Mar. 3, 1877	119,500 00
Do	Mar. 3, 1857	50,000 00	§Surveys of the Northern and Northwestern Lakes and Mississippi River.	June 20, 1878	99,000 00
Do	June 12, 1858	75,000 00			
Do	Mar. 3, 1859	75,000 00	Surveys of the Northern and Northwestern Lakes.	Mar. 3, 1879	85,000 00
Do	June 21, 1860	75,000 00			
Do	Mar. 2, 1861	75,000 00	Do	June 16, 1880	40,000 00
Do	July 5, 1862	105,000 00	‖Do	Mar. 3, 1881	18,000 00

*Appropriation $155,000, with provision that there shall not be over fifty per cent. expended during the fiscal year ending June 30, 1868, and the residue thereof shall not be expended till otherwise ordered. Restriction subsequently withdrawn by resolution No. 17, March 2, 1868.

† Part of appropriation applied to Survey of Mississippi River, as provided by the act. About $16,000 was so expended.

‡ Appropriation $110,000, of which $25,000 should be expended in continuing Survey of Mississippi River. The $9,500 was received from sales of steamers and applied to the Lake Survey, as provided in the act.

§ One-half the amount applied to Survey of Mississippi River as provided in the act.

‖ An additional amount not exceeding $8,000 was reappropriated from the unexpended balance of the appropriation of June 16, 1880.

§ 12. *Officers in charge of the Survey of the Northern and Northwestern Lakes.*

TABLE III.

Name and rank.	Assumed command.	Relieved.
Capt. W. G. Williams, Topographical Engineers	May 17, 1841	Fall of 1845.
Lieut. Col. James Kearney, Topographical Engineers	Fall of 1845	Apr. 9, 1851.
Capt. J. N. Macomb, Topographical Engineers	Apr. 9, 1851	Sept., 1856.
Lieut. Col. James Kearney, Topographical Engineers	Sept., 1856	May 20, 1857.
Capt. George G. Meade, Topographical Engineers	May 20, 1857	Sept. 1, 1861.
* Col. James D. Graham, Corps of Engineers	Sept. 1, 1861	Apr. 15, 1864.
† Lieut. Col. W. F. Raynolds, Corps of Engineers, brevet brigadier-general, U. S. A	Apr. 15, 1864	May 12, 1870.
‖ Maj. Cyrus B. Comstock, Corps of Engineers, brevet brigadier-general, U. S. A.	May 12, 1870	Close of survey.
‡ Capt. Henry M. Adams, Corps of Engineers in temporary charge	Aug. 14, 1874 May 24, 1877	Nov. 20, 1874. June 25, 1878.

* Col. James D. Graham became lieutenant-colonel in the Corps of Engineers when the Corps of Topographical Engineers was consolidated with the Corps of Engineers March 3, 1863, and was promoted to be colonel of engineers on June 1, 1863.

† In April, 1864, General Raynolds was a major of engineers, with the rank of colonel and additional aide-de-camp. He was promoted to be lieutenant-colonel of engineers March 7, 1867.

‖ Maj. C. B. Comstock was promoted to the rank of lieutenant-colonel of engineers July 19, 1881.

‡ Captain Adams was in charge of the Survey during the absence in Europe of General Comstock.

§ 13. *Officers who have served as assistants on the Lake Survey.*

[Those who served previous to March 3, 1863, were, at the time of their service, officers of the Corps of Topographical Engineers. Those serving since March 3, 1863, have been officers of the Corps of Engineers.]

TABLE IV.

Name.	Rank and remarks.	Joined.	Relieved.
Howard Stansbury	Captain	1841	1842
Joseph E. Johnston	First lieutenant and brevet captain	1841	1842
James H. Simpson	First lieutenant	1841	1845
I. Carle Woodruff	Second lieutenant; first lieutenant March 31, 1842	1841	1847
John N. Macomb	First lieutenant; placed in charge of the Survey April 9, 1851; captain August 4, 1851.	1842	Sept. 1856
William H. Warner	First lieutenant	1842	1843
John W. Gunnison	Second lieutenant; first lieutenant May 9, 1846	1842 1851	1849 Apr. 1853
J. D. Webster	Lieutenant	1842	1843
James W. Abert	Brevet second lieutenant	1843	1844
William B. Franklin	Brevet second lieutenant	1843	1845
E. Parker Scammon	First lieutenant; captain March 3, 1853	1847	Nov. 1854
William F. Raynolds	Second lieutenant; first lieutenant March 3, 1853	1851	Apr. 1856
George H. Mendell	Brevet second lieutenant	Dec. 1852	1854
George W. Rose	Brevet second lieutenant; second lieutenant March, 1855	Nov. 1852	Mar. 1856
George G. Meade	Captain; appointed superintendent of the Survey May 20, 1857	June, 1856	Sept. 1, 1861
Charles N. Turnbull	Second lieutenant	June, 1856	1859
Orlando M. Poe	Second lieutenant; first lieutenant July 1, 1860	Oct. 1, 1856	May 1, 1861
William Proctor Smith	Brevet second lieutenant	Nov. 12, 1857	Feb. 14, 1861
J. L. Kirby Smith	Second lieutenant	Nov. 12, 1859	June 13, 1861
Robert F. Beckham	Brevet second lieutenant	Oct. 1, 1859	May 3, 1861
Micah R. Brown	First lieutenant	Oct. 1, 1865	Mar. 12, 1867
James F. Gregory	First lieutenant	July 1, 1866	Nov. 19, 1869
Francis U. Farquhar	Captain; brevet lieutenant-colonel, U. S. A.	Mar. 4, 1867	Nov. 9, 1868
James Mercur	Second lieutenant; first lieutenant March 7, 1867	Oct. 1, 1866	Aug. 23, 1867
Benjamin D. Greene	Second lieutenant; first lieutenant March 7, 1867	Oct. 1, 1866	May 5, 1869
Ernest H. Ruffner	First lieutenant	Aug. 1867	Nov. 30, 1870
John C. Mallery	First lieutenant	Aug. 1867	Oct. 1, 1869
Joseph E. Griffith	Second lieutenant	Aug. 17, 1867	May 5, 1869
William E. Rogers	Second lieutenant	Sept. 1867	Aug. 6, 1869
Lewis M. Haupt	Second lieutenant	Aug. 9, 1867	Jan. 31, 1869
Jared A. Smith	Captain; brevet major, U. S. A.	June 5, 1869	Apr. 1, 1871
William R. Livermore	First lieutenant; captain January 22, 1870	Sept. 24, 1869	May 26, 1874
A. Nisbet Lee	First lieutenant; captain July 11, 1871	May 1, 1870	Feb. 28, 1874
John H. Weeden	First lieutenant	May, 1870	June 7, 1872
Edward Maguire	First lieutenant	May 6, 1871	May 12, 1875
Charles F. Powell	First lieutenant	May 6, 1871	Mar. 18, 1879
Daniel W. Lockwood	First lieutenant; captain June 30, 1879	July 10, 1872	Feb. 14, 1880
Thomas N. Bailey	Second lieutenant	June 1, 1874	Aug. 23, 1876
Henry M. Adams	First lieutenant; captain September 2, 1874; in charge of the Survey during absence in Europe of General Comstock from August 14, 1874, to November 20, 1874, and from May 24, 1877, to June 25, 1878.	June 8, 1874	Dec. 31, 1878
Philip M. Price	First lieutenant	May 13, 1875	Dec. 7, 1880

§ 14. *Civil assistants employed on the Survey.*

[For the years previous to 1870 this list is necessarily incomplete, the office records not containing the data for making a complete list.]

TABLE V.

Name.	Occupation.	Years of service.	Remarks.
R. W. Burgess	Assistant engineer	1843–1849	
J. F. Peter	do	———	Years not known, but were on the Survey during part of Mr. Burgess' term.
J. H. Forster	do	—— 1857	Years not known, but were on the Survey during part of Mr. Burgess' term.
J. Houghton, jr	do	1849 ——	Duration of service unknown.

§ 14. *Civil assistants employed on the Survey*—Continued.

[For the years previous to 1870 this list is necessarily incomplete, the office records not containing the data for making a complete list.]

TABLE V—Continued.

Name.	Occupation.	Years of service.	Remarks.
J. H. Hearding	Assistant engineer	1849–1850	
J. A. Potter	do	1849–1861	
W. H. Hearding	do	1851–1864	
F. W. Wilmotte	do	1851——	Duration of service not known
Henry Gillman	do	1851–1869	
G. W. Lameon	do	1851–1860	
I. L. Beghein	do	1853–1859	
C. S. Cole	do	1853——	Do.
J. U. Mueller	Assistant engineer and draughtsman	1854–1880	
H. C. Penny	Assistant engineer	1855–1866	
R. Brown	do	1854——	Do.
D. F. Henry	do	1854–1871	
C. Pinney	do	1855——	Do.
O. N. Chaffee	do	1855–1869	Was away one year.
E. B. Wright	Sub-assistant engineer	1854–1861	Field seasons only.
J. Wallace	do	1855——	Duration of service not known
G. Wallace	do	1855——	Do.
J. Carr	Assistant engineer	1857–1863	
W. T. Casgrain	do	1857–1865	
A. C. Lamson	do	1856–1878	
J. E. Goodell	do	1857——	Do.
J. Clague	Assistant and computer	1857–1863	
C. P. Rabaut	Assistant engineer	1858–1864	
E. P. Austin	do	1859–1863	
William Noble	do	1859——	
W. I. Thorne	do	1859–1864	
G. E. Swinecoe	do	1859–1864	
A. M. Adams	do	1859——	
J. M. Bigelow	do	1860–1867	
Henry Clague	Assistant and clerk	1860–1870	
A. F. Chaffee	Assistant engineer	1862–1871	
J. R. Mayer	do	1862–1876	
J. Paul Mayer	do	1862–1871	
E. S. Wheeler	do	1863–1882	
G. A. Marr	do	1863–1877	
A. R. Flint	do	1863–1882	
F. M. Towar	do	1863–1878	
A. W. Unthank	do	1863–1864	
Lewis Foote	do	1863–1874	
A. Molitor	do	1863–1869	
S. W. Robinson	do	1863–1866	Also in 1873, field season only
J. H. Booth	Recorder	1863–1865	
O. B. Wheeler	Assistant engineer	1862–1882	Was away two years.
William Le Baron	Recorder and assistant	1863–1864	Field-work only.
Joshua Barney	do	1864–1866	
T. C. Raynolds	Recorder	1864–1865	
William Donovan	Assistant engineer	1864–1865	
H. M. Wright	Property clerk	1864–1872	
George Y. Wiener	Assistant engineer	1864–1880	
Edward Molitor	Draughtsman	1866–1881	Away about two years.
John Whiting	Assistant engineer	1868–1870	
William A. Butler, jr	Recorder	1869——	Field-work only.
E. J. Knight	do	1870–1871	Do.
J. C. Jones	Recorder and observer	1870–1871	Do.
J. H. Southall	Chief clerk	1870–1878	
Henry Custer	Assistant engineer	1871–1874	
Thomas Russell	do	1871–1882	
Charles H. Kümmell	do	1871–1880	Office work only; computation
John Eisenmann	do	1871–1879	Away about two years.

§ 14. *Civil assistants employed on the Survey*—Continued.

[For the years previous to 1870 this list is necessarily incomplete, the office records not containing the data for making a complete list.]

TABLE V—Continued.

Name.	Occupation.	Years of service.	Remarks.
R. S. Woodward	Recorder and assistant	1871–1882	
T. W. Wright	Assistant engineer	1872–1882	
Frederick Terry	do	1871–1878	
J. A. Ockerson	Recorder and assistant	1871–1879	
William A. Metcalf	Assistant engineer	1871–1876	
Clark Olds	Recorder and assistant	1870–1875	Field-work only.
F. G. Bulkley	do	1871–1876	Except 1874.
William Voigt	do	1871–1882	Away about three years.
F. B. Martin	Recorder and clerk	1871–1878	
E. T. Tappey	do	1871–1878	
S. C. Austin	Recorder	1871–1872	Field-work only.
C. R. Wells	do	1871–1872	Do.
J. W. Perkins	do	1871–1872	Do.
A. T. Morrow	Recorder and assistant	1871–1878	
C. F. Burton	do	1871–1874	Except 1873.
Charles Pratt	do	1871–1877	
V. M. Spalding	Recorder	1871, 1874	Field-work only
Dr. V. T. McGillicuddy	Assistant engineer	1872–1874	
Charles S. Wilson	Recorder	1872, 1874	Do.
J. Teeple	Recorder and assistant	1872–1878	
Z. D. Scott	Recorder	1872–1873	Do.
A. de Witzleben	Draughtsman	1872–1880	
Julius Lohman	Property clerk	1872–1882	
C. M. Winchell	Recorder and assistant	1873–1879	
George C. Comstock	do	1873–1877	Do.
Horace Holmes	Recorder	1873–1874	Do.
A. N. Darrow	Recorder and assistant	1873–1880	Except 1878–1879.
J. C. Eaton	Recorder	1873–1878	Except 1877.
J. H. Darling	Assistant engineer	1873–1882	
Walter S. Russel	Recorder and assistant	1873, 1877	
G. W. Carmen	Recorder	1875, 1879	Field-work only.
A. Beebe	do	1874–1877	Do.
Charles A. Marshall	do	1874–1876	Do.
C. Donovan	Recorder and assistant	1874–1876	
R. B. Hostetler	Recorder	1874–1877	Do.
L. L. Wheeler	Assistant engineer	1874–1881	Away one year.
F. A. Fisher	Draughtsman	1874–1879	
M. K. Ross	Clerk	1874–1878	
William A. Wansleben	Draughtsman	1875–1878	
M. von Hippel	do	1875–1878	
J. B. Johnson	Recorder and assistant	1875–1880	
C. W. Clark	do	1875–1880	Except 1878–1879.
Max Franke	Draughtsman	1875–1880	
F. W. Lehnartz	Assistant engineer	1875–1878	
J. S. Polhemus	do	1875–1877	
J. H. Morrison	Recorder and assistant	1875–1877	
L. D. Wines	Recorder	1875–1877	Field-work only.
W. H. Lightner	do	1875–1877	Do.
O. C. Simonds	do	1875–1877	Do.
E. S. Davis	do	1875–1879	Do.
R. R. Bourland	do	1875–1877	Do.
Archie Powell	do	1875–1877	Do.
James A. Paige	do	1875–1879	Do.
J. B. Spitler	do	1875–1876	Do.
C. W. Walton	do	1875–1877	Do.
E. Donovan	do	1875–1876	Do.
E. B. Wilson	do	1875–1876	Do.
B. H. Colby	do	1876–1879	Do.
H. C. Gould	Recorder and assistant	1877–1880	

§ 14. *Civil assistants employed on the Survey*—Continued.

[For the years previous to 1870, this list is necessarily incomplete, the office records not containing the data for making a complete list.]

TABLE V—Continued.

Name.	Occupation.	Years of service.	Remarks.
N. S. Fisher	Chief clerk	1878–1881	
W. M. Childs	Recorder and assistant	1878–1880	
C. V. Mersereau	......do	1879–1881	
B. D. King	Assistant engineer	1879–1881	
Charles C. Brown	......do	1879–1882	
Alexander Ziwet	Recorder and assistant	1880–1882	

§ 15. *Annual issue of charts of the Northern and Northwestern Lakes.*

TABLE VI.

	No.
Prior to October 1, 1857	2,500
October 1, 1857, to October 1, 1858	1,675
October 1, 1858, to October 1, 1859	2,600
October 1, 1859, to October 1, 1860	4,890
October 1, 1860, to October 1, 1861	3,254
October 1, 1861, to October 1, 1862	5,245
October 1, 1862, to October 1, 1863	4,084
October 1, 1863, to October 1, 1864	3,283
October 1, 1864, to October 1, 1865	2,589
October 1, 1865, to July 1, 1866	2,082
July 1, 1866, to July 1, 1867	5,464
July 1, 1867, to July 1, 1868	6,354
July 1, 1868, to July 1, 1869	5,634
July 1, 1869, to July 1, 1870	4,597
July 1, 1870, to July 1, 1871	5,328
July 1, 1871, to July 1, 1872	3,649
July 1, 1872, to July 1, 1873	5,546
July 1, 1873, to July 1, 1874	7,701
July 1, 1874, to July 1, 1875	5,039
July 1, 1875, to July 1, 1876	4,101
July 1, 1876, to July 1, 1877	3,156
July 1, 1877, to July 1, 1878	6,623
July 1, 1878, to July 1, 1879	4,499
July 1, 1879, to July 1, 1880	4,402
July 1, 1880, to July 1, 1881	6,561

PART II.

STANDARDS OF LENGTH, BASES, AND BASE-APPARATUS.

CHAPTER II.

STANDARDS OF LENGTH OF THE UNITED STATES LAKE SURVEY, DEPENDING ON THE ENGLISH YARD.

§ **1.** These standards of length, depending on the Ordnance-Survey yard, Y_{55}, are—

1. Two steel end-measure yards, known as Clarke yards *A* and *B*. They have the letters *A* and *B* engraved on them.
2. An inch graduated to tenths, and one-tenth to hundreds, on a slip of platinum let into the upper surface of a steel bar 1 inch square and $1\frac{1}{2}$ inches long.
3. On the preceding, which are the standards, depend the lengths of a brass bar 15 feet long; and,
4. The lengths of five brass yards used in determining the length of the 15-foot brass bar.

CLARKE YARDS *A* AND *B*.

§ **2.** These yards are bars of steel 0.73 inch in depth and 0.50 inch in thickness. The ends are cylinders, 0.35 inch in length and 0.25 inch in diameter, co-axial with the bars.

The ends of the cylinders have agates firmly set in each, the surfaces of these stones being spherical surfaces whose centers are at the opposite ends of the yards.

The standard distance is that between the central points of these small end-surfaces.

Each yard is inclosed in an iron box slightly larger than itself, the box having a hinged cover and sliding ends. In the bottom of each box are two equal levers moving about horizontal axes at their centers, the ends of the levers thus giving four points of support for the yards resting on them and bearing equal pressures, these points of support being at 5.2 inches and 10.9 inches from either end of yard. Four Fahrenheit mercurial thermometers, whose bulbs lie in niches for them in the interior sides of the iron boxes, accompany each yard. Their stems are bent at right angles and are read through slits in the iron lids.*

These yards are from a design suggested to me by Lieutenant-Colonel A. R. Clarke, Royal Engineers. They were constructed by James Simms, of London, in 1873.

Yard *A* was compared by Lieutenant-Colonel Clarke at the Ordnance-Survey office at South, ampton with the standard yard, Y_{55} of the Ordnance Survey, eight times in four days in May, 1874- the temperature being between 51° and 52° Fahr., and ten times in July, 1874, in four days, the temperature being between 62° and 64° Fahr.

Yard *B* was compared eight times with Y_{55} in three days of May, 1874, at temperatures near 53°, and ten times in four days of June, 1874, at temperatures between 61°.7 and 62°.8.

The coefficients of expansion of both *A* and *B* were determined by Colonel Clarke. The method used was to compare the yards with each other, one being kept at a high temperature by being placed in a box between tanks filled with hot water at a steady temperature, while the other was

* For a more detailed description of these yards see Chapter VIII, § 25.

between tanks filled with cold water. The methods were essentially the same as those described in Colonel Clarke's work on Comparisons of Standards of Length.

A at about 91° F. was compared ten times in three days with B at about 34°, and ten times in two days with B at about 57°. It was compared ten times in two days at about 34° F. with B at about 94°; ten times in two days at 55° with B at 91°; and ten times in two days at 56° with B at 57°.

These expansion-comparisons extended from April 18 to May 1, 1874.

The four thermometers accompanying each yard were carefully compared by Colonel Clarke with the Ordnance-Survey standard 3241, and their errors determined.

The comparisons of A and B with the Ordnance-Survey standard Y_{55} were each combined by least squares to obtain the lengths of A and B at 62°. The expansion-comparisons were combined in the same way, and from them the coefficients of expansion of the two yards were obtained.

The following paper, by Colonel Clarke, gives the details of these comparisons:

CONSTANTS OF CLARKE YARDS A AND B, AS GIVEN BY LIEUTENANT-COLONEL CLARKE.

The rates of expansion of the new American standard yards A and B, constructed by Messrs. Troughton & Simms, were determined in a manner similar to that described in the Comparisons of Standards, pp. 180–186.

With the exception that the tank-boxes were only 4 feet in length, they were otherwise similar in form and dimensions with those used for the 10-feet standards.

There was one other departure from exact similarity to the 10-feet bar expansion-determinations, viz., the cold bar was kept during part of the comparisons at or near the temperature 32°.

In order to manage this the tanks of one of the boxes, instead of being fitted with a system of supply and waste pipes, had openings in the upper surfaces supplied with close-fitting covers, and through these openings the tanks were packed with broken ice, for which a proper system of drainage was adopted below.

These tanks also served for holding cold water when the bars were compared at about 57°.

The supply of hot water to the other box was managed in the same manner as in the original comparisons of the 10-feet bars, with some trifling improvements.

The thermometers did not allow comparisons at temperatures above 94°, consequently the bars were never heated above that.

The comparisons were commenced on 18th April and continued until May 1, when they were completed, the number of comparisons being fifty.

The whole of the observations were made by Lieut.-Col. A. R. Clarke, R. E., the series involving 800 thermometer readings and 1,200 micrometer-readings.

Of the thermometers supplied by Messrs. Troughton & Simms, four were marked A_1, A_2, A_3, A_4; these were used in the bar A; and four were marked B_1, B_2, B_3, B_4; these were used in the bar B.

The mean of the readings of the four thermometers A, when in finely-powdered ice, was—

	April 15.	May 2.
A	32°.30	32°.28

and the mean of the four B-thermometers, on the same days—

B	32°.28	32°.24

The whole eight thermometers were compared, with the greatest care, with the Ordnance-Survey standard 3241.

The errors of this standard are known with considerable precision. (See Comparisons of Standards, pp. 186–193.)

The result of the comparisons was that the absolute errors of the mean of the four A's at different temperatures were—

At 52°, error of mean of A's + 0°.22; correction required, —0°.22
At 55°, error of mean of A's + 0 .18; correction required, —0 .18
At 57°, error of mean of A's + 0 .17; correction required, —0 .17
At 62°, error of mean of A's + 0 .22; correction required, —0 .22
At 93°, error of mean of A's + 0 .17; correction required, —0 .17

Similarly for the B thermometers:

At 52°, error of mean of B's + 0°.13; correction required, —0°.13
At 55°, error of mean of B's + 0 .11; correction required, —0 .11
At 57°, error of mean of B's + 0 .13; correction required, —0 .13
At 62°, error of mean of B's + 0 .17; correction required, —0 .17
At 93°, error of mean of B's + 0 .13; correction required, —0 .13

The circumstance of the two yards being end measures introduced a difficulty into the process of determining the rates of expansion. This was overcome by engraving a fine dot on a platinum pin let into the upper surface of the terminal cylinder of the bar, one at each end.

In the bar A the distance between the dots was found to be 0.1670 inch shorter than a yard; and in the bar B the distance between the dots was less than a yard by 0.1678 inch.

In order, then, to determine the rates of expansion of the bars, the distance between the dots on the one bar was compared with the distance between the dots on the other bar, the bars being at various temperatures.

The resulting rates of expansion have, of course, to be increased in the proportion of a yard to the distance between the dots.

The comparisons are given in the usual form in the following table:

The values of the micrometer divisions i, j, are—

$$i = 0.81032 \qquad j = 0.80270$$

the unit being the one-millionth of a yard.

American yards.—Expansion-experiments.

Date.	A. Temperature corrected.	A. Micrometer-readings.	B. Temperature corrected.	B. Micrometer-readings.	Difference.	Equivalent.
1874.	°		°			
April 18	90.50	$29.77i+36.60j$	50.86	$182.97i+201.77j$	$153.20i+165.17j$	256.73
20	91.39	$25.43i+39.73j$	35.29	$244.85i+249.00j$	$219.42i+209.27j$	345.79
20	91.26	$34.60i+32.97j$	35.23	$246.22i+249.30j$	$211.62i+216.33j$	345.13
21	91.34	$28.35i+29.12j$	34.16	$251.78i+249.83j$	$223.43i+220.71j$	358.22
21	92.09	$24.02i+37.92j$	34.14	$248.12i+256.53j$	$224.10i+218.61j$	357.08
21	91.21	$30.77i+38.12j$	34.19	$250.72i+254.55j$	$219.95i+216.43j$	351.96
21	91.22	$31.37i+38.43j$	34.23	$256.80i+249.97j$	$225.43i+211.54j$	352.48
21	91.68	$25.17i+42.27j$	34.31	$240.52i+266.93j$	$215.35i+224.66j$	354.84
21	91.66	$33.48i+33.70j$	34.38	$259.90i+246.63j$	$226.42i+212.93j$	354.40
21	91.66	$26.13i+43.55j$	34.46	$252.55i+256.73j$	$226.42i+213.18j$	354.60
22	34.22	$238.62i+241.08j$	93.31	$40.52i+45.03j$	$-198.10i-196.05j$	−317.90
22	34.37	$242.65i+237.48j$	93.46	$40.78i+43.17j$	$-201.87i-194.31j$	−319.56
22	34.51	$240.40i+237.65j$	93.78	$26.05i+55.23j$	$-214.35i-182.42j$	−320.13
22	34.46	$236.87i+243.33j$	94.07	$27.23i+50.42j$	$-209.64i-192.91j$	−324.73
22	34.56	$244.12i+237.88j$	94.02	$32.98i+48.27j$	$-211.14i-189.61j$	−323.30
23	34.14	$242.18i+238.58j$	92.40	$50.88i+42.15j$	$-191.30i-196.43j$	−312.69
23	34.35	$237.53i+243.47j$	92.46	$42.68i+49.55j$	$-194.85i-193.92j$	−313.55
23	34.56	$235.55i+247.42j$	92.49	$45.32i+47.77j$	$-190.23i-199.65j$	−314.41
23	34.06	$242.60i+242.07j$	92.12	$39.13i+59.02j$	$-203.47i-183.05j$	−311.81
23	34.09	$240.82i+244.85j$	92.00	$49.27i+50.80j$	$-191.55i-194.05j$	−310.98
24	54.78	$162.05i+172.37j$	91.72	$50.45i+47.50j$	$-111.60i-124.87j$	−190.67
24	54.84	$172.57i+163.70j$	91.68	$45.52i+53.98j$	$-127.05i-109.72j$	−191.02
24	54.89	$169.77i+168.78j$	91.64	$56.77i+46.43j$	$-113.00i-122.35j$	−189.78
24	55.27	$173.50i+161.83j$	91.88	$32.65i+70.32j$	$-140.85i-91.51j$	−187.59
24	55.32	$170.53i+165.10j$	91.93	$44.13i+58.17j$	$-126.40i-106.93j$	−188.26
25	55.82	$167.10i+165.65j$	91.53	$49.28i+51.90j$	$-117.82i-113.75j$	−186.78
25	55.84	$167.03i+166.00j$	90.84	$51.80i+54.05j$	$-115.23i-111.95j$	−183.24
25	55.87	$166.78i+167.10j$	90.25	$64.27i+49.05j$	$-102.51i-118.05j$	−177.83
25	56.16	$163.12i+168.80j$	93.17	$44.98i+48.68j$	$-118.14i-120.12j$	−192.15
25	56.21	$168.87i+164.15j$	93.24	$40.50i+54.95j$	$-128.37i-109.20j$	−191.68
27	56.23	$155.87i+171.35j$	57.62	$177.82i+169.25j$	$21.95i-2.10j$	16.10
27	56.25	$144.15i+183.80j$	57.72	$169.37i+179.55j$	$25.22i-4.25j$	17.02
27	56.28	$159.82i+168.62j$	57.78	$173.98i+175.80j$	$14.16i+7.18j$	17.24
27	56.44	$155.82i+167.97j$	58.00	$174.90i+170.57j$	$19.08i+2.60j$	17.55
27	56.48	$161.83i+163.27j$	58.03	$175.02i+170.45j$	$13.19i+7.18j$	16.45
28	56.86	$165.95i+157.37j$	57.64	$183.43i+165.75j$	$17.48i+8.38j$	20.89
28	56.86	$157.98i+166.75j$	57.62	$172.08i+176.08j$	$14.10i+9.33j$	18.91
28	56.87	$162.67i+161.80j$	57.62	$174.05i+177.22j$	$11.38i+15.42j$	21.60
28	57.01	$158.07i+161.43j$	57.74	$176.20i+172.52j$	$18.13i+11.09j$	23.59
28	57.02	$162.25i+159.72j$	57.77	$174.03i+176.17j$	$11.78i+16.45j$	22.75
30	91.74	$45.10i+26.38j$	57.30	$176.00i+175.80j$	$130.90i+149.42j$	226.01
30	91.88	$31.62i+36.30j$	57.37	$179.62i+171.15j$	$148.00i+134.85j$	228.17
30	91.81	$28.17i+42.83j$	57.44	$178.50i+174.48j$	$150.33i+131.65j$	227.49
30	91.64	$27.33i+44.68j$	57.54	$175.08i+178.33j$	$147.75i+133.65j$	227.01
30	91.55	$31.60i+42.68j$	57.59	$177.20i+178.40j$	$145.60i+135.72j$	226.93
May 1	91.46	$20.68i+50.73j$	57.25	$171.40i+180.37j$	$150.72i+129.64j$	226.20
1	91.44	$38.58i+33.20j$	57.26	$180.42i+171.28j$	$141.84i+138.08j$	225.78
1	91.31	$42.35i+31.97j$	57.28	$176.37i+177.22j$	$134.02i+145.25j$	225.19
1	91.08	$38.92i+38.40j$	57.29	$174.53i+180.62j$	$135.61i+142.22j$	224.05
1	90.82	$37.62i+42.77j$	57.30	$180.07i+176.12j$	$142.45i+133.35j$	222.47

In order to reduce these comparisons by the method of least squares, let the lengths of the bars at the temperatures t and t' be respectively—

$$A = u + (t - 57^\circ)\, y \qquad B = v + (t' - 57^\circ)\, z$$

so that u and v are the lengths at the temperature of 57° F., and y and z are the expansions, in millionths of a yard, for one degree.

Then the difference of length at any temperature is expressed by—

$$u - v + (t - 57^\circ)\, y + (57^\circ - t')\, z = A - B$$

Put x for $u - v$ and c for the measured difference of length, then we have 50 equations of the form—

$$x + ay + bz - c = 0$$

Performing the necessary multiplications, the resulting equations are—

$$\begin{aligned} 50\ x + 443.36\, y - 510.40\, z - 834.57 &= 0 \\ 443.36\, x + 28886.99\, y + 15756.84\, z - 270609.89 &= 0 \\ -510.40\, x + 15756.84\, y + 29691.00\, z - 251647.17 &= 0 \end{aligned}$$

or, writing A, B, C for the absolute terms of these equations—

$$\begin{aligned} x + .06107850\ A - .002125409\ B + .002177907\ C &= 0 \\ y - .002125409\ A + .0001226812\ B - .00010164285\ C &= 0 \\ z + .002177906\ A - .00010164283\ B + .00012506059\ C &= 0 \end{aligned}$$

Restoring the values of A, B, C, we have—

$$x = 23.88$$
$$y = 5.8468$$
$$z = 5.7832$$

and these values give the following system of errors to the comparisons:

Date.	Error.	Date.	Error.	Date.	Error.	Date.	Error.	Date.	Error.
1874.		1874.		1874.		1874.		1874.	
April 18	−1.47	April 22	−1.39	April 24	+0.78	April 27	−0.32	April 30	−0.74
April 20	+4.72	April 22	+0.27	April 24	+1.72	April 27	−1.69	April 30	−2.49
April 20	+4.97	April 22	−0.19	April 24	+1.00	April 27	−2.08	April 30	−2.62
April 21	−1.47	April 22	+2.44	April 24	−0.36	April 27	−2.72	April 30	−3.71
April 21	+4.17	April 22	+1.88	April 24	+0.32	April 27	−1.57	April 30	−4.45
April 21	+3.86	April 23	−1.81	April 25	+4.07	April 28	−1.53	May 1	−2.28
April 21	+3.16	April 23	−0.07	April 25	+4.64	April 28	+0.56	May 1	−2.03
April 21	+3.03	April 23	+1.84	April 25	+2.82	April 28	−2.07	May 1	−2.32
April 21	+2.95	April 23	−1.54	April 25	−1.95	April 28	−3.93	May 1	−2.58
April 21	+2.29	April 23	−1.49	April 25	+1.36	April 28	−3.20	May 1	−2.57

The sum of the squares of these errors being 320.90, the probable error of a single comparison is found $= \pm\ 1.76$, and the probable errors of x, y, z,

x............$\pm$ 0.43
y............$\pm$ 0.0195
z............$\pm$ 0.0197

In order finally to get the expansion of the whole yard in each case, we have to divide the values of y and z by the distance expressed in yards between the dots in the respective bars; thus:

$$\text{Expansion of } A = \frac{y}{.995362} = 5.8740$$

$$\text{Expansion of } B = \frac{z}{.995338} = 5.8103$$

The bars A, B, were compared each eighteen times with the standard yard Y_{55}. The comparisons were made in the usual manner by means of the contact-apparatus.

This small space was carefully remeasured, but no change found in it from the previously adopted length, 565.65.* The yard Y_{55} was also compared with the bronze standard Y_{29}, and no change found in their adopted lengths, or rather in their difference of length.

*The space referred to here is the maximum distance between the graduation lines on the terminal semi-cylinders of the contact-apparatus, when these lines are parallel and the semi-cylinders are in contact. (See Comparisons of Standards of Length, pp. 10–13.)

The comparisons are given in detail in the following tables. The values of the micrometer divisions h and k are

$$h=0.79568 \qquad k=0.79868$$

Date.	Temperature corrected.	*Y*.	*A*.	Difference.	Equivalent, —565.65.
1874.	°				
May 13	51.06	399.15 h + 400.00 k	55.25 h + 54.65 k	343.90 h + 345.35 k	16.19
14	51.45	399.92 h + 399.28 k	54.57 h + 54.45 k	345.35 h + 344.83 k	15.45
14	51.54	400.10 h + 399.22 k	53.88 h + 54.65 k	346.22 h + 344.57 k	14.97
14	51.82	397.77 h + 399.12 k	53.77 h + 53.20 k	344.00 h + 345.92 k	15.65
15	51.94	398.23 h + 398.85 k	53.48 h + 54.10 k	345.75 h + 344.75 k	15.99
15	51.99	398.38 h + 398.05 k	54.25 h + 53.80 k	344.13 h + 344.25 k	16.89
16	51.73	399.65 h + 398.42 k	54.80 h + 53.62 k	344.85 h + 344.80 k	15.87
16	51.90	396.25 h + 400.40 k	54.37 h + 52.62 k	341.88 h + 347.78 k	15.86
July 4	62.09	387.60 h + 390.53 k	48.38 h + 48.43 k	339.22 h + 342.10 k	22.50
4	62.13	389.68 h + 388.48 k	49.00 h + 49.03 k	340.68 h + 339.45 k	23.47
6	61.35	389.70 h + 390.90 k	49.58 h + 50.58 k	340.12 h + 340.32 k	23.21
6	61.40	390.88 h + 389.43 k	48.35 h + 51.55 k	342.53 h + 337.88 k	23.24
9	62.42	388.33 h + 387.73 k	44.60 h + 50.43 k	343.73 h + 337.30 k	22.76
9	62.44	387.50 h + 387.33 k	48.82 h + 45.75 k	338.68 h + 341.58 k	23.35
9	62.90	382.80 h + 389.88 k	47.40 h + 44.78 k	335.40 h + 345.10 k	23.15
9	62.92	385.72 h + 387.18 k	44.63 h + 47.65 k	341.09 h + 339.53 k	23.07
10	63.87	386.63 h + 382.40 k	46.65 h + 41.15 k	339.98 h + 341.25 k	22.59
10	63.89	384.48 h + 384.50 k	45.25 h + 43.28 k	339.23 h + 341.22 k	— 23.21

Date.	Temperature corrected.	*Y*.	*B*.	Difference.	Equivalent, —565.65.
1874.	°				
May 18	52.43	412.53 h + 412.03 k	70.65 h + 72.70 k	341.88 h + 339.33 k	— 22.60
18	52.49	411.18 h + 413.23 k	71.33 h + 72.53 k	339.85 h + 340.70 k	23.13
18	52.80	407.25 h + 415.93 k	72.00 h + 72.02 k	335.25 h + 343.91 k	24.22
19	52.82	411.40 h + 411.88 k	72.70 h + 71.08 k	338.70 h + 340.80 k	23.97
19	52.90	410.82 h + 411.95 k	71.25 h + 72.43 k	339.57 h + 339.52 k	24.29
19	52.95	407.92 h + 414.10 k	71.72 h + 71.80 k	336.20 h + 342.30 k	24.76
19	53.25	410.92 h + 410.50 k	72.15 h + 69.80 k	338.77 h + 340.70 k	23.99
21	53.33	409.78 h + 410.48 k	71.15 h + 70.15 k	338.63 h + 340.33 k	24.40
June 10	63.41	386.15 h + 386.18 k	48.05 h + 49.73 k	338.10 h + 336.45 k	27.91
11	63.15	384.18 h + 387.42 k	49.25 h + 48.83 k	334.93 h + 338.59 k	28.73
11	63.19	385.72 h + 386.35 k	48.98 h + 49.40 k	336.74 h + 336.95 k	28.60
12	62.65	388.05 h + 386.68 k	50.42 h + 50.30 k	337.63 h + 336.38 k	28.34
12	62.73	388.38 h + 386.08 k	51.03 h + 49.98 k	337.35 h + 336.10 k	28.79
12	62.94	386.45 h + 386.50 k	50.35 h + 48.68 k	336.10 h + 337.82 k	28.41
12	62.98	386.85 h + 385.20 k	48.80 h + 49.88 k	338.05 h + 335.32 k	28.86
13	61.77	388.18 h + 388.75 k	52.08 h + 52.25 k	336.10 h + 336.50 k	29.46
13	61.81	387.37 h + 388.88 k	52.30 h + 52.55 k	335.07 h + 336.33 k	30.42
13	62.09	387.45 h + 388.55 k	48.45 h + 54.43 k	339.00 h + 334.12 k	— 29.05

By treating these measurements in the usual manner we get—

$$Y_{55}-A=22.65+0.652\,(t-62^\circ)$$
$$Y_{55}-B=28.49+0.495\,(t-62^\circ)$$

or, at the temperature of 62°—

$$Y_{55}=A+22.65\pm0.13$$
$$Y_{55}=B+28.49\pm0.16$$

The probable error of a single comparison in the case of the bar A, being ± 0.44, and in the case of the bar B, ± 0.55.
The length of Y_{55} at 62° is 0.99999960 yard, consequently, at 62°—

$$A = 0.99997695 \pm .00000013 \qquad B = 0.99997111 \pm .00000016$$

A. R. CLARKE,
Lieutenant-Colonel, R. E.

The value of Y_{55} with reference to the standard yard of England is given in Colonel Clarke's Comparisons of Standards of Length, p. 162. It will be noticed that no probable error is assigned to the value of Y_{55} in terms of the standard of England, which amounts to assuming that the standard is $\frac{4}{10000000}$ yard longer than Y_{55} and virtually makes Y_{55} the standard.

The following are Colonel Clarke's results:

Length of A at 62° F. in yards, $A = 0^{y}.99997695 \pm 0.00000013$
Length of B at 62° F. in yards, $B = 0^{y}.99997111 \pm 0.00000016$
Expansion for 1° F. of A in yards, $= 0^{y}.0000058740 \pm 0.0000000195$
Expansion for 1° F. of B in yards, $= 0^{y}.0000058103 \pm 0.0000000197$

In inches, the above values become—

Length of A at 62° F., $A = 35^{in}.999170 \pm 0.0000047$
Length of B at 62° F., $B = 35^{in}.998960 \pm 0.0000058$
Expansion for 1° F. of $A = 0^{in}.00021146 \pm 0.00000070$
Expansion for 1° F. of $B = 0^{in}.00020917 \pm 0.00000071$

STANDARD INCH.

§ **3.** As accurate values of the micrometer-screws used in comparisons of lengths are indispensable, a standard inch was obtained from Mr. Simms, of London, for such determinations. It is divided into tenths, and the last tenth (from $0^{in}.9$ to $1^{in}.0$) is divided into hundredths. Each of the tenths and each of the hundredths had its value determined by comparison with the Ordnance-Survey standard foot, by Colonel Clarke. On this inch depend the values of the micrometer-screws used in all comparisons.

The following letter gives Colonel Clarke's determinations.

THE STANDARD INCH.

VALUES OF THE SPACES DETERMINED BY LIEUT.-COL. A. R. CLARKE, R. E.

ORDNANCE-SURVEY OFFICE,
Southampton, June 19, 1875.

MY DEAR SIR: I send you herewith the determinations of the spaces on your standard inch. I hope you received the inch safely.

Believe me yours, very truly,

A. R. CLARKE.

General COMSTOCK.

Values of the spaces on the standard inch. Tenths; $\left(unit = \frac{yard}{1,000,000}\right)$

$$\begin{aligned}
(0.1) &= \tfrac{1}{120} F + 2.78 \pm 0.14\\
(0.2) &= \tfrac{2}{120} F + 7.01 \pm 0.13\\
(0.3) &= \tfrac{3}{120} F + 1.06 \pm 0.10\\
(0.4) &= \tfrac{4}{120} F - 0.47 \pm 0.12\\
(0.5) &= \tfrac{5}{120} F - 2.81 \pm 0.08\\
(0.6) &= \tfrac{6}{120} F - 5.04 \pm 0.09\\
(0.7) &= \tfrac{7}{120} F - 9.86 \pm 0.12\\
(0.8) &= \tfrac{8}{120} F - 6.69 \pm 0.12\\
(0.9) &= \tfrac{9}{120} F - 0.59 \pm 0.18\\
(0.10) &= \tfrac{10}{120} F + 4.40 \pm 0.13
\end{aligned}$$

Where F is the length at 62° of the Ordnance standard foot, viz:

$$F = \tfrac{1}{3} Y - 0.49$$

Y being the true length of a yard, or

$$\tfrac{1}{12} F = \tfrac{1}{36} Y - 0.04$$

The values of the 10 spaces of $\frac{1}{100}$ inch, composing (9.10), are as follows, counting from 9 towards 10:

$$\begin{aligned}
\text{Space 1st} &= \tfrac{1}{10}\,(9.10) + 3.64 \pm 0.08 \\
\text{Space 2d} &= \tfrac{1}{10}\,(9.10) - 1.44 \pm 0.06 \\
\text{Space 3d} &= \tfrac{1}{10}\,(9.10) - 1.59 \pm 0.10 \\
\text{Space 4th} &= \tfrac{1}{10}\,(9.10) + 1.50 \pm 0.06 \\
\text{Space 5th} &= \tfrac{1}{10}\,(9.10) + 0.52 \pm 0.10 \\
\text{Space 6th} &= \tfrac{1}{10}\,(9.10) + 0.36 \pm 0.10 \\
\text{Space 7th} &= \tfrac{1}{10}\,(9.10) - 1.31 \pm 0.09 \\
\text{Space 8th} &= \tfrac{1}{10}\,(9.10) + 1.06 \pm 0.10 \\
\text{Space 9th} &= \tfrac{1}{10}\,(9.10) - 0.22 \pm 0.08 \\
\text{Space 10th} &= \tfrac{1}{10}\,(9.10) - 2.53 \pm 0.06
\end{aligned}$$

N. B.—The probable error of the space (9.10) is this:

$$(9.10) = \tfrac{1}{120}F + 4.99 \pm 0.13$$

Total number of observations, 1,032.

A. R. CLARKE,
Lieutenant-Colonel, R. E.

NOTE.—In the comparisons referred to in this chapter Colonel Clarke's values of the spaces on the standard inch have been used. Subsequently to his determinations, a good deal of material accumulated in the determination of the values of micrometer-screws and in direct comparisons in the Lake-Survey office. This work being completed, and being quite extended, was reduced. It consisted in comparing different spaces on the inch with a common space on a comparator-screw, the comparator-screw being used to move a slide bearing the inch, under a fixed microscope. This gave relative values of the tenths of the inch in terms of the whole inch. A similar process applied to the hundredths of an inch between $0^{in}.90$ and $1^{in}.00$ gave the relative values of these hundredths in terms of this tenth. As the Lake-Survey determinations were in great number, it was decided to utilize them by combining them with equal weight with those of Colonel Clarke, and the resulting means have been used as the values for the parts of the inch whenever the inch has been used to determine values of graduations on the 15-feet brass bar, or on any part of the Repsold base-apparatus.

The following table gives the values of the tenths and of certain hundredths of an inch as found by Colonel Clarke; those found in the Lake-Survey office; their differences; and their weighted means. For the hundredths of an inch, as there were two independent determinations, double weight was given to the results of the Lake-Survey office.

Values of spaces on the standard inch.

Spaces on standard inch.	Clarke's values of spaces.	Lake-Survey values of spaces.	Clarke's values minus Lake-Survey values.	Weighted mean values of spaces.
in. *in.*	*in.*	*in.*	*in.*	*in.*
0.00 to 0.10	0.100100	0.100096	+ 0.000004	0.100098
0.10 to 0.20	0.100152	0.100166	− 0.000014	0.100159
0.20 to 0.30	0.099786	0.099783	+ 0.000003	0.099785
0.30 to 0.40	0.099945	0.099958	− 0.000013	0.099951
0.40 to 0.50	0.099916	0.099888	+ 0.000028	0.099902
0.50 to 0.60	0.099920	0.099906	+ 0.000014	0.099913
0.60 to 0.70	0.099826	0.099871	− 0.000045	0.099849
0.70 to 0.80	0.109114	0.100124	− 0.000010	0.100119
0.80 to 0.90	0.100219	0.100191	+ 0.000028	0.100205
0.90 to 1.00	0.100179	0.100176	+ 0.000003	0.100177
0.00 to 1.00	1.000157	1.000159	− 0.000002	1.000158
0.90 to 0.91	0.010149	0.010120	+ 0.000029	0.010130
0.91 to 0.92	0.009966	0.009996	− 0.000030	0.009986
0.92 to 0.93	0.009961	0.009970	− 0.000009	0.009967
0.93 to 0.94	0.010072	0.010080	− 0.000008	0.010077
0.94 to 0.95	0.010037	0.010000	+ 0.000037	0.010013
0.95 to 0.96	0.010031	0.010090	− 0.000059	0.010070
0.96 to 0.97	0.009971	0.009944	+ 0.000027	0.009953
0.97 to 0.98	0.010056	0.010052	+ 0.000004	0.010053
0.98 to 0.99	0.010010	0.009964	+ 0.000046	0.009979
0.99 to 1.00	0.009927	0.009961	− 0.000034	0.009950
0.90 to 1.00	0.100180	0.100177	+ 0.000003	0.100178

The probable errors of the Lake-Survey work have not been computed. Colonel Clarke's probable errors are small, and may be safely used as small enough in computing the probable error of the space $0^{in}.80$ to $0^{in}.99$, which enters the determination of R 1876 in terms of Clarke yard A.

$$\begin{aligned}\text{Space } 0^{in}.80 \text{ to } 0^{in}.99 &= \text{space } 0^{in}.80 \text{ to } 1^{in}.00 \text{ minus space } 0^{in}.99 \text{ to } 1^{in}.00\\ &= 0^{in}.200382 \pm 0^{in}.0000065 - 0^{in}.009950 \pm 0^{in}.0000023\\ &= 0^{in}.190432 \pm 0^{in}.0000069\\ &= 4^{mm}.83692 \pm 0^{mm}.00018\end{aligned}$$

FIFTEEN-FEET STANDARD BAR.

§ **4.** This bar is a brass bar about 15 feet in length, its cross-section being $1^{in}.1$ by $0^{in}.33$. In each end an agate plate has been inserted in a dovetail, and is held in position by burnishing the brass down upon it. The outer face of the agate plate coincides with the end of the bar.

On one broad face and near one end there is stamped "U. S. standard, 15 feet 0.0018 inches at 62° Fah. From yards Nos. 41, 42, 43, 44, 45. Expansion for 1° Fah. 0.0017 of an inch." The length of this bar is the distance between the middle points of the agate planes, when, the stamped face being vertical and the inscription right side up, that top edge which is farthest from an observer reading the inscription is a straight line.

This 15-feet bar appears to have been received by the Lake Survey in 1852 with the Bache-Würdemann base-apparatus.

The agate planes were not put in the ends of the bar till the winter of 1865–'66. They of course gave a new length to the bar. Nothing is known of the method by which the expansion of the bar for 1° F., which is stamped on it, was obtained. It is only an approximation to the true value. The length of the bar stamped on it, of course, is now incorrect.

In reference to the term "U. S. standard," it may be remarked that at one time the United States Treasury Department adopted the distance at 62° F. between the 27th and 63d inch, on a brass 82-inch scale, by Troughton and Simms, belonging to the Coast Survey, as a United States standard yard. Subsequently it was found that this so-called United States standard differed by about one-thousandth of an inch from the standard yard of England.

By law of 28th July, 1866, Congress, in legalizing the metrical system, adopted in round numbers the ratios which exist between the English and French standards of length, thus assuming that the yard, foot, and inch of the United States are the same as those of Great Britain. Hence, whenever these units are used in this chapter the standards to which they refer are those of Great Britain.

The length of the 15-feet brass bar was derived from comparison with the five brass yards L. S., Nos. 6, 7, 8, 9, 10, placed end to end in a right line, and inclosed in a long narrow box, to prevent rapid changes of temperature, the 15-feet bar being in the same box beside and within 2 inches of the yards.

FIVE BRASS YARDS.

§ **5.** These yards, used in determining the length of the 15-feet bar, were constructed for the Lake Survey at the Office of Weights and Measures in Washington, in 1871, under the direction of Mr. J. E. Hilgard. They are all similar brass bars, and are marked, respectively, L. S., No. 6, 7, 8, 9, 10.

These yards are prisms one inch wide by six-tenths of an inch thick and thirty four and seven-tenths inches long, having at each end an axial cylinder four-tenths of an inch in diameter and six-tenths of an inch long. In the ends of these cylinders agates are held by the brass, which is burnished down on them. The end surfaces of the agates are ground to a radius of 4 inches, the center of the sphere being in the axis of the bar. The distance between the middle points of these agate disks is taken as the length of the yard. These points are by construction in the prolongations of the axes of the end cylinders.

The values of these yards and their relative expansions have been determined by comparisons with each other and with the Clarke yards A and B at various times and at widely-differing temperatures.

COMPARATORS.

§ **6.** The comparators used are contact-level comparators, one by Würdemann and one by Stackpole. In each, a micrometer-screw moves in its own direction, a slide carrying the supports of a level having horizontal trunnions, which turn on the points of two steel screws; a vertical arm projecting from the under side of the level is in contact with one end of a sliding piece of steel, which at its other end abuts against the standard undergoing comparison, and moves parallel to the micrometer-screw on grooved rollers.

One end of the level is the heavier by an ounce or two, so that by the vertical arm underneath it the sliding piece is pressed with constant pressure against the end of the standard under comparison. Any slight motions of this sliding piece can be read on the level by reading its bubble; larger motions up to 1 inch are read by moving the slide which carries the level by turning the micrometer-screw till the bubble is near the middle of the level, and then reading the level, the micrometer-head, and an inch scale on the slide. The value of one division of the micrometer-head is approximately $0^{in}.0001$, and of the level about $0^{in}.00001$ for the Würdemann comparator. One turn of the Würdemann screw is approximately $0^{in}.02$ and of the Stackpole screw approximately $0^{in}.01$.

In comparisons of two end-measure yards, the two comparators are mounted on a stout timber, so that their sliding pieces shall be horizontal with axes in the same straight line, the outer ends of the sliding pieces being nearly a yard apart. The centers of the ends of the yards are then brought alternately between these sliding pieces, and the micrometer-heads are turned till the bubble is in the middle; both levels and micrometers are then read. The same being done for the second yard, the difference of readings gives the difference of length. The Stackpole comparator is kept as nearly at a constant reading as possible, thus throwing the measurement on the Würdemann comparator, which has the best level. For comparisons of 15-feet bars the same process is used, save that the comparators are then mounted on stable stone posts entirely disconnected with the floor on which the observer stands.

As in expansion-experiments quantities of nearly $0^{in}.1$ have to be measured, it is necessary to know the absolute values of one turn of the micrometer-screws and their periodic errors, with precision. These have been determined for the parts of the screws used, and the resulting values have been used in reductions.

The absolute values of one turn of the micrometer-screws were determined by mounting the standard inch on the comparator-slide and observing its successive divisions with a microscope, as the micrometer-screw, which was read at each observation, moved them through the field.

The Würdemann comparator should be used only in the vicinity of or between the scale-readings, $0^{in}.1$ and $0^{in}.2$.

At $0^{in}.11$ its indications are to be multiplied by 1.004 to reduce them to inches. At this point the value of one revolution increases with increased reading at the rate of $\frac{1}{16700}$ part of its value per turn. Periodic error between $0^{in}.1$ and $0^{in}.2$ scale-reading has a maximum effect on either side of the mean of $0^{in}.00003$ or a total range of $0^{in}.00006$.

The periodic error was first determined by Bessel's method by Lieutenant Lockwood, in 1874, and a later redetermination gave nearly the same value.

The readings of the Stackpole comparator at $0^{in}.47$ are to be multiplied by 1.003 to reduce them to inches.

The value of one turn diminishes at the rate of $\frac{1}{62700}$ part of its value per turn as the readings increase.

The periodic error of the screw amounts to $0^{in}.000019$ on each side of the mean, or has a total range of $0^{in}.000038$.

This screw is mainly used between the scale-readings $0^{in}.4$ and $0^{in}.5$.

THERMOMETERS.

§ **7.** The thermometers on which the results given herewith depend are the following:

1st. Standard No. 230, made by Troughton & Simms, 1871.

2d. Casella standards Nos. 21472, 21473, 21474, 21475, 21476.

3d. Four thermometers, marked A_1, A_2, A_3, A_4, made by Simms, to accompany Clarke yard A.

4th. Four thermometers, marked B_1, B_2, B_3, B_4, made by Simms, to accompany Clarke yard *B*.

Standard 230 has a bulb 0in.88 long and 0in.22 diameter. It is graduated from 20° to 220° F., in degrees, one degree having a length of 0in.067.

In the Casella thermometers, the length of the bulb is 1.0 inch and its external diameter is 0in.31. The stem is graduated from 21° to 129° F., into half-degrees, and a degree is 0in.12 in length.

In the *A*- and *B*-thermometers, the stem is bent at right angles to the bulb, which is 0in.6 long and 0in.2 in diameter. It is graduated to degrees, from 22° F. to 110°, the length of one degree being 0in.055.

The Casella thermometers having greater sensibility, and one of them, No. 21472, having been carefully studied by Professor H. Ste. Claire Deville, it is taken as the standard. Professor Deville states that it has no errors of calibration exceeding 0°.1; that at 16° C. (= 60°.8 F.) it differed from a tested standard of great perfection by but 0°.02 and but 0°.03 at 33° C. (= 91°.4 F.)

No. 230 was compared with a standard at Kew Observatory in January, 1872, and the following corrections were found to the scale-readings, the stem being vertical:

Scale-readings,	32°	42°	52°	62°	72°	82°	92°	212°
Corrections,	— 0°.0	— 0°.0	— 0°.1	— 0°.0	+ 0°.1	+ 0°.2	+ 0°.3	+ 0°.1

In September, 1873, its freezing-point was redetermined; the correction to 32° was then — 0°.16. On November 17, 1873, its boiling-point was tested. The correction to 212° was + 0°.07, and on November 18 the correction to 32° was + 0.02.

December 19, 1873, it was	— 0°. 06
January 6, 1874, it was	— 0°. 07
March 8, 1875, it was	— 0°. 24
April 2, 1877, it was	— 0°. 31

Regnault's methods were carefully followed in all determinations, and numerous determinations were made at each date. The stem was always vertical.

As this thermometer is sometimes used in a horizontal position, its error at 32° was determined in that position at the same time; —0°.08 should be added to the vertical correction at 32° to give the correction at 32° with stem horizontal.

The following tables give the corrections, determined at the Kew Observatory in January, 1875, for the five Casella thermometers, the stems being horizontal and vertical, respectively, and also the subsequent determinations of the freezing-points made at Detroit:

CASELLA THERMOMETERS, FAHRENHEIT. (HORIZONTAL.)

Corrections to be applied to scale-readings.

Date.	Scale-readings.	21472.	21473.	21474.	21475.	21476.
	°	°	°	°	°	°
Kew, January, 1875	30	— 0. 0	....	....	....	....
Do.	32	....	+ 0. 1	+ 0. 1	+ 0. 1	+ 0. 1
Do.	35	— 0. 0	+ 0. 1	+ 0. 1	+ 0. 1	+ 0. 1
Do.	40	— 0. 0	+ 0. 1	+ 0. 1	+ 0. 1	+ 0. 1
Do.	45	+ 0. 1	+ 0. 1	+ 0. 1	+ 0. 1	+ 0. 1
Do.	50	+ 0. 1	+ 0. 1	+ 0. 1	+ 0. 1	+ 0. 1
Do.	55	— 0. 0	+ 0. 1	+ 0. 1	+ 0. 1	+ 0. 1
Do.	60	— 0. 0	+ 0. 1	+ 0. 1	+ 0. 1	+ 0. 1
Do.	65	— 0. 0	+ 0. 1	+ 0. 1	+ 0. 1	+ 0. 1
Do.	70	— 0. 0	+ 0. 1	+ 0. 1	+ 0. 1	+ 0. 1
Do.	75	— 0. 0	+ 0. 1	+ 0. 1	+ 0. 1	+ 0. 1
Do.	80	— 0. 0	+ 0. 1	+ 0. 1	+ 0. 1	+ 0. 1
Do.	85	— 0. 0	+ 0. 1	+ 0. 1	— 0. 0	+ 0. 1
Do.	90	— 0. 0	+ 0. 1	+ 0. 1	— 0. 0	— 0. 0
Do.	95	0. 0	+ 0. 1	+ 0. 1	+ 0. 1	+ 0. 1
Do.	100	0. 0	+ 0. 1	+ 0. 1	+ 0. 1	+ 0. 1
Detroit, March, 1875	32	+ 0. 06	+ 0. 13	+ 0. 14	+ 0. 15	+ 0. 13
Detroit, December 23, 1875	32	— 0. 01	+ 0. 08	+ 0. 06	+ 0. 07	+ 0. 08
Detroit, December 30, 1876	32	— 0. 04	....	....	....	....

CASELLA THERMOMETERS, FAHRENHEIT (VERTICAL).

Corrections to be applied to scale-readings.

Date.	Scale-readings.	21472.	21473.	21474.	21475.	21476.
	°	°	°	°	°	°
Kew, January, 1875	32	+ 0.1	+ 0.2	+ 0.2	+ 0.2	+ 0.2
Do	35	+ 0.1	+ 0.2	+ 0.2	+ 0.2	+ 0.2
Do	40	+ 0.1	+ 0.2	+ 0.2	+ 0.2	+ 0.2
Do	45	+ 0.2	+ 0.2	+ 0.2	+ 0.2	+ 0.2
Do	50	+ 0.2	+ 0.2	+ 0.2	+ 0.2	+ 0.2
Do	55	+ 0.1	+ 0.2	+ 0.2	+ 0.2	+ 0.2
Do	60	+ 0.1	+ 0.2	+ 0.2	+ 0.2	+ 0.2
Do	65	+ 0.1	+ 0.2	+ 0.2	+ 0.2	+ 0.2
Do	70	+ 0.1	+ 0.2	+ 0.2	+ 0.2	+ 0.2
Do	75	+ 0.1	+ 0.2	+ 0.2	+ 0.2	+ 0.2
Do	80	+ 0.1	+ 0.2	+ 0.2	+ 0.2	+ 0.2
Do	85	+ 0.1	+ 0.2	+ 0.2	+ 0.1	+ 0.2
Do	90	+ 0.1	+ 0.2	+ 0.2	+ 0.1	+ 0.1
Do	95	+ 0.1	+ 0.2	+ 0.2	+ 0.2	+ 0.2
Do	100	+ 0.1	+ 0.2	+ 0.2	+ 0.2	+ 0.2
Do	105	+ 0.1	+ 0.2	+ 0.2	+ 0.1	+ 0.1
Do	110	+ 0.1	+ 0.2	+ 0.2	+ 0.2	+ 0.2
Detroit, March 25, 1875	32	+ 0.10	+ 0.18	+ 0.18	+ 0.20	+ 0.16
Detroit, January 18, 1877	32	− 0.01	....	....	....	....

The following table gives the corrections to be applied to the mean of the four A-thermometers, and to the mean of the four B-thermometers, to reduce their mean readings to Ordnance-Survey standard 3241, a thermometer which has been very thoroughly studied by Lieutenant-Colonel Clarke, who has given his methods and results in his Comparisons of Standards of Length.

Colonel Clarke states that the A- and B-thermometers were compared with 3241 with the greatest care. The table contains, besides Colonel Clarke's results, the results of later freezing-point deter minations at the Lake-Survey office.

TROUGHTON & SIMMS A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4 (HORIZONTAL).

Corrections to be applied to scale-readings.

Date.		Mean of A_1, A_2, A_3, A_4.	Mean of B_1, B_2, B_3, B_4.
	°	°	°
Lieutenant-Colonel Clarke, April 15, 1874	32	−0.30	−0.28
Lieutenant-Colonel Clarke, May 2, 1874	32	−0.28	−0.24
Lieutenant-Colonel Clarke, April, 1874	52	−0.22	−0.13
Do	55	−0.18	−0.11
Do	57	−0.17	−0.13
Do	62	−0.22	−0.17
Do	93	−0.17	−0.13
Detroit, March, 1875	32	−0.37	−0.34
Detroit, December, 1875	32	−0 40	−0.38

The comparisons which fix the length of the 15-feet brass bar and its expansion depend almost entirely on the five Casella thermometers, and on the A- and B-thermometers. As the corrections of the A- and B-thermometers depend on Ordnance-Survey standard 3241, while those of the five Casella thermometers depend on a Kew standard, a comparison between the A- and B-thermometers and the Casella thermometers becomes of interest. The freezing-point determinations, of course, give the absolute errors of the 32° line. Comparisons were made between the standard 230 and the A- and B-thermometers and between 230 and 21472 in January, 1876, at 59°.5 F. This work gives us an indirect comparison between the A- and B-thermometers and 21472.

The results are as follows:

At 59°.5, the stems of the thermometers being horizontal, the mean of the readings of A_1, A_2, A_3, A_4, having had Colonel Clarke's corrections, and also the corrections for subsequent change of freezing-point applied to it, and 21472 having had the Kew correction and also the correction for subsequent change of freezing-point applied to it, 21472 then read 0°.06 F. greater than the mean of A_1, A_2, A_3, A_4. Similarly, 21472 corrected reads 0°.02 F. greater than the corrected mean of B_1, B_2, B_3, B_4, at 59°.5. As the Kew corrections are only given to the nearest tenth of a degree, this agreement, taken in connection with Professor Sainte Claire Deville's comparisons with another standard which gave no greater discrepancies, indicates that Casella No. 21472 is very accurately constructed, and that, so far as a mercurial thermometer is concerned, the probable error of its corrected indications is but a few hundredths of a degree. (21472 was also compared with the other Casella thermometers at 59°.5. The greatest discrepancy between the results and the Kew corrections was 0°.04.)*

COMPARISON OF YARDS NUMBERS 6, 7, 8, 9, 10, WITH EACH OTHER.

§ **8.** For comparisons of the Lake-Survey yards Nos. 6 to 10 with each other and with Clarke yards A and B, the cylindrical ends of the brass yards rested in wyes, with a pressure of three or four ounces, nearly all their weight being carried by supporting spiral springs attached at one-fourth and three-fourths of the length of the yard. The comparators were firmly attached to strong timbers. The two yards under comparison were inclosed in a box with glass windows in its cover through which to read thermometers, and with slits with sliding covers through which the suspending wires passed, by whose aid, without opening the box, the yards under comparison were alternately placed in the wyes, which brought the centers of their ends in line with the axes of the sliding pieces of the comparators, and gave central contact for the end of the yard and the end of the sliding piece. The Clarke yards A and B remained in their cases, and were suspended and handled in the same way, the cases resting against adjustable stops, when exactly in the right position with reference to the comparators. Their cases were slightly inclined sidewise, so that these yards should slip so as to always take the same position in the case. Nearly all the comparisons were made in the cellar of the Lake-Survey office, where the daily range of the air-temperature, caused mainly by the presence of the observers, rarely exceeded 2° or 3° F. Readings were usually taken at about 9½ a. m. and 4 p. m., the room at other times being kept carefully closed when the temperature of the cellar was below 40° F. The rise of temperature in the box containing the yards under comparison, from the presence of the observers, during the time (about ten minutes) required for two or three comparisons, was usually about 0°.3 F., as indicated by the thermometers under the wrapping of three thicknesses of flannel which incased the brass yards. For higher temperatures the effect of the presence of the observers was less.

If the temperatures of the brass yards had risen by the same amount as the temperatures of their thermometers, namely, 0°.3 F., the lengths of the yards would have increased by $0^{in}.00010$. It is safe to say that they did not increase in length by more than one-half this amount, or $0^{in}.00005$, which must have been nearly the same for both; so that there is no probability that, during the ten or fifteen minutes of comparison of two brass yards, the difference of their lengths changed by so much as $0^{in}.00002$ on an average. As the yards were systematically alternated in position with reference to the observer, this error was eliminated in the result.

From the insignificant effect that this small rise in temperature would have on the difference in length of two similar brass bars, all comparisons made on two brass yards are used, until the thermometer has risen by 0°.3 F. In early comparisons, at low temperatures, as many as five were obtained, occupying thirty minutes. Of these, only the first two or three have been used, as the thermometers rose 0°.6 or 0°.7 during the thirty minutes, and the temperatures of the two similar yards may not have changed equally.

In later work but two comparisons were usually made, each occupying from six to ten minutes, at 9:30 a. m. and 3 p. m.

In reducing the comparisons, those at the same period, and, therefore, at about the same temperature, have been combined so as to give a single equation of condition, to which a weight is assigned equal to the number of days of comparisons. For this purpose, the mean of the observed

* See Note A, Chapter II, § 15.

differences of length is taken, and the mean of the first readings of the thermometers in each set of comparisons, as the thermometers after the first reading are slightly affected by the presence of the observers, while the temperature of the yard has probably not sensibly changed.

The form of an equation of condition for yards Nos. 6 and 7 will be—

$$(6-7)_{t^\circ}=(6-7)_{62^\circ}-(62^\circ-t^\circ)\,(e_6-e_7)$$

where 6 and 7 are the lengths of the two yards, t° the observed temperature, and e_6 and e_7 the expansions of 6 and 7 for 1° F.

Or, placing $(6-7)_{t^\circ}=n=$observed difference of lengths;
$(6-7)_{62^\circ}=x=$difference of lengths at 62°;
$e_6-e_7=y=$difference of expansions for 1° F.;
$62^\circ-t^\circ=b$;

the equation of condition takes, for 6—7, the form—

$$n-x+by=0.$$

For 6—8, 6—9, &c., x and y would receive one, two, &c., primes.

The first column of the following table gives the dates of comparisons, the second the number of sets of comparisons, the third the number of days of comparisons, or the weights, the fourth the total range in the observed values of n, the fifth, sixth, seventh, and eighth the equations of condition, and the ninth their residuals. The unit is one-millionth of an inch. The values of b show how far below 62° F. the mean temperature of each group of comparisons was.

Date.	Number of sets.	Number of days.	Range.	n	x	by	0	v	Temperature.
									° °
1874—Jan. 14 to Feb. 12	18	9	84	-1333	$-x$	$+23.84\,y$	$=0$	$+21$	37–41
June 18, 19	8	2	92	-1339	$-x$	$+1.74\,y$	$=0$	-8	60–61
1875—Dec. 7 to 13	12	6	121	-1368	$-x$	$+20.10\,y$	$=0$	-18	41–43
Nov. 29 to Jan. 5, 1876	24	14	122	$+111$	$-x'$	$+21.43\,y'$	$=0$	0	43–39
1876—Aug. 7 to 19	10	4	103	$+89$	$-x'$	$-0.28\,y'$	$=0$	0	61–63
1874—Jan. 19 to Feb. 19	15	7	85	$+1472$	$-x''$	$+23.08\,y''$	$=0$	$+3$	39–40
June 23 to 30	8	3	106	$+1509$	$-x''$	$-2.19\,y''$	$=0$	$+3$	63–65
1875—Nov. 22 to 26	8	4	141	$+1476$	$-x''$	$+18.47\,y''$	$=0$	0	44–43
1874—Jan. 22 to Feb. 21	11	4	63	-1596	$-x'''$	$+21.82\,y'''$	$=0$	$+3$	39–41
July 1, 2	8	2	83	-1532	$-x'''$	$-1.97\,y'''$	$=0$	$+11$	63–64
1875—Dec. 14 to 21	12	6	109	-1624	$-x'''$	$+23.27\,y'''$	$=0$	-21	40–36
1874—Jan. 27 to Feb. 27	10	4	110	$+2830$	$-(x''-x)$	$+23.50\,(y''-y)$	$=0$	$+8$	38–40
July 7 to 15	8	3	79	$+2830$	$-(x''-x)$	$-2.03\,(y''-y)$	$=0$	-3	63–64
Jan. 29 to Mar. 2	9	4	51	-234	$-(x'''-x)$	$+22.63\,(y'''-y)$	$=0$	$+15$	39–40
July 16, 17	8	2	101	-227	$-(x'''-x)$	$-2.63\,(y'''-y)$	$=0$	-12	64–65
Feb. 4 to Mar. 7	9	4	96	-3057	$-(x'''-x'')$	$+23.37\,(y'''-y'')$	$=0$	$+14$	37–39

Solving the equations of condition by the method of least squares, and deducing the values of the unknowns and their probable errors, the following values result:

$$(6-7)_{62^\circ}=-1329\pm12 \qquad e_6-e_7=+1.02\pm0.57$$
$$(6-8)_{62^\circ}=+89\pm13 \qquad e_6-e_8=-1.01\pm0.66$$
$$(6-9)_{62^\circ}=+1503\pm11 \qquad e_6-e_9=+1.47\pm0.56$$
$$(6-10)_{62^\circ}=-1548\pm13 \qquad e_6-e_{10}=+2.36\pm0.62$$

In determining these values no comparisons of No. 8 with another yard made prior to August 7, 1875, are used, as that yard had previously changed its length.

Five days' comparisons, March 1, 1874, at 39° F., gave $6-8=-0^{\text{in}}.00010$
Two days' comparisons, June 1, 1874, at 61°.2 F., gave $6-8=-0^{\text{in}}.00013$
Two days' comparisons, July 20, 1875, at 64° F., gave $6-8=+0^{\text{in}}.00020$

In the interval between the last two comparisons, which show an evident change of length, as their difference far exceeds any possible error of comparison, the five yards placed end to end had been compared with the 15-feet brass bar.

During these comparisons the yards had been suspended by spiral springs, which allowed the end-cylinders of the yards to rest in their guiding-wyes with but a few ounces weight, and so left them with great ease of motion in the direction of their lengths. The idea at once suggests itself that, in bringing the ends of the yards in contact, the shock of contact due to the considerable mass of the yards had been sufficient to force in one of the agates at the ends of the yard No. 8 where the fitting had not been firm before, thus shortening the yard. These agates, in constructing the yards, were simply pressed into a cavity cut for them and the brass at the ends of the cylinders was burnished down around the agates. On August 2, 1875, I heated the ends of yard No. 8 to about 150° F., and then holding the yard vertical pressed the lower agate into a piece of soft wood with a pressure of 20 or 30 pounds. Then the agate and end of the cylinder were rotated under the same pressure in a hole a little larger than the agate, the upper end of the hole having been reamed out so as to bear on the brass immediately around the agate and press it against the agate. Finally, the brass was burnished down upon the agate with a hand-burnisher. Both agates of No. 8 were treated in the same way.

Two days' comparisons, on August 7 and 9, 1875, gave, at 62°, $6 - 8 = +0^{in}.00012$, which would indicate a lengthening of No. 8 by $0^{in}.00008$ since July 20, 1875. But in the previous comparisons the comparing-room was visited three or four times during the day, which makes it uncertain whether the temperatures of the two yards were precisely the same.

As the burnishing could hardly have lengthened the yard, it seems possible that the small apparent change in length subsequent to July 20, 1875, was due simply to temperature errors in the previous comparisons, and that the length of yard No. 8 was not sensibly affected by the burnishing process; or, in other words, that its agates are now stable.

The comparisons of the other yards at different dates give no indications of change in lengths. The following are the results of comparisons of Nos. 6 and 8 at different dates:

1. Nov. 30, 1871.	Temp., 55°.5 F.,	1 day's comparisons,	$6 - 8 = 0^{in}.00000$
2. March, 1874.	Temp., 39° F.,	5 days' comparisons,	$6 - 8 = -0^{in}.00010$
3. June, 1874.	Temp., 62° F.,	2 days' comparisons,	$6 - 8 = -0^{in}.00014$
4. July 20, 1875.	Temp., 64° F.,	2 days' comparisons,	$6 - 8 = +0^{in}.00019$
Aug. 2, 1875.	Both agates of No. 8 burnished down.		
5. Aug. 9, 1875.	Temp., 62° F.,	2 days' comparisons,	$6 - 8 = +0^{in}.00012$
6. Aug. 9, 1875.	Temp., 63° F.,	2 days' comparisons,	$6 - 8 = +0^{in}.00009$
7. Dec., 1876.	Temp., 41° F.,	14 days' comparisons,	$6 - 8 = +0^{in}.00011$

The comparisons of November 30, 1871, were not made under temperature conditions which could make precision certain, and their discrepancy with the second comparisons does not make it sure that yard 8 had changed length in the interval. It certainly changed length largely between June, 1874, and July, 1875, and may have changed very slightly while being burnished, although the difference of lengths on July 20, 1875, and August 9, 1875, while larger than the probable is not larger than the possible errors in the comparisons made on only two days.

COMPARISON OF BRASS YARD NUMBER 6 WITH CLARKE YARDS *A* AND *B*.

§ **9.** Having given the lengths and expansions of the other brass yards relatively to No. 6, it remains to explain how the absolute length and expansion of No. 6 were obtained from the Clarke yards *A* and *B*.

No. 6 was compared with *A* and *B*, and No. 7 with *A*. The yard *A* or *B* remained in its iron case, and, with its thermometers in their places, was put with the brass yard in the wooden box in which the brass yards had been compared with each other. The larger part of its weight was carried by suspending-springs, and its outer iron case rested against stops, adjustable so that the centers of its end surfaces should be in line with the axes of the sliding pieces or quills of the comparators. The case was slightly inclined, so that the yard within, under its own weight, should always assume the same position with reference to the case, and so with reference to the comparators. The brass yard was also suspended, resting its end cylinders in their supporting wyes. The two yards were alternately placed between the comparators, and comparator-readings taken on both ends.

In the comparisons of No. 6 with *A* and *B*, at temperatures near or below 40° F., the three comparisons making a set required about twenty minutes. In this time the thermometer beneath the flannel wrapping of the brass yard usually rose about 0°.3 F., while those inside the cases of the yards *A* and *B* only rose about 0°.05 F.

In comparisons near 62° F. the rise in thermometers was about one-half the above amounts. Hence, as the thermometer-temperatures of No. 6 and *A* or *B* change very unequally, and as their expansions are very different, only the first comparisons and the first temperatures, on entering the comparing-room after an absence of several hours, were used in the reductions.

No comparisons were used in which the thermometer in contact with the brass yard and under three thicknesses of flannel differed by more than 0°.2 F. from the mean of the four thermometers inside the iron case of the Clarke yard; and as the presence of the observers for the two to four minutes before the thermometers were read may have slightly affected the thermometer with the brass yard, while its effect on those with the steel yards was insensible, the thermometers with the latter yards have been alone used in fixing the temperature of comparisons. No comparisons were used where the thermometers with the Clarke yards changed by more than 0°.4 F. between morning and afternoon comparisons (9:30 a. m. and 4 p. m.).

No. 6 and *A* were compared with each other at three different periods, at temperatures varying from 37°.6 to 62°.8.

No. 7 and *A* were compared at about 33°. As the length and expansion of No. 7, with reference to No. 6, are known with great precision, the comparisons of 7 with *A* were reduced to those of 6 with *A*, giving a fourth group.

Each of these four temperature-groups of comparisons gave an equation of condition of the form

$$(6-A)_{t^\circ}=(6-A)_{62^\circ}-(62^\circ-t^\circ)\,(e_6-e_A)$$

in which t° is the observed temperature, e_A and e_6 the expansions of No. 6 and *A* for 1° F., and $(6-A)_{62^\circ}$, and (e_6-e_A), the unknowns, provided the temperatures were correctly measured.

But in some thermometers, which agree with an air-thermometer at 32° and 212° F., and which have no sensible errors of construction, it is known, in consequence of the dilatations of glass and mercury depending on both the first and second powers of the temperature, that at other readings they may differ sensibly from an air-thermometer. An attempt has been made to have one of the Lake-Survey thermometers compared with an air-thermometer, but as yet no results have been received.* In the comparisons with each other of the brass yards Nos. 6, 7, 8, 9, 10, as their relative expansions are very small, no sensible error is introduced into their relative lengths by an error of 0°.1 or 0°.2 F. in measuring their common temperature. But in determining the relative lengths of *A* and No. 6, an error of 0°.2 F., in determining their common temperature, would introduce an error of $0^{in}.00002$, a quantity larger than the probable error of comparison.

If such a systematic correction should be needed to make the Lake-Survey thermometers agree with an air-thermometer, it may be taken with sufficient accuracy between 32° and 62° F., as proportional to the excess of temperature over 32°, and its form will be $\Delta\,(t^\circ-32^\circ)$.

The equation of condition from a group of comparisons will then take the form

$$(6-A)_{t^\circ}=(6-A)_{62^\circ}-\left\{62^\circ-[t^\circ+(t^\circ-32^\circ)\Delta]\right\}(e_6-e_A)$$

which may be written

$$-n+x-(a-b\Delta)\,y=0$$

in which n is the mean difference of lengths of the two yards derived from a group of comparisons at nearly the same temperatures, of which t° is the mean, x is the unknown difference of length at 62°, and y the unknown relative expansion. The weight of each equation is equal to the number of its comparisons.

* See note A, Chapter II, § 15.

The following table gives the date of comparisons, the number of days of comparisons, the number of comparisons, the temperature-range during the comparisons, the equations of condition, and their residuals, Δ being neglected. The unit is the millionth of an inch.

Comparisons of No. 6 with A.

Date.	Number of days.	Number of comparisons.	Temperature.	Equations of condition.	Residuals.
1874—December 24 to January 4, 1875	6	6	42° to 37°	$-2847-x+(22.25-\ 7.75\ \Delta)\ y=0$	−4
1875—August 20 to 23	3	11	63° to 60°	$+\ 405-x+(\ 0.09-29.91\ \Delta)\ y=0$	−8
October 2 to 6	5	9	54° to 55°	$-\ 538-x+(\ 6.65-23.35\ \Delta)\ y=0$	+13
January 18 to 23 (No. 7 and A)	6	10	33° to 34°	$-3820-x+(28.89-\ 1.11\ \Delta)\ y=0$	−1

The following table gives similar data for comparisons of No. 6 and Clarke yard B:

Comparisons of No. 6 with B.

Date.	Number of days.	Number of comparisons.	Temperature.	Equations of condition.	Residuals.
1875—January 6 to 15	6	8	36° to 33°	$-3473-x'+(27.72-\ 2.28\ \Delta)\ y'=0$	+1
October 7 to 8	2	4	55° to 54°	$-\ 414-x'+(\ 6.81-23.19\ \Delta)\ y'=0$	+4
November 3 to 9	6	8	48° to 47°	$-1508-x'+(14.25-15.75\ \Delta)\ y'=0$	−3

In this table x' and y' have replaced the x and y of the preceding table, as yard B has replaced yard A.

Solving the above equations by least squares, the following values result:

$$x=(6-A)_{62^\circ}=+426\quad-\quad4408\ \Delta\pm6$$

$$x'=(6-B)_{62^\circ}=+578\quad-\quad4385\ \Delta\pm4$$

$$y=e_6-e_A\quad=+146.94-146.94\ \Delta\pm0.32$$

$$y'=e_6-e_B\quad=+146.17-146.17\ \Delta\pm0.21$$

the unit being a millionth of an inch, and the probable errors being derived from the equations of condition when Δ is taken as zero.

If the value of x above be subtracted from that of x', we have $A-B=152$, the terms in Δ being neglected, as Δ is very small and its coefficients nearly equal.

Referring to Colonel Clarke's values of A and B, (§ 2), we find $A-B=211$, differing from the value just found by 59. Both Colonel Clarke's and the Lake-Survey determination of this difference are indirect; that is, they are each obtained by comparisons with a third yard. Still the difference is larger than would be supposed from the probable errors of the comparisons, and it is hoped hereafter to make a direct determination of its value.*

If equal weights were given to the two determinations of the difference, the two results would differ from their mean by but $\frac{1}{1200000}$ of a yard, a quantity that is not large in such work.

Subtracting y' from y we have

$$e_B-e_A=+0.77$$

Colonel Clarke's value (§ 2) is

$$e_B-e_A=-2.33$$

* See note B, Chapter II, § 20.

Were the mean of these two values to be adopted, it would require a change in the values for the expansions of A and B, found by Colonel Clarke, of $\frac{1}{136}$ part.

Adding now the value of $(6-A)_{62^\circ}$ to the length of A at 62° F., given by Colonel Clarke (§ 2), there results the length of No. 6 at 62° F.:

$$\text{No. } 6 = 35^{\text{in}}.999596 \pm 8 - 0.004408\, \varDelta$$

Adding the value of $(6-B)_{62^\circ}$, above, to the length of B at 62° F., given by Colonel Clarke (§ 2), there results the length of No. 6 at 62° F.:

$$\text{No. } 6 = 35^{\text{in}}.999538 \pm 7 - 0.004385\, \varDelta$$

the probable errors having, in both cases, been obtained by neglecting $\varDelta$ as insignificant. These values differ by $0^{\text{in}}.000058$, a quantity larger than the probable errors of the separate values would indicate. The mean of the two values must be taken, and the probable error of the mean must be derived from its differences from the two values.

There results, then, for the length of No. 6 at 62°, from the data at present (April, 1877) available,

$$\text{No. } 6 = 35^{\text{in}}.999567 \pm 0^{\text{in}}.000020 - 0.004396\, \varDelta$$

the probable error being obtained by supposing $\varDelta = 0$, and being $\frac{1}{1800000}$ part of the length.

From § 2, e_A, the expansion of yard A for 1° F. is $0^{\text{in}}.00021146$. From § 9, $e_6 - e_A$, is $0^{\text{in}}.00014694$ $(1 - \varDelta)$, and adding, the value

$$e_6 = 0^{\text{in}}.0003584 - 0.00014694\, \varDelta \text{ results.}$$

Again, deriving e_6 from $e_B + (e_6 - e_B)$ we have

$$e_6 = 0^{\text{in}}.00035534 - 0.00014617\, \varDelta$$

Taking the mean of these two values, derived from yards A and B, respectively, we have

$$e_6 = 0^{\text{in}}.0003569 - 0.00014656\, \varDelta \pm 0^{\text{in}}.00000103.$$

Subtracting from the length of No. 6 its excesses over Nos. 7, 8, 9, 10 (§ 8), there result the following lengths of these yards at 62° F.:

$$\begin{aligned}
\text{No. } 7 &= 36^{\text{in}}.000896 \pm 0^{\text{in}}.000023 - 0.004396\, \varDelta \\
\text{No. } 8 &= 35^{\text{in}}.999478 \pm 0^{\text{in}}.000024 - 0.004396\, \varDelta \\
\text{No. } 9 &= 35^{\text{in}}.998064 \pm 0^{\text{in}}.000023 - 0.004396\, \varDelta \\
\text{No. } 10 &= 36^{\text{in}}.001115 \pm 0^{\text{in}}.000024 - 0.004396\, \varDelta
\end{aligned}$$

Summing these values and deducing the probable error, we have

$$\text{Yds. } (6+7+8+9+10) = 179^{\text{in}}.999120 \pm 0^{\text{in}}.000103 - 0.02198\, \varDelta$$

If the relative expansions, $e_6 - e_7$, $e_6 - e_8$, $e_6 - e_9$, and $e_6 - e_{10}$, be subtracted in succession from the value of e_6, derived from the Clarke yards, we should have values for the absolute expansions of those yards. Summing those values, we have a value for the expansion of the five yards when placed in contact, end to end, their axes being in the same right line. That value is

$$e_{6+7+8+9+10} = 0^{\text{in}}.0017807 \pm 0^{\text{in}}.0000053 - 0.0007328\, \varDelta$$

COMPARISONS OF FIVE LAKE-SURVEY YARDS (NOS. 6 TO 10), PLACED END TO END, WITH 15-FEET BRASS BAR.

§ **10.** In these comparisons the two comparators were mounted on stone posts, about 15 feet apart, their sliding pieces being in the same right line. The bar and yards, in the comparisons prior to June, 1875, were placed side by side in a closed wooden box, parallel to each other, and 2 inches apart. The 15-feet bar rested on rollers 18 inches apart, and was provided with side guide-screws to limit its side motion and to aid by slight pressure in its alignment. By raising or lowering its rollers, and by moving its guide-screws, the bar, while remaining free to move, was made straight;

its straightness being tested with a silver wire strained by a known weight, the wire being vertically over one of the upper edges of the bar. The side of the 15-feet bar was brought into the vertical plane of the wire to within $0^{in}.01$, and the top of the bar made parallel to the wire after the latter was corrected for its computed sag within the same error. The 5 yards placed end to end were each supported at $\frac{1}{4}$ and $\frac{3}{4}$ of their lengths by spiral springs, so adjusted as to carry nearly all the weight of the yards, leaving only three or four ounces to be supported by the wyes in which the end cylinders of the yards rested. The spiral springs supporting the end yards were so inclined that these yards each pressed toward the central yard with a force of about one pound, thus securing contact between the ends of the yards. The axes of the end cylinders of the yards were brought into the same right line by adjusting the wyes in which they rested. To bring them into the proper position, with reference to a vertical plane, a small piece of a semi-cylinder of the same diameter as the end cylinders of the yards had a point in its axis marked on its diametral plane surface. This semi-cylinder was placed in the wyes in succession, and each of the latter was moved sidewise till the point was directly under a stretched fine silver wire, 16 feet long, which vibrated just above the point.

To bring them into the proper position with reference to a horizontal plane, a level was fastened to the upper surface of one of the yards which at once gave the means of determining the difference of height of two wyes a yard apart, and of adjusting them to the proper height by means of their adjusting-screws.

As where two yards met there were two wyes very near each other, they were adjusted with reference to each other by a smaller level. Prior to July 1, 1875, the box containing the bar and yards, parallel to each other and about 2 inches apart, was mounted on two trestles of the base-apparatus, which gave lateral motion sufficient to bring the 15-feet bar and the yards alternately between the comparators.

Subsequent to that date the bar and yards were mounted on a T-shaped iron beam about an inch apart, the whole being inclosed in one of the long boxes of the expansion-apparatus, the necessary lateral motion for comparisons being obtained by running the car of that apparatus sidewise.

The first comparisons of 15-feet bar with Lake-Survey yards Nos. 6, 7, 8, 9, 10, were made in a room on the first floor of the Lake-Survey office, on 17 days, between February 26, 1872, and June 24, 1873. The daily temperature-range in this room is large, though less than that of the external air. As the cross-sections of the yards are 1^{in} by $0^{in}.6$, while that of the 15-feet bar is $1^{in}.1$ by $0^{in}.33$, the cross-section of the yards is two-thirds greater than that of the bar.

In changing temperatures that of the yards will always lag behind the temperature of the bar, so that the temperature-changes should be kept far below those of the ordinary diurnal ones. For this reason these comparisons were not considered satisfactory. They were given in the Lake-Survey Report for 1874; the difference at 62° F., bar − yards, being found $0^{in}.00839$.

A comparing-room was subsequently established in the cellar of the Lake-Survey office, where the daily range in the air-temperature outside the closed box containing the yards and bar was only 2° or 3° F., this being partly caused by the presence of the observers.

Fourteen days' comparisons, between December 10 and December 30, 1873, at a mean temperature of 42°.81 F., gave bar − yards $= 0^{in}.00829$.

Three days' comparisons, between July 21 and 24, 1874, gave at a mean temperature of 64°.7 F., bar − yards $= 0^{in}.00837$.

Between June 1, 1874, and July 20, 1875, yard No. 8, as compared with No. 6, appeared to have shortened by $0^{in}.00033$, there having been two days' comparisons at the first and two at the last of those dates, the first giving at 61°.2 F., $6 - 8 = -0^{in}.00013$, and the second giving at 63°.9 F., $6 - 8 = +0^{in}.00020$.

In both sets of comparisons the room was visited too frequently (three or four times a day) for the best work, but it is improbable that either of these values is erroneous by so much as $0^{in}.00005$.

The first value agrees with a value determined by five days' comparisons on March 1, 1874.

It is uncertain at what time the change in length of yard No. 8 occurred. It may have been before March, 1874, or after June, 1874, and it therefore throws uncertainty on all comparisons of the 15-feet bar with the five yards prior to August 2, 1875.

On that date strong pressure was applied to the end agates of No. 8, the brass around them was burnished down, and No. 8 was recompared with No. 6; the resulting difference $(6-8)_{62^\circ} = +0^{in}.000089$ has already been given in § 7.

The 15-feet bar and the five yards were again compared on August 12, 13, 14, 1875, at a temperature of 63° F., and on 26 days between February 28, 1876, and April 7, 1876, at temperatures varying from 34°.7 to 38°.5 F.

From these two sets of comparisons the difference at 62° F. (bar — yards) has been found and the relative expansion of the bar and the five yards.

Each comparison gave an equation of condition of the form—

$$D_{62^\circ} + (t^\circ - 62^\circ)\, e - n = 0$$

in which D_{62° is the excess of length of bar over that of the five yards in contact in a straight line; t° is the corrected temperature of the comparison; e is the excess of expansion of bar over yards for 1° F., and n is the observed difference of length of bar and 5 yards corrected for periodic error of comparator-screw.

The following are the equations of condition, with dates and residuals:

Date.		Equations of condition.	Residuals.
1875.		*in.*	*in.*
Aug.	12	$D_{62}+0.8e-0.00872=0$	+0.00012
	13	$D_{62}+0.9e-0.00875=0$	+9
	14	$D_{62}+0.5e-0.00905=0$	−21
1876.			
Feb.	28	$D_{62}-26.0e-0.00883=0$	−12
	29	$D_{62}-25.8e-0.00878=0$	−7
Mar.	1	$D_{62}-26.0e-0.00873=0$	−2
	2	$D_{62}-26.4e-0.00871=0$	−1
	3	$D_{62}-26.6e-0.00872=0$	−2
	4	$D_{62}-26.5e-0.00888=0$	−18
	6	$D_{62}-25.2e-0.00890=0$	−19
	7	$D_{62}-23.8e-0.00891=0$	−19
	9	$D_{62}-24.1e-0.00880=0$	−8
	10	$D_{62}-24.0e-0.00863=0$	+9
	15	$D_{62}-25.3e-0.00855=0$	+16
	16	$D_{62}-25.4e-0.00861=0$	+10
	18	$D_{62}-26.4e-0.00866=0$	+4
	21	$D_{62}-27.3e-0.00865=0$	+5
	22	$D_{62}-27.2e-0.00859=0$	+11
	23	$D_{62}-26.8e-0.00871=0$	−1
	24	$D_{62}-26.3e-0.00858=0$	+12
	25	$D_{62}-25.8e-0.00865=0$	+6
	27	$D_{62}-25.4e-0.00860=0$	+11
	30	$D_{62}-26.0e-0.00868=0$	+3
Apr.	4	$D_{62}-25.0e-0.00864=0$	+7
	5	$D_{62}-24.3e-0.00874=0$	−3
	6	$D_{62}-24.1e-0.00876=0$	−4
	7	$D_{62}-23.6e-0.00876=0$	−4

From February 28 to March 10, 1876, inclusive, the yards were next the observer; from March 15 to April 7 the bar was next the observer.

Solving these equations of condition by least squares, we have: bar longer than the five yards, at 62°, or

$$D_{62^\circ} = +0^{in}.008843 \pm 0^{in}.00004$$

and expansion of 15-feet bar for 1° F. greater than that for five yards, by

$$0^{in}.0000052 \pm 0^{in}.0000017$$

In § 8 the sum of the expansions of the five brass yards, as derived from their comparisons with Clarke yards A and B, and each other, and from Colonel Clarke's values of the expansions of A and B, was given—

$$e_{6+7+8+9+10} = 0^{in}.0017807 \pm 0^{in}.0000053 - 0.0007328\,\Delta$$

Adding to this the value of excess of expansion of bar over five yards just given, we obtain a value for the expansion of the 15-feet bar for 1° F. derived from that of the Clarke yards; it is—

$$0^{in}.001786 \pm 0^{in}.0000056 - 0.0007328\,\varDelta$$

The value derived from the direct expansion-experiments given in § 11 is—

$$0^{in}.001795 \pm 0^{in}.0000016$$

When it is remembered that the first value is affected by the errors of comparisons of seven different yards, its agreement with the direct value may be considered satisfactory.

Adding to the sum of the lengths of the yards Nos. 6, 7, 8, 9, 10, given in § 8, namely, $6+7+8+9+10=179^{in}.999120 \pm 0^{in}.000103 - 0.02198\,\varDelta$, the value—

$$D_{62^\circ} = +\,0^{in}.008843 \pm 0^{in}.000040$$

there results for length of 15-feet brass bar at 62°—

$$\text{bar}_{62} = 180^{in}.00796 \pm 0^{in}.000111 - 0.02198\,\varDelta$$

It has previously been stated that 14 days' comparisons of 15-feet bar with the five yards in December, 1873, at a mean temperature of 42°.81 gave: bar — yards $= 0^{in}.00829$.

The 24 days' comparisons, beginning February 28, 1876, gave at 36°.2: bar — yards $= 0^{in}.00871$, giving, when reduced to the same temperature, a relative increase in length of bar of $0^{in}.00045$. Of this increase, $0^{in}.00023$ is due to the shortening of brass yard No. 8 between June, 1874, and August, 1875. See § 8. The rest, $0^{in}.00022$, may be accounted for by supposing the shortening of No. 8 to have partly occurred before June, 1874, by supposing all the yards to have shortened slightly, or by supposing that the brass 15-feet bar has lengthened. But as it amounts to but $\frac{1}{820000}$ part of the length of the bar, it may possibly be due to errors of comparisons.

In discussing the expansion of the 15-feet brass bar (§ 11), it will be seen that the comparisons of the brass bar with the iron bar packed in ice also indicate an increase in length in the brass bar after it had been heated from 32° to about 100°, which temperature it probably reached, of about the same amount.

If the modulus of elasticity of cast brass be taken as 6450 kilograms per square millimeter, and its breaking strength as 12 kilograms, the force needed to stretch it by $\frac{68}{100000}$ of its length would be about 4.4 kilograms per square millimeter, or more than one-third of its ultimate strength. But heating a brass bar from 32° Fahr. to 100° Fahr. stretches it by this amount ($\frac{68}{100000}$ of its length), and the question arises whether this heating may not produce a temporary or permanent change of length. No positive conclusions can be drawn, as the apparent change is so small that it may possibly be due to errors of comparison, its proportional amount little exceeding that of the errors in good comparisons of two yards *à trait*.

EXPANSION OF 15-FEET BRASS BAR FOR 1° FAHRENHEIT.

§ **11.** A determination of the expansion for 1° F. of the standard 15-feet brass bar was made in the winter of 1870–'71. Two methods were adopted. In the first the comparators being firmly fixed in the tops of stone piers, their stability was assumed. On consecutive days the bar was brought between the comparators, the room and bar on the first day being at the natural temperature, and on the next the room being heated by a stove, so that its temperature and that of the bar were from 25° to 45° F. higher. The distance between the middle points of the comparators being taken as unchanged, the difference of the comparator-readings for hot bar and cold bar, allowance being made for the expansion of half of each comparator, gave the change of length of the bar.

Temperatures of the bar were given by six thermometers.

The differences in lengths of the bar, divided by the corresponding differences of temperatures, gave the expansion for 1° F.

§ **12.** The second method was to clamp five yards together, end to end, thus forming a compound bar about 15 feet long, to surround this compound bar with broken ice, thus making its temperature and length constant, and then to compare the 15-feet bar with it, both at the natural temperature of the room and with that temperature raised about 30° F.

The value obtained for the expansion of the 15-feet brass bar by both methods was $0^{in}.00174$. (See Lake-Survey Report for 1871.)

The objection to the first method is that the perfect stability of the piers during changing temperatures is uncertain, and to both methods that the temperature of a brass bar like the standard cannot be determined from mercurial thermometers in contact with it with any such accuracy as 0°.1 F. or 0°.2 F. unless the temperature of the bar and its surroundings has been kept within a few tenths of a degree of a stationary temperature for several hours. At the high temperatures this could not be effected with means then at command. When the air-temperature rises through many degrees to a maximum, the temperature of the bar lags behind that shown by the thermometers, so that the indicated temperature-change is too great, giving too small an expansion. Subsequent work on this bar shows that at the high temperatures (70° to 80° F.), when the bar had its maximum length, its temperature was about 1° below that of the thermometers with it.

Later experience threw doubt on the accuracy of the value, and a few trials in July, 1874, of a method slightly differing from the first, made it nearly certain that the above value was somewhat too small. This new method was to place the 15-feet bar between the comparators, the temperature of the comparing-room being steady at about 64° F., to let it remain at least 16 hours, so that its thermometers might give its true temperature, then to read the comparators, and immediately to remove it to another room, when, for 4 hours, it was packed in ice. At the end of this time it was, while still packed in ice, placed between the comparators, which were again read. This process assumed the stability of the comparators for 4 hours of pretty steady temperature.

Results were obtained on five days for the expansion of the 15-feet bar for 1° F., which varied from $0^{in}.001781$ to $0^{in}.001809$, their mean being $0^{in}.0017959$.

§ **13.** It being then pretty certain that the value found for the expansion of this bar in 1871 was not sufficiently precise, I decided to attempt the careful redetermination of this important constant, and to use the method described by Captain A. R. Clarke, Royal Engineers, in Comparisons of Standards of Length, making such modifications as would adapt it to the inconvenient cellar in which we had to work.

The important point in Captain Clarke's method is, that he secures steady temperatures for each of two bars for several hours by placing them in closed boxes, whose sides are copper tanks, through which water of any desired constant temperature runs.

The method adopted, differing somewhat from Captain Clarke's, was to use a second bar constantly packed in ice, and to compare the 15-feet brass bar at temperatures of 32° and 90° F. with this second bar.

For the second bar, an iron bar, whose cross-section was $1^{in}.1$ in depth by $0^{in}.33$ in thickness, had milled cylindrical steel pins firmly screwed into its ends; the outer ends of these cylinders were planes, very nearly at right angles with the axis of the bar, and at such a distance apart as to give the two bars the same length at about 91° F.

This iron bar was mounted on ten adjustable rollers, in a semi-cylinder of quarter-inch boiler-iron, which was 15 feet long and 4 inches in diameter. This semi-cylinder was packed with pounded ice so that the bar should be entirely covered, one upper edge of the bar being tested after the ice was in to see that it did not deviate from a right line by more than one-hundredth of an inch. When the ice was well rounded over the bar, fifty pounds were required. It was replaced when the weight had diminished by about twelve pounds.

The semi-cylinder was supported at one-fourth and three-fourths of its length. To test its stiffness, five pounds were hung at one end, giving a deflection of but $0^{in}.007$. Accordingly it was assumed that the varying ice-load produced no injurious flexure of the bar due to change of form of the supporting semi-cylinder.

As the points of support for the iron bar were but 19 inches apart, and the greatest ice-load on top of the bar between two points of support could scarcely exceed half a pound, the length of the axis of the bar could not be sensibly changed by variations in this load.

For cold-comparisons, the brass bar was also packed in ice, while resting on its supporting T-shaped iron bar, which was stiff like the semi-cylinder already described.

For comparisons of brass bar hot with iron bar in ice, the brass bar was placed in a long, narrow, closed box, its ends only projecting very slightly from the ends of the box. The box had a cloth-lined wooden cover, and its sides were formed by two long metal tanks, each 5 inches in depth by 2^{in}.7 in thickness, the faces next the bar being of copper. In the interior of each tank was a 2-inch iron pipe running along the bottom of the tank, and then returning along the top; through this pipe the hot water flowed.

The brass bar was supported on adjustable rollers, on top of a stout T-shaped iron bar, nearly 15 feet long, lying between the tanks.

The two bars in their boxes, parallel to each other and about 3 feet apart, rested on a very stable, heavy truck, whose smooth lateral motion on iron rails brought, alternately, the centers of the end of one bar or the other between the comparators, mounted on their stone posts.

The bars were adjustable both horizontally and vertically, so that when once adjusted each could at once be brought into proper position with reference to the comparators, by motion of the truck alone, whose displacement was limited by adjustable stops, and a comparison of the two bars could be made within five minutes.

Hot and cold water were run into a large cask at such rates as to give a pretty steady temperature when well stirred; from this, with a slight head, the water flowed through flexible tubes into the iron pipes contained in the tanks.

In the first experiments, although the temperature of the water in the mixing-cask could be kept at a temperature varying from 99° to 101° F. by properly adjusting the flow into it of hot and cold water, yet the flow through the pipes in the tanks was not steady, the slit-opening of the stop-cock being clogged by the impurities in the water. The form of this opening was changed and the water passed through a cloth strainer. Afterward there was no difficulty in keeping the temperature of the hot-box at a temperature of about 90° F., without a greater variation for the middle thermometer than 0°.9 in eight hours.

The thermometer near the end where the water entered and left the tank was usually 0°.2 or 0°.3 greater than at the corresponding place at the other end.

At first some vapor from the wood of the hot-box condensed on the under side of the glass windows in the cover through which the thermometers beside the brass bar in the hot-box were read, making the reading uncertain. Chloride of lime was used to absorb this vapor until the boxes were dry, although it is objectionable on account of danger of rust.

The temperature of the cellar being about 35° F. below that of the hot bar, which was about 90° F., and the ends of the brass bar being exposed for comparison, the temperature of the ends of this hot bar was somewhat affected by the cold air outside.

The interior of the hot-box near its ends was stuffed with cotton, but a thermometer within 3 inches of the ends of the bar still stood at from 2° to 3° below those at the middle. While these thermometers doubtless gave a temperature lower than that of the brass they touched, it was not deemed safe to neglect so large a difference. A stove was brought into the cellar, and during hot-comparisons the temperature of the room was raised to about 85°. The temperature of the water as it left the tanks was lowered to 90° F., giving about 89° for the temperature of the bar. After this, the thermometer near the end of the bar gave a temperature but about 0°.5 lower than that near the middle.

The thermometers used were the five Casella standards Nos. 21472 to 21476, already described. They can be read to 0°.02 F.

The mean temperature of the 15-feet bar during hot-comparisons was determined as follows: The early experiments with thermometers in various positions had shown that thermometers distributed along the bar agreed within two or three tenths of a degree until they approached within 18 inches of the ends of the bar, when they began to fall, standing at the ends of the bar 2° or 3° lower. By heating the air in the cellar to 85° this fall was reduced to about 0°.5. After this, if the thermometer-readings on the bar are plotted as ordinates, the distances along the bar being abscissas, the curve will be nearly straight to within 12 or 15 inches of the ends of the bar. In the comparisons used the thermometers were placed on each side of the middle of the bar at 22½ and 72 inches from it, and a fifth at 3 inches from one end. Means of the readings of each thermometer were used for each comparison to plot the temperature-curve (whose general form was already known) for that comparison. The mean ordinate of this curve was taken as the temperature of the bar for this comparison.

The brass 15-feet bar and the iron bar were compared with each other, both being packed in ice, on ten days between April 21 and May 1, 1875, as shown in the following table:

Date.	Iron bar — brass bar, both in melting ice.
	in.
April 21, 1875	0. 038801
April 22, 1875	0. 038991
April 23, 1875	0. 038794
April 24, 1875	0. 038794
April 26, 1875	0. 038880
April 27, 1875	0. 038803
April 28, 1875	0. 038873
April 29, 1875	0. 038954
April 30, 1875	0. 038955
May 1, 1875	0. 038966
Mean	0. 038881 ± 0. 000017

The resulting excess of length of the iron bar is $0^{in}.038881 \pm 0^{in}.000017$ at 32°, the probable error being derived from the discrepancies between the daily results and their mean just given.

After the hot-comparisons had been made the two bars, both packed on ice, were again compared with each other on four days from June 21 to June 25, 1875, as shown in the following table:

Date.	Iron bar — brass bar, both in melting ice.
	in.
June 21, 1875	0. 038595
June 23, 1875	0. 038711
June 24, 1875	0. 038535
June 25, 1875	0. 038664
Mean	0. 038626 ± 0. 000026

The resulting excess of length of the iron bar was $0^{in}.038626 \pm 0^{in}.000026$, differing from the previous value by $0^{in}.000255$. This difference is larger than the probable errors would lead one to expect. But it is only $\frac{1}{720000}$ part of the length of the bar, a quantity which, in view of the many difficulties in such work, is too small to make it certain that the bar changed length during the hot-comparisons. As such a change is possible, however, equal weights have been given to the values obtained for excess of length of iron bar in ice over brass bar in ice, before and after hot-comparisons, and their mean $0^{in}.038754 \pm 0^{in}.000086$ has been adopted as the most probable value of their difference in length at 32° F.

The comparisons of the brass 15-feet standard bar with the five brass yards, Nos. 6 to 10, given in § 10, also indicate a lengthening of the brass 15-feet standard at about this time and by about the same amount, but, as already stated, the apparent discrepancy scarcely exceeds the possible errors in good work.

The comparisons between the iron bar packed in ice and the 15-feet standard brass bar at a temperature of about 90° were 19 in number on ten days, between May 26 and June 18, 1875, but only those on or after June 10 were used in obtaining the final value of the expansion. They were 12 in number, and on reducing them to a common temperature the residuals gave a probable error for their mean of $\pm 0^{in}.000003$. The resulting expansion of brass bar for 1° F. from each hot-comparison is given below under the heading e.

It has previously been stated that the early hot-comparisons were not considered satisfactory because, at first, the flow of hot water was not steady or the temperature of the air in the cellar was not near enough to that of the exposed ends of the brass bar. For these reasons no hot-comparisons prior to June 10, 1875, were used in adopting a final value for the expansion, but the values resulting from the previous comparisons are given in the following table to show how little effect those causes of error produced. The table gives the expansion resulting from each comparison of brass bar, hot, with iron bar in ice.

Date.	Temperature-range.	e.
1875.	°	*in.*
May 26	59.20	0.0018088
May 28	59.92	17964
May 28	59.24	17946
June 2	59.25	17948
June 2	59.33	17993
June 4	61.33	17997
June 8	65.22	17975
June 10	56.96	17939
June 10	56.96	17923
June 10	56.70	17947
June 14	57.68	17914
June 14	57.51	17930
June 14	57.73	17946
June 16	57.26	17962
June 16	57.41	17967
June 16	57.07	17960
June 18	56.59	17984
June 18	57.05	17996
June 18	57.10	17984

The results of the hot-comparisons on and subsequent to June 10, 12 in all, were combined with the results of cold-comparisons, and the mean value of the resulting expansions was taken as the most probable value. That value is, mean expansion of 15-feet standard brass bar between 32° and 89°.15 F. for 1° F.,

$$e = 0^{in}.001795 \pm 0^{in}.0000016.$$

This probable error is obtained by dividing the probable error in the determination of the change of length of the brass bar, namely, $\sqrt{(.000086)^2 + (.000030)^2}$, by 57°.15, and assumes that there is no constant error in the determination of this difference of temperature.

As the temperature of 32° was obtained directly from melting ice, there is no thermometer-error in it. But at the high temperatures, whose mean was 89°.15 F., there may be a small correction necessary to reduce the standard thermometer to an air-thermometer. Suppose it of the form $(t^\circ - 32^\circ)\Delta$. The corresponding correction to the expansion will be, closely enough,

$$de = 0^{in}.001795\,\Delta$$

so that we have for the mean expansion of the 15-feet brass bar between 32° and 89°.15 F.,

$$e = 0^{in}.001795\,(1 - \Delta) \pm 0^{in}.0000016$$

§ **14.** In May, 1876, an additional determination of the expansion of the 15-feet brass bar for 1° F. was made, using a temperature-range from 32° to 65° F.

The end surfaces of the steel pins in the ends of the iron bar having become slightly rusty, they were polished off, thus making the iron bar shorter than in the preceding expansion-work.

The following table gives the results of the comparisons between the iron 15-feet bar, always packed in ice, and the brass 15-feet bar, either packed in ice or heated to about 62°.

The same method and the same care was used as in the determinations previously made. The column $I—B$ gives the observed excess of length of iron over brass bar:

Date.	Iron-bar temperature.	Brass-bar temperature.	$I—B$.	Residuals, computed—observed.
1876.	° *F.*	° *F.*	*in.*	*in.*
May 6	32.00	32.00	+0.034373	−0.000067
May 9	32.00	65.18	−0.024697	−0.000256
May 10	32.00	32.00	+0.034298	+0.000008
May 11	32.00	61.60	−0.018595	+0.000036
May 13, 12 m	32.00	61.04	−0.017501	−0.000058
May 13, 3 p. m	32.00	61.06	−0.017499	−0.000096
May 15, 3 p. m	32.00	32.00	+0.034316	+0.000010
May 15, 4 p. m	32.00	32.00	+0.034282	−0.000024
May 16, 3.30 p. m	32.00	61.22	−0.017945	+0.000065
May 16, 4 p. m	32.00	61.00	−0.017561	+0.000073
May 16, 8.20 p. m	32.00	61.45	−0.018541	+0.000250
May 17	32.00	32.00	+0.034394	+0.000088
May 18	32.00	32.00	+0.034245	−0.000061

Each observation gives an equation of condition of weight 1. Solving these by least squares, we have for mean expansion of 15-feet brass bar from 32° to 62° F. for 1° F.

$$e = 0^{in}.001786 \pm 0^{in}.0000015 - 0.001786\,\Delta$$

This value is identical with that derived from the Clarke yards, through the brass yards, § 9, except that its probable error is much less.

The discrepancy, amounting to $\frac{1}{197}$ part of the whole value, between this and the value $0^{in}.001795$, obtained for the mean expansion between 32° and 89°.15, may be accounted for in three ways:

1st. It may be attributed solely to errors of observation, and the work would still be good, as the discrepancy is small.

2d. The mean expansion of the 15-feet bar may be greater between 32° and 89° than between 32° and 62°. Working on small specimens, Fizeau has found for the expansion of a unit's length of brass for t centigrade degrees—

$$(1034 \times 10^{-8})\, t + (76 \times 10^{-10})\, t^2$$

the expansion depending on both t and t^2.

3d. If thermometer 21472 should have a very small systematic position-correction at 62° to reduce it to an air-thermometer, and one of $+\,0°.3$ at 89°, the two expansions would agree.

The length of the 15-feet brass bar at 62° has already been given. As it was compared with the brass yards Nos. 6, 7, 8, 9, 10, at 62° on three days, as these yards were compared with each other and the Clarke yards at temperatures between 54° and 63°, and as the Clarke yards were compared with the Ordnance-Survey standard between 62° and 64°, any slight errors in the values of the relative or absolute expansions can have little effect on the resulting length of the 15-feet bar at 62°. But as the Keweenaw, Sandy Creek, and Buffalo bases depend on the length of this bar packed in melting ice, the question arises how its length at 32° should be derived from the data already given.

1st. Using the expansions of the Clarke yards given by Colonel Clarke, their lengths at or near 32° can be derived from their comparisons with the Ordnance-Survey standard at temperatures between 51° and 64° F. The Clarke yards, the brass yards, Nos. 6, 7, 8, 9, 10, and the 15-feet brass bar have been intercompared at temperatures between 33° and 44° F.

Hence the length of the 15-feet brass bar at 32° F. can be obtained without using any expansions, except those of the Clarke yards, over a range sufficient to introduce much error.

2d. The length of the 15-feet brass bar at 32° may be derived from the length adopted at 62° by using the mean expansion from 32° to 89°.15 F. already given, namely, $0^{in}.001795$.

3d. The length of the 15-feet brass bar at 32° F. may be derived from its adopted length at 62° by using the mean expansion from 32° to 62° F. for 1° already given, namely, $0^{in}.001786$.

In view of the probability that the expansion of the brass bar increases with the temperature, and of the fact that the expansion derived from the Clarke yards agrees with that obtained directly from expansion-experiments between 32° and 62°, it is deemed advisable to adopt, for the present, for the length of the 15-feet brass bar at 32° the value obtained by the first method, namely:

$$15 \text{ feet brass bar at } 32^\circ = 179^{in}.95438 \pm 0^{in}.000120 - 0.02198 \text{ ⅃}$$

The probable error is derived from the probable error in the length of the bar at 62°, and from the probable error in the value of the expansion from 32° to 62°, directly determined. The same value would result from the adopted length at 62°, namely,

$$180^{in}.00796 \pm 0^{in}.000111 - 0.02198 \text{ ⅃}$$

if the mean expansion between 32° and 62°, directly obtained, namely, $0^{in}.001786$, were used. If the mean expansion between 32° and 89°, directly obtained, had been used it would have made the length of the 15-feet brass bar at 32° $\frac{1}{660000}$ less.

The results obtained for the 15-feet brass bar up to May, 1877, may now be given:

Length of 15-feet brass bar at 62° F.	$= 180^{in}.00796 \pm 0^{in}.000111 - 0.02198$ ⅃
Length of 15-feet brass bar at 32° F.	$= 179^{in}.95438 \pm 0^{in}.000120 + 0.03160$ ⅃
Mean expansion between 32° and 89° for 1° F. =	$0^{in}.001795 \pm 0^{in}.0000016 - 0.001795$ ⅃
Mean expansion between 32° and 62° for 1° F. =	$0^{in}.001786 \pm 0^{in}.0000015 - 0.001786$ ⅃

NOTE A, MAY, 1880.

§ **15.** Since the preceding was written, Casella thermometer 21472 has been compared, under the supervision of Professor H. A. Rowland, with two Baudin thermometers, Nos. 6163 and 6165, belonging to the Johns Hopkins University, of Baltimore. These comparisons were made at the temperatures 45°, 62°, 75°, and 90° F., and comparisons at all these temperatures, as well as freezing-point determinations, were made on each of four days, the thermometers being vertical. Thermometers 6163 and 6165 have been carefully and repeatedly compared, when vertical, with an air-thermometer. A description of them and of the method of comparing them with an air-thermometer, as well as the final corrections needed to reduce their readings when vertical to a perfect-gas thermometer, may be found in Professor Rowland's paper on The Mechanical Equivalent of Heat in the Proceedings of the American Academy of Arts and Sciences for 1879.

The following tables give for each day's determinations the corrections derived through 6163 and 6165 needed to reduce the readings of 21472 to those of a perfect-gas thermometer, and the means of the results. They are derived from the inclosures in a letter to me from Professor Rowland, of March 8, 1880, and from Mr. W. W. Jacques, of January 30, 1880. In the reduction, as 6163 is the best thermometer, double weight has been attributed to the results derived from it in obtaining the mean corrections.

CASELLA THERMOMETER 21472 FAHRENHEIT. (VERTICAL.)

TABLE 1.—*Corrections to scale-readings to reduce to a perfect-gas thermometer, as derived from Baudin 6163, vertical.*

Date.	No. of experiment.	32°.	45°.	62°.	75°.	90°.
1879.		°	°	°	°	°
Nov. 4	1st	....	−0.11	−0.06	−0.05	−0.09
Nov. 6	2d	−0.07	−0.10	−0.10	−0.09	−0.10
Nov. 7	3d	−0.06	−0.10	−0.08	−0.09	−0.05
Nov. 10, 11	4th	−0.04	−0.04	−0.05	−0.05	−0.04
Means	......	−0.06	−0.09	−0.07	−0.07	−0.07

TABLE 2.—*Corrections to scale-readings to reduce to a perfect-gas thermometer, as derived from Baudin 6165, vertical.*

Date.	No. of experiment.	32°.	45°.	62°.	75°.	90°.
1879.			°	°	°	°
Nov. 4	1st	....	−0.12	−0.09	−0.05	−0.06
Nov. 6	2d	....	−0.08	−0.13	−0.11	−0.04
Nov. 7	3d	....	−0.12	−0.12	−0.08	−0.03
Nov. 10, 11	4th	....	−0.06	−0.06	−0.05	+0.04
Means		....	−0.00	−0.10	−0.07	−0.02

TABLE 3.—*Mean corrections to scale-readings to reduce to a perfect-gas thermometer, Baudin* 6163 *being given double weight. November* 4–11, 1879.

Temperatures.	Total corrections.	*b*-corrections.
°	°	°
32	−0.06	0.00
45	−0.08	−0.02
62	−0.08	−0.02
75	−0.07	−0.01
90	−0.06	0.00

If we call the freezing-point correction of a thermometer the *a*-correction, and the correction at any temperature, which remains when the freezing-point correction is subtracted from the total correction at that temperature, the *b*-correction, we shall have for the *b*-corrections of 21472, vertical, to reduce to a perfect-gas thermometer, the last column of the above Table 3.

For small changes in the freezing-point correction, the total correction at any time may be found by determining the freezing-point correction at the time and adding to it the *b*-correction. The values of the *a*-corrections and of the *b*-corrections will differ as the thermometer is horizontal or vertical. These corrections are for 21472 vertical, but as it has been habitually used in comparisons of standards in a position nearly or quite horizontal, it is important to know its corrections in that position to a perfect-gas thermometer.

Subsequently to the comparisons of 21472, vertical, it was compared in a horizontal position with 6163 in a vertical position, but at the higher temperatures results discrepant by half a degree were obtained. On examination, a small air-bubble was found in the bulb. It was removed and more satisfactory results were obtained, although discrepancies amounting to nearly 0.°1 F. still occurred. Professor Rowland says, "For the backward motion of the mercury its weight seems necessary. Hence, I never use a thermometer in the horizontal position." After the removal of the air-bubble, 21472, horizontal, was again compared, on March 16 and 18, 1880, with 6163, vertical, by Professor Rowland. These comparisons gave the absolute errors of 21472 when in a horizontal position at certain temperatures given in the following table:

TABLE 4.—*Absolute corrections to Casella* 21472, *horizontal, derived from Baudin* 6163, *vertical.*

Dates.	21472, vertical.	21472, horizontal.				
	32° F.	32° F.	52° F.	62° F.	75° F.	90° F.
	°	°	°	°	°	°
March 16, 1880	−0.09	−0.14	−0.19	−0.18	−0.20	−0.17
March 18, 1880	−0.10	−0.16	−0.19	−0.19	−0.18	−0.13
Means	−0.09	−0.15	−0.19	−0.18	−0.19	−0.15
Horizontal minus vertical reading	....	+0.06	....	+0.07	+0.09	+0.06

The mean horizontal-corrections are given in the third line. Between November 4–11, 1879, and March 16–18, 1880, the freezing-point correction of 21472, vertical, had increased by $-0^\circ.03$. Adding this quantity to each of the total vertical-corrections given in Table 3, the results are the absolute corrections on March 16–18, 1880, to 21472, vertical. The differences at each temperature between these quantities and the means in the above table are the changes in the absolute corrections when the thermometer is changed from a vertical to a horizontal position. Those changes or differences are given in the last line of Table 4.

There is another method of determining the change of reading of 21472, when it is changed from a vertical to a horizontal position, which is more direct and which avoids the errors of comparisons. This is to read the thermometer when vertical, and then, rapidly making it horizontal, to read it again before the slow temperature-change can have produced any appreciable effect. This method was used by Professor Rowland and also at the Lake-Survey office. Professor Rowland inserted the bulb of 21472 in a bottle, the stem projecting through the cork, and then read the thermometer many times while the bottle was rapidly changed from an upright to a horizontal or inverted position at temperatures of 72°, 90°, 96°, 97°, and 100° F. In reduction he assumed that the differences were proportional to the length of the column of mercury. At the Lake-Survey office the thermometer was immersed in water in a large tank with glass sides, the temperatures of the room being kept within 5° of the temperatures of the water. Keeping the bulb in the same place, the thermometer was read many times alternately in a vertical and a horizontal position, the intervals between readings being only one or two minutes. The results agreed well. Their means and those obtained by Professor Rowland are given in the following table:

TABLE 5.—*Horizontal minus vertical readings of Casella 21472 at different temperatures.*

	32° F.	45° F.	62° F.	75° F.	90° F.
	°	°	°	°	°
Rowland, March 16–18, 1880	0.03	0.04	0.07	0.09	0.11
Lake Survey, March 31–April 1, 1880	0.04	0.04	0.05	0.08	0.08

The results agree well with each other and do not differ very widely from those given in the last line of Table 4. As the second method is the more exact, the values obtained by Professor Rowland and given in Table 5 are adopted.

The corrections to 21472 in a horizontal position to reduce it to a perfect-gas thermometer can now be obtained for November 4–11, 1879, by combining the absolute vertical-corrections found by Professor Rowland and previously given in Table 3 with the corrections just adopted to reduce horizontal to vertical readings. In the following table the total corrections thus obtained are first given and then the part of each which is independent of the freezing-point correction.

TABLE 6.—*Corrections to reduce readings of Casella 21472, horizontal, to a perfect-gas thermometer, November 4–11, 1879.*

	32° F.	45° F.	62° F.	75° F.	90° F.
	°	°	°	°	°
Total	−0.09	−0.13	−0.15	−0.16	−0.17
Partial	0.00	−0.04	−0.06	−0.07	−0.08

§ **16.** From the numerous comparisons of Casella 21472, 21473, 21474, 21475, and 21476 with each other and with a calibrated Baudin thermometer, given in the Report of the Chief of Engineers for 1879, Appendix LL, it is known that the calibration errors of all are very small, and hence interpolation may be used for any intermediate temperature. The corrections to a perfect-gas thermometer for other thermometers can now be derived from comparisons with Casella 21472.

In April, 1880, the differences between the readings of Casella thermometers 21474, 21475, and 21476, and Baudin 6131, as they were horizontal or vertical, were determined at 32°, 56°, 74°, and

94°, and of Troughton & Simms 230 at 90°, by reading them several times at the Lake-Survey office while in a large tank of water, whose temperature was kept very nearly stationary and at the temperature of the surrounding air, first when vertical and immediately after when horizontal, the interval of time being only a minute or two, the bulb remaining in the same place. The assumption that the differences were proportional to the length of the column of mercury above the bulb satisfied these observations well, and with that assumption the differences in the following table have been computed, those for 21472 being derived alone from Professor Rowland's determinations, in which 21472 was alternately vertical with bulb up and vertical with bulb down.

TABLE 7.—*Differences between readings of certain thermometers as they are horizontal or vertical.*

[Horizontal reading always greater.]

Temp.	Casella.				T. & S. 230.	Temp.	Baudin 6131.
	21472.	21474.	21475.	21476.			
F.	F.	F.	F.	F.	F.	C.	C.
°	°	°	°	°	°	°	°
32	0. 03	0. 04	0. 05	0. 03	0. 08	0	0. 000
42	0. 04	0. 05	0. 06	0. 04	0. 09	5	0. 009
52	0. 06	0. 05	0. 06	0. 05	0. 11	10	0. 018
62	0. 07	0. 06	0. 07	0. 06	0. 12	15	0. 027
72	0. 09	0. 07	0. 07	0. 07	0. 14	20	0. 036
82	0. 10	0. 07	0. 08	0. 08	0. 15	25	0. 045
92	0. 11	0. 08	0. 08	0. 09	0. 16	30	0. 054
						35	0. 063

An attempt was made to determine the same differences for Geissler thermometers 1, 2, 3, and 4, which are used with the Repsold base-tubes. The resulting differences were no larger than their probable errors, and hence are neglected.

Between March 29 and April 1, 1880, Casella 21472, was compared, first with Casella 21474, 21475, 21476, Troughton & Simms 230, and Baudin 6131; and, second, with Geissler 1, 2, 3, and 4. The Baudin and Geissler thermometers have not heretofore been described. The bulb of standard Baudin 6131 was cracked January 3, 1879, and another bulb was made by James Green, of New York. It is graduated to fifths of centigrade degrees, from $-1^\circ.0$ C. to $+101^\circ.0$ C. One degree centigrade is $0^{in}.133$ in length. The length of its bulb is $1^{in}.5$ and diameter $0^{in}.3$. The greatest calibration correction is $0^\circ.05$ C. The Geissler thermometers 1, 2, 3, and 4 are nearly alike, and consist of a bulb and an almost capillary stem. A glass cylinder incloses this delicate stem and carries fastened to it a graduated porcelain scale against which the capillary tube rests. They are graduated to fifths of a degree Fahrenheit from $+30^\circ$ F. to $+120^\circ$ F. One degree F.$=0^{in}.065$. The bulb is $0^{in}.56$ in length and $0^{in}.17$ in diameter. Comparisons with other thermometers indicate that their calibration errors are not large.

In comparing all the above thermometers with 21472, they were placed vertically in a large tank of water at about one inch from its glass wall, and were habitually read with a horizontal telescope, whose micrometer was used for the Casella thermometers. Two groups of comparisons were made, only 21472 and the Geisslers entering the second group. The water was kept well stirred by a paddle-wheel and the temperature of the air near the tank was kept habitually within 3° or 4° F. of that of the water. In each set of comparisons 21472 was first read, then the other thermometers, and last 21472. The greatest difference in the readings of 21472 in a set was in one case $0^\circ.04$. The mean of its readings was used. Six sets of readings, each occupying about an hour, were taken at each temperature, except in the case of the Geissler thermometers at 75° and at 90°, for which four sets were deemed enough. The greatest ranges in the temperature of the water during comparisons at the different temperatures were, at 45° F., $0^\circ.02$; at 62° F., $0^\circ.30$; at 75° F., $0^\circ.05$; at 90° F., $0^\circ.08$. The thermometers were very near each other, the distance between the extreme ones not exceeding 6 inches. The following tables give the mean readings of these thermometers, the readings near 32° F. being freezing-point determinations:

TABLE 8.—*Mean readings in water.*

[Thermometers vertical.]

Date.	Casella.				T. & S. 230.	Baudin 6131.
	21472.	21474.	21475.	21476.		
	F.	F.	F.	F.	F.	C.
1880.	°	°	°	°	°	°
March 16, 18, 27, 31* ...	32.08	32.00	32.00	32.04	32.48	0.356
March 29	45.31	45.21	45.25	45.27	45.91	7.648
March 29	62.48	62.44	62.45	62.46	62.80	17.100
March 30	74.66	74.59	74.61	74.61	74.79	23.780
March 30	90.37	90.31	90.36	90.37	90.44	32.450

* Casella 21472 is the mean of Professor Rowland's readings in melting ice March 16 and 18, and those made at Detroit March 27, all vertical. The other Casellas were observed March 27; T. & S. 230, March 31; and Baudin 6131, March 27 and 31, all vertical.

TABLE 9.—*Mean readings in water.*

[Thermometers vertical.]

Date.	Casella.	Geissler.			
	21472.	1.	2.	3.	4.
	F.	F.	F.	F.	F.
1880.	°	°	°	°	°
March 20*	32.08	32.48	32.40	31.46	32.32
March 29	45.36	45.80	45.60	44.72	45.60
March 29	62.18	62.86	62.49	61.66	62.53
April 1	75.27	76.08	75.70	74.77	75.58
March 30	90.40	91.34	90.90	90.00	90.80

* Casella 21472 in melting ice same as in Table 8. The others in melting ice on March 20.

If in the preceding tables the freezing-point correction for each thermometer be subtracted from all the mean readings for that thermometer, a new table, having the same form, will be obtained. If now, in this new table, the changed reading for each thermometer at each temperature be subtracted from the corresponding changed reading of 21472, these differences will be the differences needed to correct the other thermometers so as to make them agree with 21472. If to each of these corrections be added the correction at the same temperature needed to reduce 21472 to a perfect-gas thermometer, and already given, there will result the *b*-corrections needed to reduce the readings of the other thermometers, vertical, to a perfect-gas thermometer.

The following tables give these corrections, repeating those for 21472.

TABLE 10.—*b-corrections to scale readings, vertical, to reduce to a perfect-gas thermometer.*

Temp. F.	Casella.				Geissler.			
	21472.	21474.	21475.	21476.	1.	2.	3.	4.
°	°	°	°	°	°	°	°	°
32	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
45	−0.02	0.00	−0.04	−0.02	−0.06	+0.06	0.00	−0.02
62	−0.02	−0.06	−0.07	−0.04	−0.30	−0.01	−0.12	−0.13
75	−0.01	−0.02	−0.04	0.00	−0.42	−0.12	−0.13	−0.08
90	0.00	−0.02	−0.07	−0.04	−0.54	−0.18	−0.23	−0.16

TABLE 11.—*b-corrections to scale-readings of Baudin* 6131, *vertical, to reduce to a perfect-gas thermometer.*

Thermometer readings.	*b*-corrections.
° C.	° C.
0.36	0.000
7.65	+0.047
17.10	+0.134
23.78	+0.226
32.45	+0.289

These are the *b*-corrections needed when the thermometers are vertical. The excesses of the readings of these thermometers, horizontal, over those when vertical have already been given in Table 7. Subtracting from each of those excesses for each thermometer the special excess at freezing-point for that thermometer, a series of remainders will be obtained which are the parts of the corrections due to the horizontal position, which are independent of the change of freezing-point produced by the horizontal position. Adding to these changed excesses the corresponding *b*-corrections for vertical thermometers, given in Table 10, there results the following table of *b*-corrections to reduce the readings of the thermometers, horizontal, to the reading of a perfect-gas thermometer. The corrections for 21473 have been derived from comparisons with 21472, given in the Lake-Survey Report, in the Report of the Chief of Engineers for 1879.

TABLE 12.—*b-corrections to scale-readings, horizontal, to reduce to a perfect-gas thermometer.*

Temp. F.	Casella.					Geissler.			
	21472	21473.	21474.	21475.	21476.	1.	2.	3.	4.
°	°	°	°	°	°	°	°	°	°
32	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
45	−0.04	−0.04	−0.01	−0.05	−0.03	−0.06	+0.06	0.00	−0.02
62	−0.06	−0.01	−0.08	−0.09	−0.07	−0.30	−0.01	−0.12	−0.13
75	−0.07	−0.05	−0.05	−0.06	−0.04	−0.42	−0.12	−0.13	−0.08
90	−0.08	−0.08	−0.06	−0.10	−0.11	−0.54	−0.18	−0.22	−0.16

TABLE 13.—*b-corrections to scale-readings of Baudin* 6131, *horizontal, to reduce to a perfect-gas thermometer.*

Thermometer-readings.	*b*-corrections.
° C.	° C.
0.36	0.000
7.66	+0.033
17.13	+0.103
23.82	+0.183
32.51	+0.231

§ **17.** The most important of these corrections to the Casella thermometers are those at 62° F. and at 90° F., horizontal, since these thermometers were all used near these temperatures in determining the expansion of the 15-feet bar given in § 14. In obtaining those temperatures the mean *b*-correction for all of them was taken from the Kew comparisons given in § 7. Those mean *b*-corrections, horizontal, were +0°.00 at 62° F. and −0°.04 at 90° F. The mean *b*-corrections, horizontal, now found necessary to reduce to a perfect-gas thermometer, may be derived from Table 12, and are $-0^{\circ}.06$ at 62° F. and $-0^{\circ}.09$ at 90° F. Hence at these temperatures the corrected temperatures used in § 14 need a further correction of $-0^{\circ}.06$ and $-0^{\circ}.05$, respectively, to reduce to a perfect-gas thermometer.

The thermometers A_1, A_2, A_3, A_4, belonging to the Clarke yard A, were compared with 21472, horizontal, in January and May, 1879, two sets of comparisons being made, whose greatest discrepancy in means was 0°.02. Colonel Clarke's comparisons of these thermometers with the Ordnance-Survey standard, given in § 7, show the relation of the mean of the B's to the mean of the A's. The means of the comparisons of the A's with 21472, first corrected for error of freezing-point and then corrected by the b-quantities needed to reduce 21472, horizontal, to a perfect-gas thermometer, given in Table 12, give the following as the b-corrections needed to reduce the mean of the A's with stems horizontal to a perfect-gas thermometer. In a third column are given the b-corrections derived from Colonel Clarke's work for these thermometers by subtracting the freezing-point correction from the correction at each temperature. In a fourth and fifth column the same quantities are given for the mean of the B-thermometers.

TABLE 14.—*b-corrections to scale-readings of "yard" thermometers, horizontal, to reduce to a perfect-gas thermometer.*

Temp.	Mean of A_1, A_2, A_3, A_4.		Mean of B_1, B_2, B_3, B_4.	
	b-corrections.	Col. Clarke's results for same.	b-corrections.	Col. Clarke's results for same.
F.	F.	F.	F.	F.
°	°	°	°	°
32	0.00	0.00	0.00	0.00
37	0.00	....	....	....
42	+0.03	....	....	....
47	+0.04	....	....	....
52	+0.05	+0.07	+0.11	+0.13
55	+0.07	+0.11	+0.11	+0.15
57	+0.08	+0.12	+0.09	+0.13
62	+0.09	+0.07	+0.11	+0.09
67	+0.10	....	....	....
72	+0.09	....	....	....
77	+0.09	....	....	....
82	+0.10	....	....	....
87	+0.10	....	....	....
90	+0.11	+0.12	+0.12	+0.13

On comparing the b-corrections of these thermometers, required to reduce their readings to a perfect-gas thermometer, with those derived from Colonel Clarke's corrections given in § 7, it is seen that the greatest differences are 0°.04 at 55° F. and 57° F. As the length of 1° on the A's and B's is but $0^{in}.055$, this quantity scarcely exceeds the probable errors of the comparisons, while at the important temperatures, 62° F. and 90° F., the differences are less than their probable errors. The important conclusion then follows, that when the A's and B's have their freezing-point corrections applied, and the Ordnance-Survey Standard 3241 has Clarke's corrections applied, they then agree quite well with a perfect-gas thermometer.

§ **18.** Having given the b-corrections for the various thermometers both when vertical and when horizontal, it remains to give the freezing-point or a-corrections, horizontal, as determined at different dates. The values of the a-corrections for intermediate dates may be determined by interpolation. The a-corrections, vertical, at 32° when not directly observed, may be obtained from the observed a-corrections, horizontal, by adding with a negative sign the quantities opposite 32° in Table 7, remembering that the Geissler thermometers have sensibly the same readings whether horizontal or vertical. Such computed a-corrections are marked with an asterisk.

The following tables give the *a*-corrections for different dates:

TABLE 15.—*a-corrections to scale-readings of thermometers, horizontal.*

Date.	Place.	Casella. 21472.	21473.	21474.	21475.	21476.
		F.	F.	F.	F.	F.
		°	°	°	°	°
January, 1875	Kew Observatory, England	0.0	+0.1	+0.1	+0.1	+0.1
March, 1875	Detroit	+0.06	+0.13	+0.14	+0.15	+0.13
December, 1875	do	−0.01	+0.08	+0.06	+0.07	+0.08
September, 1876	Paris, Ste. Claire Deville	0.00	....	....	....	....
December, 1876	Detroit	−0.04	....	....	....	....
May, 1879	do	−0.11	−0.05	−0.05	−0.05	−0.08
November, 1879	Baltimore, Rowland	−0.09*	....	....	....	....
March 16, 18, 1880	do	−0.12*	....	....	....	....
March 27, 1880	Detroit	−0.10*	....	−0.04*	−0.05*	−0.07*

Date.	Place.	Geissler. 1.	2.	3.	4.
		F.	F.	F.	F.
		°	°	°	°
January, 1877	Detroit	−0.22	−0.13	+0.66	−0.18
January, 1879	do	−0.55†	−0.40	+0.50	−0.36
January, 1880	do	−0.50	−0.41	+0.50	−0.28
March, 1880	do	−0.50	−0.40	+0.51	−0.36
April 9, 1880	do	....	−0.08‡	....	....

† Scale loose August, 1878.
‡ Scale of No. 2 taken out to remove moisture. Refastened in a different position April 8, 1880.

Date.	Place.	"Yard" thermometers. Mean of A_1, A_2, A_3, A_4.	Mean of B_1, B_2, B_3, B_4.
		F.	F.
		°	°
April, May, 1874	England, Clarke	−0.29	−0.26
March, 1875	Detroit	−0.37	−0.34
December, 1875	do	−0.40	−0.38
May, 1879	do	−0.44	−0.42
March, 1880	do	−0.51	....

Date.	Place.	Baudin 6131.
		C.
		°
May 15, 1879	Detroit	−0.252
February 13, 14, 1880	do	−0.386
March 20, 1880	do	−0.392
March 27, 1880	do	−0.356*
March 31, 1880	do	−0.356*

There is no difference in the horizontal and vertical readings of 6131 at freezing-point.

The details of the work on which this note is based may be found in the Lake-Survey reports embraced in the Reports of the Chief of Engineers for the years 1879 and 1880.

§ **19.** The determinations of the expansions of the 15-feet brass bar, given in § 11, depend on the Kew corrections to the Casella thermometers, given in § 7. As the corrections needed to reduce the readings of these thermometers when horizontal, as in the expansion work, to the readings of a perfect gas thermometer differ from the Kew corrections, a modification of those values of the expansions of the 15-feet bar for 1° F. results. Two sets of determinations of expansions were made, the first from temperatures at 32° F. and near 62° F.; the second at 32° F. and near 89° F. In the first set the higher mean temperature after the Kew corrections were applied was 61°.79 F., giving a temperature-range of 29°.79. In the second the higher mean temperature after the Kew corrections were applied was 89°.15, giving a range of 57°.15 F. As previously stated, the mean of the five Casella thermometers at 62° after the Kew corrections have been applied needs a further correction to reduce it to the reading of a perfect-gas thermometer of $-0^\circ.06$, while at 90° it needs a correction of $-0^\circ.05$. The mean expansion between 32° and 62°, given in § 14, namely, $0^{in}.001786$, needs then a correction of $+\frac{0.06}{29.73}=+\frac{1}{496}$ part, which makes it $0^{in}.001790$. The mean expansion between 32° and 89° F., given as $0^{in}.001795$ in § 14, needs a correction of $+\frac{0.05}{57.10}=+\frac{1}{1142}$ part, which makes it $0^{in}.001797$. It has been stated in § 10 that the expansion of the 15-feet brass bar, derived from the comparisons by which its length was deduced from the Clarke yards *A* and *B* and from the expansions obtained for those yards by Colonel Clarke, was $0^{in}.001786$ for 1° F. From the direct expansion determinations between 32° F. and 62° F., after the temperatures have been corrected to agree with those of a perfect gas thermometer, the resulting value of the mean expansion is, as has just been seen, $0^{in}.001790$. It is very difficult to assign relative weights to these two values, which indeed differ but slightly; hence their mean will be taken, giving $0^{in}.001788$ for mean expansion of 15-feet brass bar between 32° F. and 62° F. for 1° F.

This change in the adopted value of the mean expansion changes the difference of lengths of the brass bar at 32° and 62° F., given in § 14, increasing it by $0^{in}.00006$, and the lengths to be adopted for the 15-feet bar at 32° and 62° must conform to this new difference. The length at 62° is least affected by expansion errors, since inter-comparisons of Ordnance-Survey standard yard Y_{55}, the Clarke yards, the brass yards, and the 15-feet bar have been made in the vicinity of 62°, so that if those comparisons alone were used, little error would arise from the errors in the different relative expansions. Besides, when the length and expansion of one bar are derived from those of another by comparisons, and subsequent direct work gives another expansion, the resulting change in the expansion does not change the difference of length of the bars at the mean temperature of comparisons. In determining the length of the 15-feet bar from yard *A* there were many pairs of bars to be compared. The mean of the mean temperatures did not differ widely from 50°. If the mean temperature of all these comparisons had been 50°, about one-third of the change would belong to the length at 62° and two-thirds to that at 32°. For these reasons, one-third of the change $0^{in}.00006$, or $+0^{in}.00002$, will be attributed to the length of the 15-feet bar at 62° given in § 14, and two-thirds, or $-0^{in}.00004$, to its length at 32°. There result then, finally, retaining the probable errors unchanged—

15-FEET BRASS BAR AT 62° F. =	$180^{in}.00798$	$\pm 0^{in}.00011$
15-FEET BRASS BAR AT 32° F. =	$179^{in}.95434$	$\pm 0^{in}.00012$
MEAN EXPANSION BETWEEN 32° AND 62° F. FOR 1° =	$0^{in}.001788$	$\pm 0^{in}.0000015$
MEAN EXPANSION BETWEEN 32° AND 89° F. FOR 1° =	$0^{in}.001797$	$\pm 0^{in}.0000016$

The Minnesota Point, Keweenaw, and Fond du Lac Bases and the connecting triangulation have already been computed with the lengths of the 15-feet brass bar at 32° and 62° F., given in § 14. As the changes in the length of the 15-feet bar at 32° F. and 62° F. amount to only $\frac{1}{4500000}$ and $\frac{1}{9000000}$ part, respectively, quantities that are very small in comparison with the probable errors in these bases arising from other sources, a recomputation of the work is unnecessary. For the Buffalo and Sandy Creek Bases the new values will be used.

Fizeau (Comptes Rendus LXVIII, p. 1125) gives for length of a brass bar at t° C.,

$$l_t=l_o\,(1+1781\times10^{-8}t+98\times10^{-10}t^2)$$

where l_0 is its length at 0° C. This expression gives for mean expansion of 15-feet brass bar for 1° F., between 32° and 62° F., $0^{in}.001797$, and between 32° and 89° F., $0^{in}.001812$. The change in the mean expansion of brass as the temperature increases, found from our comparisons, is then somewhat less than that found by Fizeau. If the rate of expansion of the bar varies with the temperature, the length of the bar would be best expressed in the form

$$(1) \qquad l_t = l_0(1 + \alpha t + \beta t^2)$$

and this should have been the form given to the equations of condition derived from the comparisons before solving them by the method of least squares. But as the two sets of expansion determinations have already been solved by least squares, and as the dependence of the rate of expansion on the temperature, if real, is very slight, the re-solution of the equations of condition would not give results of sufficiently increased value to justify the work. Since the value of l_0 in (1) and the length of the bar at 62° F. are known, while its length at 89° F. can be computed from the mean expansion from 32° F. to 89° F., if these values be substituted with their probable errors in (1), $(t-32)$ being substituted for t, two equations will result, from which the values of α and β, with their probable errors, can be obtained. The resulting values are

$$\alpha = (9878 \pm 20.2)\,10^{-9}$$
$$\beta = (1852 \pm 451)\,10^{-12}$$

and (1) may be now written, 15-feet brass bar at $t°$ F.

$$= 179^{in}.95434\,\{1 + [(9878 \pm 20.2)\,(t-32) + (1.852 \pm 0.45)\,(t-32)^2]\,10^{-9}\}$$
$$= 179^{in}.95434 + (0^{in}.0017776 \pm 0^{in}.0000036)\,(t-32) + (0^{in}.0000006666 \pm 0^{in}.0000000812)\,(t-32)^2$$

NOTE B.—MAY, 1880.

§ **20.** In § 9 it is stated that no direct comparisons had been made at the time it was written between the Clarke yards *A* and *B*, either by Colonel Clarke or at the Lake-Survey office; but between May 24 and June 7, 1879, thirty sets of comparisons were made in the Lake-Survey office on thirteen different days. The comparisons were made as usual with the Würdemann and Stackpole contact-level comparators. From one to three comparisons were made at each visit to the comparing-room, and the mean of the determinations of the set or visit was taken as a single result. Of the thirty such results obtained, none were used in which the mean temperatures of the two yards as indicated by the four thermometers which accompany each yard differed by more than 0°.1 F. This rejected two results. Of the thirty results, there were eight in which the difference of temperature exceeded 0°.08 F., but the mean of the eight results was identical with the mean of the other twenty-two, or of the twenty-eight which were used. The mean temperature of *A* minus the mean temperature of *B* for the twenty-eight results used was + 0°.03. As the thermometers can only be read to 0°.1, this quantity does not establish any difference of temperature, and in reduction it has been assumed that the yards were of the same temperature. The adjustment of the yards and comparators was changed four times, a nearly equal number of results being obtained after each adjustment. The following table gives in the successive columns the dates of comparisons, the corrected mean temperatures of yard *A*, the corrected mean temperatures of yard *B*, the differences, *A*—*B*, in inches of the Würdemann comparator, and the residuals. Würdemann inches are reduced to English inches by multiplying by 1.004.

Comparisons of Clarke yards A and B.

Date.	Corrected mean temperature.		$A-B$, Würdemann inches.	Residuals, Würdemann inches.
	Yard A.	Yard B.		
1879.	° F.	° F.	*in.*	*in.*
May 24	50.82	50.81	+0.00019	−0.00001
May 26	51.15	51.08	0.00016	+0.00002
June 3	53.11	53.04	0.00020	−0.00002
June 4	52.89	52.99	0.00011	+0.00007
June 5	53.61	53.64	0.00013	+0.00005
June 6	53.79	53.69	0.00018	0.00000
June 6	54.04	53.96	0.00020	−0.00002
June 7	53.26	53.26	0.00018	0.00000
June 9	52.00	51.95	0.00022	−0.00004
June 9	53.53	53.56	0.00020	−0.00002
June 10	54.14	54.09	0.00016	+0.00002
June 10	54.62	54.56	0.00018	0.00000
June 11	55.23	55.22	0.00017	+0.00001
June 11	55.75	55.85	0.00019	−0.00001
June 12	56.34	56.30	0.00017	+0.00001
June 12	56.64	56.64	0.00019	−0.00001
June 16	57.36	57.36	0.00018	0.00000
June 16	57.64	57.64	0.00016	+0.00002
June 17	56.14	56.04	0.00019	−0.00001
June 18	55.03	55.05	0.00017	+0.00001
June 18	55.11	55.05	0.00020	−0.00002
July 2	60.93	60.84	0.00018	0.00000
July 2	61.23	61.24	0.00018	0.00000
July 3	61.81	61.79	0.00017	+0.00001
July 3	62.13	62.03	0.00022	−0.00004
July 5	62.15	62.11	0.00022	−0.00004
July 6	62.43	62.39	0.00020	−0.00002
July 7	62.63	62.61	+0.00022	−0.00004
Means	56.27	56.24	+0.000183	....

At 56°.25 F.,

$$A - B = +\ 0.000183 \pm 0.000003 \text{ Würdemann inches}$$
$$= +\ 0.000184 \pm 0.000003 \text{ English inches.}$$

The relative expansion of A and B is very small and has been neglected in computing the probable error. If it had been considered, the probable error would have been slightly less. The mean temperature of all the comparisons used is 56°.25, and at this temperature

$$\text{Yard } A - \text{yard } B = 0^{\text{in}}.000184 \pm 0^{\text{in}}.000003$$

Using the lengths and expansions determined by Colonel Clarke and given in § 2, there results

$$\text{Yard } A - \text{Yard } B = 0^{\text{in}}.000197 \text{ at } 56^{\circ}.25 \text{ F.}$$

The agreement is very satisfactory, so close indeed as to make any change in Colonel Clarke's value in consequence of the new comparisons unnecessary.

CHAPTER III.

KEWEENAW BASE.

LOCATION—MARKINGS—MEASUREMENT.

§ **1.** Keweenaw Base is approximately parallel to the west shore of Keweenaw Bay, Lake Superior, its north end being about 100 feet from the shore and its south end about 4,000 feet from it. The length of the base is 28,992 feet approximately, or $5\frac{1}{2}$ miles. Its height above the bay varies from 25 to 80 feet, being 68 feet at North Base and 70 feet at South Base, the slope of the ground being gentle, so that the inclination of the base-tubes reached 4° in but a single case. The latitude and longitude of its north end are respectively $46^\circ\ 57'$ and $88^\circ\ 27'$. Nearly the whole length of the base is through a forest, and the surface soil is decomposed sandstone mixed with vegetable matter.

§ **2.** The following markings of the base have been made:

At South Base the end-point of the base is a cross on a piece of brass leaded into the top of a stone post 6 feet long, the top of the post being level with the ground. On the prolongation of the base southward, precisely one yard from the end of the base, is a cross on a piece of brass, set in the top of a stone post $2\frac{1}{2}$ feet long, the top of this post being 6 feet under ground. At right angles to the end of the base, and about 100 feet east of it, a 6-foot stone post rises 1 foot above the ground. There is a similar post about 100 feet west of the end of the base, its top being broken off.

At the end of the ninety-fourth tube (1,410 feet) from south end of base, and on the base line, there is a cross on brass in the top of a stone post $2\frac{1}{2}$ feet long, the top of the post being 18 inches under ground.

At the middle of the base (end of nine hundred and fifty-second tube) there is a cross on brass in the top of a stone post, the top of the post being 2 feet under ground.

The north end of the base is marked by the intersection of two lines on the end of a copper bolt 10 inches long leaded into the solid sandstone rock, which is about 4 feet below the surface of the ground. Further details are given in the Description of Stations, Chapter XIV, A.

§ **3.** This base was originally measured in 1867, but as General W. F. Raynolds, at that time in charge of the Lake Survey, had little confidence in the precision of the work, on account of the muddy condition of the ground during the measurement, giving instability to the measuring tubes, it was decided to remeasure it. This was done in the summer and fall of 1873 by Assistant Engineer E. S. Wheeler, aided by Assistant Engineers C. F. Burton and C. Pratt.

Although the ground was tolerably firm during the second measurement, greater stability for the trestles was obtained by making them rest on stout wooden stakes driven to the surface of the ground. Prior to the second measurement, the base-line had been converted into a rarely used road, and in places grass had sprung up. As stakes were used the partial soil was not removed.

The base-apparatus used is a compensating one by Würdemann, a copy of that of the Coast Survey, and is described in the Lake-Survey Report for 1868, the original being described in the Coast-Survey Report for 1854. The measuring part consists of tubes 1 and 2, each about 15 feet long.

In previous bases of the Lake Survey, the measurement had been made without tents over the tubes. In this, Assistant Engineer Wheeler was directed to make the measurement under movable tents, so as to avoid, at least in part, the large daily fluctuations in the length of the base-tubes arising from unequal temperatures of the compensating bars.

§ **4.** Comparisons of the tubes with the standard bar were made from July 16 to August 12, 1873; measurements from August 13 to September 7, 1873; comparisons from September 8 to September 11, 1873; measurements from September 12 to September 25, 1873; comparisons from September 26 to October 5, 1873; remeasurements from October 6 to October 8, 1873; comparisons from October 9 to October 15, 1873.

The first distance of ninety-four tubes was measured four times at the beginning of the measurement, and three times after the base had been finished, to ascertain the error in measurement.

The greatest number of tubes measured in one day was one hundred and thirty on September 5, in $8^h\ 43^m$, giving a speed of 224 feet per hour. Measuring was done on thirty-two days, averaging seventy-eight tubes per day.

DESCRIPTION OF APPARATUS—COMPARISONS WITH STANDARD BAR.

§ **5.** A measuring tube of the Bache-Würdemann base-apparatus consists essentially of two parallel bars of brass and iron, each 168 inches long, rigidly connected at the rear ends by an iron cross-bar, the front ends being free to move longitudinally with reference to each other. The brass bar, $1^{in}.10$ in height and $0^{in}.37$ thick, is directly below the iron bar and $0^{in}.3$ distant from it. The iron bar is $1^{in}.40$ in height and $0^{in}.27$ in thickness. Both are inclosed in a tin case. To the front end of the brass bar a vertical compensating lever is hinged, having on its rear side a knife-edge which is pressed by a spring against a steel plate on the front end of the iron bar, while, higher up, another knife-edge on the front face of the lever has constantly pressed against it a sliding rod, whose forward end, armed with an agate plane, projects from the front end of the tin case and forms one measuring end of the tube. The object of this lever is to make the distance between the end-points of the tubes during changing temperatures constant so long as the two bars have the same temperature. At the rear end of the tube a vertical arc for measuring inclinations is fixed to the lower bar, and is read by a vernier with a level on its arm, called the sector-level, the horizontal axis of the vernier being perpendicular to the plane of the bars. The vernier arm carries a small cylindrical surface, the axis of which coincides when in adjustment with that of the vernier at the instant the bubble of a small contact-level moved by the arm bearing the cylindrical surface plays. A rod with parallel motion passing near the axis of the cylindrical surface abuts against this surface, while its rear end projects out of the rear end of the tube and is covered with an agate knife-edge, which forms the rear measuring end of the tube.

§ **6.** In measurement, the base-line was divided into sections 500 feet long, by stakes carefully placed by a theodolite in the vertical plane passing through the ends of the base-line. Two tubes were used. The end-points of each tube are small agates carried by sliding rods. The forward agate was brought into the vertical plane of the base-line by means of the tri-rectangular motion with which the trestles supporting the tubes are provided, the plane being given by a theodolite set up over one of the section-stakes, and directed toward the next, the tube being intermediate. The rear agate of this tube was then brought, by means of a slow-motion screw, into contact with the front agate of the rear tube till the bubble of the contact-level could be read. Then the contact-level, the sector-level, the sector-arc, and the thermometers inside of the tube were read. The rear tube, with its tent, was then carried forward in front of the other tube, placed in line, and the work continued. When, as at night, it became necessary to stop work, a theodolite was set up about 10 feet from the front end of the forward tube and approximately in a vertical plane through the end of its agate, at right angles to the base-line. By means of this theodolite, and the one used for alignment, a mark was made directly under the agate on the head of a copper tack in a stout peg, the top of which was driven below the surface of the ground. The stability of this mark during the cessation of work was checked by other similar marks placed in a line perpendicular to the base-line. The record was kept in duplicate by two recorders, and all readings except those of the contact-level were in duplicate.

§ **7.** In an apparatus like the Bache-Würdemann, composed of many different parts, and liable to change length at each of the places of contact, frequent comparisons with a standard are necessary. The standard used was the 15-feet brass bar of the Lake Survey. To determine what the lengths of the tubes were during the measurement, the comparisons were made under circumstances as nearly the same as those of measurement as was possible. The temperature of the standard 15-

feet bar was fixed by being packed in broken ice, which was constantly renewed. The comparisons were under tents, and were continued from 8 a. m. to 6 p. m., so as to include the same diurnal fluctuations of temperature as took place during measurement.

The method of comparison was as follows: Two contact-level comparators, reading to one hundred-thousandth of an inch, one made by Würdemann and one by Stackpole, were mounted on heavy stone piers a little more than 15 feet apart. The masonry supports of these piers rested on rock, and the piers were boxed nearly to their tops to protect them from unequal changes of temperature. A tube resting on its trestles was placed between the comparators, the standard bar was placed on the tube and carefully aligned, and then, by motions of the trestles, one or the other was brought between the comparators, the Stackpole being kept at a fixed reading and the Würdemann comparator being read. The stone piers proved to be very steady. In reducing, the thermometers were corrected for their errors and then their curves plotted. The readings on the standard bar and tubes were plotted as ordinates with times for abscissas.

On inspecting the curves of comparisons of tubes with bar in ice when the temperature was above 65° F., slight depressions are seen in the tube-curve, arising from the cooling effect of the bar during the five or ten minutes it was on the tube. In extreme cases, these depressions amount to $0^{in}.00040$. As the bar in ice was placed on the tube but once in two hours, the effect is eliminated by neglecting these depressions in the curves. The differences between the tube- and bar-curves give the corresponding differences of length, and hence the length of the tube from the computed length of the bar at the observed temperature. The values of the comparator-screws have been carefully determined.

EFFECT OF TEMPERATURE-CHANGE ON APPARATUS.

§ **8.** In a compensated base-apparatus, like the Bache-Würdemann belonging to the Lake-Survey, the length of a tube may change from a change in temperature in two ways:

1. Its length may change from the unequal expansion of the brass and iron bars when their changing temperatures are constantly equal to each other, this change of length arising from incorrect proportions in the compensating lever. This change in length from imperfect compensation is called expansion. It can be reduced to almost zero, and has been made very small in the Lake-Survey apparatus.

2. In changing temperatures the bars may not gain or lose heat alike, and the length of the tube may change from the difference in their temperatures. This is the serious difficulty in any apparatus composed of two bars, whether in the form of a compensating apparatus, like the Colby and the Bache-Würdemann, or in the form of a metallic thermometer like the apparatus of Borda, of Bessel, and of Repsold. The problem of keeping the two bars at the same temperature is not well solved in the compensating apparatus of the Lake Survey, and any such apparatus has the great disadvantage of giving exaggerated changes of length for any difference in temperature between the two bars. Thus a compensated arrangement of brass and iron bars, whose expansions for 1° are b and i, will change length for a difference of 1° in temperature of its two bars, by $\frac{b}{b-i}\,i$, or by about three times the change of length of the iron bar for 1° of temperature.

§ **9.** Some results from comparisons will be of interest as showing the amount of such changes in the Lake-Survey base-apparatus.

On August 5, 1873, during comparisons of tube 1 in full sunshine, the temperature of the interior of the tube rose from 60° F. at 6 a. m. to 98° F. at 5 p. m. The tube reached its maximum length at 9 a. m. and then decreased so as to be $0^{in}.00470$ shorter at 10 p.m. The day was one of excessive range in temperature. Such change of length approaches zero when the rate of temperature-change during the day becomes very small. At the Buffalo base, in 1875, the mean length of this tube for the eleven days of comparisons was, at 1 p. m., $0^{in}.00230$ greater than at 8 a. m., the comparisons being under tents. The variation for tube 2 was less. But as a part of this change might be due to expansion of tube from imperfect compensation, while the two bars have equal but varying temperatures, that question has been re-examined. Two attempts had been made in the office at the determinations of the expansions of the tubes, and the comparisons at the base-lines also gave data;

but the quantity to be measured is so small, and is so masked by larger errors, that its determination is difficult. A combination of the various values gave for both tubes an expansion less than $0^{in}.00001$ for 1° F., a quantity less than the probable error of determination. Hence, in the computation of the base, the expansions for both tubes have been taken as zero. As the mean temperatures of the tubes during comparisons and during measurement of the Keweenaw Base did not differ by more than 1° F., the uncertainty in the precise values of the expansion does not affect their lengths by more than $0^{in}.00001$, a quantity which may be neglected.

§ **10.** The fact that the tubes change length during the day being known, and it also being known that their proper expansion (both bars having the same temperatures) is very small, it follows that the change in length given above must be due to difference of temperature of the two bars. When tube 1, on August 5, 1873, changed length during the day by $0^{in}.00470$, taking one-third of this quantity as giving the change of the length of the iron bar relatively to the brass bar, we find that the temperatures of the iron bar relatively to the brass bar must have changed by about 1°.3 F. during the day, and that on an average comparison-day at the Buffalo Base the relative temperature of the iron and brass bars must have varied by 0°.7 F.

§ **11.** The variations in length of tubes 1 and 2 during comparisons under tents are well shown graphically by the curves in Plate XXVII, in which the abscissas give the hours of the day and the ordinates the corresponding excess of the length of the tube above that of the 15-feet brass bar in melting ice. The curves for tubes 1 and 2 are each the means of daily comparison-curves for twenty-three days of comparisons at the Sandy Creek and Buffalo Bases. From these curves it will be seen, that, for an average day during comparisons at those bases, tube 1 changed length between 8 a. m. and 5 p. m. by $0^{in}.00216$ and tube 2 by $0^{in}.00154$, these changes being due almost entirely to difference of temperatures of the brass and iron bars in each tube.

In a preliminary computation of the Keweenaw Base in 1874, the lengths adopted for the tubes during measurement were derived from comparisons of the tubes with the bar in melting ice, the tubes being under a tent as in measurements, and the comparisons being repeated at short intervals during the working-day. The differences of lengths of bar and tube were plotted for each day as ordinates, the hours of the day being the abscissas, so that for each day of comparison a curve resembling those already given was obtained. The mean ordinate for the day was computed, giving the tube's mean length for that day. The mean of such mean lengths for the periods of comparisons gave the length of the tube in terms of the bar in ice, which was used in computation.

If the mean daily temperature-curve during comparisons had precisely the same form and dimensions as during measurement, thus giving the same mean curve for the daily changes of length of tube during comparisons and measurement, no error would be introduced into the resulting length of the whole base, although these mean values would not give the same length in remeasurement for a part of the base, for which the temperature-curves on the two days of measurement differed considerably. In fact, the mean temperature-range between 8 a. m. and 12 m. on the days of comparison was 6°.1 F., while on the days of measurement it was 13°.2 F., a difference too great not to need a correction if any method can be found for making it.

§ **12.** The numerous comparisons of the base-tubes with the 15-feet brass bar in ice at Sandy Creek and Buffalo Bases added largely to our knowledge of the conduct of the tubes during rapidly changing temperatures, and showed what had previously been noticed, that the mean daily length of a tube depended on the temperature-range during the morning. After some trials it was found that if the mean daily differences of length of tube and bar in ice were plotted as ordinates, the temperature-changes from 8 a. m. to 12 m. being the abscissas, the resulting curve was nearly a straight line for the Keweenaw, Sandy Creek, and Buffalo Bases, and that for each base the inclination of the line was nearly the same. That is, if we represent by l the difference of lengths of 15-feet bar in ice and tube, when its iron and brass bars have the same temperature; by x the change in the mean difference between 8 a. m. and 5 p. m., due to a change of temperature of 1° F. between 8 a. m. and 12 m.; by a the change of temperature from 8 a. m. to 12 m. on any day when the mean difference of length between 8 a. m. and 5 p. m. of bar in ice and tube is d; we shall have

$$l + ax = d$$

l is constant so long as no permanent change in the length of the tube takes place.

As already stated, the expansions of the tube, both bars being always at like temperatures, is so small and uncertain as to be neglected. If it had a definite value it would also be necessary to fix a temperature for the normal length.

The above equation expresses in effect that the mean length of a tube between 8 a. m. and 5 p. m. on any day is equal to its normal length plus the temperature-change from 8 to 12 a. m. multiplied by a constant. This equation is entirely empirical, and its constants might be somewhat changed by using other hours than 8 and 12 for determining the temperature-range. Each day of comparisons continued from 8 a. m. to 5 p. m. of a tube under tent, with the bar in ice, on Keweenaw, Sandy Creek, or Buffalo Bases, gives an equation of condition of the above form. The value of x will be the same in all. The value of l will be the same so long as there is no evidence of a permanent change in its value. Such change occurs between the measurement of different bases, and sometimes during the measurement of the same base.

§ **13.** This liability to change in length is a serious evil in the Bache-Würdemann apparatus. There are in it thirteen joints or points of contact, at any one of which change in contact by wear or by change of adjustment may change the length of the tube. As, in consequence of jars and expansions, the screws on which these joints depend frequently get loose, changes in length are unavoidable, and they are large enough to seriously diminish the accuracy of the work, unless much time is spent in comparisons, which would otherwise be unnecessary. In any base-apparatus it is very much to be desired that the points which fix the length of the apparatus should be rigidly connected with each other; or, if there are joints, that these should be very few in number and practically unchangeable. In one of the tubes of the Lake-Survey base-apparatus the length of the tube can be changed 0.003 inch by simply tightening or loosening the screw which forms the axis of rotation of the compensating lever; the tightening twisting the original plane of rotation.

§ **14.** From the comparisons at Keweenaw, Sandy Creek, and Buffalo Bases, twenty-eight equations of condition of the form

$$l + ax = d$$

for each tube have been obtained. These equations contain for tube 1 a single value of l for Keweenaw Base, and three values for each of the other bases, x being the same for all; while for tube 2 a single value for l was used for each base, x being the same for all.

For the Keweenaw Base there was no evidence of change of length of tube 1 during measurement. Tube 2 changed length, but the comparisons of September 30 and October 1, 1873, were not included in the least-square work, as the temperature-range was small. On the Sandy Creek and Buffalo Bases, while there was no evidence of change in length of tube 2 during the measurement of the base, there was evidence that tube 1 on both bases changed length between the comparisons at beginning and middle of measurement, and between the middle and end comparisons, necessitating the assumption of different lengths at each of the three periods of comparison at each base.

The following table gives the dates of comparisons, the weights, the equations of condition, and the residuals. The coefficient of x in the equations is the temperature-increase from 8 to 12 a. m. on each day, and the absolute terms are the mean excess of length of tube over that of bar in ice between 8 a. m. and 5 p. m. of each comparison-day.

TUBE 1.

KEWEENAW BASE.

Date.	Maximum temperature.	Weight.	Equations.	v.
	° F.			
August 4, 1873	80.0	0.5	$l_1+14.6x-0.01916=0$	−0.00016
August 5, 1873	98.0	0.5	$l_1+17.4x-0.01948=0$	−0.00010
August 6, 1873	72.8	1	$l_1-1.0x-0.01734=0$	−0.00047
August 7, 1873	65.0	1	$l_1+3.0x-0.01743=0$	−0.00001
August 8, 1873	66.0	1	$l_1+8.1x-0.01796=0$	+0.00015
October 2, 1873	39.3	1	$l_1+1.8x-0.01680=0$	+0.00046

TUBE 1.—Continued.

SANDY CREEK BASE.

Date.	Maximum temperature.	Weight.	Equations.	v.
	°			
August 13, 1874	74.9	1	$l_2+4.4x-0.01904=0$	+0.00061
August 14, 1874	83.0	1	$l_2+10.6x-0.02064=0$	−0.00014
August 15, 1874	77.1	1	$l_2+14.0x-0.02068=0$	+0.00028
August 17, 1874	85.6	1	$l_2+13.4x-0.02159=0$	−0.00071
August 19, 1874	76.5	1	$l_2+10.9x-0.02059=0$	−0.00005
September 15, 1874	85.0	1	$l_3+12.7x-0.01984=0$	−0.00034
September 16, 1874	63.1	1	$l_3+2.3x-0.01790=0$	+0.00010
September 17, 1874	69.0	1	$l_3+6.3x-0.01839=0$	+0.00024
October 17, 1874	61.4	1	$l_4+11.7x-0.01931=0$	−0.00045
October 19, 1874	43.4	1	$l_4+1.4x-0.01769=0$	−0.00023
October 22, 1874	53.3	1	$l_4+15.0x-0.01912=0$	+0.00018
October 23, 1874	60.3	1	$l_4+22.9x-0.01990=0$	+0.00049

BUFFALO BASE.

Date.	Maximum temperature.	Weight.	Equations.	v.
August 31, 1875	82.0	1	$l_5+17.6x-0.02932=0$	+0.00034
September 1, 1875	85.3	1	$l_5+14.2x-0.02952=0$	−0.00032
September 2, 1875	82.4	1	$l_5+6.5x-0.02823=0$	−0.00008
September 3, 1875	84.0	1	$l_5+8.8x-0.02839=0$	+0.00007
September 22, 1875	50.6	1	$l_6+7.7x-0.02778=0$	+0.00009
September 23, 1875	57.5	1	$l_6+15.4x-0.02901=0$	−0.00009
October 11, 1875	44.4	1	$l_7+4.8x-0.02662=0$	−0.00006
October 12, 1875	40.0	1	$l_7+5.5x-0.02621=0$	+0.00045
October 13, 1875	49.9	1	$l_7+12.0x-0.02797=0$	−0.00043
October 14, 1875	61.1	1	$l_7+18.7x-0.02842=0$	+0.00004

TUBE 2.

KEWEENAW BASE.

Date.	Maximum temperature.	Weight.	Equations.	v.
July 17, 1873	76.6	0.5	$l'_1+4.8x-0.04360=0$	+0.00001
July 18, 1873	60.0	1	$l'_1-4.0x-0.04254=0$	+0.00021
July 21, 1873	90.3	0.5	$l'_1+22.3x-0.04473=0$	+0.00057
July 22, 1873	94.2	0.5	$l'_1+16.3x-0.04433=0$	+0.00039
July 23, 1873	79.2	1	$l'_1+5.8x-0.04438=0$	−0.00068

SANDY CREEK BASE.

Date.	Maximum temperature.	Weight.	Equations.	v.
August 13, 1874	75.9	1	$l'_2+4.0x-0.04438=0$	+0.00007
August 14, 1874	84.0	1	$l'_2+10.8x-0.04515=0$	−0.00004
August 15, 1874	76.1	1	$l'_2+11.3x-0.04535=0$	−0.00020
August 17, 1874	85.1	1	$l'_2+12.0x-0.04579=0$	−0.00057
August 19, 1874	76.9	1	$l'_2+11.0x-0.04560=0$	−0.00047
September 15, 1874	84.2	1	$l'_2+10.1x-0.04537=0$	−0.00033
September 16, 1874	64.6	1	$l'_2+2.3x-0.04372=0$	+0.00056
September 17, 1874	69.8	1	$l'_2+6.4x-0.04422=0$	+0.00046
October 17, 1874	62.5	1	$l'_2+12.2x-0.04509=0$	+0.00015
October 19, 1874	45.0	1	$l'_2+2.0x-0.04418=0$	+0.00007
October 22, 1874	55.5	1	$l'_2+14.6x-0.04520=0$	+0.00027
October 23, 1874	61.2	1	$l'_2+22.6x-0.04619=0$	+0.00006

BUFFALO BASE.

Date.	Maximum temperature.	Weight.	Equations.	v.
August 28, 1875	86.0	1	$l'_3+16.3x-0.04755=0$	+0.00001
August 31, 1875	84.5	1	$l'_3+19.5x-0.04750=0$	+0.00037
September 1, 1875	87.3	1	$l'_3+13.6x-0.04760=0$	−0.00030
September 2, 1875	84.4	1	$l'_3+8.2x-0.04653=0$	+0.00024
September 3, 1875	86.1	1	$l'_3+10.0x-0.04636=0$	+0.00059
September 22, 1875	51.9	1	$l'_3+7.4x-0.04673=0$	−0.00003
September 23, 1875	58.7	1	$l'_3+15.5x-0.04742=0$	+0.00006
October 11, 1875	43.4	1	$l'_3+6.6x-0.04712=0$	−0.00050
October 12, 1875	39.5	1	$l'_3+6.0x-0.04651=0$	+0.00005
October 13, 1875	48.9	1	$l'_3+11.5x-0.04741=0$	−0.00032
October 14, 1875	60.5	1	$l'_3+18.0x-0.04793=0$	−0.00021

§ **15.** Solving for each tube, by least squares, the twenty-eight equations of condition resulting from twenty-eight days' comparisons, the following values of l and x are found:

TUBE 1.

Keweenaw Base,	$l_1 = 0^{in}.01701 \pm 0^{in}.00012.$	August 4 – October 2, 1873.
Sandy Creek Base,	$l_2 = 0^{in}.01905 \pm 0^{in}.00015.$	August 13 – 19, 1874.
	$l_3 = 0^{in}.01777 \pm 0^{in}.00016.$	September 15 – 17, 1874.
	$l_4 = 0^{in}.01727 \pm 0^{in}.00017.$	October 17 – 23, 1874.
Buffalo Base,	$l_5 = 0^{in}.02726 \pm 0^{in}.00017.$	August 31 – September 3, 1875.
	$l_6 = 0^{in}.02682 \pm 0^{in}.00021.$	September 22 – 23, 1875.
	$l_7 = 0^{in}.02591 \pm 0^{in}.00016.$	October 11 – 14, 1875.

For tube 1, $x = 0^{in}.000136 \pm 0^{in}.000009.$

TUBE 2.

Keweenaw Base,	$l'_1 = 0^{in}.043135 \pm 0^{in}.00014.$	July 17 – 23, 1873.
Sandy Creek Base,	$l'_2 = 0^{in}.04406 \pm 0^{in}.00011.$	August 13 – October 23, 1874.
Buffalo Base,	$l'_3 = 0^{in}.04598 \pm 0^{in}.00012.$	August 28 – October 14, 1875.

For tube 2, $x = 0^{in}.000097 \pm 0^{in}.000008.$

The diagrams in Plates XXVIII and XXIX show how closely these values of l and x represent the results of comparisons for each tube and each base. Abscissas are the changes in temperature of tube between 8 and 12 a. m. on a comparison-day, and the ordinates of the points inclosed by a circle with a date are the excesses of the mean tube-length on that comparison-day over that of the 15-feet bar in melting ice. Where tube 1 changed length during measurement, these ordinates have been corrected so as to free them from the effect of that change and to make them give only the temperature effect. The straight line is plotted from the first values for each base of the tube's normal length and from the values of x just given. The distances of the points from the right line are due to the errors of observation and theory, and for the apparatus in question must be considered as reasonably small.

LENGTHS OF TUBES FOR ANY DAY—LENGTHS USED IN COMPUTATION.

§ **16.** The method of determining the value of a tube's length to be used in the computation of any day's measurement can now be explained.

The value of the excess l of the normal length of tube 1 over bar in ice for Keweenaw Base, already given, namely, $0^{in}.01701$, was used for the whole base. For each full day of measurement (from 8 a. m. to 5 p. m.) the temperature-change a, from 8 a. m. to 12 m., was taken from the thermometer-readings inside the tube, and this, with the proper value of x previously given, enabled us to compute $l + ax$, the excess of the mean length of the tube for that day over the bar in ice. The mean length multiplied by the number of times that tube was used during the day gave the length measured by it.

When measurements were made on any day during a fraction of the period from 8 a. m. to 5 p. m. the mean length of the tube during the fraction of a day would usually differ from the mean length for the whole day, and hence the computed value, $l + ax$, would need a correction to give the mean length of the tube while being used. To obtain this correction, the mean curve of § 11, giving the average daily variations of the tube's length at Sandy Creek and Buffalo Bases, was used. The difference between the mean ordinate of this curve for the whole day and for the fraction of the day during which measurements were made, was applied to the computed value $l + ax$ as a correction, and the result was used to obtain the mean length for the fraction of a day.

The same method was followed in computing the mean length of tube 2 for each day or fraction of a day of measurement. But the normal length of this tube changed permanently during the measurement. On September 11, 1873, one end of the tube fell. It was caught by hands so that the blow on a man's shoulders below it was not very severe; hence it sustained no violent jar. It was at once taken to camp and recompared. The not very satisfactory comparisons indicated no change of length since those before the measurement of the base was begun. But comparisons after the measurement of the base, on September 30 and October 1, 1873, showed that the tube then had changed length. It must then be assumed that this change took place between September 11 and September 30, 1873. For the portion of the base measured prior to September 11, the value of l'_1 is adopted for tube 2, which results from the least square reductions of the comparisons for the three bases, which includes for this tube and this base the comparisons of July 17, 18, 21, 22, and 23, 1873. The value has already been given as $l'_1 = 0^{in}.043135$. The comparisons of September 30 and October 1, 1873, of tube 2 with 15-feet bar in ice, give the equations of condition

September 30, 1873,	$l + 4.2\,x - 0^{in}.04031 = 0$	Weight = 1
October 1, 1873,	$l + 4.0\,x - 0^{in}.04030 = 0$	Weight = 1

Using the value of x previously given for this tube, these equations give another value for l'_1 to be used in the remeasurements subsequent to October 1. That value is $l = 0^{in}.03985$. For the interval between September 11 and September 30, the mean of the two values, namely, $0^{in}.04150$, is used, the change in the length of tube 2 being supposed proportional to the number of tubes measured. Collecting these values we have for lengths of tubes to be used in computation of the portion of the base measured on any day,

Tube 1 for whole base and for remeasurements

$$= 15\text{-feet bar in melting ice} + 0^{in}.01701 \pm 0^{in}.00012 + ax.$$

Tube 2 from 1st to 1028th tube

$$= 15\text{-feet bar in melting ice} + 0^{in}.04314 \pm 0^{in}.00014 + ax.$$

Tube 2 from 1029th tube to 1933d tube

$$= 15\text{-feet bar in melting ice} + 0^{in}.04150 + ax.$$

Tube 2 for remeasurements after measurement of base

$$= 15\text{-feet bar in melting ice} + 0^{in}.03985 + ax.$$

COMPUTATION OF LENGTH OF BASE.

§ **17.** In computing the length of the base the mean length of each tube was found, as already explained, for each day of measurement, in the form:

$$\text{Tube's mean length for the day} = 15\text{-feet bar in melting ice} + l + ax.$$

When the measurement was only during a fraction of a day the quantity ax was corrected, as has been explained in § 16. Multiplying the obtained length by the number of times the tube was used during the day, we have the sum of the lengths of the tubes for one day, and in the same way for all days of measurement. These quantities are summed. In closing on the permanent marks on the base line, several small distances less than a tube's length were carefully measured with a scale, and their sum is added to the above length.

§ **18.** There are also several instrumental corrections to be applied to this value:

1. Correction for inclination of tube to horizontal plane.
2. Correction for reading of contact-level.
3. Correction for error in sector-level adjustment.
4. Correction for error in cylindrical surface.

The first of these corrections needs no explanation. The second arises from the fact that the tube is at its definite length when the bubble is at the middle of the contact-level, and changes by very minute known quantities when the bubble is not exactly in the middle. The third error arises from the fact that the sector-level is not stable in adjustment. When found in error in adjustment, the error is assumed to be proportional to the number of tubes measured since the last adjustment. It gives a correction to the grade-angle and to the tube-length, both of which are always small. The fourth error arises from a slight defect in the cylindrical surface, by which the tube was slightly shorter at 0° inclination than at other inclinations. The sum of these corrections applied to the sum of the lengths of tubes gives the length of base, as shown by the measurement.

§ **19.** A portion of the Keweenaw Base equal to ninety-four tube-lengths (about 1,410 feet) was remeasured seven times, in order to form an idea of the accuracy of the measurement. The seventh remeasurement is not included, as it is almost certain from the notes that an error of either 1° or 2° was made at one point in reading the inclination of the tube. The mean of the other six measurements is taken for the length of this part of the base.

The following table gives the number of the measure, its date, the maximum and minimum tube-temperature during the measurement, and the excess of the separate measures over their mean:

Number.	Date.	Temperature.		Excess.
		Maximum.	Minimum.	
		°	°	*in.*
1	August 16, 18, 19, 1873	82.2	66.6	+0.075
2	August 20, 1873	71.1	66.0	—0.008
3	August 21, 22, 1873	81.8	62.7	—0.003
4	August 23, 1873	65.6	54.5	—0.041
5	October 6, 7, 1873	58.5	38.6	—0.045
6	October 7, 8, 1873	71.2	40.0	+0.022

Deducing from these residuals the probable error of one measurement, it is found $0^{in}.030$, or about $\frac{1}{560000}$.

It will be noticed that while on some days the whole distance was measured, for other measurements two or even three days were required. As this involved the difficult estimate of the mean tube-length for parts of days by the approximate method already described, it seems probable that the errors in these measurements are larger than in the continuous measurement of the base when there were relatively fewer fractional days of measurement.

It will further be observed that during these measurements the temperatures varied from $38^{\circ}.6$ F. to $82^{\circ}.2$ F., and that while during the measurement of August 20 there was a change of temperature of but $5^{\circ}.1$ F., in that of October 7 and 8 there was a change of $31^{\circ}.2$ F. These great irregularities in temperature-condition test severely the method of reduction adopted.

§ **20.** Taking the mean value of all corrections ($l+ax$ included) for this portion of the base, and adding it to the corrections for the remainder of the base, we have Keweenaw Base = 1933 (15-feet bar in melting ice) + [1] + [2] + [3] + [4] + [5] + [6], where [ ] is the sign of summation for the whole base, and 1, 2, 3, and 4 are the corrections specified by those numbers in § 18, while 5 is the value of $l+ax$ for any one tube, and 6 is a fractional tube length measured with a scale.

$[1]+[2]+[3]+[4]+[5]+[6] = -55^{in}.116 - 0^{in}.034 - 0^{in}.101 + 0^{in}.108 + 60^{in}.366 + 56^{in}.206 = +61^{in}.429$. $\therefore$ Keweenaw Base = 1933 (bar in melting ice) + $61^{in}.429$, as resulting directly from measurement.

This value needs some corrections, given below, and is to be reduced to the sea-level.

PROBABLE ERRORS OF CORRECTIONS AND PROBABLE ERROR OF LENGTH OF BASE.

§ **21.** An attempt will now be made to estimate the probable errors of these corrections, and the probable error of the final value for length of base. The sums of the corrections of the second, third, and fourth classes are so small for the whole base that their probable errors are neglected.

§ **22.** The first question is as to the accuracy of the values of tube-lengths used in the computations. The probable errors in the values of l for tubes 1 and 2 as derived by the least-square reductions are given in § 15 as $\pm 0^{in}.00012$ and $\pm 0^{in}.00014$, respectively. The corresponding probable errors in the values of x are $\pm 0^{in}.000009$ and $\pm 0^{in}.000008$. As the average temperature-range between 8 and 12 a. m. on days of measurement was $13^{\circ}.2$, the average probable errors in the values of ax would be $\pm 0^{in}.00012$ and $\pm 0^{in}.00010$ for the tubes 1 and 2. Hence the probable error in the value of d for tube 1 would be approximately $\sqrt{(0^{in}.00012)^2 + (0^{in}.00012)^2} = \pm 0^{in}.00017$, and for tube 2 also $\pm 0^{in}.00017$. Multiplying the first of these by 967, the number of times tube 1 was used, and the second by 966, the number of times tube 2 was used, there result $0^{in}.164$ and $0^{in}.164$. The square root of the sum of their squares gives for the probable error in the whole base from this cause $\pm 0^{in}.232$.

In taking for tube 2 the constant probable error $\pm 0^{in}.00017$ multiplied by the number of tubes, the resulting probable error is theoretically overestimated, since it assumes that the actual errors for all tubes have the same sign. But this is partly compensated for by the greater probable error of the few final comparisons.

§ **23.** Alignment was obtained after finding that the mark at the middle base was very nearly on the line through the marks at the ends of the base by setting up a theodolite at the middle of one of the halves of the base very nearly on the line, then reading by several repetitions the angle (very nearly 180°) between the ends of this half and computing the distance the stake must be moved to bring it on the line. Each of these smaller parts was then bisected in the same way, and the process continued till stakes had been set on the line 500 feet apart. By this method it is improbable that any of those lines could have deviated from the base-line by more than one minute as the instrument used read to 10 seconds, and the probable deviation is much less. As an error of one minute would only introduce an error into that part of the base of $\frac{1}{200000000}$, and as the probable error is much less, it is insignificant in comparison with other errors and may be neglected, or, what is the same thing, these stakes 500 feet apart may be assumed to be precisely on the line.

§ **24.** To place a tube on the line the base-line transit-instrument, with a more powerful telescope than that of the theodolite, was set up on one of the stakes and pointed at the preceding, and the front end of the tube placed in the vertical plane of the telescope. When the front tube got within 150 or 200 feet of the transit the latter was moved to the next 500-feet stake. The distances from transit to tube varied then from 200 to 700 feet. The length of the base gives fifty-seven of these 500-feet intervals, and so many times will the distance of the tubes from the transit vary from 700 to 200 feet. The maximum error that could be committed in putting the agate at the end of the tube on the line at the distance of 700 feet may be taken as $0^{in}.3$, and at the distance of 200 feet as $0^{in}.05$. From the ordinary law of error, when the maximum errors in fifty-seven observations are respectively $0^{in}.3$ and $0^{in}.05$, we have the probable errors of one such observation respectively $0^{in}.085$ and $0^{in}.014$. These values give for the probable deviation from the base-line of a tube at 700 feet from the transit

$$\frac{0.085\ \sqrt{2}}{180 \sin 1''}=138''$$

and at the distance of 200 feet

$$\frac{0.014\sqrt{2}}{180 \sin 1''}=23''$$

In order to estimate the sum of the corrections to the length of the base arising from these deviations, suppose the whole number of tubes which make up the base divided into ten classes of 193.3 tubes each, the first class including the 193.3 tubes which have the smallest probable deviation, the tenth class including the 193.3 tubes having the largest probable deviation. Assume 23″ as the probable deviation for all tubes of the first class (which is somewhat too small) and 138″ (which is somewhat too large) as the probable deviation of all tubes of the tenth class, and the deviations of the intermediate classes of 193.3 tubes to increase for each class by

$$\frac{138''-23''}{9}=12''.8$$

Assuming the ordinary law of error and considering the first class of 193.3 tubes, we can compute the number of tubes of this class included in each of its ten sections. The first section of the first class is to include the number of tubes for which it is probable the deviation will be less than $23''\times 0.4$; the second will include the number of tubes for which the deviation should be between $23''\times 0.4$ and $23''\times 0.8$; the third will give the number of tubes whose deviations lie between $23''\times 0.8$ and $23''\times 1.2$, and so on, the interval for each remaining section being 0.4, so that the last will include the tubes whose deviations lie between $23''\times 3.6$ and $23''\times 4.0$. Assuming that the mean of the extreme deviations in each section may be taken as the deviation for all tubes in that section, the number of tubes in the section multiplied by the length of a tube multiplied by $2 \sin^2 \frac{1}{2} a$ (where a is the mean probable deviation for this section) will be, when taken with a negative sign, the correction to the length of the base due to the deviations in this section. Doing the same for each section in this class and summing the results, we have the correction to the length of the base arising from the probable deviation of the 193.3 tubes in this first class. Doing the same for the second class of 193.3 tubes, for which the probable error is 35″.8, and for the other 8 classes, and summing all results, we have $-0^{in}.069$ as the correction to the measured base arising from angular deviation of the tubes from the vertical surface through the base-line.

The probable error in this correction is so small in comparison with other probable errors that it may be neglected.

§ **25.** The inclination of the tubes to the horizon is read to seconds by means of the sector. As the errors in reading would only give exceedingly small corrections, whose signs would on the whole be as often positive as negative, thus being eliminated in the result, they may be neglected. The sector, however, is liable to get out of adjustment; and this adjustment was therefore examined and the correction determined every few days during the remeasurement, eighteen times in all for each tube. The maximum correction found was 115″. The error in the adopted value for any inclination will therefore be made up of two parts: first, the error in the determination of the correction; secondly, the error in the assumption that between two such determinations the correction changed uniformly.

The correction was determined by bringing the two agates at the ends of a tube into the same horizontal plane by means of a leveling instrument close at hand and then reading the sector. Repeated trials gave a range to the values of the correction of less than 20″, and as several determinations were made each time the correction was determined, if 5″ be taken as the probable error of the correction it is certainly large enough.

If the mean value of the correction as derived from its consecutive determinations were used, the resulting correction to the base would be nearly the same as if the correction was supposed to change uniformly. Taking 5″ as the probable error of the correction for any tube of a step, the error in the horizontal projection of this tube would be $0^{in}.000151$ if its inclination be taken at 2°. This value, 2°, is more than double the average inclination, and is large enough to give a correction to the horizontal projection greater than the average value. There were thirty-six determinations of the correction, hence we may consider the line as divided into steps whose average length was fifty-three tubes.

If the inclination of all the tubes in one of these steps had had the same sign, the corrections to the horizontal projection of these tube-lengths arising from the actual error in determining the index-error would have had the same sign. Assuming this, and in order to overestimate rather than underestimate its effect on the length of the base, assuming that a tube whose inclination is 2° will give the average correction, there results for the sum of the errors in the horizontal projections of the fifty-three tubes in this step $53\ (0^{in}.000151) = 0^{in}.008$. If the inclinations of all the tubes in each step had the same sign, since the errors in determining the index-error are independent in different steps, there would result for sum of errors in horizontal projections of tubes for whole base of thirty-six steps, $0^{in}.008\sqrt{36} = 0^{in}.048$. But instead of the inclinations of the tubes all having the same sign, the two ends of the base differed in elevation by but 2 feet, so that the sum of the ascents was practically equal to that of the descents. Since the error in the horizontal projection of a tube changes sign when the inclination changes sign, it follows that in fact the positive and negative values of the correction nearly cancel each other when summed, and give a sum that is insignificant and may be neglected in comparison with the $0^{in}.04$ obtained above, when all inclinations in a set were supposed to have the same sign.

Error also exists in the assumption that between two determinations of index-error of the sector its change was proportional to the time. There is no way of knowing precisely the amount of error in this assumption, but as the correction was sometimes constant for several days, once for ten days, and as the maximum change between any two determinations was 115″, its effect will certainly tainly be overestimated if a probable error of 10″ be attributed to the inclination of any tube from this cause. The actual error in the horizontal projections of tubes is not, as in the preceding case, constant for all the tubes in a set. The sum of errors in projection would therefore be less than $\left(\frac{10}{5}\right)^2$ times the $0^{in}.048$ there estimated, all tube-inclinations in a set being supposed of the same sign. But since there are approximately as many positive as negative inclinations, and hence as many positive as negative corrections, the sum of the corrections to the projections of the tubes will for the whole base be a quantity so small in comparison with $\left(\frac{10}{5}\right)^2 0^{in}.048$ that it may be neglected.

It is then concluded that no error needing consideration arises from errors in the adopted inclinations of tubes.

§ **26.** When at any time, from the approach of night or from bad weather, the measurement was suspended, the terminal point of the measurement was marked by a cut in the head of a copper tack driven into the head of a stake, this head being below the surface of the ground as described in § 6.

In thus referring the end of the tube to the ground with a good theodolite, experience shows that an error greater than $0^{in}.03$ is impossible. A similar error may arise in placing the rear end of the rear tube over the mark on the copper tack when the measurement is resumed. Assistant Engineer E. S. Wheeler is of opinion that the greatest error did not exceed $0^{in}.02$. There were forty-four references of tube to ground or from ground to tube; and assuming the ordinary law of error to hold, and that the maximum error in such references was $0^{in}.03$, we have for the probable error in one reference $\pm 0^{in}.009$, and the probable error in the length of the base arising from this cause $\pm 0^{in}.06$.

§ **27.** There is an error entering into every measurement of one tube-length, arising from the pressure of contact between consecutive tubes. This pressure, when reduced to the lowest amount consistent with certainly overcoming the friction of the sliding rod, was still four ounces. Now, supposing a standing tube to be in the right position, when the next tube is brought into contact with it, the pressure between the two forces back the rear or standing tube by a certain amount, and, of course, the front tube is moved back in contact by the same amount. When the rear tube is taken away, the front tube is relieved from the pressure of contact and moves to the rear by the same amount, and is then the standing tube, but is at a distance in rear of the position it would have had if the contact-pressure had been zero, equal to twice the disturbance produced by the contact-pressure. The amount of this disturbance will vary with the precision with which the movable parts of the trestles fit each other, and with the height to which the tube is raised by the vertical motion of the trestles. From nearly a hundred determinations of the amount of displacement, made by Assistant Engineer E. S. Wheeler and Assistant Engineer Burton in 1872 and 1873, the following mean values have been obtained for disturbance by pressure of contact, that pressure being about four ounces, the trestle being either run up to its full height or run clear down:

Trestle up, $0^{in}.00034$; trestle down, $0^{in}.00013$.

During the remeasurement of the Keweenaw Base the trestles were kept as low as possible, the average being estimated at one-fifth their total range, and hence giving $0^{in}.00017$ as the average displacement. Doubling and multiplying by 1932 the number of tubes less one, there results a correction to the length of the base of $-0^{in}.657$. Taking into consideration the varying conditions, as cleanness of the movable parts of the trestles, firmness of ground-support, and the difficulty of measuring with precision movements so small as $0^{in}.00017$, its probable error may easily be $0^{in}.00003$ or $0^{in}.00004$. Taking the latter, we have for the total probable error, $2 \times 1932 \times 0^{in}.00004 = \pm 0^{in}.155$; and for the correction to the measurement of the base, $-0^{in}.657 \pm 0^{in}.155$.

§ **28.** It has already been stated that the length of tube 2 changed between September 11, 1873, and September 30, the value of l_1' changing from $0^{in}.043135$ to $0^{in}.03985$. In the computation of the length of this part of the base it has been assumed that the rate of change with reference to the number of tubes measured was constant. The maximum error resulting from this assumption would occur if the change had been instantaneous and had happened on September 11 or on September 30. Its amount then would be $490 \times 0^{in}.001642 = 0^{in}.804$. The change probably occurred from the gradual loosening of some of the screws of the apparatus, but in view of the uncertainty that the change was uniformly distributed over the whole period, and of the fact that the probable error of tube 2 from the 1029th tube to the 1933d tube has been taken in § 22 at the same value as prior to tube 1029 (a value undoubtedly too small, though data do not exist for its determination), it is thought that the probable error from these causes may reach one-fourth of $0^{in}.804$, and $0^{in}.201$ is accordingly taken.

RÉSUMÉ.

§ **29.** The probable errors in the length of the base resulting from the various causes are then as follows:

From probable errors in lengths of tubes (§ 22)	$\pm 0^{in}.232$
From probable error in inclinations (§ 25)	$\pm 0^{in}.000$
From probable error in reference to and from the ground (§ 26)	$\pm 0^{in}.059$
From probable error in contaet displacement (§ 27)	$\pm 0^{in}.155$
From probable error in change of length of tube 2 (§ 28)	$\pm 0^{in}.201$

Taking the square root of the sum of the square of these quantities, we have for the probable error in the length of the base $\pm 0^{in}.349$ (error in length of 15-feet standard bar not included).

§ **30.** In § 19 the probable error in one measurement of ninety-four tubes deduced from the discrepancies of six measurements is given as $0^{in}.03$. These discrepancies arise from errors in adopted lengths of tubes due either to temperature or to permanent changes in length, from errors in inclination, in alignment, and in references to the ground. They would give for the probable error in the length of the whole base due to these causes $\pm 0^{in}.03\sqrt{\frac{1933}{94}} = \pm 0^{in}.136$. This does not include the errors in the value of l nor in the estimate of contact displacement, but if these were added the probable error in the length of the base would be less than that derived above.

§ **31.** The mean height above sea level of Keweenaw Base is 662 ± 1 feet, giving a correction to reduce to sea level of $-11^{in}.022 \pm 0^{in}.017$. In §§ 24 and 27 the estimated corrections to length of base for errors in alignment and for contact displacement are given as $-0^{in}.069$ and $-0^{in}.657$, respectively. Applying these corrections to the value given in § 20, namely, 1933 (15-feet bar at 32° F.) $+ 61^{in}.429$, Keweenaw Base reduced to sea level = 1933 (15-feet bar in melting ice) $+ 49^{in}.68 \pm 0^{in}.349$.

§ **32.** Taking for the length of the 15-feet bar in melting ice the value given in Chapter II, § 14, namely, $179^{in}.95438 \pm 0^{in}.00012$, we have for the length of the

KEWEENAW BASE REDUCED TO SEA LEVEL $= 347901^{in}.50 \pm 0^{in}.419$.

§ **33.** As a check on the parts of the base into which it was divided near its middle, the angles of two triangles having a common vertex, and the parts of the base for opposite sides, were read, excepting those at the middle of the base. Computing the north half of the base from the whole base, the computed length was $1^{in}.3$ less than the measured length, while the computed south half was $1^{in}.3$ greater than the measured length. As the angles of this small triangulation were not well measured, the agreement is as good as was to be expected.

An approximate reduction of the 1867 measurement of the Keweenaw Base in Note-book S. 288, gives 347,904.154 American inches for the length of the base not reduced to the sea-level. The corresponding value derived from the remeasurement as follows from the data given above is 347,912.52 British inches.

Of this correction to the older value, namely, + 8.37 inches, + 12.18 inches arises from the change in value for length of the 15-feet brass bar at 62°. It was formerly not very accurately known in terms of the uncertain American yard (which has now properly been dropped) as 180.00166 American inches at 62°. (See Lake-Survey Report, 1868.) It is now accurately known in terms of the British standard yard of the Ordnance Survey, its length at 62° being given in Chapter II, § 11, as 180.00796 British inches.

The rest of the correction to the older result, namely, − 3.81 inches, arises from differences in the two measurements, and in the methods of reduction.

CHAPTER IV.

MINNESOTA POINT BASE.

LOCATION—MARKINGS—MEASUREMENTS.

§ **1.** Minnesota Point base-line is on a sand-spit which separates the western end of Lake Superior from Superior Bay. This spit is narrow, its greatest width not exceeding 700 feet, and its highest points, which are sand-hills, are not more than 15 feet above the lake. The length of the base-line is, approximately, 19,871 feet. The north end of the base is in latitude 46° 45′ and in longitude 92° 05′.

The ends of the base are marked by crosses on brass plugs inserted in heavy stones, whose tops are 2 feet below the surface of the ground. Near each end of the base three stones similar to the marking-stone, and 15 feet from it, were set, one in the prolongation of the base-line, and one on either side at right-angles to the base-line, in order to check any disturbance of the principal stones. The dimensions of the stones are 3^{ft} by 1^{ft} by 1^{ft}. The ground is 2 feet above the lake at North Base and 3½ feet at South Base.

The base was measured under the direction of Captain (afterward General) G. G. Meade, with wooden rods, as a secondary base, in 1861. It was measured in 1870 as a primary base by General C. B. Comstock, aided by Lieutenant J. H. Weeden and Assistant Engineer E. S. Wheeler, with the Bache-Würdemann base-apparatus of the Lake-Survey.

§ **2.** Comparisons of the measuring-tubes with the 15-feet standard bar extended from July 22 to August 9 before the measurement of the base, and from August 31 to September 2, 1870, after the measurement of the base. The measurement was made between August 13 and August 31, 1870. The number of tubes measured was 1,325, each being approximately 15 feet in length.

The greatest speed of measurement was on August 27, 1870, when 200 tubes were measured in $8^h\ 12^m$, giving a speed of 366 feet per hour. This great speed was only possible from the fact that the soil was of clean sand, on which the trestles that support the tubes could be rapidly placed, and were immediately stable.

METHOD OF MEASUREMENT—DETERMINATION OF TUBE-LENGTHS.

§ **3.** A description of the method of using the Bache-Würdemann base-apparatus has been given in Chapter III. Accordingly, it will only be necessary here to refer to points in which its use at the Minnesota Point Base differed from that at the Keweenaw Base.

1. In the measurement of the Minnesota Point Base the feet of the trestles rested usually on the soil (sand) without the intervention of supporting stakes.

2. The tubes in measurement had no tent over them to protect them from the sun.

3. The pegs over which the theodolite was set to give alignment to the tubes were about 1,000 feet apart.

The experience gained on this and the Fond du Lac Base led to modifications of methods for the Keweenaw Base.

§ **4.** The comparisons of the tubes with the standard 15-feet bar, to determine the lengths which they actually had during the days of measurement, were made in a house at Superior City. The methods were similar to those used at the Keweenaw Base, described in Chapter III, save that the standard bar was in air instead of being packed in ice, and had its temperature determined by mercurial thermometers lying in its box. As there were wide fluctuations of temperature in the room used for comparisons, it could not be assumed that the bar and the thermometers, whose masses differed widely, would have very nearly the same temperatures, except at the time of maximum or minimum temperature of the bar, which time would be indicated by its length becoming a maximum or minimum. Since no comparisons at minima-temperatures were made, the comparisons used in determining the lengths of the tubes were those in the immediate vicinity of maxima-temperatures.

§ **5.** In Chapter III the method has been explained of determining for each day of measurement the quantity, $l+ax$, by which the mean length of a tube for that day exceeds the length of the 15-feet brass bar packed in melting ice. In $l+ax$, l is the excess of the tube's length, both of its component bars having the same temperature over the length of the 15-feet bar in ice, and varies for each base, while x is the change in the length of a tube arising from difference of temperature of its bars, caused by a change of temperature of 1° F., between 8 a. m. and 12 m., and is a constant for each tube, derived from comparisons, and a is the temperature-change from 8 to 12 a. m. on the day of measurement.

For tube 1, $x=0^{in}.000136\pm0^{in}.000009$. For tube 2, $x=0^{in}.000097\pm0^{in}.000008$. (See Chapter III, § 15.)

The temperature for the value l was not defined for the Keweenaw Base, as the length of a tube at its measurement was practically constant for all temperatures so long as its component bars were of equal temperatures. But early in July, 1872, prior to the measurement of the Fond du Lac and Keweenaw Bases, and subsequent to the measurement of the Minnesota Point Base, the compensating lever of tube 1 was changed so as to diminish the expansion of this tube by $0.^{in}00020$ for 1° F. Tube 2 remained unchanged.

In Chapter III, § 9, it is stated that the expansions of both tubes (the bars in each being at the same temperature) are now very nearly zero; hence at the measurement of the Minnesota Point Base the expansion of tube 1 was $0^{in}.00020$ for 1° F. The amount of this change in coefficient was computed from careful measurements of the changes in the knife-edges of the compensating-lever. Hence, in speaking of the length of tube 1 for the Minnesota Point Base, it is necessary to define its temperature. Its length when both bars are at 62° F. will be called its normal length.

§ **6.** In reducing the comparisons at Minnesota Point of the 15-feet brass bar with the tubes, the comparator-readings on each were plotted as ordinates, the times being abscissas. The portions of ordinates intercepted between the curves for a tube and for the 15-feet brass bar near the time of maximum bar-length gave differences of length between bar and tube at known temperatures. Using the expansion $0^{in}.00020$ for 1° F. for tube 1, $0^{in}.00000$ for tube 2, and $0^{in}.001786$* for the brass bar (§ 11, Chapter II,) since the reductions were from near 62° to 32°, the computed values (tube at 62° F.)—(15-feet bar at 32° F.) were obtained. These values are given in the following table:

Maxima-Comparisons at Minnesota Point Base.

Date.	Tube.	Time of maximum length of bar.	Corrected temperature at maximum length of bar.	Increase of bar-temperature in 3^h before maximum length of bar.	Tube at 62° minus bar at 32° as derived from comparisons at maximum length of bar.
1870.		*h. m.*	° F.	° F.	*in.*
Aug. 4	1	5 12 p. m.	74. 6	+0. 5	0. 02076
Aug. 9	1	5 39 p. m.	67. 2	+0. 9	0. 02118
Sept. 2	1	5 41 p. m.	58. 2	....	0. 01895
Aug. 6	2	5 30 p. m.	71. 2	+0. 9	0. 03867
Aug. 31	2	4 30 p. m.	55. 5	0. 0	0. 03821
Sept. 1	2	1 45 p. m.	55. 3	+1. 0	0. 03804

It will be noticed that the comparisons of September 2, 1870, give a less length for tube 1 than the preceding ones. The temperature-increments in three hours on August 4 and August 9 are not large, and to give the normal length the observed lengths would not need a large negative correction. But a large negative correction would be needed to give the result of September 2. Unless, therefore, the temperature curve on September 2 was such as to give a length much less than the normal length, a supposition that is quite improbable, we must conclude that tube 1 changed its length between the comparisons of August 9 and September 2, 1870, and that its length before measurement must be determined from the comparisons of August 4 and August 9, and its

* The provisional value ($0^{in}.001787$) was used instead of $0^{in}.001786$ in the computation of the table. The effect on the length of the base, however, is so slight that the original computation has been used.

length after measurement from those of September 2, 1870. This tube has frequently changed length during the measurement of bases, the cause undoubtedly being the loosening of the numerous screws which combine its parts. The comparisons of tube 2 indicate no change in length during measurement, and hence the mean of their results is taken to determine the normal length of this tube during the whole measurement.

The mean results of the maxima-comparisons are, then, as follows:

From comparisons of August 4 and August 9, 1870, before measurement of base—

Tube 1, at 62° F.=15-feet bar at 32°+0^{in}.02097.

From comparisons of September 2, 1870, after measurement of base—

Tube 1, at 62° F.=15-feet bar at 32°+0^{in}.01895.

From comparisons of August 6 and 31 and September 1, before and after measurement—

Tube 2, at 62° F.=15-feet bar, at 32°+0^{in}.03831.

§ **7.** These results include the effect of the difference of temperatures of the component bars of a tube, on the tube's length, while the quantities needed in computation of the base are the normal lengths at 62°; that is, the lengths of the tubes when the component bars in each tube are each at 62°. The data for computing the corrections to the above results to reduce them to normal lengths are not satisfactory, but an approximate estimate can be made of their values.

At the Keweenaw Base the tubes were compared with the 15-feet bar when at its maximum lengths, and the normal lengths of the tubes were also determined. The differences are quantities which, applied to the lengths of the tubes derived from maxima-comparisons, would give their normal lengths. Now, if the daily temperature-curves during maxima-comparisons at Minnesota Point Base had been identical with those of the Keweenaw Base maxima-comparisons, the difference between maximum and normal lengths would have been the same for the two bases, and the correction above referred to could be at once applied to the results of the Minnesota Point maxima-comparisons to give the normal lengths of the tubes for that base. Unfortunately, the temperature-curves for the Minnesota Point comparisons are unknown, except for two or three hours before the time of maximum length of the bar, and hence, in the absence of other information, we have to take these portions of the temperature-curves for comparison with those of the Keweenaw Base, in order to estimate the excess of length of a tube over its normal length.

The following table gives the results of comparisons of tubes with the 15-feet bar at maxima-temperatures at Keweenaw Base. The expansion of the brass bar between 32° and 62° is 0^{in}.001786 for 1° F., as given in Chapter II, § 11. It also gives for each day the temperature-increase in three hours before the maximum length of the 15 feet bar, and the excess of length of tube found from each day's maxima-comparisons over the normal lengths of these tubes, which are for Keweenaw Base (Chapter III, § 16),

Tube 1=15-feet bar at 32°+0^{in}.01701±0^{in}.00012.
Tube 2=15-feet bar at 32°+0^{in}.04313±0^{in}.00014.

Maxima-comparisons at Keweenaw Base.

Date.	Tube.	Time of maximum length of bar.	Corrected temperature at maximum length of bar.	Increase of bar temperature in 3^h before maximum length of bar.	Tube at 62° minus bar at 32°, as derived from comparisons at maximum length of bar.	Tube-length minus normal length.
1873.		*h. m.*	° F.	° F.	*in.*	*in.*
Aug. 1	1	6 08 p.m.	64.8	+3.4	0.01860	+0.00159
Aug. 2	1	4 37 p.m.	64.5	+1.3	0.01791	+0.00090
Aug. 9	1	5 09 p.m.	63.6	+1.1	0.01746	+0.00045
Oct. 13	1	3 45 p.m.	54.8	+2.3	0.01903	+0.00202
Oct. 14	1	1 19 p.m.	53.0	+5.4	0.01831	+0.00130
		Means.	....	....	0.01826	+0.00125
July 25	2	6 03 p.m.	72.0	+1.1	0.04292	−0.00022
July 26	2	4 55 p.m.	69.3	....	0.04349	+0.00035
July 29	2	6 22 p.m.	68.9	+2.3	0.04348	+0.00034
July 31	2	6 00 p.m.	61.8	+0.4	0.04338	+0.00024
		Means.	....	....	0.04332	+0.00018

§ 8. From this table the increments in temperatures for three hours before maximum at the Keweenaw comparisons are taken for each tube and plotted as abscissas, the corresponding excesses of tube-lengths over normal lengths being plotted as ordinates. A broken line will thus be obtained which gives roughly the relation that these quantities bear to each other. They indicate that generally the excess of length of tube over its normal length increased with the increase of temperature in three hours before comparison, but with one largely discrepant result, that on October 13. The morning of this day was rainy or cloudy, but it cleared in the afternoon. There was the rare phenomenon of a minimum in temperature at 10 a. m. Usually the tube reaches its maximum length from one to two hours before the maximum temperature of the bar, and then sinks to its normal length one or two hours later. On this day it did not reach its maximum length till forty-five minutes before the bar-maximum, and so had not yet nearly approached its normal length. As it is an average temperature-condition that is needed, this day's comparisons might be omitted. In that case, a right line whose slope is $0^{in}.00025$ per degree, and which cuts the ordinate of zero temperature-change at $+0^{in}.00038$, will represent the observations for tube 1. If the result of October 13 be retained, leaving the slope unaltered, the quantity $0^{in}.00038$ would become $0^{in}.00048$. Not to exclude any comparisons, this value will be used. These results seem to be the best that can be obtained, and may be expressed as follows:

When the temperature of the bar does not change in three hours before the time of its maximum length, the average excess of the length of tube 1 over its normal length is $0^{in}.00048$. This excess increases by $0^{in}.00025$ for each degree's increase in temperature in the three hours. Plotting from the same table the corresponding quantities for tube 2 at the Keweenaw Base, we obtain a line whose slope is $0^{in}.00006$, and which cuts the axis of ordinates at $+0^{in}.00007$. In the absence of better data, these lines, which are only rough approximations, must be assumed to represent the relations between the same quantities for the Minnesota Point comparisons.

From the table in § 6 we have, from comparisons at Minnesota Point Base on August 4 and 9, 1870, before measurement, mean excess of length of tube 1 over 15-feet bar at $32^\circ = +0^{in}.02097$, while the mean temperature-change in three hours before maximum was $0^\circ.7$. Hence, the correction to be applied to $+0^{in}.02097$ is $-0^{in}.00048 - 0.7\ (0^{in}.00025) = -0^{in}.00065$; and hence, before measurement—

Normal length of tube 1 at 62° F. = 15-feet brass bar at 32° F. $+ 0^{in}.02032$.

For the comparisons of September 2, 1870, there are no data for determining the temperature-increase in three hours before maximum; accordingly, the increase and correction are taken the same as in the comparisons before measurement. Subtracting the correction $0^{in}.00065$ from $0^{in}.01895$, there results for value after measurement,

Normal length of tube 1 at 62° = 15-feet bar at 32° $+ 0^{in}.01830$.

In tube 2 there is no indication of change of length during measurement, hence the mean of all comparisons is taken from the table in § 6. The mean value for (tube 2 at 62°)—(15-feet bar at 32°) is $0^{in}.03831$, and the corresponding mean temperature-change in three hours is $0^\circ.6$. Hence from values previously given the correction to reduce to normal length at 62° is $-0^{in}.00007 - 0.6\ (0^{in}.00006) = -0^{in}.00011$, giving—

Normal length of tube 2 at 62° = 15-feet brass bar at 32° $+ 0^{in}.03820$.

ESTIMATE OF ERRORS IN ADOPTED LENGTHS OF TUBES.

§ 9. The errors in these values of the tube-lengths are those which arise in the operation of comparing; from the errors in the adopted expansions of the standard bar and tubes; in the observed temperatures; and in the estimate of the correction to be applied on account of the unequal temperatures of the component bars in each tube to reduce the mean of the results of maxima-comparisons to normal lengths. The observation-errors in comparisons are insignificant in comparison with the uncertainty in the last-named correction, as the comparators, whose screws were well determined, read to $0^{in}.00001$, and hence may be neglected. The mean temperature of the tubes during comparisons differed but little from 62° F., and hence any small error in their expansions would have little effect on their mean lengths at 62°. The same is true of the effect of error in the expansion of the 15-feet bar in reducing its length to that at 62° F. Its length at 32° F. depends on that at 62° F., and the effect of error in its expansion is included in the probable error of its length at 32°, given in Chapter II, § 11. There remain for consideration the errors in the observed differences of lengths of tubes and 15-feet bar, due to uncertainties in the observed temperature of the latter, and the errors in the corrections applied to reduce the observed lengths of the tubes to their normal lengths. The first error arises from the fact that the four thermometers lying on the 15-feet bar did not have precisely the temperature of the bar. But as no comparisons were used save those when the bar had its maximum length, at which time its change of temperature as indicated by its change of length did not exceed 0°.3 F. in a period varying from half an hour to an hour, while the change of the thermometer-indications for the same time did not exceed 0°.4 F., the probable error in the observed temperature at comparison may be taken as not exceeding $\pm 0°.1$ F., corresponding to a change in the length of the 15-feet bar of $\pm 0^{in}.00018$.

But whatever the actual errors due to this cause, as they arose in the same way at the Minnesota Point and Keweenaw comparisons, and enter the normal length resulting from the Minnesota Point comparisons, first, directly with one sign; and, second, indirectly through the correction applied to give normal length with the opposite sign, the resulting error from this cause in the normal length must be a fraction of $0^{in}.00018$.

§ **10.** Considering now the probable error in the corrections adopted to give normal lengths, it may be remarked that they might have been obtained by other methods.

1. The consideration of the temperature-curves on days of maxima-comparisons at Keweenaw and Minnesota Point Bases might have been omitted, and the assumption might have been made that the correction needed to reduce the mean length of a tube derived from maxima-comparisons at Keweenaw Base to its normal length could be applied unchanged to the results of maxima-comparisons at Minnesota Point Base to give the normal lengths of the tubes at that base. This method would have given a correction for tube 1 of $-0^{in}.00125$ in place of $-0^{in}.00065$, and for tube 2 a correction of $-0^{in}.00018$ in place of $-0^{in}.00011$. In reference to the relative values of the corrections obtained by the two methods, it may be said there can be no question that on a day when the temperature-curve has its ordinary form the needed correction increases with the amount of temperature-increase in the three hours preceding the maximum length of the 15-feet bar, and, therefore, the results obtained by taking into account this increase should be the more accurate.

2. To obtain these corrections still another method might have been followed. Instead of using all the days of maxima-comparisons at Keweenaw Base, as in the preceding paragraph, only those days (August 2 and 9 and July 25 and 31) might have been used on which the temperature-increase in three hours before maximum length of bar was small, not exceeding 1°.3; since at the Minnesota Point comparisons this change did not exceed 1° on any day, and hence, so far as our information goes, the temperature-conditions were very much alike. This process would give a correction of $-0^{in}.00067$ for tube 1, and $-0^{in}.00001$ for tube 2, quantities which do not differ widely from the values $-0^{in}.00065$ and $-0^{in}.00011$ already adopted. Indeed this method might well have been used.

Notwithstanding what has been said, the assignment of probable errors to these corrections, in the absence of sufficient data, must be a matter of judgment, and so subject to large uncertainties. The least-square reduction of six days' comparisons of tube 1 and five days' comparisons of tube 2 at the Keweenaw Base gave probable errors of $\pm 0^{in}.00012$ and $\pm 0^{in}.00014$, respectively, in their normal lengths. With fewer days of comparisons and more uncertain methods the probable errors at the Minnesota Point Base must be much greater.

Tube 1 was compared on two days before the measurement of the Minnesota Point Base and on one day after, while tube 2 was compared on one day before and on two days after the measurement. In view of the small number of days of comparisons and of the uncertainties in the method of obtaining the corrections adopted, it is thought that the probable errors, arising mainly from these causes in the adopted values of the normal length of tube 1, may reach $\pm 0^{in}.00040$, and $\pm 0^{in}.00050$ before and after measurement respectively, and that the probable error in the normal length of tube 2 may reach $\pm 0^{in}.00035$, the standard 15-feet bar being supposed exact.

The values for the normal lengths of the tubes at 62° are, then, as follows:

Tube 1 before measurement = 15-feet bar at $32° + 0^{in}.02032 \pm 0^{in}.00040$.

Tube 1 after measurement = 15-feet bar at $32° + 0^{in}.01830 \pm 0^{in}.00050$.

Tube 2 before and after measurement = 15-feet bar at $32° + 0^{in}.03820 \pm 0.^{in}00035$.

It is assumed that the change in the length of tube 1 was gradual during the whole measurement and proportional to the number of tubes measured. The length of the base was 1,325 tubes, but as the last 124 tubes were remeasured the total number measured was 1,449. At the 1325th tube, therefore, $\frac{1325}{1449}$ of the whole change would have taken place, and its length then was, 15-feet bar at $32^{\circ}+0^{in}.01847$. As the remeasurement is not included in determining the length of the base, there results, mean length (between 1st and 1325th tubes) of

Tube 1 *during measurement*=15-*feet bar at* $32^{\circ}+0^{in}.01940\pm0^{in}.00038$.

The quantity $\pm0^{in}.00038$ is nearly the average of the probable errors, which are $0^{in}.00040$ and $0^{in}.00050$ at beginning and end of measurement, and at the middle $\frac{1}{2}\sqrt{(0^{in}.0004)^2+(0^{in}.0005)^2}=0^{in}.00032$. To take this value for all the times the tube entered the base as constant is theoretically to over-estimate the resulting probable error if obtained by multiplying it by the number of times tube 1 entered the base. For, to take the probable error of all the tubes as constant assumes in effect that the actual errors in the computed lengths of all tubes intermediate to the comparisons have the same signs, which is not necessarily true.

COMPUTATION OF LENGTH OF BASE.

§ **11.** Having obtained values for the normal lengths of the tubes, the mean length of a tube for any day of measurement can be obtained by the method explained in § 16 of Chapter III. But as tube 1 had at the Minnesota Point Base an expansion of $0^{in}.00020$ for 1° F. each tube-length for this tube will need a correction $(t-62)\ 0^{in}.00020$, in which t is its observed temperature.

These corrections can be summed and applied in mass instead of being applied to each tube-length for tube 1. Tube 1 was used in the measurement 663 times, and tube 2 was used 662 times.

Multiplying their normal lengths by these numbers we have—

1,325 (15-feet bar at 32° F.)$+38^{in}.151\pm0^{in}.342$.

This quantity is to be corrected by the following sums whose origin has been explained in Chapter III:

1.	Correction for inclinations of tubes	$=-21^{in}.854\pm0^{in}.000$
2.	Correction for errors in sector-level adjustment	$=-\ 0^{in}.013\pm0^{in}.000$
3.	Correction for expansion of tube 1	$=+\ 0^{in}.652\pm0^{in}.033$
4.	Correction for contact-level	$=-\ 0^{in}.021\pm0^{in}.000$
5.	Correction for cylindrical surface	$=+\ 0^{in}.039\pm0^{in}.000$
6.	Correction for ax	$=+\ 2^{in}.154\pm0^{in}.111$
7.	Correction for backward pressure	$=-\ 0^{in}.622\pm0^{in}.106$
8.	Correction for errors of alignment	$=-\ 0^{in}.179\pm0^{in}.000$
9.	Correction for reference of ends of tubes to ground	$=\ \ 0^{in}.000\pm0^{in}.047$
10.	Correction for change of length of tube 1	$=\ \ 0^{in}.000\pm0^{in}.182$
	Sum	$=-19^{in}.844\pm0^{in}.245$

The manner in which the quantities in this list and their probable errors have been obtained is as follows:

The first quantity is the sum of the corrections needed to reduce the lengths of the tubes in measurement to their horizontal projections. The errors in this quantity arise first from errors in the observed inclinations, and second in their computed corrections. The inclinations can be read with a probable error of a few seconds, and as the corrections on account of errors in reading would on the whole be as often positive as negative, these minute corrections are not cumulative and their sum, which under the usual law of error should be zero, may be neglected. But errors in the adjustment of the sector affect the inclinations of many tubes, with the same sign, and so are cumulative, when the inclinations of all tubes have the same sign. The sector of tube 1 was found out of adjustment only once during the measurement. It was in adjustment at the 1011th tube, but needed a correction of $+50''$ at the 1325th or last tube. Tube 2 was in adjustment at 2d tube and needed $-36''$ correction at the 164th; was in adjustment at the 166th and needed $0''$ correction at the 272d; was in adjustment at the 274th and needed $-67''$ correction at the 508th; was in adjustment at the 510th and needed $+22''$ correction at the 634th; was in adjustment at the 636th, and needed $+30''$ correction at the 1010th; was in adjustment at the 1012th, and needed $+10''$ correction at the 1324th. The observed inclinations have been corrected by aid of the assumption that the changes in these errors between two adjustments of the sector-level were proportional to the number of tubes measured in the intervals. The sectors were adjusted by bringing the two agates which form the ends of a tube into the same horizontal plane by the aid of a leveling instrument, and then, the sector-arc reading zero, by bringing the bubble of the sector-level to the middle of its scale. Repeated adjustments in this way give a range in the values of the corrections of less than twenty seconds, and we may take the probable error of adjustment as five seconds. Assuming five seconds as the probable error in any adopted inclination from this cause (which overestimates the probable error in intermediate tubes), and assuming that the average correction to the horizontal projection of the tube occurs when the tube has an inclination of 1° 30′ (which is also an overestimate, since but 63 tubes had an inclination exceeding 1° 30′), we have for the probable error in the reduction of one tube's length of 180 inches to the horizontal plane, $\pm\ 0^{\text{in}}.000114$. As the adjustments of each tube were examined seven times, there are on an average, $\frac{1325}{2\times 6}=110.4$ tubes (whether No. 1 or No. 2), measured between adjustments. The probable error in one such set will be $\pm\ 0^{\text{in}}.000114\ (110.4)$, and in the whole base $\pm\ 0^{\text{in}}.000114\ (110.4)\sqrt{6\times 2}=\pm\ 0^{\text{in}}.044$, provided, as previously assumed, that all inclinations and hence all corrections in a set of 110 tubes have the same sign. But, in fact, the two ends of the base differ but 6 feet in height, so that the sum of positive corrections must nearly equal the sum of negative corrections, leaving in any case an algebraic sum of corrections so small in comparison with $0^{\text{in}}.044$ found under the opposite supposition that it may be neglected. An estimate of the error in the corrected inclinations arising from the assumption that the error in adjustment between the adjustments was proportional to the number of tubes measured, is less easy. But since the maximum correction at any time was 67″, and since the error was zero after every adjustment, the effect will doubtless be overestimated if the probable constant error of inclination of all tubes between two adjustments to which the assumption is applied is taken as ten seconds. The assumption is made for tube 1 for one set of 157 tubes, and for tube 2 for five sets averaging 120 tubes each. Taking, as before, 1° 30′ as the angle giving an error at least as large as the average error, the error in the projection of one tube will be $\pm 0^{\text{in}}.000228$. Suppose, as an extreme case, that the actual error of inclination for tube 1 or 2 in each set was constant in each set, and, moreover, that all the inclinations in each set had the same sign, then for tube 1 the whole probable error would be $\pm\ 0^{\text{in}}.000228\ (157)=\pm\ 0^{\text{in}}.036$. For tube 2 the whole probable error would be $\pm\ 0^{\text{in}}.000228\ (120)\sqrt{5}=\pm\ 0^{\text{in}}.061$. Combining these quantities, there would result for the total probable error arising from this class of errors in inclinations $\pm\ \sqrt{(0^{\text{in}}.036)^2+(0^{\text{in}}.061)^2}$ $=\pm\ 0^{\text{in}}.071$. But, in fact, as the ends of the base differed only 6 feet in height, the number of positive inclinations and corrections would be approximately equal to those of the negative ones, so that the algebraic sum of the corrections would be very small in comparison with the $0^{\text{in}}.071$ found under an opposite supposition, and may be neglected. It is concluded, then, that the probable error in the base arising from errors in inclinations of tubes may be neglected.

The second quantity in the preceding list is the sum of the corrections to the lengths of tubes arising from imperfect adjustment of the sector-level. The quantity is very small, and its probable error is insignificant.

The third quantity is the sum of the corrections for expansion of tube 1 needed to give its length at temperatures differing from 62°, the expansion being taken as $0^{\text{in}}.00020$ for 1° F. The probable error in this expansion does not exceed $0^{\text{in}}.00001$, which would give a probable error in the third quantity of $\pm 0^{\text{in}}.033$.

The fourth quantity arises from the bubble of the contact-level not being always at the middle of its scale, and is a part of the measurement. Its probable error is insignificant.

The fifth quantity arises from errors in the adjustment of the cylindrical surface. It is very small and its probable error is insignificant.

The sixth quantity gives the sum of the quantities ax, which measure the effects of the unequal temperature-changes of the two bars in a tube, on the tube's length, for the whole base. Its error arises mainly from the errors in the quantities x.

For tube 1, $x=0^{\text{in}}.000136\pm 0^{\text{in}}.000009$, §5, and

For tube 2, $x=0^{\text{in}}.000097\pm 0^{\text{in}}.000008$.

For tube 1, then, the probable error of x is $\pm \frac{1}{15.1} x$ and for tube 2 $\pm \frac{1}{12.1} x$. If $[ax]$ be divided in the ratio of 136 to 97, the parts of $[ax]$ which arise from each tube will be given. They are $1^{in}.257$ and $0^{in}.897$. Dividing these quantities by 15.1 and 12.1, respectively, the resulting probable errors due to tubes 1 and 2 are $\pm 0^{in}.083$ and $\pm 0^{in}.074$, respectively, or for both tubes, $\pm \sqrt{(0^{in}.083)^2+(0^{in}.074)^2} = \pm 0.^{in}111$.

The seventh quantity arises from the fact that a tube already in position is slightly moved by the gentle pressure of contact when the next tube is placed in front of and against it, as explained in Chapter III, § 27. The mean of the two values given there for displacement of a tube, namely, $0^{in}.000235$, corresponds to an average height of the trestles. The total correction is the whole number of tubes, less one, multiplied by twice this quantity, or $1324 \times 2 \times 0^{in}.000235 = -0^{in}.622$. As the small displacement is difficult to measure with precision, and as it will vary with the cleanness of the apparatus, the probable error in its value is taken so large as $\frac{1}{6}$, or $\pm 0^{in}.00004$, which gives $\pm 1324 \times 2 \times 0^{in}.00004 = \pm 0^{in}.106$ for the probable error in the quantity $- 0^{in}.622$.

The eighth quantity is the sum of the corrections arising from the fact that a tube was never precisely in the direction of the base-line. Stakes were carefully set on the line 1,000 feet apart, their deviations from the line not being sufficient to introduce any error that need be considered. To place the tubes in line the base-line transit-instrument, having a large telescope, was set up over one stake and pointed at the next, and the front agate of the tube was then brought into the vertical plane of the telescope. The least distance of the telescope from any tube was about 200 feet, the greatest about 1,200 feet. At the first distance the maximum error which could be committed in putting the agate in the vertical plane may be taken as $0^{in}.05$; at the greatest distance it may be taken as $0^{in}.5$. As the base was 19,870 feet in length, there were twenty sections of about 1,000 feet each in length, and so many times will the distance of the tube from the transit vary from 1,200 to 200 feet. The maximum error in twenty observations at a distance of 200 feet being $0^{in}.05$, by the ordinary law of error the probable error will be $\pm 0^{in}.0172$. The probable deviation of a tube at the distance of 200 feet from the theodolite will then be

$$\pm \frac{0.0172 \sqrt{2}}{180 \sin 1''} = 27''.9.$$

In the same way, the probable error in placing the agate at a distance of 1,200 feet from the theodolite in the vertical plane through the base-line will be found to be $\pm 0^{in}.172$, and the probable deviation of a tube from that plane will be

$$\pm \frac{0.172 \sqrt{2}}{180 \sin 1''} = \pm 279''.$$

As the distance of the tube from the theodolite varies from 200 to 1,200 feet, the probable angular deviation of the tube varies from $27''.9$ to $279''$. To get an approximate value for the correction due to this deviation, divide the whole number of tubes in the base according to their probable deviations into ten classes of 132.5 tubes each, the probable deviation of any tube of the first class being $27''.9$, that of the second class twice that deviation, and so on. Now, considering the 132.5 tubes whose probable deviation is $27''.9$, from the law of errors the number of tubes whose deviations lie between $0''$ and $0.4\ (27''.9)$, between $0.4\ (27''.9)$ and $0.8\ (27''.9)$, &c., can be found. Assuming that the mean of the extreme deviations in each set may be applied to all the tubes of that set, the correction for one tube multiplied by the number of tubes in this set will give approximately the sum of the corrections of this set. Doing the same for the other sets and summing the results, we have the total correction, a, for the first class of 132.5 tubes whose probable deviation is $27''.9$. For successive classes of 132.5 tubes the corrections will increase as the squares of the probable deviations. $a = 0^{in}.000466$, so that the total correction is $0^{in}.179$. This correction is always negative. Its probable error may be neglected.

The ninth error arises from references of end of tube to or from the ground whenever work was closed or begun. When, at the approach of night or bad weather, the work was suspended, the terminal point of the measurement was marked by a cut in the head of a copper tack driven into the head of a stake, this head being below the ground. To fix the position of this mark, a theodolite was set up at a short distance from the front agate of the foremost tube in a plane through the end of the agate, normal to the base-line. After leveling the theodolite and pointing at the end of the agate, a mark was made on the head of the tack in a vertical plane through the end of the agate, the tack having been previously placed in the vertical plane through the base-line. In thus referring the end of a tube to the ground, experience has shown that a greater error than $0^{in}.03$ is impossible. There were twenty-two such references of tube to ground (or ground to tube when starting from tack in the morning). Assuming the ordinary law of error, we have $\pm 0^{in}.010$ for the probable error of one observation when the maximum error in twenty-two observations is $0^{in}.03$. The probable error for the whole base will be $\pm 0^{in}.01 \sqrt{22} = \pm 0^{in}.047$.

The tenth error is that which arises from the assumption that the change in length of tube 1 was proportional to the number of tubes measured. The maximum error that could arise from this assumption would occur if the total change in length, namely, $0^{in}.00232$, had occurred at the first tube measured. In that case, § 10, the adopted length of tube 1 during measurement would have been $0^{in}.01940 - 0^{in}.01830 = 0^{in}.00110$ too great. As this tube was used 663 times, the total error would be $0^{in}.0011\ (663) = 0^{in}.729$, giving a length of base that much too great. But as this change occurred probably from the loosening of screws connecting the different parts of the apparatus, a process which would be gradual, the resulting probable error in the length of the base will not be underestimated if we take it as one-fourth of the maximum possible error, or as $\pm 0^{in}.182$.

§ **12.** Summing these corrections and taking the square root of the sum of the squares of their probable errors, for the probable error of their sum, we have, when this sum-correction is applied to the length previously obtained, namely, 1,325 (15-feet bar at 32° F.) $+ 38^{in}.151 \pm 0^{in}.342$, Minnesota-Point Base = 1,325 (15-feet bar at 32° F.) $+ 18^{in}.307 \pm 0^{in}.421$.

This length is to be reduced to sea-level. The mean height of the base-line tubes during measurement, above mean tide at New York, was 613 feet, with a probable error of about one foot. The correction to sea-level is then $-6^{in}.993$. Applying it, there results—

Minnesota Point Base reduced to sea-level = 1,325 (15-feet bar at 32° F.) $+ 11^{in}.314 \pm 0^{in}.421$.

From § 14, Chapter II, we have the length of the 15-feet brass bar at 32° F. $= 179^{in}.95438 \pm 0^{in}.000120$. Hence,

MINNESOTA POINT BASE REDUCED TO SEA-LEVEL $= 238450^{in}.867 \pm 0^{in}.450$

in which, however, the computed probable error may be too small, on account of the uncertainties of the comparisons and in the method of obtaining the normal lengths of the tubes.

CHECK ON PARTS OF BASE BY TRIANGULATION.

§ **13.** A point was selected near the middle of the length of the base-line and the angles of the triangles formed by this point, the North and South Base stations at ends of the base and station Superior, were read. This enabled us to compute the length of either portion of the base from the whole length.

The measured length of the north part was	$114,277^{in}.822$
Length computed from whole line	$114,278^{in}.328$
Difference	$-0^{in}.506$
The measured length of the south part was	$124,173^{in}.046$
Computed length from whole line	$124,172^{in}.539$
Difference	$+0^{in}.507$

MINNESOTA POINT BASE COMPUTED FROM KEWEENAW BASE.

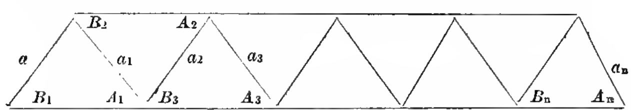
§ **14.** From the length of the Keweenaw Base, and from the adjusted angles of the triangles between the Keweenaw and Minnesota Point Bases, all the sides of these triangles can be computed, including the Minnesota Point Base itself. The length of the Keweenaw Base, when reduced to the level of the sea, is given in § 31, Chapter III, as $347,901^{in}.50$. Computing from it, with the adjusted angles in Chapter XIV, C, the length of the Minnesota Point Base, there results:

Minnesota Point Base computed from Keweenaw Base............	238, 448^{in}.32
Minnesota Point Base as measured............................	238, 450^{in}.87
Measured minus computed value........................	+2^{in}.55

The distance between the bases, measured along the axis of the triangulation, is, in round numbers, 240 miles.

The difference in the measured and computed values may arise either from errors in the measured lengths of the bases or from errors in the adjusted angles, and it is desirable to know, at least approximately, what probable error exists in the computed length of the Minnesota Point Base in consequence of the probable errors in the adjusted angles. The thorough solution of this question would require that in the process of adjustment the weights of all adjusted angles should be determined, and from them, since the Minnesota Base can be expressed as a function of the adjusted angles and the Keweenaw Base, that the weight of this function should be found. The solution involves a very large amount of labor, and it is questionable whether the value of the result is commensurate with the cost. For this reason the strict computation of the error to be expected in the computed length of the Minnesota Point Base has not been made. But a quantity which that probable error does not exceed can be found.

The probable errors of the observed angles are known. Since the observed angles are independent, the probable error in a length for the Minnesota Point Base, computed with them from the Keweenaw Base, can easily be found. Those angles have less precision before than after adjustment; hence, the probable error in the computed length of the Minnesota Base will be greater when the observed angles are used than when the adjusted angles are used, and we thus have a quantity which the probable error of the base cannot exceed, when it is computed with the adjusted angles. The following method may be used to obtain the probable error in the resulting length of the Minnesota Point Base if computed with the observed angles, from the Keweenaw Base, taken as exact:



In a triangulation in which a base a is given, from which, with the observed angles A_1, B_1, A_2, B_2, &c., the side a_n is to be computed, we have

$$a_n = a\frac{\sin B_1 \sin B_2 \sin B_3 \ldots . \sin B_n}{\sin A_1 \sin A_2 \sin A_3 \ldots . \sin A_n}$$

or, taking logarithms,

$$\log a_n = \log a + \log \sin B_1 + \log \sin B_2 \ldots . - \log \sin A_1 - \log \sin A_2 - \text{\&c.}$$

Now, when a, B_1, B_2, ... A_1, A_2, &c., are all independently observed quantities, where the angles have a common probable error ρ, and a is taken as exact, we have the probable error squared

$$\text{of } \log a_n = \left(\frac{\delta(\log \sin B_1)}{\delta B_1}\right)^2 \rho^2 + \left(\frac{\delta(\log \sin B_2)}{\delta B_2}\right)^2 \rho^2 + \ldots \left(\frac{\delta(\log \sin A_1)}{\delta A_1}\right)^2 \rho^2 +, \text{\&c.}$$

The square roots of the coefficients of ρ^2, or the rates of change of log sin for change in angle, may be obtained from tables of logarithmic sines. Denote the change of log sin for the angle B_1 for one second by β_1, for B_2 by β_2, &c., for A_1 by α_1, &c.; then the above equation becomes, probable error squared of log $a_n = \Sigma_1^n (\beta^2)\rho^2 + \Sigma_1^n (\alpha^2)\rho^2$, where a_n is the required side in the nth triangle. In applying this method, as the adjusted angles differ by a very small quantity from the observed angles, either can be used.

Taking the principal chain of triangles between the Keweenaw and Minnesota Point Bases, Chapter XIV, D, it is seen that the probable errors, ρ, of an observed angle of average weight in the sections east and west of line Split Rock-Detour are $\pm 0''.58$ and $\pm 0''.33$, respectively.

Computing $[\Sigma_1^n (\beta^2) + \Sigma_1^n (\alpha^2)]\rho^2$ for each of these sections (in which the numbers from 1 to n refer to the successive triangles in each section), and taking the square root of their sum there results for the probable error of either base computed from the other taken as exact, ± 54.1 in units of the 7th place of logarithms. Hence, the probable error in the length of the Minnesota Point Base when computed from the Keweenaw Base taken as exact, with the observed angles, is $\pm 2^{in}.97$. Since the adjusted angles are more precise than the angles resulting directly from observation, the probable error of the length of the Minnesota Point Base, computed from Keweenaw Base with the adjusted angles, would be less than $\pm 2^{in}.97$.

Comparing this probable error with the $2^{in}.55$ already found for the difference between the measured and computed lengths of the Minnesota Base, it will be seen that that difference can be attributed to the small inaccuracies in the values of the angles in the chain connecting the two bases.

In Chapter XIV, C, the approximate ratios of the probable errors of an observed and an adjusted angle, for the two sections of the triangulation between the Keweenaw and the Minnesota Point Bases are given as 0.61 and 0.60. If the adjusted angles were independent of each other, by using the mean of these two ratios the probable error in the Minnesota Point Base resulting from the probable errors of the adjusted angles would be $1^{in}.79$ in place of the $2^{in}.97$ above.

CHAPTER V.

FOND DU LAC BASE.

DESCRIPTION.

§ **1.** This base-line is situated about 10 miles southwest from Fond du Lac, Wis., in a gently rolling country with little timber. Its eastern end is approximately in latitude 43° 43′ N. and longitude 88° 29′ W. from Greenwich. The azimuth of the west end of the base from the east end is 78° 15′ west of south, and the distance approximately 24,355 feet. The stones marking the ends and middle of the base are described in Chapter XV, A, § 2. The top of the stone marking the east end of the base is 3 feet below ground, and this top is 827.3 feet above mean tide at New York. This elevation is derived from the mean elevation of Lake Michigan above tide, given in the Lake-Survey Report for 1878 as 582 feet, and from three lines of levels from Fond du Lac by river and by railroads respectively to Depere, Sheboygan, and Milwaukee. There is a range in the resulting heights above Lake Michigan of 4 feet and a probable error in the adopted value, 245.3 feet, of about 2 feet. The ground at the west end of the base is 8 feet above that at the east end. The highest point of ground on the line is 33 feet above and the lowest point 1 foot below the ground at East Base. The inclination of no tube exceeded 2°.

This base was measured in August, September and October, 1872, with the Bache-Würdemann apparatus of the Lake Survey, already partially described in Chapter III. The measurement was carried on by Assistant Engineer E. S. Wheeler, aided by Assistant Engineers Clark Olds and C. F. Burton. The method of measurement was essentially the same as for the Minnesota Point Base, given in Chapter IV.

The lengths of the measuring-tubes Nos. 1 and 2 were determined by comparisons with the 15-feet standard brass bar of the Lake Survey at the temperature of the air, and at the suggestion of Mr. Wheeler they were made at times when this bar reached its minimum temperature for the day as well as when it reached its maximum temperature. The comparisons of the measuring-tubes with the bar extended from July 29 to August 12, 1872, before the beginning of the measurement of the base; from September 7 to September 11, near the middle of the measurement; and from October 11 to October 18, after its completion. They were made in a small wooden building.

The measurement began on August 15, 1872, and closed on October 4, 1,624 tubes, each nearly 15 feet long, giving approximately the length of the base-line. The first 68 tubes were measured twice at the beginning of the measurement and four times more after the close of the main measurement, or six times in all. The measuring-tubes were not covered by tents during the measurement, and the trestles supporting the tubes rested directly on the earth, which had been rolled after removing the sod, without the intervention of supporting stakes, except for the rear trestles on the west half of the line and for the last three measurements of 68 tubes.

The expansions proper of both base-tubes are taken as zero for this base. (See Chapter IV, § 5.)

The methods of measurement and of comparisons have already been given in Chapters III and IV.

METHOD OF OBTAINING NORMAL LENGTHS OF TUBES.

§ **2.** It has previously been stated that comparisons of the measuring-tubes with the 15-feet bar were made at both maximum and minimum daily temperatures for this base. Experience makes it certain that at neither maxima- nor minima-temperatures do the tubes have their normal lengths, *i. e.*, the lengths they have when the brass and iron bars in a tube have precisely the same temperature. The first problem is to find for each tube its normal length from the comparisons at maxima- and minima-temperatures. Now, the normal lengths of these tubes were found in terms of the brass bar by an independent process at the Keweenaw Base, and the lengths of the tubes at maxima- and minima-temperatures were also found in terms of the brass bar. Taking for each tube for the Keweenaw Base the mean of its lengths as determined by comparisons at maxima-temperatures and the mean of its lengths resulting from comparisons at minima-temperatures, it was found that the mean of the two differed little from the value found for the normal length of a tube during the period in which there was no permanent change in length of either tube. For the Keweenaw Base, then, if this difference had been applied for a tube to the mean of the lengths derived from maxima- and minima-comparisons, the result would have been the normal length of a tube for that base. For the Fond du Lac Base the normal length of a tube will be found from the mean of its lengths at maxima- and minima-temperatures by applying to that value the correction which would have given at the Keweenaw Base the normal lengths from the mean of the lengths of a tube at maxima- and minima-temperatures.

The following table gives the results of comparisons of tubes 1 and 2 with the 15-feet brass bar at maxima- and minima-temperatures at Keweenaw Base. The expansion of the tubes is taken as zero (see Chapter III, § 8), and that of the brass bar as $0^{in}.001786$* for 1° F. between 32° and 62°. (See Chapter II, § 14.) With this last value the length of the 15-feet brass bar has been reduced to its length at 32°.

Comparisons at Keweenaw Base.

MAXIMA-COMPARISONS.

Date.	Time of maximum length of bar.	Tube.	Corrected temperature at max. or min. length of bar.	Tube at 62° minus bar at 32° F.
1873.	h. m.		° F.	in.
Aug. 1	6 08 p. m.	1	64. 8	0. 01860
Aug. 2	4 37 p. m.	1	64. 5	0. 01791
Aug. 9	5 09 p. m.	1	63. 6	0. 01746
Oct. 13	3 45 p. m.	1	54. 8	0. 01903
Oct. 14	1 19 p. m.	1	53. 0	0. 01831
		Mean.	60. 2	0. 01826
July 25	6 03 p. m.	2	72. 0	0. 04292
July 26	4 55 p. m.	2	69. 3	0. 04349
July 29	6 22 p. m.	2	68. 9	0. 04348
July 31	6 00 p. m.	2	61. 8	0. 04338
		Mean.	68. 0	0. 04332

MINIMA-COMPARISONS.

Aug. 11	5 37 a. m.	1	57. 96	0. 01655
Aug. 12	6 06 a. m.	1	57. 55	0. 01596
Oct. 14	6 06 a. m.	1	37. 55	0. 01451
Oct. 15	6 24 a. m.	1	45. 70	0. 01540
		Mean.	49. 69	0. 01560
July 26	5 18 a. m.	2	58. 89	0. 04250
July 30	6 20 a. m.	2	57. 93	0. 04301
		Mean.	58. 41	0. 04275

Taking from this table for tube 1 the mean of the mean maxima- and mean minima-lengths it is seen to be

$$\text{Tube } 1 = (\text{15-feet brass bar at } 32°\text{ F.}) + 0^{in}.01693$$

In the same way we have

$$\text{Tube } 2 = (\text{15-feet brass bar at } 32°\text{ F.}) + 0^{in}.04303$$

But the normal lengths of these tubes for the Keweenaw base were (Chapter III, § 15):

$$\text{Tube } 1 = (\text{15-feet brass bar at } 32°\text{ F.}) + 0^{in}.01701$$

$$\text{Tube } 2 = (\text{15-feet brass bar at } 32°\text{ F.}) + 0^{in}.04314$$

* The misprinted value $0^{in}.001787$, taken from the last line of page 1127, Lake-Survey Report for 1877, was used instead of $0^{in}.001786$. The effect on the length of the base is so slight, however, that the original computation has not been changed.

Hence, to reduce the mean of maxima- and minima-lengths of tube 1 to its normal length the correction $0^{in}.01701-0^{in}.01693=+0^{in}.00008$ must be applied to the mean of the maxima- and minima-lengths. For tube 2 the corresponding correction is $0^{in}.04314-0^{in}.04303=+0^{in}.00011$.

Applying these corrections to the mean of maxima- and minima-comparisons for each tube at the Fond du Lac Base, their normal lengths will result. If the mean temperature-curves during maxima- and during minima-comparisons could be shown to have the same form precisely, it might be expected that at maxima- and minima-temperatures the tube-length would differ from the normal length by equal amounts in opposite directions, so that the mean of the mean maxima- and mean minima-lengths might be taken as the normal length.

Possibly this method might give results as nearly correct as that followed. The small amounts found for the corrections, namely, $0^{in}.00008$ and $0^{in}.00011$, indicate that the assumption would not be much in error.

§ **3.** The following table gives the results of comparisons of tubes with 15-feet bar at the Fond du Lac Base.

Comparisons at Fond du Lac Base of tubes 1 *and* 2 *with* 15-*feet brass bar in air at maxima- and minima-temperatures.*

[Length of bar reduced to 32° F., taking expansion of bar for 1° F. as $0^{in}.001786$.]*

TUBE 1, (MAXIMA).

Date.	Hour.	Corrected bar-temperature.	Tube—bar at 32°.	Means.
1872.	h. m.	° F.	in.	in.
July 29	4 51 p. m.	80.12	0.01893	
July 31	7 00 p. m.	79.00	0.02000	
Aug. 2	6 40 p. m.	70.16	0.01910	
Aug. 3	4 57 p. m.	72.44	0.01953	0.01954
Aug. 5	6 48 p. m.	77.16	0.01929	
Aug. 6	5 33 p. m.	87.00	0.02005	
Aug. 7	5 46 p. m.	84.83	0.01986	
Sept. 7	5 24 p. m.	83.85	0.01994	
Sept. 8	4 12 a. m.	70.96	0.01831	0.01889
Sept. 8	5 42 p. m.	69.90	0.01818	
Sept. 9	3 56 p. m.	75.23	0.01912	
Oct. 11	5 06 p. m.	46.60	0.01816	
Oct. 12	4 44 p. m.	52.55	0.01870	0.01857
Oct. 14	5 42 p. m.	43.50	0.01886	

TUBE 1, (MINIMA).

July 30	7 12 a. m.	67.94	0.01747	
Aug. 1	6 27 a. m.	65.25	0.01703	
Aug. 3	5 00 a. m.	59.22	0.01748	0.01731
Aug. 5	6 34 a. m.	66.70	0.01754	
Aug. 7	3 28 a. m.	71.05	0.01715	
Aug. 8	6 33 a. m.	73.23	0.01720	
Sept. 8	8 56 a. m.	67.80	0.01759	0.01750
Sept. 10	6 18 a. m.	61.46	0.01741	
Oct. 11	7 12 a. m.	31.19	0.01645	0.01647
Oct. 14	7 54 a. m.	30.32	0.01648	

* See foot-note on page 109.

Comparisons at Fond du Lac Base of tubes 1 *and* 2 *with* 15*-feet brass bar, &c.*—Continued.

TUBE 2, (MAXIMA).

Date.	Hour.	Corrected bar-temperature.	Tube—bar at 32°.	Means.
1872.	*h. m.*	° F.	*in.*	*in.*
Aug. 10	5 54 p. m	74.83	0.03844	0.03884
Aug. 12	5 05 p. m	79.10	0.03924	
Sept. 10	5 36 p. m	*78.20	0.03926	0.03926
Oct. 15	5 34 p. m	50.09	0.03924	0.03954
Oct. 16	5 12 p. m	52.40	0.03953	
Oct. 17	5 30 p. m	53.60	0.03986	
	Mean		0.03926	

* Uncertain.

TUBE 2, (MINIMA).

Aug. 10	5 50 a. m	67.75	0.03812	0.03804
Aug. 12	6 06 a. m	61.05	0.03797	
Sept. 11	5 20 a. m	68.42	0.03841	0.03841
Oct. 16	6 12 a. m	37.94	0.03716	0.03749
Oct. 18	7 24 a. m	37.96	0.03783	
	Mean		0.03790	

From these tables it will be seen that comparisons were made at three dates, namely, before the beginning of the measurements, during the measurements between the 800th and 801st tubes, and after all measurements and remeasurements had been completed. It will also be seen that the means of the maxima-comparisons at each period show a steady decrease in the length of tube 1, while its minima-comparisons at the first and last periods show the same thing, although the length at the middle period is slightly greater than at the first. The evidence is strong that tube 1 changed length, and hence the comparisons at each period will be taken as fixing its length at that period, and it will be assumed that its change of length between two such periods of comparison was proportional to the number of tubes measured.

For tube 1 before August 8, 1872, the mean value of (tube 1—bar at 32° F.) was—

	in.
From maxima-comparisons	0.01954
From minima-comparisons	0.01731
Mean	0.01842
Correction given above to reduce to normal length	+0.00008
Normal length of tube 1 prior to August 8, 1872= 15-feet bar at 32° F.	+0.01850

From comparisons between September 7 and 10, inclusive, the mean value of (tube 1—bar at 32°) was—

From maxima-comparisons	0.01889
From minima-comparisons	0.01750
Mean	0.01820
Correction to reduce to normal length	+0.00008
Normal length of tube 1 from September 7 to September 10= 15-feet bar at 32° F.	+0.01828

From comparisons between October 11 and October 14, 1872, the mean value of (tube 1—bar at 32°) was—

From maxima-comparisons	0.01857
From minima-comparisons	0.01647
Mean	0.01752
Correction to reduce to normal length	+0.00008
Normal length of tube 1 from October 11 to October 14= 15-feet bar at 32° F.	+0.01760

If the mean results of maxima- or minima-comparisons of tube 2 at the different epochs are examined in a similar way, it will be seen that the maxima-results indicate an increase in length of tube 2, while the minima-comparisons indicate a decrease, the changes in both cases being smaller than the corresponding ones for tube 1. There is no sufficient evidence that this tube changed length during the measurement of the base, and hence the mean of all the excesses of length of tube 2 over bar at 32° F., or $0^{in}.03926$, is combined with the corresponding mean for minima-comparisons or $0^{in}.03790$, giving $0^{in}.03858$. Applying the correction $+0^{in}.00011$, previously deduced, to give the normal length, there follows for the whole base—

$$\text{Normal length of tube 2} = (\text{15-feet bar at } 32^\circ \text{ F.}) + 0^{in}.03869$$

§ 4. To obtain the mean normal length of tube 1 for any part of the measurement between the first and second sets of comparisons, the change in the tube's length for each tube measured is obtained by dividing the amount of change of length of tube 1 between the comparisons by the number of times tube 1 was used in measurement between these comparisons.

Supposing that the first time tube 1 was used its length was the same as at the first comparison, then when used the mth time its increase of length will be $(m-1)$ times the change for a single tube. Finding in this way the length of tube 1, both the first and the last times it was used in a section of the base on which there were no remeasurements, its mean length for that section will be the half sum of the two. A similar process will give the length of tube 1 for any section measured between the second and third sets of comparisons.

The following table gives the mean normal length of tube 1 for each section in which the measurement was continuous, that is, in which there were no remeasurements. It also gives for each section the product of the number of times each tube was used by the mean excess of normal length of tube for that section over the length of the 15-feet brass bar at 32°. The last column gives the sums of these quantities for each section.

Section of base: number of tubes from beginning.	Number of times tube had been used since comparisons.	Change per tube.	Mean normal length for section = 15-feet bar at 32° +	Product of excess of mean normal length over that of 15-feet bar at 32° by number of tubes.	Sum for section.
		in.	*in.*	*in.*	*in.*
1—68	Tube 1, 1—34	0.000000498	0.01849	0.62872	1.94418
	Tube 2, 1—34	0	0.03860	1.31546	
1—68	Tube 1, 35—68	0.000000498	0.01848	0.62813	1.94359
	Tube 2, 35—68	0	0.03860	1.31546	
69—259	Tube 1, 69—164	0.000000498	0.01844	1.77048	5.44603
	Tube 2, 69—163	0	0.03860	3.67555	
260—321	Tube 1, 170—200	0.000000498	0.01841	0.57066	1.77005
	Tube 2, 169—199	0	0.03860	1.19939	
322—800	Tube 1, 204—442	0.000000498	0.01834	4.38317	13.66877
	Tube 2, 203—442	0	0.03860	9.28560	
801—805	Tube 1, 13—15	0.000001201	0.01826	0.05479	0.13217
	Tube 2, 9—10	0	0.03860	0.07738	
806—809	Tube 1, 16—17	0.000001201	0.01826	0.03652	0.11390
	Tube 2, 11—12	0	0.03860	0.07738	
810—940	Tube 1, 18—82	0.000001201	0.01822	1.18437	3.73791
	Tube 2, 13—78	0	0.03860	2.55354	
941—1624	Tube 1, 89—430	0.000001201	0.01797	6.14559	19.37757
	Tube 2, 84—425	0	0.03860	13.23198	
1—68	Tube 1, 431—464	0.000001201	0.01774	0.60329	1.91875
	Tube 2, 426—459	0	0.03860	1.31546	
1—68	Tube 1, 465—498	0.000001201	0.01770	0.60190	1.91736
	Tube 2, 460—493	0	0.03860	1.31546	
1—68	Tube 1, 499—532	0.000001201	0.01766	0.60052	1.91598
	Tube 2, 494—527	0	0.03860	1.31546	
1—68	Tube 1, 533—566	0.000001201	0.01762	0.59912	1.91458
	Tube 2, 528—561	0	0.03860	1.31546	

RESULTS OF COMPUTATION OF LENGTH OF BASE.

§ 5. In the following table the first three columns have been derived from the preceding table, summations having been made in some cases. The first column shows the section of the base under consideration, expressed by the number of tubes counted from the east end of the base; the second column gives the number of the measurement of that section; the third gives the sum of the normal lengths of the tubes for that section, the symbol B representing the 15-feet brass bar at 32° F.; the fourth gives the fraction of a tube-length which was measured with a scale, considered positive when the end of a tube fell short of the point to which the measurement was made; the fifth gives the sum of the corrections arising from the inclinations of the tubes, and from the readings of the contact-level and the sector-level; they are explained in Chapter III, § 18; the correction for error in cylindrical surface is somewhat uncertain, and as its value could only amount for the whole base to a few thousandths of an inch it is neglected; the sixth column gives the sum of the corrections arising from unequal temperatures of the brass and iron bars in a tube, which must be applied to the normal length of each tube measured, to reduce it to the mean actual length of the tube for the day of measurement, as explained in Chapter III, § 16; the seventh column gives the sums of the preceding quantities; these sums still need correction for errors in alignment, for backward pressure, and for reduction to the sea-level.

To obtain the length of the base, the mean of the measurements of the first 68 tubes is used as the length of that part, and is added to the length of the rest of the base.

Section of base: number of tubes from beginning.	Number of measurement.	Sum of normal lengths of tubes.	Closure.	Instrumental corrections.	[ax].	Sums.
		in.	*in.*	*in.*	*in.*	*in.*
1—68	1	68 B+ 1.94418	0.00	− 0.97579	+0.17952	68 B+ 1.14791
1—68	2	68 B+ 1.94360	+0.07	− 0.95824	+0.17952	68 B+ 1.23488
1—68	3	68 B+ 1.91878	+0.26	− 1.02055	+0.13476	68 B+ 1.29299
1—68	4	68 B+ 1.91738	+0.25	− 1.00620	+0.14229	68 B+ 1.30347
1—68	5	68 B+ 1.91598	+0.15	− 0.96647	+0.11271	68 B+ 1.21222
1—68	6	68 B+ 1.91459	+0.35	− 1.03506	+0.10727	68 B+ 1.33680
Mean	....	68 B+ 1.92575	+0.18	− 0.99372	+0.14268	68 B+ 1.25471
69—809	....	741 B+21.13093	....	−11.06525	+1.66828	741 B+11.73396
1—809	....	809 B+23.05668	+0.18	−12.05897	+1.81096	809 B+12.98867
810—1624	....	815 B+23.11552	....	− 8.36588	+1.64025	815 B+16.38989
1—1624	....	1624 B+46.17220	+0.18	−20.42485	+3.45121	1624 B+29.37856

§ 6. To the value of the base thus obtained, namely, 1,624 (15-feet bar at 32°) + $29^{in}.379$, some corrections are to be applied which affect the whole base uniformly, or nearly so.

The first arises from the fact that the tubes were never perfectly aligned between the ends of the base. This error has been discussed in Chapter III, § 24. Since on the Keweenaw and Fond du Lac Bases the same methods of alignment were used, and since in both the base was divided into sections of 500 feet by points carefully fixed on the line, the errors from this cause will be sensibly proportional to the lengths of the two bases, and the error for the Fond du Lac Base will be $\frac{1624}{1933}$ times that at the Keweenaw Base (where it was $-0^{in}.069$), or $-0^{in}.058$.

The second correction arises from the fact that when a tube is placed in front of another, in measuring, and brought into contact with it, the slight pressure of contact, amounting to four ounces, pushes the rear tube to the rear. This error is discussed in Chapter III, § 27, where the mean of the extreme values for trestle at maximum and at minimum height is $0^{in}.00024$. Doubling this and multiplying by the number of tubes in the base, less one, or by 1,623, the resulting correction is found to be $-0^{in}.779$.

The third correction is that needed to reduce the measured base to its length at the level of the sea. The mean height of the base above its east end results from the measurement of the base. The height of the east end above Lake Michigan was derived by levels over three routes; one down the Fox River to the supposed level of Green Bay; one obtained from the officers of the railroad leading to Sheboygan; and one from the officers of the railroad leading to Milwaukee. The height of Lake Michigan (high water of 1838) above mean tide at New York, is given in the Lake-Survey Report of 1878 as 584.98 feet. There was a range of four feet in values obtained for the height of top of underground marking-stone at east end of base above Lake Michigan by the different routes, owing to the inaccuracy of the levels and to some uncertainty as to the stage of Lake Michigan to which the levels were referred. The adopted value was 245.3 feet. The mean height above tide adopted for the tubes during the measurement of the base is 854.7 feet $\pm$ 2.0 feet. It gives a correction of $-11^{in}.921$ to the measured length to obtain the length at sea-level.

Applying the first two corrections there results for the length of the base not at sea-level, 1624 (15-feet brass bar at 32°) $+ 29^{in}.379 - 0^{in}.058 - 0^{in}.779$.

In Chapter II, § 14, the length of the 15-feet brass bar at 32° F. is given as $179^{in}.95438$. Hence, the length of the Fond du Lac Base not at sea-level is $292274^{in}.455$.

Applying the reduction to sea-level ($-11^{in}.921$), there results,

FOND DU LAC BASE REDUCED TO SEA-LEVEL $= 292262^{in}.534$

ESTIMATE OF PROBABLE ERROR IN LENGTH OF BASE.

§ **7.** It remains to estimate the probable errors which enter the values just given and to combine them so as to obtain an approximate idea of the probable error in the whole base. The first error to be considered is that of the value $46^{in}.1722$, given in the third column of the table in § 5. It arises from the uncertainties in the values adopted for the normal lengths of tubes 1 and 2.

The excess of length of a tube at the hour of maximum daily temperature over its normal length arises from the changes of temperature during the day, which affect the temperatures of the brass and iron bars in the tube unequally. Its amount depends on the rate and amount of temperature-change for several hours before the time of maximum-temperature. In taking the difference between the mean of lengths at maxima- and minima-temperatures and the normal length to be the same at the Minnesota Point Base and at the Keweenaw Base, it has been tacitly assumed that comparisons at both bases, and at both maxima- and minima-temperatures, extended over days enough to determine with precision a mean maximum and a mean minimum length. The differences between the separate results and their means are due almost entirely to the fluctuations in the form of the daily temperature-curve. These fluctuations are governed by no law that we can express, and the variations in the differences must, therefore, be treated as accidental errors, in any attempt to estimate the probable error in the deduced normal length of a tube. If maxima-comparisons were made on many days, and the mean of the resulting excesses of tube-length over its normal length were taken, this mean would approach a value that further comparisons would but slightly change, inasmuch as in many days the average of the daily temperature-curves would differ little from its mean form. As comparisons were not made on many days, an estimate of the error in the assumption that the mean result of the comparisons made is the same as if they had extended over many days, is needed.

First, the probable deviation from the mean or the probable error for the result of any day's maximum-comparison is to be found for each tube. Taking the tables in §§ 2 and 3, the mean results of maxima-comparisons of tube 1 for each period at Keweenaw and Fond du Lac Bases, in which the tube did not change length, and comparing the separate results in each period with the mean for that period, a series of errors, λ, is obtained for tube 1. If these errors are m in number, and if there are n such periods of comparison, the probable error for one comparison will be $p.\ e. = 0.6745\sqrt{\frac{[\lambda\lambda]}{m-n}}$. The same process applied to both tubes, and for minima- as well as maxima-comparisons, gives the following results for one comparison:

For tube 1, maxima-comparisons, $m=19$, $n=4$; $p.\ e. = \pm 0^{in}.000387$
For tube 1, minima-comparisons, $m=14$, $n=4$; $p.\ e. = \pm 0^{in}.000337$
For tube 2, maxima-comparisons, $m=10$, $n=2$; $p.\ e. = \pm 0^{in}.000274$
For tube 2, minima-comparisons, $m=\ 7$, $n=2$; $p.\ e. = \pm 0^{in}.000301$

Taking now for either tube and for either maxima- or minima-comparisons the proper probable error just given, and dividing it by the square root of the number of comparisons made in any period under consideration, the result will be the probable error in the mean of the results for that period. From the table in § 2, the probable error in the mean result of maxima-comparisons of tube 1, from August 1, 1873, to October 14, is thus found to be $\pm 0^{in}.000173$, while for minima-comparisons between August 11 and October 15 it is found to be $\pm 0^{in}.000169$. Hence we have for the mean of mean maxima- and mean minima-comparisons of tube 1 at Keweenaw Base—

$$\text{Tube 1 minus 15-feet bar at } 32^\circ \text{ F.} = 0^{in}.01693 \pm 0^{in}.000121$$

Following the same process, there results—

$$\text{Tube 2 minus 15-feet bar at } 32^\circ \text{ F.} = 0^{in}.04303 \pm 0^{in}.000127$$

The normal lengths of these tubes for the period under consideration are given in Chapter III, § 15, as

$$\text{Tube 1 minus 15-feet bar at } 32^\circ \text{ F.} = 0^{in}.01701 \pm 0^{in}.00012$$
$$\text{Tube 2 minus 15-feet bar at } 32^\circ \text{ F.} = 0^{in}.04314 \pm 0^{in}.00014$$

Since the corrections to reduce mean of maxima- and minima-lengths to normal lengths, which have already been given in § 2 as $+0^{in}.00008$ for tube 1 and $+0^{in}.00011$ for tube 2, are obtained by taking the differences at the Keweenaw Base between the mean of maxima- and minima-lengths and normal lengths, the probable error (derived from the probable errors of these quantities) of the correction $+0^{in}.00008$, is $\pm 0^{in}.000170$, and of the correction $+0^{in}.00011$, is $\pm 0^{in}.000189$. Having found the probable errors of the corrections which are to be applied for each period at the Fond du Lac Base in which a tube did not change length, to the mean of the mean maxima- and mean minima-lengths, in order to obtain the normal lengths of the tubes for those periods, it remains to find the probable errors in these means of maxima- and minima-lengths. The probable error of the result from one maximum-comparison of tube 1 has already been found to be $\pm 0^{in}.000387$. From the table in § 3 it is seen that in the first period (July 29 to August 8) there were 7 maxima-comparisons. Hence, for the probable error in the mean result of these comparisons, $0^{in}.01954$, there results—

$$\frac{\pm 0^{in}.000387}{\sqrt{7}} = \pm 0^{in}.000146$$

Similarly, for the probable error of the result, $0^{in}.01731$, from minima-comparisons during this period there is found—

$$\frac{\pm 0^{in}.000337}{\sqrt{6}} = \pm 0^{in}.000138$$

Combining these probable errors there results for the probable error of the mean of maxima- and minima-comparisons for this period (namely, $0^{in}.01842$) the value $\pm 0^{in}.000100$. Since the normal length is obtained by adding $+0^{in}.00008 \pm 0^{in}.000170$ to the above value, the probable error in the normal length of tube 1 is $\pm 0^{in}.000197$. A similar process will give the probable error in the adopted normal lengths of tube 1 between September 7 and September 10, and between October 11 and October 14; and for tube 2 between August 10 and October 18, that is, for the whole base. The adopted normal lengths previously given may now have their probable errors added to them. Written out they are—

Tube 1 minus 15-feet bar at 32° F. (July 29 to Aug. 8) $= 0^{in}.01850 \pm 0^{in}.000197$
Tube 1 minus 15-feet bar at 32° F. (Sept. 7 to Sept. 10) $= 0^{in}.01828 \pm 0^{in}.000229$
Tube 1 minus 15-feet bar at 32° F. (Oct. 11 to Oct. 14) $= 0^{in}.01760 \pm 0^{in}.000236$
Tube 2 minus 15-feet bar at 32° F. (July 30 to Oct. 18) $= 0^{in}.03869 \pm 0^{in}.000208$

If the length of tube 1 at one of these periods of comparisons be denoted by $a \pm \alpha$ and at the next by $b \pm \beta$, if its change in length is taken as proportional to the number of times it is used, if m be the number of times tube 1 was used between these comparisons, its length when it was used the nth time would be

$$a - \frac{n}{m}(a-b) \pm \sqrt{\left(1-\frac{n}{m}\right)^2 \alpha^2 + \left(\frac{n}{m}\right)^2 \beta^2}$$

When $\frac{n}{m} = \frac{1}{2}$ this probable error becomes $\frac{1}{2}\sqrt{\alpha^2+\beta^2}$, giving for the probable error of tube 1 midway in number of times it was used, between August 8 and September 7 $\pm 0^{in}.000151$, and midway between September 10 and October 11 $\pm 0^{in}.000164$. As a sufficient approximation to the average probable error of tube 1 for this first period, it may be assumed that it varied uniformly with the number of tubes from the middle of the period to either end. Its average value will then be obtained by taking the weighted mean of its values at the beginning, middle, and end of the period, double weight being assigned to the middle value. In this way there follows for the average probable error of normal length of tube 1 during the first period (from August 9 to September 6) $\pm 0^{in}.000182$. In a similar way the corresponding quantity for the second period (September 11 to October 10) is found to be $\pm 0^{in}.000198$. The probable errors for any two tubes are not independent of each other, both depending on the same comparisons, hence to multiply the average probable error by the square root of the number of tubes measured would give too small a resulting probable error. On the other hand, to multiply it by the number of tubes would give too large a result, for the actual errors in the adopted lengths of the tubes may not all have the same sign. But to overestimate rather than underestimate the probable errors this last method will be taken. In the first period tube 1 enters the base 400 times, which multiplied by $\pm 0^{in}.000182$ gives $\pm 0^{in}.073$. In the second period tube 1 enters the base 412 times, which multiplied by $\pm 0^{in}.000198$ gives $\pm 0^{in}.082$. The two quantities $\pm 0^{in}.073$ and $\pm 0^{in}.082$ are not perfectly independent, since both depend on the errors of the middle comparisons. Hence the square root of the sum of their squares should not be taken. In order not to underestimate errors their sum $\pm 0^{in}.155$ will be taken for this tube. There is another uncertainty in the normal length at any time of tube 1. Its excess of length over the 15-feet bar at 32° F. has been taken (§ 3) as $+0^{in}.01850$ on August 8, 1872; as $0^{in}.01828$ on September 10; as $0^{in}.01760$ on October 14; and it has been assumed that its change between these comparisons was proportional to the number of times it was used. As the changes in length arise from the loosening of screws this seems a justifiable assumption. Since in effect the mean of the normal lengths obtained at two periods of comparisons has been used in computing the length of the base, the greatest possible error would arise if the total change in length had occurred during the measurement of the first or last tube of the period. The entire and greatest error for that part of the base would be obtained by taking the difference between the length of tube 1 at either beginning or end and the mean of both, and multiplying this difference by the number of times tube 1 was used in that period. For the first period we have—

$$0^{in}.01850-\frac{0^{in}.01850+0^{in}.01828}{2}=0^{in}.00011$$

This multiplied by 400, the number of times tube 1 enters this part of the base, gives $0^{in}.044$. For the second period we have—

$$412\left(0^{in}.01828-\frac{0^{in}.01828+0^{in}.01760}{2}\right)=0^{in}.140$$

These being the greatest possible errors in the two parts of the base, the probable error will be overestimated if taken as one-fourth their resultant, or as $\pm 0^{in}.036$. Taking the square root of the sum of the squares of the two errors already found, there results $\pm\sqrt{(0^{in}.155)^2+(0^{in}.036)^2}=\pm 0^{in}.159$ as the probable error in the base arising from uncertainty in the normal length of tube 1. Since tube 2 did not change length during the measurement of the base, the probable error in its normal length already given, namely, $\pm 0^{in}.000208$ multiplied by the number of times (812) it entered the base, gives $\pm 0^{in}.169$ as the probable error in the base arising from the uncertainty in the normal length of tube 2. Combining the two there results for the probable error in the base due to uncertainties in normal lengths of the two tubes, $\pm 0^{in}.232$.

§ **8.** The probable error in the mean value $(+0^{in}.18)$ of the small portions of the base measured with a scale, which are given in the fourth column of the table in § 5, is so small that it may be neglected.

The fifth column gives $-20^{in}.4248$ for the sum of the corrections arising from inclinations of the tubes, from readings of the contact-level, and from errors in sector-level adjustment. Referring to Chapter III, § 20, it will be seen that the sum of the last two corrections for the Keweenaw Base was but $-0^{in}.135$, a quantity so small that its probable error may be neglected, so that we need only consider the errors in determining the inclinations of the tubes. An error in the adjustment of the sector-level introduces error into the base in two ways: 1st, it changes the length of the tube by minute quantities; this is the quantity included in column 5 of the table; 2d, it gives slightly erroneous inclinations. The probable error in the value of the sum of the corrections needed to reduce the tube-lengths to a horizontal plane arises from the uncertainty in inclinations depending on slight errors in the adjustment of the sector-level, from changes in that adjustment, and from the error in the assumption that these changes were proportional to the time. The adjustment of the sector-level of tube 1 was examined at the 1st, 68th, 260th, 347th, 550th, 710th, 806th, 941st, 1063d, 1395th, and 1624th tubes. The sector-reading needed a correction of $+26''$ at the 260th tube and of $+20''$ at the 347th tube. At all other examinations it needed no correction. Tube 2 was examined at the same time, and was correct at the 1st, 550th, 941st, 1063d, and 1395th tubes. At the others it needed corrections, which in one case reached $50''$. There were but thirty-five tubes in the base-measurement which had an inclination exceeding $1°30'$, and the west end of the base was but 8 feet higher than the east end.

In Chapter IV, § 11, a discussion is given of the probable error in the Minnesota Base arising from error in the measurement of the inclinations, and it is found that it is so small that it may be neglected. The Fond du Lac Base has no larger errors to be corrected in its sector-readings, has but 35 tubes with inclinations exceeding $1° 30'$, and its ends are nearly at the same level. For it, then, the probable error in the measurement of the base resulting from errors in inclinations of tubes may also be neglected. The probable error of the quantity in the fifth column, namely, $-20^{in}.42485$, is therefore taken as zero.

§ **9.** The sixth column gives $+3^{in}.45121$ as the sum of the corrections to the normal lengths of all tubes, arising from the fact that from difference of temperature of the brass and iron bars in a tube its mean length for any day of measurement is greater than its normal length. The excess of mean length over normal length for any day is given by the expression ax, in which a is the temperature-increase from 8 to 12 a.m. on that day in Fahrenheit degrees and x is given in Chapter III, § 15.

$$\text{Tube 1,}\quad x = 0^{in}.000136 \pm 0^{in}.000009$$
$$\text{Tube 2,}\quad x = 0^{in}.000097 \pm 0^{in}.000008$$

These probable errors are $\frac{1}{15.1}$ and $\frac{1}{12.1}$ part of x, respectively. Since a has common values for the two tubes which are used consecutively, $\frac{136}{136+97}[ax]$ will belong to the first tube and $\frac{97}{136+97}[ax]$ will arise from the second tube. Substituting for $[ax]$ its value $3^{in}.451$, its two parts are $2^{in}.014$ and $1^{in}.437$. Dividing these by 15.1 and 12.1, respectively, there results for the probable errors arising from the tubes 1 and 2 $\pm 0^{in}.133$ and $\pm 0^{in}.119$. Hence $\sqrt{(0^{in}.133)^2 + (0^{in}.119)^2} = \pm 0^{in}.178$ is the resulting probable error for both tubes of $[ax] = 3^{in}.451$. It will be remembered that ax is always positive.

There remain to be considered the errors in the correction for alignment, for backward pressure, in the cut-offs or references of tubes to and from ground, and for height of base above mean tide.

The correction for alignment has always the same sign, and as it amounts to but $-0^{in}.058$, its probable error may be neglected.

The correction for backward pressure, explained in Chapter III, § 27, has already been given as $-0^{in}.779$, and for the reasons assigned in Chapter IV, its probable error will be taken as one-fourth its total amount, or as $\pm 0^{in}.195$.

Whenever measurement was suspended, from approach of night or other cause, the end of the tube was referred by means of a theodolite to a mark on a copper tack in the head of a stake driven into the ground. When starting again, the end of the tube was placed precisely over this mark. There were 72 such references to and from the ground. The probable error of one reference has been given in Chapter III, § 26, as $\pm 0^{in}.009$. For 72 references the probable error would be $\pm 0^{in}.076$.

There is an uncertainty of 2 feet in the height of the base above the sea-level. As the reduction to sea-level is $-11^{in}.921$ and the mean height of tubes during measurement was 854.7 feet $\pm$ 2 feet, there results a probable error of $\pm 0^{in}.028$ from this uncertainty.

During the measurement of this base on an open prairie the wind blew with such strength on many days as to cause the tubes to vibrate. On such days the measurement was stopped when to continue seemed likely to introduce serious errors, but this still left parts of the measurement effected when the wind was blowing less strongly, in which the disturbance might yet have introduced error. The wind was much stronger and steadier than on previous bases. The six measurements of the first 68 tubes were made on four days. On two of these days the wind was recorded as fresh, but it was not so strong as to stop the work. The discrepancies of these measurements should include the errors which arise from vibrations of the tubes caused by the wind. But they will also include the error in the assumed change in the length of tube 1, a part of the error in the ax correction, and the error in references of tubes to and from stakes in the ground. The first two measurements were made at the beginning of the measurement of the base, and the last four after the end. The end of the 68th tube was marked by a scratch on a copper tack in the head of a stake driven into the ground, and its stability in the six weeks' interval between the early and later remeasurements was not well assured. For this reason the two groups will be treated separately. Neglecting corrections which affect all the measurements alike, the lengths of this part of the base resulting from the different measures may be found in § 5. Taking the mean of the first two measures, and then the differeuces between this mean and the separate measures, and treating the last four measures in the same way, there results a series of residuals, λ, from which the probable error of one measure is found to be

$$\pm 0.6745\sqrt{\frac{[\lambda\lambda]}{6-2}}=\pm 0^{\text{in}}.037$$

As already stated, this error arises from several causes, of which disturbance by wind is only one, but since the work was stopped on none of these days on account of the wind, it is probable that its maximum effect is not included. On the other hand, there were numerous days on which the wind gave little or no trouble. If, then, the whole of this error be assigned to the wind, it would seem an over- rather than an under-estimate. This will be done. The effect for the whole base would be

$$\pm 0^{\text{in}}.037\sqrt{\frac{1624}{68}}=\pm 0^{\text{in}}.181$$

Thus far the 15-feet bar has been treated as exact. But as its length at 32° F. is (Chapter II, § 14) $179^{\text{in}}.95438 \pm 0^{\text{in}}.00012$, and as it is contained 1,624 times in the base, the probable error in the base arising from the probable error in its length is $\pm 0^{\text{in}}.00012 \times 1624 = \pm 0^{\text{in}}.195$.

RESULTS.

§ **10.** The different quantities which make up the length of the base can now be written out with their probable errors, and the value of the base with its probable error can be given.

		in.	*in.*
1. Sum of normal lengths of tubes	$1624\,B$	+46.17220	±0.232
2. Closures		+ 0.18000	±0.000
3. Instrumental corrections		−20.42485	±0.000
4. $[ax]$		+ 3.45121	±0.178
5. Correction for alignment errors		− 0.058	±0.000
6. Correction for backward pressure		− 0.779	±0.195
7. Correction for cut-offs		0.000	±0.076
8. Reduction to sea-level at New York		−11.921	±0.028
9. Probable error arising from wind			±0.181
10. Probable error in base arising from probable error in length of 15-feet bar at 32° F			±0.195

Substituting for B its value already given, and taking the square root of the sum of the squares of the probable errors, there results—

FOND DU LAC BASE REDUCED TO SEA-LEVEL $292262^{\text{in}}.53 \pm 0^{\text{in}}.45$

CHECKS UPON MEASUREMENT OF THE BASE.

§ **11.** The base was divided at the end of the 809th tube into two nearly equal parts. With an auxiliary station as a vertex and these two parts as bases, triangles were formed whose angles were read with the same precision as primary angles. They gave the means of computing each part of the base from the measured whole base. The results were—

East part, measured minus computed length $= +0^{\text{in}}.5$
West part, measured minus computed length $= -0^{\text{in}}.5$

the measured length of the east part being $12{,}132^{\text{ft}}.48$, and of the west part $12{,}222^{\text{ft}}.73$.

§ **12.** From the length of the Keweenaw Base, given in Chapter III, § 32, and from the adjusted angles of the triangulation (given in Chapters XIV, C, and XV, C), connecting it with the Fond du Lac Base, the length of the latter may be computed. There results

	in.
Fond du Lac Base as measured	292, 262.53
Fond du Lac Base as computed from Keweenaw Base	292, 261.37
Measured minus computed length	+1.16

The distance between the bases, measured along the triangulation, is about 320 miles. As this difference may arise either from errors in the measurement of the bases, or from errors in the values of the adjusted angles in the connecting triangulation, it is desirable to have an idea of the error which may arise from the errors in the adjusted angles alone. A method of estimating this error has already been given in Chapter IV, § 14, and the same method will be used in this case.

As seen in Chapters XIV, C, and XV, C, the triangulation connecting the Keweenaw and Fond du Lac Bases was adjusted in four parts, the first part reaching from the Keweenaw Base to the line Vulcan—Huron Mountains; the second to the line Pine Hill—Burnt Bluff; the third to the line Eldorado—Taycheedah; the fourth to the line Horicon—Minnesota Junction.

The probable error of an observed angle of the main chain in the first part, obtained from Chapter XIV, C, § 9, is $\pm0''.52$, while those of the second, third, and fourth parts are given in Chapter XV, C, § 7, as $\pm0''.42$, $\pm0''.50$, $\pm0''.43$, respectively. Using the notation in Chapter IV, § 14, there results for the different parts

$$\rho^2\Sigma_1^n\,(\beta^2+\alpha^2)\text{ for first part}=1091.68$$
$$\rho^2\Sigma_1^n\,(\beta^2+\alpha^2)\text{ for second part}=4097.10$$
$$\rho^2\Sigma_1^n\,(\beta^2+\alpha^2)\text{ for third part}=3873.82$$
$$\rho^2\Sigma_1^n\,(\beta^2+\alpha^2)\text{ for fourth part}=585.86$$

Taking the square root of the sum of these quantities, there results ±98.2 in units of the seventh place of logarithms as the probable error arising from the angles, in the length of the Fond du Lac Base when computed from the Keweenaw Base. In inches it is $6^{in}.61$. Comparing this with the difference $1^{in}.16$ actually found between the measured and computed lengths of the Fond du Lac base, it is seen that the actual discrepancy is about one-fifth its probable error arising from the uncertainties in the values of the angles alone.

In Chapter XIV, C, § 9, and Chapter XV, C, § 7, the approximate ratios of the probable errors of an observed and an adjusted angle for the four sections of the triangulation between the Keweenaw and Fond du Lac Bases are given as 0.61, 0.59, 0.56, and 0.65. If the adjusted angles were independent of each other, by using the mean of these four ratios the probable error in the Fond du Lac base resulting from the probable errors of the adjusted angles would be $3^{in}.97$ instead of the $6^{in}.61$ above.

CHAPTER VI.

SANDY CREEK BASE.

DESCRIPTION.

§ 1. This base-line is on a sand-beach about 150 feet wide, at the east end of Lake Ontario, between the Big and Little Sandy Creeks. Its direction is nearly north and south, and, as the beach is slightly concave, it was necessary to change its direction near the middle by an angle of 1° 45′, in order to obtain the needed length. It is, therefore, a broken base of two nearly equal parts. The angle between the parts was read with the same care as angles of the primary triangulation. The northern end has approximately 43° 40′ north latitude, and 76° 12′ longitude west from Greenwich. Back from the beach, sand-hills rise to the height of 80 feet, but in the measurement of the base there was no tube having an inclination greater than half a degree. On account of these sand-hills no small triangulation from one half of the base to the other was obtained. The measurement was made with the Bache-Würdemann apparatus. The ends of the base were marked by pieces of brass sunk in the heads of stones whose tops are 5 feet below the ground. These marking-stones are described in Chapter XIX, A. The base was divided into eight sections by brasses set in marking-stones. Counting from the south end of the base, these stones were at the north ends of the 138th, 266th, 405th, 523d, 661st, 797th, and 934th tubes. The length of the base is approximately 1,071 tubes, or 16,060 feet. The stone at the end of the 523d tube marks the angular point in the base-line. The sections were measured in duplicate, and counting from the south end, in the order 1, 2, 3, 4, 4, 3, 2, 1, 5, 6, 7, 8, 8, 7, 6, 5. Both the measurements and the comparisons were made under movable tents, which protected the tubes from the sun. As the soil was sand, stakes were not driven to support the foot-plates on which tube-trestles stand, as was done at the Keweenaw and Buffalo Bases.

In comparisons of the tubes with the 15-feet brass bar, the latter was kept packed in ice, and the comparisons extended through the working-day. The method of comparison has already been given in Chapter III, as well as the normal lengths resulting from these comparisons.

For alignment, points 1,000 feet apart along the base were placed with great care, by setting up a theodolite as nearly on the line between two known points as possible, and then reading the angle between the two points, from which the error in the position of the theodolite could be computed. Between these main points intermediate points 250 feet apart were lined in with a theodolite.

Comparisons extended from August 11, 1874, to August 19; measurement and remeasurement of the south half of the base from August 24 to September 12; comparisons from September 15 to September 17; measurement and remeasurement of the north half of the base from September 22 to October 15; and comparisons from October 17 to October 23. Counting only days on which some measuring was done, the average progress per day was about 1,400 feet. The measurement was made by Assistant Engineer E. S. Wheeler, assisted by Assistant Engineers C. Olds and C. Pratt. Details as to the work may be found in Assistant Engineer Wheeler's report in the Lake-Survey Report, published in the Report for 1875 of the Chief Engineer of the Army.

NORMAL LENGTHS OF MEASURING TUBES.

§ 2. As during the measurement of this base the expansion proper of the tubes was so small that it was neglected, their normal lengths at any temperature were the same as at 62° F. A tube's normal length is that when both its bars are at 62° F., as has been previously stated. From

Chapter III, § 15, the following excesses of normal lengths of tubes over that of 15-feet brass bar in melting ice for the Sandy Creek Base are taken:

TUBE 1.

Comparisons from August 13 to 19, 1874: $+ 0^{in}.01905 \pm 0^{in}.00015$
Comparisons from September 15 to 17, 1874: $+ 0^{in}.01777 \pm 0^{in}.00016$
Comparisons from October 17 to 23, 1874: $+ 0^{in}.01727 \pm 0^{in}.00017$

$$x = 0^{in}.000136 \pm 0^{in}.000009$$

TUBE 2.

Comparisons from August 13 to October 23, 1874: $+ 0^{in}.04406 \pm 0^{in}.00011$

$$x = 0^{in}.000097 \pm 0^{in}.000008$$

From Chapter III it will be seen that, while for tube 2 the results of comparisons before, after, and at the middle of the measurement agreed so closely that there was no sufficient evidence of change of length, and therefore its normal length was taken as unchanged during the measurement, on the other hand, tube 1 evidently did change length between the different sets of comparisons. In the absence of a better method, it will be assumed that the change of length of tube 1 at any instant between two comparisons was proportional to the number of times it had been used since comparison. It has been stated that this base was measured and remeasured in eight sections. With the assumption just mentioned, the normal lengths of tube 1 have been computed for each section. After the measurement of the south half of the base, in order to see over sand-hills from the station at the south end of the base, the base was prolonged southward ten tubes and another marking-stone placed. Instead of giving these tubes negative numbers, counting south from the stone first placed, they are designated by letters from A to J in order, going south.

The following table gives the excess of mean normal length of tubes over that of the 15-feet brass bar in ice for each section, and for both measurement and remeasurement, derived in the manner stated above. The first column gives the date of measurement; the second gives the numbering of the tubes which make up the base; the third gives the number of times each tube was used in the section; the fourth gives the excess for the section of the mean normal length of the tubes over that of the 15-feet brass bar at 32° F.; the fifth gives the products of the number of tubes in each section by the corresponding excesses of length; and the sixth gives the sums of these products:

Table of corrections to Sandy Creek Base due to excess of normal lengths of tubes over bar at 32°.

Date.	Section of base; number of tubes from beginning.	Number of times each tube was used.	Excess of mean normal length over that of 15-feet bar at 32° F.	Product of each excess by number of tubes.	Sum for each section.
1874.					
August 24, 25	1–128	Tube 1, 64	0.01898	1.21472	4.03456
		Tube 2, 64	0.04406	2.81984	
August 25–27	129–256	Tube 1, 64	0.01883	1.20512	4.02496
		Tube 2, 64	0.04406	2.81984	
August 28–31	257–395	Tube 1, 70	0.01868	1.30760	4.34774
		Tube 2, 69	0.04406	3.04014	
September 1, 2	396–513	Tube 1, 59	0.01852	1.09268	3.69222
		Tube 2, 59	0.04406	2.59954	
September 4–8	513–396	Tube 1, 59	0.01839	1.08501	3.68455
		Tube 2, 59	0.04406	2.59954	
September 8	395–361	Tube 1, 18	0.01830	0.32940	1.07842
		Tube 2, 17	0.04406	0.74902	
September 9	360–257	Tube 1, 52	0.01816	0.94432	3.23544
		Tube 2, 52	0.04406	2.29112	
September 10, 11	256–129	Tube 1, 64	0.01802	1.15328	3.97312
		Tube 2, 64	0.04406	2.81984	
September 11, 12	128–1	Tube 1, 69	0.01786	1.23234	4.27248
	J–A	Tube 2, 69	0.04406	3.04014	

Table of corrections to Sandy Creek Base, &c.—Continued.

Date.	Section of base; number of tubes from beginning.	Number of times each tube was used.	Excess of mean normal length over that of 15-feet bar at 32° F.	Product of such excess by number of tubes.	Sum for each section.
1874.					
September 12	A–J	Tube 1, 5	0.01778	0.08890	0.30920
		Tube 2, 5	0.04406	0.22030	
September 22, 23	514–651	Tube 1, 69	0.01774	1.22406	4.26420
		Tube 2, 69	0.04406	3.04014	
September 23–25	652–787	Tube 1, 68	0.01768	1.20224	4.19832
		Tube 2, 68	0.04406	2.99608	
September 25–October 3	788–924	Tube 1, 68	0.01762	1.19816	4.23830
		Tube 2, 69	0.04406	3.04014	
October 5	925–1061	Tube 1, 69	0.01755	1.21095	4.20703
		Tube 2, 68	0.04406	2.99608	
October 7–10	1061–925	Tube 1, 69	0.01749	1.20681	4.20289
		Tube 2, 68	0.04406	2.99608	
October 12, 13	924–788	Tube 1, 68	0.01743	1.18524	4.22538
		Tube 2, 69	0.04406	3.04014	
October 13, 14	787–652	Tube 1, 68	0.01736	1.18048	4.17656
		Tube 2, 68	0.04406	2.99608	
October 14, 15	651–514	Tube 1, 69	0.01730	1.19370	4.23384
		Tube 2, 69	0.04406	3.04014	

RESULTS OF MEASUREMENT.

§ **3.** In the following table are given the results of the measurements of the different sections of the base after applying all corrections save those arising from backward pressure of one tube upon another, from errors in alignment, and for reduction to the sea-level. These corrections being proportional to the distances measured, will be hereafter applied in mass. The origin of the corrections has been explained in §§ 16 and 18, Chapter III. The first and second columns need no explanation; the third is derived from the table in the preceding section, *B* representing the length of the 15-feet brass bar at 32° F.; the fourth gives the small distances measured with a scale in closing on a marking-stone; the fifth gives the corrections for inclinations of the tubes; the sixth gives the corrections on account of contact-level readings and non-adjustment of sector-level; the seventh gives the corrections on account of unequal temperatures of the two bars of a tube; and the eighth gives the values for the two measurements of the sections before being corrected for back pressure, for errors in alignment, or for reduction to sea-level.

Collection of corrections for Sandy Creek Base, exclusive of alignment and back-pressure corrections and reduction to sea-level.

Sections.	Direction of measurement.	Sum of normal lengths of tubes.	Closure.	Grade corrections.	Non-adjustment of sector, sector-level, and contact-level corrections.	[*ax*].	Sums.
		in.	*in.*	*in.*	*in.*	*in.*	*in.*
I	Forward	138 *B*+ 4.34376	−0.02	−0.13586	−0.00507	+0.29437	138 *B*+ 4.47720
	Backward	+ 4.27248	+0.04	−0.13466	+0.00670	+0.16295	+ 4.34747
II	Forward	128 *B*+ 4.02496	0.00	−0.20810	−0.00541	+0.25818	128 *B*+ 4.06963
	Backward	+ 3.97312	+0.04	−0.21060	+0.00794	+0.18960	+ 4.00006
III	Forward	139 *B*+ 4.34774	0.00	−0.04796	−0.00153	+0.34322	139 *B*+ 4.64147
	Backward	+ 4.31386	−0.07	−0.04794	+0.00877	+0.30258	+ 4.50727
IV	Forward	118 *B*+ 3.69222	0.00	−0.56616	+0.00243	+0.21997	118 *B*+ 3.34846
	Backward	+ 3.68455	−0.01	−0.55728	+0.00243	+0.22929	+ 3.34899
South part	Forward	523 *B*+16.40868	−0.02	−0.95808	−0.00958	+1.11574	523 *B*+16.53676
	Backward	+16.24401	0.00	−0.95048	+0.02584	+0.88442	+16.20379
Mean							523 *B*+16.37028

Collection of corrections for Sandy Creek Base, &c.—Continued.

Sections.	Direction of measurement.	Sum of normal lengths of tubes.	Closure.	Grade corrections.	Non-adjustment of sector, sector-level, and contact-level corrections.	$[ax]$.	Sums.
		in.	*in.*	*in.*	*in.*	*in.*	*in.*
V...........	Forward......	138 *B*+ 4.26420	0.00	−0.63356	−0.00736	+0.21331	138 *B*+ 3.83659
	Backward	+ 4.23384	− 0.20	−0.51880	−0.00259	+0.13737	+ 3.64982
VI...........	Forward......	136 *B*+ 4.19382	0.00	−0.30157	−0.00561	+0.22662	136 *B*+ 4.11776
	Backward	+ 4.17656	− 0.03	−0.08176	−0.00386	+0.05472	+ 4.11566
VII...........	Forward	137 *B*+ 4.23830	0.00	−0.26634	−0.00606	+0.15931	137 *B*+ 4.12521
	Backward	+ 4.22538	+ 0.20	−0.29577	−0.00269	+0.00678	+ 4.13370
VIII...........	Forward......	137 *B*+ 4.20703	−25.355	−0.41817	−0.00364	+0.24289	137 *B*−21.32689
	Backward	+ 4.20289	−25.195	−0.46007	−0.00771	+0.10122	−21.35867
North part	Forward......	548 *B*+16.90785	−25.355	−1.61964	−0.02267	+0.84213	548 *B*− 9.24733
	Backward	+16.83867	−25.225	−1.35640	−0.01685	+0.30009	− 9.45949
Mean....	....	....	....	....	....	....	548 *B*− 9.35341
Sandy Creek Base (broken)	{ Forward.....	1,071 *B*+33.31653	−25.375	−2.57772	−0.03225	+1.95787	1,071 *B*+ 7.28943
	{ Backward ...	+33.08268	−25.225	−2.30688	+0.00899	+1.18451	+ 6.74430
Mean....	....	....	....	....	....	....	1,071 *B*+ 7.01686

FURTHER CORRECTIONS.

§ **4.** The alignment of the tubes was obtained by setting stakes, 1,000 feet apart, accurately on the line. The first stake bisected one of the two parts of the base-line. The angle at it between the two ends of this part was read to within 1″, and the deviation of the stake from the line computed; then it was moved upon the line. By similar subordinate bisections, stakes 1,000 feet apart were placed on the line for the whole length of the base, and then from two such stakes intermediate stakes 250 feet apart were ranged in with theodolites. The front end of each tube was brought on the line by means of a theodolite set up on one of these stakes. The distances of the tube from the theodolite varied from 125 to 375 feet. Numerous trials were made at these two distances to determine the error in setting the end of the tube with the theodolite, the lateral deviations of the end of the tube being measured with a scale. At 125 feet distance, the mean error in setting the end of the tube was found to be $\pm 0^{\text{in}}.022$, and at 375 feet $\pm 0^{\text{in}}.083$. Assuming that this mean error varied between these limits with the distance, and remembering that the distance from 125 to 375 feet includes about 17 tubes, and also that the mean error ϵ, in setting the two ends of a tube, will give a mean error of deviation for the tube in seconds of $\pm \frac{\epsilon \sqrt{2}}{180 \sin 1''}$, the sum of the errors arising from the deviation of each tube in a set of 17 tubes can be computed. Multiplying this result by 63, the number of such sets of 17 tubes, the total correction $-0^{\text{in}}.0178$ results. Of this amount $-0^{\text{in}}.009$ belongs to the southern 523 tubes, and $-0^{\text{in}}.009$ to the northern 548 tubes.

This base being on a sand-beach had but slight variations in height, its ends differing but 7 feet in height. Hence, for the reasons assigned in Chapter III, § 25, the errors in measuring the inclinations of the tubes may be neglected.

§ **5.** The error arising from the pressure of contact between two consecutive tubes has been discussed in Chapter III, § 27. From experiments made at this base it was found that with the trestles run up six inches the rear tube was pushed to the rear by $0^{\text{in}}.00015$, and when the trestles were run up to their full height this displacement increased to $0^{\text{in}}.00031$. It is estimated that on an average the trestles were run up to a height of seven inches, and the displacement for that height is taken as $0^{\text{in}}.000175$. Multiplying this by twice the number of tubes, less one, there results for correction to whole base for backward pressure $-0^{\text{in}}.3745$, of which $-0^{\text{in}}.1830$ belongs to the southern and $-0^{\text{in}}.1915$ to the northern half, the number of tubes in the two parts being 523 and 548, respectively.

§ **6.** The mean level of Lake Ontario from 1860 to 1875, as given in Chapter XXII, § 13, was 247.25 feet above mean tide at New York. The mean height of the tubes during measurement was

9.61 feet above that plane, so that we may take 257 feet as the mean height of the base line above mean tide at New York, with a probable error not exceeding 1 foot. The values for the lengths of the southern and northern parts of the base, in terms of the 15-feet brass bar at 32° F. ($=B$), have already been given in § 3. Substituting for B its value, $179^{in}.95434 \pm 0^{in}.00012$, from Chapter II, § 19, approximate values for the lengths of the two parts are obtained. From these, the mean height of the base, its mean latitude, 43° 38′, and its azimuth, 357° 14′ west of south, the reductions to the sea-level for the southern and northern parts can be obtained. They are, respectively, $-1^{in}.158$ and $-1^{in}.213$.

RESULTING LENGTH OF BASE.

§ **7.** Collecting now the values for the southern part of the base and the additional corrections to it, there results,

	in.
Southern part of base, § 3	523 B+16.370
Correction for alignment errors, § 4	− 0.009
Correction for contact-pressure, § 5	− 0.183
Reduction to mean tide, § 6	− 1.158
Southern part of base reduced to mean tide	523 B+15.020

Or, substituting the value of B, $179^{in}.95434$, southern part of base reduced to mean tide=$94{,}131^{in}.140$.

Making a similar collection for the northern part of the base, there results,

	in.
Northern part of base, § 3....................	548 B− 9.354
Correction for alignment errors, § 4	− 0.009
Correction for contact-pressure, § 5...........	− 0.192
Reduction to mean tide, § 6....................	− 1.213
Northern part of base reduced to mean tide...	548 B−10.768

Or, substituting the value of B, northern part of base reduced to mean tide=$98{,}604^{in}.232$.

The angle between the two parts of the base measured at the Middle Base station with the same care as primary angles, was

$$178° \, 15' \, 30''.7$$

This gives a correction of $-22^{in}.244$ to the sum of the two parts of the base to obtain the distance between the end-points of the base. Hence, the length of the Sandy Creek Base reduced to mean tide is $192{,}713^{in}.128$.

PROBABLE ERROR IN ADOPTED LENGTH OF BASE.

§ **8.** In estimating the probable error of the value just given, the different sources of error will be considered in detail.

1. The probable error in the adopted length of the 15-feet brass bar is $\pm 0^{in}.00012$. Multiplying this by the number of times its length enters the base (1,071), the resulting probable error in the base is $\pm 0^{in}.129$.

2. The probable errors in the results of comparisons of tube 1 have already been given in § 2 as $\pm 0^{in}.00015$, $\pm 0^{in}.00016$, and $\pm 0^{in}.00017$ for the first, second, and third sets of comparisons. The supposition that between these comparisons the tube changed length proportionally to the number of times it had been used since comparisons (a supposition whose error will be examined hereafter), enables us to estimate the probable error in the length adopted for tube 1 at any time between comparisons, as in Chapter V, § 7. The probable error for the tube midway between the first and second comparisons would be $\pm 0^{in}.00011$, and midway between the second and third comparisons would be $\pm 0^{in}.00012$. Assuming that from these mid-values the probable errors increase toward their comparison-values proportionally to the number of tubes, we may take for the average probable error for any tube in the first measurement between the first set of comparisons and the measured tube which was midway between the first and second sets of comparisons—

$$\pm \frac{0^{in}.00015 + 0^{in}.00011}{2} = \pm 0^{in}.00013$$

In the same way for any tube of the first measurement between the second set of comparisons and the measured tube which was midway between the second and third comparisons, we have for the average probable error—

$$\pm\frac{0^{in}.00016+0^{in}.00012}{2}=\pm 0^{in}.00014$$

Multiplying $\pm 0^{in}.00013$ by 262, the number of times tube 1 entered the forward measurement of the first part of the base, there results $\pm 0^{in}.034$. Multiplying $\pm 0^{in}.00014$ by 274, the number of times tube 1 entered the second part of the base, there results $\pm 0^{in}.038$. It will be noticed that in obtaining these probable errors they have been overestimated by tacitly assuming that the actual errors in the computed lengths of all tubes have the same sign. Taking the square root of the sum of the squares of these quantities, there results $\pm 0^{in}.051$. But these two quantities are not independent, being connected by the errors of the middle comparisons, so that $\pm 0^{in}.051$ is too small; hence, their sum $\pm 0^{in}.072$ (which is too large) will be taken as the probable error in the base due to comparisons of tube 1. For tube 2, the probable error of comparisons is given in § 2 as $\pm 0^{in}.00011$. Multiplying by the number of times (535) that tube 2 entered the first measurement of the base, there results, $\pm 0^{in}.059$. Combining the results for the two tubes, there results for the probable error in the value of the base arising from comparisons, $\pm 0^{in}.093$. It will be noticed that only the forward measurement has been considered, since remeasurement could not affect quantities independent of it.

3. The correction to the normal lengths of the tubes resulting from comparisons on account of difference of temperature of the two bars in a tube is for each day of the form ax, where a is the temperature-range between 8 a. m. and 12 a. m. on that day and x a constant, whose values given in Chapter III, § 15, are for

$$\text{Tube 1,}\quad x=0^{in}.000136\pm 0.000009$$
$$\text{Tube 2,}\quad x=0^{in}.000097\pm 0.000008$$

These probable errors are $\frac{1}{15.1}$ and $\frac{1}{12.1}$ parts of the values of x for tubes 1 and 2. From § 3 the mean value of $[ax]$ for measurement and remeasurement is $+1^{in}.571$. Dividing this into two parts having the ratio of 136 to 97 (since the tubes alternate in measurement and a is common to both) the parts of $[ax]$ which arise from tube 1 and tube 2, are found to be $0^{in}.917$ and $0^{in}.654$. Dividing these respectively by 15.1 and 12.1, there result the probable errors due to tubes 1 and 2, viz., $\pm 0^{in}.061$ and $\pm 0^{in}.054$. For both tubes the probable error in $[ax]$ is then

$$\pm\sqrt{(0^{in}.061)^2+(0^{in}.054)^2}=\pm 0^{in}.081$$

4. The measurements of the amount of backward motion of a tube under the pressure of contact with the tube in front of it gave values agreeing well with those found at the Fond du Lac Base. The value adopted in § 5 is $0^{in}.000175$, and its probable error may be taken as $\pm 0^{in}.00003$. For 1,070 tubes this gives $\pm 0^{in}.064$.

5. The probable errors in the corrections for alignment-error, for reduction to sea-level, for reduction of broken base to a right line, are so small that they may be neglected.

6. The error arising from the assumption that the length of tube 1 changed proportionally to the number of times it had been used since comparison, will show its effect in the discrepancies between the measurement and remeasurement of the different sections. It is very probable that the change arose from the gradual loosening of certain connecting screws in the apparatus, and hence was some function, if not a linear one, of the number of times the tube had been used. Even if the true law was not that which has been assumed, the errors committed would in many cases have different signs in the measurement and remeasurement. As an approximation, then, it will be supposed that the effects of error in the assumption will sufficiently show themselves in the discrepancies between the measurements and remeasurements of the sections and in the resulting probable errors. This supposition will also be made for the errors in the values of the closures on the section-stones, for those in the corrections for inclination, in the corrections for contact-level and non-adjustment of sector-level, in references of end of tube to mark in ground whenever work was begun or interrupted, and for the accidental errors arising from observation, from irregular instability of the apparatus, or from other causes. The resultant, then, of all the errors enumerated under (6) will be derived from the discrepancies between measurement and remeasurement of the different sections. The mean length of a section is 134 tubes, and by reference to the table in § 2

it will be seen that for the purpose of obtaining probable errors, since they do not differ widely, they may be taken as of equal length. As there were 16 measurements of 8 sections, if the difference between two measures of a section be called d, there results for the mean error of one measure of a section $\sqrt{\frac{[d^2]}{2\,(16-8)}}=\pm 0^{in}.069$, or for the probable error, $\pm 0^{in}.047$. For a single measurement of the eight sections the probable error would be $\pm 0^{in}.131$, or for the double measurement of the whole base $\pm 0^{in}.093$.

FINAL VALUES.

§ **9.** Taking the square root of the sum of the squares of the probable errors specified, there results for the probable error in the length of the base—

$$[(0.129)^2+(0.093)^2+(0.081)^2+(0.064)^2+(0.093)^2]^{\frac{1}{2}}=\pm 0^{in}.211$$

so that we have—

SANDY CREEK BASE REDUCED TO MEAN TIDE $=192713^{in}.13\pm 0^{in}.21$

CHAPTER VII.

BUFFALO BASE.

DESCRIPTION.

§ **1.** This base-line is situated east of Buffalo, N. Y., at a distance of about 6 miles from Lake Erie. The azimuth of the line West Base – East Base is 237° 11′ west of south, and its length about 22,247 feet. Its western end has approximately 42° 57′ north latitude and 78° 53′ longitude west from Greenwich. The base runs through an open and nearly level country. The measurement was made with the Bache-Würdemann apparatus. The base was marked at the ends by stones whose description may be found in Chapter XVIII, A. Counting from the west end of the base, marking-stones were also placed at the west end of the 84th, 184th, 299th, 760th, 1029th, and 1389th tubes. There were 1,483 tubes in the whole base. The first 299 tubes were remeasured after the completion of the main measurement. The party under Assistant Engineer E. S. Wheeler, assisted by Assistant Engineers C. Pratt, C. Olds, and W. Russell, left Detroit on August 6, 1875, and returned on October 26. The base-line tubes were compared with the 15-feet standard bar packed in ice on six days before measurement, on two days after the 760th tube had been measured, and on four days after the completion of the measurement, but before the remeasurement of the first 299 tubes. The first comparisons were made between August 31 and September 3, 1875; the first 760 tubes were measured between September 7 and September 18; the second comparisons were made on September 22 and 23; the rest of the base from the 761st to the 1,483d tube was measured between September 25 and October 9; the final comparisons were between October 11 and October 14, and the first 299 tubes were remeasured between October 18 and October 22, 1875. The eastern part of the base was measured in nine days, the western in ten days, and 299 tubes were remeasured in four days. The method of measurement was the same as for Sandy Creek Base, save that at the Buffalo Base, which ran through grassy or cultivated fields, the supporting trestles for the tubes stood on iron stakes firmly driven into the ground, their heads rising a little above it. Details as to the methods of measurement, which were essentially those described in Chapter III, may be found in Assistant Engineer E. S. Wheeler's reports, published in the Lake-Survey Reports for 1875 and 1876, contained in the annual reports of the Chief of Engineers for those years.

LENGTHS OF TUBES DURING MEASUREMENT.

§ **2.** During the measurement of this base the expansion proper of the base-tubes was so small that it has been neglected, hence their normal lengths at any temperature were the same as at 62° F. As previously stated, a tube's normal length is its length when both bars are at 62° F. From Chapter III, § 15, the following excesses of normal lengths of tubes over that of 15-feet brass bar in melting ice for the Buffalo Base are taken:

Comparisons of tube 1, *Buffalo Base.*

August 31 to September 3, 1875,	$+0^{in}.02726\pm0^{in}.00017$
September 22, 23, 1875,	$+0^{in}.02682\pm0^{in}.00021$
October 11 to 14, 1875,	$+0^{in}.02591\pm0^{in}.00016$

$$x=0^{in}.000136\pm0^{in}.000009$$

Comparisons of tube 2, *Buffalo Base.*

August 28 to October 14, 1875,	$+0^{in}.04598\pm0^{in}.00012$

$$x=0^{in}.000097\pm0^{in}.000008$$

From Chapter III it is seen that while for tube 2 the results of comparisons before, during, and after the measurement of the base agreed closely, those for tube 1 differed too widely to be explained otherwise than by change of length of the tube, such as occurred during the measurements of the Keweenaw and Sandy Creek Bases. As the change in length probably arose from the loosening of connecting screws, it will be assumed to be proportional to the number of times the tube had been used since a comparison. The first set of comparisons was made before any measuring was done, the second immediately after the 760th tube had been measured, and the third immediately after the last or 1483d tube in the base had been measured. After these last comparisons the first 299 tubes were remeasured. Since comparisons of this tube (October 5–7, 1876), after its return to the office, showed that it had continued to shorten after the comparisons of October 11–14, 1875, it has been assumed that its rate of shortening during the remeasurement of these 299 tubes was the same as between the comparisons of September 22, 23, 1875, and October 11–14, 1875. Under the assumptions mentioned, the mean excess of length of tube 1 over that of brass bar in melting ice for different groups of tubes has been computed, and the mean values for each group have been multiplied by the number of times tube 1 entered the group. The same has been done for tube 2, but as it did not change its normal length, the excess of its normal length over that of the 15-feet brass bar in melting ice is constant. The results are given in the following table, in which the first column gives the date; the second column gives the section expressed in tubes counted from the west end; the third column gives the number of times the tubes were used in measurement; the fourth column gives the mean value of l, or excess of length of tube for the section; the fifth column gives the products of the excesses by the number of tubes; and the sixth column gives the sums of these products:

Table of corrections to Buffalo Base due to excess of normal lengths of tubes over bar at 32°.

Date.	Section.	Number of times each tube was used.	Mean l for section.	Product of mean by number of tubes.	Sums.
1875.					
September 7	1–84	Tube 1, 42	0.02724	1.14408	3.07524
		Tube 2, 42	0.04598	1.93116	
September 8–9	85–184	Tube 1, 50	0.02718	1.35900	3.65800
		Tube 2, 50	0.04598	2.29900	
September 11	185–299	Tube 1, 58	0.02712	1.57296	4.19382
		Tube 2, 57	0.04598	2.62086	
September 13–18	300–760	Tube 1, 230	0.02695	6.19850	16.81988
		Tube 2, 231	0.04598	10.62138	
September 25	761–766	Tube 1, 3	0.02682	0.08046	0.21840
		Tube 2, 3	0.04598	0.13794	
September 25	761–766	Tube 1, 3	0.02681	0.08043	0.21837
		Tube 2, 3	0.04598	0.13794	
September 28—October 9	767–1483	Tube 1, 359	0.02636	9.46324	25.92408
		Tube 2, 358	0.04598	16.46084	
October 18	1–84	Tube 1, 42	0.02586	1.08612	3.01728
		Tube 2, 42	0.04598	1.93116	
October 19	85–184	Tube 1, 50	0.02574	1.28700	3.58600
		Tube 2, 50	0.04598	2.29900	
October 20–22	185–299	Tube 1, 58	0.02561	1.48538	4.10624
		Tube 2, 57	0.04598	2.62086	

COMPUTATION OF LENGTH OF BASE.

§ **3.** If to the total or partial sums given in the preceding table there be added as many times the length of the 15-feet brass bar in melting ice ($=B$) as there were tubes in the whole base or in the parts, there will result a value for the length of the whole base or of its parts. These values need certain corrections whose origin has been explained in §§ 16, 18 of Chapter III.

The following table gives these corrections and applies them to the length of portions of the base obtained as just explained. It does not contain the corrections which are proportional to the distance measured and so can be applied in mass, such as the correction for backward pressure, for alignment-errors, and for reduction to mean tide. The first column gives the part of the base considered, expressed in tube-lengths from the western end; the second column gives the direction of the measurement; the third column gives the sum of the normal lengths of tubes for each section, giving a first value of lengths; the fourth column gives the small distances measured with a scale between ends of tubes and marking-stones; the fifth column gives the corrections for inclination of tubes; the sixth column gives the corrections for non-adjustment of sector-level and for contact-level; the seventh column gives the corrections for difference of temperatures of two bars in a tube; and the eighth column gives the resulting lengths of the parts of the base. As the first 299 tubes were measured twice, the mean result is taken:

Collection of corrections for Buffalo Base, except alignment and back-pressure corrections and reduction to sea-level.

Sections.	Measurement.	Sum of normal lengths of tubes.	Closure.	Grade corrections.	Non adjustment of sector-level and contact-level corrections.	[ax].	Sums.
		in.	*in.*	*in.*	*in.*	*in.*	*in.*
Tubes 1–84	Forward	84 B+ 3.07524	0.00	−0.63221	+0.00108	+0.19383	84 B+ 2.63794
	Backward	+ 3.01728	+ 0.16	−0.52168	−0.00352	+0.00336	+ 2.65544
Tubes 85–184	Forward	100 B+ 3.65800	0.00	−1.23698	+0.00570	+0.15723	100 B+ 2.58395
	Backward	+ 3.58600	+ 0.30	−1.33995	−0.00079	+0.10525	+ 2.65051
Tubes 185–299	Forward	115 B+ 4.19382	0.00	−1.08888	+0.00702	+0.21069	115 B+ 3.32265
	Backward	+ 4.10624	− 0.16	−0.70166	+0.00172	+0.21162	+ 3.45792
Tubes 1–299	Forward	299 B+10.92706	0.00	−2.95807	+0.01380	+0.50175	299 B+ 8.54454
	Backward	+10.73752	+ 0.30	−2.56329	−0.00259	+0.32023	+ 8.76387
Mean							299 B+ 8.65420
Tube 300–Mid. Base		461 B+16.81988	+115.968	−6.23570	+0.04339	+0.86064	461 B+127.45621
E. Base–Mid. Base							760 B+136.11041
Mid. Base–Tube 766	Forward	6 B+ 0.21840	−115.968	−0.16342	+0.00026	+0.00894	6 B−115.90382
	Backward	+ 0.21837	−115.968	−0.16659	−0.00027	+0.00894	6 B−115.90755
Mean							6 B−115.90568
Tube 767–W. Base		717 B+25.92408	+ 59.772	−3.79918	−0.00854	+0.71394	717 B+ 82.60230
Mid. Base–W. Base							723 B− 33.30338
Buffalo Base							1483 B+102.80703

FURTHER CORRECTIONS.

§ **4.** The other corrections to be applied will now be taken up.

1. Correction for errors of alignment. The method of alignment was the same as at the Sandy Creek Base described in Chapter VI, § 4. As this correction is proportional to the length of the base its value may be obtained by multiplying its value for the Sandy Creek Base, $-0^{in}.017$, by the ratio of the two bases ($\frac{1483}{1071}$), which gives as correction for alignment for Buffalo Base, $-0^{in}.024$.

The ends of the base differed but 12 feet in elevation, and hence, for reasons assigned in Chapter III, § 25, the correction on account of errors of inclination may be neglected.

2. The error arising from a tube's pressure against the preceding one when in contact with it has been considered in Chapter III, § 27. It is estimated that during the measurement the trestles were run up on an average seven inches above their lowest position, and the displacement as obtained by experiment was, for this height, $-0^{in}.000233$. For the whole base this gives $0^{in}.000233 \times 2 \times 1482 = -0^{in}.690$.

3. The height of the mean surface of Lake Erie between 1860 and 1875, inclusive, is 573.6 feet ±0.35 feet. (See Chapter XXII, § 13.) The mean height of the tubes during measurement of the Buffalo Base above this mean lake-surface was 35.7 feet, giving 609.3 feet as the height of the base above mean tide, with a probable error of about 1 foot. The correction for reduction to mean tide at New York is, then, $-7^{in}.770$.

The sum of the last three corrections is $-8^{in}.484$, which applied to the value for the base given in the table of § 3, gives for the base, 1,483 $B+94^{in}.32$. Substituting for B its value, $179^{in}.95434 \pm 0^{in}.00012$, from Chapter II, § 19, there results for Buffalo Base reduced to mean tide at New York, $266,966^{in}.61$.

PROBABLE ERROR IN ADOPTED LENGTH OF BASE.

§ **5.** 1. Errors in lengths of tubes arising from comparisons. The method of § 7, Chapter V, will be used in estimating these errors. The remeasurement of the first 299 tubes will be neglected. In § 2 the probable errors of comparisons of tube 1 at the beginning, middle, and end of measurement have been given as $\pm 0^{in}.00017$, $\pm 0^{in}.00021$, and $\pm 0^{in}.00016$, respectively. For a tube midway between first and second comparisons the probable error would be

$$\frac{\sqrt{(0^{in}.00017)^2+(0^{in}.00021)^2}}{2}=\pm 0^{in}.000135$$

so that

$$\frac{0^{in}.00017+2\ (0^{in}.000135)+0^{in}.00021}{4}=\pm 0^{in}.00016$$

may be taken as the average probable error of any tube measured between the first and second comparisons. As tube 1 was used in this interval 380 times, the resulting probable error may be taken as 380 $(\pm 0^{in}.00016)=\pm 0^{in}.061$, which somewhat overestimates its value for reasons stated in Chapter V, § 7. In the same way the probable error in a tube midway between the second and third comparisons is $\frac{1}{2}\sqrt{(0^{in}.00021)^2+(0^{in}.00016)^2}=\pm 0^{in}.000132$, and the average probable error for any tube measured between the second and third comparisons is

$$\frac{0^{in}.00021+2\ (0^{in}.000132)+0^{in}.00016}{4}=\pm 0^{in}.00016$$

As tube 1 was used 362 times in this interval, there results for the product of these quantities $\pm 0^{in}.058$, also an overestimate. Combining the two resulting errors for tube 1 as if they were independent, we have $\sqrt{(0^{in}.061)^2+(0^{in}.058)^2}=\pm 0^{in}.084$. But these errors both depend on the probable error of the middle comparisons, and hence are not independent of each other, and the result is too small. On the other hand, if the two quantities were added, giving $0^{in}.061+0^{in}.058=0^{in}.119$, the result would be too great. Not to underestimate errors, the greater value, $\pm 0^{in}.119$, is taken as the probable error in the length of the base arising from errors of comparisons. From § 2 the probable error in the result of comparisons of tube 2 with the 15-feet bar is given as $\pm 0^{in}.00012$. This tube entered the base 741 times. Multiplying $\pm 0^{in}.00012$ by this, there results for error in base arising from the comparisons of tube 2, $\pm 0^{in}.089$. Combining this with the corresponding quantity for tube 1, there results $\sqrt{(0^{in}.119)^2+(0^{in}.089)^2}=\pm 0^{in}.149$, as the probable error in the base resulting from errors of comparisons of tubes with 15-feet brass bar.

2. The correction to the base arising from difference of temperature of the constituent bars of the tubes is $[ax]$, and is for each day of the form ax, in which a is the temperature-range from 8 to 12 a. m. of that day, and x has the values given in Chapter III, § 15, namely:

For tube 1, $x=0^{in}.000136\pm 0^{in}.000009$
For tube 2, $x=0^{in}.000097\pm 0^{in}.000008$

These probable errors are respectively $\frac{1}{15.1}$ and $\frac{1}{12.1}$ parts of the values of x for the two tubes. Since a is common to the two tubes, which are used alternately in measurement, the part of ax $(=2^{in}.465)$ due to each tube will be found by dividing it into two parts having the ratio of 136 to 97. The part appertaining to tube 1 is $1^{in}.439$, and to tube 2, $1^{in}.026$. Dividing these by 15.1 and 12.1, the probable errors $\pm 0^{in}.095$ and $\pm 0^{in}.084$ result for tubes 1 and 2. Taking the square root of the sum of their squares, we have the probable error in $[ax]$ for the whole base, $\pm 0^{in}.128$.

3. The correction for pressure of contact of one tube against another has been given in § 4 as $-0^{in}.000233$ for a single contact and $-0^{in}.690$ for the whole base. Although the values derived for different bases agree well, it is not thought best to estimate its probable error at less than one-sixth part of its value, or $\pm 0^{in}.00004$. This gives for the whole base

$$\frac{\pm 0^{in}.686}{6}=\pm 0^{in}.114$$

4. The probable errors of the corrections for alignment-error, for contact-level and non-adjustment of sector-level, for measurements in closures, and for reduction to sea level are so small that they may be neglected.

5. The assumption that the changes of length of tube 1 between comparisons were strictly proportional to the number of times it had been used since comparison is without doubt not quite exact. If the whole change in length had taken place at once and midway between the comparisons, the errors resulting from the assumption in that part of the base would have been zero. If the whole change had taken place immediately after the first comparisons or immediately before the second, the error resulting would have been a maximum. The lengths resulting from the three sets of comparisons, § 2, being $B+0^{\text{in}}.02726$, $B+0^{\text{in}}.02682$, and $B+0^{\text{in}}.02591$, and since approximately in effect the mean value, $B+0^{\text{in}}.02704$, has been used in computation of tubes measured between first and second comparisons, and $B+0^{\text{in}}.02637$ for tubes measured between second and third comparisons, if the whole changes had taken place immediately after the first and second comparisons the values used for the tube in the two parts of the base would have been respectively $0^{\text{in}}.02704-0^{\text{in}}.02682=0^{\text{in}}.00022$, and $0^{\text{in}}.02637-0^{\text{in}}.02591=0^{\text{in}}.00046$ too small. Tube 1 was used 380 times in the first part of the base, giving $380\times 0^{\text{in}}.00022=0^{\text{in}}.084$ as the maximum error possible in that part of the base. Similarly tube 1 was used 362 times in the second part of the base, giving $362\times 0^{\text{in}}.00046=0^{\text{in}}.167$ as the maximum possible error in that part of the base, or, summing, giving $0^{\text{in}}.251$ as the maximum possible error in the whole base arising from change of length of tube 1.

By reference to Chapter VI, § 8, it will be seen that a similar assumption as to changes in length of tube 1 gives no large discrepancies between the measurement and remeasurement of the Sandy Creek Base, and as the change is doubtless due to the loosening of connecting screws, and so is gradual, if one-fourth the maximum error possible be taken as the probable error in the base from this cause it will be ample. It is $\pm 0^{\text{in}}.063$.

6. In Chapter III, § 26, the probable error in referring the end of a tube by means of a theodolite to a mark on a peg in the ground, whenever work was suspended, is given as $\pm 0^{\text{in}}.009$. In the measurement of the Buffalo Base there were 42 such references from tube to ground at stopping, or from ground to tube at starting work. $\pm 0^{\text{in}}.009\sqrt{42}$ gives for the probable error in the base from this cause, $\pm 0^{\text{in}}.058$.

7. The probable error in the length of the 15-feet standard bar at 32° (§ 4) is $\pm 0^{\text{in}}.00012$; and since its length enters the base 1,483 times, the probable error in the base resulting from the probable error in the length of the bar is $\pm 0^{\text{in}}.178$.

FINAL VALUE FOR BUFFALO BASE.

§ **6.** Collecting the probable errors specified under these seven heads, there results for the probable error in the base

$$\sqrt{(0.149)^2+(0.128)^2+(0.114)^2(+0.063)^2+(0.058)^2+(0.178)^2}=\pm 0^{\text{in}}.30$$

Hence (§ 4) the length of the

BUFFALO BASE REDUCED TO MEAN TIDE AT NEW YORK$=266966^{\text{in}}.61\pm 0^{\text{in}}.30$

CHECK ON PARTS OF BASE.

§ **7.** The marking-stone at the end of the 760th tube divided the base into two not very unequal parts. An auxiliary station was so selected that it and the two parts of the base gave well-conditioned triangles, and the angles of these two triangles were read and adjusted in the same way as the other angles of the primary triangulation. On computing from the whole base by means of these triangles the eastern part of the base,

Computed length	$136,898^{\text{in}}.13$
Measured length	$136,897^{\text{in}}.09$
Measured minus computed length	$-1^{\text{in}}.04$

Computing the western half of the base in the same way,

Computed length	$130,068^{\text{in}}.53$
Measured length	$130,069^{\text{in}}.59$
Measured minus computed length	$+1^{\text{in}}.06$

The discrepancies may be mainly attributed to errors in the angles.

COMPARISON OF MEASURED LENGTH OF BUFFALO BASE WITH ITS LENGTH COMPUTED FROM SANDY CREEK BASE.

§ 8. From the length of the Sandy Creek Base, given in Chapter VI, § 9, and with the adjusted angles of the triangulation connecting these bases, given in Chapters XVIII, C, § 4, and XIX, C, § 4, the length of the Buffalo Base can be computed. There results

Buffalo Base as measured..........	$266,966^{in}.61$
Buffalo Base as computed from Sandy Creek Base ..	$266,965^{in}.17$
Measured minus computed length	$+1^{in}.44$

The distance between these bases, measured along the axis of the triangulation, is about 210 miles.

The probable error of an observed angle in the first section of the triangulation is given in Chapter XIX, C, § 5, as $\pm 0''.52$, and for the second section is given in same chapter and § as $\pm 0''.50$.

Using the notation explained in Chapter IV, § 14, there results for the different parts:

$\rho^2\Sigma_1^n(\beta^2+\alpha^2)$ from Sandy Creek Base to Falkirk - Pekin..	3,262.6
$\rho^2\Sigma_1^n(\beta^2+\alpha^2)$ from Falkirk - Pekin to Buffalo Base	995.6

Taking the square root of the sum of these, there results ± 65.3 in units of the seventh place of logarithms as the probable error in the length of the Buffalo Base when computed from the Sandy Creek Base, arising from errors in the observed angles. In inches it is $\pm 4^{in}.01$. Comparing this with the difference actually found between the measured and computed lengths of the Buffalo Base, $1^{in}.44$, it is seen that the actual discrepancy is about one-third of the probable error derived from the uncertainties in the values of the observed angles alone.

In Chapters XVIII, § 5, and XIX, § 5, the approximate ratios of the probable errors of an observed and an adjusted angle for the two sections of the triangulation between the Sandy Creek and Buffalo Bases are given as 0.59 and 0.58. If the adjusted angles were independent of each other, by using the mean of these two ratios the probable error in the Buffalo Base resulting from the probable errors of the adjusted angles would be $2^{in}.34$ instead of the $4^{in}.01$ above.

CHAPTER VIII.

DESCRIPTION OF THE REPSOLD BASE-MEASURING AND COMPARING APPARATUS AND THE STANDARDS APPERTAINING THERETO.

§ **1.** The Repsold base-apparatus consists of a measuring-bar of steel, approximately 4 metres long. Its exact length at any temperature is known. By the side of the steel bar is a similar zinc bar. The two are fastened firmly together in the middle. Their unequal expansion is observed upon scales at both ends, making a metallic thermometer by which the temperature of the steel bar becomes known. These two bars are placed within a hollow iron cylinder called the "tube-cylinder," which supports them rigidly and protects them from sudden changes of temperature. The bars are supported in the cylinder by a system of rollers, which keeps them straight, parallel, and at a constant distance from each other. The combination of the two bars and the tube-cylinder is called a "tube." The tube is provided with a sector which indicates the deviation of the tube from the horizontal, so that a base can be measured upon slightly inclined as well as upon level surfaces. A telescope is also attached, which points in the same direction as the tube and enables consecutive tube-measurements to be kept in the same vertical plane.

In measuring a base the rear end of the tube is placed at the beginning of the line, and the position of the front end is marked. Then the tube is carried forward, and the rear end is placed at the mark and the front end is marked again, and so on, in the same way that a line is measured with a chain and pins. In order that the tube may stand firmly, it is supported upon iron stands, one at each end. These stands have three legs, which rest upon iron pins driven in the ground. To place the tube exactly in the line and at a proper height, the tops of the tube-stands are provided with movements in three directions, by means of which the tube can be moved sidewise, lengthwise, and up and down. For convenience there are four tube-stands, so that two can be placed in position while the tube is resting on the other two. The positions of the ends of the tube are marked with microscopes. Thus while the tube does the work of a chain, the microscopes do that of the pins. The microscopes are mounted upon iron stands, which, like the tube-stands, are supported upon iron pins driven into the ground. The microscope-stands are so constructed that the microscope can be placed directly over the end of the tube. The microscopes are provided with two motions, so that they can be moved a short distance along the line or at right angles to it. They also have levels attached, so that they can be made vertical. For convenience, there are four microscopes, so that two can be placed in position while two are standing over the ends of the tube.

To measure a tube-length, the rear end of the tube is placed under the microscope which marks the position of the front end of the preceding tube-length. The tube is then brought into the line by means of its telescope. Its inclination is found by reading its sector, and the temperature of the steel bar is found by microscope-readings on the scales at front and rear ends. From the temperature the length of the steel bar is found at the instant the measurement is made. From its inclination the horizontal projection of this length is found, and thus the actual advance becomes known.

Such is a general outline of the salient points in the construction and use of the Repsold base-apparatus. The details will be given by reference to the drawings in Plates VII to XII inclusive. The figures are numbered consecutively from 1 to 16.

THE TUBE.

§ **2.** Fig. 2 shows the tube supported on its stands. In the figure a portion of the middle of the tube is removed. The tube-cylinder is shown at *A*, Fig. 2. It is $0^{m}.125$ in diameter, and the iron of which it is made is $0^m.003$ thick. It contains the steel and zinc bars shown at *B*. The bars are placed side by side, with the steel on the right. They are supported at five points, viz, rear end, one metre, two metres, three metres, and front end. The supports at rear end are shown in Fig. 15. Z is the zinc bar, and S is the steel bar. Each is $0^m.027$ deep and $0^m.013$ thick. The steel rollers, *a* and *a*, prevent the bars from separating. They are prevented from approaching each other by the roller *b*, which is fastened to the top of the steel bar and presses against the plate *c*, which is fastened to the top of the zinc bar. Directly below *b* and *c* are two horizontal rollers, not shown in the figure, on which the bars rest. They are similar to the rollers shown at *a*, *a*, Fig. 6.

The supports at the front end are similar to those at the rear end. The supports at the one-metre mark are shown in Fig. 6. S and Z are cross-sections of steel and zinc bars. The horizontal rollers *a*, *a*, support the bars, and the vertical rollers *b*, *b*, prevent the bars from separating. A side view of one of these rollers is given in Fig. 12. The bars are prevented from approaching each other by an adjustable screw shown at *c*, Fig. 6, which presses against the roller in the steel bar. This roller turns about the axis *e*. The steel bar is slotted to receive the roller. The whole system of rollers and bars is supported upon the bolt *d*, *d*, which passes transversely through the center of the tube-cylinder and is keyed as shown in the figure. The supports at the three-metre mark are similar to those at the one-metre mark. The supports at the two-metre mark, or the middle of the tube, are also similar to those at the one-metre mark, except that at the middle of the tube the bars are firmly bolted together with a steel bolt about 3^{mm} in diameter, shown by the dotted lines at *a*, Fig. 11. It is entirely below the center of the bars, so that the two-metre mark, which is directly over it and not shown in this figure, can be precisely at the center of the two bars. The bars are prevented from approaching each other by a washer between them on the bolt *a*, as shown in the figure.

At each half-metre there are supports attached to the steel bar which hold the zinc bar in position, keeping it parallel to the steel bar both horizontally and vertically. One of these supports is shown in Fig. 8. A steel pin, shown at *m*, passes through both bars. It is firmly fastened to the steel, but is free from the zinc, which is slotted for the purpose. This pin is also shown by the dotted lines *f*, *e*, Fig. 9. On this pin as an axis are placed two thin steel wheels or rollers, one on each side of the zinc bar, shown at *a*, Fig. 9, and in section at *n*, *n*, Fig. 8. The diameter of the wheels is such that their upper edges come a little above the top of the zinc bar. A steel plate is screwed to the top of the zinc bar, shown at *b*, *b*, Fig. 9, and in section in Fig. 8. This plate is a little wider than the bar, so that its edges project over and rest on the steel rollers, thus preventing the zinc bar from sagging any more than the steel bar. The rollers are prevented from slipping in or out on their axis by washers and a screw shown at *m*, Fig. 8, and *a*, Fig. 9. The two bars are thus kept parallel. The bars are prevented from moving longitudinally by a device shown in Fig. 11. The small rectangle is the opening in the top of the tube-cylinder. A piece of brass is screwed firmly to the top of the steel bar. From this two lugs, *b* and *b*, rise even with the top of the tube-cylinder and carry the two capstan-headed butting screws, *c* and *c*, which press in opposite directions against sections of the tube-cylinder. The longitudinal position of the bars with respect to the tube-cylinder can be slightly adjusted by means of these screws. When in use there is no pressure on the upper surface of the bars, but when a tube is being transported from one place to another the bars are kept from shaking up and down in their supports by metal blocks which press on the bars directly over the horizontal rollers. These blocks are screwed to the under side of the plates which cover the openings in the tube-cylinder at the 0^m, 1^m, 3^m, and 4^m marks, as shown in Fig. 11. To remove the pressure the screws in the plates are loosened.

GRADUATIONS ON THE BARS.

§ **3.** The graduations on the bars are distributed as follows: on the zinc bar, at the rear end, one metre, three metres, and front end; on the steel bar at rear end, 1^m, $1^m.5$, $1^m.6$, $1^m.7$, $1^m.8$, $1^m.9$, 2^m, $2^m.1$, $2^m.2$, $2^m.3$, $2^m.4$, $2^m.5$, 3^m, and front end.

The graduations at the rear end of the steel and zinc bars are shown in Fig. 15. The upper half of both bars is cut away from the end back to the point *b*, so that a side view of the bar is as shown at *d*. The two steel plates, *c* and *e*, have their upper surfaces covered with platinum.

They are fastened firmly to the bars, and their inner edges approach within $0^{mm}.2$ of each other. The graduations are made on the upper surfaces and close to the inner edges of these plates, as shown in the figure. An enlarged view is given in Fig. 14.* The graduations on the steel and zinc bars at the front end are similar to those at the rear end. They are also shown in Fig. 14. There are 33 graduations on the zinc and 3 on the steel bar at the 0^m or rear end, and also at the 4^m or front end. The spaces between consecutive graduations are approximately $0^{mm}.1$. The graduations are numbered from the rear toward the front, as shown in Fig. 14.*

The graduations on the steel and zinc bars at 1^m are shown in Fig. 10. The steel pins, *c* and *d*, pass through the bars and very nearly meet midway between them. Their inner ends are plated with platinum, on which the graduations are made. They are smaller than the holes in the bars, are flattened on the upper and lower sides, and have broad, flat heads, which are screwed to the outside of the bars, as shown in the figure. There are 13 graduations on the zinc bar and 3 on the steel at 1^m, and also at 3^m. An enlarged view of these graduations is given in Fig. 14.

The remaining graduations on the steel bar are those which divide the middle metre into decimeters. A rib of steel one metre long is fastened to the inner side of the steel bar. Its upper surface is midway between the top and bottom of the steel bar. Its thickness is about 4^{mm}, which is a little greater than half the space between the bars. At proper intervals, small disks of platinum are set in its upper surface. On these disks the graduations are made. An enlarged view of the group at the 2^m mark is given in Fig. 14. All the others are similar.

The graduations at the rear and front ends are seen through the short tubes, *e* and *e*, Fig. 2. When not in use these tubes are covered with caps. The remaining graduations are seen through holes 1^{cm} in diameter, in the top of the tube-cylinder. When not in use these holes are stopped with short milled-head screw-caps.

THE TUBE-TELESCOPE.

§ **4.** The tube-telescope is shown at *t*, Fig. 2. Its use is to keep the tube on the line to be measured. It is adjusted so that its plane of collimation is parallel to a vertical plane through the axis of the steel bar. Then when the telescope moves in a vertical plane and is pointed to the forward end of the line, the tube also points in the same direction. The telescope is mounted a little on one side of the tube, so that it can be revolved on its horizontal axis. In the figure it is pointing backwards. The method of mounting is shown in Fig. 8. The axis, *a*, of the telescope turns in the wyes *e* and *e*. The axis of the telescope should be at right angles to the steel bar. To effect this adjustment the wyes are made of one solid piece of brass, which can be turned slightly in azimuth around the screw *d*. This motion is made with the opposite butting screws shown at *c*, Fig. 8, and *v*, Fig. 2. A clamp-screw, shown at *f*, Fig. 8, and *x*, Fig. 2, holds the axis firmly after it is adjusted. At the rear end of the tube there is a target, used to test this adjustment of the telescope. It is shown at *y*, Fig. 2, and *t*, Fig. 7. It is an adjustable disk, with a small hole in the center. After the telescope is in adjustment the disk is moved until the telescope points to the hole. When in use the telescope is from time to time turned back at the target. So long as it points at the hole it is assumed that the adjustment of the telescope remains undisturbed. When not in use the target is turned in against the tube. After the axis of the telescope is made horizontal the level *p*, Fig. 2, is adjusted so that its bubble is in the middle. Then, so long as this adjustment remains undisturbed, the axis of the telescope can be made horizontal by simply bringing the level-bubble to the center. The screw with which this is done is shown at *e*, Fig. 5.

MERCURIAL THERMOMETERS.

§ **5.** There is a mercurial thermometer at each end of the tube, having its bulb inside the tube-cylinder. The one at the rear end is on the right side of the bars, and is shown at *h*, in Fig. 2. The one at the front end is on the left of the bars and is concealed in the figure.

THE SECTOR.

§ **6.** The sector is the apparatus which measures the inclination of the tube. It consists of a graduated arc, which is fastened firmly to the tube, and an index which turns about an axis, and

*An error in drawing makes the graduations in Fig. 14 appear as though seen through an inverting microscope, except as to numbering.

can be kept vertical by means of a level. The reading of the index on the arc gives the inclination of the tube.

A front view of the sector is shown at *a*, Fig. 2, and a side view at *a*, Fig. 7. In Fig. 2, *a* is the graduated arc which is fixed with reference to the tube; *i* is the axis around which the index turns; *d* is the level; and *c* is the tangent-screw. At *a*, Fig. 7, is shown a magnifier, with which the arc is read. The arc is 15° in length, so that an inclination as great as 7° 30′, up or down, can be measured. The index is a vernier reading to 30″. In use, the vernier is usually set at an even minute and the fractional part of a minute is read from the level.

THE TUBE-STANDS.

§ **7.** The tube when in use is mounted upon two supports called tube-stands, as shown in Fig. 2. The rear end rests upon a single point which rises from the center of the tube-stand and is shown at *c*, Fig. 5. The forward end rests upon two wheels, one of which is shown at *i*, Fig. 2. One of the wheels has a grooved rim, the other one a flat rim. The grooved wheel runs on a track $0^m.1$ in length, shown at *n*, Fig. 2. The track has a raised rib to fit the wheel, shown at *a*, Fig. 5. The other wheel runs on a flat track, which is shown at *b*, Fig. 5. A bent handle of iron, shown at *k*, Fig. 2, prevents the tube from being drawn backwards off the tube-stand. The legs of the tube-stands are of iron with a T-shaped cross-section. The feet are circular disks, $0^m.2$ in diameter, and rest on iron pins driven into the ground. When the tube-stand is used at the rear end, the tube rests on the point *c*, Fig. 5. When the same stand is used at the forward end, the tube rests on the rails or tracks, *a* and *b*, Fig. 5.

The head of the tube-stand is provided with three motions, longitudinal, lateral, and vertical. The vertical motion is made with the screw *o*, Fig. 5, which pushes the piston *g* up and down in the cylinder *i*, *i*. The spiral springs *h*, *h*, are stiff enough to support the weight of the end of the tube, so that the screw, *o*, turns easily in either direction. In Fig. 2, *l* is a clamp-screw which clamps the vertical motion. A screw, shown at *n*, Fig. 5, can be removed, leaving an orifice through which the screw *o* can be oiled. The legs of the tube-stand are attached to the plate *r*, *l*, Fig. 5, shown also at *f*, Fig. 2. A circular hole is cut through this plate, with a diameter equal to *s*, *m*, Fig. 5. The lateral motion is made with the screw *u*, which carries all that part which is above the line *u*, *p*, from side to side until the cylinder *i*, *i*, comes in contact with the points *s* or *m*. The longitudinal motion is made with a similar screw, the end of which is shown at *f*, Fig. 5, and at *j*, *j*, Fig. 2. The slides for this movement are on the line *q*, *y*, Fig. 5. The amount of motion is limited by the contact of the cylinder *i*, *i*, with the front or rear edge of the hole through the plate *r*, *l*. The guides between which the head of the tube-stand slides when longitudinal motion is made are shown at *q* and *y*. The guide *q* is adjustable. The vertical screw-holes are slotted so that it can be pushed in or out with two adjusting screws, which are not shown in this figure, but similar screws are shown at *i* and *i*, Fig. 3. The upper shoulders of the screws press against the guide, forcing it in. The lateral motion has a similar adjustment. All of the large screws with which vertical, lateral, and longitudinal motions are produced are provided with milled heads for slow motion and cranks for rapid motion, as shown at *u* and *v*, Fig. 5. The screw *e*, Fig. 5, turns the tracks *a* and *b* a small amount around the center *j*. A spring, shown at *d*, keeps a constant pressure on the screw *e*. When the forward end of the tube is resting on the tracks *a* and *b*, the motion of the screw *e* rolls the tube until the level at *p*, Fig. 2, shows that the axis of the tube-telescope is horizontal. All that part of the apparatus which produces the vertical motion can be removed from the head of the tube-stand by taking out four milled head screws, two of which are shown at *k* and *w*, Fig. 5, thus leaving the tube-stand with only the lateral and longitudinal motions. This is done when the cut-off cylinder is used. The hole left in the head of the tube-stand by the removal of the vertical-motion apparatus is occupied by the cut-off cylinder, as shown in Fig. 4. The lateral and longitudinal motions are then used to make the cut-off cylinder vertical.

THE MICROSCOPES.

§ **8.** Fig. 1 is a front view of a microscope on its stand. In use the microscopes stand directly over the ends of the tube. To accomplish this, one leg of the tube-stand must be placed between the legs of the microscope-stand. Thus arranged each of the two stands rests on its own pins, but the microscope-stand cannot be removed without first removing the tube-stand. The microscope is pointed at the zero or middle graduation on the end of the steel bar, and thus marks and preserves the position of the end of the tube while it is being carried forward to a new position. The microscope at either end of the tube also measures the distance from the zero-graduation on the steel bar to the nearest graduation on the zinc bar, thus indicating the temperature. The feet, legs, and bracing of the microscope-stands are nearly similar to those of the tube-stands. The differences are sufficiently shown in the figure. A section and a side view of a microscope are shown in Fig. 3. A is the top of the microscope-stand, shown also at *a*, *b*, Fig. 1. A heavy metal frame, *y*, is attached at its lower end to the microscope-stand and leans outward, as shown in the figure. The upper and outer end carries the microscope with its slides, levels, &c. It is fastened to the microscope-stand by the bolt *b*, Fig. 3, which has a universal joint at its lower end, a section of which is shown in the figure. This joint permits the leveling of the microscope without strain or torsion of the bolt. The front leveling screws are shown at E and *d*, Fig. 1. One of them is shown at *x*, Fig. 3. The rear leveling screw is shown at *e*, Fig. 3. The spiral spring *f* and the weight of the overhanging microscope force the rear end of the metal frame *y* up against the rear leveling screw *e*. The front leveling screws are clamped with the milled-head screws *p*, *g*, Fig. 1. The rear leveling screw is clamped with the screw *D*, Fig. 3. All the leveling screws are provided with milled heads for slow motion and cranks for rapid motion. A roller is shown at *z*, Fig. 3, which runs up and down on the track *k*. There is another on the opposite side. The two prevent the frame from turning in azimuth. A front view of a microscope is shown at *i*, Fig. 1; a side view and section at *a*, *o*, Fig. 3. It is provided with lateral and longitudinal motions similar to those in the head of a tube-stand. It has no vertical motion. The lateral motion is made with the screw *c* and the longitudinal motion with the screw *v*, Fig. 3. The end of the longitudinal level is shown at *d*, Fig. 3, and the end of the transverse level is shown at *l*, Fig. 1. These two are called the fixed levels. There is a detached level shown at E, which is used only in adjusting the fixed levels. It is attached to the eye-end of the microscope, which is then revolved in azimuth and made vertical by means of its leveling screws. The fixed levels are then adjusted, after which the microscope is made vertical by means of the fixed levels alone. There is a small telescope attached to the microscope, a side view of which is given at *n*, Fig. 1, and an end view at *F*, Fig. 3. It is used to place the microscope approximately in azimuth, so that the longitudinal thread in the reticule shall be approximately parallel to the longitudinal graduation on the steel bar. This is accomplished by first making the line of collimation of the telescope parallel to the longitudinal thread in the reticule. Then, when a microscope is being placed in position, it is revolved in azimuth until its telescope points to the eye-end of the telescope next in rear. This makes the longitudinal thread and graduation parallel. The telescope is clamped to the microscope with the screw *h*, Fig. 3. There is a lug projecting from the under side of the telescope, shown at *j*, Fig. 3. It falls in a slot in the plate *t*. This plate has two butting screws, one of which is shown at *t*, which press in opposite directions against the lug and move it backwards and forwards the width of the slot, thus permitting the adjustment of the optical axis of the telescope to parallelism with the longitudinal thread in the microscope.

The microscope is turned in azimuth by the tangent-screw *w*, Fig. 3. This screw turns in the plate *j*, *w*, which is firmly attached to the microscope. The end of the screw is held by the catch *l*. When this catch is turned down from the tangent-screw the microscope can be revolved completely in azimuth, carrying with it the telescope, the plate *j*, *w*, and the tangent-screw *w*. The micro scope revolves in the cylinder *m*, *m*. There is a capstan-headed clamp-screw at *n*, which clamps the microscope in the cylinder. The object-glass *o* is held in place by four butting screws, shown at *I*, *I*, *I*, *I*, Fig. 3. The adjustment for collimation is made with these screws. The reflector to the microscope is shown at A, Fig. 1, and *I*, Fig. 3. It is a plane surface perforated and covered with plaster of Paris. The value of a revolution of the micrometer-screw is adjusted with the short tube *p*, *p*, Fig. 3, which carries the object-glass. In use it is pushed up as far as possible and clamped with the screw shown at *o*, Fig. 1. The microscope is provided with an ordinary micrometer, shown at *h*, Fig. 1, and *u*, Fig. 3. One revolution of the micrometer-screw is approximately equal to $0^{mm}.1$. The magnifying power of the microscope is approximately 27 diameters. There is a plumb-bob suspended by three strings, shown at *B*, Fig. 3, and B, Fig. 1. It is used to place the tube-stand directly under the microscope.

THE CUT-OFF APPARATUS.

§ **9.** A mark on or below the surface of the ground directly under the end of a tube is called a cut-off, and the apparatus for placing such a mark under the end of a tube, or placing the end of a tube over the mark, is called the cut-off apparatus. Cut-offs are sometimes made upon permanent marks, as ends of bases, sections, &c., and sometimes upon temporary marks on the surface of the ground to mark the close of a day's work or any short interruption.

The cut-off mark is a hemisphere about $0^{m}.01$ in diameter. When it is the extremity of a base-line, this hemisphere is usually made of rock crystal, set in the end of a brass bolt, which is in turn set in a block of granite and then surrounded with masonry. This kind of mark is usually set one metre below the surface of the ground. The placing of the end of the tube directly over the center of the hemisphere of crystal is called making a cut-off.

The cut-off cylinder is a tube of iron about $0^{m}.04$ in diameter and $1^{m}.7$ in length. In its lower end there is a conical hole which allows the cylinder to stand vertically on the hemisphere of crystal and revolve freely in azimuth. The upper end of the cut-off cylinder passes through the top of a tube-stand, which holds it firmly. It has a level, by means of which it is made vertical, and a horizontal scale, the center graduation of which is in the axis of the cut off cylinder. Therefore, when the cylinder is vertical this graduation is directly over the center of the hemispherical crystal. A microscope is then placed vertically over the scale and pointed at this graduation. Without disturbing the microscope, the cut-off cylinder is removed, and the end of a tube placed under the microscope. This brings the end of the tube exactly over the center of the hemispherical crystal. When a temporary cut-off is made there is used a cut-off plate. This plate is of cast iron, circular, and about $0^{m}.3$ in diameter. It is placed under the end of a tube near the surface of the ground, resting on three wooden stakes and held in position by nails. On it there is a smaller plate, which can be moved $0^{m}.02$ in any direction by four butting screws. In the center of the movable plate is a screw about $0^{m}.1$ in length, which has a hemispherical head of the same diameter as the crystal. This hemisphere is placed under the end of a tube in substantially the same manner as a tube is placed over a hemisphere. The cut-off cylinder is in two parts, so that one-half its length can be used when the mark is on the surface of the ground.

The several parts mentioned above will now be described in detail by reference to the drawings. The cut-off apparatus is shown in Fig. 4. The cut-off plate, *v*, rests on three wooden stakes, and is held in place by nails driven in the stakes and bent over the edge of the plate, as shown in the figure. The movable plate *u* is adjusted and held in place by four butting screws, three of which are shown at *s*, *w*, and *t*. The screw carried by the movable plate has a milled head, shown at *q*. Its hemispherical head is concealed by the lower end of the cut-off cylinder. A lock-nut, shown at *r*, holds the screw after it has been raised or lowered to the proper place. The horizontal motions of the plate *u* are used to place the cut-off cylinder and scale directly under a fixed microscope. (The cut-off cylinder is kept vertical during these movements by corresponding movements of the lateral and longitudinal slides in the head of the tube-stand.) The vertical motion of the screw *q* is used to bring the cut-off cylinder to the proper height for passing through the head of the tube-stand. The cut-off cylinder is shown at *p*, standing on the hemispherical head of the screw in the movable plate. Its upper end passes through the hole in the head of the tube-stand left by the removal of the vertical motion apparatus. A collar, shown at *h*, is fastened to the top of the tube-stand by the milled-head screws *e* and *f*, which before held the vertical-motion apparatus in place. The top of the cut-off cylinder passes through the collar, which fits it neatly and holds it firmly, yet allows it to turn easily in azimuth. The piston *d* has a motion up and down of $0^{m}.1$ in the cut-off cylinder. This movement is made with a rack and pinion, shown at *m*. It is used to bring the cut-off scale into the focus of the microscope. A clamp-screw, *n*, clamps the piston when in position. The blackened metal cone, *a*, has its apex in the prolongation of the axis of the cut-off cylinder, and is used for a target when aligning the marking-stones, &c., but is removed when the cut-off is used under a microscope. Between *b* and *c* there is a scale divided into millimeters. The middle graduation is numbered zero, and is approximately in the axis of the cylinder. There are sixty-five graduations on each side of the zero, making the total length of the scale $0^{m}.13$. The ends of the scale are marked with the letters A and B, and the graduations are distinguished by these letters. A cover, *b*, *c*, protects the scale. The cut-off level is shown at *o*. It is adjustable, so that the bubble will remain in the center while the cut-off cylinder is revolved 180° in azimuth. One division of this level is approximately two seconds.

END- AND SECTION-MARKS OF BASE-LINE.

§ **10.** The cut-off plates above described are used for temporary marks only. The different kinds of permanent marks are shown in Fig. 13. That for the end of a base-line is shown at *g*. The hemisphere is of rock crystal set in a brass bolt, which is in turn set in a block of granite, and then surrounded with masonry. An enlarged view of the upper end of the brass bolt with its protecting cap is shown at *d*. A section is marked with a single stone $0^m.6$ square and $0^m.4$ thick. In the center of the upper surface of this stone is a small brass bolt with a conical hole through its center. A steel pin with a hemispherical head fits in this hole, as shown at *e*, Fig. 13. When a cut-off is made on a section-stone the steel pin is inserted in the brass cylinder, the cut-off cylinder is placed on it, and the cut-off is made in the usual manner. After the cut-off is completed the steel pin is removed and a brass or wooden plug is put in its place. A section-stone is usually placed one metre below the surface of the ground. Besides the end-mark proper of a base, already described, there are two side-stones, one of which is shown in Fig. 13. These stones are $0^m.2$ square at top and $0^m.9$ long. They stand vertically with their tops in the same horizontal plane as the end-mark. They have marks in their tops similar to section-marks. They are placed each side of the end-mark and four metres distant, so that a line joining them passes through the end-mark and is approximately at right angles to the base-line.

ADJUSTMENTS.

§ **11.** The adjustments of the base-apparatus which are made in the field are as follows:

First. The microscopes are collimated. The point which is collimated is the intersection of the movable thread when at the tenth or middle revolution with the longitudinal thread.

Method.—The object-glass is moved by means of the four butting screws, which hold it until the point to be collimated appears to be stationary while the microscope is revolved.

Second. The microscope is made vertical.

Method.—The detached level is placed on the top of the microscope, which is then adjusted by means of its leveling screws until it will revolve without moving the bubble of the level. The microscope is then vertical. The fixed levels are then adjusted until their bubbles are in the center.

Third. The longitudinal thread should be parallel to the direction of motion of the slide. This is made so by the maker, and there is no adjustment provided. It should, however, be tested from time to time by the following—

Method.—The longitudinal thread is made to coincide with the longitudinal graduation on the bar. Then the micrometer-screw is turned through the whole amount of its motion, and if the thread does not separate from the graduation, the axis of motion and the thread are parallel.

Fourth. The optical axis of the telescope attached to the microscope is made parallel to the longitudinal thread in the microscope.

Method.—Two microscopes are placed over the ends of the tube. Their longitudinal threads are made to coincide with the longitudinal graduation on the bar. Then the forward telescope is adjusted so that it points to the eye-end of the rear telescope. This adjustment is convenient but not essential.

Fifth. The tube-telescope must be collimated.

First method.—Two microscopes are set over the ends of the tube and focused over the longitudinal graduations. The tube should be horizontal. The tube-telescope is then pointed to some distant object in the horizon. The tube is then changed end for end and again brought under the microscopes. The telescope is revolved 180°. If it point to the same distant object, then its collimation is in adjustment. If it does not, half the error is corrected by means of the collimating screws which move the reticule.

Second method.—Three points are determined in line by means of an auxiliary instrument. The tube-telescope is placed over the middle one and made to point to the other two when revolved 180°. The three points must be in the same horizontal plane. This is a convenient method to use while measuring, for when the tube is over a section-point it is in line with the next section-point ahead and the last one in the rear, and a simple revolution of the telescope will test its collimation.

Sixth. The axis of rotation of the tube-telescope must be made horizontal.

Method.—The tube is rolled until the telescope will follow a plumb-line. In this position the cross level on the forward end of the tube is adjusted so that its bubble is in the center.

Seventh. The plane of sight of the tube-telescope must be made parallel to the longitudinal graduation on the steel bar.

Method.—Two microscopes are set up in line with some distant object. To accomplish this small targets are placed in the tops of the microscopes, and a theodolite is set up about twenty metres distant, with which the targets are sighted in line with the distant object. The tube is then brought under the microscopes. In this position the tube-telescope is made to point to the same distant object by adjusting the axis of rotation. The target at the rear end of the tube is then adjusted so that the tube-telescope points at the small hole in the center.

Eighth. The reading of the sector when the tube is horizontal must be determined.

Method.—The tube is focused under two microscopes and the sector read. The tube is then changed end for end, refocused, and the sector read again. The mean of the two readings is the reading of the sector when the tube is horizontal.

Ninth. The sector arc should be in a plane parallel to the vertical plane passing through the longitudinal graduation of the steel bar. This is made so by the maker, and no adjustment is provided. It can be tested by the following—

Method.—Measure two known grade-angles and compare the values given by the sector with the known values.

Tenth. The steel and zinc bars in the tube should be straight. They were made so by the maker and no adjustment was provided. They can, however, be tested with the comparing apparatus, but not in the field. The apparatus and method are described in § 24.

MEASURING A LINE.

§ **12.** A measuring party consists of three observers, two recorders, and twelve laborers. The work is divided as follows: Two men with wooden mauls drive the iron pins on which are placed the tube- and microscope-stands. A wooden pattern is used for placing the pins properly, and the heads of the pins are brought into the same horizontal plane with a carpenter's level. Two men carry forward the microscopes on their stands and place them on the pins. Two men carry forward the tube-stands. One observer places the microscope in position and levels it. Three men carry the tube forward and place it on its stands under the microscopes. The men stand on the right-hand side of the tube and lift it with their hands. Each man has a strap around his shoulders and under the tube, for safety. The recorder at the forward end points the telescope at the target, which stands over the next section-point in advance. The observer at the forward end levels the tube. The observer at the rear end brings the tube under the microscope. These three things are done simultaneously. The observer at the forward end next brings the microscope over the zero of the steel bar and then brings the tube into the focus of the microscope. After the microscope is in position the crank which moves it longitudinally is removed, so that the observer at the rear end may not by mistake move the microscope after it has been used at the forward end and before it has been used at the rear end. The recorders have each a note-book; one is called "front record" and the other "rear record." The recorder at the rear end prepares and reads the sector, recording the reading in his book without calling it out. The rear and front observers then make simultaneous readings on the zeros of the steel bar. Each reading is recorded in its proper note-book. Then simultaneous readings are made on that graduation of the zinc bar which is nearest the zero of the steel, and properly recorded. Next the mercurial thermometers are read. The observer at the rear end then reads the sector, calling it out, and the recorder at the forward end records it in his book.

While the tube is being carried forward under the supervision of the observers, the two recorders change places and examine the readings of the microscopes to see if they agree with the records. The time at which the reading is made on the steel bar is recorded. The recorders then compare their notes. If there is a discrepancy in the last readings of the microscopes, or in the reading of the sector, it is corrected before either is disturbed. Thus every important reading is re-read by a second observer, who has no knowledge of, and is therefore not influenced by, the first reading. The readings on the steel bar are well checked by the reading on the zinc bar. The mercurial-thermometer readings are not of sufficient importance to be re-read. While the tube is being read, one man with a grub-hoe pulls up the iron pins that have been uncovered in the rear and another man carries them forward. Awnings are carried over the tube and microscopes. They are made of canvas stretched over light wooden frames. The frames are five metres long, two and a half metres wide, and two metres high. There are four such awnings used. The rear one is carried forward and placed in front as often as necessary. The four men who carry tube-stands and microscopes do this work. Platforms, on which the observers stand, are placed in front of the microscopes. These platforms are planks $2^{m}.5$ long, $0^{m}.15$ wide, and $0^{m}.05$ thick, supported at the ends upon blocks $0^{m}.1$ in thickness. The weight of the observer is thus removed at least one metre from the feet of the microscope-stands. The man who carries the pins carries these platforms forward and places them in front of the microscopes. The tube which is used in the field is covered with felt, two centimeters in thickness, and outside of the felt is a cover of canvas painted white. This is to protect the tube from sudden changes of temperature. The microscope-stands are covered in the same way.

FORM OF THE RECORD.

§ **13.** The record of measurement is kept in the following form:

Chicago Base-Line, Tube 1, *Front End, August* 25, 1877.

Number of tube.	Time.	Number of micro-scope.	Steel bar.		Zinc bar.		Sector arc.	Sector-level.		Therm. No. 2.	Remarks.
	A. M.		Div.	Micro. reading.	Div.	Micro. reading.		L.	R.		
	h. m.			rev.		rev.	° ′	div.	div.	°	
1846	10 27	3	0	10.967	13	9.052	5 41	4.8	4.9	76.0	E. S. Wheeler, observer.
1847	10 30	4	0	10.991	13	10.991	5 29	5.0	4.5	76.3	J. B. Johnson, recorder.
1848	10 41	1	0	9.018	12	9.856	5 04	5.3	4.0	77.0	Cloudiness = 2.
1849	10 44	2	0	10.950	12	9.766	6 04	5.2	4.0	77.2	
1850	10 47	3	0	9.008	12	9.820	5 42	5.7	3.7	77.5	

Chicago Base-Line, Tube 1, *Rear End, August* 25, 1877.

Number of tube.	Time.	Number of micro-scope.	Steel bar.		Zinc bar.		Sector arc.	Sector-level.		Therm. No. 1.	Remarks.
	A. M.		Div.	Micro. reading.	Div.	Micro. reading.		L.	R.		
	h. m.			rev.		rev.	° ′	div.	div.	°	
1846	10 27	2	0	9.077	17	9.557	5 41	4.8	4.9	76.8	Chas. Pratt, observer.
1847	10 30	3	0	10.901	17	9.383	5 29	5.0	4.5	77.0	E. S. Davis, recorder.
1848	10 41	4	0	9.170	17	9.830	5 04	5.5	3.8	77.8	Cloudiness = 2.
1849	10 44	1	0	9.030	17	9.743	6 04	5.3	4.0	78.1	
1850	10 47	2	0	9.073	17	9.853	5 42	5.7	3.6	78.3	

The record of cut-offs is kept as below. On stopping at night, three cut-off plates are placed under the ends of consecutive tubes. At starting in the morning the cut-offs are re-read, and the distances between the plates are remeasured. If the distances remain unchanged it is a proof of the stability of the cut-off plates. The cut-off cylinder is not usually exactly vertical. Its inclination is shown by the reading of the cut-off level. To compute the correction for this inclination, the length of the cylinder as used must be known. This varies as the scale is moved up or down

The total length is read from a scale on the side of the piston, and entered in the column of remarks as "length of cut-off cylinder," or "L. C."

Time.	Number of tube.	Cut-off scale.			Level-reading.		Remarks.
P. M.		End forward.	Gradua-tion.	Micro. reading.	Front.	Rear.	
h. m.				*rev.*	*div.*	*div.*	
4 20	1272	A	0	10. 325	15	21	L. C. = $0^m.823$.
4 22	1272	B	0	10. 331	18	18	E. S. Wheeler, observer.
4 29	1273	A	0	10. 171	4	28	L. C. = $0^m.792$.
4 30	1273	B	0	10. 230	27	5	
4 39	1274	A	0	9. 837	12	20	L. C. = $0^m.807$.
4 41	1274	B	0	9. 869	12	20	Cut-off cylinder left standing.

The rate of measuring varies much with the conditions of the ground, weather, &c. The most rapid work yet done was the last measurement of the Olney Base, which was made in 13 working-days, being an average of 507^m a day.

THE COMPARING-APPARATUS.

§ **14.** This apparatus is designed to determine the differences in length between line-measure standards. It consists of two microscopes so mounted that the distance between them remains practically constant while two standards are brought alternately under them and observed. Thus the standards are each compared with the space between the microscopes, and therefore with each other. The apparatus then is simply the means for holding firmly the microscopes and standards, adjusting and illuminating them, protecting them from rapid changes in temperature, &c. It is called the comparator, and is shown in Plates XVII to XXII, inclusive. The figures are numbered consecutively from 16 to 38, and will be described in detail.

THE COMPARING-ROOM.

§ **15.** Fig. 16 is a plan of the comparing-room and some of the adjacent walls. A is the comparing-room proper. Its interior dimensions are $3^m.25 \times 6^m.5$, height, $3^m.0$. It is lined on all sides, top, and bottom, with sawdust, $0^m.3$ in thickness. The room is entered through the double glass doors at F. B is the room in which the light is placed. The light is from an argand gas-burner in the focus of the parabolic reflector shown at G, Figs. 16 and 17, and is thrown into the comparing-room through the glass doors at F. It is then reflected to the points which require illumination, and is intensified with lenses if necessary. This room has a window at H, and a door at I. C and D are halls, $2^m.5$ and 1^m in width, respectively.

THE PIERS.

§ **16.** Fig. 17 is a vertical section through the comparing-rooms, showing the piers. The two outside ones support the microscopes. The other three support the comparator. The piers stand on the solid earth at the bottom of the cellar and pass up through the floor of the comparing-room without contact. There is a small space of about $0^m.05$ between the floor and the piers, which is lightly packed with cotton. The parts of the piers which rise above the floor are protected by boxes, sections of which are shown in Figs. 17, 18, and 19.

THE COMPARATOR-BOX.

§ **17.** This box surrounds and protects the comparator. It is $4^m.3$ long, $0^m.8$ high, and $0^m.7$ wide. It is made of mahogany. A view of the top is shown in Fig. 16, of the south side in Fig. 17, of the north side in Fig. 19, and of the west end in Fig. 24. Its longer axis is nearly east and west. The observers stand on the north side. The tubes and standards are usually placed with their zero or rear ends east. This is called the "direct" position. When they are placed with their zero-ends west they are said to be in the "reversed" position. The comparator-box is supported on the boxes which surround the piers, as shown in Figs 17, 18, and 19. The screws which support the comparator on the piers and the frame which supports the microscopes pass through holes in the bottom of the box which are so large that the box is nowhere in contact with either comparator or microscopes. In the south side of the box there are panels of plate-glass, shown at K, Fig. 17. Through these, light is thrown upon those parts of the comparator which it is necessary to illuminate. The top of the box is covered with movable panels, some of which are of plate-glass, through which the thermometers are read. The eye-ends of the microscopes project upwards through these panels, but not in contact with them, as shown in Fig. 19. The north side of the box turns down on hinges, as shown in the figure.

MOUNTING OF THE MICROSCOPES.

§ **18.** The comparing-microscopes are numbered 5 and 6. Their optical arrangements and micrometers are similar to the measuring-microscopes described in § 8. When standards 4^{m} long are to be compared, microscope 5 is mounted on the east pier and microscope 6 on the west pier, as shown at A and B, Fig. 18. The method of mounting is shown in Plate XXI. Fig. 24 shows the west end of the comparator-box, and the top of microscope 6. Fig. 26 shows west end of comparator with box removed. Fig. 25 shows side view of support for microscope. A, Figs. 24, 25, and 26, is an iron plate $0^{m}.6$ long, $0^{m}.3$ wide, and $0^{m}.03$ thick. It is held to the top of the pier by four iron bolts, and adjusted horizontally by a double set of nuts. The iron frame which supports the microscopes is firmly bolted to this plate. A side and an end view of this frame are shown at B, Figs. 25 and 26. The circular opening at B, Fig. 25, contains a lens of proper focal length, which concentrates the light upon the microscope-reflector shown at C, Figs. 25 and 26. D, D are projecting rails $0^{m}.1$ long, to which the microscopes are clamped with the screw E. These rails allow the microscopes to move longitudinally about $0^{m}.03$. F, Fig. 26, is a tangent-screw which moves the microscope transversely on its upper slide about $0^{m}.03$. G is a tangent-screw which moves the microscope vertically about $0^{m}.002$. H, H, Figs. 25 and 26, are leveling screws with which the microscope is made vertical. For this purpose a small cross-level is attached to the top of the microscope. This level is shown in Fig. 3, Plate IX. When standards of 1^{m} or less are to be compared, the microscopes are mounted on the microscope-car, as shown at C and D, Fig. 18. The method of mounting and adjusting is similar to that just given. An end-view of car with microscope is given in Fig. 22.

THE FIXED FRAME.

§ **19.** This is the heavy iron frame which supports the tube- and microscope-cars. A side view is shown at E, Fig. 18; an end view at B, Fig. 22; a top view at A, Fig. 20; and a section at C, Fig. 21. It rests on the three interior piers and is supported by the screws shown at F, Fig. 18; B, Fig. 21; and A, Fig. 22. It is $3^{m}.7$ long, $0^{m}.56$ wide, and $0^{m}.15$ deep. It weighs about 300 kilogrammes. F and G, Fig. 20, are the rails on which the microscope-car stands, and on which it is moved longitudinally. The rail at F is flat; the one at G triangular in section, as shown at C and D, Fig. 22. The cross-piece at H, Fig. 20, has a flat upper surface, on which the wheels of the tube-car run when it is moved from side to side of the fixed frame. There is a similar cross-piece at the other end, not shown in the figure. O and N are adjustable screws or stops against which the tube-car strikes. There are two other stops not shown in the figure. The stops on the north side are provided with lengthening-bars, one of which is shown at P. This bar is supported on the axis R. When not in use it is put down out of the way by turning the axis, R, 90°.

THE TUBE-CAR.

§ **20.** The tube car is supported by the fixed frame. A side-view of the car is shown in Fig. 18; a top-view in Fig. 20; an end-view in Fig. 22; and a section in Fig. 21. It is $4^{m}.1$ long, $0^{m}.33$ wide, and $0^{m}.17$ deep. It weighs about 250 kilogrammes. It moves upon four small wheels or rollers, two of which are shown at I, Figs. 20 and 21, and E, Fig. 22. These rollers allow it to move easily from side to side of the fixed frame. It carries the tubes, and thus allows them to be interchanged under the microscopes. A side-view and an end-view of the tubes mounted on the tube-car are shown at G, Fig. 18, and I, Fig. 26. The tubes are supported at their ends upon trucks, which are shown at H, Fig. 18; J, Fig. 26; and F, Fig. 22. These trucks rest upon the upper rails of the tube-car. One pair is provided with wheels, the other pair is not. The rear-end trucks are shown in Fig. 22. G is a screw with which the end of the tube can be raised or lowered. H, H are butting screws which press the screw G from side to side, thus giving the rear end of the tube a slight transverse motion. I is a clamp-screw which clamps the truck to the rail of the tube-car. The forward-end trucks are shown in Fig. 26. K, K are screws which raise and lower the end of the tube. There is no transverse motion in the forward-end trucks. The apparatus with which the tube-car is moved from side to side is shown in its proper place at S, Fig. 20. An enlarged top-view is shown in Fig. 28, and a side-view in Fig. 29. Fig. 30 is a section on the line CD, Fig. 28; and Fig. 31 is a section on the line AE, Fig. 28. K, K, Fig. 20, are levers turning around the points K and K. Their longer arms are attached to the under side of the tube-car, as shown at L. When the shorter arms are moved in one direction the longer ones move in the opposite direction and carry the tube-car with them. The shorter arms are moved by turning the crank M, Fig. 20. This movement is better shown in Fig. 28. A is the screw which the crank turns. B is a nut which is moved backwards and forwards by the turning of the screw, and carries the shorter arms of the levers with it. The screw and nut are shown in section at A and B, Fig. 31. When the tube-car is pushed to either side of the fixed frame it comes in contact with the stops described in § 19 and shown at I, Fig. 18, and N, O, Fig. 20. The levers K and K are elastic, so that when the tube-car is once pushed firmly against the stops it will remain in contact with them, while they are moved a short distance in or out so as to bring the standards exactly under the microscopes. The stops are turned with a key, which passes through holes in the comparator-box. These holes are covered with a slide when not in use, as shown in Fig. 19. F and G, Fig. 28, are friction-rollers, which prevent the tube-car from moving longitudinally. A projecting piece of the tube-car, shown at S, Fig. 21, fits between these rollers, one of which, F, Fig. 28, is adjustable and can be pressed against the tube-car. H, Figs. 28 and 29, is the screw with which this movement is accomplished.

THE MICROSCOPE-CAR.

§ **21.** The microscope-car supports the microscopes when standards of one metre or less are to be compared. A side-view is given in Fig. 18 and an end-view in Fig. 22. It stands on the fixed frame, astride of the tube-car. It is $1^{m}.1$ long, $0^{m}.6$ wide, and $0^{m}.4$ high. It weighs about 40 kilogrammes. It can be pushed along the rails from end to end of the fixed frame. It has three supports, two of them resting on the north rail of the fixed frame, as shown at K, Fig. 18, and one on the south rail, as shown at D, Fig. 22. The two on the north rail are grooved at the bottom so as to fit the rail, as shown at C. The one on the south rail is a wheel, as shown at D, and is in the middle of the south side of the car, supporting half the weight of car and microscopes. J, Fig. 18, is an iron beam $1^{m}.1$ long. An end-view is shown at Y, Fig. 22. The microscopes are clamped to this beam, and can be moved along it so as to be at any distance from each other less than 1^{m} and greater than $0^{m}.1$. M, Figs. 18 and 22, is a tangent-screw, which moves the microscope longitudinally. The adjustments of the microscopes when mounted on this car are the same as described in § 11. K, Figs. 18 and 22, is a lever with which slow longitudinal motion is communicated to the car. An enlarged view of the lower end is given in Fig. 23. It turns about the pivot L, Figs. 22 and 23. Its lower end is pressed into notches in the rail, when a movement of the upper end communicates a much slower movement to the car. I, Fig. 23, is a spring which holds the lever up out of the notches when not in use. The upper end of the lever passes through a slot in the top of the comparator-box. When not in use it is removed and the slot covered.

BEAM FOR SUPPORTING STANDARDS.

§ **22.** This beam supports the standards, which are 1 metre or less in length. It is $1^{m}.1$ long and is mounted on two of the trucks described in § 20. A side-view, with *M T* 1876 lying on it, is given at L, Fig 18. Another side-view, with standard yard lying on it, is given at H, Fig. 32. An end-view, with *M T* 1876 and *R* 1876 lying on it, is given at J, Fig. 22. A cross-section is shown at H, Fig. 33. When two standards are being compared one of them rests on the flat upper surface of the beam, the other on two shoulders which project from the side of the beam. One of these shoulders is shown at J, Fig. 22. The former standard is adjusted horizontally by means of the vertical movements in the trucks described in § 20. The latter is brought into the same horizontal plane by means of the vertical motion of the shoulders. The screws with which this motion is produced are shown at O, Fig. 22, and I, Fig. 32. The shoulders are clamped with a screw not shown in the figure. P, Fig. 22, is a screw which pushes the standard, which lies on the shoulders, laterally. Q is a screw with a small projecting point at R which draws the standard laterally. When spaces of less than $0^{m}.1$ are compared the standards are mounted end to end, their axes in the same vertical plane and their graduated surfaces in the same horizontal plane. The beam is supported upon the trucks with wheels, and the standards are pushed backwards and forwards, longitudinally, under the microscopes. To accomplish this a steel ruler is attached to the beam and projects through a slot in the end of the comparator-box. This is not shown in the figure.

APPARATUS FOR COMPARING LINE- WITH END-MEASURES.

§ **23.** The Clarke yards are end-measure standards, the Repsold metres are line-measure standards. In order to compare a yard with a metre an apparatus is required which will temporarily convert the yard into a line-measure. This apparatus is shown in Figs. 32 to 35, inclusive. It consists of two steel cylinders called "quills" and the apparatus for adjusting and holding them in contact with the yard. The quills have graduations on their upper surfaces, and, when in contact with the yard, the yard and quills together become a line-measure and can be compared with other line-measures. The lengths of the quills can be found separately; therefore the relative lengths of the two standards become known. One of the quills in half actual size is shown in Fig. 34. Agates are set in each end, one having a plane, the other a spherical surface (radius $= 10^{mm}.5$). Only each fifth graduation is shown in the figure. One side of the apparatus for holding the quills is shown in Fig. 32. The other side and an end-view are shown in Fig. 33. The apparatus is clamped to the beam with the screws O, Figs. 32 and 33. The middle one releases or tightens the hook shown at P, Fig. 33. Therefore by loosening the middle screw and tightening the end ones, or *vice versa*, the whole apparatus is carried sidewise on the beam. Also, by loosening one end-screw and tightening the other the apparatus is turned in azimuth. M, M, Fig. 33, are the wyes in which the quill lies. J, J, are screws which raise and lower the wyes. There is a small striding-level, not shown, with which the quill is made horizontal. Q is a weight which presses against the end of the quill and keeps it in contact with the end of the yard. Fig. 35 is a perforated plate which is used in adjusting the quill in azimuth. When the two quills are mounted at opposite ends of the beam with their plane ends towards each other this plate is slipped upon one of them, the end of the quill just fitting in the large hole in the center of the plate. A light is then held at one of the small holes near the end of the plate. The other quill is then turned in azimuth until the reflected light is seen through the hole at the other end of the plate. Since the holes are equidistant from the axis of the first quill, it follows that the plane surface of the second quill is at right angles to the line joining the two quills. The plate is then changed to the second quill, and the first one is similarly adjusted. If, now, the plane ends of the quills are at right angles to their axes (and trial has shown them to be very nearly so), then the axes of the quills are in the same vertical plane. They are brought into the same horizontal plane by means of a striding level one yard long, not shown in the figure.

ALIGNMENT-APPARATUS.

§ **24.** The alignment of the bars in the tubes is tested by means of a silk thread, which is stretched from one end of the comparator to the other. The apparatus for holding the thread at one end of the comparator is shown in Fig. 22. T is a pulley over which the thread runs and is strained by the weight U. The thread is adjusted sidewise and vertically by the screws V and W, respectively. The apparatus is similar at the other end. The thread used weighs 0.024 grammes per metre. It is strained with the weight U=118 grammes. The computed sag at the middle of the thread, the supports being 4^{m} apart, is $0^{mm}.3$. There is a small vertical scale on the microscope at X. To test the alignment of the bars in a horizontal plane, the thread is adjusted so as to be tangent to this scale. The microscope on its car is moved along and focused over each metre-mark in turn. The elevation of the microscope is determined each time by the reading of the thread on the vertical scale. This gives the relative distances of the several metre-marks below the thread. To test the alignment of the bars in a vertical plane the thread is stretched between the two tubes in the same horizontal plane as the graduated surfaces of the bars. This is done by removing the lower section of the standards which support the thread and using only the upper section. The thread is then focused under the microscope. The microscope on its car is pushed from one end of the comparator to the other. The distance of the microscope from the thread is determined opposite each metre-mark. The tube is then brought under the microscope, which is again pushed from end to end of the comparator. The distance of the microscope from the longitudinal graduation on the steel bar is determined at each metre-mark. Since the path of the miscroscope was the same both times, the relative distances of the graduations from the thread become known.

STANDARD YARDS *A* AND *B*.

§ **25.** These yards were made by Troughton and Simms. One of them is shown at R, Fig. 32. It is of steel, with rectangular cross-section 13^{mm} wide and 19^{mm} deep. One centimeter of each end of the yard is cylindrical and projects from the box as shown in the figure. In the ends of these cylinders agates are set to form the end-surfaces of the yard. These surfaces are convex with a radius of one yard. The yard is inclosed in an iron box, the lid of which is lifted up in the figure. The yard is supported at four points on the ends of two levers. In the figure the side of the box is removed at one point to show one of the levers. There are caps not shown in the figure which fit on the ends of the box and protect the yard when not in use. There are four thermometers, with bulbs bent at right angles to the stems, inserted in cavities in the sides of the box and read through slots in the lid. In the figure the yard is removed at one point to show one of these thermometers.

METALLIC-THERMOMETER METRE.

§ **26.** This metre was made by Repsold, in 1876, to accompany the base-apparatus. It is designated "*MT*1876." It consists of two bars, one of steel, the other of zinc, placed side by side in an iron box. A side-view of this box is shown at N, Fig. 18, and an end-view at S, Fig. 22. Both bars have rectangular cross-sections 13^{mm} wide and 27^{mm} deep. They are bolted together one decimeter from the forward end, and are supported on rollers in the same manner as the bars in the tubes described in § 2. There are two groups of decimillimeter graduations, one at the rear and the other at the front end, shown at S and T respectively, Fig. 38. The bars in Fig. 38 are about half actual size, and the intervals between the graduations are about twice their actual size. The upper halves of the bars are cut away at the ends, so that the graduations are on the neutral axis when the two bars are considered together.

STANDARD STEEL METRE.

§ **27.** This metre was made by Repsold in 1876, and is designated as "*R*1876." It is a single bar of steel with its cross-section as shown at N, Fig. 22. Its graduations are on platinum surfaces on the neutral axis. A top-view is shown in Fig. 27. All the graduations are shown except those in the first millimeter, which is divided into tenths. The numbers shown at the side of the bar in the figure are engraved on the bar opposite their respective graduations. The figure 1 and the letter m at the one-metre end are in their proper places. The space between the figure 1 and the middle stroke of the letter m has been carefully determined and used as a standard space in determining the value of the micrometers.

STANDARD DECIMETER.

§ **28.** This scale was made by Repsold in 1876, and is designated as "*D*1876." A top-view and cross-section are shown in Fig. 37. It is of steel with its graduations on platinum. All of the graduations are shown in the figure except those in the first millimeter, which is divided into tenths. Fig. 36 shows the apparatus for holding the decimeter on the beam. U and V are shoulders against which the decimeter is pressed by the spring W. X is a screw which clamps the apparatus to the beam. Y is a tangent-screw which moves the decimeter longitudinally.

CHAPTER IX.

PRINCIPAL CONSTANTS OF REPSOLD BASE-APPARATUS AND OF ITS STANDARDS.

§ **1.** In comparisons of standards of length, when great precision is desired, two things should be especially aimed at, namely, steadiness of temperature and stability of comparing apparatus. To obtain steadiness of temperature, before comparisons of the Repsold base apparatus were made, a room 22 feet long, 11 feet broad, and 12 feet high, on the ground floor of the Lake-Survey office, had its whole interior lined with a layer, 1 foot thick, of sawdust, in order to diminish its fluctuations of temperature. A double door, the two doors being 17 inches apart, was prepared, and in entering and leaving the room both doors were never open at the same time when comparisons were going on, so that no inrush or outrush of air was possible. The doors had large plates of glass in them, through which a gas-burner at the focus of a parabolic mirror, 23 inches in diameter, sent the light needed for illumination. This room is described in Chapter VIII, § 15. The result of the construction was that when the daily fluctuations of external temperature were 20° F. (they were often greater), the daily fluctuations in the room arising from those external fluctuations were only 0°.1 or 0°.2 F. When a well-marked maximum or minimum, extending over several days, occurred in the external temperature, the maximum or minimum in the comparing-room occurred from two to four days later. The usual temperature-change in a day in the comparing-box did not exceed 2° F. In a long series of observations, extending from November 15, 1879, to March 15, 1880, the temperature-change reached 4° on but two days. The fluctuations of the air of the comparing-room were greater than those in the comparing box, but they rarely exceeded 3° F.

The comparing-apparatus, as designed by Repsold, had a contrivance for fastening two microscopes, 4 metres apart, to the ends of the heavy bed-frame on which all the movable parts rest. Then, by moving the tube-car with the two tubes resting on it from side to side on the bed-frame, the ends of the tubes could be alternately brought under the microscopes for comparison. But this motion from side to side of 500 kilogrammes on the cross-rails of the bed-frame, through a distance of $0^m.155$, necessarily changed the form of the bed-frame, which is supported on six vertical screws. The first comparisons gave discrepant results, and on examining the question it was found that in consequence of these strains the microscopes changed direction by large amounts when the tube-car was run from side to side. The bed-frame of the apparatus was already mounted on brick piers. After these experiments, brick piers were also built for the microscopes, and the latter were firmly mounted on them by means of iron gallows-frames solidly fastened to iron bolts let into the brick piers. These heavy piers were founded on clay, the foundations being $0^m.5$ below the ground of the cellar beneath the comparing-room, and 1^m square, but owing to yielding of the clay they were not perfectly stable. Their motions were, however, so slow as not to affect sensibly any comparisons.

For comparisons of *R*1876 with the separate metres of the tubes, Repsold prepared a bar on which microscopes could be mounted at any distance apart from $0^m.1$ to $1^m.0$, this bar being carried by four feet, which ran on the rails of the bed-frame. The microscopes being mounted on this microscope-car, which was astride of the tube-car, the latter carrying the tube and metre *R*1876, would, when moved laterally, bring one or the other of these alternately under the microscopes for comparison, the microscope-car remaining unmoved. The early comparisons not being satisfactory, it occurred to me that since the bed frame, on whose rails the microscope-car stood, was strained by running the tube-car from side to side, and as the microscope-car rested on four feet, the pressure of the feet on the rails would vary, thus introducing strains into the microscope-car frame. Delicate levels were mounted on the microscopes, and it was found that when the tube-car was moved from side to side the microscopes changed their relative inclination by several seconds. The microscope-car was then changed so as to stand on three feet (instead of four), forming in plan an isosceles triangle. Since that time repeated attempts have been made to discover any motion of the microscopes mounted on the microscope-car with reference to each other when the movable tube-carrying frame beneath it was run from side to side. The methods have been to mount levels on the two microscopes parallel to a tube and also at right angles to a tube, and then to run the tube from side to side. The levels were delicate (one division$=1''$ or $2''$), but, though in the experiments the microscopes changed their absolute inclination by several seconds, there has never been any change in their relative inclination that could be detected. A second method was to suspend R 1876 under the microscopes and to read on it before and after the tube-carriage was run from side to side. Either of these methods should have detected a change of 1^{μ} or less in the microscope-interval. Indeed, with the bar carrying the microscopes supported on a tripod whose feet rest on the heavy bed-frame, it is difficult to see how slight strains in this frame should bend the bar. The first of these methods was also applied to detect any motion of the microscopes when mounted four metres apart on their piers. Running the tube-carriage from side to side might transmit, through the piers supporting the tube-carriage and the earth at their foundation, some strain to the foundation of the microscope piers, and so displace the microscopes. No motion, absolute or relative, could be detected in them.

Of the two measuring-tubes of the Repsold base-apparatus, No. 1 alone has been used in the measurement of bases, No. 2 having been kept in the office of the Lake Survey, as a standard, and as a reserve in case of accident to No. 1. The two tubes were identical when received from Repsold, but prior to use in the field No. 1 was covered by a coating of felt held snugly against the tube by a canvas cover. The thickness of the combined covering is 10^{mm}, and its object is to make the changes of temperature in the interior of the tube less rapid. The steel and zinc bars of tube 1 are designated by S_1 and Z_1, respectively, and those of tube 2 by S_2 and Z_2. The standard metre for these tubes is R1876, described in § 27, Chapter VIII.

The different steps in determining the length and expansion of the parts of the base-apparatus are the following:

1. Determining the length and expansion of the standard metre, R1876, in terms of the Clarke yard A.
2. Determining from R1876 the length of S_2 and its relative expansion.
3. Determining from S_2 the length and expansion for 1° F. of S_1.
4. Determining from Z_2 the length and expansion for 1° F. of Z_1.
5. Determining from S_2 the length and expansion for 1° F. of Z_2.
6. Determining from the 15-feet brass bar of the Lake Survey the expansion of S_2 for 1° F.
7. Adjustment and collection of results.
8. Foerster's data as to R1876.

With the exception of Foerster's determinations, all the work under these heads has been done in the comparing-room of the Lake Survey, a description of which has already been given in this section as well as some data as to the steadiness of its temperature.

Various constants, including periodic errors and values of micrometer-screws, graduation-errors, etc., are given in §§ 68 to 75.

DETERMINATION OF ERRORS OF GRADUATION OF R 1876.

§ **2.** Clarke yard A has been described in Chapter II, § 2; and also with reference to diagram in Chapter VIII, § 25, and R1876 in Chapter VIII, § 27. As R1876 is a line-measure, while Clarke yard A is an end-measure, two steel cylinders 110^{mm} long and $9^{mm}.5$ in diameter, each having one plane and one convex agate end, were prepared by Repsold for such comparisons. The convex ends have a radius of $10^{mm}.5$. At each end of these cylinders a portion of the cylinder has been cut away, leaving a plane surface parallel to, and $4^{mm}.4$ from, the axis, on which a scale of tenths of millimeters, 5^{mm} long, has been graduated on platinum. On each small scale the zero-division is that nearest the plane end of the cylinder. By placing the axes of these small cylinders exactly in the prolongation of the axes of the terminal cylinders of Clarke yard *A*, the convex ends of the cylinders being in contact with the ends of the yard, such graduation-lines on the cylinders can be selected as shall be very nearly the same distance apart (within less than $0^{mm}.1$) as two graduation-lines on the metre. The lines selected on the metre were the 81st millimeter and the 1000th millimeter mark. On the cylinders they were the 46th lines on the scales at the convex ends of the cylinders. After carefully comparing the distance above specified on the metre with the nearly equal distance between the 46th division lines on the cylinders, when their convex ends abut against the ends of yard *A*, the yard was removed from between the cylinders, the convex ends of the latter were brought into contact, and the small distance (about $4^{mm}.9$) then separating their 46th division marks was carefully determined by comparisons with *R* 1876. Subtracting this distance from the interval between the 81st and 1000th millimeter on *R* 1876, there results the value of yard *A* in terms of this distance. Having given a general sketch of the manner of comparison, the details will now be considered.

§ **3.** First, it is necessary to know exactly what fraction the interval on *R* 1876, between the 81^{mm} mark and the $1,000^{mm}$ mark, is of the interval between 0^{mm} and $1,000^{mm}$ mark on this metre. *R* 1876 is graduated into decimeters, and the first decimeter into millimeters. The first step was to intercompare the double decimeters. For this purpose *R* 1876 was placed on the iron beam described in § 22, Chapter VIII, and made level within 30″. The two microscopes were mounted on their carriage at a distance of $0^{m}.2$ from each other; their lines of collimation were made vertical within 42″, and the microscope-carriage remaining stationary, *R* 1876 was so arranged without disturbing its level that when the iron beam carrying the metre was moved with its supporting trucks lengthwise under the microscopes, their horizontal threads followed closely the longitudinal lines crossing the transverse graduations of *R* 1876. The comparisons were made in the closed box of the comparison-apparatus, by pointing with the microscopes at the successive double-decimeter marks, the microscope-car remaining stationary while the metre was run under the microscopes. The programme for observations was the following:

1. Read thermometer on *R* 1876.
2. Read the two microscopes twice on $0^{m}.0$ and $0^{m}.2$.
3. Move metre lengthwise and read twice on $0^{m}.2$ and $0^{m}.4$; $0^{m}.4$ and $0^{m}.6$; $0^{m}.6$ and $0^{m}.8$; $0^{m}.8$ and $1^{m}.0$; and, finally, twice again on $0^{m}.0$ and $0^{m}.2$.
4. Read thermometer.
5. Three sets to be read daily at 9 a. m., 1 p. m., and 5 p. m.

Each set required about fifteen minutes, and no other visits were made to the comparing-room during the day. After five days' observations the metre was turned end for end, and three days more of comparisons were obtained. In the following table the results of each visit are given. The mean of the first and last set of readings at each visit on the space $0^{m}.0$ to $0^{m}.2$ is used as the reading on this space. The first column gives the date of observation; the second and third the corrected temperatures at beginning and end of observation; the fourth, the excess of length of the first over the second double decimeter; the fifth, the residuals or excess of separate results over mean of all; the sixth and seventh, eighth and ninth, tenth and eleventh columns give for the other double decimeters the same information as the fourth and fifth columns give for the second double decimeter.

Results of comparisons of $0^m.2$ spaces of metre R1876 with each other.

O-END OF METRE EAST.

Date.	Corrected temperature. Before.	After.	Mean.	$(0^m.0-0^m.2)$ *minus* $(0^m.2-0^m.4)$.	*v.*	$(0^m.0-0^m.2)$ *minus* $(0^m.4-0^m.6)$.	*v.*	$(0^m.0-0^m.2)$ *minus* $(0^m.6-0^m.8)$.	*v.*	$(0^m.0-0^m.2)$ *minus* $(0^m.8-1^m.0)$.	*v.*
1.	2.	3.		4.	5.	6.	7.	8.	9.	10.	11.
1879.	° F.	° F.	° F.	μ	μ	μ	μ	μ	μ	μ	μ
April 29, 9:13–9:27 a. m	59.72	59.72	59.72	−3.2	+1.1	−3.0	+0.4	−0.4	−0.3	−3.0	+0.8
April 29, 1:03–1:17 p. m	59.52	59.52	59.52	−2.7	+0.6	−2.4	−0.2	−1.3	+0.6	−2.9	+0.7
April 29, 5:00–5:15 p. m	59.62	59.52	59.57	−2.4	+0.3	−2.0	−0.6	−0.9	+0.2	−1.6	−0.6
April 30, 8:59–9:20 a. m	58.92	59.02	58.97	−2.2	+0.1	−2.8	+0.2	−0.5	−0.2	−2.0	−0.2
April 30, 1:00–1:20 p. m	58.92	58.92	58.92	−2.2	+0.1	−2.6	0.0	+0.1	−0.8	−2.6	+0.4
April 30, 4:58–5:12 p. m	58.82	58.82	58.82	−2.2	+0.1	−2.3	−0.3	−0.9	+0.2	−2.9	+0.7
May 1, 9:23–9:36 a. m	57.22	57.22	57.22	−3.1	+1.0	−3.0	+0.4	−1.1	+0.4	−2.1	−0.1
May 1, 1:00–1:12 p. m	57.02	57.12	57.07	−2.4	+0.3	−3.7	+1.1	−0.7	0.0	−2.5	+0.3
May 1, 4:55–5:06 p. m	57.12	57.12	57.12	−2.0	−0.1	−3.6	+1.0	−1.4	+0.7	−2.8	+0.6
May 2, 9:41–9:54 a. m	56.32	56.32	56.32	−1.4	−0.7	−2.0	−0.6	−0.3	−0.4	−2.5	+0.3
May 2, 1:12–1:24 p. m	56.32	56.32	56.32	−1.8	−0.3	−3.0	+0.4	−0.8	+0.1	−1.4	−0.8
May 2, 5:00–5:14 p. m	56.22	56.22	56.22	−1.6	−0.5	−2.3	−0.3	0.0	−0.7	−2.8	+0.6
May 3, 9:03–9:14 a. m	55.22	55.22	55.22	−2.3	+0.2	−2.4	−0.2	−1.5	+0.8	−2.8	+0.6
May 3, 12:59–1:10 p. m	55.01	55.11	55.06	−2.7	+0.6	−3.3	+0.7	−0.3	−0.4	−1.7	−0.5
May 3, 4:56–5:07 p. m	55.01	55.01	55.01	−1.6	−0.5	−1.7	−0.9	−0.6	−0.1	−2.4	+0.2
O-END OF METRE WEST.											
May 8, 9:09–9:26 a. m	53.50	53.60	53.55	−0.6	−1.5	−3.9	+1.3	−1.6	+0.9	+0.2	−2.4
May 8, 1:03–1:20 p. m	53.60	53.60	53.60	−2.2	+0.1	−2.2	−0.4	0.0	−0.7	−1.7	−0.5
May 8, 5:02–5:15 p. m	53.71	53.81	53.76	−2.5	+0.4	−2.1	−0.5	−1.4	+0.7	−1.6	−0.6
May 9, 9:19–9:30 a. m	53.20	53.20	53.20	−0.8	−1.3	−2.2	−0.4	+0.1	−0.8	−2.5	+0.3
May 9, 1:07–1:17 p. m	53.30	53.40	53.35	−3.8	+1.7	−2.8	+0.2	−1.3	+0.6	−3.7	+1.5
May 9, 4:47–4:57 p. m	53.50	53.40	53.45	−1.4	−0.7	−1.8	−0.8	+0.4	−1.1	−1.3	−0.9
May 10, 9:26–9:40 a. m	53.40	53.50	53.45	−2.4	+0.3	−2.5	−0.1	−1.3	+0.6	−2.3	+0.1
May 10, 1:02–1:13 p. m	53.71	53.71	53.71	+0.3	−2.4	−2.1	−0.5	−0.3	−0.4	−0.7	−1.5
May 10, 4:46–4:55 p. m	53.91	53.91	53.91	−2.2	+0.1	−3.2	+0.6	−2.2	+1.5	−3.4	+1.2
Means				−2.1		−2.6		−0.7		−2.2	
Probable errors				±0.12		±0.08		±0.09		±0.12	

If the means of the temperatures on entering and on leaving the comparing-room be taken, the latter will be found 0°.02 F. the greater; but as on the whole the exterior temperature was rising, this quantity is not sufficient to show that the observers' presence affected the temperatures. After fifteen visits to the comparing-room the metre R1876 was turned end for end, and nine more visits were then made. The following table gives the difference in the results:

	0^m end east, 15 visits.	0^m end west, 9 visits.
m. m. m. m.	μ	μ
0.0 to 0.2 minus 0.2 to 0.4	−2.25	−1.73
0.0 to 0.2 minus 0.4 to 0.6	−2.67	−2.53
0.0 to 0.2 minus 0.6 to 0.8	−0.71	−0.84
0.0 to 0.2 minus 0.8 to 1.0	−2.40	−1.89

The discrepancies are not sufficiently large to show that any effect was produced by reversing the metre. The stability in the distance between the microscopes was examined by comparing the changes in the observed difference between their interval and $0^m.0$ to $0^m.2$ at the first observations of a visit, and at the last observations at the same visit, the metre space being supposed constant. The maximum change in the observed interval was $-1^{\mu}.2$. Ten times the microscope-interval seemed to increase and ten times to decrease, the mean increase being $0^{\mu}.05$. This indicates no relative motion of the microscopes.

Deriving the probable errors of the means from the residuals, there result for metre R1876, rounding them to the nearest decimicron—

$$(1)\qquad \begin{cases} \text{Space } 0^{m}.2 \text{ to } 0^{m}.4 = 0^{m}.0 \text{ to } 0^{m}.2 + 2^{\mu}.1 \pm 0^{\mu}.1 \\ \text{Space } 0.4 \text{ to } 0.6 = 0.0 \text{ to } 0.2 + 2.6 \pm 0.1 \\ \text{Space } 0.6 \text{ to } 0.8 = 0.0 \text{ to } 0.2 + 0.7 \pm 0.1 \\ \text{Space } 0.8 \text{ to } 1.0 = 0.0 \text{ to } 0.2 + 2.2 \pm 0.1 \\ \text{Space } 0.0 \text{ to } 0.2 = 0.0 \text{ to } 0.2 \end{cases}$$

Adding, there results—

$$\text{Space } 0^{m}.0 \text{ to } 1^{m}.0 = 5\,(\text{Space } 0^{m}.0 \text{ to } 0^{m}.2) + 7^{\mu}.6$$

Since each of the probable errors given above depends on pointings at the first double decimeter they are not independent, and the probable error of the sum $7^{\mu}.6$ is not the square root of the sum of the squares of the separate probable errors of the results of comparisons of the other double decimeters with the first. It is necessary to recur to the actual errors. The quantities measured are in fact the differences between the double decimeters and the microscope-interval. Denote the mean microscope-interval during comparisons by I, and the successive double decimeters by A, B, C, D, E, and let d_a= the mean of the observed $A-I$; d_b= the mean of the observed $B-I$, &c.; and let a, b, &c., represent the actual errors in these means, then—

$$(2)\qquad \begin{cases} A=A \\ B=A+d_b-d_a+b-a \\ C=A+d_c-d_a+c-a \\ D=A+d_d-d_a+d-a \\ E=A+d_e-d_a+e-a \end{cases}$$

Summing and transposing—

$$(3)\qquad A=\tfrac{1}{5}\,(R1876)+\tfrac{4}{5}d_a-\tfrac{1}{5}\,(d_b+d_c+d_d+d_e)+[\tfrac{4}{5}a-\tfrac{1}{5}\,(b+c+d+e)]$$

where the last term is the actual error. If the probable errors of the mean values d_a, d_b, &c., be represented by a', b', &c., when we pass from actual errors in d_a, d_b, &c., to probable errors, the value of A becomes

$$(3')\qquad A=\tfrac{1}{5}\,(R1876)+\tfrac{4}{5}d_a-\tfrac{1}{5}d_b-\tfrac{1}{5}d_c-\tfrac{1}{5}d_d-\tfrac{1}{5}d_e\pm\tfrac{1}{5}\sqrt{16a'^2+b'^2+c'^2+d'^2+e'^2}$$

We also wish to know the distances $A+B$, $A+B+C$, $A+B+C+D$, in terms of R1876. From the value of A just found, (3), and (1) there results—

$$(4)\qquad \begin{cases} A+B=\tfrac{2}{5}\,(R1876)+\tfrac{3}{5}d_a+\tfrac{3}{5}d_b-\tfrac{2}{5}d_c-\tfrac{2}{5}d_d-\tfrac{2}{5}d_e+(\tfrac{3}{5}a+\tfrac{3}{5}b-\tfrac{2}{5}c-\tfrac{2}{5}d-\tfrac{2}{5}e) \\ A+B+C=\tfrac{3}{5}\,(R1876)+\tfrac{2}{5}d_a+\tfrac{2}{5}d_b+\tfrac{2}{5}d_c-\tfrac{3}{5}d_d-\tfrac{3}{5}d_e+(\tfrac{2}{5}a+\tfrac{2}{5}b+\tfrac{2}{5}c-\tfrac{3}{5}d-\tfrac{3}{5}e) \\ A+B+C+D=\tfrac{4}{5}\,(R1876)+\tfrac{1}{5}d_a+\tfrac{1}{5}d_b+\tfrac{1}{5}d_c+\tfrac{1}{5}d_d-\tfrac{4}{5}d_e+(\tfrac{1}{5}a+\tfrac{1}{5}b+\tfrac{1}{5}c+\tfrac{1}{5}d-\tfrac{4}{5}e) \end{cases}$$

The last terms in the second members are the actual errors of the successive sums; passing to probable errors they are in order

$$\tfrac{1}{5}\sqrt{9a'^2+9b'^2+4c'^2+4d'^2+4e'^2}$$
$$\tfrac{1}{5}\sqrt{4a'^2+4b'^2+4c'^2+9d'^2+9e'^2}$$
$$\tfrac{1}{5}\sqrt{a'^2+b'^2+c'^2+d'^2+16e'^2}$$

In the numerical values, $0^{\mu}.1$, already given for the probable error in the differences of length of the first and the other double decimeters, two elements enter; the first is the probable error in comparing the microscope-interval, I, with the first double decimeter; the second is the error in comparing that interval with each of the other double decimeters. The number of comparisons of the first class was double that of the second class for each decimeter, therefore its mean difference should have double weight, and in the $(0^{\mu}.1)^2$=the square of the probable error of the comparisons of the first with each of the other double decimeters, 0.0033 must be attributed to the comparisons of

A with I and 0.0067 to the comparisons of each of the other double decimeters with I. We have, then, $a'^2=0.0033$ and $b'^2=c'^2=d'^2=e'^2=0.0067$. Substituting these values in algebraic expressions for the probable error, previously given, and substituting for (d_b-d_a), &c., in (3) and (4), their numerical values given in (1), there result finally

$$\text{Space } (\overset{m.}{0.0} \text{ to } \overset{m.}{0.2})=\tfrac{1}{5}\,(R\,1876)-\overset{\mu}{1.5}\pm\overset{\mu}{0.06}$$
$$\text{Space } (0.0 \text{ to } 0.4)=\tfrac{2}{5}\,(R\,1876)-1.0\pm0.08$$
$$\text{Space } (0.0 \text{ to } 0.6)=\tfrac{3}{5}\,(R\,1876)+0.1\pm0.09$$
$$\text{Space } (0.0 \text{ to } 0.8)=\tfrac{4}{5}\,(R\,1876)-0.7\pm0.07$$

§ **4.** The second step was to compare the space $0^m.0$ to $0^m.1$ with the space $0^m.1$ to $0^m.2$ on metre R 1876. The method of comparison was essentially the same as that just given for the intercomparison of the double decimeters. The microscopes were only one decimeter apart, and they, as well as the metre, were adjusted as in the double-decimeter comparisons. A thermometer lying on R 1876 was read on entering and leaving the comparing-room. Three visits to the comparing-room were made on each day at 9 a. m., 12:30 p. m., and 4:15 p. m. At each visit the spaces $0^m.0$ to $0^m.1$ and $0^m.1$ to $0^m.2$ were read on alternately till each had been read on three times. Each reading on a space consisted of two pointings at each of its ends, thus making 6 pointings at $0^m.0$ and $0^m.2$, and 12 at $0^m.1$ at each visit. Seventeen visits were made on six days. In reduction, each reading on one decimeter was combined with both the preceding and following one on the other decimeter, and the mean taken as the result of the visit. In the following table the first column gives the date of comparison; the second and third the corrected temperatures on entering and leaving the comparing room; the fourth the excess of length of the first over the second decimeter, and the fifth the residuals or excesses of the mean of the mean results over the individual mean results.

Results of comparisons of space ($0^m.0$–$0^m.1$) *with space* ($0^m.1$–$0^m.2$) *of metre* R 1876.

Date.	Corrected Temp.		($0^m.0$–$0^m.1$) *minus* ($0^m.1$–$0^m.2$).	Residuals.
	Before observations.	After observations.		
1879.	° F.	° F.	μ	μ
June 11, 9.21—9:35 a. m.	64.52	64.52	−0.3	
			+1.1	
			+0.6	
			+1.0	
			+2.1	
Mean			+0.9	+0.2
June 11, 12:29—12:44 p. m	64.71	64.71	+1.1	
			+0.9	
			+1.3	
			+1.4	
			+1.0	
Mean			+1.1	0.0
June 11, 4:12—4:23 p. m	64.81	64.91	−0.1	
			+1.0	
			+0.6	
			−0.6	
			+0.6	
Mean			+0.3	+0.8
June 12, 9:11—9:22 a. m.	65.41	65.41	−0.1	
			+1.0	
			+2.5	
			+2.4	
			+2.0	
Mean			+1.6	−0.5

Results of comparisons of space ($0^m.0$–$0^m.1$) *with space* ($0^m.1$–$0^m.2$) *of metre* R1876—Continued.

Date.	Corrected Temp.		($0^m.0$–$0^m.1$) *minus* ($0^m.1$–$0^m.2$).	Residuals.
	Before observations.	After observations.		
1879.	° F.	° F.	μ	μ
June 12, 12:27—12:40 p. m	65.61	65.71	+0.2	
			+0.5	
			+1.5	
			+0.8	
			+0.9	
Mean			+0.8	+0.3
June 13,— —9:43 a. m	66.51	66.51	+1.9	
			+1.4	
			+0.9	
			+1.0	
			+0.9	
Mean			+1.2	−0.1
June 13, 12:31—12:42 p. m	66.61	66.61	+0.4	
			+0.2	
			+0.7	
			+0.2	
			+0.4	
Mean			+0.4	+0.7
June 13, 4:15—4:26 p. m	66.71	66.81	+1.4	
			+1.1	
			+0.4	
			+0.7	
			+0.3	
Mean			+0.8	+0.3
June 14, 9:09—9:21 p. m	66.31	66.41	+1.1	
			+0.7	
			+1.1	
			+0.5	
			+0.5	
Mean			+0.8	+0.3
June 14, 12:29—12:38 p. m	66.61	66.61	+1.4	
			+0.6	
			+0.6	
			+0.0	
			+0.1	
Mean			+0.5	+0.6
June 14, 4:12—4:21 p. m	66.71	66.71	+2.2	
			+1.5	
			+0.6	
			+1.5	
			+2.3	
Mean			+1.6	−0.5
June 17,— —9:23 a. m	66.01	66.01	+1.2	
			+2.2	
			+1.8	
			+0.7	
			+1.5	
Mean			+1.5	0.4

Results of comparisons of space ($0^m.0-0^m.1$) *with space* ($0^m.1-0^m.2$) *of metre R* 1876—Continued.

Date.	Corrected Temp. Before observations.	Corrected Temp. After observations.	($0^m.0-0^m.1$) *minus* ($0^m.1-0^m.2$).	Residuals.
1879.	° F.	° F.	μ	μ
June 17, 12:29—12:39 p. m	66.01	66.01	+0.7	
			+1.4	
			+2.2	
			+1.2	
			+0.1	
Mean			+1.1	0.0
June 17, 4:17—4:25 p. m	66.11	66.11	+0.4	
			−0.4	
			+0.7	
			+1.3	
			+1.8	
Mean			+0.8	+0.3
June 18, 9:16—9:25 a. m	65.21	65.21	+1.4	
			+1.9	
			+2.0	
			+1.9	
			+2.1	
Mean			+1.9	−0.8
June 18, 12:25—12:33 p. m	65.21	65.21	+2.1	
			+2.9	
			+3.0	
			+2.4	
			+2.4	
Mean			+2.6	−1.5
June 18, 3:43—3:52 p. m	65.11	65.11	+1.3	
			+0.6	
			+1.1	
			+2.0	
			+1.6	
Mean			+1.3	−0.2
Means	65.78	65.80	+1.1	
Probable error of mean of mean results			±0.1	

There results from this work

$$\text{Space } (0^m.0 \text{ to } 0^m.1) = \text{space } (0^m.1 \text{ to } 0^m.2) + 1^\mu.1 \pm 0^\mu.10$$

the probable error being derived from the differences between the individual means and the general mean. This gives

$$2 \times \text{space } (0^m.0 \text{ to } 0^m.1) = \text{space } (0^m.0 \text{ to } 0^m.2) + 1^\mu.1 \pm 0^\mu.10.$$

But from § 3

$$\text{Space } (0^m.0 \text{ to } 0^m.2) = \tfrac{1}{5}\,(R\,1876) - 1^\mu.5 \pm 0^\mu.06.$$

Hence,

$$\text{Space } (0^m.0 \text{ to } 0^m.1) = \tfrac{1}{10}\,(R\,1876) - 0^\mu.2 \pm 0^\mu.06.$$

The mean of temperatures on leaving the room after a visit, usually about 12 minutes long, exceeded that on entering the room by 0°.02 F. A slight increase was to be expected, as the exterior temperature was, on the whole, rising during the period of comparisons.

§ **5.** The next step was to determine the relative lengths of the double centimeters of the first decimeter of *R*1876. As the microscopes cannot be mounted nearer to each other than about a decimeter, an auxiliary decimeter scale, *D*1876, divided to millimeters, was used. The microscopes were mounted on the microscope-car as usual at a distance apart at first of a little more than a decimeter. The metre rested horizontally, as in the preceding comparisons, on its iron beam and trucks, while the decimeter scale was placed horizontally on the same beam in prolongation of the metre and at the same height, so that when the beam carrying metre and decimeter was made to travel on its trucks lengthwise under the stationary microscopes, the longitudinal threads of the microscopes would follow the longitudinal lines engraved on metre and decimeter. The two scales being so placed that their graduations increase in the same direction, their zeros being at nearly the same distance apart as the vertical lines of collimation of the microscopes, if one microscope is pointed at the zero of one scale and the other at the zero of the other scale, and if then the beam carrying both scales is moved lengthwise till the 20^{mm} divisions of the two scales come under the two microscopes, and are pointed at, a comparison of these parts of the scales is thus effected. Having made a sufficient number of such comparisons of the space 0^{mm} to 20^{mm} on decimeter with 0^{mm} to 20^{mm} on *R*1876, one microscope was then displaced by 20^{mm} and the comparisons of 0^{mm} to 20^{mm} on decimeter with 20^{mm} to 40^{mm} on *R*1876 were made, and so on till the values of the 20^{mm} spaces on the first decimeter of *R*1876 are all found in terms of 0^{mm} to 20^{mm} on the decimeter. Three visits to the comparing room were usually made on each day at 9 a. m., 12:20 p. m., and 4:15 p. m. At each visit readings were made with the two microscopes on the two ends of the portions of the scales under comparison, the readings being alternately on the left-hand ends and right-hand ends of the scales, until three readings had been obtained on each end, thus giving three independent comparisons. Each reading was the mean of two microscope-pointings. In reduction, to eliminate, partially at least, any possible change in the relative distance of the scales while being moved under the microscopes, each reading at one end of the scales, except the first and last, was combined with the preceding and following readings on the other ends of the scales, thus giving two results. The mean of all such results was taken as the result of the visit, and the mean of these means as the final result. The following table gives the dates of comparisons, the temperature of the metre, the excess of length of the double centimeter spaces on the metre *R*1876 over the space 0^{mm} to 20^{mm} on the decimeter, the means of these results for one visit, and the residuals or excesses of general mean result over the individual mean results:

Results of comparisons of 20^{mm} *spaces on metre R*1876 *with space* (0^{mm}—20^{mm}) *on decimeter D*1876.

Date.	Temp.	Space 0^{mm} to 20^{mm} on metre *minus* space 0^{mm} to 20^{mm} on decimeter.	Means.	Residuals.
1879.	° F.	μ	μ	μ
May 16, 9:44—9:58 a. m.	64.0	+1.6		
		+1.4		
		+1.0		
		+0.0		
		+1.2	+1.0	+0.4
May 16, 12:19—12:28 p. m.	64.15	+1.0		
		+2.1		
		+0.8		
		+0.9		
		+2.0	+1.4	0.0
May 16, 4:27—4:41 p. m.	64.0	+1.5		
		+0.3		
		+1.3		
		+1.5		
		+1.7	+1.3	+0.1
May 27, 9:19—9:33 a. m.	59.3	+1.0		
		+0.4		
		+0.8		
		+0.4		
		+0.8	+0.7	+0.7

Comparisons of 20^{mm} *spaces on R*1876 *and D*1876—Continued.

Date.	Temp.	Space 0^{mm} to 20^{mm} on metre *minus* space 0^{mm} to 20^{mm} on decimeter.	Means.	Residuals.
1879.	° F.	μ	μ	μ
May 27, 12:32—12:42 p. m	59. 3	+0. 9		
		+0. 3		
		+0. 6		
		+0. 0		
		—0. 3	+0. 3	+1. 1
May 27, 4:28—4:36 p. m	50. 4	+1. 6		
		+0. 6		
		+1. 7		
		+2. 1		
		+0. 8	+1. 4	0. 0
May 29, 9:14—9:25 a. m	59. 55	+2. 8		
		+0. 8		
		—0. 3		
		+0. 7		
		+2. 8	+1. 4	0. 0
May 29, 12:38—12:48 p. m	50. 7	+3. 8		
		+3. 0		
		+0. 8		
		—1. 1		
		—0. 3	+1. 2	+0. 2
May 29, 4:27—4:37 p. m	59. 85	+2. 4		
		+0. 9		
		+1. 5		
		+1. 7		
		+2. 1	+1. 7	—0. 3
May 30, 9:20—9:40 a. m	60. 35	+2. 0		
		+3. 4		
		+2. 5		
		+2. 2		
		+2. 8	+2. 6	—1. 2
May 30, 12:40—12:50 p. m	60. 7	+1. 6		
		+1. 4		
		+1. 1		
		+1. 2		
		+2. 2	+1. 5	—0. 1
May 30, 7:03—7:20 p. m	61. 2	+2. 1		
		+3. 1		
		+2. 2		
		+2. 0		
		+2. 2	+2. 5	—1. 1
Mean of the 12 means...	....	....	+1. 4	
Probable error............	....	....	±0. 13	

Comparisons of 20^{mm} *spaces on* R1876 *and* D1876—Continued.

Date.	Temp.	Space 20^{mm} to 40^{mm} on R 1876 *minus* space 0^{mm} to 20^{mm} on D 1876.	Means.	Residuals.
1879.	° F.	μ	μ	μ
May 17, 9:14—9:29 a. m	62. 75	+2. 8		
		+2. 4		
		+0. 4		
		−0. 7		
		−0. 1	+1. 0	−0. 4
May 17, 12:25—12:44 p. m	62. 8	−0. 3		
		+1. 2		
		+1. 1		
		+0. 2		
		−0. 4	+0. 4	+0. 2
May 17, 4:15—4:31 p. m	62. 8	0. 0		
		+1. 1		
		+1. 0		
		+0. 8		
		+0. 6	+0. 7	−0. 1
May 26, 9:40—9:52 a. m	59. 8	−0. 5		
		−0. 5		
		0. 0		
		+1. 0		
		+0. 8	+0. 2	+0. 4
May 26, 12:31—12:40 p. m	59. 95	−0. 1		
		−0. 7		
		−0. 1		
		+0. 1		
		−0. 8	−0. 3	+0. 9
May 26, 4:43—4:53 p. m	59. 9	−0. 6		
		+0. 6		
		+2. 4		
		+2. 2		
		+1. 5	+1. 2	−0. 6
May 31, 9:10—9:25 a. m	62. 05	+0. 7		
		+1. 0		
		+0. 8		
		+0. 4		
		+0. 8	+0. 7	−0. 1
May 31, 12:33—12:44 p. m	62. 55	+2. 8		
		+1. 6		
		+0. 2		
		+0. 1		
		+0. 3	+1. 0	−0. 4
May 31, 4:23—4:34 p. m	62. 0	+1. 2		
		−1. 2		
		−1. 2		
		+1. 7		
		+2. 4	+0. 6	0. 0
Mean of the 9 means			+0. 6	
Probable error			±0. 10	

Comparisons of 20^{mm} *spaces on* R1876 *and* D1876—Continued.

Date.	Temp.	Space 40^{mm} to 60^{mm} on R 1876 *minus* space 0^{mm} to 20^{mm} on D1876.	Means.	Residuals.
1879.	° F.	μ	μ	μ
May 18, 10:18—10:45 a. m	61.85	+1.2		
		+0.7		
		−0.9		
		−0.1		
		+0.9	+0.4	−0.8
May 18, 3:41—3:57 p. m	62.05	−0.1		
		−0.7		
		−1.3		
		−0.7		
		+0.2	−0.5	+0.1
May 18, 7:03—7:20 p. m	62.1	0.0		
		−0.7		
		−1.5		
		−0.6		
		−1.0	−0.8	+0.4
May 25, 9:40—9:55 a. m	59.7	0.0		
		+0.1		
		−0.5		
		−0.6		
		−0.5	−0.3	−0.1
May 25, 12:46—1:00 p. m	59.95	+1.8		
		+1.4		
		−0.3		
		−0.9		
		−2.0	0.0	−0.4
May 25, 6:43—7:00 p. m	62.2	−1.4		
		−0.9		
		−0.1		
		−0.8		
		−1.0	−0.8	+0.4
June 1, 9:32—9:48 a. m	64.35	−0.8		
		−0.4		
		0.0		
		+0.2		
		+1.0	0.0	−0.4
June 1, 2:30—2:44 p. m	64.85	−0.5		
		−0.7		
		−2.0		
		−2.1		
		−2.2	−1.5	+1.1
June 1, 7:45—7:59 p. m	65.2	−0.5		
		−0.8		
		−0.2		
		−0.2		
		−0.2	−0.4	0.0
Mean of the 9 means			−0.4	
Probable error			±0.13	

Comparisons of 20mm *spaces on R*1876 *and D*1876—Continued.

Date.	Temp.	Space 60mm to 80mm on R 1876 *minus* space 0mm to 20mm on D 1876.	Means.	Residuals.
1879.	° F.	μ	μ	μ
May 19, 9:13—9:23 a. m	61.2	+0.4		
		+0.3		
		−0.4		
		+0.1		
		+0.6	+0.2	+0.2
May 19, 2:16—2:29 p. m	61.3	+0.5		
		+1.0		
		0.0		
		−0.7		
		−0.3	+0.1	+0.3
May 19, 4:26—4:36 p. m	61.4	+0.2		
		0.0		
		−0.2		
		+0.6		
		+1.4	+0.4	0.0
May 24, 9:10—9:29 a. m	59.45	−1.2		
		−1.5		
		−0.9		
		−1.5		
		−1.2	−1.3	+1.7
May 24, 12:42—12:49 p. m	59.6	−0.1		
		−0.7		
		−1.2		
		−0.7		
		0.0	−0.5	+0.9
May 24, 4:17—4:26 p. m	59.7	−1.7		
		−1.8		
		−0.8		
		+0.1		
		−0.5	−0.9	+1.3
June 2, 9:40—9:58 a. m	65.3	+3.2		
		+3.0		
		+0.8		
		+1.0		
		+2.5	+2.1	−1.7
June 2, 12:33—12:44 p. m	65.3	+0.8		
		+0.8		
		+1.2		
		+2.0		
		+1.4	+1.2	−0.8
June 2, 4:21—4:31 p. m	65.3	−0.2		
		+1.6		
		+1.8		
		−0.1		
		−0.3	+0.6	−0.2
1881.				
March 26, 10:14—10:31 a. m	42.51	+2.5		
		−0.6		
		+1.0		
		+0.4		
		+1.5		
		+0.7	+0.9	−0.5

Comparisons of 20^{mm} *spaces on R*1876 *and D*1876—Continued.

Date.	Temp.	Space 60^{mm} to 80^{mm} on *R* 1876 *minus* space 0^{mm} to 20^{mm} on *D* 1876.	Means.	Residuals.
1881.	° F.	μ	μ	μ
March 26, 12:18—12:36 p. m ..	42.71	+1.0		
		+1.1		
		+0.8		
		+0.8		
		+1.4		
		+1.2	+1.0	−0.6
March 26, 4:10—4:24 p. m	42.91	+0.7		
		+1.2		
		+0.6		
		+0.3		
		0.0		
		+1.5	+0.7	−0.3
March 27, 10:02—10:25 a. m ..	42.21	+0.8		
		+1.1		
		+1.4		
		+1.2		
		+1.0		
		+0.3	+1.0	−0.6
March 28, 9:19—9:38 a. m ...	41.86	+1.2		
		+0.5		
		+0.4		
		−0.1		
		+0.9		
		−0.4	+0.4	0.0
March 28, 12:02—12:20 p m ..	42.06	0.0		
		+0.3		
		+1.3		
		+0.8		
		+0.4		
		+0.6	+0.6	−0.2
March 28, 3:58—4:17 p. m	42.21	−0.4		
		+0.4		
		0.0		
		−0.8		
		+0.4		
		+0.5	0.0	+0.4
March 28, 7:18—7:38 p. m	42.36	+0.9		
		+1.0		
		0.0		
		+0.9		
		−0.4		
		+0.4	+0.5	−0.1
Mean of the 17 means	....	....	+0.4	
Probable error	....	....	±0.1	

Comparisons of 20^{mm} *spaces on R*1876 *and D*1876—Continued.

Date.	Temp.	Space 80^{mm} to 100^{mm} on *R* 1876 *minus* space 0^{mm} to 20^{mm} on *D* 1876.	Means.	Residuals.
1879.	° F.	μ	μ	μ
May 20, 9:02—9:13 a. m	61.2	+0.5		
		−1.0		
		−1.7		
		−0.3		
		−0.9	−0.7	−0.2
May 20, 12:36—12:44 p. m	61.3	−0.2		
		−0.4		
		−0.3		
		−0.7		
		−1.4	−0.6	−0.
May 20, 4:29—4:37 p. m	61.45	−0.8		
		+0.2		
		+0.9		
		0.0		
		−1.4	−0.2	−0.7
May 22, 9:05—9:16 a. m	60.8	−1.5		
		−2.0		
		−2.6		
		−2.1		
		−2.4	−2.1	+1.2
May 22, 12:32—12:40 p. m	60.95	−1.3		
		−2.6		
		−2.0		
		−3.0		
		−2.0	−2.2	+1.3
May 22, 4:16—4:25 p. m	61.1	−1.9		
		−0.2		
		−1.0		
		−1.8		
		−1.2	−1.2	+0.3
May 23, 9:28—9:36 a. m	60.0	−1.4		
		−2.0		
		−3.0		
		−2.2		
		−0.6	−1.8	+0.9
May 23, 12:46—12:50 p. m	60.0	−3.4		
		−2.8		
		−2.0		
		−2.2		
		−1.8	−2.4	+1.5
May 23, 4:34—4:41 p. m	60.05	−3.1		
		−1.8		
		−1.1		
		−1.3		
		−0.7	−1.6	+0.7
June 3, 9:16—9:27 a. m	64.6	−1.8		
		−2.0		
		−1.0		
		−1.0		
		−1.6	−1.5	+0.6
June 3, 12:39—12:48 p. m	64.5	−2.4		
		−2.0		
		−2.9		
		−1.8		
		−0.9	−2.0	+1.1

Comparisons of 20^{mm} *spaces on R*1876 *and D*1876—Continued.

Date.	Temp.	Space 80^{mm} to 100^{mm} on *R*1876 *minus* space 0^{mm} to 20^{mm} on *D*1876.	Means.	Residuals.
1879.	° F.	μ	μ	μ
June 3, 4:25—4:34 p. m.	64 3	−3. 0		
		−2. 3		
		−2. 5		
		−2. 1		
		−1. 1	−2. 2	+1. 3
1881.				
March 29, 9:30—9:50 a. m	42. 06	−0. 6		
		+0. 0		
		−0. 2		
		−0. 3		
		+1. 2		
		−0. 6	−0. 1	−0. 8
March 29, 12:06—12:23 p. m	42. 16	−0. 5		
		+0. 3		
		+0. 1		
		+0. 7		
		+0. 3		
		+0. 3	+0. 2	−1. 1
March 29, 4:10—4:30 p. m	42. 16	+0. 6		
		−0. 5		
		+0. 3		
		+0. 4		
		+0. 2		
		−0. 5	+0. 1	−1. 0
March 29, 7:36—7:56 p. m	42. 16	−0. 7		
		−0. 7		
		−0. 2		
		−1. 5		
		+0. 2		
		−1. 2	−0. 7	−0. 2
March 30, 9:14—9:34 a. m	41. 51	−0. 9		
		−0. 0		
		−0. 4		
		+0. 5		
		+0. 4		
		−0. 2	−0. 1	−0. 8
March 30, 12:22—12:43 p. m	41. 76	−0. 2		
		0. 0		
		+0. 7		
		−0. 0		
		+1. 0		
		+0. 5	+0. 3	−1. 2
March 30, 4:22—4:40 p. m	41. 61	+0. 6		
		+0. 8		
		+0. 4		
		0. 0		
		−0. 7		
		+1. 1	+0. 4	−1. 3
March 31, 9:56—10:12 a. m	41. 22	+0. 3		
		−0. 2		
		−0. 7		
		+0. 5		
		+0. 8		
		−0. 8	−0. 0	−0. 9
Mean of the 20 means			−0. 9	
Probable error			±0. 1	

The residuals for the earlier comparisons of the spaces 60^{mm} to 80^{mm} and from 80^{mm} to 100^{mm} being large, additional comparisons of these spaces were made in March, 1881, and are included in the preceding tables. The expansions of the two scales per degree Fahrenheit per metre can scarcely differ by 1^{μ}, hence temperatures of comparisons can be neglected. Probable errors of mean results are derived from their differences from individual mean results.

Collecting the results and rounding the probable errors to the nearest tenth of a micron, we have—

$$\begin{aligned}
0^{mm} \text{ to } 20^{mm} \text{ on metre } R1876 &= 0^{mm} \text{ to } 20^{mm} \text{ on decimeter } D1876 + 1^{\mu}.4 \pm 0^{\mu}.1 \\
20^{mm} \text{ to } 40^{mm} \text{ on metre } R1876 &= 0^{mm} \text{ to } 20^{mm} \text{ on decimeter } D1876 + 0^{\mu}.6 \pm 0^{\mu}.1 \\
40^{mm} \text{ to } 60^{mm} \text{ on metre } R1876 &= 0^{mm} \text{ to } 20^{mm} \text{ on decimeter } D1876 - 0^{\mu}.4 \pm 0^{\mu}.1 \\
60^{mm} \text{ to } 80^{mm} \text{ on metre } R1876 &= 0^{mm} \text{ to } 20^{mm} \text{ on decimeter } D1876 + 0^{\mu}.4 \pm 0^{\mu}.1 \\
80^{mm} \text{ to } 100^{mm} \text{ on metre } R1876 &= 0^{mm} \text{ to } 20^{mm} \text{ on decimeter } D1876 - 0^{\mu}.9 \pm 0^{\mu}.1
\end{aligned}$$

Adding these values, there results—

$$0^{mm} \text{ to } 100^{mm} \text{ on metre } R1876 = 5\,(0^{mm} \text{ to } 20^{mm} \text{ on decimeter } D1876) + 1^{\mu}.1 \pm 0^{\mu}.2$$

Whence

$$0^{mm} \text{ to } 20^{mm} \text{ on decimeter } D1876 = \tfrac{1}{5}\,(0^{mm} \text{ to } 100^{mm} \text{ on metre } R1876) - 0^{\mu}.22 \pm 0^{\mu}.04.$$

But substituting from § 4 the value of (0^{mm} to 100^{mm} on metre $R1876$), namely:

$$\tfrac{1}{10}\,(R1876) - 0^{\mu}.2 \pm 0^{\mu}.06,$$

there follows

$$0^{mm} \text{ to } 20^{mm} \text{ on decimeter } D1876 = \tfrac{1}{50}\,(R1876) - 0^{\mu}.26$$

and hence, from differences of 20^{mm} spaces on metre and decimeter given above—

$$\begin{aligned}
0^{mm} \text{ to } 20^{mm} \text{ on } R1876 &= \tfrac{1}{50}\,(R1876) + 1^{\mu}.1 \\
0^{mm} \text{ to } 40^{mm} \text{ on } R1876 &= \tfrac{2}{50}\,(R1876) + 1^{\mu}.5 \\
0^{mm} \text{ to } 60^{mm} \text{ on } R1876 &= \tfrac{3}{50}\,(R1876) + 0^{\mu}.8 \\
0^{mm} \text{ to } 80^{mm} \text{ on } R1876 &= \tfrac{4}{50}\,(R1876) + 1^{\mu}.0
\end{aligned}$$

From the method of comparison the probable errors in the differences between the spaces on the metre and that on the decimeter are independent of each other. But in the final values of the errors of the 20^{mm}, 40^{mm}, 60^{mm}, 80^{mm}, marks on $R1876$, a part of the error comes from the comparisons with the 20^{mm} on the decimeter, and a part from the adopted value for this 20^{mm} space depending on the same comparisons, so that the errors are entangled, and as they all depend on $0^{m}.0$ to $0^{m}.1$ of $R1876$, a part of the error in this space will enter also.

Denote the 20^{mm} space on the decimeter by A, the successive 20^{mm} spaces on the metre by I, II, III, IV, V, the observed differences of A and I, A and II, &c., by α, β, γ, δ, ε, and the actual errors in these observed quantities, or the corrections needed to make them exact, by a, b, c, d, e. The above equations may be written in the form

$$\begin{aligned}
I &= A + \alpha + a \\
II &= A + \beta + b \\
III &= A + \gamma + c \\
IV &= A + \delta + d \\
V &= A + \varepsilon + e
\end{aligned}$$

Calling the first decimeter of $R1876$, D, by summing and dividing by 5 there results—

$$A = \tfrac{1}{5}\,D - \tfrac{1}{5}\,(\alpha + \beta + \gamma + \delta + \varepsilon) - \tfrac{1}{5}\,(a + b + c + d + e)$$

Substituting in the values of I, II, &c.,

$$(1) \quad \left\{ \begin{aligned}
I &= \tfrac{1}{5}D + \tfrac{1}{5}\,(4\alpha - \beta - \gamma - \delta - \varepsilon) + \tfrac{1}{5}\,(4a - b - c - d - e) \\
II &= \tfrac{1}{5}D + \tfrac{1}{5}\,(4\beta - \alpha - \gamma - \delta - \varepsilon) + \tfrac{1}{5}\,(4b - a - c - d - e) \\
III &= \tfrac{1}{5}D + \tfrac{1}{5}\,(4\gamma - \alpha - \beta - \delta - \varepsilon) + \tfrac{1}{5}\,(4c - a - b - d - e) \\
IV &= \tfrac{1}{5}D + \tfrac{1}{5}\,(4\delta - \alpha - \beta - \gamma - \varepsilon) + \tfrac{1}{5}\,(4d - a - b - c - e) \\
V &= \tfrac{1}{5}D + \tfrac{1}{5}\,(4\varepsilon - \alpha - \beta - \gamma - \delta) + \tfrac{1}{5}\,(4e - a - b - c - d)
\end{aligned} \right.$$

Or, summing

$$
\begin{aligned}
I+II &= \tfrac{2}{5}D+\tfrac{1}{5}(3\alpha+3\beta-2\gamma-2\delta-2\varepsilon)+\tfrac{1}{5}(3a+3b-2c-2d-2e)\\
I+II+III &= \tfrac{3}{5}D+\tfrac{1}{5}(2\alpha+2\beta+2\gamma-3\delta-3\varepsilon)+\tfrac{1}{5}(2a+2b+2c-3d-3e)\\
I+II+III+IV &= \tfrac{4}{5}D+\tfrac{1}{5}(\ \alpha+\ \beta+\ \gamma+\ \delta-4\varepsilon)+\tfrac{1}{5}(\ a+\ b+\ c+\ d-4e)
\end{aligned}
$$

Passing now from the actual errors, a, b, c, d, e, of the comparisons of the space on the decimeter with those on the metre to their probable errors already given and which may be represented by a', b', c', d', e', the probable errors due to these comparisons in the different sums are:

$$
(2)\qquad \begin{cases}
\text{For } I: & \pm\tfrac{1}{5}\sqrt{16a'^2+b'^2+c'^2+d'^2+e'^2}\\
\text{For } I+II: & \pm\tfrac{1}{5}\sqrt{9a'^2+9b'^2+4c'^2+4d'^2+4e'^2}\\
\text{For } I+II+III: & \pm\tfrac{1}{5}\sqrt{4a'^2+4b'^2+4c'^2+9d'^2+9e'^2}\\
\text{For } I+II+III+IV: & \pm\tfrac{1}{5}\sqrt{a'^2+b'^2+c'^2+d'^2+16e'^2}
\end{cases}
$$

But D, whose value is given in § 4, has a probable error of $\pm\,0^{\mu}.06$; hence one-fifth of that error must be combined by summing squares and extracting square root with the first of the above values, two-fifths with the second, three-fifths with the third, and four-fifths with the last. Performing the various substitutions in (1) and (2) and the value of A, there result—

$$
\begin{aligned}
0^{\text{mm}} \text{ to } 20^{\text{mm}} \text{ on } R1876 &= \tfrac{1}{50}\,(R1876)+\overset{\mu}{1.1}\pm\overset{\mu}{0.09}\\
0^{\text{mm}} \text{ to } 40^{\text{mm}} \text{ on } R1876 &= \tfrac{2}{50}\,(R1876)+1.5\pm0.11\\
0^{\text{mm}} \text{ to } 60^{\text{mm}} \text{ on } R1876 &= \tfrac{3}{50}\,(R1876)+0.8\pm0.12\\
0^{\text{mm}} \text{ to } 80^{\text{mm}} \text{ on } R1876 &= \tfrac{4}{50}\,(R1876)+1.0\pm0.10\\
A \text{ on decimeter } D1876 &= \tfrac{1}{50}\,(R1876)-0.3\pm0.05
\end{aligned}
$$

§ **6.** The preceding work having given the error in the position of the 80^{mm} mark on metre R1876, the next step was to find the error of the 81st millimeter mark, this being the one used in comparisons with the yard. The error was obtained in two ways, of which the first was by comparing the space 80^{mm} to 81^{mm} on R1876 with the space $0^{\text{in}}.95$ to $0^{\text{in}}.99$ on the Troughton and Simms standard inch described in Chapter II, § 3. The metre and inch were both mounted on the iron beam or metre-carriage previously described. The microscopes at a distance from each other of $0^{\text{m}}.12$ on the microscope-car remained unmoved. The line of collimation was made vertical within $10''$. The metre and inch were made level within $3'$. Both were so adjusted that when the metre-car was moved under the microscope's longitudinal thread the longitudinal graduation-lines of inch and metre remained under it. The correction for error in focusing was obtained by numerous readings on millimeter spaces on inch and on metre. The order of pointings was:

Microscope 5 at $0^{\text{in}}.99$ on inch.
Microscope 6 at 80^{mm} on metre.
Microscope 5 at $0^{\text{in}}.95$ on inch.
Microscope 6 at 81^{mm} on metre.
Microscope 6 at 81^{mm} on metre.
Microscope 5 at $0^{\text{in}}.95$ on inch.
Microscope 6 at 80^{mm} on metre.
Microscope 5 at $0^{\text{in}}.99$ on inch.

Two pointings of each microscope were thus obtained at each of the division lines. Taking their means, a single value of the difference between the space $0^{\text{in}}.95$ to $0^{\text{in}}99$ on inch and the space 80^{mm} to 81^{mm} on R1876 results. This was one set of observations. Five sets were obtained at each visit to the comparing-room, and two visits per day were made, one at 9 a. m. and one at 4 p. m. In the following table are given the dates of observations, the corrected temperatures of R1876, the results of each set in the form $0^{\text{in}}.95$ to $0^{\text{in}}.99$ minus 80^{mm} to 81^{mm}, and the residuals obtained by subtracting the individual results from the mean of all of them.

Results of comparisons of space $0^{in}.95$ *to* $0^{in}.99$ *on standard inch and space* 80^{mm} *to* 81^{mm} *on metre* R1876.

Date.	Thermom. 21476 on metre, (corrected).	$0^{in}.95$ to $0^{in}.99$ *minus* 80^{mm} to 81^{mm}.	Residuals.
1879.	° F.	μ	μ
October 4, a. m	68.22	+16.8	−0.6
		+15.0	+1.2
		+15.0	+1.2
		+17.4	−1.2
	68.31	+17.2	−1.0
October 4, p. m	68.22	+16.9	−0.7
		+18.5	−2.3
		+16.5	−0.3
		+16.2	0.0
	68.27	+17.0	−0.8
October 7, a. m	68.71	+16.2	0.0
		+14.7	+1.5
		+15.6	+0.6
		+15.5	+0.7
		+15.5	+0.7
	68.81	+17.3	−1.1
October 7, p. m	69.01	+16.7	−0.5
		+16.6	−0.4
		+18.7	−2.5
		+15.7	+0.5
	69.21	+15.2	+1.0
October 9, a. m	70.19	+15.7	+6.5
		+17.5	−1.3
		+15.7	+0.5
		+17.3	−1.1
	70.29	+16.5	−0.3
October 9, p. m	70.39	+16.6	−0.4
		+14.1	+2.1
		+16.6	−0.4
		+13.6	+2.6
	70.48	+15.6	+0.6
October 11, a. m	71.37	+15.5	+0.7
		+17.6	−1.4
		+16.1	+0.1
		+16.0	+0.2
	71.47	+15.0	+1.2
October 11, p. m	71.57	+16.9	−0.7
		+18.5	−2.3
		+16.8	−0.6
		+15.3	+0.9
	71.67	+16.3	−0.1
October 14, a. m	71.57	+16.5	−0.3
		+16.7	−0.5
		+16.5	−0.3
		+18.1	−1.9
	71.67	+15.0	+1.2
October 14, p. m	71.67	+15.2	+1.0
		+12.5	+3.7
		+16.2	0.0
		+16.0	+0.2
	71.77	+16.1	+0.1
Mean of 51 results		+16.2	
Probable error		± 0.11	

In Chapter II, § 3, Colonel Clarke's value of the space $0^{in}.95$ to $0^{in}.99$ is given in terms of the space (9.10), which is again given in terms of the Ordnance standard foot, *F.*, at 62°, which is given in terms of the English yard. Deriving the value of the space $0^{in}.95$ to $0^{in}.99$ in terms of the English yard from these data, there results—

$$\text{Space } 0^{in}.95 \text{ to } 0^{in}.99 \text{ on inch} = 0^{y}.00111300 \pm 0^{y}.00000019$$

under the assumption that the probable errors of the hundredths of an inch given by Colonel Clarke are independent. Converting this into microns, with Clarke's value of the metre ($39^{in}.370432$), there results—

$$\text{Space } 0^{in}.95 \text{ to } 0^{in}.99 = 1017^{\mu}.7 \pm 0^{\mu}.17$$

But the comparisons gave—

$$\text{Space } 80^{mm} \text{ to } 81^{mm} \text{ on } R1876 = \text{space } 0^{in}.95 \text{ to } 0^{in}.99 \text{ on inch} - 16^{\mu}.2 \pm 0^{\mu}.11$$

Hence, finally,

$$\text{Space } 80^{mm} \text{ to } 81^{mm} \text{ on } R1876 = 1001^{\mu}.5 \pm 0^{\mu}.2 \text{ at } 62^{\circ}\text{ F}$$

§ **7.** The second method of determining the value of the space 80^{mm} to 81^{mm} on $R1876$ was as follows: The value of the space 80^{mm} to 100^{mm} on $R1876$ has already been given in terms of $R1876$. Each of the four 5^{mm} spaces forming an aliquot part of this distance was compared with the space 0^{mm} to 5^{mm} on the decimeter. These comparisons gave the value of the space 80^{mm} to 85^{mm} on $R1876$. Then the space 0^{mm} to 1^{mm} on the decimeter, $D1876$, was compared with each of the millimeters between the 80^{mm} and the 85^{mm} on $R1876$. This gave the value of the space 80^{mm} to 81^{mm} on $R1876$. In these comparisons the methods were the same as in comparing the 0^{mm} to 20^{mm} space on the decimeter with the similar spaces between 0^{mm} and 100^{mm} on $R1876$. Each result for difference of length obtained depended on two microscope-pointings at each end of each scale. Six such results were obtained in each visit to the comparing-room, a visit occupying about 20 minutes. Four such visits were made. The mean of the 24 results is taken as the value of the difference between the lengths of the portions of the scales compared. In the following tables of the comparisons of the 5^{mm} spaces, the first column gives the date, the second the temperatures, the third the resulting difference of lengths, the fourth the mean result of visit, and the fifth the residual or mean of results minus mean result of visit.

Results of comparisons of 5^{mm} *spaces on metre* $R1876$ *and decimeter* $D1876$.

Dates.	Temp.	80^{mm} to 85^{mm} on $R1876$ *minus* 0^{mm} to 5^{mm} on $D1876$.	Means.	Residuals.
1881.	° F.	μ	μ	
March 31, 10:19—11:02 a. m	41.42	+2.93		
		+1.68		
		+1.83		
		+2.72		
		+3.35		
	41.71	+3.46	+2.66	−0.02
March 31, 12:15—12:42 p. m	41.71	+1.26		
		+2.36		
		+2.04		
		+2.41		
		+3.09		
	42.01	+1.83	+2.16	+0.48
March 31, 2:18—2:36 p. m	42.06	+2.98		
		+2.88		
		+2.93		
		+2.77		
		+3.35		
	42.11	+1.88	+2.80	−0.16
March 31, 4:17—4:34 p. m	42.16	+3.46		
		+3.61		
		+2.57		
		+2.20		
		+2.88		
	42.21	+2.88	+2.93	−0.29
Mean of 4 means			+2.64	
Probable error			±0.11	

Comparisons of 5^{mm} *spaces on* R1876 *and* D1876—Continued.

Date.	Temp.	85^{mm} to 90^{mm} on R1876 *minus* 0^{mm} to 5^{mm} on D1876.	Means.	Residuals.
1881.	° F.	μ	μ	μ
April 1, 9:24—9:42 a. m	41. 37	+0. 37		
		+0. 21		
		+0. 68		
		0. 00		
		+0. 21		
	41. 42	+0. 37	+0. 31	−0. 32
April 1, 11:29—11:45 a. m	41. 42	+0. 26		
		0. 00		
		−0. 21		
		+0. 16		
		−0. 69		
	41. 51	+0. 47	0. 00	−0. 01
April 1, 2:18—2:33 p. m	41. 51	+0. 05		
		+0. 05		
		+0. 10		
		+0. 05		
		−0. 05		
	41. 61	−0. 21	0. 00	−0. 01
April 1, 4:10—4:27 p. m	41. 61	+0. 52		
		+0. 05		
		−0. 79		
		−0. 10		
		−1. 20		
	41. 66	−0. 58	−0. 35	+0. 34
Mean of 4 means			−0. 01	
Probable error			±0. 09	

Date.	Temp.	90^{mm} to 95^{mm} on R1876 *minus* 0^{mm} to 5^{mm} on D1876.	Means.	Residuals.
1881.	° F.	μ	μ	μ
April 2, 8:48—9:11 a. m	40. 72	−0. 05		
		−0. 58		
		+0. 10		
		−0. 89		
		+0. 84		
	40. 82	+0. 94	+0. 06	−0. 09
April 2, 11:48—12:07 p. m	40. 92	−0. 05		
		+0. 63		
		−0. 42		
		−0. 21		
		+0. 31		
	41. 02	+0. 37	+0. 10	−0. 13
April 2, 2:26—2:45 p. m	41. 17	−0. 79		
		−0. 42		
		0. 00		
		−0. 79		
		−0. 10		
	41. 22	−0. 37	−0. 41	+0. 38
April 2, 4:25—4:39 p. m	41. 22	+0. 47		
		−0. 37		
		+0. 89		
		+0. 47		
		−0. 31		
	41. 32	−0. 37	+0. 13	−0. 10
Mean of 4 means			−0. 03	
Probable error			±0. 09	

Comparisons of 5^{mm} *spaces on* $R1876$ *and* $D1876$—Continued.

Date.	Temp.	95^{mm} to 100^{mm} on $R1876$ *minus* 0^{mm} to 5^{mm} on $D1876$.	Means.	Residuals.
1881.	° F.	μ	μ	μ
April 4, 9:28–9:44 a. m	39.52	−1.41		
		−0.05		
		−1.52		
		−1.78		
		−0.73		
	39.62	−0.63	−1.02	−0.01
April 4, 11:24–11:42 a. m	39.67	−0.73		
		−0.63		
		−1.68		
		−0.10		
		−1.73		
	39.77	−1.20	−1.01	−0.02
April 4, 2:22—2:37 p. m	39.87	−0.68		
		−0.58		
		−1.47		
		−1.15		
		−0.42		
	39.87	−2.25	−1.09	+0.06
April 4, 4:09–4:25 p. m	39.92	−1.41		
		−0.52		
		−1.10		
		−1.05		
		−0.84		
	40.02	−0.99	−0.98	−0.05
Mean of 4 means			−1.03	
Probable error			±0.02	

Collecting the results and giving to the nearest tenth of a micron the probable errors which have been derived from the discrepancies between the mean and individual mean results, we have—

$$80^{mm} \text{ to } 85^{mm} \text{ on } R1876 = 0^{mm} \text{ to } 5^{mm} \text{ on } D1876 + 2^{\mu}.6 \pm 0^{\mu}.1$$
$$85^{mm} \text{ to } 90^{mm} \text{ on } R1876 = 0^{mm} \text{ to } 5^{mm} \text{ on } D1876 - 0^{\mu}.0 \pm 0^{\mu}.1$$
$$90^{mm} \text{ to } 95^{mm} \text{ on } R1876 = 0^{mm} \text{ to } 5^{mm} \text{ on } D1876 - 0^{\mu}.0 \pm 0^{\mu}.1$$
$$95^{mm} \text{ to } 100^{mm} \text{ on } R1876 = 0^{mm} \text{ to } 5^{mm} \text{ on } D1876 - 1^{\mu}.0 \pm 0^{\mu}.0$$

Adding these values, there results after division by 4,

$$0^{mm} \text{ to } 5^{mm} \text{ on } D1876 = \tfrac{1}{4}\,(80^{mm} \text{ to } 100^{mm} \text{ on } R1876) - 0^{\mu}.4 \pm 0^{\mu}.05$$

Substituting the value of 80^{mm} to 100^{mm} on $R1876$, derived from the correction to the 80^{mm} mark and the 100^{mm} mark previously given, this space being $\frac{1}{50}$ $(R1876)-1^{\mu}.2$, there follows:

$$0^{mm} \text{ to } 5^{mm} \text{ on } D1876 = \tfrac{1}{200}\,(R1876) - 0^{\mu}.7 \pm 0^{\mu}.05$$

and hence from differences of 5^{mm} spaces on $R1876$ and $D1876$, given above,

$$80^{mm} \text{ to } 85^{mm} \text{ on } R1876 = \tfrac{1}{200}\,(R1876) + 1^{\mu}.9 \pm 0^{\mu}.1$$
$$85^{mm} \text{ to } 90^{mm} \text{ on } R1876 = \tfrac{1}{200}\,(R1876) - 0^{\mu}.7 \pm 0^{\mu}.1$$
$$90^{mm} \text{ to } 95^{mm} \text{ on } R1876 = \tfrac{1}{200}\,(R1876) - 0^{\mu}.7 \pm 0^{\mu}.1$$
$$95^{mm} \text{ to } 100^{mm} \text{ on } R1876 = \tfrac{1}{200}\,(R1876) - 1^{\mu}.7 \pm 0^{\mu}.1$$

The probable errors have been derived in the same way as those of the successive double decimeter marks, save that the space 80^{mm} to 100^{mm} on $R1876$ has itself a probable error. From (1) of § 5 the actual error in the space 80^{mm} to 100^{mm} on $R1876$ is $\frac{1}{5}\,(4e-a-b-c-d)$, provided a, b, c, d, e are now the actual errors of the comparisons of 0^{mm} to 20^{mm} on $D1876$ with the successive 20^{mm} spaces between 0^{mm} and 100^{mm} on $R1876$, or if the same letters primed denote the probable errors, the probable error in the value of the space 80^{mm} to 100^{mm} on $R1876$ will be

$$\tfrac{1}{5}\sqrt{a'^2+b'^2+c'^2+d'^2+16e'^2}$$

in which a', b', &c., have already been given. This result is combined with the probable errors in the values of the 5^{mm} spaces expressed in terms of the space 80^{mm} to 100^{mm} on R1876.

The next step in finding the value of the space 80^{mm} to 81^{mm} on R1876 was to compare each of the millimeters in this space with the space 0^{mm} to 1^{mm} on D1876. The methods and number of comparisons were the same as in the determination of the space 80^{mm} to 85^{mm} on R1876.

The tables have the same arrangement, and the probable errors have been derived in a similar way.

Results of comparisons of millimeter spaces on metre R1876 and decimeter D1876.

Date.	Temp.	80^{mm} to 81^{mm} on R 1876 *minus* 0^{mm} to 1^{mm} on D 1876.	Means.	Residuals.
1881.	° F.	μ	μ	μ
April 5, 10:26—10:40 a. m	39. 32	+1. 47		
		+1. 10		
		+0. 89		
		+0. 90		
		+0. 68		
	39. 42	+1. 47	+1. 10	−0. 14
April 5, 12:25—12:40 p. m	39. 42	+0. 94		
		+0. 10		
		+1. 57		
		+0. 89		
		+1. 47		
	39. 42	+0. 84	+0. 97	−0. 01
April 5, 2:27—2:39 p. m	39. 47	+1. 41		
		+0. 10		
		+0. 26		
		+1. 78		
		+0. 84		
	39. 52	+1. 26	+0. 94	+0. 02
April 5, 4:13—4:26 p. m	39. 62	+1. 05		
		+1. 05		
		+1. 31		
		+0. 58		
		+0. 73		
	39. 62	+0. 26	+0. 83	+0. 13
Mean of 4 means			+0. 96	
Probable error			±0. 04	

Date.	Temp.	81^{mm} to 82^{mm} on R 1876 *minus* 0^{mm} to 1^{mm} on D 1876.	Means.	Residuals.
1881.	° F.	μ	μ	μ
April 6, 9:37—9:53 a. m	38. 92	−0. 47		
		+0. 05		
		−0. 79		
		−0. 21		
		−0. 05		
	38. 07	+0. 31	−0. 19	−0. 26
April 6, 12:20—12:33 p. m	39. 02	−0. 21		
		−0. 52		
		−0. 42		
		−0. 94		
		+0. 10		
	39. 12	−0. 63	−0. 44	−0. 01

Results of comparisons of millimeter spaces on metre R1876 and decimeter D1876—Continued.

Date.	Temp.	81mm to 82mm on *R*1876 *minus* 0mm to 1mm on *D*1876.	Means.	Residuals.
1881.	° F.	μ	μ	μ
April 6, 2:10—2:21 p. m	39.17	−0.84		
		−0.79		
		−0.63		
		−0.42		
		−1.20		
	39.17	−0.31	−0.70	+0.25
April 6, 4:20—4:30 p. m	39.22	+0.05		
		−0.26		
		−1.52		
		−0.05		
		−1.47		
	39.27	+0.37	−0.48	+0.03
Mean of 4 means			−0.45	
Probable error			±0.07	

Date.	Temp.	82mm to 83mm on *R*1876 *minus* 0mm to 1mm on *D*1876.	Means.	Residuals.
1881.	° F.	μ	μ	μ
April 7, 9:19—9:31 a. m	38.92	+1.41		
		+0.99		
		+0.99		
		+1.62		
		+1.15		
	39.02	+0.58	+1.12	+0.19
April 7, 12:42—12:53 p. m	39.12	+2.25		
		+0.99		
		+2.62		
		+1.88		
		+1.57		
	39.17	+1.20	+1.75	−0.44
April 7, 2:15—2:26 p. m	39.22	+1.15		
		+0.52		
		+1.10		
		+1.20		
		+1.36		
	39.27	+1.15	+1.08	+0.23
April 7, 4:19—4:30 p. m	39.42	+1.56		
		+0.63		
		+1.78		
		+0.52		
		+1.99		
	39.42	+1.15	+1.27	+0.04
Mean of 4 means			+1.31	
Probable error			±0.10	

Date.	Temp.	83mm to 84mm on *R*1876 *minus* 0mm to 1mm on *D*1876.	Means.	Residuals.
1881.	° F.	μ	μ	μ
April 8, 9:09—9:22 a. m	39.42	+0.10		
		−0.26		
		−0.10		
		−0.31		
		−0.68		
	39.42	−0.84	−0.35	+0.09

Results of comparisons of millimeter spaces on metre R 1876 and decimeter D 1876—Continued.

Date.	Temp.	83^{mm} to 84^{mm} on *R* 1870 *minus* 0^{mm} to 1^{mm} on *D* 1876.	Means.	Residuals.
1881.	° F.	μ	μ	μ
April 8, 12:22—12:33 p. m.	39.62	+0.10		
		−0.63		
		−0.79		
		−0.89		
		−1.05		
	39.67	−0.05	−0.55	+0.29
April 8, 2:21—2:33 p. m.	39.72	−0.63		
		+0.10		
		−0.31		
		+0.21		
		+0.37		
	39.82	−0.31	−0.09	−0.17
April 8, 4:14—4:27 p. m.	39.92	−0.58		
		−0.16		
		−0.26		
		+0.05		
		0.00		
	39.97	+0.79	−0.03	−0.23
Mean of 4 means			−0.26	
Probable error			±0.08	

Date.	Temp.	84^{mm} to 85^{mm} on *R* 1876 *minus* 0^{mm} to 1^{mm} on *D* 1876.	Means.	Residuals.
1881.	° F.	μ	μ	μ
April 9, 9:02—9:10 a. m.	40.17	+1.26		
		+0.89		
		+0.73		
		+0.47		
		+0.84		
	40.22	+1.05	+0.87	+0.05
April 9, 12:26—12:38 p. m.	40.42	+0.47		
		+0.63		
		+0.94		
		+0.79		
		+0.89		
	40.47	+0.10	+0.64	+0.28
April 9, 1:24—1:37 p. m.	40.62	+1.31		
		+0.94		
		+0.94		
		+0.94		
		+1.36		
	40.72	−0.52	+0.83	+0.09
April 9, 4:15—4:26 p. m.	40.87	+1.41		
		+1.62		
		+1.78		
		+1.05		
		+0.73		
	40.92	+1.41	+1.33	−0.41
Mean of 4 means			+0.92	
Probable error			±0.10	

Collecting the results we have—

$$80^{mm} \text{ to } 81^{mm} \text{ on } R1876 = 0^{mm} \text{ to } 1^{mm} \text{ on } D1876 + 0^{\mu}.96 \pm 0^{\mu}.04$$
$$81^{mm} \text{ to } 82^{mm} \text{ on } R1876 = 0^{mm} \text{ to } 1^{mm} \text{ on } D1876 - 0^{\mu}.45 \pm 0^{\mu}.07$$
$$82^{mm} \text{ to } 83^{mm} \text{ on } R1876 = 0^{mm} \text{ to } 1^{mm} \text{ on } D1876 + 1^{\mu}.31 \pm 0^{\mu}.10$$
$$83^{mm} \text{ to } 84^{mm} \text{ on } R1876 = 0^{mm} \text{ to } 1^{mm} \text{ on } D1876 - 0^{\mu}.26 \pm 0^{\mu}.08$$
$$84^{mm} \text{ to } 85^{mm} \text{ on } R1876 = 0^{mm} \text{ to } 1^{mm} \text{ on } D1876 + 0^{\mu}.92 \pm 0^{\mu}.10$$

Adding these values, and dividing by 5, we have—

$$0^{mm} \text{ to } 1^{mm} \text{ on } D1876 = \tfrac{1}{5}\,(80^{mm} \text{ to } 85^{mm} \text{ on } R1876) - 0^{\mu}.5 \pm 0^{\mu}.04$$

Substituting the value of the space 80^{mm} to 85^{mm}, derived in this section, there results—

$$0^{mm} \text{ to } 1^{mm} \text{ on } D1876 = \tfrac{1}{1000}\,(R1876) - 0^{\mu}.1 \pm 0^{\mu}.04$$

and hence, from the differences between 0^{mm} to 1^{mm} on $D1876$ and the millimeter spaces on $R1876$, there follows:

$$80^{mm} \text{ to } 81^{mm} \text{ on } R1876 = \tfrac{1}{1000}\,(R1876) + 0^{\mu}.9 \pm 0^{\mu}.05$$
$$81^{mm} \text{ to } 82^{mm} \text{ on } R1876 = \tfrac{1}{1000}\,(R1876) - 0^{\mu}.6 \pm 0^{\mu}.07$$
$$82^{mm} \text{ to } 83^{mm} \text{ on } R1876 = \tfrac{1}{1000}\,(R1876) + 1^{\mu}.2 \pm 0^{\mu}.09$$
$$83^{mm} \text{ to } 84^{mm} \text{ on } R1876 = \tfrac{1}{1000}\,(R1876) - 0^{\mu}.4 \pm 0^{\mu}.07$$
$$84^{mm} \text{ to } 85^{mm} \text{ on } R1876 = \tfrac{1}{1000}\,(R1876) + 0^{\mu}.8 \pm 0^{\mu}.09$$

The probable errors are obtained in a manner similar to that used in obtaining those of the 5^{mm} spaces just given.

§ **8.** Professor Foerster (letter of June 20, 1879) § 67, gives as a closely approximate value—

$$R1876 = 1^{m} + 248^{\mu}.89 + 10^{\mu}.31\,(t-15)$$

in which t is temperature in Centigrade degrees. This gives—

$$80^{mm} \text{ to } 81^{mm} \text{ on } R1876 = 1{,}001^{\mu}.1 \pm 0^{\mu}.05$$

The value found from the inch, § 6, is—

$$80^{mm} \text{ to } 81^{mm} \text{ on } R1876 = 1001^{\mu}.5 \pm 0^{\mu}.2$$

If the two values were combined with weights depending on the probable errors, the first value would still result. It may therefore be adopted.

We now have the data for determining the corrections to certain graduation-marks needed to change their nominal values into exact values in terms of the interval 0^{mm} to $1{,}000^{mm}$ on $R1876$. Collecting them from §§ 3, 4, 5, 7, we have—

Graduation ...	0	20	40	60	80	81	82	83	84	85	90	95	100	200	400	600	800	1,000
		μ	μ	μ	μ	μ	μ		μ	μ	μ	μ	μ	μ	μ	μ	μ	
Correction	0	+1.1	+1.5	+0.8	+1.0	+1.9	+1.3	+2.5	+2.1	+2.9	+2.2	+1.5	−0.2	−1.5	−1.0	+0.1	−0.7	0

The probable errors of these corrections resulting from the comparisons have already been given for the most of them. They are usually about $\pm 0^{\mu}.1$, and for none do they exceed $\pm 0^{\mu}.2$. For the 81^{mm} mark, which is a very important one, it has been specially computed and found to be $\pm 0^{\mu}.11$.

The determinations made in the Lake-Survey office of the errors of certain graduation-marks on $R1876$ in terms of the interval 0^{mm} to $1{,}000^{mm}$, considered as exact, have now been given. This metre was for some time in the hands of the Kaiserliche Normal-Eichungs-Kommission at Berlin, by whom its graduation-errors were also determined. They are given in § 67.

These results were not received from Berlin until after the determinations at the Lake-Survey office were far advanced, and the work at Detroit was completed prior to the reception of the details of the work at Berlin, which indeed, August 1, 1881, have not yet arrived. Accordingly, there are two entirely independent determinations of the errors of some of the graduation-lines on $R1876$, made at Detroit and Berlin respectively. Their accordance indicates the degree of accuracy of the work. In the following table the first line gives the name of the graduation-line on $R1876$; the second gives its correction in terms of the length from 0^{mm} to $1{,}000^{mm}$, as determined by the Kaiserliche Normal-Eichungs-Kommission, the correction being positive when the graduation-line is too far from 0^{mm}; and the third gives the same corrections as determined in the Lake-Survey office.

Corrections to graduations of R1876, the interval between 0^{mm} *and* $1{,}000^{mm}$ *being taken as one metre.*

Graduation ...	0	20	40	60	80	81	82	83	84	85	90	95	100	200	400	600	800	1,000
Eichungs-Amt correction ..	0	μ +2.4	μ +2.4	μ +0.6	μ −0.5	μ 0	μ 0	μ +1	μ +1	μ +1	μ +1.0	μ +1.0	μ −0.5	μ −2.5	μ −1.1	μ +0.5	μ −0.1	0
Lake-Survey correction ..	0	+1.1	+1.5	+0.8	+1.0	+1.9	+1.3	+2.5	+2.1	+2.9	+2.2	+1.5	−0.2	−1.5	−1.0	+0.1	−0.7	0
Differences..	..	+1.3	+0.9	−0.2	−1.5	......	......	......	......	......	......	......	−0.3	−1.0	−0.1	+0.4	+0.6	......

To the graduation-lines in the table which are multiples of 10^{mm} the Normal-Eichungs-Kommission assigns an accuracy of $0^{\mu}.2$ to $0^{\mu}.3$ (*verbürgbar*), and to the others an accuracy of $1^{\mu}.0$ to $1^{\mu}.5$. For the Lake-Survey corrections the probable errors do not exceed $0^{\mu}.2$. If the graduation-lines whose values are multiples of 20^{mm} be examined, it will be seen that the greatest discrepancy between the independent values is at 80^{mm}, where it is $1^{\mu}.5$. If the means of these values for each of such graduation-lines be taken, and the probable error of one mean be derived from the discrepancies between the means and the individual results, it is found to be $\pm 0^{\mu}.27$. This process of combination attributes equal weights to the results obtained for these special graduations by the Normal-Eichungs-Kommission and the Lake Survey, although the probable errors of the latter seem to be the less.

Since the comparisons of yard *A* depend on the 81^{mm} mark of *R*1876, its error is of great importance. It must be obtained by taking the mean of the corrections found at Berlin and in the Lake-Survey office for the 80^{mm} mark, attributing to it the probable error of one such mean, already given, and then adding to it the value of the interval 80^{mm} to 81^{mm}, which has been doubly determined in the Lake-Survey office. We thus have—

$$0^{mm} \text{ to } 80^{mm} \text{ on } R1876 = \tfrac{8}{100}\,(R1876) + 0^{\mu}.2 \pm 0^{\mu}.27$$

$$80^{mm} \text{ to } 81^{mm} \text{ on } R1876 = \tfrac{1}{1000}\,(R1876) + 0^{\mu}.9 \pm 0^{\mu}.05 \qquad (\S\ 7)$$

or, adding—

$$0^{mm} \text{ to } 81^{mm} \text{ on } R1876 = \tfrac{81}{1000}\,(R1876) + 1^{\mu}.1 \pm 0^{\mu}.3$$

The probable error here is considerably larger than that which results from the Lake-Survey work alone, namely, $\pm 0^{\mu}.11$, but depending on entirely independent determinations, the larger value is probably the more accurate. Hence the interval—

$$81^{mm} \text{ to } 1{,}000^{mm} \text{ on } R1876 = \tfrac{919}{1000}\,(R1876) - 1^{\mu}.1 \pm 0^{\mu}.3$$

COMPARISONS OF *R*1876 WITH CLARKE YARD *A*.

§ **9.** Having thus obtained the value of the space between the 81st and the 100th millimeter-mark on *R*1876, which space will be designated by "*R'* 1876," the details of the comparison of this space with that between the $4^{mm}.6$ marks on the auxiliary cylinders (or quills as they have been called) when their axes are in the axis of yard *A* and their convex ends abut against its ends, may be given.

The Clarke yard *A* was mounted on the iron beam of the comparing-apparatus described in Chapter VIII, § 22. The metre *R*1876 was mounted parallel to yard *A* at a distance of 30^{mm} or 40^{mm} on the two supports carried by the iron beam, which were adjustable vertically. The yard *A* having been adjusted, *R*1876 could thus be made parallel to it and its graduations brought to the same height as those on the auxiliary cylinders of the yard, nearly enough for microscope-readings. Yard *A* lay on the flat surface of the iron bar. Two dove-tail slides with spring contacts could be moved along the iron beam so as to admit the yard between them and be clamped to the beam at any place. Each carried wyes 70^{mm} apart, in which accurately turned equal rings on the auxiliary cylinders rested. These wyes were separately adjustable vertically and laterally, so that any elevation, inclination, or azimuth, within certain limits, could be given to the auxiliary cylinders with reference to the axis of the yard, and the wyes were so situated with reference to the slides that when the auxiliary cylinders were laid in them they were nearly in the prolongation of the axis of the end-cylinders of yard *A* when lying on the flat surface of the iron beam.

The conditions for the yard A and its auxiliary cylinders to fulfill for precise comparisons were, first, that yard A should be horizontal; second, that the axes of its auxiliary cylinders should lie in the prolongation of the axis of the end-cylinders of the yard.

Yard A was easily made horizontal within 1′ by the adjustments of the iron bar on which it rested.

An examination of the errors arising from not fulfilling the second condition led to the following results calculated from the most unfavorable case in which the auxiliary cylinders are used, namely, when their convex ends are in contact as in measuring the interval of $4^{mm}.93$ between their 46th graduations.

Since the ends of yard A have less curvature, the errors in using the auxiliary cylinders with it will be less.

a. To avoid errors greater than $0^{\mu}.1$ in measuring the distance between the 46th graduations on the two auxiliary cylinders or quills, the axes of the quills must have the same azimuth within 15′. This is effected as follows: A piece of brass plate 112^{mm} by 30^{mm} has a central hole by which it can be slipped snugly upon the end of the auxiliary cylinders, its plane then being normal to the cylinder-axis. There are two small holes in the plate 44^{mm} distant from the center and in line with it. The two slides with their auxiliary cylinders having been placed so that the cylinders are nearly in the axis of the yard and abut against its ends, the yard is then removed, the brass plate is put on one of the cylinders, its two holes being at the same height, a light is held behind one of its small holes while the eye is placed at the other looking at the plane end of the distant auxiliary cylinder. The two cylinders are then adjusted till the eye sees the reflection of the light in the plane surface of either when viewed from the other. The auxiliary cylinders when a metre apart can thus be brought into the same azimuth within about 1′. When yard A was removed from between the quills, and the latter were brought in contact so that the interval $4^{mm}.9$ between their $4^{mm}.6$ graduations might be measured, this method could not so well be used, and the following was adopted: A silk thread was fastened to the bar carrying the quills, and adjusted so that it could be run under the longitudinal microscope-wires for a distance of over 100^{mm} without deviating from them by $0^{mm}.05$. The microscope was then pointed at the side of the quill at the places where the rings gave it accurately equal diameters. This could be done with an accuracy of $0^{mm}.01$. These edges of the quills were then made to run along the longitudinal thread of the microscope. In this way the azimuth of the quills could be made the same within about 3′. Examination showed that the plane ends of the cylinders were normal to its axis within about 40″.

b. To avoid errors greater than $0^{\mu}.1$, when the cylinders have the same azimuth but differing inclinations the difference in their vertical inclinations must not exceed 6″. The adjustment is effected with a short, delicate striding level (one division = 7″), which can be set on the rings of the cylinders. Examination showed that these rings had so nearly the same diameter that the difference can be neglected.

c. The auxiliary cylinders having closely the same azimuths and inclinations to the yard, to avoid errors greater than $0^{\mu}.1$ the contacts of cylinders and yard must be central on their end surfaces within $0^{mm}.07$. This can be effected within about $0^{mm}.12$ by looking at the point of contact with a lens.

d. The graduated surfaces on the cylinders should be nearly horizontal. This is effected by rolling the auxiliary cylinders till a light and its reflection on the graduated surface are both covered by a plumb-line.

In comparisons the adjustment-errors were brought within these limits.

The programme for comparisons of R1876 and Clarke yard A was as follows: Two visits, twelve hours apart, to comparing-room on each of three days. Then let metre and yard change sides and readjust everything. At a visit the following readings were made:

1. Thermometer-reading on R1876.
2. Thermometers on yard A.
3. Microscope-pointings at metre.
4. Double microscope-pointing at yard.
5. Microscope-pointing at metre.
6. Thermometer-reading on metre.

The means of all thermometer-readings on each bar were taken and the proper corrections applied for thermometer-errors. The time occupied by a visit was usually from ten to fifteen minutes. There were four A-thermometers inside the iron case of the Clarke yard A, and a Casella thermometer lay on R1876. All have been described in Chapter II, §§ 2 and 7. Both bars were inside the comparing-box.

In reduction ninety comparisons on sixty-one days were used. No comparisons were used when indicated temperatures of R1876 and Clarke yard A differed by more than $0^\circ.15$ F. This rejected eight comparisons. It was assumed that yard and metre had the same temperature, namely, the mean of the observed temperatures of yard and metre. As both rising and falling temperatures entered the work, any slight error in the assumption of equal temperatures tends to eliminate itself. The mean temperature, t_0, of the comparisons used was $57^\circ.918$ F., and the corresponding mean value of yard $A+$quill interval $- R'1876 = 61^{\mu}.36$. This will be designated as $(A'-R'1876)_0$. The observation-equations may be written in the form—

$$(A'-R'1876)_0+(t_0-t)\ (E_{R'1876}-E_{Y'})-(A'-R'1876)=v$$

where t is the mean of the temperatures of yard and metre, $E_{R'1876}$ and $E_{Y'}$, the expansions for 1° F. of $R'1876$ and A', respectively; and $(A'-R'1876)$ is the difference of lengths resulting from a single visit to the comparing-room.

The following table gives the results of the comparisons. The first column gives the date of comparison; the second, the position of R1876 as north (next observer) or south; the third, the corrected mean thermometer-reading for yard A; the fourth, the corrected thermometer-reading for the metre; the fifth gives t_0-t; the sixth gives the value of $(A'-R'\,1876)$ resulting from a visit to the comparing-room; and the seventh gives the residuals in the sense computed minus observed.

Results of comparisons of R'1876 and Clarke yard A plus quill-interval.

Date.	Position of R 1876 as north (next observer) or south.	Corrected mean thermometer-reading for yard A.	Corrected thermometer-reading for R 1876.	t_0-t	Value of $(A'-R'1876)$ resulting from a visit to the comparing-room.	Residuals.
1879.		° F.	° F.	° F.	μ	μ
Mar. 2, 9:28 a. m	North	34. 91	34. 91	+22. 998	+62. 3	−0. 81
8:45 p. m	do	35. 06	34. 91	+22. 928	+63. 3	−1. 81
3, 9:20 a. m	do	35. 11	35. 11	+22. 798	+62. 8	−1. 31
9:12 p. m	do	35. 58	35. 61	+22. 308	+62. 6	−1. 11
4, 9:37 a. m	do	35. 84	35. 81	+22. 088	+61. 0	+0. 49
9:28 p. m	do	36. 21	36. 21	+21. 698	+61. 6	−0. 11
6, 9:22 a. m	South	39. 15	39. 10	+18. 788	+62. 6	−1. 13
10:46 p. m	do	39. 25	39. 15	+18. 708	+61. 8	−0. 33
7, 9:24 a. m	do	39. 37	39. 25	+18. 598	+60. 9	+0. 57
8:58 p. m	do	39. 50	39. 50	+18. 408	+60. 7	+0. 67
8, 9:14 a. m	do	39. 73	39. 70	+18. 188	+61. 9	−0. 43
10:02 p. m	do	40. 18	40. 10	+17. 768	+61. 6	−0. 14
10, 9:00 a. m	North	42. 77	42. 79	+15. 128	+61. 4	+0. 05
9:02 p. m	do	43. 41	43. 39	+14. 508	+61. 7	−0. 22
11, 9:08 a. m	do	44. 29	44. 14	+13. 688	+60. 6	+0. 84
8:50 p. m	do	44. 68	44. 59	+13. 268	+59. 6	+1. 84
12, 9:45 a. m	do	44. 78	44. 69	+13. 168	+59. 6	+1. 84
9:07 p. m	do	45. 10	45. 09	+12. 808	+61. 8	−0. 37
14, 9:18 p. m	South	45. 30	45. 19	+12. 668	+61. 1	+0. 33
16, 10:29 a. m	do	41. 99	42. 10	+15. 868	+62. 8	−1. 35
9:21 p. m	do	41. 67	41. 60	+16. 268	+62. 8	−1. 35
17, 9:06 a. m	do	40. 88	40. 85	+17. 048	+60. 5	+0. 86
9:06 p. m	do	39. 98	40. 09	+17. 918	+63. 4	−1. 94
18, 9:17 a. m	do	30. 37	39. 50	+18. 468	+62. 2	−0. 73
9:16 p. m	do	38. 79	38. 80	+19. 108	+60. 0	+1. 47

Results of comparisons of R′ 1876 and Clarke yard A plus quill-interval—Continued.

Date.	Position of R1876 as north (next observer) or south.	Corrected mean thermometer-reading for yard A.	Corrected thermometer-reading for R 1876.	t_0-t	Value of $(A'-R'1876)$ resulting from a visit to the comparing-room.	Residuals.
1879.		° F.	° F.	° F.	μ	μ
Aug. 22, 9:08 a. m.	South	70.71	70.72	−12.812	+61.9	−0.61
23, 9:20 a. m.	do	71.28	71.17	−13.312	+61.9	−0.62
9:03 p. m.	do	71.28	71.42	−13.442	+61.8	−0.52
26, 9:04 p. m.	North	69.95	69.83	−11.982	+62.1	−0.81
27, 9:20 a. m.	do	69.44	69.43	−11.532	+63.0	−1.71
9:08 p. m.	do	69.24	69.23	−11.332	+61.2	+0.09
28, 9:21 a. m.	do	68.96	69.03	−11.092	+64.4	−3.10
9:03 p. m.	do	68.84	68.83	−10.932	+63.4	−2.10
Sept. 3, 9:28 a. m.	South	73.08	73.02	−15.142	+61.8	−0.53
9:08 p. m.	do	73.01	73.02	−15.112	+61.3	−0.03
4, 9:31 a. m.	do	72.75	72.72	−14.832	+60.8	+0.47
9:10 p. m.	do	72.33	72.32	−14.412	+59.8	+1.48
5, 9:15 a. m.	do	71.65	71.72	−13.772	+63.2	−1.92
8:51 p. m.	do	71.18	71.12	−13.242	+60.5	+0.78
7, 9:36 a. m.	North	68.74	68.68	−10.802	+61.8	−0.50
9:11 p. m.	do	68.44	68.38	−10.502	+61.9	−0.60
8, 9:10 a. m.	do	67.92	67.83	−9.972	+62.5	−1.20
9:00 p. m.	do	67.34	67.23	−9.372	+61.6	−0.29
9, 9:15 a. m.	do	66.62	66.58	−8.692	+63.0	−1.69
9:11 p. m.	do	65.86	65.94	−7.992	+65.0	−3.69
10, 9:28 a. m.	do	65.24	65.24	−7.332	+62.3	−0.98
11, 9:20 a. m.	South	63.98	63.94	− 6.052	+61.4	−0.08
12, 9:18 a. m.	do	63.45	63.49	− 5.562	+60.4	+0.93
9:12 p. m.	do	63.51	63.44	− 5.572	+61.4	−0.07
13, 9:18 a. m.	do	63.55	63.44	− 5.592	+62.7	−1.37
14, 9:20 a. m.	do	63.55	63.59	− 5.662	+60.3	+1.03
Sept. 16, 9:26 a. m.	North	62.63	62.65	− 4.732	+60.1	+1.23
9:10 p. m.	do	62.63	62.65	− 4.732	+58.9	+2.43
17, 9:32 a. m.	do	62.49	62.55	− 4.612	+60.1	+1.23
18, 9:10 p. m.	do	62.31	62.25	− 4.372	+60.7	+0.63
19, 12:27 p. m.	do	61.79	61.70	− 3.832	+61.7	−0.36
20, 9:25 a. m.	do	61.11	61.15	− 3.222	+62.0	−0.66
21, 9:45 a. m.	South	60.51	60.55	− 2.622	+61.9	−0.56
9:10 p. m.	do	60.31	60.30	− 2.392	+61.4	−0.05
22, 0:24 a. m.	do	60.19	60.15	− 2.262	+62.4	−1.05
9:11 p. m.	do	60.23	60.25	− 2.332	+62.5	−1.15
23, 9:23 a. m.	do	60.39	60.45	− 2.512	+63.8	−2.45
9:06 p. m.	do	60.69	60.65	− 2.762	+62.2	−0.86
25, 9:26 a. m.	North	59.32	59.34	− 1.422	+59.7	+1.65
9:05 p. m.	do	59.30	59.24	− 1.362	+60.8	+0.55
26, 9:20 a. m.	do	58.90	58.84	− 0.962	+59.7	+1.65
9:07 p. m.	do	58.72	58.69	− 0.792	+61.6	−0.24
27, 9:13 a. m.	do	58.58	58.59	− 0.672	+61.2	+0.16
9:05 p. m.	do	58.68	58.64	− 0.752	+60.4	+0.96
1880.						
May 6, 11:17 a. m.	South	59.89	59.98	− 2.032	+57.9	+3.45
7, 9:48 a. m.	do	60.14	60.11	− 2.212	+59.3	+2.05
8, 10:00 a. m.	do	61.09	61.14	− 3.212	+57.7	+3.64
10, 10:35 a. m.	do	63.98	63.91	− 6.032	+60.2	+1.12
11, 10:41 a. m.	North	64.93	65.07	− 7.092	+61.7	−0.38
12, 9:33 a. m.	do	64.60	64.69	− 6.732	+63.4	−2.08
13, 10:16 a. m.	do	63.47	63.58	− 5.612	+62.2	−0.87
14, 10:21 a. m.	do	61.57	61.51	− 3.632	+61.6	−0.26

Results of comparisons of R′ 1876 and Clarke yard A plus quill-interval—Continued.

Date.	Position of *R* 1876 as north (next observer) or south.	Corrected mean thermometer-reading for yard *A*.	Corrected thermometer-reading for *R* 1876.	t_0-t	Value of $(A'-R'\,1876)$ resulting from a visit to the comparing-room.	Residuals.
1880.		° F.	° F.	° F.	μ	μ
May 15, 12:20 p. m	South	59.54	59.66	− 1.692	+59.0	+2.35
17, 9:40 a. m	...do	58.89	58.82	− 0.952	+58.9	+2.45
18, 9:56 a. m	...do	59.62	59.71	− 1.752	+59.7	+1.65
19, 11:50 a. m	...do	61.49	61.51	− 3.592	+59.4	+1.94
22, 9:24 a. m	...do	64.70	64.82	− 6.852	+61.0	+0.32
24, 9:48 a. m	...do	64.41	64.36	− 6.472	+60.5	+0.82
25, 9:58 a. m	...do	65.18	65.12	− 7.242	+62.2	−0.88
26, 9:25 a. m	...do	67.50	67.47	− 9.572	+61.7	−0.40
28, 9:17 a. m	...do	69.15	69.13	−11.232	+60.3	+0.99
29, 9:17 a. m	...do	68.36	68.45	−10.492	+59.9	+1.40
30, 9:16 a. m	...do	67.63	67.52	− 9.672	+61.0	+0.30
31, 8:36 a. m	...do	66.90	66.97	− 9.032	+61.9	−0.59
June 1, 9:18 a. m	...do	66.60	66.57	− 8.672	+60.1	+1.21

The results obtained from the foregoing comparisons are as follows:

$$t_0 = 57^\circ.918$$

$$(A'-R'\,1876)_0 = +61^\mu.36 \pm 0^\mu.097$$

$$E_{R'\,1876} - E_{A'} = +0^\mu.0058 \pm 0^\mu.0082$$

$$(A'-R'\,1876) \text{ at } 59^\circ \text{ F.} = +61^\mu.354 \pm 0^\mu.097$$

§ **10.** In fifty-one visits the thermometers with yard *A* had the higher temperature, and in thirty-four visits that on metre had the higher. The mean residual for the first class is $+0^\mu.017$, and for the second $+0^\mu.035$. There is, then, no sufficient evidence that the slight thermometer-differences indicated a corresponding difference in the temperature of *R* 1876 and Clarke yard *A*.

In eighteen visits the temperature of yard had risen more than $0^\circ.5$ F. in the preceding twenty-four hours. The mean residual for these visits was $+0^\mu.42$. In twenty-nine visits the temperature of yard had fallen more than $0^\circ.5$ F. in the preceding 24 hours. The mean residual for these visits was $-0^\mu.34$. Since the residual is computed $(A'-R'\,1876)$ minus observed $(A'-R'\,1876)$, a positive residual indicates that A' was relatively too short for rising temperatures and relatively too long for falling temperatures. Since *R* 1876 was freely exposed in the air of the comparing-box, while Clarke yard *A* was inclosed in its iron case, having walls and cover about 3^{mm} thick, it was to be expected that *A* would change temperature more slowly.

Further examination will show that in thirty-one visits the temperature had risen in the preceding twenty-four hours, there being 14 plus residuals and 17 minus residuals, the mean of all being $+0^\mu.23$. In forty-five visits the temperature had fallen in the preceding twenty-four hours, and there were 17 positive to 28 negative residuals, the mean residual being $-0^\mu.26$. Hence this examination also indicates that the temperature of Clarke yard *A* lagged behind that of *R* 1876 Had the number of visits with rising and falling temperatures been equal, an elimination of any resulting error would have probably occurred. But even if the whole of the mean residuals, $+0^\mu.23$ and $-0^\mu.26$, be attributed to temperature differences of the two standards, the inequality of the number of visits (thirty-one) at rising and (forty-five) at falling temperatures would give in the result for difference in lengths of A' and R' 1876 an uneliminated error of only $0^\mu.05$, which, in view of the uncertainty as to its true value, may be neglected.

An examination has also been made to see if there was any connection between the values of the residuals and the time, varying from six to fifteen minutes, occupied in a visit to the comparing-room. No connection could be perceived.

§ **11.** The determination of the interval of about $4^{mm}.93$ between the 46th graduations on the two auxiliary cylinders when their convex ends abutted against each other was made by comparing this interval at one time with the interval between $0^{in}.80$ and $0^{in}.99$ on the standard inch described in Chapter II, § 3, and at another with the interval between 0^{mm} and 5^{mm} on *D*1876. The auxiliary cylinders were mounted and adjusted as in the comparisons of Clarke yard *A* with *R*1876, save that the convex ends of these cylinders abutted against each other, instead of against the ends of yard *A*. The standard with which the interval was compared was mounted in the prolongation of the auxiliary cylinders, was made level, and aligned so that its longitudinal graduation when its supporting ear was moved would run along the longitudinal microscope-thread. The two microscopes were pointed at the left-hand ends of the intervals on the standard and on the auxiliary cylinders. The car carrying both was then moved longitudinally under the microscopes till the right-hand ends of the two intervals could be pointed at. These pointings gave a comparison of the lengths.

In the comparisons with the inch, which gave twenty-two values for the interval $4^{mm}.93$ on two days, there was no readjustment and no known space was read by which to determine the error of run of microscope due to focusing. The adjustments were as in comparisons with *D*1876.

In the comparisons of the interval $4^{mm}.93$ with the space from 0^{mm} to 5^{mm} on *D*1876, the following method was followed. The auxiliary cylinders or quills were brought to the same azimuth within 10′; to the same inclination within 6″; the contact of convex ends was made central, both horizontally and vertically, within $0^{mm}.1$. After each adjustment spaces of known values about 1^{mm} were read on with both microscopes to determine the error in run due to focusing. By running the truck carrying the auxiliary cylinders and *D*1876 alternately backward and forward, 24 pointings to each end of each of the intervals under comparison were made at each visit. After each visit the quills were disarranged in all their adjustments and readjusted. Six visits to the comparing-room were made in six days.

In the following table are given the results of the comparisons of the interval between the 46th graduations at the convex ends of the quills when the convex ends abut, with the space $0^{in}.80$ to $0^{in}.99$ on the standard inch. Each result is the mean derived from two pointings at each graduation. The first column gives the date; the second, the temperature; the third, the interval on quills minus the interval on inch as observed; and the fourth, the residuals for this interval in the sense computed minus observed—each is obtained by subtracting the results of individual comparisons from the mean of the mean results derived from the six visits.

Comparison of interval between 46th graduations on quills with space $0^{in}.80$ to $0^{in}.99$ on standard inch.

Date.	Temperature.	Observed (interval on quills *minus* interval on inch).	Residuals.
1879.	° F.	μ	μ
June 7, 2:50 p. m	64	+91.33	−0.35
		+92.23	−1.25
3:16 p. m		+90.23	+0.75
		+92.13	−1.15
		+91.93	−0.95
		+90.43	+0.55
3:32 p. m		+89.73	+1.25
		+90.83	+0.15
		+90.73	+0.25
3:43 p. m		+90.73	+0.25
		+90.13	+0.85
3:58 p. m		+90.73	+0.25
3:59 p. m		+92.93	−1.95
		+91.43	−0.45
		+92.33	−1.35
4:14 p. m	65	+89.63	+1.35
June 9, 3:10 p. m	63	+90.63	+0.35
		+91.93	−0.75
		+90.93	+0.05
		+90.53	+0.45
		+89.03	+1.95
3:32 p. m		+91.33	−0.35
Mean		+90.98	

The following table gives the results of comparisons of the interval between the 46th graduations on the auxiliary cylinders with the space 0^{mm} to 5^{mm} on *D*1876. The arrangement of the table is the same as that of the preceding one.

Comparison of interval between 46th graduations on quills with space 0^{mm} *to* 5^{mm} *on* *D*1876.

Date.	Temperature.	Observed (interval on quills *minus* interval on *D* 1876).	Residuals from means of each separate set.	Residuals of means from general mean.
1881.	° F.	μ	μ	μ
May 28, 3:08—3:37 p. m.	67. 45	—74. 5	—0. 5	
		—74. 2	—0. 2	
		—73. 6	+0. 4	
		—73. 5	+0. 5	
		—73. 8	+0. 2	
	67. 64	—74. 7	0. 7	
Means	67. 54	—74. 0		—1. 34
June 1, 2:50—3:15 p. m.	68. 68	—72. 7	+0. 5	
		—73. 9	—0. 7	
		—74. 2	—1. 0	
		—72. 5	+0. 7	
		—72. 8	+0. 4	
	68. 73	—73. 1	+0. 1	
Means	68. 70	—73. 2		—0. 54
June 2, 10:15—10:30 a. m.	68. 44	—71. 8	+0. 2	
		—71. 8	+0. 2	
		—71. 8	+0. 2	
		—71. 3	+0. 7	
		—73. 0	—1. 0	
	68. 44	—72. 5	—0. 5	
Means	68. 44	—72. 0		+0. 66
June 3, 10:30—11:00 a. m.	66. 85	—71. 6	+0. 3	
		—70. 7	+1. 2	
		—71. 8	+0. 1	
		—72. 8	—0. 9	
		—72. 8	—0. 9	
	66. 85	—72. 0	—0. 1	
Means	66. 85	—71. 9		+0. 76
June 4, 11:45—11:58 a. m.	65. 66	—73. 2	—1. 1	
		—73. 5	—1. 4	
		—71. 5	+0. 6	
		—72. 4	—0. 3	
		—70. 6	+1. 5	
	65. 66	—71. 3	+0. 8	
Means	65. 66	—72. 1		+0. 56
June 6, 10:43—11:48 a. m.	63. 48	—72. 8	—0. 2	
		—71. 9	+0. 7	
		—72. 2	+0. 4	
		—72. 8	—0. 2	
		—72. 7	—0. 1	
	63. 48	—73. 5	—0. 9	
Means	63. 48	—72. 6		+0. 06
Mean of mean values for each visit	66. 78	—72. 66		

If in comparisons of the interval with 0^{mm} to 5^{mm} on D1876 the differences between the individual results of a visit and the mean result be denoted by λ, there will result for the probable error of an individual result due to pointing and reading only,

$$0.6745\sqrt{\frac{\lambda\lambda}{36-6}}=\pm 0^{\mu}.50, \text{ which gives } \frac{0^{\mu}.50}{\sqrt{6}}=\pm 0^{\mu}.20$$

as the probable error from these causes in the result of a set. Again, if the probable error of the result of a visit in comparisons of interval with D1876 be derived from the discrepancies between the results of one visit and the mean of all, there results, *p. e.* of result of one visit $=\pm 0^{\mu}.56$, and of mean of six visits, $\pm 0^{\mu}.23$. The probable error of the result of a visit is made up of the probable error of pointing and reading already found as $\pm 0^{\mu}.20$, and of the probable error due to change in adjustment by readjustment, which will therefore be $\sqrt{(0^{\mu}.56)^2-(0^{\mu}.20)^2}=\pm 0^{\mu}.52$. Large errors would occur in adjustment most easily if the quills had differing inclinations; these errors would as probably be positive as negative. The errors which would arise from the contact of the convex ends of the quills not being precisely central, or from their differing in azimuth, are most dangerous, as they would always have the same sign. A series of experiments has shown that the probable error in a single attempt at focusing a microscope of the Repsold apparatus by the eye alone on a graduated scale produces a probable error in $0^{mm}.1$ distance measured of about $\pm 0^{\mu}.35$. As the interval measured with the microscope-screw was about $0^{mm}.091$, $\pm 0^{\mu}.32$ may be taken as the probable error due to focusing in comparisons of the interval with the inch.

Taking the mean of the results of the comparisons of the interval on the quills with the space $0^{in}.80$ to $0^{in}.99$ on the inch, and deriving its probable error from the differences between the mean and the individual results, we have—

Interval on quills minus $0^{in}.80$ to $0^{in}.99$ on inch $=+90^{\mu}.98\pm 0^{\mu}.14$

Taking the mean of the results of each visit in the comparisons of the interval on quills with space 0^{mm} to 5^{mm} on D1876, and then taking the mean of these means as the final difference, and deriving its probable error from the differences between the general mean and the individual means, there results for the final difference—

Interval on quills minus space 0^{mm} to 5^{mm} on $D1876=-72^{\mu}.66\pm 0^{\mu}.23$

Since, in the comparisons of the interval with the space on the inch, the probable error already found, $\pm 0^{\mu}.14$, was due to pointings and readings alone, this probable error must be combined with the $\pm 0^{\mu}.52$ from error of adjustment and the $\pm 0^{\mu}.32$ due to focusing, so that the difference between interval on quills and on inch becomes $+90^{\mu}.98\pm 0^{\mu}.63$. From the values of the spaces on the inch and their probable errors given in Chapter II, § 3, there results at 62° F.—

$0^{in}.80$ to $0^{in}.99$ on inch $=4{,}836^{\mu}.9\pm 0^{\mu}.17$

and combining this with the value of the space on quills minus space on inch,

$=+90^{\mu}.98\pm 0^{\mu}.63$

there results—

Interval between 46th divisions on quills at 62° F., when their convex ends abut, $=4{,}927^{\mu}.88\pm 0^{\mu}.65$

From §§ 7 and 8, at 59° F.,

Space 0^{mm} to 5^{mm} on $D1876=5{,}000^{\mu}.55\pm 0^{\mu}.05$

Combining this with the difference between interval on quills and space on D1876, namely,

$-72^{\mu}.66\pm 0^{\mu}.23$

there results—

Interval between 46th divisions on quills at 59° F., when their convex ends abut, $=4{,}927^{\mu}.89\pm 0^{\mu}.24$

Reducing the first value of this interval to 59° F., with the rate of expansion $0^{\mu}.00588$ per 1^{mm}, we have—

Interval on quills at 59° F. $=4927^{\mu}.79\pm 0^{\mu}.65$

and combining the two values for this interval with weights proportional to their probable errors, finally—

Interval on quills at 59° F. $=4^{mm}.92788\pm 0^{\mu}.23$

Reducing this to the mean temperature of comparisons of yard A and R1876,

Interval on quills at 57°.92 $=4^{mm}.9279\pm 0^{\mu}.23$

Of this interval about $4^{mm}.4$ is steel and $0^{mm}.5$ is apparently quartz crystal. The mean of the two expansions of quartz differs very little from that of steel, so that the assumption tacitly made that the rate of expansion of the $4^{mm}.93$ interval was the same as that of the steel standards with which it was compared can introduce no error of importance.

§ **12.** From § 9,

(1) $$A'-R'1876=+61^{\mu}.36\pm0^{\mu}.097 \text{ at } 57^{\circ}.92 \text{ F.}$$

But

$$A'=\text{Clarke yard } A+\text{interval on quills}=A+4^{mm}.9279\pm0^{\mu}.23$$

$$R'1876=\tfrac{919}{1000}R1876-1^{\mu}.1\pm0^{\mu}.27. \quad (\S\ 8)$$

Substituting these values in (1)

$$A=\tfrac{919}{1000}R1876-4^{mm}.8676\pm0^{\mu}.37 \text{ at } 57^{\circ}.92 \text{ F.}$$

or,

$$R1876=\tfrac{1000}{919}A+5^{mm}.2966\pm0^{\mu}.40 \text{ at } 57^{\circ}.92 \text{ F.}$$

Substituting Colonel Clarke's value of yard A, Chapter II, § 2, namely,

$$A=0^{y}.99995298\pm0^{y}.00000010 \text{ at } 57^{\circ}.92 \text{ F.}$$

and reducing millimeters to yards with Colonel Clarke's value of the metre, namely,

$$1^{m}=1^{y}.093623$$

since the error in this value would produce no appreciable error, there results—

$$R1876=1^{y}.09388063\pm0^{y}.00000045 \text{ at } 57^{\circ}.92 \text{ F.}$$

The adjusted rate of expansion of $R1876$ is, § 58,

$$5^{\mu}.885\pm0^{\mu}.043+(0^{\mu}.0037\pm0^{\mu}.0009)\,(t-62)$$

or,

$$E_{R\,1876}=0^{y}.000006420+0^{y}.000000004\,(t-57.92)$$

and for any temperature there may be written

$$R\,1876=1^{y}.09388063+0^{y}.000006420\,(t-57.92)+0^{y}.000000002\,(t-57.92)^2$$

DETERMINATION OF LENGTH OF STEEL BAR IN TUBE 2 (DESIGNATED BY S_2) AND OF ZINC BAR (Z_2) IN TERMS OF THE METRE $R1876$.

§ **13.** The four successive metres into which S_2 is divided by graduation-lines will be designated (beginning at the rear end of the tube) by $S_{2.1}$, $S_{2.2}$, $S_{2.3}$, $S_{2.4}$.

In the comparisons $R1876$ was placed in the comparing-box, parallel to the metre of S_2, with which it was to be compared. In the earlier comparisons it rested on a flat surface, but in the later ones it rested on supports at about one-fourth and three-fourths of its length. As this metre has great stiffness, and its graduations are in its neutral axis, the change in method of support could have no effect on its length. Temperatures were determined by one Casella thermometer laid on $R1876$, one at the middle of tube 2, and one Geissler thermometer in the nearest end of tube 2. Unit weight was assigned to the thermometer on $R1876$, and to the nearest one in the tube; to the most distant one the weight $\frac{1}{2}$ was given, and the weighted mean was taken as the temperature of the comparison. This method was adopted in order to obtain in changing temperatures a resulting mercurial temperature which should lie between the true temperatures of $S_{2.n}$ and $R1876$. Since $R1876$ and its thermometer are in the open air inside the comparing-box, while S_2 and its thermometers are inside the heavy iron cylinder which contains S_2, it seems probable that in changing temperatures the thermometer on $R1876$ preceded its temperature, while those in the tube followed the temperature of $R1876$, but slightly preceded that of S_2.

In comparisons the tube was made horizontal within 1′ or 2′. Metre $R1876$ was made to have the same inclination by bringing it to the focus of the microscopes, which could be done so that the maximum difference of inclinations should not exceed 3′, the probable difference being much less.

The microscopes were set one metre apart on the microscope-car, their line of collimation was made vertical within 40″, and they then remained undisturbed, while by the lateral motion of the tube-car the tube-metre and $R1876$ were alternately brought under the microscopes and were read on.

A part of the comparisons were made by a single observer and a part by two, under the following programmes:

One observer.

Read thermometers.
1 pointing at 0^m on R1876.
2 pointings at 1^m on R1876.
1 pointing at 0^m on R1876.
1 pointing at 0^m on zinc bar.
1 pointing at 0^m on steel bar.
2 pointings at 1^m on steel bar.
1 pointing at 0^m on steel bar.
1 pointing at 0^m on zinc bar.
Read thermometers.

In next comparison begin with the other microscope. In a part of this work a second set of readings on R1876 was added.

Two observers.

Read thermometers.
1 pointing at each end of R1876.
2 pointings at each end of S_2 metre.
2 pointings at each end of Z_2 metre.
1 pointing at each end of R1876.
Read thermometers on metre.

Alternate tube and R1876 in first pointings.

Run of microscopes to be determined by reading known spaces at each end of each metre after each adjustment of position of R1876 and tube.

§ **14.** In the reduction, since there is, as stated in § 21, some danger that the bars in tube 2 prior to October 23, 1878, may have been partially clamped, and so not perfectly free to expand and contract, no comparisons made before that date are used. The length of a visit to the comparing-room was usually about fourteen minutes and there were usually two visits a day, about twelve hours apart. No comparisons have been used when the length of the visit exceeded twenty-five minutes. This rejected two comparisons of $S_{2.1}$, one of $S_{2.2}$, and one of $S_{2.4}$. No comparisons have been used in which the difference of temperature of R1876 and the metre of the tube, as indicated by the mercurial thermometers, differed by more than $0^\circ.25$ F. This rejected five comparisons of $S_{2.3}$. No comparisons were used in which the tube changed temperature in the preceding twenty-four hours by more than $1^\circ.5$ F. This rejected eight comparisons of $S_{2.1}$ and two of $S_{2.3}$.

The microscope-readings were corrected for run and for non-rectangularity of microscope-threads; and when the zero or full-metre graduation was not read on, the reading was referred to the zero of graduation by means of the values of the graduation-intervals given in § 72.

Habitually the zero and full-metre graduations on the steel bar were read on, but sometimes those $0^{mm}.1$ distant from these were also read on.

Each visit gave a mean mercurial temperature, a mean difference of length of R1876, and one of the metres of S_2, and also a mean difference of length of R1876 and one of the metres of Z_2. These mean values enter the observation-equations. These mean values depend usually on two (or sometimes four) pointings at each end of each metre. A weight unity is assigned to the result of each visit, whether it includes two or four pointings at each graduation, since the errors most to be feared are those of temperature, not those of observation. In reduction each visit gave an observation- or error-equation of the form

$$(R1876-S_{2.n})_{t_0}+(t-t_0)\,(E_{R\,1876}-E_{S_{2.n}})-(R1876-S_{2.n})=v$$

in which the first term is the excess of length of R1876 over the n^{th} metre of S_2 at a chosen temperature t_0; t is the weighted mean temperature of comparison, derived as previously stated; $E_{R\,1876}$ and $E_{S_{2.n}}$ are the expansions for 1° F. of R1876 and of the n^{th} metre of S_2 respectively; $(R1876-S_{2.n})$ is the observed difference of length; and v is the residual in the sense computed minus observed. For the comparisons of each metre of S_2 with R1876 the value of the first term is obtained by taking the mean of all the results of the comparisons, and the value of t_0 is obtained by taking the corresponding mean of the observed temperatures. This leaves the difference of expansions as the single unknown to be determined. By taking the mean difference and mean temperature $(R1876-S_{2.1})_{t_0}$ is found to be $+45^{\mu}.54$ and t_0 is found to be $54^{\circ}.16$ F.

The following tables give the results of the comparisons. The first column gives the date of comparison; the second shows whether $R1876$ was north (next observer) or south; the third gives the weighted mean temperature of tube and metre; the fourth gives the temperature of $R1876$ minus the weighted mean tube-temperature; the fifth gives the number of observers; the sixth gives $(t-t_0)$ the coefficient of the unknown $(E_{R\,1876}-E_{S_{2.n}})$; the seventh gives the observed absolute term $(R1876-S_{2.n})$; and the eighth gives the residuals, v.

§ **15.** I.—*Comparisons of metres R 1876 and $S_{2.1}$.*

Date.	Position of metre.	Weighted mean temperature of tube and metre.	Temperature of R1876 *minus* weighted mean tube-temperature.	No. of observers.	$(t-t_0)$.	$(R\,1876-S_{2.1})$.	Residuals, computed *minus* observed.
1879.		° F.	° F.		° F.	μ	
Jan. 15, a. m.	South side	35.85	+0.02	1	−18.31	+48.3	+3.3
15, p. m.	....do	36.05	+0.02	....	−18.11	+49.2	+2.4
16, p. m.	....do	36.23	+0.05	....	−17.93	+47.4	+4.1
17, a. m.	....do	36.31	−0.08	....	−17.85	+47.9	+3.6
17, p. m.	....do	36.37	−0.02	....	−17.79	+47.4	+4.1
18, a. m.	....do	36.55	+0.02	....	−17.61	+47.0	+4.4
18, p. m.	....do	36.99	+0.12	....	−17.17	+46.7	+4.6
Jan. 30, a. m.	North side	40.24	−0.16	1	−13.92	+50.4	−0.2
30, p. m.	....do	40.44	+0.01	....	−13.72	+48.5	+1.6
31, a. m.	....do	40.60	+0.08	....	−13.56	+51.4	−1.3
31, p. m.	....do	40.76	−0.02	....	−13.40	+50.3	−0.3
Feb. 1, a. m.	....do	40.84	+0.01	....	−13.32	+49.4	+0.6
1, p. m.	....do	40.34	+0.01	....	−13.82	+49.4	+0.7
2, a. m.	....do	39.64	+0.01	....	−14.52	+49.0	+1.4
2, p. m.	....do	39.16	−0.02	....	−15.00	+49.4	+1.1
1880.							
Apr. 15, a. m.	South side	50.12	−0.07	2	− 4.04	+49.2	−2.3
17, a. m.	....do	51.47	−0.06	....	− 2.69	+51.3	−4.9
19, a. m.	....do	51.55	−0.03	....	− 2.61	+49.7	−3.3
Apr. 20, a. m.	North side	51.81	−0.13	2	− 2.35	+48.5	−2.2
21, a. m.	....do	52.13	−0.08	....	− 2.03	+51.6	−5.4
22, a. m.	....do	53.24	−0.10	....	− 0.92	+43.9	+1.9
23, a. m.	....do	54.02	−0.07	....	− 0.14	+48.0	−2.5
Apr. 24, a. m.	South side	53.44	−0.10	2	− 0.72	+49.6	−3.9
26, a. m.	....do	52.46	−0.13	....	− 1.70	+48.6	−2.5
27, a. m.	....do	51.97	+0.02	....	− 2.19	+51.3	−5.1
28, a. m.	....do	51.69	−0.10	....	− 2.47	+52.0	−5.7
June 19, a. m.	North side	66.96	+0.03	2	+12.80	+44.5	−3.3
20, a. m.	....do	67.36	+0.02	....	+13.20	+42.9	−1.8
21, a. m.	....do	67.87	0.00	....	+13.71	+44.2	−3.3
21, p. m.	....do	68.40	+0.04	....	+14.24	+42.8	−2.1
22, a. m.	....do	68.96	+0.03	....	+14.80	+41.9	−1.4
22, p. m.	....do	69.39	−0.02	....	+15.23	+43.7	−3.3
23, a. m.	....do	70.13	0.00	....	+15.97	+43.3	−3.1
23, p. m.	....do	70.56	−0.05	....	+16.40	+42.0	−2.0
24, a. m.	....do	71.23	+0.08	....	+17.07	+38.2	+1.6
24, p. m.	....do	71.48	+0.17	....	+17.32	+41.0	−1.3
June 28, a. m.	South side	74.37	+0.04	2	+20.21	+39.0	−0.3
28, p. m.	....do	74.53	+0.02	....	+20.37	+40.2	−1.5
29, a. m.	....do	74.09	+0.09	....	+19.93	+36.9	+1.9
29, p. m.	....do	74.14	+0.01	....	+19.98	+36.9	+1.9
30, a. m.	....do	73.13	−0.06	....	+18.97	+37.8	+1.3
30, p. m.	....do	72.97	+0.04	....	+18.81	+37.5	+1.7
July 1, a. m.	....do	72.22	+0.10	....	+18.06	+37.5	+1.9
1, p. m.	....do	72.26	+0.03	....	+18.10	+38.4	+1.0
2, a. m.	....do	71.78	0.00	....	+17.62	+36.9	+2.7

I.—*Comparisons of metres R* 1876 *and* $S_{2.1}$—Continued.

Date.	Position of metre.	Weighted mean temperature of tube and metre.	Temperature of *R* 1876 *minus* weighted mean tube-temperature.	No. of observers.	$(t-t_0)$.	$(R\,1876-S_{2.1})$.	Residuals, computed *minus* observed.
1880.		° F.	° F.		° F.	μ	μ
Aug. 25, a. m....	North side....	72. 94	+0. 01	2	+18. 78	+42. 2	−3. 0
25, p. m....	....do	73. 04	−0. 08	....	+18. 88	+44. 2	−5. 0
26, a. m....	....do	72. 71	−0. 05	....	+18. 55	+41. 6	−2. 3
26, p. m....	....do	72. 44	+0. 08	....	+18. 28	+41. 8	+2. 4
27, a. m....	....do	72. 15	−0. 03	....	+17. 99	+43. 4	+3. 9
27, p. m....	....do	72. 10	−0. 12	....	+17. 94	+42. 4	+2. 9
28, a. m....	....do	72. 37	+0. 02	....	+18. 21	+42. 5	+3. 1
Aug. 30, a. m....	South side	73. 59	+0. 01	2	+19. 43	+37. 0	+2. 0
30, p. m....	....do	73. 42	−0. 04	....	+19. 26	+35. 6	+3. 4
31, a. m....	....do	73. 13	−0. 06	....	+18. 97	+35. 4	+3. 7
31, p. m....	....do	72. 98	+0. 02	....	+18. 82	+37. 7	+1. 5
Sept. 1, a. m....	....do	72. 93	−0. 06	....	+18. 77	+35. 0	+4. 2
1, p. m....	....do	73. 05	−0. 01	....	+18. 89	+32. 7	+6. 5
2, a. m....	....do	73. 39	+0. 01	....	+19. 23	+34. 4	+4. 6
2, p. m....	....do	73. 89	+0. 01	....	+19. 73	+33. 5	+5. 4
3, a. m....	....do	74. 25	−0. 09	....	+20. 09	+33. 5	+5. 2
Oct. 14, a. m....	South side	60. 88	−0. 03	1	+ 6. 72	+41. 4	+1. 8
14, p. m....	....do	60. 52	−0. 09	....	+ 6. 36	+41. 0	+2. 4
15, a. m....	....do	60. 38	−0. 03	....	+ 6. 22	+40. 6	+2. 8
15, p. m....	....do	60. 42	−0. 09	....	+ 6. 26	+40. 9	+2. 5
16, a. m....	....do	60. 52	−0. 09	....	+ 6. 36	+40. 8	+2. 6
17, a. m....	....do	59. 18	−0. 01	....	+ 5. 02	+41. 8	+2. 0
21, a. m....	....do	50. 91	+0. 06	....	− 3. 25	+47. 1	−0. 5
21, p. m....	....do	50. 01	−0. 02	....	− 4. 15	+45. 6	+1. 3
22, a. m....	....do	51. 18	+0. 02	....	− 2. 98	+47. 1	−0. 6
22, p. m....	....do	51. 29	−0. 07	....	− 2. 87	+45. 2	+1. 3
23, a. m....	....do	51. 26	−0. 03	....	− 2. 90	+44. 7	+1. 8
Oct. 29, a. m....	North side	50. 04	−0. 08	1	− 4. 12	+49. 3	−2. 4
29, p. m....	....do	50. 04	−0. 08	....	− 4. 12	+48. 7	−1. 8
30, a. m....	....do	50. 05	−0. 01	....	− 4. 11	+49. 6	−2. 7
31, a. m....	....do	50. 17	−0. 04	....	− 3. 99	+48. 5	−1. 7
31, p. m....	....do	50. 03	−0. 06	....	− 4. 13	+47. 3	−0. 4
Nov. 1, a. m....	....do	49. 69	−0. 00	....	− 4. 47	+50. 4	−3. 4
1, p. m....	....do	49. 48	−0. 15	....	− 4. 68	+47. 2	−0. 1
2, a. m....	....do	49. 47	−0. 13	....	− 4. 69	+50. 8	−3. 7
1881.							
Jan. 6, a. m....	South side	31. 71	....	2	−22. 45	+56. 6	−3. 6
7, a. m....	....do	32. 73	+0. 23	....	−21. 43	+54. 1	−1. 4
8, a. m....	....do	32. 19	+0. 21	....	−21. 97	+51. 4	+1. 5
Jan. 10, a. m....	North side....	30. 09	+0. 05	2	−24. 07	+53. 2	+0. 4
11, a. m....	....do	29. 37	+0. 01	....	−24. 79	+54. 0	−0. 2
12, a. m....	....do	29. 03	+0. 15	....	−25. 13	+55. 7	−1. 8
13, a. m....	....do	29. 83	−0. 02	....	−24. 33	+52. 3	+1. 4
14, a. m....	....do	30. 47	+0. 08	....	−23. 69	+53. 8	−0. 3
17, a. m....	....do	28. 78	+0. 07	....	−25. 38	+53. 1	+0. 9
18, a. m....	....do	28 82	0. 00	....	−25. 34	+54. 8	−0. 8
19, a. m....	....do	29. 26	−0. 07	....	−24. 90	+59. 3	−5. 4
20, a. m....	....do	29. 92	0. 00	....	−24. 24	+55. 5	−1. 9
21, a. m....	....do	30. 83	−0. 02	....	−23. 33	+50. 5	+2. 8
22, a. m....	....do	32. 02	0. 00	....	−22. 14	+50. 7	+2. 2
24, a. m....	....do	33. 99	−0. 05	....	−20. 17	+49. 7	+2. 6
95	....	5145. 04	....	....	....	+4, 326. 6	
Means	....	54. 158	....	....	....	+45. 54	

$(R\ 1876-S_{2.1})=+45^{\mu}.54\pm 0^{\mu}.20$ at $54^{\circ}.16$ F.

$(E_{R\,1876}-E_{S_{2.1}})=-0^{\mu}.3355\pm 0^{\mu}.0126$

Probable error of result from single visit $=\pm 1^{\mu}.94$

§ 16. II.—*Comparisons of metres $R1876$ and $S_{2.2}$.*

Date.	Position of metre.	Weighted mean temperature of tube and metre.	Temperature of $R1876$ *minus* weighted mean tube-temperature.	No. of observers.	$(t-t_0)$	$(R1876-S_{2.2})$	Residuals, computed *minus* observed.
1878.		° F.	° F.		° F.	μ	μ
Oct. 24, p. m.	South side	59.38	−0.18	1	+ 1.53	+ 6.0	−1.9
25, p. m.	....do	59.05	+0.05	....	+ 1.20	+ 3.9	+0.3
26, a. m.	....do	59.13	+0.09	....	+ 1.28	+ 4.3	−0.2
26, p. m.	....do	59.05	−0.11	....	+ 1.20	+ 4.2	−0.0
27, a. m.	....do	58.67	+0.02	....	+ 0.82	+ 4.0	+0.3
27, p. m.	....do	58.47	+0.02	....	+ 0.62	+ 3.6	+0.7
Oct. 28, a. m.	South side	58.17	+0.01	1	+ 0.32	+ 5.2	−0.8
28, p. m.	....do	57.75	+0.05	....	− 0.10	+ 4.5	−0.0
29, a. m.	....do	56.79	+0.15	....	− 1.06	+ 4.3	+0.5
Oct. 31, p. m.	North side	54.22	−0.06	1	− 3.63	+ 5.5	+0.1
Nov. 1, a. m.	....do	54.04	−0.09	....	− 3.81	+ 5.8	−0.2
1, p. m.	....do	53.72	−0.06	....	− 4.13	+ 6.2	−0.5
2, a. m.	....do	53.56	+0.04	......	− 4.29	+ 6.1	−0.4
2, p. m.	....do	53.46	+0.04	....	− 4.39	+ 4.6	+1.2
3, a. m.	....do	53.22	+0.11	....	− 4.63	+ 5.0	+0.8
3, p. m.	....do	53.07	+0.04	....	− 4.78	+ 7.2	−1.3
4, a. m.	....do	52.83	−0.06	....	− 5.02	+ 7.2	−1.2
4, p. m.	....do	52.55	−0.09	....	− 5.30	+ 5.4	+0.6
5, a. m.	....do	52.21	−0.03	....	− 5.64	+ 7.5	−1.4
5, p. m.	....do	52.05	−0.10	....	− 5.80	+ 5.4	+0.8
6, a. m.	....do	51.83	+0.10	....	− 6.02	+ 7.9	−1.7
6, p. m.	....do	51.69	0.00	....	− 6.16	+ 6.3	−0.0
7, a. m.	....do	51.59	0.00	....	− 6.26	+ 7.1	−0.8
7, p m.	....do	51.49	0.00	....	− 6.36	+ 6.9	−0.6
8, a. m.	....do	51.45	−0.10	....	− 6.40	+ 6.5	−0.1
1879.							
Feb. 4, a. m.	North side	38.22	+0.07	1	−19.63	+ 9.7	+0.5
4, p. m.	....do	38.12	−0.09	....	−19.73	+10.7	−0.5
5, a. m.	....do	37.98	−0.03	....	−19.87	+10.5	−0.2
5, p. m.	....do	38.04	−0.13	....	−19.81	+ 9.1	+1.2
6, a. m.	....do	38.12	−0.09	....	−19.73	+ 8.6	+1.6
6, p. m.	....do	38.30	−0.06	....	−19.55	+10.7	−0.5
7, a. m.	....do	38.26	+0.01	....	−19.59	+ 9.6	+0.6
7, p. m.	....do	38.26	+0.01	....	−19.59	+ 9.0	+1.2
1880.							
Aug. 12, a. m.	South side	71.97	−0.06	2	+14.12	+ 3.1	−2.7
12, p. m.	....do	72.03	+0.01	....	+14.18	+ 1.0	−0.6
13, a. m.	....do	71.63	−0.08	....	+13.78	+ 1.3	−0.8
13, p. m.	....do	71.51	−0.04	....	+13.66	+ 1.4	−0.9
14, a. m.	....do	71.41	−0.04	....	+13.56	+ 0.3	+0.3
14, p. m.	....do	71.43	+0.01	....	+13.58	+ 2.2	−1.6
15, a. m.	....do	71.09	−0.01	....	+13.24	+ 1.9	−1.2
15, p. m.	....do	70.83	+0.09	....	+12.98	+ 1.0	−0.2
16, a. m.	....do	69.86	+0.04	....	+12.01	+ 3.1	−2.1
Aug. 18, a. m.	North side	69.09	−0.01	2	+11.24	+ 0.7	+0.6
18, p. m.	....do	69.17	+0.02	....	+11.32	+ 1.3	−0.1
19, a. m.	....do	69.61	−0.04	....	+11.76	− 1.0	+2.1
19, p. m.	....do	70.13	−0.08	....	+12.28	+ 0.7	+0.3
20, a. m.	....do	70.56	−0.04	....	+12.71	− 1.3	+2.1
20, p. m.	....do	71.04	−0.01	....	+13.19	− 2.1	+2.8
21, a. m.	....do	71.33	+0.01	....	+13.48	− 1.5	+2.1
21, p. m.	....do	71.49	−0.01	....	+13.64	− 2.4	+3.0
23, a. m.	....do	71.23	+0.01	....	+13.38	− 0.6	+1.2
51	....	2950.15	....	....	....	+227.6	
Means	....	57.846	....	....	....	+ 4.46	

$(R1876-S_{2.2}) = +4^{\mu}.46 \pm 0^{\mu}.12$ at $57^{\circ}.85$ F.

$(E_{R\,1876}-E_{S\,2.2}) = -0^{\mu}.2896 \pm 0^{\mu}.0102$

Probable error of result from single visit $= \pm 0^{\mu}.83$

§ **17.** III.—*Comparisons of metres R1876 and $S_{2.3}$.*

Date.	Position of metre.	Weighted mean temperature of tube and metre.	Temperature of R1876 *minus* weighted mean tube-temperature.	No. of observers.	$(t-t_0)$	$(R1876-S_{2.3})$	Residuals, computed *minus* observed.
1878.		° F.	° F.		° F.	μ	μ
Nov. 9, p. m ...	North side....	50. 84	+0. 10	1	− 3. 27	+12. 2	+1. 7
10, a. m....	....do	50. 98	+0. 03	....	− 3. 13	+13. 8	+0. 0
10, p. m....	....do	51. 08	+0. 03	....	− 3. 03	+12. 8	+1. 0
11, a. m....	....do	51. 18	+0. 04	....	− 2. 93	+12. 2	+1. 5
11, p. m ...	....do	51. 32	+0. 14	....	− 2. 79	+11. 3	+2. 4
12, a. m....	....do	51. 67	+0. 03	....	− 2. 44	+11. 6	+2. 0
12, p. m....	....do	51. 91	−0. 04	....	− 2. 20	+12. 4	+1. 1
13, a. m....	....do	52. 07	−0. 13	....	− 2. 04	+11. 8	+1. 7
Nov. 25, p. m....	South side ..	51. 22	−0. 03	1	− 2. 89	+16. 2	−2. 5
26, a. m....	....do	50. 96	+0. 07	....	− 3. 15	+15. 8	−2. 0
26, p. m....	....do	50. 64	−0. 07	....	− 3. 47	+15. 6	−1. 7
27, a. m....	....do	50. 18	+0. 03	....	− 3. 93	+14. 2	−0. 1
27, p. m....	....do	49. 92	+0. 04	....	− 4. 19	+16. 3	−2. 1
28, a. m....	....do	49. 72	−0. 04	....	− 4. 39	+12. 7	+1. 5
28, p. m....	....do	49. 32	+0. 13	....	− 4. 79	+15. 4	−1. 0
29, a. m....	....do	48. 92	−0. 04	....	− 5. 19	+16. 4	−1. 9
1879.							
Aug. 3, a. m....	North side....	76. 31	+0. 17	1	+22. 20	+ 4. 6	+1. 0
3, p. m....	....do	76. 35	+0. 11	....	+22. 24	+ 3. 6	+2. 0
4, a. m...	....do	76. 23	+0. 14	....	+22. 12	+ 4. 1	+1. 5
4, p. m....	....do	76. 39	+0. 04	....	+22. 28	+ 4. 7	+0. 9
5, a. m....	....do	76. 25	+0. 11	....	+22. 14	+ 4. 2	+1. 4
Aug. 7, p. m....	South side ...	75. 61	+0. 17	1	+21. 50	+ 5. 7	+0. 1
8, a. m....	....do	75. 13	+0. 14	....	+21. 02	+ 5. 6	+0. 4
8, p. m....	....do	74. 75	+0. 10	....	+20. 64	+ 6. 8	−0. 7
10, p. m....	....do	71. 19	+0. 22	....	+17. 08	+ 9. 1	−1. 8
11, a. m....	....do	70. 53	+0. 15	....	+16. 42	+ 9. 5	−2. 0
Aug. 12, p. m....	South side....	70. 27	+0. 09	1	+16. 16	+ 8. 0	−0. 4
13, a. m....	....do	70. 35	+0. 12	....	+16. 24	+ 7. 5	+0. 1
13, p. m....	....do	70. 49	+0. 22	....	+16. 38	+ 8. 8	−1. 3
14, a. m....	....do	70. 51	+0. 18	....	+16. 40	+ 9. 2	−1. 7
14, p. m....	....do	70. 67	+0. 08	....	+16. 56	+ 8. 0	−0. 6
Aug. 16, a. m....	North side...	68. 85	+0. 13	1	+14. 74	+ 9. 5	−1. 5
16, p. m....	....do	68. 83	+0. 16	....	+14. 72	+ 7. 3	+0. 7
17, a. m....	....do	68. 35	+0. 14	....	+14. 24	+ 7. 7	+0. 5
17, p. m....	....do	68. 11	+0. 03	....	+14. 00	+ 7. 4	+0. 9
18, a. m....	....do	67. 63	+0. 17	....	+13. 52	+ 8. 7	−0. 3
18, p. m...	....do	67. 43	+0. 17	....	+13. 32	+ 8. 6	−0. 1
1881.							
Jan. 27, a. m....	North side....	33. 89	+0. 05	2	−20. 22	+23. 9	−4. 6
27, p. m....	....do	33. 85	+0. 14	....	−20. 26	+21. 4	−2. 0
28, a. m....	....do	33. 35	+0. 12	....	−20. 76	+20. 8	−1. 3
28, p. m....	....do	32. 91	+0. 10	....	−21. 20	+22. 3	−2. 6
29, a. m....	....do	32. 69	+0. 05	....	−21. 42	+20. 4	−0. 7
29, p. m ..	....do	32. 55	+0. 04	....	−21. 56	+23. 0	−3. 2
30, p. m....	....do	33. 27	+0. 09	....	−20. 84	+20. 5	−1. 0
31, a. m ..	....do	33. 43	+0. 07	....	−20. 68	+20. 8	−1. 3
Feb. 1, a. m....	South side....	34. 44	+0. 11	2	−19. 67	+17. 5	+1. 7
1, p. m....	....do	34. 13	+0. 07	....	−19. 98	+14. 1	+5. 2
2, a. m....	....do	33. 45	+0. 12	....	−20. 66	+18. 4	+1. 1
2, p. m....	....do	32. 85	+0. 12	....	−21. 26	+19. 6	+0. 1
4, a. m....	....do	30. 57	+0. 09	....	−23. 54	+16. 6	+3. 8
4, p. m....	....do	30. 31	+0. 10	....	−23. 80	+18. 7	+1. 8
5, a. m....	....do	29. 95	+0. 04	....	−24. 16	+17. 1	+3. 5
52	....	2813. 80	....	....	....	664. 4	
Means	....	54. 111	....	....	....	+ 12. 78	

$(R1876-S_{2.3})=+12^{\mu}.78\pm0^{\mu}.18$ at $54^{\circ}.11$ F.

$(E_{R\,1876}-E_{S_{2.3}})=-0^{\mu}.3236\pm0^{\mu}.0108$

Probable error of result from single visit $=\pm1^{\mu}.28$

§ 18. IV.—*Comparisons of metres R 1876 and $S_{2.4}$.*

Date.	Position of metre.	Weighted mean temperature of tube and metre.	Temperature of *R* 1876 *minus* weighted mean tube-temperature.	No. of observers.	$(t-t_0)$	$R\,1876-S_{2.4}$	Residuals, computed *minus* observed.
1878.		° F.	° F.		° F.	μ	μ
Nov. 14, p. m.	North side	52.63	−0.07	1	− 4.23	+ 8.2	+3.1
15, a. m.	do	52.39	0.00		− 4.47	+ 9.8	+1.6
15, p. m.	do	52.27	+0.03		− 4.59	+10.3	+1.2
16, a. m.	do	52.13	−0.07		− 4.73	+ 9.9	+1.6
16, p. m.	do	51.93	−0.07		− 4.93	+12.6	−1.0
17, p. m.	do	52.03	−0.07		− 4.83	+11.1	+0.5
18, a. m.	do	52.23	−0.07		− 4.63	+10.7	+0.8
18, p. m.	do	52.39	0.00		− 4.47	+ 9.5	+1.9
19, a. m.	do	52.59	0.00		− 4.27	+10.3	+1.0
Nov. 20, p. m.	South side	53.13	−0.06	1	− 3.73	+ 9.9	+1.2
21, a. m.	do	53.19	+0.01		− 3.67	+ 9.5	+1.6
21, p. m.	do	53.19	+0.01		− 3.67	+11.3	−0.2
22, a. m.	do	53.09	+0.01		− 3.77	+14.0	−2.8
22, p. m.	do	52.97	+0.03		− 3.89	+11.8	−0.6
23, a. m.	do	52.81	−0.03		− 4.05	+11.6	−0.3
23, p. m.	do	52.59	0.00		− 4.27	+ 9.1	+2.2
24, a. m.	do	52.15	+0.07		− 4.71	+10.7	+0.8
1879.							
Jan. 20, a. m.	South side	37.21	−0.12	1	−19.65	+19.4	−2.1
20, p. m.	do	37.23	+0.08		−19.63	+17.6	−0.3
21, a. m.	do	37.15	+0.18		−19.71	+18.2	−0.9
21, p. m.	do	36.59	−0.06		−20.27	+18.7	−1.2
22, a. m.	do	36.57	−0.02		−20.29	+19.6	−2.1
22, p. m.	do	36.47	−0.02		−20.39	+20.0	−2.4
23, a. m.	do	36.57	−0.02		−20.29	+19.9	−2.4
23, p. m.	do	36.71	+0.08		−20.15	+20.9	−3.4
Jan. 25, a. m.	North side	37.17	−0.02	1	−19.69	+18.5	−1.2
25, p. m.	do	37.49	−0.06		−19.37	+17.2	−0.0
26, a. m.	do	37.71	−0.09		−19.15	+15.3	+1.8
26, p. m.	do	37.95	+0.01		−18.91	+15.0	+2.0
27, a. m.	do	38.09	−0.06		−18.77	+15.3	+1.7
27, p. m.	do	38.41	−0.09		−18.45	+17.2	−0.4
28, a. m.	do	38.75	−0.16		−18.11	+14.9	+1.8
28, p. m.	do	39.11	−0.10		−17.75	+14.7	+1.8
July 8, a. m.	North side	73.52	(*)	1	+16.66	+ 5.7	−2.4
8, p. m.	do	73.75			+16.89	+ 6.1	−2.9
9, a. m.	do	73.72			+16.86	+ 5.8	−2.6
9, p. m.	do	73.89			+17.03	+ 5.1	−2.0
10, p. m.	do	74.15			+17.29	+ 5.3	−2.3
11, a. m.	do	74.25			+17.39	+ 4.3	−1.3
11, p. m.	do	74.59			+17.73	+ 4.9	−2.0
July 21, a. m.	South side	72.48		1	+15.62	+ 4.5	−0.8
21, p. m.	do	72.71			+15.85	+ 2.3	+1.3
22, a. m.	do	72.71			+15.85	+ 1.9	+1.7
22, p. m.	do	72.65			+15.79	+ 1.7	+1.9
23, a. m.	do	72.78			+15.92	+ 2.2	+1.4
23, p. m.	do	73.09			+16.23	+ 1.2	+2.2
24, a. m.	do	73.29			+16.43	+ 1.8	+1.6
24, p. m.	do	73.32			+16.46	+ 1.6	+1.7
July 27, a. m.	South side	74.09		1	+17.23	+ 3.5	−0.4
27, p. m.	do	74.09			+17.23	+ 3.6	−0.5
28, a. m.	do	73.72			+16.86	+ 4.3	−1.1
28, p. m.	do	73.72			+16.86	+ 3.6	−0.4

*Readings on the thermometer on *R* 1876 were rejected from July 8 to 31, 1879. Discrepancies in its readings, varying between 0°.4 and 0°.9 were probably due to an air-bubble in its bulb.

IV.—*Comparisons of metres R 1876 and $S_{2.4}$*—Continued.

Date.	Position of metre.	Weighted mean temperature of tube and metre.	Temperature of R 1876 *minus* weighted mean tube-temperature.	No. of observers.	$(t-t_0)$	$R\,1876-S_{2.4}$	Residuals, computed *minus* observed.
1879.		° F.	° F.		° F.	μ	μ
July 30, a. m....	North side....	74.12	....	1	+17.26	+ 2.5	+0.5
30, p. m....	....do	73.19	....	....	+16.33	+ 2.9	+0.5
31, a. m....	....do	73.75	....	....	+16.89	+ 3.2	+0.0
31, p. m...	....do	73.89	....	....	+17.03	+ 2.3	+0.8
56	....	3,184.36	....	....	....	543.0	....
Means......	....	56.86	....	....		+ 9.70	

$(R\,1876-S_{2.4})=+9^{\mu}.70\pm0^{\mu}.15$ at $56^{\circ}.86$ F.
$(E_{R\,1876}-E_{S_{2.4}})=-0^{\mu}.3862\pm0^{\mu}.0101$
Probable error of result from single visit$=\pm1^{\mu}.14$

Collecting these results and reducing them to temperature of 59° F., they are—

$(R\,1876-S_{2.1})$ at $59^{\circ}=+43.92^{\mu}\pm0.21^{\mu}$	$(E_{R\,1876}-E_{S_{2.1}})=-0.3355^{\mu}\pm0.0126^{\mu}$
$(R\,1876-S_{2.2})$ at $59^{\circ}=+\ 4.13\pm0.12$	$(E_{R\,1876}-E_{S_{2.2}})=-0.2896\pm0.0102$
$(R\,1876-S_{2.3})$ at $59^{\circ}=+11.20\pm0.19$	$(E_{R\,1876}-E_{S_{2.3}})=-0.3236\pm0.0108$
$(R\,1876-S_{2.4})$ at $59^{\circ}=+\ 8.87\pm0.15$	$(E_{R\,1876}-E_{S_{2.4}})=-0.3862\pm0.0101$
$(4\,R\,1876-S_2)$ at $59^{\circ}=+68.12\pm0.34$	$(4E_{R\,1876}-E_{S_2})=-1.3349\pm0.0219$

Combining them, there results—

$$S_2=4\,R\,1876-68^{\mu}.12+1^{\mu}.3349\,(t-59)$$

These values, which result directly from the relative expansions, are slightly modified by the adjustment of the expansions given in § 58.

§ **19.** Since S_2 is inclosed in a heavy iron tube, while R1876 in comparisons is freely exposed to the air inside the comparing-box, it might be anticipated that R1876 would be hotter in rising temperatures, giving negative residuals, and the reverse in falling temperatures.

The following table gives the results for adopted temperature-changes of from 0°.3 to 0°.7, in twenty-four hours; from 0°.7 to 1°.1; and from 1°.1 to 1°.5:

Temperature-rise in twenty-four hours	0°.3 to 0°.7 F.	0°.7 to 1°.1	1°.1 to 1°.5
Number of visits	26	11	7
Mean of residuals..	μ +0.38	μ +0.97	μ +0.07
Temperature-fall in twenty-four hours.................	0°.3 to 0°.7	0°.7 to 1°.1	1°.1 to 1°.5
Number of visits..	48	11	7
Mean of residuals	μ +0.07	μ −0.14	μ +0.67

This table includes comparisons with all the metres of S_2. The residuals will be positive when the observed length of R1876 is too small, as would be the case when it was cooler than $S_{2.n}$. An examination of the table does not, on the whole, show any connection between the values of the mean residuals and the values of the temperature-changes in twenty-four hours. The first two columns would seem to indicate that in rising temperatures R1876 was somewhat shorter relatively to the tube-metre than in falling, which is the reverse of what would have been expected, namely, that R1876, heating the faster, would be too long in rising and too short in falling temperatures.

When the metre R1876 is on the north side of the comparing-box it is next the observer's body for about ten minutes, and the question arises whether the heat of the observer's body, penetrating the box, acted more powerfully on R1876 when it was next the observer than when the tube was next the observer. In the following table the first column gives the tube-metre with which R1876 was compared; the second gives the side on which R1876 was; the third gives the number of visits; and the fourth gives the corresponding algebraic mean residual:

Tube-metre.	Position of R 1876.	No. of visits.	Mean residual.
			μ
$S_{2.1}$	North	49	−1.4
$S_{2.1}$	South	46	+1.3
$S_{2.2}$	North	33	+0.4
$S_{2.2}$	South	18	−0.6
$S_{2.3}$	North	27	−0.1
$S_{2.3}$	South	25	+0.0
$S_{2.4}$	North	28	+0.2
$S_{2.4}$	South	28	−0.2

If the observer's body produced a perceptible effect, it would be by heating R1876 most when it was north or next the observer, thus giving a negative residual. The table gives no mean residuals sufficiently large to indicate this, unless in the comparisons of $S_{2.1}$, where they are of the signs that would have been expected, and of considerable magnitude. In the forty-nine residuals for this metre when R1876 was north, seventeen are positive and thirty-two negative; while of the forty-six when R1876 was south, thirty-three were positive and thirteen negative. The distribution of residual-signs indicates, then, like the mean residuals, that the observer affected the temperature of R1876.

An examination of the cases in which the temperatures indicated by the thermometer lying on R1876 differed by 0°.1 F. or more from the weighted mean of the two nearest thermometers inside the tube, has shown that in thirty-six visits out of a total of two hundred and fifty-four the thermometer on R1876 was hottest by 0°.1 F. or more; that the mean excess was 0°.14 F.; that in nineteen of these cases the residual had the positive sign, which would indicate that R1876 was too short; that in seventeen cases the residual had the negative sign; and that the algebraic mean residual was $+ {}^{\mu}.07$.

In eighteen visits the thermometer on R1876 was 0°.1 F. or more cooler than the weighted mean temperature of the thermometers in the tube, the average difference being 0°.12 F., and the corresponding algebraic mean residual being $-0^{\mu}.78$. Of the residuals, seven were positive and eleven negative.

From these results it will be seen that when the thermometer on R1876 was hottest, neither the mean residual nor the preponderance of residuals of one sign indicates any connection between the length of R1876 and its greater thermometer temperature. When the thermometer temperature of R1876 was least, it would be expected that R1876 would be relatively too short; in fact, the mean residual, $-0^{\mu}.78$, indicates that it was too long. But this residual depends largely on a few large negative residuals in the comparisons of R1876 with $S_{2.1}$, which for some reason are anomalous, as has already been stated. From what has been stated, it may be concluded that the residuals do not indicate that any sensible error was introduced by assuming R1876 and the metre of S_2 under comparison as of the same temperature.

LENGTHS OF METRES ON Z_2 IN TERMS OF R1876.

§ **20.** Habitually, whenever R1876 was compared with one of the metres which make up S_2, it was at the same time compared with the corresponding metre on Z_2. These comparisons have been reduced. They were generally made in the same way as those of R1876 and S_2, and the same rules for rejection of comparisons were applied to them, and the method of reduction was the same. A few of the comparisons were made by moving the microscope-car along the tube and comparing the metre on Z_2 with the corresponding one on S_2, whose value was known. Since no other lengths depend on those of the metres on Z_2 it is not deemed necessary to give the results in detail. The following are the means of the observed differences of length of $R1876$ and each metre of Z_2, and the corresponding mean temperatures:

$$Z_{2.1} = R\,1876 + 956^{\mu}.71 \pm 0^{\mu}.33, \text{ at } 53^{\circ}.78 \text{ F.}$$
$$Z_{2.2} = R\,1876 + 415.63 \pm 0.25, \text{ at } 52.21 \text{ F.}$$
$$Z_{2.3} = R\,1876 - 621.79 \pm 0.37, \text{ at } 47.64 \text{ F.}$$
$$Z_{2.4} = R\,1876 - 1067.70 \pm 0.31, \text{ at } 51.60 \text{ F.}$$

The following are the relative expansions for 1° F., derived from the least-square reductions:

$$E_{Z_{2.1}} - E_{R\,1876} = 10^{\mu}.47 \pm 0^{\mu}.021$$
$$E_{Z_{2.2}} - E_{R\,1876} = 10.51 \pm 0.017$$
$$E_{Z_{2.3}} - E_{R\,1876} = 10.85 \pm 0.022$$
$$E_{Z_{2.4}} - E_{R\,1876} = 8.15 \pm 0.018$$

Since some of the metres of Z_2 differed in length at the mean temperature of comparisons by $\frac{1}{1000}$ part of $R1876$, the preceding values of relative expansions would need a correction not exceeding $\frac{1}{500}$ part to make them relative expansions for bars 1^m long.

If the relative expansions be summed, there results—

$$E_{Z_2} - 4\,E_{R\,1876} = 39^{\mu}.98.$$

But from § 58, $E_{S_2} - 4\,E_{R\,1876} = 1^{\mu}.39$. Hence, $E_{Z_2} - E_S = 38^{\mu}.59$, when derived through $R1876$ and the zinc metres of tube 2. The entirely independent determination of this quantity obtained by comparing the whole length of Z_2 with the whole length of S_2, § 26, is

$$E_{Z_2} - E_{S_2} = 38^{\mu}.465$$

The agreement is satisfactory.

The wide variation in the relative expansion of $Z_{2.4}$ from that of the others will be noticed. The comparisons of Z_1 indicate a similar decrease in the rate of expansion at the front end of that bar also. Two theories have been suggested for this variation in the properties of Z_2 at different portions of its length. First, there is great difficulty in casting homogeneous zinc bars of considerable length, and the end of the bar is likely to be imperfect. If Z_1 and Z_2 were sawn from the same casting, they would behave similarly. Second, A. Repsold has suggested that the bars before being rolled to the proper size may have been smaller at one end than at the other, and so may have undergone different amounts of compression.

The mean of the mean temperatures for the comparisons of the different metres of Z_2 is 51°.31. Using the relative expansion for 1° F. to reduce the mean differences of length resulting directly from comparisons to this mean temperature, we have—

$$Z_{2.1} = R1876 + 930^{\mu}.85, \text{ at } 51^{\circ}.31 \text{ F.}$$
$$Z_{2.2} = R1876 + 406^{\mu}.17, \text{ at } 51^{\circ}.31 \text{ F.}$$
$$Z_{2.3} = R1876 - 581^{\mu}.97, \text{ at } 51^{\circ}.31 \text{ F.}$$
$$Z_{2.4} = R1876 - 1070^{\mu}.06, \text{ at } 51^{\circ}.31 \text{ F.}$$

Adding

$$Z_2 = 4R1876 - 315^{\mu}.01, \text{ at } 51^{\circ}.31 \text{ F.}$$

as derived from the separate metres of Z_2. But the length of Z_2 is given from direct comparisons with S_2 in § 26 as

$$Z_2 = S_2 - 305^{\mu}.55 + 38^{\mu}.465\ (t - 49.50)$$

which gives

$$Z_2 = S_2 - 235^{\mu}.93, \text{ at } 51^{\circ}.31 \text{ F.}$$

and, § 60,

$$S_2 - 4\,R1876 = -72^{\mu}.47 + 1^{\mu}.387\ (t - 55.74)$$

whence

$$S_2 = 4R1876 - 78^{\mu}.61, \text{ at } 51^{\circ}.31 \text{ F.}$$

Combining these values at 51°.31 F., there results—

$$Z_2 = 4R1876 - 314^{\mu}.54, \text{ at } 51^{\circ}.31 \text{ F.}$$

which differs but $0^{\mu}.47$ from the value for Z_2 derived from comparisons of the metres $Z_{2.1}$, $Z_{2.2}$, $Z_{2.3}$, $Z_{2.4}$

with R1876. As these determinations of the length of Z_2 are entirely independent of each other, and were made at different times, their close and satisfactory agreement seems to indicate that in considerable periods mean values of lengths on the zinc bar may be obtained which are free from the effects of the large regular residuals for the values of $Z_2 - S_2$, described in § 27.

DETERMINATION OF DIFFERENCES OF LENGTH AND RELATIVE EXPANSIONS OF S_1 AND S_2 AND OF Z_1 AND Z_2, OR OF THE STEEL BARS AND ZINC BARS IN TUBES 1 AND 2 OF THE REPSOLD BASE-APPARATUS.

§ **21.** The comparisons were made inside the comparing-box in the comparing-room. Two microscopes, 4^m apart, were mounted on their brick piers, and the two tubes on their carriage inside the comparing-box were leveled to within 1′ or 2′ and their ends were then alternately brought under the microscopes (previously made vertical within 40″) which were read. The temperatures of the tubes were given by Casella and Geissler thermometers, previously described, inserted one at the middle and one at either end of each tube. In obtaining the mean temperature of a tube, its middle thermometer was given double weight.

The method of leveling the tubes was as follows: The main iron bed-frame of the comparator was adjusted by its leveling screws till one of its rails was, throughout its four metres of length, horizontal within about 1′. The other rail was then so adjusted that a transverse level on the microscope-car changed reading by not more than a minute when the microscope-car was run from end to end of the rails. A microscope was then mounted on the microscope-car and the 0^m and 3^m (or the 1^m and 4^m) marks on the tube were brought into focus, which could be done with a probable error of about $0^{mm}.15$. This made the tube closely parallel to the rail. The microscopes were then mounted on their piers, collimated to within $0^{mm}.05$, made vertical within 40″, and adjusted for focus on the adjusted tube. The ends of the other tube were then run under the microscope and adjusted to their foci.

The order of observations was as follows:

1. Reading of thermometers.
2. Simultaneous readings at the two microscopes on both steel and zinc bars of first tube.
3. Second tube was brought under microscopes and like readings made.
4. First tube was again brought under microscopes, as at first, and read on.
5. Reading of thermometers.

On the steel bars the 0^m and 4^m graduations were always pointed at; on the zinc bars the graduations nearest to those pointed at on the steel were used, but were reduced to the zeros on the zinc bars by means of the known values of the zinc graduations. (See § 72.)

Since, in placing the tubes in the comparator-box, the double door of the comparing-room was usually open for several hours, in order to avoid the effect of irregular temperatures no comparisons were used in reduction which were made within three days after the doors were long open. Subsequent experience indicated that one day would have been quite enough.

In reduction no comparisons have been used where the temperature of the tubes changed more than 2° F. in twenty-four hours. This rejected 11 days of comparisons. No comparisons were used in the reduction where the mean temperatures of tubes, as indicated by mercurial thermometers, differed more than 0°.15 F. This rejected no comparisons. Nor where the ends of either tube differed in temperature by more than 0°.19 F. This rejected two days. No comparisons were used when the length of visit to comparing-room exceeded 30^m. This rejected one day of comparisons.

Since the errors most to be feared are those of temperature, and such errors change little in a day, the mean results of comparisons on any day have received equal weight, whether there was but one or more than one visit to the comparing-room on that day. Usually there were two, one at 8:15 a. m. and one at 4 p. m. or 9 p. m.

Numerous comparisons made prior to October 23, 1878, have not been used in this reduction for the following reason:

Previous to that day it was noticed that the small removable plates on the tubes at 0^m, 1^m, 3^m, and 4^m pressed on the upper surface of the bars at some of these points. On October 11, 1878, when this clamping was first noticed, a pressure of 15 or 20 pounds on the end of the steel bar of tube 2 was insufficient to move it endwise in the tube. At what times previous to this date clamping had existed is unknown for either tube. The plates had been removed and replaced several times since the tubes were received, and the amount of the clamping would at any time depend largely on the tightness with which the plates were screwed down. If but lightly screwed down, it would be small. No positive evidence that the clamping affected the expansion of the bars has been discovered, although in some of the earlier comparisons the greater irregularities in the residuals and some sudden changes might be accounted for by such clamping. Subsequent to October 23, 1878, it has always been practicable to move the bars lengthwise by a pressure of from two to four pounds.

In the reductions of the comparisons of S_1 and S_2 it was assumed that the mercurial thermometers gave the true temperatures of S_1 and S_2, and the length of S_1 was reduced with an approximate value of the expansion to the temperature of S_2, which is taken as the temperature of the comparison. The mean temperature of the comparisons and the corresponding mean value of $S_1—S_2$ were then obtained. This mean temperature was 42°.747 F. and will be represented by t_0. The corresponding mean difference of length was 32$^{\mu}$.09, and this will be designated by $(S_1—S_2)_0$. The observation-equations may then be written in the form

$$(S_1-S_2)_0+(t-t_0)\ (E_{S_1}-E_{S_2})-(S_1-S_2)=v$$

where $(S_1—S_2)$ is any observed difference of length of S_1 and S_2 at the observed temperature t, and v is the residual.

In the following table the first column gives the date of comparison; the second gives the number of comparisons on that day; the third shows whether tube 1 was north or south; the fourth gives the observed temperature; the fifth gives $(t—t_0)$ or excess of observed temperature above the mean of all observed temperatures of comparison, 42°.747 F., $(t—t_0)$ being the coefficient of the unknown $(E_{S_1}—E_{S_2})$ or difference of expansions of the two bars; the sixth gives, with a negative sign, the observed difference $(S_1—S_2)$; and the seventh gives the residuals in the sense computed minus observed.

Results of comparisons of S_1 and S_2.

Date.	No. of comparisons.	Position of tube 1.	Observed temperature.	$(t-t_0)$	$-(S_1-S_2)$	Residuals, computed *minus* observed.
1878.			° F.	° F.	μ	μ
Dec. 30	1	North side..	34.03	− 8.7	−33.2	−0.6
1879.						
Jan. 1	1	....do	34.90	− 7.8	−32.7	−0.1
2	2	....do	35.28	− 7.5	−32.9	−0.4
3	1	....do	35.35	− 7.4	−32.1	+0.4
9	1	South side ..	33.63	− 9.1	−30.8	+1.8
10	2	....do	34.46	− 8.3	−30.6	+2.0
11	2	....do	34.51	− 8.2	−31.1	+1.5
12	1	....do	34.46	− 8.3	−30.8	+1.8
13	2	....do	34.44	− 8.3	−31.0	+1.6
June 26	2	....do	71.20	+28.4	−31.2	−0.8
27	2	....do	72.42	+29.7	−31.6	−1.3
28	1	....do	73.42	+30.7	−32.0	−1.7
29	2	North side..	73.45	+30.7	−29.9	+0.4
30	2	....do	72.95	+30.2	−31.8	−1.5
Oct. 29	2	....do	54.15	+11.4	−29.4	+2.0
30	2	....do	54.02	+11.3	−30.5	+0.9
31	2	....do	53.14	+10.4	−30.2	+1.3
Nov. 1	2	....do	51.71	+ 9.0	−28.4	+3.2
3	1	....do	48.88	+ 6.1	−31.4	+0.3
7	2	....do	42.70	0.0	−32.0	+0.1
8	1	....do	44.28	+ 1.5	−29.7	+2.3
11	2	....do	52.72	+10.0	−30.8	+0.7
12	1	....do	53.94	+11.2	−28.0	+3.4
13	1	....do	55.74	+13.0	−30.6	+0.7

Results of comparisons of S_1 and S_2—Continued.

Date.	No. of comparisons.	Position of tube 1.	Observed temperature.	$(t-t_0)$	$-(S_1-S_2)$	Residuals, computed *minus* observed.
			° F.	° F.	μ	μ
1880.						
Nov. 13	2	North side..	48.62	+ 5.9	−32.7	−1.1
14	2	....do	47.42	+ 4.7	−31.8	0.0
15	2	....do	45.84	+ 3.1	−31.6	+0.3
16	2	....do	44.22	+ 1.5	−31.0	+1.0
17	2	....do	42.52	− 0.2	−32.8	−0.7
20	2	....do	36.53	− 6.2	−32.6	−0.1
21	2	....do	34.44	− 8.3	−30.7	+1.0
23	1	....do	31.29	−11.5	−34.8	−2.0
24	1	....do	31.38	−11.4	−28.4	+4.1
25	2	....do	32.50	−10.2	−32.6	+0.1
26	2	....do	33.08	− 9.7	−30.9	+1.8
28	1	....do	34.58	− 8.2	−32.4	+0.4
29	1	....do	35.33	− 7.4	−33.7	−1.2
30	1	....do	35.93	− 6.8	−35.0	−2.5
Dec. 1	1	....do	36.28	− 6.5	−36.2	−3.7
2	1	....do	36.45	− 6.3	−36.3	−3.8
3	1	....do	37.70	− 5.0	−36.8	−4.5
4	1	....do	37.95	− 4.8	−33.7	−1.3
5	1	....do	39.61	− 3.1	−31.1	+1.2
6	1	....do	38.84	− 3.9	−28.9	+3.4
10	1	....do	31.26	−11.5	−33.3	−0.5
11	1	....do	30.93	−11.8	−33.9	−1.2
14	1	....do	35.13	− 7.6	−33.8	−1.2
15	1	....do	36.79	− 6.0	−32.4	+0.1
16	1	....do	37.90	− 4.8	−34.0	−1.6
17	1	....do	38.38	− 4.4	−36.1	−3.7
18	1	....do	38.47	− 4.3	−33.6	−1.2
19	1	....do	38.20	− 4.6	−35.8	−3.4
20	1	....do	37.88	− 4.9	−31.3	+1.1
21	1	....do	38.13	− 4.6	−31.9	+0.5
54	....	....	2308.36		−1732.8	
	Means..	....	42.747		−32.09	

$(S_1-S_2)_0=+32^{\mu}.09\pm0^{\mu}.18$ $\qquad$ (S_1-S_2) at 32° F. $=+32^{\mu}.74\pm0^{\mu}.24$

$(E_{S_1}-E_{S_2})=-0^{\mu}.0605\pm0^{\mu}.0151$ $\qquad$ (S_1-S_2) at t° F. $=+32^{\mu}.74-0^{\mu}.0605\ (t-32)$

Probable error of result from single visit $=\pm1^{\mu}.30$

§ **22.** The mean temperatures of the two tubes have been compared to see if the tubes heated and cooled at the same rate, the data for such comparison being given in the following table:

No. of days of comparison.	Daily temperature-increase.	Mean temperature, tube 1.	Mean temperature, tube 2.
	° °	°	°
13	+0.6 to +1.0	46.35	46.36
4	−0.6 to −1.0	50.21	50.19
9	+1.1 to +1.5	53.49	53.52
9	−1.1 to −1.5	51.34	51.28
6	+1.6 to +2.0	45.78	45.86
6	−1.6 to −2.0	45.43	45.36

When tubes 1 and 2 were received from Repsold they were very closely identical, but previous to using tube 1 in the field it was covered with a layer of felt 10^{mm} thick, held on by a layer of stout sail-cloth. The table shows, as was to be expected, that the thermometers in tube 2 changed temperature most rapidly in changing temperatures, the difference of temperatures for daily temperature-changes of 1°.6 to 2°.0 reaching 0°.08. A less difference of temperature doubtless existed between S_1 and S_2. Taking all days (seventy-four) of comparison, in the forty-four days of rising temperature as indicated by thermometers, tube 2 was 0°.01 the hotter in mean, while in the thirty days of falling temperature it was 0°.04 the cooler in mean. An examination of the residuals shows that on thirty-three days, when the temperature-increase varied from 0°.3 to 1°.8 F. in twenty-four hours, the mean residual was $-0^{\mu}.32$, being positive on sixteen days and negative on seventeen. On twenty-one days, when the temperature-decrease in twenty-four hours varied between 0°.02 and 2°.2 F., the mean residual was $+0^{\mu}.50$, the residuals being positive on thirteen days and negative on eight. These residuals are small, but so far as they go they indicate that S_2 heats and cools more slowly than S_1. This is the reverse of what would have been expected, and of what is indicated by the thermometers. It may be due to over-correction in reducing S_1 to temperature of S_2. An examination of the residuals shows that the mean residual for tube 1 in its two positions with respect to the observer (who was always on the north side of the comparing-box) was—

	μ
December 30, 1878, to January 3, 1879, four days, tube 1 north.........	−0.18
January 9, 1879, to January 13, 1879, five days, tube 1 south...........	+1.74
June 26, 1879, to June 28, 1879, three days, tube 1 south	−1.27
June 29, 1879, to June 30, 1879, two days, tube 1 north.........	−0.55

The number of days is too small to draw any conclusion. Subsequent to June 30, 1879, tube 1 was always on the north side. An examination of the temperatures and residuals shows that on thirty-nine days the mercurial temperature of tube 1 was the higher by a mean amount of 0°.05 F., the corresponding mean residual being $+0^{\mu}.39$. On twenty-eight days the temperature of tube 1 was the lower, the mean amount being 0°.049. The corresponding mean residual was $-0^{\mu}.63$. Since a positive residual indicates that S_1 was too short, the mean residuals do not correspond in sign to the thermometer indications. But a mean correction of $-0.05\times25^{\mu}=-1^{\mu}.25$ has already been applied to S_1 for the times when it had the higher temperature. The residuals would then indicate that S_1 had been slightly over-corrected, as would be expected, since the thermometers probably changed temperature more rapidly than the bars. The mean residual for the cases in which S_1 was the cooler indicates the same thing, but in both cases the amounts are too small to give certainty. The mean residuals of Z_1-Z_2, given hereafter, have the same relation to the excess of temperature of S_1 over S_2, but are greater in amount, being for the thirty-nine days of rising temperature $+1^{\mu}.68$, and for the twenty-eight days of falling temperature $-2^{\mu}.54$. Since the length of Z_1 has for the observations now under consideration been corrected by about $0.05\times63^{\mu}$, to reduce it to the temperature of Z_2, these residuals, like those of S_1-S_2, indicate that an over-correction was applied, and that in changing temperatures the thermometer difference did not give the correct difference of temperature of the bars by some such quantity as 0°.02 F.

§ **23.** At every comparison of the steel bars S_1 and S_2, in tubes 1 and 2, the zinc bars Z_1 and Z_2 were also compared. The method of reducing the comparisons is the same as for the steel bars. Substituting Z_1 and Z_2 for S_1 and S_2 in the observation-equations for the steel bars, the observation-equations for the zinc bars take the form—

$$(Z_1-Z_2)_0+(t-t_0)(E_{Z_1}-E_{Z_2})-(Z_1-Z_2)=v$$

in which t_0 has the same value as for the steel bars, namely 42°.747 F., and $(Z_1-Z_2)_0=-70^{\mu}.69$.

In the following table the first column gives the date of comparison; the second gives the number of comparisons on that day; the third shows whether tube 1 was north (next observer) or south; the fourth gives the observed temperature; the fifth gives $(t-t_0)$ or excess of temperature of comparison above the mean of all observed temperatures of comparison, namely, 42°.747 F., $(t-t_0)$ being the coefficient of the unknown $(E_{Z_1}-E_{Z_2})$; the sixth gives, with a negative sign, the observed difference (Z_1-Z_2); and the seventh gives the residuals in the sense computed minus observed.

Results of comparisons of Z_1 and Z_2.

Date.	No. of comparisons.	Position of tube.	Observed temperature.	$(t-t_0)$	$-(Z_1-Z_2)$	Residuals, computed *minus* observed.
1878.			° F.	° F.	μ	μ
Dec. 30	1	North side..	34.03	− 8.7	+78.5	+11.6
1879.						
Jan. 1	1	....do	34.90	− 7.8	+82.1	+14.8
2	2	....do	35.28	− 7.5	+83.0	+15.6
3	1	....do	35.35	− 7.4	+80.5	+13.0
9	1	South side ..	33.63	− 9.1	+73.2	+ 6.5
10	2	....do	34.46	− 8.3	+76.8	+ 9.7
11	2	....do	34.51	− 8.2	+72.0	+ 4.9
12	1	....do	34.46	− 8.3	+75.9	+ 8.8
13	2	....do	34.44	− 8.3	+76.6	+ 9.5
June 26	2	....do	71.29	+28.4	+84.2	+ 1.1
27	2	....do	72.42	+29.7	+84.3	+ 0.7
28	1	....do	73.42	+30.7	+83.0	− 1.1
29	2	North side..	73.45	+30.7	+82.6	− 1.5
30	2	....do	72.95	+30.2	+82.2	− 1.7
Oct. 29	2	....do	54.15	+11.4	+78.6	+ 2.9
30	2	....do	54.02	+11.3	+80.2	+ 4.6
31	2	....do	53.14	+10.4	+78.1	+ 2.9
Nov. 1	2	... do	51.71	+ 9.0	+79.0	+ 4.4
3	1	....do	48.88	+ 6.1	+71.2	− 2.2
7	2	....do	42.70	+ 0.0	+65.9	− 4.8
8	1	....do	44.28	+ 1.5	+71.9	+ 0.5
11	2	....do	52.72	+10.0	+73.6	− 1.5
12	1	....do	53.94	+11.2	+75.6	0.0
13	1	....do	55.74	+13.0	+74.3	− 2.1
1880.						
Nov. 13	2	....do	48.62	+ 5.9	+76.8	+ 3.5
14	2	....do	47.42	+ 4.7	+77.5	+ 5.1
15	2	....do	45.84	+ 3.1	+75.4	+ 3.3
16	2	....do	44.22	+ 1.5	+75.0	+ 3.6
17	2	....do	42.52	− 0.2	+70.6	0.0
20	2	....do	36.53	− 6.2	+65.2	− 2.9
21	2	....do	34.44	− 8.3	+61.4	− 5.7
23	1	....do	31.29	−11.5	+60.4	− 5.3
24	1	....do	31.38	−11.4	+74.5	+ 8.8
25	2	....do	32.50	−10.2	+65.9	− 0.4
26	2	....do	33.08	− 9.7	+69.3	+ 2.8
28	1	....do	34.58	− 8.2	+63.2	− 3.3
29	1	....do	35.33	− 7.4	+65.2	− 2.3
30	1	....do	35.93	− 6.8	+60.7	− 7.0
Dec. 1	1	....do	36.28	− 6.5	+58.4	− 9.5
2	1	....do	36.45	− 6.3	+60.5	− 7.5
3	1	....do	37.70	− 5.0	+58.0	−10.6
4	1	....do	37.95	− 4.8	+61.5	− 7.1
5	1	....do	39.61	− 3.1	+68.9	− 0.4
6	1	....do	38.84	− 3.9	+60.3	− 8.7
10	1	....do	31.26	−11.5	+57.2	− 8.5
11	1	....do	30.93	−11.8	+60.3	− 5.3
14	1	....do	35.13	− 7.6	+61.7	− 5.7
15	1	....do	36.79	− 6.0	+70.0	+ 1.9
16	1	....do	37.90	− 4.8	+63.6	− 5.0
17	1	....do	38.38	− 4.4	+61.2	− 7.6
18	1	....do	38 47	− 4.3	+62.4	− 6.4
19	1	....do	38.20	− 4.6	+61.0	− 7.7
20	1	....do	37.88	− 4.9	+65.9	− 2.7
21	1	....do	38.13	− 4.6	+62.3	− 6.4
54	....	....	2308.36	....	+3817.5	
	Means..	....	42.747	....	+70.69	

$(Z_1-Z_2)_0=-70^{\mu}.69\pm0^{\mu}.60$ (Z_1-Z_2) at 32° F. $=-66^{\mu}.00\pm0^{\mu}.31$

$(E_{Z_1}-E_{Z_2})=-0^{\mu}.4366\pm0^{\mu}.0513$ (Z_1-Z_2) at t° F. $=-66^{\mu}.00-0^{\mu}.4366\ (t-32)$

Probable error of result from single visit $=\pm4^{\mu}.42$

§ **24.** An examination of the residuals shows that on thirty-three days, when the temperature-increase in twenty-four hours varied between 0°.03 and 1°.8 F., the residuals were positive on thirteen days, giving a sum of residuals of $85^\mu.4$; and negative on twenty days, giving a residual sum of $-98^\mu.7$. For the thirty-three days of rising temperature the algebraic mean residual was $-0^\mu.40$. On twenty-one days the temperature-decrease in twenty-four hours varied between 0°.02 and 2°.2 F. On twelve days the residuals were positive, giving in sum $+55^\mu.1$; on nine days they were negative, giving in sum $-4[illegible]^\mu.2$. The algebraic mean residual for twenty-one days of falling temperature was $+0^\mu.60$. The residuals for the zinc bars change in the same way, and by nearly the same amounts in increase and decrease of temperature as the steel bars. But these mean residuals are but about $\frac{1}{8000000}$ part of the length of the bars, and are too small to make any certain connection between their values and the sign of the temperature-change.

If the residuals of S_1-S_2 be examined, it will be seen that there are eleven residuals numerically greater than 2^μ, while the probable error of one day's comparisons is but $1^\mu.3$. The residuals of Z_1-Z_2 had the same sign as those of S_1-S_2, except on one of the eleven days.

Omitting the residuals of both steel and zinc bars for this one day, the average S_1-S_2 residual for the remaining days, regardless of sign, was $3^\mu.1$, while that of Z_1-Z_2 was $6^\mu.3$, or the residuals had similar signs and a ratio nearly that of the expansion of zinc to the expansion of steel. This points strongly for these days to a difference between the true temperatures of the bars in the tubes and those given by the thermometers, an erroneous temperature of one of the tubes of $\frac{3^\mu.1}{2^\mu} \times 1°$ F., or about 0°.12, accounting for the mean of the large residuals.

In the zinc-bar comparisons from December 30, 1878, to January 3, 1879, there are extremely large residuals, amounting to $\frac{1}{250000}$ part of the length of the bars, while in the comparisons of the steel bars at the same time the residuals are very small. Nothing is known that throws any light on the cause of this discrepancy. If these four comparisons were rejected, the sum of the squares of the residuals would be reduced about one-third, and the probable error of one day of comparison, now very large ($4^\mu.4$), would be reduced. Since the probable error of pointing of one of the microscopes is only $0^\mu.4$, and but four pointings are required for a comparison, the probable error from this cause would be for a comparison but $0^\mu.8$. The fact that the comparisons of the steel bars give for the probable error of one day's comparison of the same character and at the same time as those of the zinc bars and including temperature and all other errors, but $1^\mu.3$, indicates that the probable error of comparison due to errors of measurement alone can little exceed $1^\mu.0$, and that the rest of the probable error in the difference Z_1-Z_2 must be due to changes in that difference from temperature or other causes.

The range in the residuals of S_1-S_2 in § 21 is $7^\mu.9$. If this range were due largely to differences of temperature of the two tubes, since the expansion of Z_2 is about 2.5 times that of S_2, residuals of $2.5\times 7^\mu.9=20^\mu$ would be expected in Z_1-Z_2. If the days from December 30, 1878, to January 3, 1879, be omitted in the comparisons of Z_1 and Z_2, the range of the residuals is reduced from $26^\mu.2$ to $21^\mu.2$. It therefore is probable that the large residuals are mainly due to differences of temperature of Z_1 and Z_2.

Collecting the values found, they are

$$(S_1-S_2) \text{ at } 42°.747 \text{ F.} = +32^\mu.09 \pm 0^\mu.18$$
$$E_{S_1}-E_{S_2} = -0^\mu.0605 \pm 0^\mu.0151$$
$$(S_1-S_2) \text{ at } t° = +32^\mu.74 \pm 0^\mu.24 - (0^\mu.0605 \pm 0^\mu.0151)(t-32)$$
$$(Z_1-Z_2) \text{ at } 42°.747 \text{ F.} = -70^\mu.69 \pm 0^\mu.60$$
$$E_{Z_1}-E_{Z_2} = -0^\mu.4366 \pm 0^\mu.0513$$
$$(Z_1-Z_2) \text{ at } t° = -66^\mu.00 \pm 0^\mu.81 - (0^\mu.4366 \pm 0^\mu.0513)(t-32)$$

If the value of S_2 in terms of R1876, depending solely on its comparisons with R1876, given in § 18, be substituted in the value just given for S_1-S_2, there results, after proper reduction,

$$S_1 = 4\,R1876 - 37^\mu.01 + 1^\mu.2744\,(t-59)$$

The square of the probable error of this value of S_1, excluding the probable error of R1876, is, with 1^μ as unit,

$$0.1480 + 0.000480\,(t-59)^2 + 0.000228\,(t-42.75)^2$$

DIFFERENCE OF LENGTH AND RELATIVE EXPANSION OF ZINC AND STEEL BARS IN TUBE 2 OF REPSOLD BASE-APPARATUS.

§ **25.** These bars are designated respectively by Z_2 and S_2. A description of them and of the graduation-lines, about 4 metres apart, which limit the parts considered, may be found in Chapter VIII, § 3.

Tube 2 has been repeatedly compared at widely differing temperatures, both with tube 1 and with the 15-feet brass bar of the Lake Survey. Such comparisons were made in the comparing-room of the Lake-Survey office, where the daily temperature-change, as indicated by mercurial thermometers in tube 2, rarely exceeded 2° F. Among other observations, those comparisons included the reading with microscopes of the distance between contiguous graduations on Z_2 and S_2 at both ends of the bars, and also the reading of three of the mercurial thermometers, whose errors are given in Chapter II, §§ 15–19, Geissler thermometers being inserted in the tube at its ends and a Casella thermometer at its middle. These microscope- and thermometer-readings for tube 2 complete the observations so far as the present investigation is concerned. In each visit to the comparing-room the thermometers were read on entering and again on leaving the room, about fifteen minutes later. The room was not entered again till the next set of observations was made. In the majority of the observations used in reduction, but one visit to the comparing-room was made in a day, to avoid irregular temperatures, but in some cases the number of visits rose to three. In reduction, however, the mean result for the day was used and the same weight was attributed to it whether it was derived from three or fewer visits in that day, the errors to be feared from temperature-differences being much larger than those of observation.

After a preliminary reduction, which gave approximate values for the difference of length of Z_2 and S_2 at a mean temperature, and for their relative expansion, the residuals of the observations were computed and plotted with dates as abscissæ. Below this curve the mercurial temperatures of the tube were plotted with the same abscissæ. A comparison of the two curves showed at once that, whenever the temperature of the tube rose, the residuals (which were computed values of Z_2-S_2 minus observed values) increased algebraically with the temperatures, and *vice versa*. This preliminary reduction, in which all comparisons prior to its date were included, showed that the residual errors increased as the temperature-change increased, yet never exceeded about 25^{μ}; that a temperature-change of about 10° or 15° F. would produce this amount of change in the residuals, and that a further temperature-change did not increase it; that the rate of residual-change was approximately 2^{μ} per degree of temperature-change. The closeness with which each fluctuation of any importance in the temperature-curve was repeated in the curve of residuals made it certain that the large residuals were not due to accidental errors of comparisons, but were actually measured quantities.

In the earlier comparisons of Z_2 and S_2 in 1877 it is possible that the two bars were not perfectly free to expand with reference to each other, although the comparisons themselves do not give any positive indication of it. For this reason these comparisons have not been used in the final reductions. With this uncertainty it may be said that the comparisons of 1877 and those of 1881 show that Z_2 has retained unchanged its length with reference to S_2 within the limits of uncertainty due to the large regular residual errors. Thus in a series of comparisons on twenty-four days between October 22, 1877, and November 17, 1877, the mean temperature of the comparisons being 61°.98, the mean value of Z_2-S_2 was $+177^{\mu}.4$, while the value for this temperature resulting from the final reduction given immediately hereafter is $174^{\mu}.5$. The difference is too small to indicate change of length, but may be attributed entirely to the cause which gives large regular residuals for Z_2-S_2.

If, then, Z_2-S_2 does not permanently change its value at any temperature, but fluctuates in the results of comparisons, from some cause, about a mean value, the question is as to the method of obtaining a mean value for Z_2-S_2 at some temperature, and a mean relative expansion. If we had a long period of daily comparisons at low temperatures fluctuating about a mean temperature nearly the same for the whole period, and a similar long series for high temperatures, we might hope that from such series mean values of Z_2-S_2 for the high and low temperatures and a mean relative expansion might be obtained which should have a good degree of accuracy. This idea governed in the selection of comparisons to be used in the final reductions.

It has already been stated that when the Repsold base-apparatus was received from its maker in November, 1876, it was noticed that removable plates at 0^m, 1^m, 3^m, and 4^m, in the tops of the tubes, might be screwed down so tightly as to clamp the steel and zinc bars against their supporting rollers, and thus prevent their free expansion. This clamping was first noticed on October 12, 1878, at which time the plates were screwed down so tightly that a pressure of 8 or 10 kilogrammes on the ends of the steel and zinc bars in tube 2 would not move them in the tube. The plates were loosened, and they then moved under a pressure of 1 kilogramme. A stress of 1 kilogramme should elongate S_2 by $0^{\mu}.5$ and Z_2 by $1^{\mu}.1$. These plates had been repeatedly taken off and replaced before this date. The prior comparisons, which were numerous, give no certain evidence that clamping existed at any time. If it did exist, then in expansion, since the zinc bar has a greater expansion than the steel bar and the cast-iron tube which have nearly equal expansions, the zinc bar should be compressed and relatively too short. On passing a maximum temperature, a few tenths of a degree of fall in temperature should entirely remove this compression, and the residual curve of Z_2-S_2 should show an abrupt jump at this point. In none of the many maxima observed does this occur, so that it is not probable that clamping existed for any long period. But as the other comparisons of Z_2 and S_2 are very numerous, none prior to October 12, 1878, have been used in the final reductions.

§ **26.** It has been stated that to get a mean value for Z_2-S_2 and the relative expansion, two long series of comparisons with temperatures fluctuating about steady general mean temperatures are desirable, one of these mean temperatures being high and the other low. The comparisons give an excellent low-temperature series of this character, extending from November 25, 1879, to March 17, 1880, with temperatures between 38° F. and 50° F., and including comparisons on ninety days. No such favorable high-temperature series can be selected from the observations, and therefore all comparisons at temperatures above 55° F. not rejected by the following conditions have been used for the high-temperature series. The condition that no comparisons should be used in which the thermometers indicated a difference of temperature of the ends of tube 2, exceeding 0°.2 F., rejected thirteen days; that when the mean temperature of tube 2 differed by more than 0°.15 F. from the temperature of tube 1 or from that of the brass 15-feet bar, if it was being compared with the latter, rejected four days; and three days were rejected because the observers were in the comparing-room for more than forty minutes. These conditions rejected so many of the group of comparisons between January 6 and 22, 1881, as to leave no good distribution of the remaining comparisons with reference to the temperature-changes; hence all were rejected. There remained after these rejections one hundred and fifty days of comparisons.

In some cases thermometers were read on entering and on leaving the comparing-room. The first reading was taken, the second being treated as a check. A visit usually occupied about fifteen minutes, and in the majority of days there was but one visit a day to the comparing-room. If more than one comparison was made on a day, the mean of the results and of the temperatures was taken for that day and used in the observation-equations. The form of the observation-equation is

$$(Z_2-S_2)_0+(t-t_0)\,(E_{Z_2}-E_{S_2})-(Z_2-S_2)=v,$$

where (Z_2-S_2) is the mean difference of lengths of Z_2 and S_2 resulting from one day's comparisons; $(Z_2-S_2)_0$ is the mean of all such quantities; t is the mean temperature of tube 2 for a day's comparisons; t_0 is the mean of all such temperatures, and v is the residual error in the sense computed (Z_2-S_2) minus observed (Z_2-S_2). The value of t_0 is 49°.504 F., and of $(Z_2-S_2)_0$ is $-305^{\mu}.6$.

In the following table the first column gives the date of the comparison; the second shows when tube 2 was north (next observer); the third, the mean temperature of tube 2; the fourth, the mean of the observed values of Z_2-S_2 for that day; and the fifth, the residuals. The results of the least-square reduction follow the tables.

Comparisons of the steel and zinc bars of tube 2.

Date.	Position of tube 2.	Mean temperature of tube 2.	Mean (Z_2-S_2) for each day.	Residuals.
1879.		° F.	μ	μ
June 26	North side	71.20	+ 514.1	+14.89
27	....do	72.42	+ 560.0	+15.91
28	....do	73.41	+ 598.6	+15.39
29	South side	73.44	+ 601.2	+13.95
30	....do	72.95	+ 588.3	+ 8.00
Nov. 15	South side	59.68	+ 71.6	+14.27
16	....do	58.76	+ 41.2	+ 9.29
17	....do	57.27	− 13.5	+ 6.67
18	....do	55.38	− 83.5	+ 3.97
Nov. 25	South side	42.15	− 580.6	− 7.82
26	....do	42.30	− 577.0	− 5.65
27	....do	43.01	− 552.2	− 3.14
28	....do	44.58	− 497.2	+ 2.25
29	....do	45.61	− 456.7	+ 1.37
30	....do	45.05	− 476.6	− 0.27
Dec. 1	....do	44.82	− 487.6	+ 1.88
2	....do	44.92	− 483.4	+ 1.53
3	....do	45.70	− 455.8	+ 3.93
4	....do	46.43	− 426.2	+ 2.41
5	....do	47.22	− 398.8	+ 5.40
6	... do	48.46	− 350.7	+ 4.99
7	....do	49.34	− 318.8	+ 6.94
8	....do	49.22	− 323.3	+ 6.83
9	....do	48.86	− 338.0	+ 7.68
10	....do	49.35	− 317.2	+ 5.73
11	....do	48.74	− 339.6	+ 4.66
13	....do	45.01	− 473.8	− 4.61
14	....do	43.60	− 529.3	− 3.35
15	....do	43.26	− 542.1	− 3.62
16	....do	42.04	− 585.4	− 7.25
17	....do	40.94	− 627.3	− 7.66
18	....do	40.13	− 656.6	− 9.52
19	....do	39.79	− 667.0	−12.20
22	....do	39.46	− 685.8	− 6.09
23	....do	39.76	− 675.5	− 4 85
24	....do	40.14	− 660.2	− 5.53
25	....do	40.54	− 645.5	− 4.85
26	....do	40.42	− 649.8	− 5.16
27	... do	39.52	− 684.1	− 5.48
29	....do	38.82	− 707.6	− 8.91
30	....do	39.41	− 689.8	− 4.01
31	....do	39.72	− 676.4	− 5.49
1880.				
Jan. 1	....do	40.22	− 657.8	− 4.86
2	....do	40.74	− 642.3	− 0.36
3	....do	41.29	− 620.2	− 1.30
5	....do	43.31	− 548.8	+ 5.00
7	....do	44.94	− 489.2	+ 8.10
8	....do	45.29	− 476.0	+ 8.36
9	....do	45.91	− 455.7	+11.91
10	....do	46.96	− 416.7	+13.30
12	....do	47.75	− 383.1	+10.08
13	....do	47.45	− 395.3	+10.74
14	....do	46.75	− 419.0	+ 7.52
16	....do	45.56	− 465.7	+ 8.44
17	....do	45.58	− 461.8	+ 5.31
19	....do	46.60	− 424.7	+ 7.45
20	....do	46.95	− 413.6	+ 9.81
21	....do	46.93	− 414.1	+ 9.54
22	....do	46.58	− 426.4	+ 8.38

Comparisons of the steel and zinc bars of tube 2—Continued.

Date.	Position of tube 2.	Mean temperature of tube 2.	Mean (Z_2-S_2) for each day.	Residuals.
1880.		° F.	μ	μ
Jan. 23	South side	46.45	− 432.2	+ 9.18
26	....do	45.76	− 456.4	+ 6.84
27	....do	45.88	− 452.2	+ 7.25
28	....do	46.46	− 432.1	+ 9.46
29	....do	46.53	− 424.1	+ 4.16
30	....do	45.71	− 455.0	+ 3.51
31	....do	45.58	− 458.1	+ 1.61
Feb. 2	....do	43.61	− 531.4	− 0.86
3	....do	42.04	− 589.4	− 3.25
6	....do	40.38	− 651.5	− 5.06
7	....do	40.14	− 661.2	− 4.53
9	....do	40.02	− 663.5	− 6.85
10	....do	39.76	− 677.1	− 5.56
11	....do	39.97	− 664.8	− 7.47
13	....do	42.06	− 591.2	− 0.68
16	....do	43.23	− 546.2	− 0.68
17	....do	43.66	− 534.9	+ 4.56
18	....do	44.72	− 492.0	+ 2.43
19	....do	45.63	− 461.0	+ 6.44
20	....do	44.26	− 511.0	+ 3.74
21	....do	43.23	− 551.7	+ 4.82
24	....do	42.86	− 564.6	+ 3.49
25	....do	43.41	− 542.0	+ 2.05
26	....do	44.96	− 480.5	+ 6.17
27	....do	46.12	− 442.6	+ 6.89
28	....do	47.18	− 404.0	+ 9.06
Mar. 1	....do	45.17	− 478.1	+ 5.84
2	....do	43.70	− 530.4	+ 1.60
3	....do	43.68	− 530.7	+ 1.13
4	....do	44.66	− 496.2	+ 4.33
5	....do	45.95	− 447.3	+ 5.05
6	....do	45.58	− 465.0	+ 8.51
8	....do	45.10	− 480.7	+ 5.75
9	....do	43.53	− 538.0	+ 2.66
10	....do	42.13	− 587.1	− 2.69
11	....do	40.96	− 628.4	− 5.79
12	....do	40.41	− 647.2	− 8.15
13	....do	39.46	− 685.2	− 6.69
15	....do	38.14	− 733.3	− 9.36
17	....do	37.92	− 739.2	−11.93
July 10	South side	75.91	+ 704.2	+ 5.95
11	....do	75.83	+ 701.4	+ 5.67
12	....do	76.11	+ 713.0	+ 4.84
13	....do	76.72	+ 734.4	+ 6.91
14	....do	77.27	+ 754.8	+ 7.66
15	....do	77.36	+ 760.0	+ 5.93
July 22	North side	68.97	+ 448.1	− 4.89
23	....do	68.48	+ 431.7	− 7.34
24	....do	68.53	+ 433.1	− 6.82
25	....do	68.94	+ 448.9	− 6.85
26	....do	69.80	+ 477.6	− 2.47
27	....do	70.68	+ 509.0	− 0.02
28	....do	70.91	+ 521.3	− 3.47
29	....do	70.61	+ 508.4	− 2.11
30	....do	70.26	+ 495.4	− 2.57
31	....do	70.19	+ 492.2	− 2.07
Aug. 1	....do	70.56	+ 505.6	− 0.64
2	....do	71.13	+ 524.3	+ 1.99
Sept. 24	South side	63.70	+ 251.5	−11.00
25	....do	63.49	+ 244.3	−11.88

Comparisons of the steel and zinc bars of tube 2—Continued.

Date.	Position of tube 2.	Mean temperature of tube 2.	Mean (Z_2-S_2) for each day.	Residuals.
1880.		° F.	μ	μ
Sept. 26	South side	63.98	+ 261.0	− 9.73
27	... do	64.59	+ 284.1	− 9.37
28	....do	64.08	+ 266.7	−11.59
29	....do	62.81	+ 220.4	−14.14
30	....do	61.01	+ 156.6	−19.57
Oct. 1	North side	59.66	+ 103.8	−18.70
2	....do	59.39	+ 93.2	−18.49
3	....do	59.62	+ 101.0	−17.44
4	....do	59.82	+ 110.0	−18.75
5	....do	59.38	+ 91.6	−17.27
6	....do	58.97	+ 73.8	−15.24
7	....do	58.20	+ 48.8	−19.86
8	....do	57.73	+ 29.0	−18.14
9	....do	57.60	+ 23.7	−17.84
Dec. 4	South side	37.95	− 753.8	+ 3.83
5	....do	39.61	− 690.9	+ 4.78
6	....do	38.84	− 724.3	+ 8.56
7	....do	36.73	− 797.1	+ 0.20
8	....do	34.18	− 889.1	− 5.88
9	... do	32.02	− 970.1	− 7.97
10	....do	31.26	− 999.6	− 7.70
11	....do	30.93	−1010.3	− 9.69
13	....do	33.39	− 920.6	− 4.77
14	....do	35.13	− 860.4	+ 1.96
15	....do	36.79	− 796.7	+ 2.11
16	... do	37.90	− 759.2	+ 7.30
17	....do	38.38	− 743.1	+ 9.67
18	... do	38.47	− 737.7	+ 7.73
19	....do	38.20	− 752.4	+12.04
20	... do	37.88	− 763.8	+11.14
21	....do	38.13	− 754.1	+11.05
150		7425.66	−45832.7	+ 2.61
	Means	49.504	−305.551	

$$(Z_2-S_2) = -305^{\mu}.551 \ \pm 0^{\mu}.463 \text{ at } 49^{\circ}.504 \text{ F.}$$
$$(E_{Z_2}-E_{S_2}) = + \ 38^{\mu}.4648 \pm 0^{\mu}.0388 \text{ for } 1^{\circ} \text{ F.}$$
$$(Z_2-S_2) = -978^{\mu}.857 \ \pm 0^{\mu}.823 \text{ at } 32^{\circ} \text{ F.}$$

Probable error of result from single day's observations $= \pm 5^{\mu}.67$.

§ **27.** The connection between the changes in the residuals and those in the temperatures is best shown graphically. Accordingly, in Plate XXIV the temperatures and residuals have been plotted with times as abscissas for the long series of comparisons from November 25, 1879, to March 17, 1880. An examination of the curves shows at once the closest connection between the temperature-changes and the residual-changes, and that there is some regular cause other than accidental errors of observation for these residuals, which are often large. The other residuals, not plotted in Plate XXIV, are for shorter periods of comparisons, but their indications are the same. In general, all show that when the temperature rises, the observed value of $Z_2—S_2$ diminishes with reference to the computed value at the rate of about 2^{μ} per degree Fahrenheit, or Z_2 becomes relatively shorter than was to be expected. In falling temperatures the reverse is the case.

§ **28.** A number of theories to account for the phenomenon have been examined.

A. The graduation-lines pointed at on the ends of Z_2 and S_2 are in the horizontal plane through the neutral axes of the bars; but to bring their ends closely together for microscopic reading at the same time, the tubes are so made that the portions of these lines pointed at are 8^{mm} distant from the vertical planes through the neutral axes of the two bars. If, then, the bars bend laterally, the lengths of Z_2 and S_2 fixed by these lines will change by $2^{\mu}.3$ for a change of one minute in the relative inclination in a horizontal plane of the ends of a bar. Now, the zinc bar, Z_2, has a large expansion relatively to the steel bar, S_2, and is supported by rollers and pins carried by S_2 at $0^m.5$, $1^m.5$, $2^m.5$, and $3^m.5$. If Z_2 expands and is in some degree constrained by friction in its free motion with reference to S_2, it would tend, if laterally free to bend, to become concave towards S_2, which would tend to become convex towards Z_2, so that the distance between the graduation-marks on Z_2, which lie between its neutral axis and S_2, will be less than if measured on its neutral axis, and Z_2 will become too short with reference to S_2 in rising temperatures. Possibly this effect might be aided by the rollers with vertical axes which confine the two bars between them at each metre of the length of the bars. Should the temperature fall the reverse effect would take place. But this supposes that Z_2 and S_2 are free to bend laterally. In fact, at each metre they are confined by rollers which do not allow $0^{mm}.1$ play. The question as to the amount of the effect of bending has been examined in two ways. In the first place, in all the work since October 12, 1878 (and only this has been used), it has been made certain that the bars in the tubes moved freely in the tube under an end-pressure of one or two kilogrammes. Since a stress of 1^{kg} extends Z_2 by only $1^{\mu}.1$ and S_2 by $0^{\mu}.5$, no serious errors are to be feared from direct compression or extension without bending. The end metres of each bar are under a slight constraint from friction of vertical rollers confining them laterally at 0^m and 1^m, and at 3^m and 4^m. Under this slight resistance to expansion (about $\frac{1}{2}^{kg}$) may the end zinc metres bend outward slightly? Computation gave no appreciable bending effect for such a force of compression. The zinc metres of Z_2 are supported at their middles on thin rollers 28^{mm} in diameter, carried by horizontal pins projecting from S_2. May the friction of the end zinc metres in expansion on these rollers or pins slightly bend the steel pins which will carry with them the guiding-wheels turning upon them? In this case, if the zinc bar were in contact with the edge of the wheel at the height of the center of the wheel, before the bending of the pin, the bending would, by the pressure of the edge of the wheel on the zinc bar in expansion, press the middle of the zinc bar away from the steel bar and change the azimuth of the end of the zinc bar. Computation indicates that a push of 1^{kg} on this pin at $0^m.5$ could not shorten Z_2 by so much as 3^{μ} in this way. One metre of the zinc bar weighs $2^{kg}.5$. An attempt was made to bend the rear-end zinc metre at its middle slightly away from the steel bar. No change in the difference of readings on Z_2 and S_2 could be observed. These theoretical estimates of possible errors, especially of the last, are not very precise, on account of the uncertainty of the data used.

§ **29.** The second method of examining the question of bending of Z_2 and S_2 when expanding and contracting was as follows: Such slight bendings as those now considered, even if they actually existed, would not change the length of the line joining the ends of the neutral axis of either the steel or zinc bars by quantities that need to be considered. If, then, lines be drawn at right angles to the neutral axis at each end of each bar, the interval between them in the vertical plane through the neutral axis may be taken as invariable so far as horizontal bending is concerned, and the comparisons of this interval on a bar with the corresponding interval on the bar used in measuring (called S_1, S_2, Z_1, or Z_2) will give the changes in the values of the latter. Accordingly, such lines were graduated on the ends of the four bars, approximately 4^m apart. The graduations were successfully ruled with a diamond by Assistant Engineer E. S. Wheeler on the small steel plates which carry the platinum plates on which are the ordinary graduations. Unfortunately, the portions of the steel plates nearest the neutral axes of the bars are about 1^{mm} lower than the platinum plates carrying the ordinary graduations. The microscopes had, therefore, to be focused for an intermediate height, $0^{mm}.5$ from either surface, and so a little outside the range of distinct vision, which is about $0^{mm}.4$ on each side of perfect focus. This rendered the errors of pointing much larger than in ordinary work, giving ranges of 4^{μ} in 10 pointings. The 4^m intervals which are used in measuring being denoted by S_1, Z_1, &c., these new 4^m intervals will be denoted by S_1', Z_1', S_2', Z_2'. Strictly speaking, these new intervals are in the vertical plane through the neutral axis and about $1^{mm}.0$ from it, so that they are slightly above the neutral axis. But it is horizontal bending that is feared, and the lengths of the new intervals will not be affected by that. Indeed, their lengths at 1^{mm} distance in a vertical plane from the neutral axis would not be sensibly affected by any probable bending.

The following table gives the dates of comparisons of intervals of about 4^m in the neutral axes of steel and zinc bars of tube 2 with the intervals on the same bars, also of 4 metres used in base measuring; the temperatures; the observed differences of length of S_1 and S_2; Z_1 and Z_2; S_2 and S_2'; Z_1 and Z_2'; and the residuals for $Z_1 - S_1$ and $Z_2 - S_2$.

Synopsis of results of comparisons of tubes 1 *and* 2 *in the comparing-room Nov.* 10 *to Dec.* 21, 1880.

Date.	Mercurial temperature.		S_1-S_2	Z_1-Z_2	S_2-S_2'	Z_2-Z_2'	Computed (Z_1-S_1) *minus* observed (Z_1-S_1).	Computed (Z_2-S_2) *minus* observed (Z_2-S_2).
	Tube 1.	Tube 2.						
1880.	° F.	° F.	μ	μ	μ	μ	μ	μ
Nov. 10, 3:26—3:58 p. m	51.79	51.84	+32.7	−79.1	....	....	− 1.6	− 5.2
11, 3:08—3:32 p. m	52.58	52.65	+26.7	−89.9	+778.2	+653.2	− 1.6	− 9.0
12, 9:16—9:38 a. m	50.98	50.90	+35.0	−70.3	+777.8	+655.7	−11.6	−14.2
12, 8:18—8:35 p. m	49.86	49.89	+35.1	−72.7	+776.9	+655.8	−10.5	−11.7
13, 9:43—9:56 a. m	48.96	48.93	+35.3	−72.7	+777.4	+656.1	−11.6	−15.7
13, 8:50—9:10 p. m	48.37	48.31	+32.4	−75.4	+782.7	+661.4	−12.0	−17.3
14, 9:55—10:15 a. m	47.75	47.71	+30.5	−78.5	+779.9	+655.5	−11.0	−16.8
14, 8:22—8:36 p. m	47.17	47.15	+34.4	−72.8	+776.5	+659.2	−16.4	−19.9
15, 9:13—9:32 a. m	46.43	46.38	+33.8	−73.2	+781.0	+656.4	−12.9	−17.6
15, 8:27—8:41 p. m	45.42	45.29	+33.6	−67.0	+779.5	+657.0	−17.3	−19.2
16, 9:05—9:21 a. m	44.48	44.42	+34.9	−71.5	+781.9	+657.9	−16.4	−21.8
16, 8:38—9:00 p. m	44.07	44.01	+30.1	−71.1	+774.6	+658.6	−20.1	−20.4
17, 9:06—9:21 a. m	43.10	42.98	+34.5	−63.3	+779.4	+656.7	−20.8	−20.3
17, 8:30—8:43 p. m	42.15	42.05	+36.3	−64.9	+781.2	+663.1	−16.4	−18.9
Means	....	....	....	....	+779.0	+657.4	−12.9	−16.3
Nov. 18, 9:08—9:20 a. m	40.84	40.74	+36.8	−57.6	+778.8	+660.4	−25.0	−21.2
18, 8:31—8:44 p. m	39.51	39.37	+36.2	−51.3	+778.4	+664.1	−30.0	−21.4
19, 9:17—9:36 a. m	38.10	37.99	+39.1	−54.7	+780.5	+664.8	−26.4	−23.5
19, 8:28—8:53 p. m	37.52	37.43	+37.7	−56.1	+778.2	+665.2	−23.6	−20.1
20, 9:02—9:18 a. m	36.87	36.80	+38.7	−58.9	+779.3	+664.2	−25.2	−25.0
20, 8:24—8:47 p. m	36.38	36.27	+30.7	−60.2	+776.1	+661.3	−23.0	−17.9
21, 10:04—10:20 a. m ...	35.15	35.06	+31.5	−55.4	+777.1	+663.4	−28.5	−19.0
21, 8:30—8:48 p. m	33.98	33.84	+35.9	−52.7	+781.4	+660.7	−24.9	−19.6
22, 9:12—9:36 a. m	32.78	32.66	+34.4	−53.0	+780.3	+663.4	−24.2	−17.3
22, 8:26—8:41 p. m	31.96	31.86	+35.4	−58.0	+777.8	+666.5	−24.1	−22.8
23, 9:18—9:36 a. m	31.33	31.29	+35.8	−57.9	+778.0	+668.6	−23.2	−20.0
24, 9:18—9:47 a. m	31.39	31.38	+28.9	−71.5	+781.3	+659.2	−16.0	−18.3
25, 9:32—9:57 a. m	32.42	32.38	+34.3	−66.9	+779.3	+670.4	− 9.4	−13.3
Means	....	....	....	....	+779.0	+664.0	−23.3	−20.0
Nov. 25, 8:30—8:45 p. m	32.61	32.62	+31.8	−63.2	+781.5	+667.4	−15.0	−10.7
26, 9:11—9:25 a. m	32.91	32.93	+31.8	−65.5	+773.3	+665.6	−11.6	− 9.1
26, 8:41—9:06 p. m	33.34	33.26	+31.8	−68.9	+778.7	+667.7	− 8.7	−13.3
27, 9:25—9:43 a. m	33.63	33.73	+30.8	−67.2	+777.7	+664.9	−11.4	− 6.1
28, 9:56—10:12 a. m	34.53	34.58	+31.4	−66.4	+780.3	+664.9	− 7.9	− 4.1
29, 9:11——a. m	35.30	35.33	+33.0	−67.3	+773.4	+659.0	− 3.4	− 2.6
30, 9:07—9:26 a. m	35.95	35.93	+35.5	−59.6	+775.2	+664.2	− 7.5	− 3.2
Dec. 1, 9:05—9:23 a. m	36.30	36.28	+36.7	−57.3	+771.4	+656.3	− 4.4	+ 1.1
2, 9:34—9:55 a. m	36.40	36.45	+35.4	−63.1	+774.1	+659.0	− 5.8	− 2.1
3, 2:36—2:54 p. m	37.56	37.70	+33.2	−66.6	+775.3	+660.6	− 1.3	+ 5.1
4, 9:33—9:50 a. m	37.92	37.95	+33.0	−63.4	+774.5	+661.6	− 3.4	+ 2.2
5, 3:05—3:30 p. m	39.62	39.60	+31.3	−68.3	+771.7	+653.5	+ 1.8	+ 2.9
6, 9:21—9:38 a. m	38.83	38.84	+28.7	−60.9	+778.8	+663.3	− 5.1	+ 6.9
Means	....	....	....	....	+775.8	+662.2	− 6.4	− 2.5
Dec. 7, 9:35—9:52 a. m	36.82	36.73	+34.4	−56.9	+780.9	+661.8	− 7.3	− 1.5
8, 9:27—9:41 a. m	34.25	34.18	+32.1	−58.4	+773.9	+664.6	−14.2	− 7.8
9, 9:30—9:44 a. m	32.08	32.02	+37.4	−54.0	+776.6	+665.0	−15.2	−10.1
10, 9:15—9:34 a. m	31.23	31.26	+32.6	−59.1	+777.3	+665.5	−17.9	−10.0
11, 9:17—9:38 a. m	30.91	30.93	+33.4	−61.5	+780.2	+665.1	−16.1	−12.0
13, 9:06—9:20 a. m	33.33	33.39	+32.3	−69.9	+777.9	+664.4	− 6.1	− 6.7
14, 9:10—9:22 a. m	35.05	35.13	+31.6	−67.3	+777.1	+663.4	− 4.0	+ 0.1
15, 9:53—10:12 a. m	36.80	36.79	+32.6	−69.4	+776.1	+661.5	+ 2.2	+ 0.4
16, 9:28—9:48 a. m	37.85	37.90	+32.8	−66.7	+776.8	+659.5	+ 2.3	+ 5.6
17, 9:44—9:58 a. m	38.34	38.38	+34.9	−64.3	+776.3	+660.8	+ 4.7	+ 8.1
18, 10:06—10:16 a. m ...	38.47	38.47	+33.6	−62.4	+778.3	+662.8	+ 1.1	+ 6.1
19, 11:22—11:52 a. m ...	38.20	38.20	+35.8	−61.0	+780.1	+661.1	+ 6.2	+10.4
20, 9:27—9:38 a. m	37.88	37.85	+30.6	−67.8	+786.3	+664.3	+ 7.0	+ 8.3
21, 10:27—10:40 a. m ...	38.12	38.13	+31.7	−62.9	+781.2	+661.2	+ 2.7	+ 9.5
Means	....	....	....	....	+778.5	+662.9	− 3.9	+ 0.0

If there were no bending and no error of observation, the quantities in the columns headed S_2-S_2', and Z_2-Z_2', during the temperature-changes, which, as is seen, gave residuals for Z_2-S_2 of 25^μ, should remain absolutely constant. But as the focusing was on the very limit or slightly outside the limit of distinct vision, the errors of pointing become large. Dividing the series into 4 groups, of 13 or 14 observations each, the resulting means for the four periods are for—

	1st group.	2d group.	3d group.	4th group.
$S_2-S_2'=$	$+779^\mu.0$	$+779^\mu.0$	$+775^\mu.8$	$+778^\mu.5$
$Z_2-Z_2'=$	$+657^\mu.4$	$+664^\mu.0$	$+662^\mu.2$	$+662^\mu.9$

while the corresponding means for the residuals of Z_2-S_2 are for the several groups—

$$-\ 16^\mu.3 \quad -\ 20^\mu.0 \quad -\ 2^\mu.5 \quad +\ 0^\mu.0$$

The values found for S_2-S_2' and Z_2-Z_2' from many other comparisons besides these are $+777^\mu.9$ and $660^\mu.0$, §33. The extreme range in the mean values of S_2-S_2' is but $3^\mu.2$, and in those of Z_2-Z_2' but $6^\mu.6$, while the range in the mean values of the residuals of Z_2-S_2 is 20^μ. Bending does not then account for any large part of the range in residuals, and, indeed, the variation in the mean values of S_2-S_2' and Z_2-Z_2' is too small (about $\frac{1}{1000000}$ and $\frac{1}{500000}$ of the length of the bars, respectively) to indicate with positiveness anything more than errors of observation, due to the necessarily bad focusing of the microscopes.

§ **30.** B. If the theory of bending or of longitudinal strain in the bars does not account for any large part of the residuals, may not the theory of difference of temperatures of Z_2 and S_2 do so? Although the volumes of the two bars and the coatings of their surfaces are alike, and though their specific heats and conductivities do not differ widely, yet it is very improbable that they retain precisely equal temperatures when the temperature is rapidly changing.

If, during a steady temperature-rise of 2° F. per day, Z_2 heated more slowly than S_2, its temperature would lag behind that of S_2 till the difference became so great that Z_2 would receive heat enough from its surroundings to raise its temperature at the same rate as that at which the temperature of S_2 rose. This would be the maximum difference in temperature, and the continuance of the steady temperature-rise of 2° per day would not increase it. If there were a steady temperature-fall the reverse would be the case. This difference of temperature would show itself as a residual for the observed lengths of Z_2-S_2; when Z_2 was the hotter its observed would be greater than its computed length, and the reverse would be true when it was the cooler. In the results of comparisons already given in the final reductions, no residual exceeds 20^μ; if this were due solely to Z_2 being cooler or hotter than S_2, since Z_2 expands about 63^μ for 1° F., a temperature-difference of about 0°.3 F. would be necessary to produce it. While this theory would account in part for the dependence of the residuals on the temperature-changes, and would give the residual-curves fluctuations corresponding to those of the temperature-curves, it does not always account for the values of the residuals. Thus, on September 30, 1880, the residual was -19^μ, and would be accounted for by supposing Z_2 0°.3 F. the hotter. But from September 30 to October 9 the temperature fell but 3°.4, or at the rate of 0°.38 per day, and yet the residual was in the vicinity of -18^μ during the whole period. That is, we must suppose that with a daily temperature-fall of 0°.38, Z_2 and S_2, side by side, could differ about 0°.3 in temperature for nine days while inclosed in a heavy iron tube. This supposition is very improbable.

Again, from January 10, 1880, to January 28, 1880, the residuals varied from $+13^\mu$ to $+10^\mu$ falling once to $+5^\mu.3$. In this period the total temperature-range was 2°.2 F., the temperature being 46°.96 on the first and 46°.46 on the last day. With a temperature-range of but 2°.2 F. in eighteen days, it seems very improbable that Z_2 could, on account of a different rate of heating, have differed in temperature from S_2 by the amount needed to account for the residnals, namely, by from 0°.2 F. to 0°.1 F., for this period.

If these differences of temperature between Z_2 and S_2 actually exist, it may be asked if it is not due to unequal exposure to sources of heat. The side walls of the comparing-room are about $1^m.3$ distant from the comparing-box. One of these walls separates the comparing-room from the adjoining house, is of brick, and has a passage-way on the opposite side communicating freely with a kitchen and warmed by it; the other side wall is a lathed and plastered partition-wall separating the comparing-room from a hall (which is never heated) of the Lake-Survey office. Fearing that these walls might differ in temperature and affect, through the wooden sides of the comparing-box, the standards within it, many experiments and investigations were made to detect such an effect.

It will be remembered that the whole interior of the comparing-room is lined with a layer of sawdust $0^m.3$ thick. Thermometers sunk 5^{cm} in this sawdust on the side walls sometimes showed a difference of temperature of as much as $4^\circ.5$ F. But thermometers on the comparing-room surface of these walls showed no certain difference of temperature. No difference could be detected between thermometers in the interior of the comparing-box at its two sides. In the tubes a comparison of the readings of thermometers on opposite sides of the bars gave no evidence of temperature-difference between the sides. Two or three canvas screens a decimeter apart were set up each side of the comparing-box, so as to cut off the radiation from the walls, and comparisons were carried on at a time of large residuals with these screens up for many days. The presence or absence of the screens had, so far as could be discovered, no effect on the results of the comparisons.

It is seen, then, that all attempts to establish the existence of a difference of temperature between the bars under comparison, due to external sources, failed. Moreover, it will be noticed that if such an effect was sensible, it would extend over long periods, and its changes would be slow, while in fact the residuals follow closely small temperature-changes in the comparing-box from day to day. In comparing the two tubes the order of their bars from north to south was: 1, steel; 2, zinc; 3, steel; 4, zinc. If heat had come from the south wall it would have tended to heat the two bars nearest it the most, especially the zinc bar. But, in fact, the two tubes varied together quite closely. The heating should also have affected for long periods the south thermometer, but this was not observed. The bars under comparison could not of course have absolutely equal exposure to heat on the two sides of the comparing-box, but from the examinations made it would seem that the effects of such unequal exposure were either so small as not to be detected or were masked by other errors.

§ **31.** There is another indication that the theory of difference of temperatures of the two bars is not sufficient to account for the residuals of Z_2-S_2. In comparisons of tube 1 with the 15-feet brass bar packed in ice, described in §§ 42–46, made on the Cass farm in August and September, 1880, the times of maximum lengths of Z_1 and S_1 and the corresponding readings of thermometers in the tube, resulted directly from the observations. The first series of these comparisons extended continuously from 8 a. m. August 24, 1880, to 7:40 p. m. August 26; the second extended from 8 a. m. August 31 to 5 p. m. September 3, 1880. The comparisons were made once in about 20^m during the periods, and gave the results in the following table, in which dates near maxima- or minima-temperatures are given in the first column, corrected mercurial temperatures in the second, differences of observed lengths of S_1 and the brass bar B in the third, and differences of length of Z_1 and B in the fourth.

Continuous comparisons of tube 1 *and B.*

I.—FOR MAXIMA.

[Maxima- and minima-values are in black type.]

Date.	Corrected mercurial temp.	$S_1-B_{32^\circ}$	$Z_1-B_{32^\circ}$
1880.	° F.	μ	μ
Aug. 23, 2:00 p. m.	87.60	....	....
2:20 p. m.	87.93	2906	3880
2:26 p. m.	**87.96**	2906	3883
2:40 p. m.	87.86	2908	3891
3:00 p. m.	87.65	2913	3900
3:21 p. m.	87.49	**2913**	3905
3:31 p. m.	87.35	2912	**3906**
3:40 p. m.	87.22	2911	3905
4:00 p. m.	86.74	2907	3898
4:20 p. m.	86.22	2902	3881

Continuous comparisons of tube 1 *and B*—Continued.

I.—FOR MAXIMA—Continued.

Date.	Corrected mercurial temp.	$S_1-B_{32°}$	$Z_1-B_{32°}$
1880.	° F.	μ	μ
Aug. 24, 12:20 p. m.	88.32	2866	3802
12:40 p. m.	89.21	2894	3863
12:59 p. m.	**89.90**	2915	3926
1:00 p. m.	89.87	2916	3927
1:04 p. m.	89.25	**2917**	3934
1:06 p. m.	88.85	2916	**3935**
1:20 p. m.	86.07	2905	3901
1:40 p. m.	....	2875	3802
2:00 p. m.	....	2821	3659
Aug. 25, 1:20 p. m.	64.49	....	....
1:40 p. m.	64.78	....	....
2:00 p. m.	65.00	....	....
2:20 p. m.	65.14	....	....
2:40 p. m.	65.20	2375	2587
2:44 p. m.	**65.20**	2375	2588
3:00 p. m.	65.17	2377	2591
3:20 p. m.	65.14	2378	2593
3:29 p. m.	65.10	**2378**	2594
3:36 p. m.	65.09	2378	**2594**
3:40 p. m.	65.08	2378	2594
4:00 p. m.	65.03	2377	2592
4:20 p. m.	64.94	2375	2589
Aug. 26, 3:00 p. m.	71.32	....	....
3:20 p. m.	71.50	....	....
3:40 p. m.	71.74	....	....
4:00 p. m.	71.94	....	....
4:20 p. m.	72.12	2535	2962
4:40 p. m.	72.18	2538	2970
4:44 p. m.	**72.18**	2539	2971
5:00 p. m.	72.14	3541	2975
5:20 p. m.	72.04	2542	2979
5:40 p. m.	71.90	2543	2983
5:58 p. m.	71.75	**2544**	2984
5:59 p. m.	71.75	2544	**2984**
6:20 p. m.	71.55	2542	2977
6:40 p. m.	71.27	2534	2963
7:00 p. m.	....	2522	2937

I.—FOR MINIMA.

Date.	Corrected mercurial temp.	$S_1-B_{32°}$	$Z_1-B_{32°}$
Aug. 24, 4:00 a. m.	61.15	....	....
4:20 a. m.	60.73	....	....
4:40 a. m.	60.43	....	....
5:00 a. m.	**60.32**	2283	2358
5:20 a. m.	60.32	2279	2347
5:38 a. m.	60.37	2276	**2343**
5:40 a. m.	60.38	2276	2343
5:54 a. m.	60.57	**2275**	2345
6:00 a. m.	60.73	2275	2347
6:20 a. m.	61.51	2283	2361
6:40 a. m.	....	2298	2398
Aug. 25, 11:40 a. m.	....	2373	2590
12:00 a. m.	64.08	2366	2574
12:20 p. m.	63.74	2360	2555
12:38 p. m.	**63.60**	2355	2544
12:40 p. m.	63.60	2355	2543
12:45 p. m.	63.63	**2355**	2542
12:54 p. m.	63.74	2356	**2541**
1:00 p. m.	63.87	2357	2541
1:20 p. m.	64.49	2362	2551
1:40 p. m.	....	2367	2563

Continuous comparisons of tube 1 *and B*—Continued.

I.—FOR MINIMA—Continued.

Date.		Corrected mercurial temp.	$S_1 - B_{32^\circ}$	$Z_1 - B_{32^\circ}$
1880.		° F.	μ	μ
Aug. 26,	1:20 a. m.	58.40	....	....
	1:40 a. m.	58.25	....	....
	2:00 a. m.	58.14	....	....
	2:20 a. m.	58.05	2205	2161
	2:40 a. m.	57.98	2201	2152
	2:56 a. m.	**57.95**	2198	2145
	3:24 a. m.	58.01	**2196**	2141
	3:30 a. m.	58.05	2196	**2140**
	3:40 a. m.	58.09	2197	2141
	4:00 a. m.	58.16	2199	2144
	4:20 a. m.	58.27	2198	2143
	4:40 a. m.	58.35	2198	2142
	5:00 a. m.	58.44	2200	2144
	5:20 a. m.	58.55	2202	2152

II.—FOR MAXIMA.

Aug. 31,	1:00 p. m	82.04	....	....
	1:20 p. m	82.26	....	....
	1:40 p. m	82.37	....	....
	1:52 p. m	**82.40**	....	....
	2:00 p. m	82.40	2784	3588
	2:20 p. m	82.35	2789	3599
	2:40 p. m	82.25	2793	3606
	2:56 p. m	82.10	2794	**3608**
	3:09 p. m	82.02	**2794**	3608
	3:20 p. m	81.90	2794	3607
	3:40 p. m	81.76	2793	3605
	4:00 p. m	....	2792	3602
Sept. 1,	1:20 p. m	89.85	2950	3989
	1:40 p. m	89.99	2960	4017
	2:00 p. m	90.15	2967	4038
	2:16 p. m	**90.25**	2972	4050
	2:30 p. m	90.15	2974	**4053**
	2:38 p. m	90.00	**2975**	4052
	3:00 p. m	89.35	2971	4044
	3:20 p. m	....	2963	4023
	3:40 p. m	....	2951	3993
Sept. 2,	1:00 p. m	82.90	....	....
	1:20 p. m	84.08	2812	3652
	1:40 p. m	84.86	2832	3705
	2:00 p. m	85.48	2851	3748
	2:08 p. m	**85.51**	2858	3764
	2:25 p. m	85.32	**2864**	**3782**
	2:40 p. m	84.77	2863	3777
	3:00 p. m	84.26	2859	3766
	3:20 p. m	....	2854	3753
Sept. 3,	2:20 p. m	82.10	2793	3605
	2:40 p. m	82.33	2798	3618
	3:00 p. m	82.54	2804	3631
	3:20 p. m	**82.75**	2809	3643
	3:40 p. m	82.55	**2812**	3648
	3:48 p. m	82.49	2812	**3649**
	4:00 p. m	82.45	2810	3648
	4:20 p. m	82.51	2807	3644
	4:40 p. m	82.45	2804	3630

Continuous comparisons of tube 1 *and B*—Continued.

II.—FOR MINIMA.

Date.		Corrected mercurial temp.	$S_1-B_{32°}$	$Z_1-B_{32°}$
1880.		° F.	μ	μ
Sept. 1,	4:40 a. m	68.57	....	....
	5:00 a. m	68.40	2472	2825
	5:20 a. m	68.29	2470	2816
	5:36 a. m	68.25	2468	2810
	5:53 a. m	68.32	2466	2807
	6:02 a. m	68.45	2465	2809
	6:20 a. m	69.25	2471	2823
	6:40 a. m	70.22	2485	2852
Sept. 2,	5:00 a. m	73.11	2589	3116
	5:20 a. m	72.98	2585	3107
	5:40 a. m	72.87	2582	3099
	5:56 a. m	72.80	2579	3095
	6:06 a. m	72.90	2578	3094
	6:20 a. m	73.23	2581	3096
	6:40 a. m	73.78	2587	3107
	7:00 a. m	....	2594	3127

It should be remarked that the residuals of Z_1-S_1 follow the same law as those of Z_2-S_2, so that the conclusions concerning one tube may be applied to the other. This is seen in § 29, where the residuals of S_1-S_2 and Z_1-Z_2 are given. There the residuals of S_1-S_2 are no larger than are to be expected from errors of temperature and of observation, and the residuals of Z_1-Z_2, which are about 2.5 times larger than those of S_1-S_2, will be in the main accounted for by the supposition that the residuals in S_1-S_2 are largely due to small differences of temperature of the two tubes. Moreover, the numerous office comparisons of tubes 1 and 2 give a curve of residuals for Z_1-S_1, which follows quite closely the residual-curve for Z_2-S_2, showing that the two tubes behave essentially in the same way. An examination of these tables shows that in eight well-marked maxima of S_1-B, the time of the corresponding maximum of Z_1-B was later by from -13 to $+10$ minutes, averaging $+1$ minute; that the average change of S_1-B in the intervals was $-0^{\mu}.13$, corresponding to $-0^{\circ}.005$ F. in its true temperature; that the average change in Z_1-B in the intervals was $+0^{\mu}.25$, corresponding to $+0^{\circ}.004$ F.; and that in these intervals the thermometer-fall varied between $-0^{\circ}.15$ F. and $+0^{\circ}.40$ F., averaging $+0^{\circ}.04$ F. The intervals between the time of the maximum thermometer-reading and the time of maximum value of Z_1-B varied from 7^{min} to 74^{min}, averaging 40^{min}. The thermometer-fall in this interval varied from $0^{\circ}.09$ F. to $1^{\circ}.05$ F., averaging $0^{\circ}.38$ F. There were five well-marked minima. The average time of minimum of S_1-B was 2^{min} later than that of Z_1-B. The average change in S_1-B in this interval was $-0^{\mu}.05$, corresponding to $-0^{\circ}.002$ F.; the average change in Z_1-B was $+0^{\mu}.55$, corresponding to $+0^{\circ}.009$ F.; the average rise of thermometer in the interval between the minimum of Z_1-B and the minimum of S_1-B was $0^{\circ}.03$ F. The intervals between the time of minimum thermometer-reading and minimum value of Z_1-B averaged 23^{min}; the average thermometer-rise in this interval was $0^{\circ}.09$ F.

When the temperature rises, if Z_1 heats more slowly than S_1 (and this supposition is necessary in order to account for the residuals of Z_1-S_1 by the supposition that Z_1 and S_1 are not at the same temperature), the temperature of the place will first reach a maximum and then begin to fall; a little later, S_1, still rising in temperature, will equal the falling temperature of the place, and the temperature of S_1 will then be at its maximum, and will begin to fall; still later, the temperature of Z_1, still rising, will equal the falling temperature of the place, and the temperature of Z_1 will then be at its maximum, and will begin to fall. At the time of the maximum of Z_1 it will be colder than S_1, since Z_1 is at the temperature of the place and S_1 has already lagged behind that temperature. By temperature of the place is meant that which would be indicated by a perfect thermometer of infinitely small mass, lying in the place of the steel and zinc bars.

Now since the average intervals of time between the maximum and minimum of the thermometer and the maximum and minimum length of Z_1 are $+40^{\mathrm{min}}$ and $+23^{\mathrm{min}}$, and the corresponding average thermometer-changes are 0°.38 F. and 0°.09 F.; while the average intervals of time between the maxima or minima of Z_1 and those of S_1 are 1^{min} and 2^{min}, and the corresponding average thermometer-changes are 0°.04 F. and 0°.03 F., we may assume for this short interval that even if the thermometers do not give the true temperatures of the bars at the beginning and end of the interval, the difference in the temperature-errors at these two instants of time is small in comparison with the temperature-change, 0°.04 F., in that interval, not exceeding, say, 0°.01 or 0°.02 F.; so that within that degree of accuracy the thermometers measure the change of temperature of the place in the interval. Hence, since at the maximum or minimum of S_1 it had the temperature of the place, and at the maximum or minimum of Z_1 it had the temperature of the place, and the place in the average changed temperature in this interval by but 0°.04 F., it follows that in the average at the later of the times of their maxima or minima Z_1 and S_1 did not differ in temperature by more than 0°.04 F.

Now, the daily temperature-range in these comparisons from minimum to maximum or *vice versa*, in these cases varied between 2° F. and 30° F., averaging 17° F. If such a change in about eight and a half hours produced less than 0°.04 F. difference of temperature in the two bars at maxima- or minima-temperatures, it seems improbable that in the office comparisons a change of 10° distributed over five or ten days should give a difference of temperature of 0°.3 F. to the bars.

It is concluded, then, that the theory of unequal rates of heating of the steel and zinc bars does not satisfactorily account for the large residuals.

§ **32.** C. Can the large residuals be accounted for by the theory that the temperature of the thermometers is greater than that of the bars in rising temperatures and less in falling temperatures? Since Z_2-S_2 increases in value by about 38^{μ} per degree F. of temperature-increase, if the thermometers in rising temperatures were hotter than Z_2 and S_2, the value of Z_2-S_2, computed with these thermometer-temperatures, would be too great, and the residual, which is computed Z_2-S_2 minus observed Z_2-S_2, would be positive in rising temperatures and negative in falling temperatures. If the thermometers preceded the temperature of the bars by 0°.5 F., the residual would be $+19^{\mu}$. Experiments have been made to ascertain the rate of cooling of the thermometers. Two thermometers were placed in a metal water-cooler of about 20^{kg} capacity. To cut off radiation from the heated thermometer they were separated by a board, which allowed communication of air above and below it between the two sides of the screen. The thermometers, Casella 21476 and Baudin 6131, were inserted through a closely-fitting cover till their bulbs were at the same level. After a few readings on each, Casella 21476 was taken out of the water-cooler, and its temperature raised by from 9° to 18° F., when it was replaced in the cooler, and both were then read at short intervals till their temperatures became the same within 0°.1 F. The experiments were repeated on three days. After heating on November 3, 1880, the two differed at $2^{\text{h}}\ 21^{\text{m}}\ 30^{\text{s}}$ by +1°.65 F., Casella 21476 being the hotter. This difference gradually diminished to +0°.06 at 3 p. m., the temperature then being 51°.32 F. On November 4 the difference diminished gradually from +1°.22 F. to −0°.03 F. in one hour. On November 5 the difference diminished gradually from +1°.20 F. to +0°.17 F. in eighteen minutes. These experiments show that when this thermometer differs from the temperature due to its surroundings by 1° F., it may be expected to approach that temperature within 0°.1 F. in less than an hour.

The bulbs of the thermometers other than Casellas, used in determining the temperatures of the tubes, had masses considerably less than these, and hence may be expected to take the temperature of the space where they may be more rapidly than the Casellas. Now, in the comparisons of the tubes in the Lake-Survey office, the temperature very rarely rose at the rate of 0°.1 F. per hour or 2°.4 per day. Since in less than one hour the thermometers reduce their temperature-difference from the space they are in from 1° to 0°.1 F., we may expect that with a difference of 0°.5 at the beginning of an hour in which the temperature-rise is 0°.1, they will be within 0°.1 F. of the temperature of the space at the end of the hour, or, in other words, that the thermometers give the temperature of the space occupied by them without greater errors than 0°.1 F. in the comparisons under consideration. But if the thermometers are very nearly of the temperature of the space occupied by them, it does not follow that the zinc and steel bars are of this temperature. In rising temperatures they derive their heat from the tube-walls and from the air inside of the tubes, and, as their masses are considerable, their temperature inevitably lags behind that of the tube-wall and of the interior air, depending mainly, it is probable, on the tube-wall, since the interior air communicates with the exterior only by three small openings, the sum of whose areas does not exceed twenty-six square centimeters. But when in comparisons a maximum temperature of a bar is passed, this bar is cooler than the space it is in immediately before the maximum, and is hotter immediately after. Hence, before such a maximum the spaces occupied by the thermometers (or the thermometers themselves within 0°.1 F.) are hotter than the bar, and are cooler afterward.

If, then, the large residuals are due to the lagging of the temperature of the bars behind that of the thermometers, these residuals should change sign at the maxima- or minima-temperatures (or lengths) of the bars. An examination of the comparisons of tube 1 with the 15-feet brass bar in ice, made at the Cass farm in August and September, 1880, has shown that Z and S reach their maxima on the average within about one minute of each other, even when the daily temperature-fluctuations are 15° or 20° F., and in this interval neither changes temperature on the average more than 0°.04 F. In office-comparisons with daily temperature-ranges rarely exceeding 2°.5, it may be supposed that the differences of maxima-temperatures of Z_2 and S_2 will be still less, so that the time of the maximum value of Z_2-S_2 may be taken as the time of maximum length of either Z_2 or S_2 without important error. Now, an examination of the table of comparisons of Z_2 and S_2, already given in § 29, shows that in no case did the times of maxima or minima values of Z_2-S_2, and of the thermometer-readings, differ by twenty-four hours, while in no case did the residuals change sign within as little as twenty-four hours after a maximum or minimum, and in several cases they did not change sign for many days after a maximum or minimum temperature. If the large residuals were due to lagging of temperatures of the bars behind those given by the thermometers in rapidly changing temperatures, residuals of one-third the size running through many days would be expected in the comparisons of S_2 and B. None such were observed.

It may be concluded, then, that while the theory that the temperature of the bars when they are heating or cooling lags behind that given by the thermometers is undoubtedly true, yet the amount of the lagging is not sufficient to account for the large residuals.

§ **33.** D. Assistant Engineer E. S. Wheeler first called attention to the fact that the residuals might be accounted for by supposing that Z_2 when heated or cooled took a set so that its length at a given date and temperature might depend on its previous temperatures. When a thermometer of glass which has long remained at ordinary temperatures is heated to 212° F., and is then allowed to cool in air, it is well known that its freezing-point will be found to have sunk several tenths of a degree, indicating an increase in the volume of its bulb, and that during many weeks there is a slow return towards the original volume. The melting-points of glass and zinc do not differ very widely. May zinc in this respect behave when heated and then cooled in some degree like glass?

To test this question it was decided to compare the zinc and steel bars in tube 2 with those in tube 1; then leaving tube 2 in the steady temperature of the comparing-room, to place tube 1 in another room and heat it through 20° or 30° F., and afterward to recompare its bars with those of tube 2, to see if any change could be detected in their relative lengths. In order to eliminate any errors which might arise from lateral bending (if possibly any slight bending existed) during the considerable expansion and contraction, the graduations at the ends of the 4-metre zinc and steel bars lying nearly in the neutral axes of the bars were used in comparisons. The 4-metre intervals on these bars used in measurements of bases have already been designated for tube 1 by S_1 and Z_1, and for tube 2 by S_2 and Z_2. These intervals are parallel to the neutral axes of the bars at a distance of 8^{mm} from them in the same horizontal plane. The 4-metre intervals between the new graduations are parallel to the neutral axes of the bars at a distance of 1^{mm} below them in the same vertical plane. These intervals are designated by S_1', Z_1', S_2', Z_2'.

The new and old graduations differing about 1^{mm} in level, the microscopes were focused for an intermediate level. This gave some indistinctness to all pointings, increased the probable error of a microscope-pointing from $0^{\mu}.4$ to about 1^{μ}, gave for the probable error of one comparison due to pointing errors alone 2^{μ}, and a probable range in fifty results due to pointing errors alone of about 14^{μ}. Otherwise the comparisons of S_1' and S_2', and of Z_1' and Z_2', were made like the comparisons of S_1 and Z_1 already described. But in the comparisons on and after June 10, 1881, only S_1', S_2',

Z_1', Z_2' were compared. This permitted accurate focusing in all pointings of the microscopes. Temperatures during comparisons were determined as usual by three thermometers in each tube, the mean of the corrected readings in each tube (the thermometer at the middle of the tube having double weight) being taken as the temperature of the bars in that tube. The values of S_1-S_2 and Z_1-Z_2 and the relative expansions have already been given in § 24 as—

$$S_1-S_2=+32\overset{\mu}{.}74\pm0\overset{\mu}{.}24-0\overset{\mu}{.}0605\ (t-32)$$
$$Z_1-Z_2=-66.00\pm0.81-0.4366\ (t-32)$$

The values of S_1-S_1', S_2-S_2', Z_1-Z_1', Z_2-Z_2', were derived from numbers of comparisons varying between 48 and 114. Since each of these differences is between intervals on the same bar, no temperature-errors or expansion-errors are to be feared, and the results should be accurate, provided the conclusion already reached, that no serious lateral bending is to be feared, is true.

From 48 comparisons on 35 days, $S_1-S_1'=-707\overset{\mu}{.}07\pm0\overset{\mu}{.}41$
From 114 comparisons on 76 days, $S_2-S_2'=+777.94\pm0.21$
From 48 comparisons on 35 days, $Z_1-Z_1'=+400.14\pm0.30$
From 114 comparisons on 76 days, $Z_2-Z_2'=+660.03\pm0.27$

Combining these values with those of S_1-S_2 and Z_1-Z_2, there result—

$$S_1'-S_2'=+1517\overset{\mu}{.}7\pm0\overset{\mu}{.}5-0\overset{\mu}{.}0605\ (t-32)$$
$$Z_1'-Z_2'=+\ 193.9\pm0.9-0.4366\ (t-32)$$

With these equations, computed values of $S_1'-S_2'$ and $Z_1'-Z_2'$ have been deduced for the observed temperatures of the tubes. The residuals are obtained by subtracting observed values from the computed values.

§ **34.** In the first heating-experiments the bars in tube 1 were compared on four days with those in tube 2, then tube 1 was taken from the comparing-room to a distant room, where it remained twenty-four hours, the temperature of this room being kept steadily at between 70° and 80° F. Tube 1 was then taken back to the comparing-room and placed in the comparing-box by the side of tube 2; comparisons were then begun and were continued through six days. In reduction, when mercurial temperatures of the two tubes are different, the lengths of S_1' and of Z_1' are reduced to the temperature of S_2' and Z_2' by means of their coefficients of expansion, namely, $E_{S_1}'=24^{\mu}.866$ and $E_{Z_1}'=62^{\mu}.955$ for 1° F. It is this corrected observed difference of lengths which is subtracted from the computed difference of length for that common temperature to obtain the residuals in the following table.

The first column gives the date of comparison; the second and third the temperatures, t_1 and t_2, of tubes 1 and 2; the fourth gives the observed $S_1'-S_2'$ corrected for difference of temperature of S_1 and S_2, the temperature t_2 being taken as the standard; the fifth gives the residuals of $S_1'-S_2'$, computed as already explained; the sixth gives the observed $Z_1'-Z_2'$ corrected for difference of temperature of the two bars to the temperature t_2; and the seventh gives the residuals of $Z_1'-Z_2'$:

Comparisons of S_1', S_2', Z_1', and Z_2'.

BEFORE HEATING OF TUBE 1.

Date.	t_1	t_2	$S_1'-S_2'$ corrected.	Residuals, $S_1'-S_2'$ computed *minus* observed.	$Z_1'-Z_2'$ corrected.	Residuals, $Z_1'-Z_2'$ computed *minus* observed.
1881.	° F.	° F.	μ	μ	μ	μ
March 14, 9:40 a. m.	39. 74	39. 82	+1517. 4	−0. 2	+195. 4	−5. 0
15, 9:39 a. m.	40. 58	40. 08	+1520. 4	−3. 2	+198. 7	−8. 6
16, 3:36 p. m.	41. 79	41. 80	+1525. 2	−8. 1	+194. 8	−5. 2
17, 9:27 a. m.	41. 90	41. 90	+1526. 0	−8. 9	+194. 7	−5. 2
Means				−5. 1		−6. 0

At 9:30 a. m. on March 17 tube 1 was placed in a room the temperature of which was kept constantly between 70° and 80° F. It remained in this room until March 18, 9:15 a. m., when it was again placed in the comparing-box.

Comparisons of S_1', S_2', Z_1', and Z_2'—Continued.

AFTER HEATING OF TUBE 1.

Date.	t_1	t_2	$S_1'-S_2'$ corrected.	Residuals, $S_1'-S_2'$ computed *minus* observed.	$Z_1'-Z_2'$ corrected.	Residuals, $Z_1'-Z_2'$ computed *minus* observed.
1881.	° F.	° F.	μ	μ	μ	μ
March 18, 10:07 a. m.	73. 33	43. 64	+1616. 7	−99. 7	+357. 4	−168. 6
18, 10:33 a. m.	67. 42	44. 46	+1594. 7	−77. 5	+458. 2	−269. 8
18, 11:23 a. m.	61. 10	45. 08	+1602. 1	−85. 2	+389. 4	−201. 3
18, 12:31 p. m.	55. 81	45. 76	+1556. 0	−39. 1	+376. 5	−188. 6
18, 3:10 p. m.	49. 76	45. 54	+1532. 4	−15. 5	+282. 5	− 94. 5
18, 4:48 p. m.	47. 91	45. 47	+1529. 4	−12. 5	+287. 3	− 99. 3
18, 8:15 p. m.	46. 51	45. 12	+1526. 2	− 9. 3	+249. 8	− 61. 7
19, 9:44 a. m.	44. 25	43. 68	+1524. 3	− 7. 3	+237. 7	− 49. 0
19, 2:31 p. m.	44. 02	43. 52	+1515. 3	+ 1. 7	+230. 4	− 41. 6
19, 8:08 p. m.	43. 88	43. 50	+1513. 5	+ 3. 5	+233. 7	− 44. 9
March 20, 9:39 a. m.	43. 66	43. 38	+1511. 8	+ 5. 2	+225. 9	− 37. 0
20, 8:35 p. m.	43. 70	43. 46	+1522. 8	− 5. 8	+231. 1	− 42. 3
21, 10:23 a. m.	43. 48	43. 33	+1512. 5	+ 4. 5	+226. 4	− 37. 5
21, 8:17 p. m.	43. 52	43. 32	+1514. 8	+ 2. 2	+224. 4	− 35. 5
22, 9:12 a. m.	43. 33	43. 12	+1513. 6	+ 3. 4	+220. 6	− 31. 6
22, 8:43 p. m.	43. 03	42. 90	+1521. 1	− 4. 0	+218. 4	− 29. 3
23, 9:18 a. m.	42. 76	42. 59	+1516. 9	+ 0. 2	+220. 7	− 31. 5
Means for dates March 20, 9:39 a. m. to March 23, 9:18 a. m.				+ 0. 8	....	− 35. 0

§ **35.** From March 23, 1881, to April 14, 1881, both tubes remained in the comparing-room, whose temperature rose gradually to 45° F. The tubes were then compared from April 14 to April 18, 1881, and on April 18 tube 1 was again taken into the heated room, whose temperature was kept for twenty-four hours at between 70° and 80° F. Tube 1 was then replaced in the comparing-box and comparisons were made daily until May 5, 1881. The results are given in the following table, which is arranged like the preceding one:

Comparisons of S_1', S_2', Z_1', and Z_2'—Continued.

BEFORE HEATING OF TUBE 1.

Date.	t_1	t_2	$S_1'-S_2'$ corrected.	Residuals, $S_1'-S_2'$ computed *minus* observed.	$Z_1'-Z_2'$ corrected.	Residuals, $Z_1'-Z_2'$ computed *minus* observed.
1881.	° F.	° F.	μ	μ	μ	μ
April 14, 8:17 p. m.	45. 06	45. 11	+1513. 4	+3. 5	+218. 3	−30. 2
15, 9:17 a. m.	44. 90	44. 92	+1524. 9	−8. 0	+216. 2	−28. 0
15, 8:17 p. m.	45. 00	45. 04	+1511. 9	+5. 0	+213. 0	−24. 8
16, 9:42 a. m.	45. 10	45. 13	+1523. 6	−6. 7	+212. 6	−24. 5
16, 8:05 p. m.	45. 60	45. 76*	+1517. 2	−0. 3	+221. 0	−33. 2
17, 11:14 a. m.	46. 00	46. 07	+1524. 5	−7. 6	+210. 5	−22. 8
18, 11:57 a. m.	46. 24	46. 21	+1522. 9	−6. 0	+204. 2	−12. 8
Means	....	....	....	−2. 9	....	−25. 2

* Thermometers disturbed by the proximity of the hand in adjusting tube.

At 12:30 p. m., April 18, tube 1 was placed in a room, where it remained until 9:30 a. m., April 19, when it was replaced in the comparing-box. The temperature of the room remained constantly between 70° and 80° F.

Comparisons of S_1', S_2', Z_1', and Z_2'—Continued.

AFTER HEATING OF TUBE 1.

Date.	t_1	t_2	$S_1'-S_2'$ corrected.	Residuals, $S_1'-S_2'$ computed *minus* observed.	$Z_1'-Z_2'$ corrected.	Residuals, $Z_1'-Z_2'$ computed *minus* observed.
1881.	° F.	° F.	μ	μ	μ	μ
April 19, 10:12 a. m.	78.32	48.28	+1609.1	−92.4	+337.2	−150.5
19, 11:16 a. m.	68.40	50.12	+1607.4	−90.8	+487.3	−301.4
19, 12:11 p. m.	63.22	50.76	+1584.3	−67.7	+384.0	−198.4
19, 2:36 p. m.	56.07	50.63	+1543.7	−27.1	+325.3	−139.6
19, 4:16 p. m.	53.75	50.33	+1527.8	−11.2	+288.3	−102.5
19, 8:09 p. m.	51.22	49.57	+1515.6	+ 1.1	+261.5	− 75.3
20, 9:10 a. m.	49.08	48.45	+1514.1	+ 2.6	+241.7	− 55.1
20, 2:30 p. m.	49.12	48.67	+1525.8	− 9.1	+252.4	− 65.9
20, 8:09 p. m.	49.23	48.80	+1521.1	− 4.4	+239.7	− 53.2
April 21, 9:15 a. m.	48.72	48.40	+1515.3	+ 1.4	+236.5	− 49.8
21, 8:16 p. m.	48.89	48.62	+1522.7	− 6.0	+232.3	− 45.7
22, 9:42 a. m.	49.02	48.87	+1517.3	− 0.6	+233.3	− 46.8
22, 8:10 p. m.	49.82	49.67	+1521.9	− 5.3	+232.8	− 46.7
23, 9:22 a. m.	49.88	49.72	+1522.8	− 6.2	+228.1	− 42.0
23, 8:12 p. m.	50.64	50.50	+1515.9	+ 0.7	+221.7	− 35.9
24, 10:09 a. m.	51.30	50.94	+1516.5	+ 0.1	+221.8	− 36.2
25, 9:19 a. m.	52.54	52.58	+1515.1	+ 1.4	+227.2	− 42.4
27, 9:24 a. m.	55.96	55.96	+1511.0	+ 5.3	+214.2	− 30.9
29, 10:30 a. m.	56.91	56.89	+1513.7	+ 2.5	+213.7	− 30.8
May 1, 11:31 a. m.	54.35	54.24	+1523.4	− 7.0	+204.2	− 20.1
3, 9:14 a. m.	52.96	52.89	+1512.3	+ 4.2	+213.5	− 28.8
5, 11:04 a. m.	51.54	51.50	+1512.6	+ 3.9	+207.8	− 22.5
Means for dates April 21, 9:15 a. m. to May 5, 11:04 a. m.				− 0.5	....	− 36.8

At 11:30 a. m., May 5, tube 1 was placed in a room, where it remained until May 7, 9:30 a. m., when it was replaced in the comparing-box. The temperature-range in the room was indicated by maximum and minimum thermometers, whose readings were 58° and 50° F., respectively. It was hoped that the temperature would fall by a larger amount during the night than it did.

Comparisons of S_1', S_2', Z_1', and Z_2'—Continued.

AFTER COOLING OF TUBE 1.

Date.	t_1	t_2	$S_1'-S_2'$ corrected.	Residuals, $S_1'-S_2'$ computed *minus* observed.	$Z_1'-Z_2'$ corrected.	Residuals, $Z_1'-Z_2'$ computed *minus* observed.
1881.	° F.	° F.	μ	μ	μ	μ
May 7, 10:16 a. m.	55.52	55.16*	+1515.4	+0.9	+198.7	−15.0
7, 2:50 p. m.	55.72	55.88	+1521.6	−5.3	+212.9	−29.5
8, (†)	55.85	55.92	+1520.8	−4.5	+202.6	−19.3
May 9, (†)	57.38	57.46	+1518.7	−2.5	+210.1	−27.4
10, 9:25 a. m.	59.54	59.66	+1519.5	−3.4	+210.3	−28.6
11, 9:40 a. m.	61.73	61.85	+1515.3	+0.6	+206.2	−25.5
14, 10:52 a. m.	65.27	65.26	+1513.5	+2.2	+196.5	−17.2
17, 9:14 a. m.	62.88	62.83	+1512.1	+3.8	+196.2	−15.9
20, 12:14 p. m.	59.40	59.38	+1520.6	−4.5	+193.6	−11.8
23, 12:08 p. m.	59.53	59.52	+1519.9	−3.8	+191.6	− 9.8
25, 9:39 a. m.	60.88	60.86	+1517.7	−1.7	+191.9	−10.7
Means for dates May 9 to 25, inclusive				−1.2	....	−18.4

*This observation not to be used, as the thermometers were disturbed by contact with the hand.
† Time not recorded.

§ **36.** From May 25, 1881, to June 10, 1881, both tubes remained in the comparing-room, when further comparisons were made before and after heating. Prior to June 10, 1881, readings were made both on the proper graduations of the bars and on those nearer the neutral axes, which are about 1^{mm} lower. Hence, as the microscopes were focused for an intermediate point $0^{mm}.5$ distant from that of most distinct vision and near the limit of distinctness, considerable pointing-errors are to be expected. From June 10, 1881, to July 2, 1881, only the neutral-axis graduations were pointed at and the focusing was good.

Comparisons of S_1', S_2', Z_1', and Z_2'—Continued.

BEFORE HEATING OF TUBE 1.

Date.	t_1	t_2	$S_1'-S_2'$ corrected.	Residuals, $S_1'-S_2'$ computed *minus* observed.	$Z_1'-Z_2'$ corrected.	Residuals, $Z_1'-Z_2'$ computed *minus* observed.
1881.	° F.	° F.	μ	μ	μ	μ
June 10, 10:22 a. m	61.38	61.34	+1523.7	− 7.7	+195.3	−14.2
11, 9:52 a. m	61.01	60.96	+1526.8	−10.8	+194.0	−12.9
13, 10:26 a. m	61.88	61.90	+1522.9	− 7.0	+191.6	−10.9
14, 9:40 a. m	63.56	63.60	+1522.8	− 7.0	+191.3	−11.3
15, 9:08 a. m	65.40	65.40	+1522.9	− 7.2	+192.1	−13.0
Means				− 7.9		−12.5

At 9:16 a. m., June 15, tube 1 was placed in a room the temperature of which was kept constantly between 85° and 95° F., and there remained until June 16, 9:40 a. m., when it was replaced in the comparing-box.

Comparisons of S_1', S_2', Z_1', and Z_2'—Continued.

AFTER HEATING OF TUBE 1.

Date.	t_1	t_2	$S_1'-S_2'$ corrected.	Residuals, $S_1'-S_2'$ computed *minus* observed.	$Z_1'-Z_2'$ corrected.	Residuals, $Z_1'-Z_2'$ computed *minus* observed.
1881.	° F.	° F.	μ	μ	μ	μ
June 16, 10:15 a. m	89.94	66.44	+1595.9	−80.2	+313.7	−135.0
16, 11:30 a. m	80.32	67.72	+1581.7	−66.1	+334.3	−156.1
16, 12:20 p. m	77.13	67.94	+1569.1	−53.5	+313.1	−135.1
16, 2:20 p. m	72.72	67.86	+1539.6	−24.0	+271.9	− 93.9
16, 4:10 p. m	70.68	67.56	+1530.6	−15.0	+251.2	− 73.0
16, 8:05 p. m	68.63	67.09	+1524.0	− 8.4	+227.0	− 48.6
17, 9:05 a. m	67.16	66.58	+1524.4	− 8.8	+227.7	− 49.0
June 18, 10:10 a. m	66.80	66.54	+1526.0	−10.4	+220.0	− 41.3
20, 9:21 a. m	67.51	67.42	+1524.9	− 9.3	+216.0	− 37.7
21, 10:20 a. m	67.05	66.96	+1524.7	− 9.1	+219.6	− 41.1
22, 10:40 a. m	65.46	65.36	+1524.5	− 8.8	+215.9	− 36.7
22, 3:53 p. m	65.22	65.08	+1524.0	− 8.3	+211.2	− 31.8
Means for dates June 18, 10:10 a. m. to June 22, 3:53 p. m., inclusive				− 9.2		− 37.7

At 4:15 p. m., June 22, tube 1 was placed in a room for the purpose of cooling it. Both ends of the tube were opened. It remained until June 23, 9 a. m., when it was replaced in the comparing-box, where it remained until 4 p. m. June 23. It was then taken back to the cooling-room, where it remained until June 24, 9 a. m. The minimum temperature attained for the two nights was 48°.8 F. It was replaced in the comparing-box June 24, 9 a. m.

Comparisons of S_1', S_2', Z_1, and Z_2'—Continued.

AFTER COOLING OF TUBE 1.

Date.	t_1	t_2	$S_1'-S_2'$ corrected.	Residuals, $S_1'-S_2'$ computed *minus* observed.	$Z_1'-Z_2'$ corrected.	Residuals, $Z_1'-Z_2'$ computed *minus* observed.
1881.	° F.	° F.	μ	μ	μ	μ
June 24, 3:45 p. m	61.60	62.10	+1521.5	−5.6	+188.8	− 8.1
25, 10:18 a. m	61.56	61.50	+1521.9	−6.0	+191.9	−11.0
June 27, 11:45 a. m	61.50	61.49	+1522.3	−6.4	+190.4	− 9.5
28, 11:24 a. m	62.55	62.56	+1520.5	−4.7	+190.1	− 9.6
29, 9:56 a. m	64.62	64.72	+1522.1	−6.4	+193.2	−13.7
30, 10:34 a. m	66.56	66.61	+1522.1	−6.5	+194.4	−15.7
July 1, 9:54 a. m	66.98	66.96	+1524.5	−8.9	+196.3	−17.8
2, 9:58 a. m	66.96	66.91	+1520.8	−5.2	+196.6	−18.1
Means for dates June 27 to July 2, inclusive				−6.4	....	−14.1

§ **37.** An examination of the preceding tables shows that when tube 1 was either heated above or cooled below the temperature of tube 2, within twenty-four hours after tube 1 was placed again beside tube 2 in the comparing-box and allowed to approach its temperature, S_1' had reached the length relatively to S_2' that it had before the heating or cooling, within about 5^μ, a quantity that may be attributed to errors of comparison. A change of length of 5^μ in S_1' is produced by a change of temperature of 0°.2 F. If, then, the heating or cooling has produced no temporary change of length in S_1', twenty-four hours appears to be sufficient for it to cool from a temperature 20° or 30° F. above that of S_2' to a temperature within 0°.2 F. of that of S_2'. After forty-eight hours from the time tube 1 is replaced in the comparing-box, it would seem, then, judging from the relative lengths of S_1' and S_2', that the relative temperatures of the two tubes would no longer be influenced by quantities larger than the errors of comparison by the preceding heating, the difference of temperature after that time depending mainly on the surroundings of the two tubes. Accordingly comparisons about forty-eight hours after the heating ceases and subsequent thereto will be used. But at about forty-eight hours after each of the three heatings ended, it will be seen that the mercurial thermometers indicated that tube 1 was the hotter by from 0°.26 F. to 0°.32 F., and that this excess gradually diminished in the following four to ten days without in any case entirely disappearing. It was at first thought that these excesses might in part be due to a temporary change of the freezing-points of the thermometers by the heatings and coolings. The freezing-points were accordingly redetermined, but they were found unchanged. A part of the excesses may be due to unequal action of the surroundings of the tubes on their temperatures, as similar differences of temperature, but of smaller amounts, are found in other comparisons; but it seems also possible that it may take several days for the whole mass of tube 1, after being heated, to reach the temperature of tube 2 within 0°.1 F.

An examination of the residuals of $S_1'-S_2'$ for dates before or forty-eight hours or more after heating, prior to June 10, 1881, shows irregularities in them much greater than those of the comparisons of S_1 and S_2 given in § 21. It is thought that these irregularities are due, at least in part, to the necessity previously mentioned of reading nearly at the limit of distinct vision of the microscopes.

Referring now to § 34, it is seen that prior to the heating of tube 1 on March 17 through 35° F., the mean residual of $S_1'-S_2'$ was $-5^\mu.1$, while, rejecting the first two days' comparisons after heating, the mean residual of $S_1'-S_2'$ after heating was $+0^\mu.8$. The change in the residuals is too small to indicate any change in the length of S_1 due to the heating. But the mean residuals of $Z_1'-Z_2'$ for the same periods were $-6^\mu.0$ and $-35^\mu.0$, showing a change of -29^μ. Since a negative residual indicates that $Z_1'-Z_2'$ observed is too great, if the change be attributed entirely to the heated bar Z_1', a negative residual will indicate that Z_1' is too long. It would appear, then, that the heating had left Z_1' 29^μ longer at a given temperature than it was before heating.

Examining the tables in § 35 in the same way, it will be seen that the mean residuals of $S_1'-S_2'$ before and after heating through 30° F., were $-2^\mu.9$ and $-0^\mu.5$, quantities too small to indicate any change in the length of S_1'. But the corresponding residuals of $Z_1'-Z_2'$ were $-25^{\mu}.2$ and $-36^{\mu}.8$, indicating that although Z_1' before heating was $25^{\mu}.2$ longer than its normal length, the heating increased that excess of length by $11^{\mu}.6$.

From the tables in § 35, giving the results of an attempt at cooling on May 5, 1881, no positive conclusion can be drawn, as only a few degrees of cooling were obtained.

From the tables in § 36 it is seen that prior to the heating on June 15 of tube 1 through about 30° F., the mean residual of $S_1'-S_2'$ was $-7^{\mu}.9$, while after the heating it was $-9^{\mu}.2$, indicating no change in the length of S_1' at a given temperature. But the residual of $Z_1'-Z_2'$ changed from $-12^{\mu}.5$ to $-37^{\mu}.7$, indicating an increase of length of Z_1' at a given temperature of $25^{\mu}.2$. The subsequent cooling of tube 1 through about 18° F., on June 23, 1881, changed the residual of $S_1'-S_2'$ only from $-9^{\mu}.2$ to $-6^{\mu}.4$, while the residual of $Z_1'-Z_2'$ changed from $-37^{\mu}.7$ to $-14^{\mu}.1$; that is, Z_1' seems to have been shortened $23^{\mu}.6$ by the cooling.

Considering now the three cases in which tube 1 was heated through about 30° F., and then allowed to cool to the temperature of tube 2, it appears that the mean change of residual for $S_1'-S_2'$ following the heating of S_1' was $+2^{\mu}.3$. For $Z_1'-Z_2'$ it was $-21^{\mu}.9$. There is no reason known why the errors of comparison of $Z_1'-Z_2'$ should exceed those of $S_1'-S_2'$, nor why from this cause there should be any greater change in the residuals before and after heating. Since Z_1' expands about 62^{μ} for 1° F., an increase in relative temperature of $\frac{1}{3}$° F. in Z_1' would give a change in residual of -21^{μ}. But it is very difficult to suppose an average change of temperature of $\frac{1}{3}$° between Z_1' and Z_2' without supposing a like change between S_1' and S_2'. Since S_1 expands about 25^{μ} per degree F., this would require an increase of $-8^{\mu}.3$ in the mean residual of $S_1'-S_2'$ during the heatings, while the actual mean increase is $+2^{\mu}.3$.

Since then errors of comparison and difference of temperature of the bars $Z_1'-Z_2'$ are alike inadequate to explain the change in the residuals of $Z_1'-Z_2'$ before and after heating, it seems that the larger part of this change must be attributed to a change in the length of Z_1' at the same temperature, due to the heating.

§ **38.** If the length of Z_1' is changed by heating it to about 30° F. above its original temperature and then allowing it to cool to its original temperature, the question arises as to the permanence of this change.

The mean residual for Z_1'—Z_2' from March 20 to March 23 (§ 34) was —$35^{\mu}.0$. From March 23 to April 14 both tubes remained in the comparing-room without large temperature-changes. Between April 14 and 18 the mean residual was —$25^{\mu}.2$, indicating that Z_1' had shortened towards its permanent length by $9^{\mu}.8$ in about twenty-six days.

In the comparisons extending from April 21, 1881, to May 5, 1881, the mean residual from April 21 to 25, inclusive, was —$43^{\mu}.2$, while from April 29 to May 5, inclusive, it was —$25^{\mu}.5$, indicating a shortening towards its permanent length of $17^{\mu}.7$ in about nine days. The two results differ widely. They agree in indicating the gradual loss of the increase of length which followed the heating. The heating of March 17 through about 35°, changed the mean residual of Z_1'—Z_2' from —$6^{\mu}.0$ to —$35^{\mu}.0$, while an equal but less prolonged heating on April 18 changed the mean residual only from —$25^{\mu}.2$ to —$36^{\mu}.8$, and on June 15 it changed the residual from —$12^{\mu}.5$ to —$37^{\mu}.7$. It would appear, then, that when Z_1' has already a length considerably greater than its normal length, a heating and subsequent cooling produces a less increase in length than would have occurred had Z_1' at beginning been at or near its normal length. This would indicate that repeated heatings and coolings would not give to Z_1' a length greatly in excess of that given by the first one or two heatings and coolings. The change of 29^{μ} in the mean residual of Z_1'—Z_2', in consequence of the heating through about 35° on March 17, corresponds to a change of $0^{\mu}.21$ per metre per degree F. of heating.

§ **39.** In addition to these experiments on tube 1, similar experiments were tried with the standard metre *MT*1876, which has been described in Chapter VIII, § 26. *MT*1876 was subjected to cooling as well as heating. In order to cool it from the temperature of the comparing-room, which was about 36° F. to —3° F., it was placed in another room in a tin case containing a minimum and maximum thermometer to give temperatures. This tin case was packed in a mixture of snow, or broken ice, and salt, and was allowed to remain there for at least twenty hours. Then it was placed in the comparing-box and compared with *R*1876, which throughout the heating and cooling experiments remained in the comparing-box at the temperature of the comparing-room. *MT*1876 was heated by being placed in a different room from the comparing-room, the temperature of this other room being raised by a stove. When taken from this room it was again placed in the comparing-box and comparisons were begun with *R*1876. A thermometer lay in contact with the top-surfaces of the zinc and steel bars of *MT*1876, and another lay on the top-surface of *R*1876. Indicating the steel and zinc bars of *MT*1876 by subscripts *S* and *Z*, from §§ 58, 65—

$$\text{Expansion of } R1876 = 5^{\mu}.885 \text{ for } 1^{\circ}\text{ F.}$$
$$(MT1876)_S = R1876 - 47^{\mu}.6 + 0^{\mu}.451\ (t-32)$$
$$(MT1876)_Z = R1876 + 218^{\mu}.9 + 10^{\mu}.639\ (t-32)$$

in which t is the temperature in Fahrenheit degrees. From these expressions and from the corrected temperatures given by the thermometer which lay on the two bars of *MT*1876, values of the difference of length of $(MT1876)_S$ and *R*1876 have been computed. Subtracting the observed values of this difference, the residuals result. The residuals of $R1876-(MT1876)_Z$ are obtained in the same way. The temperatures of *R*1876 and *MT*1876 are taken as given by thermometers lying on them, the thermometer-readings being corrected for their errors.

In the following tables the first column gives the date of comparison; the second and third give the corrected temperatures of the thermometers lying on the two metres; the fourth gives the residual errors of $R1876-(MT1876)_S$ in the sense computed minus observed; and the fifth gives similar errors for $R1876-(MT1876)_Z$.

1. *MT*1876, on February 7, 1881, was placed in a room whose temperature was kept constantly between 70° and 80° F. until February 14, 10:50 a. m., when it was placed in the comparing-box, and comparisons with *R*1876 were made.

Comparisons of R1876 and MT1876.

AFTER HEATING OF *MT*1876.

Date.	Temperature of *R*1876.	Temperature of *MT*1876.	Residuals, $R1876-(MT1876)_S$ computed *minus* observed.	Residuals, $R1876-(MT1876)_Z$ computed *minus* observed.
1881.	° F.	° F.	μ	μ
Feb. 14, 10:50 a. m	40.44	51.16	+9.9	+53.2
14, 11:32 a. m	40.54	46.39	+5.3	+41.1
14, 2:24 p. m	40.04	41.01	+4.8	+35.6
14, 4:02 p. m	39.69	40.01	+2.4	+28.5
14, 4:44 p. m	39.54	39.71	+0.1	+25.7
14, 8:14 p. m	39.03	39.21	+0.4	+22.7
Feb. 15, 9:06 a. m	37.82	37.81	+3.6	+23.9
15, 8:58 p. m	37.32	37.21	+3.7	+25.4
16, 9:10 a. m	37.02	36.91	+2.0	+21.4
16, 8:19 p. m	36.92	36.81	+0.6	+19.3
17, 9:12 a. m	36.52	36.41	+1.9	+20.4
17, 7:58 p. m	36.32	36.21	+1.9	+18.5
18, 9:35 a. m	36.12	36.21	+1.4	+16.6
18, 8:49 p. m	36.32	36.21	+3.5	+21.8
19, 9:25 a. m	36.37	36.31	+4.9	+22.1
19, 8:05 p. m	36.42	36.41	+2.1	+17.4
20, 10:38 a. m	36.32	36.21	+4.0	+20.4
20, 8:37 p. m	36.42	36.41	+2.9	+16.5
21, 9:56 a. m	36.52	36.41	+5.5	+20.6
21, 8:09 p. m	36.62	36.61	+2.7	+17.5
22, 10:12 a. m	36.72	36.71	+0.4	+14.9
22, 8:44 p. m	37.12	37.21	+2.0	+15.2
23, 9:22 a. m	37.32	37.26	+1.0	+17.1
23, 7:35 p. m	37.07	37.01	+3.1	+18.7
24, 9:17 a. m	36.52	36.51	+3.1	+17.7
Means of residuals from Feb. 15, 9:06 a. m			+2.6	+19.2

2. *MT*1876 was, on February 24, 10 a. m., placed in the freezing-mixture already described and kept constantly at temperatures between —1° F. and —6° F. until February 25, 9:30 a. m., when it was replaced in the comparing-box, and comparisons with *R*1876 were begun, resulting as follows:

Comparisons of R1876 and MT1876—Continued.

AFTER COOLING OF *MT*1876.

Date.	Temperature of *R*1876.	Temperature of *MT*1876.	Residuals, *R*1876—(*MT*1876)*s* computed *minus* observed.	Residuals, *R*1876—(*MT*1876)*z* computed *minus* observed.
1881.	° F.	° F.	μ	μ
Feb. 25, 9:53 a. m	35. 52	Unknown		
25, 10:44 a. m	35. 32	....do		
25, 11:42 a. m	35. 46	31. 07	—2. 4	—12. 7
25, 12:40 p. m	35. 52	33. 07	+1. 3	— 4. 0
25, 2:19 p. m	35. 52	34. 47	+1. 1	— 1. 4
25, 4:18 p. m	35. 52	35. 22	+4. 1	+ 2. 3
25, 7:22 p. m	35. 52	35. 42	+4. 4	+ 2. 8
Feb. 26, 9:03 a. m	34. 91	34. 82	+4. 7	+ 3. 3
26, 9:38 a. m	34. 91	34. 82	+6. 3	+ 5. 6
27, 10:22 a. m	35. 31	35. 22	+2. 7	+ 3. 3
27, 7:43 p. m	35. 91	35. 81	+1. 6	+ 0. 8
28, 9:05 a. m	36. 52	36. 51	+1. 8	+ 2. 5
Means of residuals from Feb. 26, 9:03 a. m			+3. 4	+ 3. 1

3. *MT*1876 was taken to a room and heated for 22 hours, February 28, 11:30 a. m., to March 1, 9:10 a. m., being kept constantly at temperatures between 70° and 80° F. It was then replaced in comparing-box and comparisons were made with *R*1876.

Comparisons of R1876 and MT1876—Continued.

AFTER SECOND HEATING OF *MT*1876.

Date.	Temperature of *R*1876.	Temperature of *MT*1876.	Residuals, *R*1876—(*MT*1876)*s* computed *minus* observed.	Residuals, *R*1876—(*MT*1876)*z* computed *minus* observed.
1881.	° F.	° F.	μ	μ
March 1, 9:25 a. m	37. 53	65. 71	+26. 4	+70. 6
1, 10:46 a. m	37. 93	46. 89	+ 5. 5	+30. 1
1, 11:36 a. m	38. 13	42. 15	+ 3. 0	+28. 3
1, 12:31 p. m	38. 13	40. 01	— 0. 2	+25. 6
1, 2:08 p. m	37. 93	38. 41	+ 2. 4	+24. 5
1, 3:05 p. m	37. 93	38. 16	+ 1. 0	+23. 7
1, 4:21 p. m	37. 88	37. 91	+ 2. 8	+23. 1
1, 7:51 p. m	37. 63	37. 61	+ 1. 4	+21. 6
March 2, 9:11 a. m	37. 02	36. 96	+ 2. 0	+18. 5
2, 9:04 p. m	36. 72	36. 61	+ 2. 6	+18. 7
3, 9:08 a. m	36. 32	36. 21	+ 2. 2	+19. 0
3, 8:51 p. m	36. 32	36. 21	+ 2. 4	+18. 9
4, 9:10 a. m	36. 32	36. 21	0. 0	+15. 1
Means of residuals from March 2, 9:11 a. m			+ 1. 8	+18. 0

4. *MT*1876 was again cooled by means of the freezing-mixture for twenty-four hours, from March 4, 9:30 a. m., to March 5, 9:30 a. m., being kept constantly at temperatures between —2° and —5° F. It was replaced in comparing-box March 5, 9:30 a. m., and comparisons with *R*1876 were resumed.

Comparisons of R1876 and MT1876—Continued.

AFTER SECOND COOLING OF *MT*1876.

Date.	Temperature of *R*1876.	Temperature of *MT*1876.	Residuals, *R*1876—(*MT*1876)$_S$ computed *minus* observed.	Residuals, *R*1876—(*MT*1876)$_Z$ computed *minus* observed.
1881.	° F.	° F.	μ	μ
March 5, 9:51 a. m.	36.32	Unknown		
5, 10:57 a. m.	36.17	....do		
5, 11:46 a. m.	36.17	31.62	—0.4	—9.4
5, 12:43 p. m	36.32	33.82	+1.3	—9.7
5, 2:14 p. m	36.37	35.22	+4.2	—4.9
5, 3:22 p. m	36.52	35.91	+3.1	—3.5
5, 4:29 p. m	36.57	36.26	+3.8	—0.1
5, 8:38 p. m	36.72	36.61	+4.9	—0.9
March 6, 9:50 a. m.	36.82	36.71	+4.5	—1.1
6, 8:04 p. m	37.22	37.16	+3.1	—2.1
7, 8:52 a. m	37.32	37.21	+3.7	—0.6
7, 7:55 p. m	37.58	37.51	+4.9	+0.3
8, 9:03 a. m	37.88	37.81	+5.0	+0.5
Means of residuals from March 6, 9:50 a. m.			+4.2	—0.6

§ **40.** An examination of these tables shows that in twenty-four hours after *MT*1876 had been replaced in the comparing-box, the thermometer lying on the upper surfaces of its two bars (those surfaces being freely exposed to the air of the comparing-box) indicated the same temperature as that lying on *R*1876 within about 0°.1 F.; that the residuals of the two steel bars after twenty-four hours reached 4$^\mu$.7 in one case, but that the range in these residuals at twenty-four hours after the several heatings and coolings was but from +2$^\mu$ to +4$^\mu$.7. If the temperature indicated by the thermometer on *MT*1876 was greater than the true temperature of the bars of this metre, then the computed *R*1876—*MT*1876 would be too small and the residuals too small. If the thermometer indicated too low a temperature for *MT*1876, the residuals would be too great. An examination of the tables shows that during the periods of rapid cooling and heating, when *MT*1876 would naturally lag behind the thermometers, the residuals are such as would be anticipated, and that within six hours after *MT*1876 was replaced in the comparing-box, although the thermometers still differed considerably, they gave the differences of temperature of the bars nearly enough to give no large residuals for the steel bars.

But excluding the first twenty-four hours after heating or cooling, by which time, judging from the steadiness of the thermometer-differences and the residuals, the bars *R*1876 and (*MT*1876)$_S$ had reached very nearly the same temperature, the mean residual of *R*1876 minus (*MT*1876)$_S$ was after heating, February 15–24, 1881, +2$^\mu$.6; after cooling, February 26–28, was +3$^\mu$.4; after heating, March 2–4, was +1$^\mu$.8; after cooling, March 6–8, was +4$^\mu$.2; a plus residual indicating that (*MT*1876)$_S$ observed was shorter than its computed value. These positive mean residuals are larger than was to be expected. They may be due in part to errors in the value of (*MT*1876)$_S$ in terms of *R*1876, used in computing residuals, which has not been very precisely determined, and in part to some unknown cause which made *R*1876 a little cooler than (*MT*1876)$_S$, although the thermometers indicated the opposite. The first supposition seems most important, although the fact that the residuals of *R*1876 and the two bars of *MT*1876 had simultaneously large residuals on February 18, 19, 20, and small residuals on February 22, seems to point to a temperature-disturbance or to errors in pointing at *R*1876. But the range in these mean residuals, which is only from +4$^\mu$.2 to +1$^\mu$.8, shows that heating and cooling (*MT*1876)$_S$ through 30° or 40° F. did not affect its length at the same temperature by quantities greater than 2$^\mu$.4, and such quantities may easily be due solely to errors in the short series of comparisons.

If the mean residuals of *R*1876 and (*MT*1876)$_Z$ be compared, they are—

February 15–24, after heating through 40° F., +19$^\mu$.2
February 26–28, after cooling through 40° F., + 3$^\mu$.1
March 2– 4, after heating through 40° F., +18$^\mu$.0
March 6– 8, after cooling through 40° F., — 0$^\mu$.6

It is thus seen that $(MT1876)_z$, having had its temperature raised for seven days from 37° F. to 77° F., on being allowed then to cool to the vicinity of its original temperature, was found to be $19^{\mu}.2$ longer than was to have been expected; that when, the next day after comparisons, it was cooled for twenty-four hours to about −3° F., or 40° below the temperature of the comparing-room, and was then allowed to return to the temperature of the comparing-room, it was found to be $+3^{\mu}.1$ longer than its normal length, having shortened in consequence of the cooling by $16^{\mu}.1$; that when after comparisons it was again heated to 77° F., or 40° above the temperature of the comparing-room, for twenty-four hours, and afterwards allowed to return to the temperature of the comparing-room, its length was then $18^{\mu}.0$ greater than its normal length, having increased its length during the heating experiment by $14^{\mu}.9$; that when it was again cooled for twenty-four hours to −3° F., or 40° below the temperature of the comparing-room, and then allowed to return to the temperature of the comparing-room, it was found to be $-0^{\mu}.6$ longer than its normal length, having shortened during the cooling experiment by $18^{\mu}.6$. It seems, then, that heating $(MT1876)_z$ through 40° F. and then allowing it to cool to its original temperature may increase its length by about 15^{μ}, and while in this condition, if it be cooled through about 40° and then be allowed to return to the temperature of the comparing-room, its length will be diminished by about 17^{μ}. Since the zinc metre, $(MT1876)_z$, expands $16^{\mu}.5$ per degree F., if this apparent change in length were due to differences of temperature of $R1876$ and $(MT1876)_z$, it would require an error in determining relative temperatures of the bars of 1° F., an error that is entirely out of the question.

As there are no other known sources of error that could produce such an apparent change in length, it seems to follow that this zinc bar one metre long changes its length, when its temperature is raised and afterwards reduced to the original temperature, in the same way that the four-metre zinc bar Z_1 does. On February 26 the zinc bar was near its normal length. The heating through about 40° on February 28 changed its residual from $+3^{\mu}.1$ to $+18^{\mu}.0$, or at the rate of $0^{\mu}.37$ per metre per degree Fahrenheit. In § 27 it is stated that during office comparisons in rising temperatures the residual of Z_2-S_2 increased at the rate of about 2^{μ} per degree of temperature rise, which for a zinc bar a metre long would be $0^{\mu}.5$ per degree.

§ **41.** Assuming that the steel bars have very nearly the same lengths at the same temperatures, as we are justified in doing, and attributing changes in length at the same temperature to the zinc bars alone, the following includes the observed facts:

When zinc bars having their normal lengths are heated they do not at once take the lengths for the new temperatures which they would ultimately take. By normal length is understood the mean length at a given temperature about which actual lengths may be made to fluctuate. The differences for changes of temperature of 10° or 15° F., may be $0^{\mu}.5$ per metre per degree of temperature-rise. When after being heated they are cooled to their original temperatures, they do not necessarily have their original lengths. The deviations may amount to $0^{\mu}.37$ per metre per degree F. of heating.

The phenomena are accounted for by Despretz's theory that when the form of a body is changed by the action of any cause, it does not at once take its old form when the cause is removed, and by the assumption that in changing form it does not at once take the new form due to the cause acting.

CORRECTION TO BE APPLIED TO LENGTH OF STEEL BAR S_1 ON ACCOUNT OF THE OBSERVED Z_1-S_1 NOT GIVING THE TRUE TEMPERATURE OF S_1.

§ **42.** Various suppositions have now been examined as to the causes of the large residuals for the observed values of Z_2-S_2 in the office comparisons, and the conclusion has been reached that they are due, at least in large part, to variations at the same temperature in the length of the zinc bar, Z_2. This, of course, is not their sole cause. The facts that the mercurial thermometers, which are depended on in the office comparisons to give the temperatures of the bars in a tube, do not exactly do so; that even with equal exposures the steel and zinc bars in a tube cannot heat at precisely the same rate; that the exposures of the two bars to external heating and cooling are not identical; that there may be some slight lateral flexure in the bars; all these causes combine to make up the residuals. The largest of these residuals is $-19^{\mu}.9$. Since Z_2-S_2 changes by about 38^{μ} for 1° F., this residual would give an error of about $\frac{1}{2}$° F. in the temperature of S_2, corresponding to $\frac{1}{300000}$ part of its length.

The numerous comparisons of Z_1 and Z_2 and S_1 and S_2 given in §§ 21, 23, show that those bars in expanding follow each other very closely, showing no special connection between temperature-changes and residuals, and comparisons of tubes 1 and 2 give residual-curves for observed Z_1-S_1 which follow closely those for Z_2-S_2. It may then be safely assumed that Z_1 and Z_2 behave alike as regards their regular residuals. Since tube 1 has been used in all base-measurements, it is very important to know whether during the hours in which base-measurement is usually carried on the observed Z_1-S_1 is in error, so that the true length of S_1 at any instant cannot be correctly obtained from it, and if it is, what the amount of this error is at any instant. As in the office comparisons, so too in base-measurement, this error will be the result of many separate causes, and since the temperature-variation during the portion of a day used for the measurement of a base may be ten times larger than the maximum daily temperature-changes which occur in the office comparisons, it might natually be feared that errors in Z_1-S_1 resulting from difference of temperature of Z_1 and S_1, or from inconstancy in the length of Z_1, would be largely increased.

From whatever source the errors arise, they would be detected by comparing once in an hour, during the time of base-measuring, both Z_1 and S_1 with some standard of known length. These comparisons would give directly the length at each comparison of S_1. Comparing this with the length computed for S_1 from the observed value of Z_1-S_1 and the values of the lengths and rates of expansion of Z_1 and S_1 derived from the office comparisons, the difference would be the error in the computed length of S_1; that is, in the length used in base-reductions as a first approximation. Such a system of comparisons during the measurement of a base is entirely impracticable. A tolerable approximation to its results can be obtained by selecting the part of the year in which base-measurements have usually been made, and comparing tube 1 under the same conditions of exposure to sun, air, and temperature-changes as those which exist during base-measurement with a known length, these comparisons being made at short intervals of time during the portion of the day ordinarily used in base-measurement so as to give the errors in S_1 or Z_1-S_1 throughout the measurement day; and being repeated on so many days that it may be safely assumed that the mean results of these days will approximate to the mean results which would have been obtained if the comparisons and the base-measurement had been carried on simultaneously throughout the measurement of a base.

§ **43.** Such comparisons were made between August 2 and September 17, 1880, by a party in charge of Assistant Engineer E. S. Wheeler, at Detroit. The awnings used in base-measurement to protect the tubes from the sun were put up in an open field on the Cass farm. Under them two microscopes, mounted on their stands, were set on brick piers 4^{m} apart sunk below the surface of the ground and disconnected with its surface, while beams of timber sunk in the ground supported the trestles carrying tube 1. The piers stood on an east and west line, as this has been the general direction of bases measured with the Repsold apparatus. The interval between the microscopes was very steady, the average daily variation between 8 a. m. and 5 p. m. being but $11^{\mu}.3$, and the maximum daily variation being $27^{\mu}.5$. The average variation during the interval of one hour between successive pointings at the 15-feet brass bar packed in ice was $3^{\mu}.5$, and the maximum $15^{\mu}.0$. This interval between the zeroes of the two microscopes was measured once in an hour, by putting the 15-feet brass bar, kept at 32° F., under the microscopes and reading on the 0^{m} and the 4^{m} graduations on the upper surface of the bar. This distance was the nearly invariable length with which S_1 and Z_1 were compared. The straightness of the bar was repeatedly tested by a stretched thread. No variations from straightness, sufficient to make it necessary to readjust it with reference to the stiff wrought-iron semi-cylinder in which it is carried, were observed during the work. It was mounted in this semi-cylinder precisely as described in Chapter II, § 13. The 15-feet brass bar was kept at 32° F. by being surrounded throughout its whole length by finely-broken ice, which covered the bar to the depth of 2^{cm} or 3^{cm}, free opportunity for the escape of the melted ice being given. The bar was packed in ice an hour before observations on it began in the morning, and was kept continuously buried in ice while the observations were continued. Experiments had been made on the Buffalo base-line to test the rapidity with which the brass bar packed in ice reached its minimum length, and hence the temperature of the melting ice. In these experiments the 15-feet bar in its receiver was placed between the comparators and read on before ice was put in the receiver. Ice was then rapidly placed in the receiver, this process requiring seventeen minutes to complete it. Readings of the comparators showed that the bar had reached an unchanging length in twelve minutes, or five minutes before the bar was completely and deeply covered. A similar experiment was made at the Cass farm August 14, 1880. At 9 a. m. the filling of the receiver with ice was begun, and it was completed at 9:10 a. m. The bar was then placed under the microscopes and read on at 9:29 a. m. and subsequently. At 9:29 a. m. it had already reached its permanent length. As the bar was always kept covered with ice, it may be assumed, therefore, that its temperature was very closely 32° F. If it had varied from this temperature at any time when being read on, the change, unless very small, would have shown itself in the difference between bar-length and microscope-interval.

The brass bar packed in ice was kept under the microscopes in comparisons for as short a period as practicable (usually about 4.6 minutes), to avoid unequal cooling effects on the microscopes and microscope-stands which might give rise to slight changes in the distance between the microscope's zeroes in the interval of about ten minutes between readings on the brass bar and on tube 1 in ordinary comparisons, since if the temperature of the microscope-stand was affected by the iced bar it would slowly return to the proper temperature when the bar was removed. To investigate the cooling effect of the ice on the microscope-interval, experiments were made on six days, when the temperature of the tube varied between 77° and 89° F. The brass bar in ice in these experiments was placed under the microscopes and read on, and then was allowed to remain for a time varying from four to thirteen minutes, and averaging nine minutes, and then again read on. In one case no change was observed; in four cases the microscope-interval appeared to diminish by from 0^{μ} to 3^{μ}, and in one case it appeared to increase by 2^{μ}. These quantities are not larger than possible errors of reading, and their smallness shows that the presence of the iced bar for these periods produced no sensible change in the interval between the microscope zeroes, and hence that for the interval of about 4.6 minutes, during which the iced bar in the ordinary comparisons was under the microscopes, such change is still less to be feared.

§ **44.** The necessity of knowing the relative inclination of the top-surfaces of the 15-feet bar at the 0^{m} and the 4^{m} graduations, whenever this interval is used in comparisons, is fully explained in § 54. Immediately before and immediately after the bar was placed under the microscopes, and while resting on supports at the same distance apart as when it was under the microscopes, a level (one division $=1''.01$) was read at each end of the bar, the 0^{m} or 4^{m} graduation bisecting the interval of 18 inches between its feet. Since, in the comparisons of S_2 and B in the office, a level whose feet were only $4\frac{3}{4}$ inches apart had been used, and since the upper surface of the bar was not perfectly plane even for distances of $0^{m}.5$, a careful series of comparisons was made, so as to be able to reduce the difference of inclinations of the two ends of the bar at the 0^{m} and 4^{m} marks given by one level to those given by the other. The weight of the ice around the 15-feet brass bar averaged about 68 pounds, but varied from 56 to 76 pounds. As the ice melted away fresh ice was added, say once in ten to twenty minutes, the amount added being usually 10 pounds or less, but sometimes rising to 20 pounds. An experiment was tried to ascertain approximately the change in form of the brass bar and its supporting receiver when loaded. Fifty pounds' weight was distributed uniformly along the receiver when there was no ice in it. Before the loading the relative inclination of the ends of the bar was $-204''$; while loaded it was $+34''$. The variations in the weight of the ice produced considerable flexures in the brass bar. The average daily variation in the relative inclination of the ends of the bar between 8 a. m. and 5 p. m. was $88''$, while the maximum and minimum variations were $223''$ and $32''$, respectively. A variation of $1'$ corresponds to $4^{\mu}.06$ change in the interval from 0^{m} to 4^{m}, since the half height of the bar is 14^{mm}. Since no ice was added while the bar was under the microscopes, its inclination at the instant of pointing was derived by interpolation from the preceding and following determinations of inclination. Since the levels on the 15-feet brass bar could not be read while it was under the microscopes, they were read immediately before and after, while its receiver was supported at the same two points as when under the microscopes, in order that the flexures of the bar might be the same.

§ **45.** The question whether in the Cass-farm work or during the measurement of a base with tubes of the Repsold apparatus the relative length of the bars in a tube is sensibly influenced by the position of the sun with reference to the tube is an interesting one, inasmuch as the base-apparatus used in the Indian Survey (G. T. Survey of India, Vol. I, Chap. VIII) is so affected. Since the bases of the Lake Survey measured with the Repsold apparatus have all been measured in both directions, if such an error existed it should be partially eliminated in the mean of the two measurements.

An investigation into the effect of the sun's position on the relative lengths of the steel and zinc bars of a tube was made in September and October, 1877. The tubes were compared in the open air under the awning used in base-measurement with the interval between two firmly-mounted microscopes, this interval being checked once in two hours by pointings at graduations on the brass bar kept at 32° F. by ice. In comparisons they were a part of the time in the "direct" position and a part of the time turned end for end, or, in the "reversed" position. In the direct position the forward end of the tube pointed South 55° West, and the zinc bar was on the southeast side. In the reversed position the zinc bar was on the northwest side of the tube. If the sun heated the southeast side of the tube and the bar next it more than the bar on the northwest side, Z_1 would be too long relatively to S_1 when the tube was in the direct position, and too short when it was in the reversed position. The direction of the tube was such that for the time covered by the comparisons (8 a. m. to 5 p. m.) the sun was practically on the same side of the tube. If a series of comparisons with tube direct be made giving a mean value of S_1-B and a mean value for Z_1-S_1, and if then the tube be reversed and another series of comparisons be made in which the mean value of S_1-B is the same as before, it might be expected that if the sun's position was without influence the mean value of Z_1-S_1 would also be the same as before. As it was not practicable to get mean temperatures or mean S_1-B exactly the same for the direct and reversed positions, the mean Z_1-B for the reversed position was reduced with approximate expansions to its proper value for the mean S_1-B before reversal. As this reduction was in any case for less than 5° F., any slight error in the expansions would not affect seriously the result.

The values of Z_1-S_1 and S_1-B were taken out for each hour between 8 a. m. and 5 p. m. for days in which tube 1 was compared in the reversed position, and tube 2 in the direct position. For the opposite positions such days and parts of days were selected as would give nearly the same mean temperatures as for the first positions. For tube 1 direct comparisons on eight days were used and when reversed, five days. The result was that for the same S_1-B, Z_1-S_1 was $0^{\mu}.5\pm3^{\mu}.8$ longer in the direct position, or when the zinc bar was next the sun. For tube 2, three days' comparisons were used with the tube direct, and five days with the tube reversed. The result was that for the same S_1-B, Z_2-S_2 was $1^{\mu}.7\pm2^{\mu}$ longer when the zinc bar faced the sun.

These results are so small that they give no evidence of any relation between the relative length of the bars as used in base-measuring and the position of the sun. Much larger differences might be ascribed to the varying length of the zinc bars at the same temperature, and the resulting effect of such variation on the means of $Z-S$ for the few days of comparisons.

§ **46.** The routine of ordinary comparisons was as follows: The 15-feet brass bar packed in ice was put under the microscopes and read on at each full hour between 8 a. m. and 5 p. m. The inclinations of both ends were read with a level immediately before and after its being under the microscopes. At other times tube 1 was kept under the microscopes, and both its steel and zinc bars were pointed at with the microscopes once in twenty minutes. Thermometers inserted in the ends of the tubes were read at the same time. The order of pointings at tube 1 was: 1, steel bar; 2, zinc bar; 3, steel bar; 4, thermometers. The interval between the microscopes at each hour resulted from the pointings at the 0^{m} and 4^{m} marks on the brass bar, that distance being known. Between two hours it is assumed that the change in the microscope-interval is proportional to the time. The lengths of steel and zinc bars result from the microscope-pointings at them and from the known interval between the microscope-zeroes.

Care was taken that the water from the melting ice drained rapidly from the receiver. Besides the daily comparisons between 8 a. m. and 5 p. m. there were periods in which comparisons were kept up continuously during the night in order to examine the conduct of the zinc bar at minimum temperatures. The periods of continuous comparisons were from 8 a. m. August 23, 1880, to 7:20 p. m. August 26, 1880, and from 8 a. m. August 31 to 5 p. m. September 3, 1880. Comparisons were made on twenty-six days. The maximum temperature indicated by the thermometers in tube 1 during the work was 91° F., and the minimum temperature was 38°.5 F. The minimum temperature-range in twenty-four hours on days of comparisons was 10° F., and the maximum was 30° F.

§ 17. In reducing the Cass-farm work the following method was used: The microscope-readings on the brass bar were corrected for run and for errors of graduation of the short scales on the brass bar, so that the microscope-readings were referred to the 0^m and 4^m graduations on the brass bar. The observed relative inclination of the top-surface of the bar at 0^m and 4^m graduations gave, with the half height of the bar (14^{mm}), the correction needed to reduce its length at the observed relative inclination of the ends to what it would have been at zero inclination as indicated by the level (feet $0^m.45$ apart). From these corrections the difference of the interval between the microscope-zeroes and the interval between 0^m and 4^m on the brass bar, when at these points its surfaces had the same inclination longitudinally, at once resulted. These differences were plotted on a scale of $0^{\mu}.2=1^{mm}$, or $\frac{5000}{1}$, times of observation being the abscissas. The differences between the microscope-interval and the length of the steel bar S_1 were obtained and plotted in the same way on a scale of $2^{\mu}.5=1^{mm}$, or $\frac{1000}{2.5}$. So, too, the differences between the microscope-interval and the length of the zinc bar Z_1 were computed in the same way, the correction for non-rectangularity of microscope-threads being included, and then were plotted on a scale of $4^{\mu}=1^{mm}$, or $\frac{1000}{4}$. The scale of the abscissas was the same for the three curves, namely, 2 minutes = 1^{mm}. Through the three series of points obtained by these plottings curves were drawn, which, while fairly representing the points, yet deviated from them in no case by quantities greater than 14^{μ} for the brass bar, than $2^{\mu}.5$ for the steel bar, or than 4^{μ} for the zinc bar. The average deviations were less than 2^{μ} for the brass bar and less than 1^{μ} for the others.

As the times of observation on the different bars differed by a few minutes, in order to get results corresponding to simultaneous readings the ordinates of each of these curves were read from the curves for each twenty minutes between 8 a. m. and 5 p. m. and tabulated. From these tables the values of S_1-B_{32}, and of Z_1-S_1, for every twenty minutes during comparisons can be at once derived, and are to be considered as observed quantities. The scales used for plotting were so large that for the smallest of them 1^{μ} could be estimated. Hence no errors that are serious were introduced by the graphical process.

When, as in office comparisons, the relative inclination of the top-surfaces of the 15-feet brass bar about the 0^m and 4^m marks is measured with the $4\frac{3}{4}$-inch level, we have—

$$S_2-B \text{ at } 32° \text{ F.}=+1498^{\mu}.88. \quad § 56.$$
$$S_1-S_2 \text{ at } 32° \text{ F.}=+\quad 32^{\mu}.74. \quad § 24.$$

Hence,

$$S_1-B \text{ at } 32° \text{ F.}=+1531^{\mu}.62$$

Now in the Cass-farm work all observations on the brass bar have been corrected so as to reduce them to the values they would have had if the relative inclinations of the ends of the brass bar had been zero when measured with the 18-inch level. But when the 18-inch level indicates zero difference of inclination of the two ends, the $4\frac{3}{4}$-inch level used in office comparisons indicates $+270''$, the plus sign being used when the effect of the relative inclination of the ends of the bar is to shorten B. Then, since the half height of the brass bar is $14^{mm}.2$, B is shorter for zero inclination of the 18-inch level than in the office comparisons by $14^{mm}.2 \tan 270''=18^{\mu}.5$, and hence for this inclination $(S_1-B)_{32°}=1550^{\mu}$. From §§ 62, 64,

$$S_1=S_{1_{32°}}+24^{\mu}.42\,(t-32)+0^{\mu}.0074\,(t-32)^2$$
$$E_{Z_1}-E_{S_1}=38^{\mu}.0887 \text{ for } 1° \text{ F., and}$$
$$Z_1=S_1 \text{ at } 60°.292 \text{ F.}$$

From these values we can compute for any observed value of $S_1-B_{32°}$ the corresponding value of Z_1-S_1 as it should result from the office determinations of relative lengths and expansions of these bars. Reciprocally, using this computed value of Z_1-S_1, and the relative lengths and expansions of the bars determined by the office comparisons, the true or observed length of S_1 can be computed for any twenty minutes. Hence, the observed values of Z_1-S_1 in the Cass-farm work need a correction before they can be used to obtain at any instant the true length of S_1. That correction is, computed Z_1-S_1 minus observed Z_1-S_1.

These differences between the computed and observed values of Z_1-S_1 are the net results of the Cass-farm work, and in the following tables are given for each day of the comparisons, and for every twenty minutes of those days between 8 a. m. and 5 p. m. The tables also give for every twenty minutes the means of the corresponding residuals for the whole twenty-six days of comparisons, and also the means of the residuals for twenty-one selected days, these alone being used in correcting a measurement. These selected days were such as would have permitted base-measurement on them. The other five days, which are rejected in these last means, were too stormy to have permitted base-measurement. These five days were rejected in order that the mean residuals thus obtained might represent as closely as possible those which would be obtained from comparisons carried on daily during the measurement of a base, and in using these mean residuals, as will be done, to correct the observed Z_1-S_1 in base-measurements, it is assumed that they do fairly represent the errors in the observed Z_1-S_1 in such measurements with the Repsold apparatus.

Cass-farm work. Residuals, (Z_1-S_1) computed minus (Z_1-S_1) observed.

Date.	8 a. m.	8:20 a. m.	8:40 a. m.	9 a. m.	9:20 a. m.	9:40 a. m.	10 a. m.	10:20 a. m.	10:40 a. m.	11 a. m.	11:20 a. m.	11:40 a. m.	12 m.	12:20 p. m.
1880.	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ
Aug. 16	−15.8	−20.9	−14.5	−10.8	−10.9	−7.0	−3.8	−5.4	−4.4	−4 3	+0.3	+3.6	−2.7	−5.1
18	−2.1	−2.5	−11.4	+0.7	−7.8	−1.0	+1.3	−1.8	−4.0	−0.5	+5.2	+10.9	+7.5	+0.3
19, rainy	−17.0	−20.2	−23.4	−18.1	−6.2	−9.9	−5.1	−3.6	−6.9	−3.1	−2.9	+0.6	−5.3	+3.1
20	−14.8	−24.5	−26.1	−17.3	−13.8	−9.2	−13.2	−17.2	−14.4	−11.6	−14.4	−14.3	−1.7	+0.4
21	−32.6	−33.7	−27.0	−29.7	−26.5	−26.9	−19.4	−17.5	−14.7	−17.4	−16.0	−12.7	−8.9	−8.6
23	−9.9	−6.2	−14.4	−3.2	−1.6	−1.9	−3.6	+0.1	+8.2	+9.1	+8.4	+16.6	+16.9	+17.5
24, rainy	−12.2	−16.7	−14.3	−18.6	−27.2	−17.0	−15.5	+6.9	−2.3	−9.2	−8.2	−1.8	−2.5	+5.4
25, rainy	−12.6	−23.1	−17.3	−11.9	−14.5	−14.2	−19.0	−14.5	−13.6	−18.2	−18.3	−14.4	−18.6	−22.4
26, rainy	−12.6	−13.3	−5.5	−9.3	−6.6	+3.5	+1.9	+1.1	+1.8	−0.0	−3.7	+3.9	+4.6	+9.1
27	+17.2	+18.1	+12.2	+13.4	+4.9	+5.1	+8.7	+0.6	+16.0	+18.3	+17.5	+13.2	+11.4	+16.7
28, rainy	−4.8	−9.8	−4.3	+2.8	−3.7	+0.8	−7.9	+4.0	+8.2	+12.4	+13.5	+21.9	+20.2	+14.0
31	−1.2	−3.4	−7.1	−3.8	−1.5	+4.8	+2.8	−25.1	−4.0	+0.7	+4.2	+7.0	+3.0	+11.3
Sept. 1	3.6	−7.1	+6.0	−6.5	−1.9	+1.7	+8.7	+7.6	+11.0	+9.8	+10.5	+16.2	+17.5	+15.2
2	+6.7	+0.2	−0.1	+2.2	+3.2	+6.6	+6.6	+13.2	+14.9	+17.5	+14.6	+16.3	+16.9	+13.0
3	−2.2	+1.4	−0.0	−0.3	+1.4	+0.0	−1.2	−5.6	−4.8	−3.9	−3.6	+26.3	−0.4	+1.9
4	−5.1	−3.5	−0.8	+1.4	−1.2	−3.5	+11.1	+2.9	+7.9	+7.0	+6.7	+12.8	+12.9	+7.5
6	+8.5	−4.2	+8.4	+8.7	+8.3	+12.7	+13.4	+9.3	+12.8	+12.5	+10.4	+7.4	+11.8	+14.7
7	−24.7	−12.6	−13.6	−10.3	−10.6	−7.5	−5.0	−10.3	−9.9	−5.2	−4.6	−3.9	−2.1	−1.3
8	−22.7	−21.4	−15.8	−13.7	−15.0	−12.7	−5.3	+0.3	−1.2	+1.0	−10.2	+3.0	+8.0	+3.4
9	−15.0	−8.6	−10.1	−9.4	−6.2	−2.0	−2.3	−4.5	−7.5	+6.9	+5.7	+7.0	+6.2	+18.3
10	−15.4	−12.7	−10.2	−7.9	−3.6	+5.5	+1.6	+4.1	+4.7	+5.6	+18.7	+10.0	+11.4	+13.7
11	−19.1	−29.6	−16.2	−6.6	−8.9	+5.9	+9.1	+6.4	+6.6	+6.2	+13.4	+18.8	+24.0	+17.6
13	−12.1	−12.2	−9.3	−12.9	−12.5	−4.3	−6.6	−3.7	−3.3	−0.4	+2.5	+5.9	+3.4	+2.9
14	−14.6	−19.7	−17.5	−21.8	−9.4	−10.8	−7.2	−9.6	−4.8	−3.2	+7.0	+11.3	+7.0	+8.7
15	−10.5	−11.0	−12.2	−14.1	−9.8	−7.8	+0.3	−3.6	+10.9	+9.7	+9.8	+6.2	+8.2	+17.5
16	−11.0	−2.2	+9.0	+7.2	+5.1	+8.7	+9.5	+3.4	+4.1	+5.0	+13.8	+16.1	+22.5	+19.4
Mean of 21 fair days.	−9.5	−10.3	−8.1	−6.4	−5.6	−2.1	+0.3	−2.7	+1.1	+3.0	+4.8	+8.5	+8.2	+8.8
Mean of all the 26 days	−10.0	−11.5	−9.1	−7.3	−6.8	3.1	−1.5	−2.4	+0.4	+1.7	+3.1	+7.2	+6.6	+7.5

Cass-farm work. Residuals, (Z_1-S_1) *computed minus* (Z_1-S_1) *observed*—Continued.

Date.	12:40 p. m.	1 p. m.	1:20 p. m.	1:40 p. m.	2 p. m.	2:20 p. m.	2:40 p. m.	3 p. m.	3:20 p. m.	3:40 p. m.	4 p. m.	4:20 p. m.	4:40 p. m.	5 p. m.
1880.	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ
Aug. 16	+ 1.6	+ 5.5	+ 4.0	+ 7.1	+ 1.2	+ 8.3	+ 8.3	+12.7	+11.3	+ 8.0	+10.0	+ 6.6	+ 9.8	+11.0
18	+14.6	+12.0	+11.3	+ 3.6	+ 6.9	+ 4.1	+16.8	+16.0	+10.0	+13.6	+11.6	+ 8.2	+ 9.3	+ 6.3
19, rainy	+ 1.9	− 1.4	− 8.8	− 8.5	− 7.9	− 3.4	− 4.8	− 0.1	− 5.7	− 8.4	−10.8	−13.7	−12.1	−15.1
20	− 6.8	− 0.6	+ 3.3	+ 0.1	− 2.4	+ 6.1	+ 5.7	− 5.1	+ 2.3	+ 6.2	+ 8.0	+ 8.6	+ 9.9	+ 4.8
21	− 8.3	− 8.1	− 2.7	− 1.6	+ 4.2	− 0.6	− 1.0	− 2.0	− 3.9	+ 4.7	+ 0.7	+ 2.2	+ 0.7	+ 2.7
23	+18.2	+15.9	+18.5	+21.2	+13.7	+33.0	+13.1	+20.4	+20.4	+15.4	+12.3	+16.9	+19.3	+11.6
24, rainy	+15.4	+ 5.9	+ 4.3	+28.0	+35.5	+27.9	+21.9	+14.9	+24.1	+23.1	+16.2	+ 8.4	+ 5.7	+ 5.2
25, rainy	−23.1	−16.0	−13.4	−12.7	−15.1	−15.4	−17.9	−15.3	−19.8	−15.8	−17.4	−18.4	−17.5	−20.1
26, rainy	+ 7.9	+ 7.8	+ 3.8	+ 7.3	+ 7.9	+ 0.9	+ 6.5	+14.2	+ 5.8	+ 8.8	+13.9	+12.7	+11.7	+14.8
27	+20.4	+26.0	+22.7	+28.8	+28.3	+32.4	+30.5	+31.5	+26.5	+31.5	+33.0	+29.4	+32.8	+34.8
28, rainy	+24.5	+30.4	+22.9	+16.1	+20.3	+30.8	+39.3	+27.9	+21.4	+18.0	+ 8.2	+13.0	+ 8.4	+ 2.7
31	+ 9.5	+10.7	+14.4	+14.0	+13.6	+15.1	+18.1	+18.6	+19.6	+19.1	+15.6	+22.1	+17.5	+21.1
Sept. 1	+16.9	+25.7	+29.3	+25.4	+27.0	+24.1	+29.1	+27.0	+39.5	+27.8	+30.7	+31.0	+24.3	+13.3
2	+18.6	+18.1	+19.8	+17.0	+21.9	+22.4	+22.9	+25.8	+28.3	+26.7	+26.6	+22.5	+19.9	+14.8
3	+ 3.6	−20.2	+10.1	+15.0	+12.0	+11.6	+29.7	+18.2	+18.3	+23.8	+18.8	+ 7.7	+10.2	+ 9.7
4	+11.1	+16.3	+17.5	+18.1	+14.2	+19.3	+16.9	+16.4	+18.4	+16.9	+22.0	+20.0	+19.9	+29.4
6	+13.1	+11.8	+20.6	+23.1	+13.6	+19.1	+16.1	+14.5	+10.3	+ 6.6	+ 8.2	+ 3.4	− 6.5	+ 3.4
7	+ 1.1	+ 4.3	+ 3.6	+ 6.7	− 3.8	+ 3.2	− 1.1	+ 8.8	+ 7.0	+ 5.7	+ 7.7	+ 8.8	+ 6.3	+ 5.4
8	+ 5.7	+ 6.2	+ 5.1	+11.0	+ 9.0	+16.5	+12.5	+ 9.5	+10.0	+12.6	+13.0	+ 9.0	+14.6	+ 6.2
9	+19.7	+17.7	+15.2	+22.0	+19.6	+20.9	+21.4	+19.4	+19.7	+13.9	+12.8	+24.3	+27.7	+27.2
10	+17.4	+20.9	+22.8	+22.4	+26.7	+27.6	+24.4	+24.4	+30.1	+30.6	+26.6	+26.1	+25.9	+21.6
11	+22.2	+24.0	+32.3	+27.4	+25.1	+28.3	+25.4	+28.0	+26.0	+28.0	+25.8	+20.1	+18.9	+29.3
13	+ 4.4	+ 0.9	+ 5.0	+ 4.0	+ 6.5	+ 3.0	+ 2.5	− 3.4	− 4.7	+ 0.2	+ 2.3	+ 4.8	− 3.5	− 0.5
14	+10.6	+19.2	+19.2	+12.5	+16.8	+19.5	+17.2	+22.3	+26.3	+24.2	+22.0	+23.2	+15.7	+25.8
15	+25.5	+19.7	+20.1	+22.1	+25.0	+22.4	+27.7	+31.1	+25.8	+23.8	+29.2	+21.8	+23.0	+18.4
16	+27.5	+30.4	+24.3	+31.0	+27.2	+37.2	+34.8	+35.5	+36.0	+32.7	+30.7	+31.7	+31.5	+37.5
Mean of 21 fair days.	+11.7	+12.2	+15.1	+15.8	+14.6	+17.5	+17.8	+17.6	+18.0	+17.7	+17.5	+16.6	+15.6	+15.9
Mean of all the 26 days	+10.5	+10.9	+12.5	+13.9	+13.3	+15.9	+15.8	+15.8	+15.5	+15.3	+14.5	+13.5	+12.4	+12.4

Mean temperatures F. of twenty-six days of comparisons, made on the Cass farm, for every twenty minutes from 8 a. m. to 5 p. m., derived from the mean observed metallic temperatures for the same times.

Time.	Temperature.	Time.	Temperature.
	° F.		° F.
8:00 a. m.	61.8	12:40 p. m.	72.9
8:20 a. m.	62.5	1:00 p. m.	73.3
8:40 a. m.	63.2	1:20 p. m.	73.7
9:00 a. m.	64.1	1:40 p. m.	74.0
9:20 a. m.	65.0	2:00 p. m.	74.2
9:40 a. m.	65.9	2:20 p. m.	74.2
10:00 a. m.	66.8	2:40 p. m.	74.3
10:20 a. m.	67.9	3:00 p. m.	74.3
10:40 a. m.	68.8	3:20 p. m.	74.2
11:00 a. m.	69.7	3:40 p. m.	74.1
11:20 a. m.	70.5	4:00 p. m.	74.0
11:40 a. m.	71.1	4:20 p. m.	73.9
12:00 m.	71.8	4:40 p. m.	73.7
12:20 p. m.	72.4	5:00 p. m.	73.5

§ **48.** Although individual days give widely varying residuals, yet the means of all days for each twenty minutes are quite regular. When plotted with times as abscissas on such scales that two minutes or $0^{\mu}.1$ were easily measured, a regular curve somewhat resembling an hyperbola could be traced among the plotted points, such that no point deviated from the curve by more than three. microns. This curve is given in Plate XXV, and was assumed to represent the mean residuals The intervals of time in which the ordinates of this curve changed from half a micron less than an entire number of microns to half a micron more were then taken from the curve and tabulated, this entire number of microns being considered the correction for that period. The following is that table.

Table giving periods of time corresponding to corrections to $Z_1 - S_1$.

[Derived from Cass-farm comparisons.]

Time.	Correction.	Time.	Correction.
	μ		μ
7:58 a. m.	−11	11:15 a. m.	+ 5
8:10 a. m.	−10	11:28 a. m.	+ 6
8:22 a. m.	− 9	11:41 a. m.	+ 7
8:35 a. m.	− 8	11:53 a. m.	+ 8
8:47 a. m.	− 7	12:06 p. m.	+ 9
8:59 a. m.	− 6	12:18 p. m.	+10
9:11 a. m.	− 5	12:30 p. m.	+11
9:24 a. m.	− 4	12:43 p. m.	+12
9:36 a. m.	− 3	12:57 p. m.	+13
9:49 a. m.	− 2	1:09 p. m.	+14
10:01 a. m.	− 1	1:23 p. m.	+15
10:14 a. m.	− 0	1:39 p. m.	+16
10:26 a. m.	+ 1	1:58 p. m.	+17
10:38 a. m.	+ 2	2:06 p. m.	+18
10:50 a. m.	+ 3	3:49 p. m.	+17
11:03 a. m.	+ 4	4:29 p. m.	+16
11:15 a. m.		5:01 p. m.	

§ **49.** By aid of this table to find the correction to any section of the base on account of the values of $Z_1 - S_1$ observed during the measurement of the base not being precisely those which give the correct length of S_1, the following method is adopted: For each interval of time in the table, the number of tubes in the section which were measured in that interval is counted and multiplied by the corresponding value of the correction. These products are summed so as to include all tubes measured in that section. The sum, when multiplied by $\frac{E_{S_1}}{E_{Z_1} - E_{S_1}}$, will give the sum of the corrections for this section to the length of the section, which has already been computed without taking the correction into account. Since the sum of the corrections is small, sufficient accuracy will be obtained by using the first term in the value of $\frac{E_{S_1}}{E_{Z_1} - E_{S}}$, namely (Chapter X, § 6), 0.6522. It is not to be expected, then, that the application of the correction to the observed $Z_1 - S_1$ will on all days increase the accuracy of the work. On some days it will doubtless diminish it. But in the whole measurement of the base, the regularity of the correction-curve is too great, and its form is too certain to leave any doubt of its substantial accuracy, or that in many days of measurement it will increase the precision of the work. Since the largest correction to $Z_1 - S_1$ from the table in § 48, between 8 a. m. and 5 p. m., is 18^{μ}, and since this is multiplied by 0.6522 to obtain the correction to S_1, this last correction at its maximum is only 12^{μ} or $\frac{1}{330000}$ part of S_1. On the average for the Chicago, Sandusky, and Olney Bases the measurement ceased for dinner at noon from 11:27 a. m. to 1:18 p. m., or for $1^{h}\ 51^{m}$. Since the evening placing of the cut-off plates and the starting from them in the morning took a good deal of time, rapid measurement did not in the average begin before 8:29 a. m., nor continue after 3:52 p. m. If the mean of the mean residuals be taken for the period of actual measurement, it is found to be 7^{μ}. Multiplying it by 0.6522, there results $4^{\mu}.6$ as an approximation to the average correction to the length of S_1 during the measurement of a base or $\frac{1}{870000}$ part.

As already remarked, while the mean residuals for twenty-one days give a very regular curve, the series of residuals for a given day may give a very irregular curve, or it may approach a straight line as on August 25, which was the second of three rainy days, with residuals all of the same sign. The residuals for a given day evidently depend on the temperature-fluctuations for each day and also on the value of the residual at the beginning of the day.

If a simple relation was known, connecting the residuals on a given day with the temperatures on that day and the preceding day, then, instead of using in reduction mean residuals for all days of measurement, the special residuals for each day might be computed. This would undoubtedly give a more accurate value for the part of the measurement executed on each day, though it would probably make little change in the value for the whole base. But at present it is useless to attempt such a detailed correction.

§ **50.** It has already been stated that from 8 a. m., August 23, 1880, to 7:40 p. m., August 26, and from 8 a. m., August 31, to 5 p. m., September 3, 1880, the comparisons of the 15-feet brass bar packed in ice with tube 1 at the Cass farm were continuous. These observations have been reduced, and the values (Z_1-S_1) computed minus (Z_1-S_1) observed have been obtained for each twenty minutes of these periods, precisely as in the daily periods from 8 a. m. to 5 p. m. Those values are given in the following table and also the mean of the residuals for each twenty minutes.

The mean residuals will be found plotted in Plate XXVI.

Table of residuals of (Z_1-S_1) from continuous comparisons made on Cass farm.

Date.	8 a. m.	8:20 a. m.	8:40 a. m.	9 a. m.	9:20 a. m.	9:40 a. m.	10 a. m.	10:20 a. m.	10:40 a. m.	11 a. m.	11:20 a. m.	11:40 a. m.
1880.	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ
Aug. 23	−9.9	−6.2	−14.4	−3.2	−1.6	−1.9	−3.6	+0.1	+8.2	+9.1	+8.4	+16.6
24	−12.1	−16.7	−14.3	−18.6	−27.2	−17.1	−15.4	+6.9	−2.2	−9.2	−8.2	−1.8
25	−12.5	−23.1	−17.3	−11.8	−14.3	−14.2	−19.0	−14.5	−13.6	−18.2	−18.3	−14.5
26	−12.6	−13.3	−5.5	−9.3	−6.7	+3.5	+2.0	+1.2	+1.8	−0.1	−3.7	+3.9
31	−1.2	−3.4	−7.1	−3.8	−1.5	+4.8	+2.8	−25.1	−4.0	+0.7	+4.2	+7.0
Sept. 1	−3.6	−7.1	+6.1	−6.5	−1.9	+1.8	+8.7	+7.6	+11.0	+9.8	+10.5	+16.2
2	+6.8	−0.2	−0.1	+2.2	+3.2	+6.6	+6.4	+13.2	+14.9	+17.5	+14.6	+16.3
3	−2.2	+1.4	+0.1	−0.2	+1.4	+0.1	−1.2	−5.6	−4.8	−3.9	−3.6	+26.3
Means	−5.91	−8.58	−6.56	−6.40	−6.08	−2.05	−2.41	−2.03	+1.41	+0.71	+0.49	+8.75

Date.	12 m.	12:20 p. m.	12:40 p. m.	1 p. m.	1:20 p. m.	1:40 p. m.	2 p. m.	2:20 p. m.	2:40 p. m.	3 p. m.	3:20 p. m.	3:40 p. m.
1880.	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ
Aug. 23	+16.9	+17.5	+18.2	+15.9	+18.5	+21.2	+13.7	+27.8	+21.8	+20.4	+20.4	+15.4
24	−2.5	+5.4	+15.4	+5.9	+4.3	+28.0	+35.5	+33.0	+13.1	+14.9	+24.1	+23.1
25	−18.6	−22.4	−23.0	−15.9	−13.4	−12.6	−15.1	−15.5	−18.0	−15.3	−19.8	−15.8
26	+4.6	+9.1	+7.9	+7.8	+3.8	+7.3	+7.9	+0.9	+6.5	+14.2	+5.8	+8.8
31	+3.0	+11.3	+9.5	+10.7	+14.4	+14.0	+13.6	+15.1	+18.1	+18.6	+19.6	+19.1
Sept. 1	+17.5	+15.2	+16.9	+25.7	+29.3	+25.4	+27.0	+24.1	+29.1	+27.0	+39.5	+27.8
2	+16.9	+13.0	+18.6	+18.1	+19.8	+17.0	+21.7	+22.4	+22.9	+25.8	+28.3	+26.7
3	−0.4	+1.9	+3.6	−20.2	+10.1	+15.0	+12.0	+11.6	+29.7	+18.2	+18.3	+23.8
Means	+4.68	+6.38	+8.39	+6.00	+10.85	+14.41	+14.54	+14.92	+15.40	+15.48	+17.02	+16.11

Date.	4 p. m.	4:20 p. m.	4:40 p. m.	5 p. m.	5:20 p. m.	5:40 p. m.	6 p. m.	6:20 p. m.	6:40 p. m.	7 p. m.	7:20 p. m.	7:40 p. m.
1880.	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ
Aug. 23	+12.3	+16.9	+19.3	+11.6	+17.0	+11.8	+8.6	+8.9	+0.1	+3.8	+16.1	+7.7
24	+16.2	+8.4	+5.7	+5.2	+3.5	+4.0	+0.4	−4.6	−9.7	−6.2	−6.9	−10.6
25	−17.3	−18.3	−17.5	−20.1	−22.2	−23.3	−20.4	−16.4	−20.6	−17.8	−21.0	−23.7
26	+13.9	+12.7	+11.7	+14.8	+14.3	+12.8	+13.3	+15.3	+9.1	+4.8	+9.1	+11.8
31	+15.6	+22.1	+17.5	+21.1	+19.4	+12.8	+21.7	+10.1	+12.4	+11.2	+10.4	+9.6
Sept. 1	+30.7	+31.0	+24.3	+13.3	+16.6	+7.4	+10.8	+25.2	+23.9	+19.3	+20.1	+15.0
2	+26.6	+22.4	+19.9	+14.8	+14.1	+15.1	+11.4	+15.8	+17.1	+17.8	+14.0	+12.1
3	+18.8	+7.7	+10.2	+9.7	......	......	......	......	......	......	......	......
Means	+14.60	+12.86	+11.39	+8.80	+8.96	+5.80	+6.54	+7.76	+4.61	+4.70	+5.97	+3.13

Table of residuals of $(Z_1 - S_1)$ *from continuous comparisons made on Cass farm*—Continued.

Date.	8 p. m.	8:20 p. m.	8:40 p. m.	9 p. m.	9:20 p. m.	9:40 p. m.	10 p. m.	10:20 p. m.	10:40 p. m.	11 p. m.	11:20 p. m.	11:40 p. m.
1880.	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ
Aug. 23	+ 6.7	+ 4.6	+ 5.5	+ 0.9	—12.2	—10.3	— 9.9	—14.1	—17.1	—14.7	—12.5	—13.1
24	—12.8	—13.9	—21.4	—21.8	—23.0	—15.5	—27.5	—25.1	—24.3	—28.7	—28.9	—31.1
25	—27.1	—19.8	—20.9	—25.2	—31.5	—34.6	—33.5	—23.5	—27.8	—27.6	—24.5	—29.5
26	....	....	....	....	....	....	....	....	....	....	....	....
31	+ 6.9	+ 4.4	+ 3.8	+ 2.6	+12.4	+ 2.3	— 1.5	— 8.2	+ 0.1	— 2.1	+ 1.3	— 4.2
Sept. 1	+10.0	+ 9.6	+ 6.9	— 2.5	+ 8.0	+ 5.5	+ 8.3	+ 7.3	+ 5.3	+ 3.0	+ 1.6	+ 1.4
2	+ 7.0	+ 5.6	— 0.7	— 1.3	— 0.2	— 4.8	+ 3.8	— 8.5	—15.8	—13.8	—10.5	—17.2
3	....	....	....	....	....	....	....	....	....	....	....	....
Means	— 1.55	— 1.58	— 4.47	— 7.88	— 7.75	— 9.57	—10.05	—12.02	—13.27	—13.98	—12.25	—15.62

Date.	12 mid.	12:20 a. m.	12:40 a. m.	1 a. m.	1:20 a. m.	1:40 a. m.	2 a. m.	2:20 a. m.	2:40 a. m.	3 a. m.	3:20 a. m.	3:40 a. m.
1880.	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ
Aug. 23	—14.6	—11.4	—16.3	—14.7	—16.7	—15.3	—17.0	—21.4	—16.4	—19.7	—12.6	—14.1
24	—29.5	—26.8	—26.1	—23.8	—29.8	—27.9	—23.7	—24.1	—25.1	—32.1	—28.0	—26.2
25	—30.8	—23.5	—19.4	—20.6	—18.2	—22.6	—23.4	—22.4	—21.7	—18.2	—32.5	—19.9
26	....	....	....	....	....	....	....	....	....	....	....	....
31	— 9.2	—10.0	— 6.6	— 3.9	— 4.5	—10.2	— 8.4	— 7.1	—10.6	— 7.8	— 7.4	— 9.2
Sept. 1	— 2.1	— 5.5	— 3.1	— 2.1	— 3.2	— 1.9	— 2.3	— 8.6	— 6.4	+ 3.5	+ 0.2	+ 5.3
2	—14.1	—12.5	—17.2	—12.5	—15.4	—13.7	—15.7	—10.0	—11.9	—10.4	—10.9	—11.6
3	....	....	....	....	....	....	....	....	....	....	....	....
Means	—16.72	—14.95	—14.78	—12.93	—14.63	—15.27	—15.08	—15.60	—15.35	—14.12	—15.20	—12.62

Date.	4 a. m.	4:20 a. m.	4:40 a. m.	5 a. m.	5:20 a. m.	5:40 a. m.	6 a. m.	6:20 a. m.	6:40 a. m.	7 a. m.	7:20 a. m.	7:40 a. m.	Means of days.
1880.	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ	μ
Aug. 23	—15.4	—24.8	—25.7	—20.6	—12.1	—23.3	—28.4	—23.2	—23.5	—21.3	—19.0	—11.6	— 2.25
24	—22.7	—26.3	—16.8	—16.4	—13.6	—22.4	—22.6	—22.5	—18.4	—15.7	—23.6	—18.5	—10.65
25	—17.7	—19.2	—19.4	—16.1	—17.5	—21.4	—15.6	—14.0	—12.4	— 7.2	— 7.0	—16.4	—19.69
26	....	....	....	....	....	....	....	....	....	....	....	....	....
31	— 8.3	— 8.4	— 7.3	— 9.3	— 5.4	— 6.1	— 8.7	— 8.6	— 2.1	— 9.3	— 5.5	— 4.7	+ 2.27
Sept. 1	— 1.6	— 1.4	— 2.7	— 2.7	— 5.3	— 5.7	— 9.8	— 5.8	— 1.0	— 3.6	+ 1.7	+ 5.2	+ 8.33
2	— 6.8	— 5.6	— 2.7	— 5.4	—11.0	— 7.3	— 5.6	— 5.5	— 3.5	— 8.4	— 2.9	— 2.1	+ 3.68
3	....	....	....	....	....	....	....	....	....	....	....	....	....
Means	—12.08	—14.28	—12.43	—11.75	—10.82	—14.37	—15.12	—13.27	—10.15	—10.92	— 9.38	— 8.02	....

§ **51.** From the observed $Z_1 - S_1$ for each entire hour covered by the observations in the table of the preceding section the corresponding mercurial temperature of S_1 can be computed with sufficient accuracy from the value

$$E_{z_1} - E_{s_1} = 38^{\mu}.0887$$

derived from results given in §§ 21, 23, 26.

The following table gives to the nearest tenth of a degree these temperatures. A comparison of it with the table of residuals in the preceding section will show the connection between the temperatures and the residuals. The curve of the mean temperatures is plotted in Plate XXVI.

Table of temperatures, derived from the observed $(Z_1 - S_1)$ *of every hour in the continuous comparisons made on the Cass farm.*

Date.	8 a. m.	9 a. m.	10 a. m.	11 a. m.	12 m.	1 p. m.	2 p. m.	3 p. m.	4 p. m.	5 p. m.	6 p. m.	7 p. m.	8 p. m.
1880.	° F.	° F.	° F.	° F.	° F.	° F.	° F.	° F.	° F.	° F.	° F.	° F.	° F.
Aug. 23	68.3	72.2	76.4	79.9	82.5	84.4	85.7	86.4	86.3	85.3	83.7	80.8	77.5
24	66.8	71.1	75.0	79.6	83.8	86.8	82.3	79.1	76.4	75.8	75.5	75.2	74.6
25	67.7	67.1	66.6	66.2	65.7	65.1	65.6	65.9	66.0	65.8	65.4	64.8	64.2
26	60.4	61.1	62.7	65.3	67.9	69.9	70.3	70.7	71.3	71.7	71.8	71.2	....
31	67.7	69.0	71.2	75.0	78.5	80.3	81.4	81.7	81.7	81.1	80.0	78.6	76.9
Sept. 1	72.6	75.8	78.8	82.4	84.7	87.0	88.3	88.5	86.9	85.8	84.3	82.2	80.4
2	74.9	76.1	78.2	79.4	80.6	81.7	83.8	84.0	83.4	82.2	80.9	78.8	77.0
3	73.4	74.2	75.2	76.8	78.7	81.3	81.6	82.0	82.3	82.2	....	....	....
Means	69.0	70.8	73.0	75.8	77.8	79.6	79.9	79.8	79.3	78.7	77.4	76.0	75.1

Table of temperatures, derived from the observed (Z_1-S_1) *of every hour, &c.*—Continued.

Date.	9 p. m.	10 p. m.	11 p. m.	12 mid.	1 a. m.	2 a. m.	3 a. m.	4 a. m.	5 a. m.	6 a. m.	7 a. m.	Means for 24 hours.
1880.	° F.	° F.	° F.	° F.	° F.	° F.	° F.	° F.	° F.	° F.	° F.	° F.
Aug. 23	75.0	73.4	71.7	70.0	67.8	66.1	64.8	63.4	62.3	62.2	63.8	74.6
24	73.9	73.2	72.5	72.1	71.8	71.7	71.8	71.2	70.6	69.5	68.4	74.5
25	63.2	62.5	61.5	60.8	59.8	59.4	58.8	58.8	58.8	59.1	59.4	63.3
26	....	....	....	....	....	....	....	....	....	....	....	....
31	75.3	73.8	72.7	71.9	71.2	70.8	70.2	69.9	69.6	69.2	70.7	74.5
Sept. 1	79.2	77.4	76.2	75.3	74.8	74.4	74.4	74.4	74.1	73.8	74.3	79.4
2	75.8	74.9	74.7	74.2	73.7	73.4	73.1	73.1	73.0	72.9	73.0	77.2
3	....	....	....	....	....	....	....	....	....	....	....	....
Means	73.7	72.5	71.6	70.7	69.9	69.3	68.9	68.5	68.1	67.8	68.2	

§ **52.** If the mean residuals for twenty-one days in § 47 be compared with the mean residuals for the periods of continuous observations given in § 50, it will be seen that at 8 a. m. the mean residual for twenty-one days was —$9^{\mu}.5$, while for the eight days in two periods of continuous comparisons it was —$5^{\mu}.9$. For the twenty-one days the time when the mean residual was zero was 9:57 a. m., while for the continuous work it was 10:32 a. m. For the twenty-one days the greatest positive residual was $+18^{\mu}.0$ at 3:20 p. m., while for the continuous work it was $+17^{\mu}.0$ at 3:20 p. m. During these parts of the day, the curve of the mean residuals is, therefore, nearly the same as for the twenty-one days, and hence leads to the assumption that if for all the twenty-one days the work had been continuous through the twenty-four hours, the rest of the daily curve of residuals would have resembled that of the continuous work, which gives another zero residual at 7:54 p. m., and greatest negative residuals of —$16^{\mu}.7$ at 12 midnight and of —$15^{\mu}.6$ at 2:20 a. m. It is thought that these mean residuals are chiefly due to the zinc bar's length differing at the different temperatures from its mean or normal length for that temperature. An examination of the mean temperatures of the continuous work, § 51, shows that at 8 a. m. the mean temperature was 69°; that the maximum temperature of 79°.9 was reached at 2 p. m. and the minimum temperature of the day, 67 at 6 a. m.

If the residuals be attributed entirely to the difference between temporary and normal lengths, then for the continuous work, at 8 a. m. Z_1 was about 6^{μ} too long; it shortened to its normal length at 10:32 a. m.; became too short by the maximum amount of 17^{μ} at 3:20 p. m.; again had its normal length at 7:54 p. m.; and became too long by $16^{\mu}.7$, its maximum amount, at 12 midnight.

It has already been stated in § 27 that the office comparisons indicated a change in residuals of Z_1-S_1 of about 2^{μ} per degree F. of temperature-change. Since in the continuous work from the time of zero residual to the time when the residual had its maximum value, $+17^{\mu}$, the temperature rose but 5°.5, the 2^{μ} rate would give a maximum residual of but 11^{μ} instead of 17^{μ}. From the time of the second zero residual in the means till the time of the greatest negative residual, $-16^{\mu}.7$, the temperature fell but 4°.5, which at 2^{μ} per degree would give a residual -9^{μ} instead of $-16^{\mu}.7$. The residuals are, then, of the signs that would be expected from the office residuals, but are larger numerically. But it must be remembered that the residuals of the Cass-farm work arise from all causes that would make Z_1-S_1 actually differ from the value to be expected from office comparisons, and it is to be expected that the difference of temperature between Z_1 and S_1 will in some degree affect these residuals, when there are, relatively to the office comparisons, large fluctuations in temperature. On August 24, 1 p. m., a temperature-fall occurred, which was rapid at first and then more gradual, and which continued till 5 p. m., August 25, the total fall being 18° F. Since at 1 p. m., August 24, the residual was only $+5^{\mu}.9$, if the residuals depended solely on temporary changes in length of Z_1 it would be supposed that Z_1 not shortening to its new normal length at once would be steadily too long and give negative residuals for the whole period. In fact its residuals only became negative at 6 p. m., August 24, and then remained steadily so till 9:35 a. m., August 26, reaching at one time $-34^{\mu}.6$. At the beginning of this temperature-fall other causes than temporary anomalous variations in the length of Z_1 must have acted. The supposition that in the first rapid temperature-fall Z_1 was cooler than S_1, and that this temperature-difference nearly disappeared when the rate of temperature-fall became small, would account for the conduct of the residuals. Again, if the residuals were due alone to temporary sets in the zinc bar, the maximum mean-residual would be looked for at the time of the maximum mean-temperature, but it occurs $1^h\ 20^m$ later, although the change in residual is but $2^\mu.5$ in this interval.

If it be true that in changing temperatures the zinc bar only after some time takes the length due to a new temperature, then if we suppose at the time of maximum temperature the zinc and steel bars had nearly the same temperature, as seems probable from § 31, the mean residual of $Z_1 - S_1$ at that time, 2 p. m., namely $+14^\mu.5$, would be principally due to the temporary set in the zinc bar. As the temperature fell it would be expected that these positive residuals would gradually become negative and would continue so till after the minimum temperature had been passed, after which they should again become positive and continue so till the maximum temperature of the following day had been passed. The mean residuals of the continuous work vary in this way. Whatever fluctuations the residuals go through in the course of a day, which has the usual variations in temperature, since the bar does not steadily change its length from day to day in the same direction, it would be expected that the residuals should not vary much from zero at the times of day when the bars had the mean temperature of the day. In the continuous work this mean temperature (excluding August 25 on account of there being little temperature-fluctuation) occurred on the average at about 10 a. m. and 8:45 p. m. The corresponding residuals, $-2^\mu.4$ and -5^μ, differ from zero by quantities that are not large for the means of so few days.

An error in the computed value of $Z_1 - S_1$ due to errors in the rates of expansion adopted for Z_1 and S_1 would give zero residuals at some other time than that when the temperature of S_1 has its mean daily value, and would give maximum positive and negative mean residuals which would differ in numerical amount. The fact that these residuals differ but slightly in amount, and also that (rejecting August 25, for the reasons already stated) the mean of the daily mean residuals is small (less than 1^μ), indicates that there can be no large errors in the values for the relative lengths and rates of expansion of Z_1 and S_1 derived from the office comparisons.

§ **53.** It has been stated in § 49 that the average correction to the length of each measured tube derived from the residuals of the Cass-farm work is about $4^\mu.6$. It is difficult to assign a probable error to this value, since it depends on the accuracy with which the mean residuals for the working portions of twenty-one days on the Cass farm represent the mean residuals which would be obtained by similar comparisons on each day during which the measurement of the base is carried on. The curves for twenty-one and for twenty-six days differ but slightly, although the latter include five abnormal days, and both are very regular; they were obtained at the time of the year when all the bases have been measured, and cover so many days that it seems quite sufficient to attribute to the $4^\mu.6$ average correction to S_1, a probable error of $\pm 1^\mu.5$, and this will accordingly be done.

DETERMINATION OF THE RELATIVE EXPANSION OF S_2 AND THE 15-FEET BRASS BAR OF THE LAKE SURVEY.

§ **54.** The groups of transverse graduations on the top of the 15-feet brass bar giving tenths of millimeters, the zeroes of the two principal groups being nearly four metres apart, are described in § 71. Since the expansion of the 15 feet brass bar is well known (see § 14, Chapter II), the comparison at different temperatures of S_2 with this 4-metre space, which may be designated by B, gives the absolute expansion of S_2. In making these comparisons the brass bar was mounted in its iron receiver, a semi-cylinder 90^{mm} in diameter with walls 5^{mm} thick, described in § 13, Chapter II, and then was placed in the comparing-box by the side of tube 2, both being mounted on the movable carriage of the comparing-apparatus so that they could be brought alternately under the microscopes mounted on their brick piers. They were leveled within $1'$ or $2'$. The deviations of the controlling lines on the graduated surfaces of S_2 from a vertical and from a horizontal plane can be measured with apparatus provided for the purpose. In the first the limit of accuracy is about $0^{mm}.05$ and in the second that of focusing of microscope, which in no case would exceed $0^{mm}.5$. This was done February 20, 1877, and July 1, 1879, and no change in deviations was found from either a vertical or a nearly horizontal plane, exceeding the above limits.

The absolute amount of deviation at 1^m, 2^m, or 3^m did not exceed $0^{mm}.3$ from either vertical or horizontal plane. As the graduations on S_2 are at its mid-height, small vertical bendings will be without sensible influence on its length. But as the controlling longitudinal lines of graduation on both steel and zinc bars are at a horizontal distance of 7^{mm} from the axes of these bars, if the azimuth of either bar at its 4-metre mark, with reference to its azimuth at the 0-metre mark, varied by one minute, the interval between its 0^m and 4^m marks would vary by 2^μ. For a change in azimuth of one minute at end of bar, there would be required a motion of $0^{mm}.30$ at the rollers 1^m distant. But the lateral motion of the bars between the guide-rollers with vertical axes rigidly fixed to the tube at each metre is very slight, not exceeding, probably, $0^{mm}.04$, and as the tube is very rigid the S_2 cannot change form laterally from loosening of guide-rollers or by lateral bending of the tube. The 15-feet brass bar has its graduations 4 metres apart at the middle of its top-surface and near its ends. Its deviations from a vertical plane are measured from a stretched fine silk thread, to within $0^{mm}.3$, and if larger deviations are found, the bar, whose cross-section is $28^{mm} \times 8^{mm}$, is so flexible laterally that it can be brought into line by gentle lateral pressure at its adjustable roller supports, which are 18 inches apart, and loosely confined there by guide-screws. As the graduations lie in the vertical plane through the neutral axis of the bar, such slight lateral bending does not sensibly change their interval. In the earlier comparisons the deviations of the top surface of the bar from a horizontal plane were also measured from a thread, but on reducing these measurements it was found that they did not give changes in relative inclination of the bar at the 0^m and 4^m marks with sufficient accuracy, and accordingly a level was substituted for the thread. In the later work no attempt was made to adjust the bar in the receiver so as to make its top-surface horizontal. The receiver kept its form well and the inclinations of the graduated surfaces were measured with the level which was always carefully placed in the same positions near the graduations on the bar. Such a bar, when bent into gentle curves of a metre or so in length and with versed sines of half the arc of not more than $0^{mm}.25$, does not have the length of its neutral axis differ by more than $0^\mu.7$ from that of a line joining the end-points of its neutral axis; but if its bending is in a vertical plane, the change in length of its upper surface will be very closely equal to the half-height of the bar multiplied by the tangent of the change of relative inclination between the surfaces of the bar at 0^m and at 4^m. For such a change in inclination of one minute, the top-surface of the bar changes length by about 4^μ. The relative inclinations of the end of the bar should then, if possible, be known within $15''$. These inclinations were at first read with a short level, whose supports are 9^{cm} apart, one division being $52''$, but later with a striding level 13^{cm} between supports, one division being $4''$. Of the numerous comparisons made in which the form of the upper surface of the bar depended only on the thread, none have been used. Had the position of the bar in the receiver remained unchanged, since the receiver is very stiff and no change in its form has been detected, the comparisons might have been used, but when tests with a thread were made the bar was sometimes adjusted to a straight line, the only measure of the change being the measurements from the thread. In the following reduction no comparisons prior to November 14, 1879, have been used, the inclinations of the graduated surfaces before that date being somewhat uncertain. Since that date the bar has not been adjusted vertically in the receiver. The slight horizontal adjustments, which would also have been better omitted, amounting only to $0^{mm}.2$ or $0^{mm}.3$, could have no sensible effect on the vertical curve of the bar. The numerous measurements of the relative inclinations of the top-surface of the brass bar at 0^m and at 4^m have an extreme range of but $68''$ in the comparisons since November 14, 1879. As this range is no larger than was to be expected from errors of observation, it is assumed that its surfaces about the graduations did not change their relative inclination in that period. Counting the inclination of the surfaces in the direction of the bar as positive when such as to make the top-surface convex upwards, the mean inclination of these surfaces during these comparisons when measured with a striding level whose feet were $2\frac{3}{8}$ inches in front and in rear of the graduations was $+2''.8$, or practically zero.

The temperatures of the brass bar were determined by three of the Casella thermometers already described, one being at its middle and one near each end. The temperatures of S_2 were determined by a Casella thermometer inserted at the middle opening of tube 2, and by two Geissler thermometers already described, one being inserted at each end of the tube.

§ **55.** The programme for observations was somewhat varied from time to time, since other investigations were going on at the same time, but it always included the following work:

1. Reading three thermometers in tube and three on bar, these thermometers being at ends and middle of tube and bar, respectively.
2. Pointings at ends of bar (or tube).
3. Pointings at ends of tube (or bar).
4. Pointings at ends of bar (or tube).
5. Reading of thermometers on bar.

To accomplish this programme usually required from fifteen to twenty minutes. In the majority of the observations but one visit was made to the comparing-room in a day, but a part of the time there were two visits, either six or twelve hours apart. The ends of Z_2 as well as of S_2 were read on. As S_2 has but three graduation lines at its ends, the middle ones, which are taken as the 0^m and 4^m marks, were always read on. As the brass bar expanded relatively to S_2, it was necessary to use differing graduations on the brass bar. The values of these graduations are given in § 71, and by their aid the readings on the brass bar have been reduced to what they would have been had the zero-graduations on the brass bar been pointed at.

In reduction, no comparisons were used in which the temperature-change in twenty-four hours, shown by the thermometers, exceeded $1^\circ.5$ F.; this rejected eleven days' comparisons. Nor when the mean temperature indicated by the thermometers with S_2 and with B differed by more than $0^\circ.1$; this rejected eight more days. Nor when the indicated temperatures of the ends of S_2 or of B differed by more than $0^\circ.20$ F.; this rejected four more days. Nor when the length of the visit to the comparing-room exceeded twenty-five minutes; this rejected twenty-three days. Ninety nine days of comparisons still remained. In obtaining the mean temperature of B or of S_2, the middle thermometer was given double weight. While an examination of temperatures shows no certain inequality in temperature-indications of thermometers in tube, and with bar depending on rising and falling temperatures, yet as some such inequality should be expected, the indications of the thermometers in the tube have been taken as giving the temperatures of both S_2 and B. In rising temperatures the thermometers in the tube, surrounded by an iron wall 3^{mm} thick, would probably lag behind the brass bar freely exposed to the air in the comparing-box, while they should precede S_2, which is like themselves inside the tube-wall. Hence, they should give a temperature lying between those of S_2 and B. On the other hand, the thermometers with B might be expected to precede the temperature of B when rising, and still more that of S_2. The small errors in the assumption that the thermometers in the tube give the correct temperatures of S_2 and B is partially eliminated in comparisons made at both falling and rising temperatures.

The number of days of comparisons (99) is much greater than is needed to obtain a satisfactory result, but it has arisen in part from the uncertainties as to the causes of the irregular variations in the lengths of the zinc bar, Z_2.

In reduction, since temperature-errors are the ones mainly to be feared, equal weight has been given to the mean result of a day's comparisons, whether one visit or two was made to the comparing-room on that day. When two visits were made, the means of the temperatures and of the observed differences of length for that day are used to give a single equation of condition. One day's comparisons gave an equation of condition of the following form:

$$(S_2-B)_0+(t_0-t)(E_B-E_{S_2})-(S_2-B)=v$$

where $(S_2-B)_0$ is the required difference of length of the steel and brass bars at an assumed temperature t_0; (S_2-B) is the observed difference of length of the two bars at the observed temperature t; $E_B-E_{S_2}$ is the required difference of expansion per degree Fahrenheit of the brass and steel bars. To simplify the least-square work the mean temperature of all the comparisons, namely, $51^\circ.687$ F., was used for t_0. The mean of all the observed differences of lengths (S_2-B) was $1206^\mu.88\pm0^\mu.14$, and is, therefore, the excess of length of the steel bar S_2 over that of the brass bar at $51^\circ.687$ F., or $1206^\mu.88$ is the value of $(S_2-B)_0$.

In the following table the first column gives the date of the comparison; the second, the number of visits to comparing-room on that day; the third, the mean of the observed temperatures of the tube for that day; the fourth, the value of the coefficient t_0-t of the unknown $(E_B-E_{S_2})$; the fifth, the absolute term, which is the observed difference of length of the bars; the sixth, the residual in the sense computed minus observed.

Results of comparisons of S_2 and B.

BRASS BAR ON NORTH SIDE, NEXT TO OBSERVER.

Dates.	No. of visits to comparing-room each day.	Mean of observed temperatures of tube for each day.	t_0-t	(S_2-B)	Residuals, (computed —observed).
1879.		° F.	° F.	μ	μ
Nov. 15	1	59.66	− 7.973	+1089.9	−1.28
17	1	57.64	− 5.953	+1118.5	+0.08
22	1	44.07	+ 7.617	+1323.1	−3.24
23	2	43.39	+ 8.297	+1330.3	−0.36
24	2	42.46	+ 9.227	+1346.5	−2.76
25	1	42.15	+ 9.537	+1348.2	+0.14
26	2	42.32	+ 9.367	+1345.8	+0.01
27	2	43.03	+ 8.657	+1334.5	+0.78
29	1	45.62	+ 6.067	+1295.3	+1.57
30	2	45.06	+ 6.627	+1303.9	+1.27
Dec. 1	2	44.82	+ 6.867	+1308.2	+0.53
2	2	44.93	+ 6.757	+1306.5	+0.60
3	2	45.71	+ 5.977	+1295.5	+0.03
4	2	46.45	+ 5.237	+1283.0	+1.56
5	2	47.22	+ 4.467	+1273.0	+0.14
6	2	48.49	+ 3.197	+1250.9	+3.40
7	2	49.35	+ 2.337	+1240.1	+1.44
8	2	49.22	+ 2.467	+1242.0	+1.47
9	1	48.77	+ 2.917	+1249.5	+0.65
10	2	49.35	+ 2.337	+1240.3	+1.24
11	1	49.34	+ 2.347	+1241.6	+0.00
14	2	43.60	+ 8.087	+1329.2	−2.38
15	2	43.26	+ 8.427	+1333.2	−1.33
16	2	42.02	+ 9.667	+1350.0	+0.26
17	2	40.95	+10.737	+1362.7	+3.43
18	2	40.12	+11.567	+1378.2	+0.25
19	1	39.82	+11.867	+1383.0	−0.11
22	1	39.59	+12.097	+1389.1	−2.79
23	1	39.74	+11.947	+1384.8	−0.72
26	1	40.43	+11.257	+1375.8	−1.95
27	1	39.49	+12.197	+1388.9	−1.11
29	1	38.85	+12.837	+1399.3	−2.02
30	1	39.42	+12.267	+1390.7	−1.87
1880.					
Jan. 1	1	40.23	+11.457	+1375.4	+1.41
2	1	40.74	+10.947	+1369.7	−0.45
5	1	43.32	+ 8.367	+1330.9	+0.08
7	1	44.95	+ 6.737	+1306.8	+0.01
8	1	45.32	+ 6.367	+1302.0	−0.68
9	1	45.92	+ 5.767	+1292.2	+0.22
14	1	46.76	+ 4.927	+1278.2	+1.76
19	1	46.61	+ 5.077	+1280.1	+2.08
23	1	46.44	+ 5.247	+1281.2	+3.51
26	1	45.73	+ 5.957	+1294.8	+0.44
27	1	45.92	+ 5.767	+1293.2	−0.78
28	1	46.47	+ 5.217	+1285.0	−0.74
Feb. 2	1	43.61	+ 8.077	+1325.9	+0.78
7	1	40.14	+11.547	+1376.5	+1.65
9	1	40.02	+11.667	+1378.6	+1.33
11	1	40.00	+11.687	+1378.8	+1.43
13	1	42.09	+ 9.597	+1347.6	+1.63
19	1	45.64	+ 6.047	+1297.3	−0.73
20	1	44.26	+ 7.427	+1317.8	−0.76
21	1	43.23	+ 8.457	+1333.0	−0.68
24	1	42.83	+ 8.857	+1338.3	−0.05

Results of comparisons of S_2 and B—Continued.

BRASS BAR ON NORTH SIDE, NEXT TO OBSERVER—Continued.

Dates.	No. of visits to comparing-room each day.	Mean of observed temperatures of tube for each day.	t_0-t	(S_2-B)	Residuals, (computed − observed).
1880.		° F.	° F.	μ	μ
Feb. 25	1	43.44	+ 8.247	+1327.9	+1.30
26	1	44.97	+ 6.717	+1307.6	−1.09
Mar. 1	1	45.17	+ 6.517	+1302.9	+0.64
2	1	43.67	+ 8.017	+1325.0	+0.79
3	1	43.68	+ 8.007	+1324.9	+0.74
4	1	44.67	+ 7.017	+1308.4	+2.56
6	1	45.58	+ 6.107	+1296.8	+0.66
8		45.08	+ 6.607	+1305.0	−0.12
10	1	42.13	+ 9.557	+1347.5	+1.13
12	1	40.41	+11.277	+1372.2	+1.94
13	1	39.47	+12.217	+1388.7	−0.61
15	1	38.16	+13.527	+1406.6	+0.92
17	1	37.95	+13.737	+1411.1	−0.47
July 10	2	75.91	−24.223	+ 848.9	−1.30
11	2	75.83	−24.143	+ 849.3	−0.52
12	2	76.13	−24.443	+ 843.9	+0.43
13	2	76.73	−25.043	+ 835.0	+0.43
14	2	77.27	−25.583	+ 828.1	−0.67
15	2	77.39	−25.703	+ 827.0	−1.35
16	1	77.07	−25.383	+ 828.7	+1.69

BRASS BAR ON SOUTH SIDE.

July 22	1	69.02	−17.333	+ 949.4	+0.39
23	1	68.50	−16.813	+ 953.6	+3.90
24	2	68.53	−16.843	+ 957.1	−0.04
25	2	68.95	−17.263	+ 950.5	+0.33
26	2	69.82	−18.133	+ 938.5	−0.57
27	2	70.69	−19.003	+ 922.2	+2.82
28	1	70.91	−19.223	+ 916.3	+5.46
29	1	70.59	−18.903	+ 921.2	+5.31
30	2	70.26	−18.573	+ 929.3	+2.10
31	2	70.19	−18.503	+ 930.1	+2.34
Aug. 1	2	70.56	−18.873	+ 924.3	+2.65
2	1	71.14	−19.453	+ 918.8	−0.45

BRASS BAR ON NORTH SIDE, NEXT TO OBSERVER.

Sept. 24	1	63.70	−12.013	+1033.4	−4.70
25	2	63.49	−11.803	+1029.2	+2.61
26	2	63.98	−12.293	+1026.3	−1.75
27	2	64.59	−12.903	+1018.7	−3.20
28	2	64.09	−12.403	+1026.4	−3.49

BRASS BAR ON SOUTH SIDE.

Oct. 2	2	59.40	− 7.713	+1096.5	−4.02
3	2	59.61	− 7.923	+1091.4	−2.04
4	2	59.82	− 8.133	+1089.6	−3.35
5	2	59.37	− 7.683	+1096.0	−3.68
6	2	58.97	− 7.283	+1103.6	−4.74
7	2	58.20	− 6.513	+1113.8	−3.52
8	2	57.72	− 6.033	+1120.1	−2.70
9	1	57.59	− 5.903	+1124.3	−4.98
99		5117.02	− 0.007	+119481.5	+0.28
	Means	51.687		+1206.88	

§ **56.** The residuals in this table have been examined, 1st, to see if there was any connection between their values and the sign of the daily temperature change in the comparing-box. Since the brass bar was exposed in its supporting semi-cylinder of iron freely to the air of the box, while S_2 was almost entirely separated from this air by inclosure in its heavy iron tube, it is to be expected that in rising temperatures B will heat most rapidly, so as to have a higher temperature than S_2, and that the reverse will be the case in falling temperatures. Since the residuals are computed values of S_2-B minus observed values, if B were hotter than S_2 the residual would tend to be positive. Taking those residuals, forty-eight in number, for which the daily temperature was increasing by known amounts, the mean residual is $+0^{\mu}.53$. Doing the same for falling temperatures, forty-seven cases, the mean residual is $-0^{\mu}.56$. These quantities are too small to indicate certainly the effect in question, but they point in that direction. They would indicate that S_2 was on the average about 0°.02 F. cooler than B in rising temperatures, and as much hotter in falling temperatures.

The residuals have been examined, 2d, to see if it made any difference whether tube or brass bar was on the north side (next the observers). In seventy-five cases the brass bar was north, with a mean residual of $+0^{\mu}.05$; in twenty cases it was south, with a mean residual of $-0^{\mu}.24$. A little heating of the bar nearest the observers might be expected in the fifteen or twenty minutes occupied in a visit, and the signs of these mean residuals are in the direction of such an effect, but their amounts are too small for any certain conclusions.

An examination of the residuals, 3d, to see if the relative expansions depended on the second as well as on the first power of the temperature led to no conclusion save that the part depending on the second power must be too small to be detected by these comparisons.

4th. The thermometers on B were read for a part of the time, both when entering and when leaving the comparing-room. After allowing for the daily rate of temperature-rise in the comparing-room, the remaining mean rise during a visit was 0°.01 F, which might be attributed to the presence of the observers. The maximum change was +0°.12, and the minimum −0°.06 F. The thermometers on tube 2 were always read on entering and on leaving the comparing-room. They give, also, a mean rise of 0°.01 F., with a maximum rise of +0°.10 F. and a minimum of −0°.05. The thermometers, then, as well as the residuals, indicate that the presence of the observers had an effect on the relative temperatures of the bars, which is very small in comparison with other errors.

We may adopt, then, for the differences of lengths and expansions of S_2 and B, the values derived from the least-square reduction of the comparisons, namely,

$$
\begin{aligned}
S_2-B,\ \text{at } 51^{\circ}.687\ \text{F.} &= +1206^{\mu}.88 \pm 0^{\mu}.14 \\
S_2-B,\ \text{at } 32^{\circ}\ \text{F.} &= +1498.88 \pm 0.27 \\
E_B-E_{S_2} &= +\ \ 14.833 \pm 0.012
\end{aligned}
$$

A preliminary reduction (of May 1, 1880), which embraced thirty-three days' comparisons prior to November 14, 1879, not included in this reduction, and with a temperature-range from 34° to 84° F., and which did not include thirty-two days of comparisons subsequent to the preliminary reduction and to July 10, 1880, which are included in this reduction, gave for S_2-B, at 32° F., a value $2^{\mu}.4$. less than that given above, and a value for $E_B-E_{S_2}$ $0^{\mu}.155$ less than that given above.

In computing the probable error of S_2-B, no notice has been taken of the errors in reducing the different graduations pointed at on the brass bar to the zero-graduations. As the probable error in the position of one of these lines with reference to the zero-line was but about $0^{\mu}.4$, and as many of these lines were used, the resulting error in the reduction will be small and has been neglected.

COLLECTION OF RESULTS.

§ **57.** It had been hoped to express the bases measured with the Repsold apparatus in terms of $R1876$ and of the metre, but in the absence of all details as to the derivation of the value for $R1876$ given by Professor Foerster, § 67, namely, $R1876 = 1\ \text{metre} + 247^{\mu} + (10^{\mu}.31 \pm 0^{\mu}.034)(t^{\circ}-15^{\circ}\ \text{C})$, this will not be done, but all lengths will be derived from the English yard through the Clarke yard A.

The lengths of S_2 and $R1876$ are derived simply from comparisons with Clarke yard A. The expansions of these bars are not obtained so simply, since they depend on three independent absolute-expansion determinations. The absolute expansion of S_2 may be derived from the absolute expansion of B as determined in the Lake-Survey office, from the absolute expansion of $R1876$ given by Professor Foerster, or through $R1876$ from the absolute expansion of yard A as determined by Colonel Clarke. The expansion of $R1876$ can also be obtained in three ways. This makes an adjustment of the different values necessary to obtain the best results and to avoid discrepancies in them. In adjusting the expansions, to avoid the question of their dependence on the second power of the temperature, a common temperature should be used. By reference to Chapter II, § 2, it will be seen that in Colonel Clarke's determination of the absolute expansion, E_A, of yard A, the mean of his low temperatures was about 34° F., and of his high temperatures about 91° F., so that his expansion corresponds to a temperature of about 62° F. Of the mean temperature to which the expansion for $R1876$ given by Professor Foerster, § 67, namely, in Fahrenheit degrees for 1°, $5^\mu.728 \pm 0^\mu.019$, nothing is known, but as expansion-determinations usually have a mean temperature of about 62° F., that will be assumed as the temperature in this case. The relative-expansion determinations $E_B - E_S$, $4E_{R1876} - E_{S_2}$, and $E_{R1876} - 1.0937\ E_A$ have shown no dependence of the rate of expansion on the temperature. Moreover, their mean temperature has been in the vicinity of 62°. Accordingly, 62° F. will be taken as the temperature for which the expansions are to be adjusted, so as to obtain their most probable values at that temperature.

Chapter II, § 19 gives for the length of the 15-feet brass bar (which may be designated by B' to distinguish it from B, which is the length between the 0^m and 4^m graduations on its top-surface),

$$B' = 179^{in}.95434 + 0^{in}.0017776\ (t-32) + 0^{in}.000000333\ (t-32)^2$$

This may be written in the form

$$B' = 179^{in}.95434\ [1 + 9878\ (10)^{-9}\ (t-32) + 1852\ (10)^{-12}\ (t-32)^2]$$

In this expression, $179^{in}.95434$ is the length of B' at 32° F., or is B'_{32}. Similarly, if we assume that the rate of expansion of the B part of the brass bar (this part being 4^m lying in the middle of the bar) is the same as for the whole of the bar, B might be written

$$B = B_{32}\ [1 + 9878\ (10)^{-9}(t-32) + 1852\ (10)^{-12}(t-32)^2]$$

From § 56:

$$S_2 - B = +1498^\mu.88 \text{ at } 32° \text{ F.}$$

From § 18:

$$S_2 = 4\,R1876 - 104^\mu.26 \text{ at } 32° \text{ F.}$$

From § 67:

$$R1876 = 1^m.000092 \text{ at } 32°$$

Hence,

$$B_{32} = 3^m.998765$$

very nearly, and

$$B = 3^m.998765\ [1 + 9878\ (10)^{-9}\ (t-32) + 1852\ (10)^{-12}\ (t-32)^2]$$

Differentiating, there results—

$$\frac{dB}{dt} = E_B = 3^m.998765\ [9878\ (10)^{-9} + 3704\ (10)^{-12}(t-32)]$$

whence,

$$E_B = 39^\mu.945 \text{ at } 62° \text{ F.}$$

The relative expansion of a part of $R1876$ and of the Clarke yard A slightly prolonged by the auxiliary cylinders, § 9, is

$$E_{R'1876} - E_{A'} = +\,0^\mu.0058 \pm 0^\mu.0082$$

Assuming that the rate of expansion of the portion ($4^{mm}.9$) of the auxiliary cylinders used is the same as that of yard A, and that the rate of expansion of the interval from 81^{mm} to 1000^{mm} on $R1876$ is the same as for the whole metre, the difference of expansion per metre of length will be obtained with sufficient accuracy by multiplying the value $E_{R'1876} - E_{A'} = +\,0^\mu.0058 \pm 0^\mu.0082$ by the ratio of the prolonged yard to $R1876$, namely, 1.0937. This gives $E_{R1876} - 1.0937\ E_A = 0^\mu.006 \pm 0^\mu.009$ There is doubtless some error in the assumption that the rate of expansion of the part of $R1876$ between 0^{mm} and 81^{mm} is the same as for the rest of the bar. This length is about $\frac{1}{12}$ of the whole length of the bar. The comparisons of $R1876$ with each of the four metres of S_2 give as the greatest difference of expansions of any two metres about $0^{\mu}.10$, while the absolute expansion is about $6^{\mu}.2$, or the greatest range in expansions is $\frac{1}{60}$ part. If so great a difference as this were to exist between the interval from 0^{mm} to 81^{mm} on R1876 and the rest of the metre, it would introduce an error of $\frac{1}{12}\times\frac{1}{60}=\frac{1}{720}$ part into the derived expansion of R1876.

In deriving the expansion of the B part of the 15-feet brass bar from that of the whole bar, a similar assumption of equal rates of expansion for B and B' has been made. The ratio of B to B' is $\frac{4^{m}}{4^{m}.57}=\frac{10}{11}$ approximately. We have no data as to the variations of the rate of expansion of different parts of the brass bar. Since the $0^{m}.57$ is half at one end of the bar and half at the other, the chances would be even that if one half had a greater rate of expansion than the whole bar, the other half would have a less rate and the errors would be eliminated. Even if they both varied in the same direction and by $\frac{1}{30}$ of their absolute values, an error of but $\frac{1}{11}\times\frac{1}{30}=\frac{1}{330}$ would be introduced into the expansion of B.

Collecting results, the equations of condition for the values of E_B, E_{R1876}, E_A, and E_{S_2} may now be written.

		Weights from probable errors.	Adopted weights.
From § 57,	$E_B-39^{\mu}.945\pm0^{\mu}.038$ (at 62° F.) $=v_1$	0. 7	1
From § 67,	$E_{R1876}-5^{\mu}.728\pm0^{\mu}.019$ (at 62° F.) $=v_2$	2. 8	4
From Chapter II, § 2,	$E_A-5^{\mu}.371\pm0^{\mu}.018$ (at 62° F.) $=v_3$	3. 1	4
From § 57,	$E_{R1876}-1^{\mu}.0937\,E_A-0^{\mu}.006\pm0^{\mu}.009$ (at 62° F.) $=v_4$	12. 3	8
From § 18,	$4E_{R1876}-E_{S_2}+1^{\mu}.335\pm0^{\mu}.022$ (at 62° F.) $=v_5$	2. 1	3
From § 56,	$E_B-E_{S_2}-14^{\mu}.833\pm0^{\mu}.012$ (at 62° F.) $=v_6$	7. 0	6

The probable error of $39^{\mu}.945$ in the first equation has been derived from the probable errors of the two independent values of the expansion E_B for different temperature-ranges, given in Chapter II, § 14, by expressing the coefficients of the first and second powers of the temperature in the general value for the length of the brass bar in terms of these independent expansions. In the fourth equation the probable error of $0^{\mu}.006$ is obtained by multiplying the probable error of the relative expansion resulting from the comparisons of the yard with a part of R1876, given in § 9, by the ratio of the prolonged yard to the metre. The probable errors of the other constants are given in the sections from which the values are derived.

§ **58.** The solution by least squares of the equations of condition just given, the adopted weights being applied, gives

$$(1)\qquad \begin{cases} E_B \;\;\;\;\, = \overset{\mu}{39}.787\pm\overset{\mu}{0}.152, \text{ at } 62^\circ \text{ F. for } 1^\circ \\ E_{R1876} = \;\;5.885\pm0.043, \text{ at } 62^\circ \text{ F. for } 1^\circ \\ E_A \;\;\;\;\, = \;\;5.374\pm0.055, \text{ at } 62^\circ \text{ F. for } 1^\circ \\ E_{S_2} \;\;\;= 24.927\pm0.160, \text{ at } 62^\circ \text{ F. for } 1^\circ \end{cases}$$

The facts that the absolute-expansion determinations of the 15-feet brass bar indicate that the rate of expansion changes with the temperature, while the comparisons of S_2 and B, of R1876 and S_2, and of R1876 and Clarke yard A indicate no dependence of the relative rates of expansion on the temperature, show, so far as the comparisons serve to settle the question, that the other bars change their rate of expansion with the temperature by the same amount as does the brass bar.

The length of any bar at the temperature t° F. may be written in terms of its length L_{32}, at 32° F., in the form

$$(2)\qquad L=L_{32}\,[1+\alpha\,(t-32)+\beta\,(t-32)^2]$$

Its rate of expansion will be

$$(3)\qquad \frac{dL}{dt}=E_L=L_{32}[\alpha+2\beta\,(t-32)]$$

The sum of its expansions between 62° and $t°$ will be—

$$\int_{62}^{t}\frac{dL}{dt}dt=L_{32}[(\alpha+60\beta)(t-62)+\beta(t-62)^2]$$

and its length will be—

(4) $$L=L_{62}+L_{32}[(\alpha+60\beta)(t-62)+\beta(t-62)^2]$$

The conclusion that the bars S_2, $R1876$, and Clarke yard A change their rate of expansion with the temperature at the same rate as does B in

$$B=B_{32}[1+9878(10)^{-9}(t-32)+1852(10)^{-12}(t-32)^2]$$

§ 57, carries with it the conclusion that when their lengths are written in the form (2), $\beta=1852(10)^{-12}$ for them all. The values of the expansions in (1) for any temperature may be obtained as follows: For $t=62°$, (3) and (1) give—

(5) $$\begin{cases} E_B \text{ at } 62°=39^{\mu}.787\pm0^{\mu}.152=B_{32}(\alpha_1+60\beta) \\ E_{R1876} \text{ at } 62°=5.885\pm0.043=R_{32}(\alpha_2+60\beta) \\ E_A \text{ at } 62°=5.374\pm0.055=A_{32}(\alpha_3+60\beta) \\ E_{S_2} \text{ at } 62°=24.927\pm0.160=S_{2.32}(\alpha_4+60\beta) \end{cases}$$

The lengths at 32° F. in metres of these bars are sufficiently well known (as will be seen by comparing the values substituted below with the final values in § 66 following) to write them in the above equations without introducing sensible errors in the α; the value of β is known; and hence from the above equations the values of the α may be deduced. Substituting them in (3) there result the final values—

(6) $$E_B=3^{m}.998765\,[9839(10)^{-9}+2\times1852(10)^{-12}(t-32)]$$
(7) $$E_{R1876}=1^{m}.000092\,[5773(10)^{-9}+2\times1852(10)^{-12}(t-32)]$$
(8) $$E_A=0^{m}.914209\,[5767(10)^{-9}+2\times1852(10)^{-12}(t-32)]$$
(9) $$E_{S_2}=4^{m}.000264\,[6120(10)^{-9}+2\times1852(10)^{-12}(t-32)]$$

The coefficients in metres, or the lengths of the different bars at 32° F. in metres, are derived as follows:

In (6), from S_2, § 18; $S_2-B=1499^{\mu}$, § 56; and Foerster's value of $R1876$, § 67.

In (7), from Foerster's value of the metre, $R1876=1^{m}+247^{\mu}+10^{\mu}.31\,(t°-15°\text{ C.})$.

In (8), from Clarke's value of yard A, Chapter II, § 2, and his ratio of yard to metre, namely, 1.09362311.

In (9), from Foerster's value of $R1876$, § 67, and the value of S_2 in terms of $R1876$, § 18, namely, $S_2=4R1876-68^{\mu}.12+1^{\mu}.3349\,(t°-59°\text{ F.})$.

The difference in computed lengths of S_2 at 92° F., when derived first from its length at 62° F. with a constant expansion, namely, that for 62°, and when derived with the varying expansion given above is but $6^{\mu}.7$, the latter value being the larger. Now, the probable error in the value of E_{S_2} at 62°, namely, $\pm0^{\mu}.160$, would give a probable error of $4^{\mu}.8$ at 92° F. in the length of S_2 when computed from its length at 62° F. It is then apparent that there is but little precision in the term which gives the variation of E with temperature, and were it not that Fizeau's work makes such a term probable *a priori*, it would be better to reject it entirely for all the expansions given above.

§ **59.** Having obtained the necessary rates of expansion, the lengths of the different bars may now be deduced.

From § 9 the value of the mean temperature of the comparisons in the Lake-Survey office of Clarke yard A and $R1876$ is 57°.92 F. Hence, at this temperature the relative length of the two is independent of errors in the relative coefficient of expansion. Colonel Clarke's value for the length of yard A, derived from Ordnance-Survey standard Y_{55} is, Chapter II, § 2—

Clarke yard A at 62° F. $=0^{y}.99997695\pm0^{y}.00000013$
Expansion $=0^{y}.000005874\pm0^{y}.0000000195$ for 1° F.

The mean temperature of Colonel Clarke's comparisons of yard A and Y_{55} was 57°.71 F. In the computation of probable errors, to simplify the work it is best to count the length of Clarke yard A from this temperature. Its length follows at once. The probable error of its length at 57°.71 is derived from that at 62° by allowing for the effect of the probable error in the expansion for this difference of temperature, and we have—

$$\text{Clarke yard } A \text{ at } 57°.71 \text{ F.} = 0^y.99995175 \pm 0^y.00000010$$

At 57°.92 this value becomes, using his expansion,

$$\text{Clarke yard } A \text{ at } 57°.92 \text{ F.} = 0^y.99995298 \pm 0^y.00000010$$

But, § 12, at 57°.92 F.,

$$R1876 = \tfrac{1000}{919} \text{ Clarke yard } A + 5296^\mu.6 \pm 0^\mu.40$$

Hence,

$$\text{At } 57°.92,\ R1876 = 1^y.09388063 \pm 0^y.00000045$$

From this value of $R1876$, at the temperature 57°.92, and the general expression for $R1876$, namely,

$$R1876 = R1876_{32}\,[1 + \alpha\,(t-32) + \beta\,(t-32)^2]$$

where, § 58, $\alpha = 5773\,(10)^{-9}$ and $\beta = 1.852\,(10)^{-9}$, there results,

$$R1876_{32} = 1^y.09371561$$

and finally,

$$R1876 = 1^y.09371561\,[1 + 5773\,(10)^{-9}\,(t-32) + 1.852\,(10)^{-9}\,(t-32)^2]$$

The uncertainty in the length of $R1876$ is least for a temperature near 57°.92, the mean temperature of comparisons of Clarke yard A with it, since the mean temperature of Colonel Clarke's comparisons of Clarke yard A with Ordnance-Survey standard yard Y_{55} was 57°.71, so that slight errors in the values of the rates of expansion have little influence on the value for $R1876$ at this temperature.

§ **60.** The values of S_2 in yards and in terms of $R1876$ may next be found. From § 18

$$\begin{aligned}
R1876 &= S_{2.1} + \overset{\mu}{45.54} \pm \overset{\mu}{0.20} - \overset{\mu}{0.3485}\,(t-54.16)\\
R1876 &= S_{2.2} + 4.46 \pm 0.12 - 0.3027\,(t-57.85)\\
R1876 &= S_{2.3} + 12.78 \pm 0.18 - 0.3366\,(t-54.11)\\
R1876 &= S_{2.4} + 9.70 \pm 0.15 - 0.3992\,(t-56.86)
\end{aligned}$$

where the relative expansions have been adjusted to conform to the adjusted values of E_{R1876} and E_{S_2} and the numerical temperatures are the mean temperatures of the comparisons of the different metres of S_2. Taking the means of these separate means and then, by aid of the separate expansion terms, reducing the equations to this mean temperature, which, as the reduction is small, will involve but slight errors arising from the errors in the expansions and insensible changes in probable errors, there result—

$$\begin{aligned}
R1876 &= S_{2.1} + 44^\mu.99 \pm 0^\mu.20 \text{ at } 55°.74 \text{ F.}\\
R1876 &= S_{2.2} + 5^\mu.10 \pm 0^\mu.12 \text{ at } 55°.74 \text{ F.}\\
R1876 &= S_{2.3} + 12^\mu.23 \pm 0^\mu.18 \text{ at } 55°.74 \text{ F.}\\
R1876 &= S_{2.4} + 10^\mu.15 \pm 0^\mu.15 \text{ at } 55°.74 \text{ F.}
\end{aligned}$$

and adding

$$S_2 = 4\,R1876 - 72^\mu.47 \pm 0^\mu.33 \text{ at } 55°.74 \text{ F.}$$

and since the difference of the adjusted expansions of S_2 and $4\,R1876$, § 58, is $+1^\mu.387$ per degree F. we may write

$$S_2 = 4\,R1876 - 72^\mu.47 \pm 0^\mu.33 + 1^\mu.387\,(t-55.74)$$

as the value of S_2, not as obtained from direct comparisons of S_1, S_2, and $R1876$, but from an adjusted value of the relative expansion of S_2 and $R1876$. From the value of $R1876$ in § 59 there results

$$R1876 \text{ at } 55°.74 \text{ F.} = 1^y.09386664$$

Substituting this in the value of S_2 there results

$$S_2 \text{ at } 55°.74 \text{ F.} = 4^y.37538730$$

From this value and the value for E_{S_2} at any temperature in § 58, may be written for any temperature

$$S_2 = 4^y.37474713\,[1 + 6120\,(10)^{-9}\,(t-32) + 1.852\,(10)^{-9}\,(t-32)^2]$$

Clarke yard A was derived from Ordnance-Survey standard Y_{55} by comparisons whose mean temperature was 57°.71 F. R1876 was derived from Clarke yard A, § 9, by comparisons whose mean temperature was 57°.92 F. S_2 was derived from R1876 by comparisons of each metre of S_2 with R1876. The mean of the mean temperatures of comparisons of the several metres was 55°.74. Hence it is seen that the length of S_2 at 55°.74, as derived from Y_{55}, is very nearly free from the effect of errors in the rates of expansion of the different bars.

§ **61.** The mean temperature of the comparisons of S_2 and B, § 56, was 51°.687 F., for which temperature

$$S_2-B=+1206^{\mu}.88\pm0^{\mu}.14$$

The difference between the adjusted values of E_{S_2} and E_B, § 58, is

$$E_B-E_{S_2}=14^{\mu}.860$$

Hence

$$B=S_2-1206^{\mu}.88\pm0^{\mu}.14+14^{\mu}.860\ (t-51.687)$$

or, substituting for S_2 its value in terms of R1876 from § 60,

$$B=4R1876-1284^{\mu}.97 \text{ at } 51^{\circ}.69 \text{ F.}$$

and substituting the value of R1876 in terms of the yard, § 59,

$$B=4^{y}.37310732\ [1+9839\ (10)^{-9}\ (t-32)+1.852\ (10)^{-9}\ (t-32)^2]$$

§ **62.** The values of S_1, Z_1, Z_2 will be derived directly from comparisons with each other and with S_2.

From § 60,

$$S_2 \text{ at } 55^{\circ}.74=4^{y}.37538730$$

From § 58,

$$E_{S_2}=24^{\mu}.927+0^{\mu}.0148\ (t-62)$$

From § 21,

$$E_{S_2}-E_{S_1}=+0^{\mu}.061$$

The mean temperature of comparisons of S_1 and S_2 was 42°.75 F. From the value at any temperature of S_2, given in § 60, there results

$$S_2 \text{ at } 42^{\circ}.75=4^{y}.37503588.$$

But, § 21,

$$(1)\qquad S_1=S_2+32^{\mu}.09\pm0^{\mu}.18-(0^{\mu}.0605\pm0^{\mu}.015)\ (t-42.75)$$

Substituting the value of S_2 at 42°.75 F.

$$S_1 \text{ at } 42^{\circ}.75 \text{ F.}=4^{y}.37507085$$

Combining the values of E_{S_2} and $E_{S_2}-E_{S_1}$ given above, there results

$$E_{S_1}=24^{\mu}.866+0^{\mu}.0148\ (t-62)$$

The expansion of S_1 between 42°.75 and any temperature t will be—

$$\int_{42.75}^{t} E_{S_1}dt=24^{\mu}.581\ (t-42.75)+0^{\mu}.0074\ (t-42.75)^2$$

Hence at any temperature,

$$S_1=4^{y}.37507085+0^{y}.00002688\ (t-42.75)+0^{y}.00000000809\ (t-42.75)^2$$

or,

$$S_1=4^{y}.37478294\ [1+6105\ (10)^{-9}\ (t-32)+1.852\ (10)^{-9}\ (t-32)^2]$$

In § 60 the value of S_2 at any temperature in terms of R1876 is given. Substituting that value in (1) and neglecting the probable errors, there results:

$$S_1=4R1876-58^{\mu}.40+1^{\mu}.326\ (t-42.75)$$

§ **63.** Since the lengths of the bases measured with the Repsold apparatus all depend on the length of S_1 it is important to know the probable error in this length. But the error in its length depends on the errors in the value of the Clarke yard A; on the errors of intercomparisons of Clarke yard A, R1876, S_2, and S_1; and since the intercomparisons were not all at the same temperature, the errors of the rates of expansion also enter. These rates of expansion have been adjusted in § 58 so that their values are not independent. The errors are therefore entangled, and to obtain the probable error in the value of S_1 it is necessary to express this value in terms of the independently observed quantities on which it depends, and then derive the probable error of S_1 from the function of these observed quantities so obtained. The computation is somewhat long and hence is not given; the following is the resulting value of the probable error squared for the value of S_1 at t° F., the unit being the millionth of a yard:

$$(p.\ e.)^2 S_1 = 3.56 - 0.1109\,(t-t_2) + 0.0306\,(t-t_2)^2 + 0.00027(t-t_4)^2$$
$$+0.00016\,(t+t_2-124)\,(t-t_2) + 0.00000389\,(t+t_2-124)^2\,(t-t_2)^2$$

in which $t_2=55^\circ.74$ F. is the mean temperature of comparisons of S_2 and R 1876, and $t_4=42^\circ.75$ F., the mean temperature of comparisons of S_2 and S_1.

The value of S_1 is also given in terms of R 1876 in § 62. The error in this value depends also on entangled errors which are less complex than in the preceding case, as there are fewer steps connecting the standards. Computing the probable error squared of S_1 at any temperature expressed in terms of R 1876 it is found to be, the unit being 1^μ:

$$(p.\ e.)^2\ S_1 = 0.1413 + 0.00952(t-t_2)^2 + 0.00023(t-t_4)^2$$

This is merely the probable error in the derivation of S_1 from R 1876 through S_2; and t_2 and t_4 are the same as stated above, namely, $t_2=55^\circ.74$ F., and $t_4=42^\circ.75$ F.

§ **64.** From § 26

$$Z_2 = S_2 - 305^\mu.55 \pm 0^\mu.46 \text{ at } 49^\circ.50 \text{ F.}$$

or substituting for S_2 its value from § 60,

$$Z_2 = 4^y.37488398 \text{ at } 49^\circ.50$$

Since, § 26,

$$E_{Z_2} - E_{S_2} = +38^\mu.465 \pm 0^\mu.039$$

it follows that

$$Z_2 - S_2 = 0 \text{ at } 57^\circ.44 \text{ F.}$$

The rate of expansion of Z_2 is, therefore,

$$E_{Z_2} = E_{S_2} + 38^\mu.465$$

or, by § 58,

$$E_{Z_2} = 63^\mu.392 + 0^\mu.0148\,(t-62)$$

and hence at any temperature

$$Z_2 = 4^y.37488398 + 0^y.00006912(t-49.50) + 0^y.00000000809\,(t-49.50)^2$$

or, in a different form,

$$Z_2 = 4^y.37367686\ [1 + 15740\ (10)^{-9}\ (t-32) + 1.852\ (10)^{-9}\ (t-32)^2]$$

From § 23

$$Z_1 = Z_2 - 70^\mu.69 \pm 0^\mu.60 \text{ at } 42^\circ.75 \text{ F.}$$

Substituting for Z_2 its value above,

$$Z_1 = 4^y.37434045 \text{ at } 42^\circ.75 \text{ F.}$$

Since, § 23,

$$E_{Z_1} - E_{Z_2} = -0^\mu.437$$

$$E_{Z_1} = 62^\mu.955 + 0^\mu.0148\ (t-62)$$

Hence, at any temperature

$$Z_1 = 4^y.37434045 + 0^y.00006854\ (t-42.75) + 0^y.00000000809\ (t-42.75)^2$$

or, in a different form,

$$Z_1 = 4^y.37360458\ [1 + 15632\ (10)^{-9}\ (t-32) + 1.852\ (10)^{-9}\ (t-32)^2]$$

From the values of S_1 and Z_1, §§ 62, 64, it follows that

$$Z_1 - S_1 = 0 \text{ at } 60^\circ.292 \text{ F.}$$

§ **65.** Approximate values for the lengths of the steel and zinc bars of the standard metre described in Chapter VIII, § 26, and designated as $MT1876$ have been derived in the following way: Between May 28 and June 28, 1878, both bars of $MT1876$ were compared at high temperatures with each of the metres $S_{2.1}$, $S_{2.2}$, $S_{2.3}$, and $S_{2.4}$, into which the steel bar S_2 is divided. Comparisons with $S_{2.4}$ were made on ten days and with each of the other metres on five days. The number of comparisons made on a day varied from one to four, but was usually two. The values of the several metres making up the length of S_2 in terms of $R1876$ are given in §60. Combining the observed differences of length between $(MT1876)_s$ and these metres of S_2 with the values of these metres, the values of $(MT1876)_s$ in terms of $R1876$ resulted. The mean of the comparisons of each day was used as a single result, no matter how many comparisons were made on that day, and the corresponding mean temperature was taken. The temperatures varied between 67°.40 F. and 75°.90 F. Taking general means of differences of length and of temperatures, there resulted,

$$(MT1876)_s = R1876 - 30^{\mu}.06 \text{ at } 70^{\circ}.90 \text{ F.}$$

By the same number of comparisons there was derived in the same way,

$$(MT1876)_z = R1876 + 632^{\mu}.78 \text{ at } 70^{\circ}.90 \text{ F.}$$

Both bars of $MT1876$ were compared directly with $R1876$ at low temperatures between January 8 and February 5, 1881, in connection with some comparisons of $R1876$ with S_2. Comparisons were made on twenty-four days, two comparisons usually being made on each day. The temperatures varied between 28°.72 F. and 34°.44 F. The mean results are as follows:

$$(MT1876)_s = R1876 - 47^{\mu}.96 \text{ at } 31^{\circ}.21 \text{ F.}$$
$$(MT1876)_z = R1876 + 210^{\mu}.48 \text{ at } 31^{\circ}.21 \text{ F.}$$

Combining the results of the high- and low-temperature comparisons, we have:

$$\text{Expansion of } (MT1876)_s = \text{expansion of } R1876 + 0^{\mu}.451 \text{ for } 1^{\circ} \text{ F.}$$
$$\text{Expansion of } (MT1876)_z = \text{expansion of } R1876 + 10^{\mu}.639 \text{ for } 1^{\circ} \text{ F.}$$

From these may be written:

$$(MT1876)_s = R1876 - 47^{\mu}.6 + 0^{\mu}.451(t-32)$$
$$(MT1876)_z = R1876 + 218^{\mu}.9 + 10^{\mu}.639(t-32)$$

From these values the residuals of the daily mean observed differences from $R1876$ have been computed. For the high-temperature comparisons the residuals vary between $+3^{\mu}.1$ and $-4^{\mu}.4$ for $(MT1876)_s$ and between $+3^{\mu}.7$ and $-3^{\mu}.6$ for $(MT1876)_z$. For the low-temperature comparisons the residuals for $(MT1876)_z$ vary between $+11^{\mu}.6$ and $-10^{\mu}.8$, while for $(MT1876)_s$ they vary between $+9^{\mu}.7$ and $-5^{\mu}.3$. The comparisons of both bars of $MT1876$ with $R1876$ give plus residuals amounting to from $+6^{\mu}$ to 11^{μ} on February 1, 2, 3, 4, 5. The fact that these residuals are of the same sign for both bars during these days points to a temperature or other error in one of the standards, as this would affect both bars of $MT1876$ in the same way. As the temperatures were very low at this time, and there were three persons in the comparing-room, it is probable that the large residuals are due in part to differences of temperature of $R1876$ and $MT1876$. If those days had been rejected, the length of $(MT1876)_s$ at 32° would have resulted $2^{\mu}.1$ less than that given above, and of $(MT1876)_z$ $2^{\mu}.4$ less, and the range in the residuals, for either, would have been much reduced. An uncertainty of about 1^{μ} therefore exists in the values of these bars in terms of $R1876$ at the temperature of 31°.21 F.

§ **66.** The resulting lengths and expansion of the different bars may now be collected.

Clarke Yard A at 62° F.=

$$0^{y}.99997695 \pm 0^{y}.00000013. \quad \text{(Chapter II, § 2.)}$$

Clarke Yard A, adjusted rate of expansion,

$$E_A = 0^{m}.914209[5767(10)^{-9} + 3.704(10)^{-9}\,(t-32)] \quad (\text{§ } 58.)$$
$$R1876 = 1^{y}.09371561[1 + 5773(10)^{-9}(t-32) + 1.852(10)^{-9}(t-32)^2] \quad (\text{§ } 59.)$$
$$S_2 = 4^{y}.37474713[1 + 6120(10)^{-9}(t-32) + 1.852(10)^{-9}(t-32)^2]. \quad (\text{§ } 60.)$$

Rate of expansion of S_2=

$$E_{S_2}=4^{\mathrm{m}}.000264\,[6120(10)^{-9}+3.704\,(10)^{-9}(t-32)].\quad(\S\ 58.)$$

$$S_1\ =4^{\mathrm{y}}.37478294[1+6105(10)^{-9}(t-32)+1.852(10)^{-9}(t-32)^2].\quad(\S\ 62.)$$

$$(S_1)_{Z_1=S_1}=4^{\mathrm{y}}.37554481 \text{ at } 60^\circ.292 \text{ F.}=13^{\mathrm{ft}}.12663443.$$

$$E_{S_1}=24^{\mu}.866+0^{\mu}.0148\,(t-62).\quad(\S\ 62.)$$

$$Z_2\ =4^{\mathrm{y}}.37367686[1+15740(10)^{-9}(t-32)+1.852(10)^{-9}(t-32)^2].\quad(\S\ 64.)$$

$$E_{Z_2}=63^{\mu}.392+0^{\mu}.0148\,(t-62).\quad(\S\ 64.)$$

$$Z_1\ =4^{\mathrm{y}}.37360458\,[1+15632\,(10)^{-9}(t-32)+1.852\,(10)^{-9}(t-32)^2].\quad(\S\ 64.)$$

$$E_{Z_1}=62^{\mu}.955+0^{\mu}.0148\,(t-62).\quad(\S\ 64.)$$

$$B\ =4^{\mathrm{y}}.37310732\,[1+9839\,(10)^{-9}(t-32)+1.852\,(10)^{-9}(t-32)^2].\quad(\S\ 61.)$$

The probable error squared in millionths of a yard in the above value of S_1 is

$$(p.\ e.)^2\ S_1=3.56-0.1109\,(t-t_2)+0.0306\,(t-t_2)^2+0.00027\,(t-t_4)^2$$
$$+0.00016\,(t+t_2-124)\,(t-t_2)+0.00000389\,(t+t_2-124)^2\,(t-t_2)^2$$

in which $t_2=55^\circ.74$ F. and $t_4=42^\circ.75$ F. (§ 63.)

In terms of R1876, § 62,

$$S_1=4\ R1876-58^{\mu}.40+1^{\mu}.326\,(t-42.75)$$

this value depending on the adjusted relative expansion of R 1876 and S_2; and the square of the probable error of this value, § 63, is in microns,

$$(p.\ e.)^2\ S_1=0.1413+0.00952\,(t-t_2)^2+0.00023\,(t-t_4)^2$$

in which $t_2=55^\circ.74$ F. and $t_4=42^\circ.75$ F.

The value of S_1 may also be derived solely from the results of comparisons of R1876 with S_2, § 18, namely,

$$S_2=4\,R1876-68^{\mu}.12+1^{\mu}.3349\,(t-59)$$

and of S_1 with S_2, § 21, namely,

$$S_1=S_2+32^{\mu}.09-0^{\mu}.0605\,(t-42.75)$$

These give

$$S_1=4\,R1876-37^{\mu}.01+1^{\mu}.2744\,(t-59)$$

a value which is independent of the absolute expansion of R1876 derived by adjustment.

The square of the probable error of this value of S_1, derived from the probable errors in the results of comparisons of S_1 and S_2, and of S_2 and R1876 is, in microns,

$$(p.\ e.)^2\ S_1=0.1480+0.00048\,(t-59)^2+0.00023\,(t-42.75)^2$$

From § 65 we have for the lengths of the bars in metre MT1876,

$$(MT\,1876)_S=R\,1876-\ 47^{\mu}.6+\ 0^{\mu}.451\,(t-32)$$
$$(MT\,1876)_Z=R\,1876+218^{\mu}.9+10^{\mu}.639\,(t-32)$$

§ **67.** The lengths of the bases, measured with the Repsold apparatus, will be expressed in terms of S_1. The value of S_1 in terms of R1876, and the probable error of this value, have been given in § 66. Hence, when a precise value of R1876 is obtained, the bases can be accurately expressed in terms of the metre. In the mean time the following gives the present information about R1876.

Through the kindness of Professor Foerster of the Kaiserliche Normal-Eichungs-Kommission of Berlin, which it is desired warmly to acknowledge, R1876 was compared in 1878–'79 with an entirely similar steel metre made for the Kommission by Repsold, and designated as R1878. The errors of the subordinate graduations of R1876 and of the decimeter D1876 were also determined by the Normal-Eichungs-Kommission. It was hoped to obtain in time for insertion in this report the details of these comparisons, but they have not yet been received.

R1878 has since been compared by the Comité International des Poids et Mesures with a unit which will probably not differ by more than 1 or 2 microns from the metrical prototype (Comité International des Poids et Mesures, Procès Verbaux, 1880, p. 106), and it is supposed that Professor Foerster's statements in his letter of September 15, 1880, refer to these comparisons.

It is hoped ultimately to obtain and publish the comparisons of *R* 1876 with *R* 1878 and its value in terms of the prototype metre. The following letters give all the information that has thus far been received from the Normal-Eichungs-Kommission with reference to our standards depending on the metre.

RESULTS OF COMPARISONS OF LAKE-SURVEY STANDARD METRE (*R* 1876), BY PROFESSOR W. FOERSTER, BERLIN.

BERLIN, *den* 16. *April* 1879.

In Beantwortung des gefälligen Schreibens vom 17. v. M. erlaube ich mir, Ihnen zunächst ein Verzeichniss der blossen Eintheilungsfehler Ihres stählernen Meterstabes von Repsold und des zugehörigen Decimeters zu übersenden, indem ich bezüglich der noch restirenden, von Ihnen dringend gewünschten anderweitigen Festsetzungen Folgendes ergebenst bemerke:

Der hauptsächliche Grund der Verzögerung besteht darin, dass wir durch dringliche laufende Aufgaben bisher noch immer verhindert worden sind, absolute Ausdehnungsbestimmungen zu machen. Indessen sind wir wenigstens im letzten Winter dazu gelangt, gute relative Ausdehnungsbestimmungen eines Stahlmeters und eines Messingmeters gegen einen Platinstab zu machen, dessen absolute Ausdehnung ziemlich nahe bekannt ist. Die Ergebnisse dieser Bestimmungen werden in zwei bis drei Wochen so weit abgeschlossen sein können, dass ich hoffe in 4 bis 5 Wochen Ihnen die Längen Ihrer Maassstäbe gegen den erwähnten Platinstab und die zugehörigen Ausdehnungswerthe zu übersenden. Einige ungefähre Bestimmungen werden vielleicht schon früher Ihnen zugehen können.

Wir werden zur Beschleunigung der Sache von Ihrer Ermächtigung einer Kostenliquidation, die wir bisher in allegemein wissenschaftlichem Interesse zu unterlassen beabsichtigt hatten, nunmehr vollen Gebrauch machen.

Kaiserliche Normal-Eichungs-Kommission.

FOERSTER.

To the OFFICE OF UNITED STATES LAKE SURVEY,
Detroit, Mich.

Beglaubigtes Fehlerverzeichniss der sämmtlichen Striche eines biegungsfreien Strichmaasses von Stahl mit Theilung von 1 Meter Länge auf Platin und einer zugehörigen gleichfalls auf Platin getheilten Decimeterskala von Stahl. Verfertigt von A. Repsold Söhne in Hamburg, zur Prüfung eingereicht von dem Office of United States Lake Survey, in Detroit, Mich.

I. DAS STRICHMAASS VON 1 METER LÄNGE.

a. Die Zehntel-millimeter-striche.		*b.* Die Millimeterstriche.								*c.* Die Centimeterstriche.		*d.* Die Decimeterstriche.	
Theilstrich.	Fehler.	Theilstrich.	Fehler.	Theilstrich.	Fehler.	Theilstrich.	Fehler.	Theilstrich.	Fehler.	Theilstrich.	Fehler.	Theilstrich.	Fehler.
—0.1	0	0	0	25	+2	50	+1	75	0	0	0	0	0
0.0	0	1	—1	26	+3	51	+1	76	0	1	+2.6	1	—0.5
+0.1	—1	2	—1	27	+2	52	+2	77	0	2	+2.4	2	—2.5
0.2	+1	3	0	28	+2	53	+1	78	0	3	+2.9	3	—1.5
0.3	+1	4	0	29	+3	54	—1	79	0	4	+2.4	4	—1.1
0.4	+2	5	+1	30	+3	55	+1	80	—1	5	+1.1	5	—1.6
0.5	—1	6	0	31	+4	56	0	81	0	6	+0.6	6	+0.5
0.6	+1	7	0	32	+3	57	0	82	0	7	+1.2	7	—1.4
0.7	0	8	+1	33	+3	58	0	83	+1	8	—0.5	8	—0.1
0.8	0	9	+1	34	+2	59	+1	84	+1	9	+1.0	9	—0.5
0.9	—2	10	+3	35	+3	60	+1	85	+1	10*	—0.6	10	0
1.0	—1	11	+2	36	+2	61	+2	86	+1				
1.1	+1	12	+2	37	+3	62	+1	87	+1				
		13	+2	38	+2	63	+1	88	+3				
		14	+2	39	+1	64	+1	89	—1				
		15	+3	40	+2	65	+3	90	+1				
		16	+2	41	+1	66	+2	91	0				
		17	+3	42	+2	67	+2	92	—1				
		18	+3	43	+1	68	+2	93	+1				
		19	+3	44	+1	69	+2	94	0				
		20	+2	45	+1	70	+1	95	+1				
		21	+2	46	+1	71	+2	96	+1				
		22	+2	47	+1	72	+1	97	0				
		23	+3	48	+1	73	+1	98	+1				
		24	+1	49	0	74	0	99	0				
		25	+2	50	+1	75	0	100*	+1				

Verbörgbar,
bei *a* und *b* $\overset{\mu}{1.0}$ bis $\overset{\mu}{1.5}$
bei *c* und *d* 0.2 bis 0.3

Die Gesammtlänge ist noch nicht bestimmt worden.

* Foerster's letter of June 20, 1879 (following), changes correction at 100^{mm} from +1 to —1, and at 10^{cm} from —0.6 to —0.5.

Beglaubigtes Fehlerverzeichniss, &c.—Continued.

II. DIE DECIMETERSKALA.

a. Die Zehntel-millimeter-striche.

Theil-strich.	Fehler.
−0.1	+0.6
0.0	0
+0.1	+0.3
0.2	+0.9
0.3	+0.4
0.4	+0.9
0.5	−0.1
0.6	+0.4
0.7	+0.2
0.8	−0.1
0.9	+0.1
1.0	−0.2
1.1	+1.6

Verbürgbar, 0.3 bis 0.5 μ.

b. Die Millimeterstriche.

Theil-strich.	Fehler.	Theil-strich.	Fehler.	Theil-strich.	Fehler.	Theil-strich.	Fehler.
0	0	25	0.0	50	+1.0	75	0.0
1	0.0	26	+0.5	51	+1.0	76	0.0
2	−0.5	27	0.0	52	+1.0	77	0.0
3	0.0	28	−0.5	53	+1.0	78	0.0
4	0.0	29	−1.0	54	+1.5	79	0.0
5	0.0	30	−0.5	55	+1.0	80	+1.0
6	+1.5	31	0.0	56	+1.0	81	0.0
7	+1.0	32	0.0	57	+0.5	82	0.0
8	+1.0	33	0.0	58	+1.5	83	−0.5
9	0.0	34	−1.5	59	+0.5	84	+1.0
10	+1.0	35	+0.5	60	+1.0	85	+2.0
11	0.0	36	0.0	61	+1.5	86	+1.0
12	+1.0	37	−0.5	62	+1.0	87	+2.0
13	+0.5	38	−1.0	63	0.0	88	+2.5
14	−0.5	39	+0.5	64	+1.0	89	+0.5
15	−0.5	40	0.0	65	+1.5	90	+1.0
16	−1.5	41	0.0	66	+1.0	91	0.0
17	−1.0	42	0.0	67	+1.5	92	−0.5
18	−0.5	43	−1.0	68	0.0	93	0.0
19	−0.5	44	0.0	69	+1.0	94	−1.5
20	−0.5	45	+2.0	70	+1.5	95	0.0
21	−0.5	46	+0.5	71	+0.5	96	0.0
22	−0.5	47	0.0	72	0.0	97	+1.0
23	−0.5	48	+0.5	73	+0.5	98	0.0
24	−0.5	49	+1.5	74	−0.5	99	−0.5
25	0.0	50	+1.0	75	0.0	100	0

Verbürgbar, 0.5 bis 1 μ.

Die Gesammtlänge ist nicht bestimmt worden.

In vorstehenden Verzeichnissen bedeutet das {positive / negative} Zeichen, dass das Intervall zwischen dem Nullstrich und dem betreffenden Theilstrich um die danebenstehende Anzahl von Tausendtheilen des Millimeter {grösser / kleiner} ist, als das in der Bezifferung ausgedrückte nominelle Verhältniss dieses Intervalles zu der Gesammtlänge, d. h. zu demjenigen Abstande, welcher auf dem Meterstabe zwischen dem Null- und 1000-Millimeter-Strich auf der beigegebenen Decimeterskala zwischen dem Null- und 100-Millimeter-Strich enthalten ist.

BEISPIEL.

Für den Centimeterstrich 7 findet man in dem Verzeichniss die Zahlenangabe + 1.2, d. h. das Intervall zwischen dem Null- und dem 7-Centimeterstrich beträgt 1.2 Tausendtheile des Millimeter mehr als $\frac{70}{1000}$ des Abstandes des Nullstriches des Maassstabes von dem 1000-Millimeter-Strich.

Der wahre metrische Werth jedes Intervalles kann erst gefunden werden, wenn die oben definirte Gesammtlänge und der Ausdehnungs-Coefficient des Stabes, bezw. der Decimeterskala bekannt sein wird.

Berlin, den 16. April 1879.

Kaiserliche Normal-Eichungs-Kommission.
Im Auftrage.
BAUMANN I.

[TRANSLATION.]

BERLIN, *April* 16, 1879.

In reply to your kind letter of March 17, permit me to send you a list of the errors of graduation of the steel metre made for you by Repsold, and also of the accompanying decimeter scale.

With reference to the further determinations, urgently desired by you, I wish to say that the chief cause of delay in making absolute-expansion determinations was pressing current business. Last winter, however, we succeeded in making good relative-expansion determinations of a steel metre and of a brass metre with a platinum bar whose absolute expansion is known pretty nearly. The results of these determinations will be so far completed in two or three weeks that I hope to be able in four or five weeks to send to you the lengths of your measures with reference to the platinum bar above mentioned, and also their respective expansion-values. Some approximate determinations will perhaps be sent sooner.

We will, in order to hasten your business, make full use of the authority given for payment of expenses which we have hitherto been in the habit of neglecting in general scientific interests.

Kaiserliche Normal-Eichungs-Kommission.

FOERSTER.

To the OFFICE OF U. S. LAKE SURVEY,
Detroit, Mich.

Certified list of errors of each one of the graduations of a line-measure metre made of steel, graduated on platinum and free from flexure; also a similarly constructed decimeter scale; both made by A. Repsold's Sons, Hamburg. Submitted for examination by office of United States Lake Survey, Detroit, Mich.

I.—THE LINE-MEASURE 1 METRE IN LENGTH.

a. The tenth-millimeter graduations.		*b.* The millimeter graduations.								*c.* The centimeter graduations.		*d.* The decimeter graduations.	
Graduations.	Errors.	Graduations.	Errors.	Graduations.	Errors.	Graduations.	Errors.	Graduations.	Errors.	Graduations.	Errors.	Graduations.	Errors.
-0.1	0	0	0	25	+2	50	+1	75	0	0	0	0	0
0.0	0	1	-1	26	+3	51	+1	76	0	1	+2.6	1	-0.5
+0.1	-1	2	-1	27	+2	52	+2	77	0	2	+2.4	2	-2.5
0.2	+1	3	0	28	+2	53	+1	78	0	3	+2.9	3	-1.5
0.3	+1	4	0	29	+3	54	-1	79	0	4	+2.4	4	-1.1
0.4	+2	5	+1	30	+3	55	+1	80	-1	5	+1.1	5	-1.6
0.5	-1	6	0	31	+4	56	0	81	0	6	+0.6	6	+0.5
0.6	+1	7	0	32	+3	57	0	82	0	7	+1.2	7	-1.4
0.7	0	8	+1	33	+3	58	0	83	+1	8	-0.5	8	-0.1
0.8	0	9	+1	34	+2	59	+1	84	+1	9	+1.0	9	-0.5
0.9	-2	10	+3	35	+3	60	+1	85	+1	10*	-0.6	10	0
1.0	-1	11	+2	36	+2	61	+2	86	+1				
1.1	+1	12	+2	37	+3	62	+1	87	+1				
		13	+2	38	+2	63	+1	88	+3				
		14	+2	39	+1	64	+1	89	-1				
		15	+3	40	+2	65	+3	90	+1				
		16	+2	41	+1	66	+2	91	0				
		17	+3	42	+2	67	+2	92	-1				
		18	+3	43	+1	68	+2	93	+1				
		19	+3	44	+1	69	+2	94	0				
		20	+2	45	+1	70	+1	95	+1				
		21	+2	46	+1	71	+2	96	+1				
		22	+2	47	+1	72	+1	97	0				
		23	+3	48	+1	73	+1	98	+1				
		24	+1	49	0	74	0	99	0				
		25	+2	50	+1	75	0	100*	+1				

Reliability,

a and *b*, $\overset{\mu}{1.0}$ to $\overset{\mu}{1.5}$
c and *d*, 0.2 to 0.3

The total length has not yet been determined.

*Foerster's letter of June 20, 1879 (following), changes correction at 100^{mm} from +1 to -1, and at 10^{cm} from -0.6 to -0.5.

Certified list of errors, &c.—Continued.

II.—THE DECIMETER SCALE.

a. The tenth-millimeter graduations.		*b.* The millimeter graduations.							
Graduations.	Errors.	Graduations.	Errors.	Graduations.	Errors.	Graduations.	Errors.	Graduations.	Errors.
−0.1	+0.6	0	0	25	0.0	50	+1.0	75	0.0
0.0	0	1	0.0	26	+0.5	51	+1.0	76	0.0
+0.1	+0.3	2	−0.5	27	0.0	52	+1.0	77	0.0
0.2	+0.9	3	0.0	28	−0.5	53	+1.0	78	0.0
0.3	+0.4	4	0.0	29	−1.0	54	+1.5	79	0.0
0.4	+0.9	5	0.0	30	−0.5	55	+1.0	80	+1.0
0.5	−0.1	6	+1.5	31	0.0	56	+1.0	81	0.0
0.6	+0.4	7	+1.0	32	0.0	57	+0.5	82	0.0
0.7	+0.2	8	+1.0	33	0.0	58	+1.5	83	−0.5
0.8	−0.1	9	0.0	34	−1.5	59	+0.5	84	+1.0
0.9	+0.1	10	+1.0	35	+0.5	60	+1.0	85	+2.0
1.0	−0.2	11	0.0	36	0.0	61	+1.5	86	+1.0
1.1	+1.6	12	+1.0	37	−0.5	62	+1.0	87	+2.0
		13	+0.5	38	−1.0	63	0.0	88	+2.5
Reliability, 0.3^{μ} to 0.5^{μ}		14	−0.5	39	+0.5	64	+1.0	89	+0.5
		15	−0.5	40	0.0	65	+1.5	90	+1.0
		16	−1.5	41	0.0	66	+1.0	91	0.0
		17	−1.0	42	0.0	67	+1.5	92	−0.5
		18	−0.5	43	−1.0	68	0.0	93	0.0
		19	−0.5	44	0.0	69	+1.0	94	−1.5
		20	−0.5	45	+2.0	70	+1.5	95	0.0
		21	−0.5	46	+0.5	71	+0.5	96	0.0
		22	−0.5	47	0.0	72	0.0	97	+1.0
		23	−0.5	48	+0.5	73	+0.5	98	0.0
		24	−0.5	49	+1.5	74	−0.5	99	−0.5
		25	0.0	50	+1.0	75	0.0	100	0
		Reliability, 0.5^{μ} to 1^{μ}							

The total length has not yet been determined.

In the preceding tables the + and — signs show that the value of the space between the zero and the graduation to be considered is greater or less than the nominal proportional value of this space to the whole length indicated by the number of the graduation, by the amount which stands opposite to the number of the graduation and is expressed in thousandths of a millimeter.

EXAMPLE.

Opposite the centimeter-mark 7 will be found in the table the figures +1.2, that is, the interval between the 0- and the 7-centimeter mark amounts to 1.2 thousandths parts of a millimeter more than .07 of the distance of the 0-mark from the 1000-millimeter mark.

The true metric value of each space can only be found when the total lengths and expansion-coefficients of the metre- and decimeter-scales are known.

Berlin, April 16, 1879.

Kaiserliche Normal-Eichungs-Kommission.

By order.

BAUMANN I.

BERLIN, *den* 20. *Juni* 1879.

In Beantwortung des gefälligen Schreibens vom 12. v. Mts. und im Anschluss an das diesseitige Schreiben vom 16. April d. J. theilt die Kommission Ihnen hierdurch ergebenst mit, dass sich für das Ihnen gehörige stählerne Normalmeter von Repsold (R1876) auf Grund der gesetzlichen Beziehungen, welche zwischen unserm Platinmeter und dem metrischen Urmaass angenommen sind, nunmehr folgende Gleichung ergeben hat, in welcher t in Centigraden ausgedrückt ist:

(1) $$R1876 = 1\text{ Meter} + 248^{\mu}.89 \pm 0^{\mu}.25 + (10^{\mu}.31 \pm 0^{\mu}.034)\,(t-15)$$

Ausserdem liegt eine vorläufige Berechnung der Vergleichungen eines dem Ihrigen vollständig entsprechenden stählernen Normalmeters von Repsold mit einer gutbestimmten Kopie der Bessel'schen Toise vor, aus welcher sich unter der Annahme, dass das metrische Urmaass die bekannte Beziehung zu der durch die Bessel'sche Toise vertretenen altfranzösischen Einheit hat, für Ihr Normalmeter folgende Gleichung in altfranzösischem Metermaass ergeben würde:

(2) $$R\,1876 = 1 \text{ Meter} + 245^{\mu}.6 + 10^{\mu}.31\ (t-15)$$

Beide Gleichungen dürften noch um mehrere Tausendtheile des Millimeter unsicher sein, da in Gleichung (1) die sehr unsichere Vergleichung unseres Urmaasses mit dem Mètre des Archives, in der Gleichung (2) dagegen die Unsicherheit der Beziehungen zwischen der Bessel'schen Toise und der Toise du Pérou, sowie zwischen letzterer und dem Mètre des Archives enthalten ist. Indessen dürfte doch wohl auf 0,01 mm. sicher anzunehmen sein:

$$R\,1876 = 1 \text{ Meter} + 247^{\mu} + 10^{\mu}.31\ (t-15)$$

Die absolute Länge Ihres *Decimeterstabes* ist leider nicht ausreichend diesseits bestimmt worden, wenigstens nicht entfernt mit derjenigen Schärfe, mit welcher die *Eintheilungsfehler* dieser Hülfsskale ermittelt worden sind. Wir bedauern dies und müssen Ihnen daher anheimgeben, die absolute Länge dieser Skale durch Vergleichung mit einem der Decimeter-Intervalle Ihres Normalmeters ermitteln zu wollen, welche durch die obige Angabe der absoluten Länge des ganzen Stabes in Verbindung mit den in dem Schreiben vom 16. April Ihnen mitgetheilten Bestimmungen der inneren Eintheilungsfehler mit entsprechender Zuverlässigkeit angegeben werden können. Wir behalten uns vor, Ihnen demnächst weitere Mittheilungen betreffend Ihren Pendelmaassstab zu senden und die Liquidationsangelegenheit danach entsprechend zu regeln.

Zu dem unter dem 16. April d. J. gesandten Fehlerverzeichniss sind folgende Verbesserungen nachzutragen:

$$R\,1876 \begin{cases} \text{Theilungsfehler bei } 100^{\text{mm}} -1^{\mu} \text{ statt } +1^{\mu} \\ \text{Theilungsfehler bei } 10^{\text{cm}} \quad -0.5 \text{ statt} -0.6 \end{cases}$$

Ausserdem wird bemerkt dass in dem Theilungsfehler-Verzeichnisse der Decimeterskale die Fehler der Millimeter auf $0^{\mu}.5$ abgerundet sind, während diejenigen der Zehntelmillimeter in $0^{\mu}.1$ angegeben werden könnten, und dass hierauf die scheinbare Verschiedenheit der für den 1 mm-Strich in den beiden Reihen angegebenen Fehler zurückzuführen ist.

Kaiserliche Normal-Eichungs-Kommission.
FOERSTER.

To the OFFICE OF THE U. S. LAKE SURVEY, *Detroit, Mich.*

[TRANSLATION.]

BERLIN, *June* 20, 1879.

In answer to your kind letter of the 12th of last month, and in connection with our letter of April 16 of this year, the Commission has the honor to inform you that your steel standard metre (R1876), on the basis of the legal relations which have been taken between our platinum metre and the metrical standard, has the following equation, in which t is expressed in centigrade degrees:

(1) $$R\,1876 = 1^{m} + 248^{\mu}.89 \pm 0^{\mu}.25 + (10^{\mu}.31 \pm 0^{\mu}.034)\ (t-15)$$

A preliminary computation of the comparisons of a steel standard metre by Repsold perfectly corresponding with yours, with a well-determined copy of Bessel's toise, has been made; which computation, under the assumption that the metrical standard has the known relation to the old French unit represented by the Bessel toise, would give for your standard metre the following equation in old French metre-measure:

(2) $$R\,1876 = 1^{m} + 245^{\mu}.6 + 10^{\mu}.31\ (t-15)$$

Both equations are to be regarded as uncertain by several thousandths of a millimeter, since in equation (1) is contained the very uncertain comparison of our original measure with the metre of the Archives, and in equation (2) the uncertainty of the relations between the Bessel toise and the toise of Peru, as also between the last and the metre of the Archives.

We may, however, certainly assume within $0^{\text{mm}}.01$

$$R\,1876 = 1^{m} + 247^{\mu} + 10^{\mu}.31\ (t-15)$$

The absolute length of your decimeter-bar has unfortunately not been determined sufficiently here, at least not nearly with the same precision with which the graduation-errors of this auxiliary scale have been determined. We regret this, and must therefore leave it to you to determine the absolute length of this scale by comparison with one of the decimeter-spaces of your standard metre, which by the absolute length given above of the whole bar, in connection with the determination of the relative graduation-errors, transmitted to you in our letter of April 16, can be given with corresponding precision.

We reserve sending to you further communications regarding your pendulum, and also the expense account.

To the list of errors sent April 16 of this year the following corrections are to be carried in:

$$R\,1876 \begin{cases} \text{Graduation error at } 100^{\text{mm}}, -1^{\mu} \text{ instead of } +1^{\mu} \\ \text{Graduation error at } 10^{\text{cm}}, \quad -0.5 \text{ instead of } -0.6 \end{cases}$$

Besides, we may remark that in the list of graduation-errors of the decimeter-scale the errors of the millimeter are rounded to $0^{\mu}.5$, while those of the tenths of the millimeter can be taken to $0^{\mu}.1$, and that the apparent difference of the errors given in the two series for the 1^{mm} mark is to be attributed to this.

Kaiserliche Normal-Eichungs-Kommission.
FOERSTER.

BERLIN, *den* 15. *September* 1880.

In Beantwortung des gefälligen Schreibens vom 24. v. M. erwidert die Kommission hiermit ergebenst, dass die Details der Vergleichungen Ihres Stahlmeters (*R* 1876) mit unserem entsprechenden Stahlmeter (*R* 1878) Ihnen in kürzester Frist mitgetheilt werden sollen. Publicirt sind dieselben bisher noch nicht, weil einige Elemente der Messung und Rechnung noch nicht sicher genug erschienen waren.

Wie Sie voraussetzen, hängen in der That die sämmtlichen Bestimmungen Ihres Stahlmeters (*R* 1876) von dem in unserem Besitz befindlichen mit ersterem nahezu identischen Stahlmeter (R 1878) ab, und dieses letztere Normalmeter ist es auch, welches mittelbar mit der Bessel'schen Toise und mit der Toise No. 10 verglichen worden ist. Auch ist das Normalmeter (*R* 1878) dasselbe, welches im vorigen Herbst dem internationalen Bureau für Maasse und Gewichte zu Paris übersendet und dort mit den neuesten und besten Kopien des französischen Urmeters verglichen worden ist.

Spätestens gegen Ende Oktober hoffen wir Ihnen alle Details über die bis jetzt von uns ermittelten oder zu unserer Kenntniss gekommenen Ergebnisse der bezüglichen Vergleichungen zu senden.

Von den Pariser Ergebnissen, auf welche wir noch warten, werden Ihnen ebenfalls alle wünschenswerthen Details mitgetheilt werden, sobald dieselben in unsern Händen sind.

Einstweilen können Sie nach vorläufigen Mittheilungen, die wir aus Paris erhalten haben, annehmen, dass die von uns vorläufig gemachten Angaben über die Gleichung von (*R* 1876) im wahren metrischen Maass höchstens einen Fehler von 1 bis 2 Tausendtheilen des Millimeter haben, und dass der innerhalb dieser Grenzen liegende definitive Werth bis auf Bruchtheile eines Tausendtheils baldigst sicher gestellt sein wird.

Bezüglich der Vergleichung mit dem altfranzösischen System können wir einstweilen nur genähert angeben, dass das durch (*R* 1876) und seine Gleichung definirte Meter um etwa 8 Tausendtheile des Millimeter kleiner ist als das gleich 443,296 Pariser Linien definirte Meter, wie es sich aus Toise No. 10 ergeben würde.

In Betreff eines Maassstabes mit Metallthermometer, welcher, wie wir jetzt in Erfahrung gebracht haben, ebenfalls dem von Ihnen geleiteten Unternehmen angehört, welcher aber von uns längere Zeit hindurch irrthümlich als ein für Pendelbeobachtungen bestimmtes Normalmeter bezeichnet worden ist, würden wir Ihnen auch in kürzester Frist die Ergebnisse der hiesigen Vergleichungen mittheilen können, wenn Sie einige Unbestimmtheiten, die sich nachträglich bei der definitiven Berechnung der Messungen herausgestellt haben, und über welche die Herren Repsold selbst uns keine sichere Auskunft geben können, dadurch beseitigen wollten, dass Sie uns bald möglichst eine nähere Angabe in Betreff der Einrichtung der auf dem erwähnten Maassstabe enthaltenen Eintheilungen senden, und zwar wohl am zweckmässigsten in Form einer kleinen graphischen Darstellung der Lage der Endpunkte des Maasses und der Lage der Nullpunkte der Metallthermometer, sowie der Art und Bezifferung der Hilfseintheilungen, welche an den beiden Enden sowohl des Stabes als des Metallthermometers angebracht sind.

Kaiserliche Normal-Eichungs-Kommission.
FOERSTER.

An Major COMSTOCK, *Office of United States Lake Survey, Detroit, Mich.*

[TRANSLATION.]

BERLIN, *September* 15, 1880.

In reply to your letter of 24th ultimo, the Commission has respectfully to state that the details of comparisons of your steel metre (*R* 1876) with our corresponding steel metre (*R* 1878) will be communicated to you shortly. They are not yet published because several elements of measurement and computation do not yet appear sufficiently certain.

As you suppose, all the determinations of your steel metre (*R* 1876) depend indeed upon the steel metre (*R* 1878) in our possession, which is nearly identical with yours, and this normal metre (*R* 1878) was the one which was compared indirectly with Bessel's toise and with Toise No. 10. The normal metre (*R* 1878) is also the same one which was sent last fall to the International Bureau of Weights and Measures at Paris, and was there compared with the newest and best copies of the original French metre.

We hope to send you towards the end of October all the details of the results of the respective comparisons determined by us or which have come to our knowledge.

Of the Paris results, for which we are yet waiting, all desirable details will be communicated to you as soon as we receive them. In the mean time, you may assume, according to preliminary information received from Paris, that our preliminary statements relative to the equation of (*R* 1876) in the true metric measure have at the greatest an error of 1 or 2 thousandths of a millimeter, and the definitive value lying within this range will soon be determined to fractions of a thousandth part.

Relative to the comparison with the old French system, we can for the present only state approximately that the metre defined through (*R* 1876) and its equation is about 8 thousandths of a millimeter shorter than the metre defined =443,296 Parisian Lines, as it would result from Toise No. 10.

Concerning a measure (Maassstab) with metallic thermometer which, as we have now learned, also belongs to the work under your direction, but which has been for some time erroneously designated by us as a standard intended for pendulum observations, we could also shortly communicate to you the results of our comparisons, if you would remove some uncertainties which have shown themselves in making the final computations of measurements, and upon which Messrs. Repsold themselves could not give us any positive information, by sending us as soon as practicable a statement relative to the arrangement of the graduations of the measure mentioned, most practically in the form of a small graphic representation of the position of the end-points of the measure, and the positions of the zero-points of the metallic thermometer, as well as the kind and numbering of the subdivisions on both ends of the measure, and of the metallic thermometer.

Kaiserliche Normal-Eichungs-Kommission.
FOERSTER.

To Major COMSTOCK, *Office of United States Lake Survey, Detroit, Mich.*

PERIODIC ERROR OF MICROMETER-SCREWS.

§ **68.** This error was determined by measuring the same space on different parts of a micrometer-head. First, a space nearly equal to one-half a revolution of the screw was measured 100 times. Each measurement began one division of the micrometer-head in advance of the preceding one. There being 100 divisions, each one was used once as a starting-point. Next, a space nearly equal to a quarter of a revolution was measured 100 times in the same way. Each measurement of a space gives an observation-equation of the following form:

$$\begin{aligned}(u'-u-f) = &+2a_1 \sin\frac{f}{2} \sin\left(u+\frac{f}{2}\right) - 2b_1 \sin\frac{f}{2} \cos\left(u+\frac{f}{2}\right)\\ &+2a_2 \sin f \sin(2u+f) - 2b_2 \sin f \cos(2u+f)\end{aligned}$$

where f is the space measured, u and u' are the first and second readings on the space, a_1, a_2, b_1, and b_2 are unknown quantities to be determined. Since f is measured on many parts of the micrometer-head, the periodic error is very nearly eliminated in the mean, and the mean of all the observed values of f is sufficiently near its true value. The observation-equations when reduced give normal equations of the following form:

$$\begin{aligned}100a_1 \sin\frac{f}{2} &= +\Sigma(u'-u-f)\sin\left(u+\frac{f}{2}\right)\\ 100a_2 \sin\frac{f}{2} &= -\Sigma(u'-u-f)\cos\left(u+\frac{f}{2}\right)\\ 100b_1 \mathrm{siu}\ f &= +\Sigma(u'-u-f)\sin(2u+f)\\ 100b_2 \sin f &= -\Sigma(u'-u-f)\cos(2u+f)\end{aligned}$$

From these normals the values of a_1, a_2, b_1, and b_2 were determined. With them the correction to each graduation of the micrometer was computed. The corrections are given for each microscope in the following table. The first column contains the number of the graduation on the micrometer-head. The remaining columns contain the corresponding corrections for the several micrometers. The corrections are applied to the micrometer-readings with the given signs.

Table of corrections for periodic errors of micrometers.

Graduation.	Mic. No. 1.	Mic. No. 2.	Mic. No. 3.	Mic. No. 4.	Mic. No. 5.	Mic. No. 6.
	μ	μ	μ	μ	μ	μ
0	−0.08	+0.02	+0.11	−0.12	+0.19	−0.09
1	−0.09	+0.01	+0.11	−0.11	+0.18	−0.09
2	−0.10	−0.01	+0.10	−0.11	+0.16	−0.09
3	−0.11	−0.02	+0.10	−0.11	+0.15	−0.08
4	−0.12	−0.04	+0.09	−0.11	+0.14	−0.08
5	−0.13	−0.05	+0.08	−0.10	+0.12	−0.07
6	−0.13	−0.07	+0.07	−0.10	+0.10	−0.06
7	−0.14	−0.08	+0.06	−0.10	+0.08	−0.06
8	−0.15	−0.09	+0.05	−0.10	+0.06	−0.05
9	−0.15	−0.11	+0.03	−0.10	+0.05	−0.05
10	−0.16	−0.12	+0.02	−0.09	+0.02	−0.04
11	−0.17	−0.13	+0.00	−0.09	+0.00	−0.04
12	−0.17	−0.15	−0.01	−0.09	−0.02	−0.03
13	−0.18	−0.16	−0.03	−0.09	−0.04	−0.03
14	−0.18	−0.17	−0.05	−0.09	−0.07	−0.02
15	−0.18	−0.18	−0.07	−0.10	−0.09	−0.02
16	−0.19	−0.18	−0.08	−0.10	−0.11	−0.02
17	−0.19	−0.19	−0.10	−0.10	−0.14	−0.01
18	−0.19	−0.19	−0.12	−0.10	−0.16	−0.01
19	−0.19	−0.19	−0.14	−0.10	−0.19	−0.00

Table of corrections for periodic errors of micrometers—Continued.

Graduation.	Mic. No. 1.	Mic. No. 2.	Mic. No. 3.	Mic. No. 4.	Mic. No. 5.	Mic. No. 6.
	μ	μ	μ	μ	μ	μ
20	—0.20	—0.19	—0.15	—0.10	—0.21	—0.00
21	—0.20	—0.19	—0.17	—0.10	—0.23	—0.00
22	—0.20	—0.19	—0.18	—0.10	—0.25	—0.00
23	—0.20	—0.18	—0.19	—0.11	—0.27	—0.00
24	—0.20	—0.17	—0.21	—0.11	—0.29	+0.01
25	—0.20	—0.16	—0.22	—0.11	—0.30	+0.01
26	—0.20	—0.15	—0.23	—0.10	—0.32	+0.01
27	—0.20	—0.14	—0.23	—0.10	—0.33	+0.01
28	—0.20	—0.13	—0.23	—0.10	—0.34	+0.01
29	—0.20	—0.12	—0.23	—0.10	—0.35	+0.01
30	—0.20	—0.10	—0.24	—0.09	—0.35	+0.01
31	—0.19	-0.09	—0.23	—0.09	—0.36	+0.02
32	—0.19	—0.07	—0.23	—0.08	—0.36	+0.02
33	—0.18	—0.06	—0.22	—0.07	—0.36	+0.02
34	—0.17	—0.04	—0.21	—0.07	—0.35	+0.02
35	—0.17	—0.03	—0.21	—0.06	—0.35	+0.02
36	—0.16	—0.01	—0.19	—0.05	—0.34	+0.03
37	—0.15	—0.00	—0.18	—0.03	—0.33	+0.03
38	—0.14	+0.01	—0.17	—0.02	—0.32	+0.03
39	—0.13	+0.03	—0.15	—0.01	—0.31	+0.03
40	—0.12	+0.04	—0.13	—0.00	—0.29	+0.04
41	—0.11	+0.05	—0.11	+0.02	—0.27	+0.04
42	—0.10	+0.06	—0.10	+0.03	—0.25	+0.04
43	—0.08	+0.07	—0.08	+0.05	—0.23	+0.05
44	—0.07	+0.08	—0.06	+0.06	—0.21	+0.05
45	—0.06	+0.09	—0.04	+0.08	—0.19	+0.06
46	—0.04	+0.09	—0.02	+0.09	—0.17	+0.06
47	—0.02	+0.10	—0.01	+0.11	—0.15	+0.06
48	—0.01	+0.10	+0.01	+0.12	—0.13	+0.07
49	+0.01	+0.10	+0.03	+0.14	—0.10	+0.07
50	+0.03	+0.10	+0.04	+0.15	—0.08	+0.08
51	+0.04	+0.10	+0.06	+0.17	—0.06	+0.08
52	+0.06	+0.10	+0.07	+0.18	—0.04	+0.08
53	+0.08	+0.10	+0.08	+0.19	—0.02	+0.09
54	+0.10	+0.09	+0.09	+0.20	—0.00	+0.09
55	+0.12	+0.09	+0.10	+0.21	+0.02	+0.09
56	+0.13	+0.08	+0.11	+0.21	+0.04	+0.09
57	+0.15	+0.08	+0.12	+0.22	+0.06	+0.09
58	+0.16	+0.07	+0.12	+0.22	+0.07	+0.10
59	+0.18	+0.07	+0.13	+0.22	+0.09	+0.10
60	+0.20	+0.06	+0.13	+0.22	+0.11	+0.10
61	+0.21	+0.06	+0.13	+0.22	+0.12	+0.10
62	+0.22	+0.05	+0.13	+0.22	+0.13	+0.09
63	+0.23	+0.05	+0.12	+0.21	+0.14	+0.09
64	+0.24	+0.04	+0.12	+0.21	+0.15	+0.09
65	+0.25	+0.04	+0.12	+0.20	+0.16	+0.08
66	+0.26	+0.03	+0.11	+0.19	+0.16	+0.08
67	+0.26	+0.03	+0.10	+0.18	+0.17	+0.07
68	+0.26	+0.03	+0.10	+0.17	+0.18	+0.07
69	+0.27	+0.03	+0.09	+0.16	+0.18	+0.06
70	+0.27	+0.03	+0.09	+0.14	+0.19	+0.05
71	+0.27	+0.03	+0.08	+0.13	+0.19	+0.04
72	+0.27	+0.03	+0.08	+0.11	+0.19	+0.03
73	+0.26	+0.03	+0.07	+0.10	+0.20	+0.03
74	+0.26	+0.04	+0.06	+0.08	+0.20	+0.02
75	+0.25	+0.04	+0.06	+0.07	+0.20	+0.01
76	+0.25	+0.05	+0.05	+0.05	+0.20	+0.00
77	+0.24	+0.05	+0.05	+0.04	+0.20	—0.01
78	+0.23	+0.06	+0.05	+0.02	+0.21	—0.02
79	+0.22	+0.06	+0.05	+0.01	+0.21	—0.03
80	+0.21	+0.07	+0.05	—0.01	+0.21	—0.04

Table of corrections for periodic errors of micrometers—Continued.

Graduation.	Mic. No. 1.	Mic. No. 2.	Mic. No. 3.	Mic. No. 4.	Mic. No. 5.	Mic. No. 6.
	μ	μ	μ	μ	μ	μ
81	+0.20	+0.07	+0.05	−0.02	+0.21	−0.05
82	+0.18	+0.08	+0.05	−0.03	+0.21	−0.05
83	+0.17	+0.08	+0.05	−0.05	+0.21	−0.06
84	+0.15	+0.09	+0.05	−0.06	+0.22	−0.07
85	+0.14	+0.09	+0.06	−0.07	+0.22	−0.08
86	+0.12	+0.09	+0.06	−0.08	+0.22	−0.08
87	+0.10	+0.10	+0.07	−0.09	+0.22	−0.09
88	+0.09	+0.10	+0.07	−0.10	+0.22	−0.10
89	+0.07	+0.10	+0.08	−0.10	+0.22	−0.10
90	+0.06	+0.10	+0.08	−0.11	+0.22	−0.10
91	+0.04	+0.10	+0.09	−0.11	+0.22	−0.11
92	+0.03	+0.09	+0.09	−0.12	+0.22	−0.11
93	+0.02	+0.09	+0.10	−0.12	+0.22	−0.11
94	+0.00	+0.08	+0.10	−0.12	+0.22	−0.11
95	−0.01	+0.07	+0.11	−0.12	+0.22	−0.11
96	−0.03	+0.06	+0.11	−0.12	+0.22	−0.11
97	−0.04	+0.05	+0.11	−0.12	+0.21	−0.10
98	−0.05	+0.04	+0.11	−0.12	+0.20	−0.10
99	−0.07	+0.03	+0.11	−0.12	+0.20	−0.10

In measuring a space 100 times it was found convenient to divide the observations into sets of 20, each set being symmetrically distributed around the micrometer-head. Thus, if the first pointing of a measurement was at zero, the first pointing of the next measurement would be at 5, and the next at 10, and so on around the head. Care was always taken to complete a set of 20 without interruption, thus diminishing the danger of error from change of focus, light, &c. It was found difficult to measure a space as small as ¼ of a revolution, the parallel threads in the microscope being about that distance apart. A space of 1¼ revolutions was, therefore, measured instead. Since the quantities a_1, a_2, b_1, and b_2 are nearly the same for several consecutive revolutions, no appreciable error was introduced. A graduated space equal to one revolution was made equal to one and one-fourth revolution by drawing out and clamping the object-glass tube at the proper distance.

VALUES OF ONE REVOLUTION OF MICROMETER-SCREWS.

§ **69.** The value of a revolution on each of the micrometers was determined by measuring a known space on a standard. The tube which carries the object-glass was pushed as far in as possible and clamped. It was found by trial that the microscope could be moved 1^{mm} to and from the scale without sensibly affecting the distinctness of vision. A large number of experiments showed that such a movement changed the value of the micrometer-screw about $\frac{1}{80}$ part. The value was, therefore, determined midway between the upper and lower limits of distinct vision. Each screw was determined in four different parts. The values are as follows. The middle revolution is called 10 revolutions, so that there shall be no minus readings.

Mean values of one revolution of micrometer-screws.

Part of screw.	Mic. No. 1.	Mic. No. 2.	Mic. No. 3.	Mic. No. 4.	Mic. No. 5.	Mic. No. 6.
	μ	μ	μ	μ	μ	μ
Between rev. 5 and rev. 7.5	101.4	101.1	101.0	100.9	104.6	104.5
Between rev. 7.5 and rev. 10	101.6	100.9	101.2	101.3	104.8	104.4
Between rev. 10 and rev. 12.5	101.6	101.0	101.0	101.2	105.0	104.8
Between rev. 12.5 and rev. 15	101.4	100.6	101.9	101.1	104.7	104.6
Between rev. 5 and rev. 15	101.5	100.9	101.0	101.1	104.8	104.6

It will be observed that the middle values of the screws are greater than the extreme ones. The difference, however, is very slight, and the mean value (from $5^{rev.}$ to $15^{rev.}$) has been used. When the highest accuracy is required, special values of the screws are determined for each focus.

The magnifying power of each microscope is 27 diameters.

NON-RECTANGULARITY OF THREADS IN MICROSCOPES.

§ **70**. When microscope-readings are made on the steel and zinc bars of the tubes, the longitudinal thread in the microscope is made to coincide with the longitudinal graduation on the steel bar. The longitudinal graduation on the zinc bar is about $0^{mm}.5$ from that on the steel. If the transverse threads do not cross the bars at right angles the readings on the zinc will be erroneous; therefore the angle which these threads make with the longitudinal threads was determined, and the correction to the zinc-reading was computed. The method of determination was as follows: The microscope was mounted over a theodolite. A suitable scale was placed on the center of the theodolite. One of the threads in the microscope was made to coincide with a graduation on the scale. The theodolite was then revolved until the graduation coincided with the other thread. The angle through which the theodolite revolved was the angle between the threads. The lower right-hand angle in the microscope-field was measured in every case. The several values are as follows:

	° ′
Microscope 1. Lower right-hand angle	= 89 53
Microscope 2. Lower right-hand angle	= 89 52
Microscope 3. Lower right-hand angle	= 90 00
Microscope 4. Lower right-hand angle	= 90 01
Microscope 5. Lower right-hand angle	= 90 02
Microscope 6. Lower right-hand angle	= 90 05

The probable error of each of the above values is about $\pm 1'$. Since the distance of the longitudinal graduation on the zinc bar from that on the steel bar is $0^{mm}.5$, the corrections to readings on zinc bar for non-rectangularity of threads are as follows:

	μ
Microscope 1	$=-1.1$
Microscope 2	$=-1.2$
Microscope 3	$=-0.0$
Microscope 4	$=+0.1$
Microscope 5	$=+0.3$
Microscope 6	$=+0.7$

VALUES OF GRADUATED SPACES ON BRASS BAR.

§ **71.** The values of the spaces at the 4^m or non-lettered end of brass bar were determined as follows: Each half-millimeter space on the bar was compared ten or more times with the space from $0^{in}.90$ to $0^{in}.92$ on T. and S. inch in the following manner. The inch was mounted in the prolongation of the brass bar, the graduated surfaces of inch and bar being in the same horizontal plane and separated from each other about $0^{in}.4$. The comparing-microscopes were mounted on the microscope-car, collimated, and made vertical. Microscope No. 5 was pointed at $0^{in}.90$ on the inch; microscope No. 6 was pointed at $0^{mm}.0$ on the bar. In this position micrometer-readings were made on the inch and bar. The microscope-car was then moved by means of its lever until microscope No. 5 pointed at $0^{in}.92$ on the inch and microscope No. 6 pointed very nearly at $0^{mm}.5$ on the bar. A second set of micrometer-readings was then made on the inch and bar. These two sets of readings give the difference between the space $0^{in}.90$ to $0^{in}.92$ on the inch and the space $0^{mm}.0$ to $0^{mm}.5$ on the bar. The other half-millimeter spaces on the bar were compared in the same way. The whole space on the bar ($=3^{mm}$) was compared twenty times with space $0^{in}.80$ to $0^{in}.92$ on the T. and S. inch.

The following are the observed differences expressed in micrometer-divisions:

Comparisons of spaces on brass bar at 4-metre end with spaces on T. and S. inch.

$(0^{in}.90-0^{in}.92)$ on inch *minus* (0—5) on brass bar.	v	$(0^{in}.90-0^{in}.92)$ on inch *minus* (5—10) on brass bar.	v	$(0^{in}.90-0^{in}.92)$ on inch *minus* (10—15) on brass bar.	v	$(0^{in}.90-0^{in}.92)$ on inch *minus* (15—20) on brass bar.	v	$(0^{in}.90-0^{in}.92)$ on inch *minus* (20—25) on brass bar.	v	$(0^{in}.90-0^{in}.92)$ on inch *minus* (25—30) on brass bar.	v	$(0^{in}.80-0^{in}.92)$ on inch *minus* (0—30) on brass bar.	v
Div.		*Div.*		*Div.*		*Div.*		*Div.*		*Div.*		*Div.*	
11.2	0.1	16.1	0.5	12.3	1.2	19.5	3.2	10.0	0.9	13.9	0.5	67.9	0.8
11.3	0.0	14.9	0.7	14.5	1.0	16.8	0.5	10.1	0.8	9.4	4.0	67.8	0.7
10.5	0.8	15.9	0.3	12.9	0.6	14.5	1.8	12.0	1.1	10.5	2.9	65.9	1.2
11.3	0.0	15.3	0.3	12.3	1.2	14.8	1.5	9.2	1.7	15.3	1.9	65.9	1.2
10.9	0.4	14.8	0.8	12.1	1.4	13.5	2.8	9.0	1.9	16.1	2.7	68.9	1.8
9.7	1.6	17.1	1.5	13.6	0.1	15.1	1.2	10.7	0.2	12.6	0.8	69.9	2.8
10.1	1.2	15.5	0.1	17.6	4.1	18.2	1.9	13.2	2.3	13.0	0.4	65.1	2.0
11.4	0.1	15.4	0.2	14.8	1.3	17.8	1.5	13.5	2.6	14.6	1.2	65.5	1.6
13.9	2.6	15.9	0.3	12.6	0.9	14.7	1.6	10.3	0.6	13.5	0.1	68.2	1.1
12.9	1.6	15.5	0.1	12.3	1.2	15.9	0.4	11.0	0.1	15.1	1.7	66.9	0.2
11.32		15.64		13.50		18.4	2.1	10.90		13.40		67.5	0.4
$=11^{\mu}.80$		$=16^{\mu}.30$		$=14^{\mu}.07$		18.0	1.7	$=11^{\mu}.36$		$=13^{\mu}.96$		67.3	0.2
						17.7	1.4					67.4	0.3
						16.1	0.2					67.8	0.7
						16.2	0.1					67.5	0.4
						17.2	0.9					67.3	0.2
						15.4	0.9					67.8	0.7
						13.9	2.4					66.4	0.7
						15.2	1.1					64.8	2.3
						17.7	1.4					66.2	0.9
						16.33						67.10	
						$=17^{\mu}.02$						$=69^{\mu}.92$	

Since the conditions of the comparisons of the several spaces are essentially the same, the probable error of a single comparison can be found from all the residuals. The sum of the squares of the 90 residuals is 192.04. The number of unknown quantities is 7. Therefore, the probable error of a single comparison is $\pm 1^{div}.03$, and the probable error of the mean of 10 comparisons is $\pm 0^{div}.32$, and of 20 is $\pm 0^{div}.23$. Therefore, after changing micrometer-divisions to microns, we have the following observed values:

$$
\begin{aligned}
&(0^{in}.90 \text{ to } 0^{in}.92) \text{ on inch } minus\ (\ 0 \text{ to } \ 5) \text{ at } 4^{m} \text{ on brass bar} = \overset{\mu}{11.80} \pm \overset{\mu}{0.34}\\
&(0^{in}.90 \text{ to } 0^{in}.92) \text{ on inch } minus\ (\ 5 \text{ to } 10) \text{ at } 4^{m} \text{ on brass bar} = 16.30 \pm 0.34\\
&(0^{in}.90 \text{ to } 0^{in}.92) \text{ on inch } minus\ (10 \text{ to } 15) \text{ at } 4^{m} \text{ on brass bar} = 14.07 \pm 0.34\\
&(0^{in}.90 \text{ to } 0^{in}.92) \text{ on inch } minus\ (15 \text{ to } 20) \text{ at } 4^{m} \text{ on brass bar} = 17.02 \pm 0.24\\
&(0^{in}.90 \text{ to } 0^{in}.92) \text{ on inch } minus\ (20 \text{ to } 25) \text{ at } 4^{m} \text{ on brass bar} = 11.36 \pm 0.34\\
&(0^{in}.90 \text{ to } 0^{in}.92) \text{ on inch } minus\ (25 \text{ to } 30) \text{ at } 4^{m} \text{ on brass bar} = 13.96 \pm 0.34\\
&(0^{in}.80 \text{ to } 0^{in}.92) \text{ on inch } minus\ (\ 0 \text{ to } 30) \text{ at } 4^{m} \text{ on brass bar} = 69.92 \pm 0.24
\end{aligned}
$$

The value of space $(0^{in}.80 \text{ to } 0^{in}.92)$ on inch is $3056^{\mu}.12 \pm 0^{\mu}.26$ (Chapter II, § 3, note). The value of space $(0^{in}.90 \text{ to } 0^{in}.92)$ is not so well known. It has therefore been treated simply as a constant with which the relative values of the $0^{mm}.5$ spaces are determined. Substituting the value of space $(0^{in}.80 \text{ to } 0^{in}.92)$ there results:

$$(0 \text{ to } 30) = 2986^{\mu}.21 \pm 0^{\mu}.36$$

Then solving for space $(0^{in}.90 \text{ to } 0^{in}.92)$ it is found:

$$(0^{in}.90 \text{ to } 0^{in}.92) = \tfrac{1}{6}\left[(0 \text{ to } 30) + 11^{\mu}.80 + 16^{\mu}.30 + 14^{\mu}.07 + 17^{\mu}.02 + 11^{\mu}.36 + 13^{\mu}.96\right]$$

Substituting this value in the table just preceding, there results:

$$
\begin{aligned}
(0 \text{ to } 5\ ) &= \tfrac{1}{6}\left[(0 \text{ to } 30) - 5 \times 11^{\mu}.80 + 16^{\mu}.30 + 14^{\mu}.07 + 17^{\mu}.02 + 11^{\mu}.36 + 13^{\mu}.96\right]\\
(5 \text{ to } 10) &= \tfrac{1}{6}\left[(0 \text{ to } 30) + 11^{\mu}.80 - 5 \times 16^{\mu}.30 + 14^{\mu}.07 + 17^{\mu}.02 + 11^{\mu}.36 + 13^{\mu}.96\right]
\end{aligned}
$$

$$(10 \text{ to } 15)=\tfrac{1}{6}\,[(0 \text{ to } 30)+11^\mu.80+16^\mu.30-5\times14^\mu.07+17^\mu.02+11^\mu.36+13^\mu.96]$$
$$(15 \text{ to } 20)=\tfrac{1}{6}\,[(0 \text{ to } 30)+11^\mu.80+16^\mu.30+14^\mu.07-5\times17^\mu.02+11^\mu.36+13^\mu.96]$$
$$(20 \text{ to } 25)=\tfrac{1}{6}\,[(0 \text{ to } 30)+11^\mu.80+16^\mu.30+14^\mu.07+17^\mu.02-5\times11^\mu.36+13^\mu.96]$$
$$(25 \text{ to } 30)=\tfrac{1}{6}\,[(0 \text{ to } 30)+11^\mu.80+16^\mu.30+14^\mu.07+17^\mu.02+11^\mu 36-5\times13^\mu.96]$$

By adding the preceding values the following are obtained:

$$(0 \text{ to } 5)=\tfrac{1}{6}(0 \text{ to } 30)+\tfrac{1}{6}(-5\times11^\mu.80+16^\mu.30+14^\mu.07+17^\mu.02+11^\mu.36+13^\mu.96)$$
$$(0 \text{ to } 10)=\tfrac{1}{3}(0 \text{ to } 30)+\tfrac{1}{6}(-4\times11^\mu.80-4\times16^\mu.30+2\times14^\mu.07+2\times17^\mu.02+2\times11^\mu.36+2\times13^\mu.96)$$
$$(0 \text{ to } 15)=\tfrac{1}{2}(0 \text{ to } 30)+\tfrac{1}{6}(-3\times11^\mu.80-3\times16^\mu.30-3\times14^\mu.07+3\times17^\mu.02+3\times11^\mu.36+3\times13^\mu.96)$$
$$(0 \text{ to } 20)=\tfrac{2}{3}(0 \text{ to } 30)+\tfrac{1}{6}(-2\times11^\mu.80-2\times16^\mu.30-2\times14^\mu.07-2\times17^\mu.02+4\times11^\mu.36+4\times13^\mu.96)$$
$$(0 \text{ to } 25)=\tfrac{5}{6}(0 \text{ to } 30)+\tfrac{1}{6}(-11^\mu.80-16^\mu.30-14^\mu.07-17^\mu.02-11^\mu.36+5\times13^\mu.96)$$

And the probable errors are as follows:

$$p.\ e. \text{ of } (0 \text{ to } 5)=\pm\sqrt{.06^2+.28^2+.06^2+.06^2+.04^2+.06^2+.06^2}=\pm0^\mu.31$$
$$p.\ e. \text{ of } (0 \text{ to } 10)=\pm\sqrt{.12^2+.23^2+.23^2+.11^2+.08^2+.11^2+.11^2}=\pm0^\mu.40$$
$$p.\ e. \text{ of } (0 \text{ to } 15)=\pm\sqrt{.18^2+.17^2+.17^2+.17^2+.12^2+.17^2+.17^2}=\pm0^\mu.44$$
$$p.\ e. \text{ of } (0 \text{ to } 20)=\pm\sqrt{.24^2+.11^2+.11^2+.11^2+.08^2+.23^2+.23^2}=\pm0^\mu.45$$
$$p.\ e. \text{ of } (0 \text{ to } 25)=\pm\sqrt{.30^2+.06^2+.06^2+.06^2+.04^2+.06^2+.28^2}=\pm0^\mu.43$$

Computing values of spaces, and tabulating,

$$\begin{aligned}
(0 \text{ to } 5)&=\ \ 500.0\pm0.31 \ (\mu)\\
(0 \text{ to } 10)&=\ \ 995.5\pm0.40\\
(0 \text{ to } 15)&=1493.2\pm0.44\\
(0 \text{ to } 20)&=1988.0\pm0.45\\
(0 \text{ to } 25)&=2488.0\pm0.43\\
(0 \text{ to } 30)&=2986.0\pm0.36
\end{aligned}$$

The graduation at 35 was made after the other spaces were determined. The value of the space (30 to 35) was found by comparing it with other spaces on the scale. The *p. e.* of this work is $\pm0^\mu.28$, and of the scale with which it was compared, $\pm0^\mu.31$. Therefore, the probable error of (30 to 35) is $\pm\sqrt{.28^2+.31^2}=\pm0^\mu.42$ and the *p. e.* of (0 to 35)$=\pm0^\mu.55$. The thirty-fifth graduation was used only in the open-air work, never in the comparing-room. Its accuracy is therefore not essential.

The relative values of the intermediate spaces were found by mounting a microscope over them, and reading on each graduation consecutively from 0 to 5, forward and backward ten times, then moving the microscope and reading in the same way from 5 to 10, and so on through the whole scale. A discussion of all the work at both ends of the bar shows that this gave the value of each space in terms of the micrometer with a *p. e.* of $\pm0^\mu.20$. The value of the micrometer was determined in each case from the known values of the spaces (0 to 5), (5 to 10), &c., as given above. Thus the absolute value of each intermediate space became known. The probable error of the distance of each intermediate graduation-mark from the zero has been accurately computed. It depends upon the probable errors of the graduation-marks given above, and also upon the relative microscope-work. This makes the computation somewhat long, and it is therefore omitted, the results only being given.

§ **7**. The term involving $(Z_1—S_1)^2$ is very small and its value may be computed for values of $Z_1—S_1$, 100^μ, 300^μ, 500^μ, &c., differing by 200^μ. Then in reducing a base the number of tubes in each section for which $Z_1—S_1$ is between 0^μ and 200^μ can be counted. This number multiplied by the tabular correction will give the correction for this set of tubes. Tubes for which $Z_1—S_1$ lies between 200^μ and 400^μ, &c., will be treated in the same way. The following table gives the values of the second term in question, that is, of corrections to the length of S_1 for any value of Z_1-S_1 when computed with $e_0=0.6522$ from its length when $S_1—Z_1=0$. The first line gives the limits of the different classes into which the tubes are separated according to the observed values of Z_1-S_1, and the second line gives the corresponding corrections to the length of S_1 computed with e_0.

Z_1-S_1	$\overset{\mu}{0}$ to $\overset{\mu}{200}$	$\overset{\mu}{200}$ to $\overset{\mu}{400}$	$\overset{\mu}{400}$ to $\overset{\mu}{600}$	$\overset{\mu}{600}$ to $\overset{\mu}{800}$	$\overset{\mu}{800}$ to $\overset{\mu}{1000}$	$\overset{\mu}{1000}$ to $\overset{\mu}{1200}$
Correction	$\overset{\mu}{0}.051$	$\overset{\mu}{0}.460$	$\overset{\mu}{1}.276$	$\overset{\mu}{2}.502$	$\overset{\mu}{4}.136$	$\overset{\mu}{6}.178$

This method gives the correction to the length of S_1 accurately for that tube in a class which has the middle temperature, and least accurately for that tube whose temperature is 100^μ greater or less. But the errors on the two sides of the middle temperature differ in sign and the maximum error does not exceed $\frac{1}{10000000}$ of S_1, so that the resulting error in the length of a section of the base is entirely insignificant.

§ **8**. 6. Of the sum of the corrections for inclination of the different tubes or parts of a tube. The inclination is read by a sector and accompanying level. (See Chapter VIII, § 6.) The sector gives 30″, and one division of the level is 33″. The level was read to tenths of a division. Seven determinations of index-error at different periods of the measurement had a range of but 26″, and their mean, 5° 30′ 27″, was used as the index-error throughout. Habitually the sector-arc was clamped at the nearest minute and the level read. The level-reading was applied to correct the sector-reading. Subtracting the index-error from this corrected reading, the inclination resulted. Denoting the inclination by i, the approximate correction was taken from a table computed from

$$\text{Correction} = -2\times 4^{\text{m}} \sin^2 \tfrac{1}{2} i$$

These corrections were summed for each section and, obtaining from the reduction the mean value of $Z_1—S_1$ for the section, a sufficiently accurate value of the average length of S_1 for the section results from

$$S_1=4^{\text{m}}.000978+0.6632\ [\text{mean}\ (Z_1—S_1)]$$

The sum of the approximate corrections for the section was then multiplied by the ratio of this value of S_1 to 4^{m}, giving the final value of the correction. In the duplicate records the sector-reading of tube No. 398 was recorded in one as 4° 36′ and in the other as 5° 36′. An examination of the relative heights of the two section-stones resulting from using one or the other angle makes it probable that 5° 36′ is correct, and it has been used; 4° 36′ would give a correction about $0^{\text{mm}}.6$ greater. For tube No. 1114 the records gave sector-readings differing by 10′. For tube No. 1202 the records gave sector readings differing by 1′. As there is no way of judging which is correct, the mean has been taken, introducing in the case of No. 1114 a error of about $0^{\text{mm}}.1$ into the length of the base.

§ **9**. 7. Of the sum of the intervals between the tubes measured with microscopes. This quantity arises in the following way. A microscope having been pointed at the 4^{m}-mark at the front end of the steel bar, when the tube is carried forward and the rear end of the steel bar is placed under the same microscope, it is difficult to put the 0^{m} mark of the steel bar precisely under the wires of the microscope by moving the tube; hence this is only done approximately, and the precise pointing at the steel-bar zero is effected by the micrometer-screw of the microscope. The change of reading of the microscope measures the interval in the direction of the base between the positions occupied at first by the front zero-mark of the steel bar and the position occupied later by the rear zero-mark of the steel bar. A similar quantity is measured in starting from cut-off plates. The quantities read with the microscopes are in reduction corrected for run.

§ **10**. 8. Of the sum of the movements of the front cut-off plate. As already stated, on closing the work of the day, three marks, described in Chapter VIII, § 9, are placed on the ground under the front ends of the last three tubes measured. When work is recommenced the next morn-

The values of the constants in these equations are given in Chap. IX, § 66. Since Z_1^{32} is but about $\frac{1}{4000}$ shorter than S_1^{32}, the latter may be substituted for the former without introducing any error approaching the size of the probable error in the value of E_{Z_1}. Then

$$\frac{dZ_1}{dt'}=E_{Z_1}=S_1^{32}(a'+2\beta t')$$

and

$$\frac{\frac{dS_1}{dt'}}{\frac{dZ_1}{dt'}-\frac{dS_1}{dt'}}=\frac{E_{S_1}}{E_{Z_1}-E_{S_1}}=\left(\frac{a}{a'-a}+\frac{2\beta}{a'-a}t'\right)$$

Substituting the known constants from Chapter IX, § 66 and for t', $(t-32)$, this becomes

$$\frac{E_{S_1}}{E_{Z_1}-E_{S_1}}=0.64119+0.000389\ (t-32)$$

If we wish to count temperatures from the temperature 60°.292, for which $Z_1=S_1$, we may write

$$\frac{E_{S_1}}{E_{Z_1}-E_{S_1}}=\left(\frac{a}{a'-a}+\frac{2\beta}{a'-a}28.29\right)+\frac{2\beta}{a'-a}t''$$

where t'' is the temperature counted from 60°.292 F. The constants in this expression are known, and it may be written in the form—

$$\frac{E_{S_1}}{E_{Z_1}-E_{S_1}}=a+bt''$$

This value may be represented by e, and we may write—

$$dS_1=\frac{dS_1}{dZ_1-dS_1}\ d(Z_1-S_1)$$

or dividing both terms of the fraction by dt and replacing the result by its equivalent,

$$dS_1=(a+bt'')\ d(Z_1-S_1)$$

But from Chapter IX, 66,

$$E_{Z_1}-E_{S_1}=38^{\mu}.089$$

which is the relative change in length of Z_1 and S_1 for 1° F. Hence

$$\frac{Z_1-S_1}{38^{\mu}.089}=t''$$

and

$$dS_1=\left(a+b\frac{Z_1-S_1}{38^{\mu}.089}\right)d(Z_1-S_1)$$

Integrating from $Z_1-S_1=0$ to Z_1-S_1,

$$\int_0^{Z_1-S_1}dS_1=a(Z_1-S_1)+\tfrac{1}{2}\frac{b}{38^{\mu}.089}(Z_1-S_1)^2$$

or substituting for a and b their values,

$$\int_0^{Z_1-S_1}dS_1=0.6522\ (Z_1-S_1)+0.000005106\ (Z_1-S_1)^2$$

This expression gives for any metallic temperature, Z_1-S_1, the excess of the length of S_1 over its length at 60°.292 or when $Z_1=S_1$. That is, the length of S_1 for any measured tube is—

$$S_1=(S_1)_{Z_1=S_1}+0.6522(Z_1-S_1)+0.000005106\ (Z_1-S_1)^2$$

and the sum of the lengths of the bars S_1 which enter a section of the base containing n tubes will be

$$\Sigma S_1=n(S_1)_{Z_1=S_1}+0.6522\ \Sigma\ (Z_1-S_1)+0.000005106\ \Sigma\ (Z_1-S_1)^2$$

cut-off marks. The average distance measured per day was 292 metres. The greatest distance measured in one day was 500 metres. The base was remeasured in the opposite direction. The record was in duplicate, as were the principal observations. (See Chapter VIII, §§12 and 13.) The party reached the base-line May 31, 1877, and completed the second measurement August 25. Further details as to the work may be found in the Report of the Chief of Engineers, U. S. Army, for 1878. The computation of the length of the base has been duplicated in all parts where no other efficient check could be obtained. The details of the method of measuring a base are given in Chapter VIII, §12.

§ **2.** In computing the length of a section of a base-line measured with the Repsold apparatus, it may be considered as composed of the following parts:

1. Of as many times the zero-length of the steel bar S_1, as it enters the section, the zero-length of S_1, being its length when equal to that of the zinc bar Z_1 by its side; that is, its length when the metallic temperature, $Z_1 - S_1 = 0$. The value of the zero-length of S_1 is given in Chapter IX, § 66, as $4^y.37554481$. Multiplying the number of times the whole length of S_1, at temperature $Z_1 - S_1 = 0$, entered the section by this value, this part of the length of the section results.

§ **3.** 2. Of the sum of the lengths each less than one tube-length, measured with the tube in closing on a marking-stone at the end of a section.

3. Of the sums of the intervals measured with the cut-off scale, either in closing on a section-stone or in stopping at a cut-off plate, corrected for error and temperature of cut-off scale.

§ **4.** 4. Of the sum of the excesses of the actual lengths of S_1 due to temperatures during measurements over its zero-length, on the assumption that the metallic temperature of S_1 at any instant of measurement is correctly given by $Z_1 - S_1$, the difference of lengths of the zinc and steel bars, and on the further assumption that for all temperatures the rate of expansion of the S_1 divided by the difference of rates of expansion of Z_1 and S_1 has the same value, 0.6522 (§ 6), as when the metallic temperature or $Z_1 - S_1$ is zero.

§ **5.** 5. Of the sum of small corrections to the length of a section arising from the fact that the ratio of the difference of rates of expansion of zinc and steel bars to that of steel bar differs slightly from the constant 0.6522 at metallic temperatures different from $Z_1 - S_1 = 0$.

§ **6.** The value of $Z_1^m - S_1^m$, or the metallic temperature of any tube, results from the corrected observations made during the base-measurement. At each end of the tube the distance (less than $0^{mm}.1$) by which the 0^m or 4^m graduation-mark on the steel bar precedes a graduation-mark on the zinc bar is read with the microscope and the number of the graduation pointed at on the zinc bar is noted. The graduation of the zinc bar, though very good, is not perfect, hence the number of the graduation-mark does not give precisely the number of tenths of millimeters between the 0^m or 4^m mark on the zinc bar and the graduation-mark pointed at. Hence a small correction for error of graduation is needed. One turn of the microscope-screw is not exactly $0^{mm}.1$, hence there is a correction for run. The data for both corrections are given in Chapter IX, §§ 69, 72. Applying these corrections, there result the true distances by which the 0^m and 4^m graduation-marks on the steel bar precede the same marks on the zinc bar. The difference of these distances is $Z_1 - S_1$, the difference of lengths of the two bars, or the metallic temperature. In reducing the base-measurements, these metallic temperatures are obtained for every tube measured. From this metallic temperature and the known length of S_1 when $S_1 - Z_1 = 0$, the length of S_1 for any metallic temperature, $Z_1 - S_1$, is derived in the following way:

When S_1 and Z_1 are expressed in terms of their lengths, S_1^{32} and Z_1^{32}, at 32° F., if $t' = t - 32$, where t is the Fahrenheit temperature, we have—

$$S_1 = S_1^{32}\,(1 + at' + \beta t'^2) \tag{1}$$

$$Z_1 = Z_1^{32}\,(1 + a't' + \beta t'^2) \tag{2}$$

$$\frac{dS_1}{dt'} = E_S = S_1^{32}\,(a + 2\,\beta t') \tag{3}$$

$$\frac{dZ_1}{dt'} = E_{Z_1} = Z_1^{32}\,(a' + 2\,\beta t') \tag{4}$$

CHAPTER X.

CHICAGO BASE.

§ **1.** This straight base was measured in duplicate with the Repsold base-apparatus in 1877 by a party under the charge of Assistant Engineer E. S. Wheeler, who had with him Assistant Engineers Charles Pratt and F. W. Lehnartz. The description of the Repsold base-apparatus and of the method of using it has already been given in Chapter VIII. The length of the base is about $4\frac{5}{8}$ miles, its west end is in latitude 41° 47′ and in longitude 87° 49′ west from Greenwich. The azimuth of the base from the west end is 290° 59′. The mean elevation of the tube during measurement was 584.3 feet above mean tide in New York Harbor.

The line lies in an open prairie; no points ot it differ in level by more than 4 feet; the soil is stiff blue clay covered by six or eight inches of black loam. A part of the line ran through cultivated fields and a part over grassy prairie. The sod was not removed.

The ends of the base were marked by small agate hemispheres set in brass cylinders, which were leaded into the tops of granite posts set in brick-work, the agate hemispheres being three feet below the surface of the ground. A description of these marks may be found in the Lake-Survey Report for 1878, and a sketch is given in Plate XII.

The base was divided into eight nearly equal sections by marks on stones, as described in Chapter VIII, § 10. Counting from the west end of the base, these stones were approximately at 228, 458, 692, 924, 1154, 1382, and 1682 tube-lengths from it.

During the measurement of the base the tube was kept covered by an awning of sail-cloth stretched over a hut-like frame-work, which was moved with it and protected it from the sun. The microscope-stands were protected in the same way except when being moved forward.

In measurement the axes of the microscopes are placed in the vertical plane through the base-line by means of a telescope attached to the measuring-tube and made to move in a vertical plane. The forward microscope having been pointed at the front-end zero of the steel bar in the measuring-tube, the tube is then carried forward, and its rear-end zero having been placed under the same microscope it is pointed at. This method assumes that in the interval between the pointings of the same microscope at the front and rear ends of the tube, that is, while the tube is being carried forward, the microscope is perfectly stable. To increase the stability of the microscopes, the feet of their tripod-stands were placed on iron pins 15 inches long and 2 inches in diameter, driven into the ground until their heads were only an inch or two above it. The iron legs of the stands were wrapped with felt and covered with canvas to reduce temperature-changes. Experiment having shown that the observers' weight resting on the ground in changing positions near the iron pins might disturb the microscope, a plank about 8 feet long, having low supports at its ends was used in all cases for the observers to stand on. Details as to the stability of the microscope-stands will be given in the discussion of the sources of error in the base-measurement. The apparatus, partially protected by awnings, proved to have great stability in winds, so that it was possible to measure on many days on which it would have been impossible with the old apparatus. At the close of work on each day marks were placed under the front ends of the last two or three tubes measured, to which the ends of the tubes were referred by the cut-off apparatus (called by Repsold the *Absetz Cylinder*). (Chapter VIII, §§ 9 and 13.) On recommencing work the next day, measurements were begun from all these marks, a weighted-mean result being used in computation. The last three microscope-stands are allowed to retain their position, protected by the awnings, during the suspension of work. Their stability is found to be nearly equal to that of the

VALUES OF DIVISIONS OF LEVELS OF BASE-APPARATUS.

§ **74.** The value of one division on each of the levels was determined with a level-trier. The level-tubes are all numbered. The values are as follows:

Hand level No. 14	$1^{div}=51''$
Fixed levels on measuring-microscopes, Nos. 15 to 22, inclusive	$1^{div}=35''$
Sector level of tube 1, No. 23	$1^{div}=33''$
Sector level of tube 2, No. 24	$1^{div}=38''$
Detached level of measuring microscopes, No. 25	$1^{div}=42''$
Cut-off level, No. 26	$1^{div}=1''.6$
Striding level for quills, No. 27	$1^{div}=7''.3$
Cross level on tube 1, No. 40	$1^{div}=52''$
Cross level on tube 2, No. 41	$1^{div}=52''$

§ 75. MISCELLANEOUS CONSTANTS.

Leugth of cut-off cylinder	$=0^m.781$
Length, with lengthening bar attached	$=1^m.698$
Distance between axis of tube-telescope and longitudinal graduation on steel bar	$=0^m.1062$
Value of space between 45 and 50, convex end of quill A	$=500^{\mu}.0$
Value of space between 45 and 50, convex end of quill B	$=499^{\mu}.3$
Value of space between figure 1 and middle stroke of letter m on metre *R*1876	$=511^{\mu}.4$

Values of graduated spaces on zinc bar of tube 2.

Spaces at 0^m	Spaces at 1^m	Spaces at 4^m	On neutral axis. Spaces at 0^m.
mm. μ	*mm.* μ	*mm.* μ	*mm.* μ
0 to 0.1 = 102.1	0 to 0.1 = 101.0	0 to 0.1 = 100.9	0 to 0.1 = 102.7
0 to 0.2 = 200.1	0 to 0.2 = 201.4	0 to 0.2 = 200.2	0 to 0.2 = 201.3
0 to 0.3 = 301.3	0 to 0.3 = 301.1	0 to 0.3 = 300.7	0 to 0.3 = 298.0
0 to 0.4 = 401.4	0 to 0.4 = 401.9	0 to 0.4 = 401.5	0 to 0.4 = 400.4
0 to 0.5 = 501.4	0 to 0.5 = 500.7	0 to 0.5 = 500.6	0 to 0.5 = 503.8
0 to 0.6 = 602.3	0 to 0.6 = 602.1	0 to 0.6 = 601.0	0 to 0.6 = 602.3
0 to 0.7 = 702.5	0 to 0.7 = 702.1	0 to 0.7 = 700.4	0 to 0.7 = 702.2
0 to 0.8 = 800.9	0 to 0.8 = 801.5	0 to 0.8 = 800.8	0 to 0.8 = 797.2
0 to 0.9 = 900.5	0 to 0.9 = 901.8	0 to 0.9 = 900.4	0 to 0.9 = 902.7
0 to 1.0 = 1001.6	0 to 1.0 = 1000.4	0 to 1.0 = 1000.8	0 to 1.0 = 999.3
0 to 1.1 = 1101.4		0 to 1.1 = 1100.7	
0 to 1.2 = 1200.7		0 to 1.2 = 1201.3	
0 to 1.3 = 1301.1		0 to 1.3 = 1301.7	
0 to 1.4 = 1401.0		0 to 1.4 = 1400.9	
0 to 1.5 = 1501.7		0 to 1.5 = 1501.2	
0 to 1.6 = 1601.3		0 to 1.6 = 1601.7	
0 to 1.7 = 1701.4		0 to 1.7 = 1701.0	
0 to 1.8 = 1801.3	Spaces at 3^m.	0 to 1.8 = 1801.7	Spaces at 4^m
0 to 1.9 = 1901.6		0 to 1.9 = 1901.1	
0 to 2.0 = 2001.6	*mm.* μ	0 to 2.0 = 2001.6	*mm.* μ
0 to 2.1 = 2102.3	0 to 0.1 = 100.2	0 to 2.1 = 2102.0	0 to 0.1 = 104.5
0 to 2.2 = 2202.2	0 to 0.2 = 199.9	0 to 2.2 = 2201.7	0 to 0.2 = 207.0
0 to 2.3 = 2302.2	0 to 0.3 = 300.3	0 to 2.3 = 2302.6	0 to 0.3 = 310.4
0 to 2.4 = 2401.6	0 to 0.4 = 399.9	0 to 2.4 = 2402.6	0 to 0.4 = 407.2
0 to 2.5 = 2502.3	0 to 0.5 = 499.6	0 to 2.5 = 2501.7	0 to 0.5 = 506.9
0 to 2.6 = 2602.7	0 to 0.6 = 599.8	0 to 2.6 = 2603.1	0 to 0.6 = 608.0
0 to 2.7 = 2702.8	0 to 0.7 = 699.1	0 to 2.7 = 2702.7	0 to 0.7 = 706.6
0 to 2.8 = 2802.7	0 to 0.8 = 800.6	0 to 2.8 = 2802.6	0 to 0.8 = 806.2
0 to 2.9 = 2901.4	0 to 0.9 = 900.0	0 to 2.9 = 2902.3	0 to 0.9 = 904.7
0 to 3.0 = 3001.6	0 to 1.0 = 1000.1	0 to 3.0 = 3002.5	0 to 1.0 = 1008.5

Values of graduated spaces on zinc bar of MT1876.

Spaces at 0^m.	Spaces at 0^m.	Spaces at 1^m.
mm μ	*mm* μ	*mm* μ
0 to 0.1 = 101.1	0 to 1.1 = 1101.3	0 to 0.1 = 100.7
0 to 0.2 = 199.4	0 to 1.2 = 1200.8	0 to 0.2 = 200.5
0 to 0.3 = 299.6	0 to 1.3 = 1301.0	0 to 0.3 = 300.8
0 to 0.4 = 399.6	0 to 1.4 = 1400.9	0 to 0.4 = 400.7
0 to 0.5 = 499.5	0 to 1.5 = 1501.1	0 to 0.5 = 500.7
0 to 0.6 = 599.2	0 to 1.6 = 1600.9	0 to 0.6 = 601.2
0 to 0.7 = 699.5	0 to 1.7 = 1701.0	0 to 0.7 = 702.3
0 to 0.8 = 800.1	0 to 1.8 = 1800.9	0 to 0.8 = 801.2
0 to 0.9 = 900.1	0 to 1.9 = 1901.1	0 to 0.9 = 901.9
0 to 1.0 = 999.9	0 to 2.0 = 2000.5	0 to 1.0 = 1001.9

VALUES OF SPACES ON CUT-OFF SCALE.

§ **73.** This scale is $0^m.13$ long and is divided into millimeters. It is described in Chapter VIII, § 9. Its whole length was compared with the space $0^m.07$ to $0^m.20$ on *R*1876 and found to be correct at a temperature of about 38° F. The relative positions of the intermediate graduations were determined, and no errors greater than 3μ were found. Since no great accuracy is required in this scale, it is assumed to be correct at 38° F.

Values of graduated spaces on steel bars.

		On tube 1.	On tube 2.			On tube 1.	On tube 2.
	mm.	μ	μ		*mm.*	μ	μ
Spaces at 0^{m}	0 to − 0.1	101.9	101.2	Spaces at $2^{m}.2$	0 to − 0.1	100.5	100.2
	0 to + 0.1	100.1	99.4		0 to + 0.1	97.7	99.1
Spaces at $1^{m}.0$	0 to − 0.1	98.9	98.8	Spaces at $2^{m}.3$	0 to − 0.1	99.6	99.4
	0 to + 0.1	98.6	98.5		0 to + 0.1	98.7	99.0
Spaces at $1^{m}.5$	0 to − 0.1	100.8	100.3	Spaces at $2^{m}.4$	0 to − 0.1	99.7	100.1
	0 to + 0.1	99.0	99.2		0 to + 0.1	98.7	99.0
Spaces at $1^{m}.6$	0 to − 0.1	99.5	100.1	Spaces at $2^{m}.5$	0 to − 0.1	101.0	100.8
	0 to + 0.1	99.5	98.4		0 to + 0.1	100.6	98.9
Spaces at $1^{m}.7$	0 to − 0.1	99.3	101.0	Spaces at $3^{m}.0$	0 to − 0.1	99.8	100.2
	0 to + 0.1	99.2	98.1		0 to + 0.1	99.7	99.8
Spaces at $1^{m}.8$	0 to − 0.1	98.8	100.2	Spaces at $4^{m}.0$	0 to − 0.1	99.9	100.4
	0 to + 0.1	99.0	99.6		0 to + 0.1	100.2	98.9
Spaces at $1^{m}.9$	0 to − 0.1	99.2	99.0	ON NEUTRAL AXIS.			
	0 to + 0.1	98.6	100.3				
Spaces at $2^{m}.0$	0 to − 0.1	99.9	99.7	Spaces at 0^{m}	− 1 to + 1	197.1	206.6
	0 to + 0.1	98.8	98.9	Spaces at 4^{m}	− 1 to + 1	200.6	198.7
Spaces at $2^{m}.1$	0 to − 0.1	98.8	101.1				
	0 to + 0.1	98.8	100.6				

Values of graduated spaces on zinc bar of tube **1.**

Spaces at 0^{m}.	Spaces at 1^{m}.	Spaces at 4^{m}.	On neutral axis.	
			Spaces at 0^{m}.	Spaces at 4^{m}.
mm. μ	*mm.* μ	*mm.* μ	*mm.* μ	*mm.* μ
0 to 0.1 = 101.8	0 to 0.1 = 100.2	0 to 0.1 = 100.6	0 to 0.1 = 98.4	0 to 0.1 = 96.2
0 to 0.2 = 202.4	0 to 0.2 = 199.1	0 to 0.2 = 200.9	0 to 0.2 = 199.8	0 to 0.2 = 198.1
0 to 0.3 = 302.3	0 to 0.3 = 299.2	0 to 0.3 = 300.4	0 to 0.3 = 296.2	0 to 0.3 = 295.4
0 to 0.4 = 401.6	0 to 0.4 = 398.6	0 to 0.4 = 400.6	0 to 0.4 = 396.0	0 to 0.4 = 397.5
0 to 0.5 = 501.0	0 to 0.5 = 499.1	0 to 0.5 = 500.4	0 to 0.5 = 497.1	0 to 0.5 = 495.2
0 to 0.6 = 602.6	0 to 0.6 = 597.1	0 to 0.6 = 601.2	0 to 0.6 = 594.7	0 to 0.6 = 593.8
0 to 0.7 = 701.9	0 to 0.7 = 697.1	0 to 0.7 = 701.1	0 to 0.7 = 696.1	0 to 0.7 = 697.2
0 to 0.8 = 802.4	0 to 0.8 = 796.8	0 to 0.8 = 800.6	0 to 0.8 = 800.5	0 to 0.8 = 795.3
0 to 0.9 = 902.5	0 to 0.9 = 896.3	0 to 0.9 = 901.7	0 to 0.9 = 892.2	0 to 0.9 = 895.3
0 to 1.0 = 1001.7	0 to 1.0 = 994.8	0 to 1.0 = 1000.2	0 to 1.0 = 994.7	0 to 1.0 = 994.2
0 to 1.1 = 1102.0		0 to 1.1 = 1101.0	0 to 1.1 = 1094.6	0 to 1.1 = 1092.0
0 to 1.2 = 1203.3		0 to 1.2 = 1202.3	0 to 1.2 = 1192.8	0 to 1.2 = 1190.9
0 to 1.3 = 1302.8		0 to 1.3 = 1302.4	0 to 1.3 = 1291.0	0 to 1.3 = 1292.3
0 to 1.4 = 1401.7		0 to 1.4 = 1401.6	0 to 1.4 = 1389.2	0 to 1.4 = 1395.2
0 to 1.5 = 1501.2		0 to 1.5 = 1500.8	0 to 1.5 = 1490.3	0 to 1.5 = 1489.1
0 to 1.6 = 1602.0		0 to 1.6 = 1601.8	0 to 1.6 = 1590.5	0 to 1.6 = 1588.3
0 to 1.7 = 1700.2		0 to 1.7 = 1700.6	0 to 1.7 = 1692.3	0 to 1.7 = 1685.4
0 to 1.8 = 1801.1	Spaces at 3^{m}.	0 to 1.8 = 1801.1	0 to 1.8 = 1790.9	0 to 1.8 = 1785.7
0 to 1.9 = 1901.6		0 to 1.9 = 1900.2	0 to 1.9 = 1891.8	0 to 1.9 = 1888.7
0 to 2.0 = 2001.8	*mm.* μ	0 to 2.0 = 2001.0	0 to 2.0 = 1995.4	0 to 2.0 = 1983.8
0 to 2.1 = 2102.8	0 to 0.1 = 99.7	0 to 2.1 = 2102.5	0 to 2.1 = 2095.5	0 to 2.1 = 2086.8
0 to 2.2 = 2201.2	0 to 0.2 = 199.7	0 to 2.2 = 2200.7	0 to 2.2 = 2194.6	0 to 2.2 = 2182.5
0 to 2.3 = 2302.8	0 to 0.3 = 298.3	0 to 2.3 = 2301.8	0 to 2.3 = 2294.9	0 to 2.3 = 2287.4
0 to 2.4 = 2402.2	0 to 0.4 = 395.9	0 to 2.4 = 2400.9	0 to 2.4 = 2395.9	0 to 2.4 = 2385.7
0 to 2.5 = 2502.2	0 to 0.5 = 496.1	0 to 2.5 = 2501.8	0 to 2.5 = 2493.2	0 to 2.5 = 2485.9
0 to 2.6 = 2602.2	0 to 0.6 = 594.8	0 to 2.6 = 2600.5	0 to 2.6 = 2596.4	0 to 2.6 = 2582.8
0 to 2.7 = 2702.6	0 to 0.7 = 693.1	0 to 2.7 = 2701.7	0 to 2.7 = 2693.4	0 to 2.7 = 2683.2
0 to 2.8 = 2801.7	0 to 0.8 = 792.8	0 to 2.8 = 2801.5	0 to 2.8 = 2792.6	0 to 2.8 = 2788.9
0 to 2.9 = 2902.4	0 to 0.9 = 891.1	0 to 2.9 = 2901.8	0 to 2.9 = 2893.8	0 to 2.9 = 2887.0
0 to 3.0 = 3002.4	0 to 1.0 = 990.5	0 to 3.0 = 3001.7	0 to 3.0 = 2991.7	0 to 3.0 = 2985.4

Table of values of graduated spaces at 4^{m} or non-lettered end of brass bar.

Space.	Value.	Space.	Value.	Space.	Value.
	μ μ		μ μ		μ μ
0 to 1	101.0 ± 0.19	0 to 12	1196.5 ± 0.44	0 to 23	2288.4 ± 0.46
0 to 2	202.2 ± 0.22	0 to 13	1294.7 ± 0.45	0 to 24	2387.6 ± 0.47
0 to 3	298.8 ± 0.26	0 to 14	1394.8 ± 0.47	0 to 25	2488.4 ± 0.43
0 to 4	399.6 ± 0.32	0 to 15	1493.2 ± 0.44	0 to 26	2582.8 ± 0.45
0 to 5	500.0 ± 0.31	0 to 16	1591.5 ± 0.48	0 to 27	2687.0 ± 0.42
0 to 6	596.0 ± 0.37	0 to 17	1693.4 ± 0.48	0 to 28	2784.9 ± 0.40
0 to 7	696.1 ± 0.38	0 to 18	1789.5 ± 0.48	0 to 29	2885.6 ± 0.39
0 to 8	794.3 ± 0.39	0 to 19	1888.2 ± 0.50	0 to 30	2986.2 ± 0.36
0 to 9	894.5 ± 0.43	0 to 20	1988.0 ± 0.45	0 to 35	3485.6 ± 0.63
0 to 10	995.5 ± 0.40	0 to 21	2086.6 ± 0.48		
0 to 11	1096.0 ± 0.45	0 to 22	2188.2 ± 0.46		

The values of the graduated spaces at the 0^{m} or lettered end of the brass bar were determined in the same way as those at the non-lettered end.

After the first determination, a second was made using the space ($0^{in}.92$ to $0^{in}.94$) on T. and S. inch. Thus, twice as much work was done on the half-millimeter spaces at the lettered end as on those at the non-lettered end. The values and probable errors of the spaces have been accurately computed in the same manner as for the non-lettered end. The computation is quite long and is omitted, the results only being given in the following table:

Table of values of graduated spaces at 0^{m} or lettered end of brass bar.

Space.	Value.	Space.	Value.	Space.	Value.
	μ μ		μ μ		μ μ
0 to 1	101.4 ± 0.18	0 to 11	1100.1 ± 0.26	0 to 21	2100.7 ± 0.31
0 to 2	200.5 ± 0.18	0 to 12	1200.8 ± 0.26	0 to 22	2200.9 ± 0.31
0 to 3	301.2 ± 0.19	0 to 13	1300.4 ± 0.26	0 to 23	2298.2 ± 0.32
0 to 4	402.3 ± 0.21	0 to 14	1400.6 ± 0.28	0 to 24	2398.9 ± 0.33
0 to 5	500.0 ± 0.13	0 to 15	1496.4 ± 0.22	0 to 25	2497.5 ± 0.29
0 to 6	600.6 ± 0.22	0 to 16	1599.5 ± 0.29	0 to 26	2597.9 ± 0.34
0 to 7	700.8 ± 0.22	0 to 17	1702.4 ± 0.27	0 to 27	2698.1 ± 0.34
0 to 8	799.9 ± 0.23	0 to 18	1800.4 ± 0.29	0 to 28	2796.3 ± 0.34
0 to 9	900.4 ± 0.22	0 to 19	1901.5 ± 0.30	0 to 29	2896.5 ± 0.35
0 to 10	1000.1 ± 0.18	0 to 20	2000.3 ± 0.26	0 to 30	2995.9 ± 0.31

VALUES OF GRADUATED SPACES ON S_1, S_2, Z_1, Z_2, AND $(MT1876)_Z$.

§ **72.** The spaces on Z_1, Z_2, and $(MT1876)_z$ were determined in the same manner as those on the 4^{m} end of the brass bar. The values are given in the following tables. The probable errors have not been computed, but since the work was of the same character and equal in amount to that done at the 4^{m} end of brass bar, it is likely that the probable errors do not exceed $\pm 0^{\mu}.5$.

The spaces on S_1 and S_2 were determined with a microscope only and therefore have an uncertainty of at least 1^{μ}. These spaces are unimportant since the zero-mark is always used. In 1880 graduations were made on the neutral axes of the steel and zinc bars of both tubes. Their values have also been determined, and are given below.

The values of the spaces on the zinc bar enter the determination of temperatures by the metallic thermometer formed by the steel and zinc bars. But no very great accuracy is needed for the values of the zinc spaces, since an error of 1^{μ} in them would give an error of but $\frac{1}{38}$ of a degree Fahrenheit in temperature.

ing, the measurement is made from each of these three marks. Since the intervals between the marks when measured the second time will not agree precisely with those obtained on the preceding day, either from errors of observation or from slight changes in the positions of the marks, it is assumed in computation that the center of gravity of the three marks has not moved in the direction of the base-line, and with this condition all the measurements are referred to the mark last placed.

If a, b, c are the distances from the beginning of the base of three cut-off marks made when stopping work, and $a+\delta a$, $b+\delta b$, and $c+\delta c$ the corresponding distances when beginning work again, the abscissa of c, with reference to the center of gravity of the three marks will be at stopping

$$c-\frac{a+b+c}{3}$$

and at starting

$$c+\delta c-\frac{a+\delta a+b+\delta b+c+\delta c}{3}$$

Subtracting the first quantity from the second there results for the motion of c in the direction of measurement with reference to the center of gravity

$$+\frac{\delta\,(c-a)+\delta(c-b)}{3}$$

These variations follow directly from the two measurements of the intervals between the cut-off marks, and the resulting motion of c is to be added to the length of the base, since the condition that the center of gravity of the three marks remains unchanged requires that the front mark c shall have moved forward by that amount during the suspension of the work. The sum of these movements is the cut-off correction for the whole base. Habitually the microscopes were left in their positions over the cut-off marks, so that they also could be treated as stable marks in the same way as the cut-off marks were. Such treatment showed that they had about the same stability as the marks, and also that when the cut-off marks showed a good deal of relative motion, the microscope showed the same thing in the same direction. This would indicate that the motion of the cut-off marks when large was not due to slight motions in the ground of the stakes supporting the cut-off marks, but to a motion of the ground itself, which carried with it not only the stakes supporting the cut-off plate but also the heavy iron pins several decimeters distant, which supported the microscope-stands and penetrated the ground about 30 centimeters. In stopping for short periods, as at noon for dinner, but a single cut-off mark was placed. A discussion of these displacements is given later in §§ 24–28.

9. Of the sum of corrections to S_1 arising from the fact that the observed Z_1-S_1 in base-measurements does not at all times give the true metallic temperature of S_1, and hence does not give its true length when taken in connection with the known expansions of the zinc and steel bars. The derivation of these corrections has been fully explained in Chapter IX, §§ 42–53, and the method of applying them has been given.

§ **11.** As an example of the method of reduction, copies of the notes of measurement on the Chicago Base from the 1784th to the 1791st tube are given, including the notes of a cut-off. Following the record of measurement, a reduction of the measurement with explanatory notes and a reduction of the cut-off with explanatory notes are given.

CHICAGO BASE—FIRST MEASUREMENT.

1. *Notes of measurement of line.*

Date.	Number of tube.	Grade.			Rear end.					Mean time.	Front end.				
		Angle.	Level.		Reading on zero of steel bar.	Reading on zinc bar.	No. of graduation on zinc bar.	Thermometer, Geissler No. 1.	No. of microscope.		Reading on zero of steel bar.	Reading on zinc bar.	No. of graduation on zinc bar.	Thermometer, Geissler No. 2.	No. of microscope.
1877.		° ′	*div.*	*div.*	*rev.*	*rev.*		°			*rev.*	*rev.*		°	
July 21	1784	5 36	6.6	3.8	10.110	10.110	17	73.7	4	11:22 a. m.	10.000	9.588	13	73.4	1
	1785	5 00	6.1	4.1	9.987	9.891	17	74.2	1	11:30 a. m.	9.987	9.692	13	73.8	2
	1786	4 49	5.3	5.0	10.034	9.858	17	74.6	2	11:38 a. m.	9.990	9.777	13	74.2	3
	1787	5 29	4.5	5.8	9.835	9.617	17	74.8	3	11:45 a. m.	10.000	9.810	13	74.5	4
	1788	5 53	4.9	5.4	9.960	9.669	17	75.0	4	11:48 a. m.	10.012	9.870	13	74.5	1

2. *Notes of cut-offs at stopping and starting.*

Date.	No. of tube.	Mean time.	No. of microscope.	Microscope over end of tube.	End of scale forward.	Division of scale.	Reading of microscope.	Level. Left (rear).	Level. Right (front).	Elevation of slow-motion of cut-off cylinder.
1877.							*rev.*	*div.*	*div.*	*in.*
July 21	1786	12:00 m ...	3	Front.......	B	0	9.925	12.2	11.0	1.5
					A	0	9.669	7.9	15.4	
	1787	12:14 p. m.	4	Front......	B	0	9.535	23.6	0.7	1.2
					A	0	9.254	19.0	5.0	
	1788	12:21 p. m.	1	Front.......	A	0	13.555	13.0	11.0	1.4
					B	0	13.810	14.0	10.0	
July 23	1788	9:08 a. m.	1	Front.......	B	0	12.625	15.0	9.0	
					A	0	12.294	13.2	10.8	
	1787	9:13 a. m	4	Front.......	A	0	11.565	7.2	11.1	
					B	0	11.898	12.3	6.9	
	1786	9:18 a. m.	3	Front.......	B	0	10.306	12.7	5.6	
					A	0	10.000	12.0	6.0	

3. *Measurement of line*—Continued.

Date.	Number of tube.	Grade. Angle.	Grade. Level.		Rear end. Reading on zero of steel bar.	Rear end. Reading on zinc bar.	Rear end. No. of graduation on zinc bar.	Rear end. Thermometer, Geissler No. 1.	Rear end. No. of microscope.	Mean time.	Front end. Reading on zero of steel bar.	Front end. Reading on zinc bar.	Front end. No. of graduation on zinc bar.	Front end. Thermometer, Geissler No. 2.	Front end. No. of microscope.
1877.		° ′	*div.*	*div.*	*rev.*	*rev.*		°			*rev.*	*rev.*		°	
July 23	1787	5 29	5.0	5.1	9.997	9.137	16	73.3	3	9:20 a. m ..	10.788	10.263	13	72.8	4
	1788	5 53	4.0	5.9	10.350	9.436	16	73.5	4	9:22 a. m ..	8.396	7.806	13	73.2	1
	1789	6 10	6.9	3.0	11.132	10.100	16	73.9	1	9:26 a. m ..	10.030	9.648	13	73.5	2
	1790	6 01	5.0	5.0	10.015	10.015	17	74.2	2	9:29 a. m ..	9.995	9.704	13	73.8	3
	1791	4 46	5.0	5.0	10.015	9.906	17	74.5	3	9:33 a. m ..	9.996	9.706	13	74.2	4

REDUCTION OF THE NOTES OF THE PART OF THE BASE-LINE GIVEN IN THE PRECEDING TABLE.

Column 2 in the following table gives the measured value of the grade-angle. The value of one division of the level is 33″, and the level correction is positive when the front end of the bubble is the higher.

Column 3 gives the grade-angle as found from measured value of grade-angle minus index-correction. The index-correction was 5° 30′ 27″.

Column 4 gives the correction for grade as found from the formula

$$-2 \times \text{length of tube} \times \sin^2 \tfrac{1}{2} \text{ grade-angle, or} -8^{m} \sin^2 \tfrac{1}{2} \text{ grade-angle.}$$

Column 5 gives the difference between the reading on the zero of the steel bar, and the reading on the graduation of the zinc bar pointed at, expressed in terms of a revolution of the micrometer-screws.

Column 6 gives the sum of the two corrections; (*a*) the correction for the graduation-errors of the divisions of the zinc bar, and (*b*) the correction to one-tenth the quantity in column 5 to reduce it to millimeters.

Column 7 gives the sum of (*a*) the zinc-bar graduation read on, (*b*) one-tenth the quantity in column 5, (*c*) the quantity in column 6 for each tube.

Columns 8, 9, and 10 give the quantities for the front end of the tube corresponding to those that columns 5, 6, and 7 give for the rear or zero-end.

Column 11 gives the difference between the quantities in columns 7 and 10 for each tube, which is the $Z-S$ for the whole tube.

Reduction of the notes of the measurement of the line.

1. BEFORE THE CUT-OFF.

No. of tube.	Grade.			Rear end.			Front end.			Metallic temperature.
	Reading corrected for level.	Grade angle.	Correction for grade.	Difference of micrometer-readings on steel and zinc.	Correction to zinc *minus* steel.	Corrected zinc *minus* steel.	Difference of micrometer-readings on steel and zinc.	Correction to zinc *minus* steel.	Corrected zinc *minus* steel.	
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
	° ′ ″	° ′ ″	*mm.*	*rev.*	*mm.*	*mm.*	*rev.*	*mm.*	*mm.*	*mm.*
1784	5 35 14	0 04 47	−0.0039	0.000	+0.0002	1.7002	+0.412	+0.0030	1.3442	+0.3560
1785	4 59 27	0 31 00	−0.1626	+0.096	+0.0003	1.7099	+0.295	+0.0027	1.3322	+0.3777
1786	4 48 55	0 41 32	−0.2920	+0.176	+0.0004	1.7180	+0.213	+0.0026	1.3239	+0.3941
1787	5 29 21	0 01 06	−0.0002	+0.218	+0.0004	1.7222	+0.190	+0.0026	1.3216	+0.4006
1788	5 53 08	0 22 41	−0.0871	+0.291	+0.0005	1.7296	+0.142	+0.0026	1.3168	+0.4128

2. AFTER THE CUT-OFF.

1787	5 29 00	0 01 27	−0.0004	+0.860	+0.0029	1.6889	+0.525	+0.0030	1.3555	+0.3334
1788	5 53 31	0 23 04	−0.0900	+0.814	+0.0030	1.6944	+0.500	+0.0032	1.3532	+0.3412
1789	6 08 56	0 38 29	−0.2507	+1.032	+0.0036	1.7068	+0.382	+0.0027	1.3409	+0.3659
1790	6 01 00	0 30 33	−0.1580	0.000	+0.0002	1.7002	+0.291	+0.0027	1.3318	+0.3684
1791	4 46 00	0 44 27	−0.3344	+0.109	+0.0003	1.7112	+0.290	+0.0027	1.3317	+0.3795

REDUCTION OF NOTES OF CUT-OFF MEASUREMENTS.

Column 1 in the following table gives the number of the cut-off set, the numbers running in the direction of the measurement.

Column 2 gives the level-correction to the sector-reading, which is positive when the rear end of the bubble is the higher, the axis of the cut-off being then in advance of the cut-off sphere.

Column 3 gives the quantities in column 2 reduced to seconds of arc. The value of one division of the level is 1″.6.

Column 4 gives the measured elevation of the slow motion of the cut-off cylinder reduced to millimeters.

Column 5 gives the height of the cut-off scale above the center of the sphere of the cut-off plate, found by adding $0^{m}.781$ (the length of the cut-off cylinder) to the quantity in column 4.

Column 6 gives the distance of the zero of the micrometer from the axis of the cut-off cylinder, found by subtracting the mean of the two micrometer-readings on the cut-off scale, from ten revolutions, and reduced to millimeters by the relation, 1 revolution of micrometer-screw $=0^{mm}.1011$.

Column 7 gives the product of the quantity in column 5 by the sine of the angle given in column 3. This is the horizontal distance of the intersection of the axis of the cut-off cylinder with the cut-off scale from the vertical through the cut-off sphere.

Column 8 gives the sum of the quantities in columns 6 and 7. It gives, therefore, the horizontal distance from the zero of the micrometer to the point vertically over the cut-off sphere. The sign is positive when the zero of the micrometer is in advance of the sphere.

Column 9 gives the difference between the rear and front readings on the steel bar in the tube, that is, the difference of the distance between the microscopes and $[S_{1_{S_1=Z_1}}+(Z-S)e]$.

Columns 10 and 11 contain the distances to be added to $S_{1_{S_1=Z_1}}$ to obtain the whole distance between the cut-off spheres. These quantities are formed from columns 8 and 9, with the addition of the terms $(Z-S)\,e$, for the tube measured between the cut-offs obtained from column 11 of the preceding table.

The correction to the base given underneath the table is the movement of the forward cut-off sphere plus the movement of the forward microscope with reference to that sphere. It is, therefore, formed from columns 8, 10, and 11, the first two terms being taken from column 8, and the last two being obtained from columns 10 and 11, by the formula given in § 10. The value of e is formed from the equation

$$e = \frac{E_{S_1}}{E_{Z_1} - E_{S_1}}$$

and is numerically equal to +0.6522.

Reduction of the notes of cut-off measurements.

No. of cut-off.	Level correction. Divisions.	Level correction. Seconds.	Elevation of the slow motion.	Total length of cut-off cylinder.	10 revolutions *minus* mean of micrometer-readings.	Scale in advance of cut-off.	Micrometer in advance of cut-off.	Rear *minus* front readings on steel bar in tube reduced to millimeters.	Distance between cut-off spheres. 1 and 2.	Distance between cut-off spheres. 2 and 3.
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
			m.	*m.*	*mm.*	*mm.*	*mm.*	*mm.*	*mm.*	*mm.*
1	−1.6	− 2.6	0.038	0.819	+0.0205	−0.0103	+0.0102	....	+0.0102	+0.1192
2	+9.2	+14.7	0.030	0.811	+0.0613	+0.0579	+0.1192	−0.0167	−0.1192	+0.3627
3	+1.5	+ 2.4	0.036	0.817	−0.3722	+0.0095	−0.3627	−0.0052	−0.0167	−0.0052
									−0.1257	+0.4767
									+0.4006e	+0.4128e
1	+3.3	+ 5.3	0.038	0.819	−0.0155	+0.0211	+0.0056	....	+0.0056	−0.1726
2	+0.4	+ 0.6	0.030	0.811	−0.1750	+0.0024	−0.1726	−0.0800	+0.1726	+0.2351
3	+2.1	+ 3.4	0.036	0.817	−0.2486	+0.0135	−0.2351	+0.2067	−0.0800	+0.2067
									+0.0982	+0.2692
									+0.3334e	+0.3412e

Correction to base = + 0.3627 mm. — 0.2351 mm. — 0.0637 mm. — 0.0701e mm.

§ **12.** The method of computing the measurement of a base measured with the Repsold apparatus having been explained, the results of the measurement and remeasurement of the different sections of the Chicago Base may be given.

In the following table the first column gives the number of the section of the base; the second shows by the numbers 1 and 2 whether the first measurement or the remeasurement in the opposite direction is given on that horizontal line; the third gives the entire number of times that the steel bar at the temperature 60°.292 F. $(S_{1_{S_1=Z_1}} = S_1^0)$ entered into the measurement of the section; the fourth gives the fractional parts of S_1 which entered the measurement when closing on the permanent mark at the end of a section, this interval being measured by means of the intermediate graduations on S_1, except in a single case, when it was measured with an accurately divided leveling-rod, the value of whose unit was determined by office comparisons with a metre of known length, while other office comparisons have shown that the intermediate graduations on S_1 give the fractional parts of the whole tube which they represent without a greater error than 10^{μ}, a quantity that can be neglected in comparison with other errors in the length of a section of the base; the fifth gives the sums of the parts of the base measured with the cut-off scale in closing on section-stones; (the graduations of this cut-off scale have been examined and found sufficiently accurate to introduce no error that need be considered); the sixth gives for each section the sums of the corrections to S_1^0 for metallic temperature, on the assumption that $\frac{E_{S_1}}{E_{Z_1} - E_{S_1}}$ equals its value when $Z_1 = S_1$ or 0.6522; the seventh gives the sums of corrections to the length of each section on account of the inaccuracy of this last supposition; the eighth gives the sums of the corrections for the sections on account of inclination of S_1; the ninth gives the sums of the small distances measured with the microscopes in each section; the tenth gives the sums of the movements of the front cut-off plate in each section; the eleventh gives the sums of the corrections for each section on account of the observed $Z_1 - S_1$ not being the correct metallic temperature of S_1; the twelfth gives the sum of the different corrections for each section.

The duplicate computation gave a maximum difference in the lengths of sections of $3^{\mu}.4$, a quantity which can be entirely neglected.

Results of measurement of Chicago Base-line.

Section.	Measurement.	Number of tubes.	Fractional tubes.	Cut-off scale measurements.	Temperature-correction to $S_{S_1-Z_1}$.	Correction to S_1 for difference of e at $Z_1-S_1=n$ and at $Z_1-S_1=0$.	Correction for inclination.	Sum of intervals between tubes, measured with micrometers.	Sum of movements of front cut-off plates.	Correction for regular residuals.	Sums of the corrections.
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
			T.	μ	μ	μ	μ	μ	μ	μ	μ
I....	1	228	−0.75	+ 60020.0	+ 73257.2	+ 362.1	−119484.0	+ 402.0	+ 66.4	+1266.3	+ 15890.0
	2	227	+0.25	+ 64028.5	+ 46112.1	+ 157.5	− 94030.6	+ 499.4	− 25.2	+ 477.4	+ 17219.1
II....	1	230	+0.25	+ 4005.5	+ 43480.5	+ 181.0	− 30256.9	+ 161.6	− 205.4	+ 896.1	+ 18262.4
	2	230	+0.25	− 57022.4	+103751.6	+ 661.0	− 34525.6	+2005.4	+ 218.3	+ 717.5	+ 15805.8
III....	1	234	+0.50	− 99039.4	+ 97214.5	+ 536.0	−106907.8	+ 603.3	− 140.8	+1310.8	−106423.4
	2	234	+0.50	− 54018.0	+ 70797.5	+ 339.0	−126204.7	− 35.0	+ 29.1	+ 391.3	−108700.8
IV....	1	232	mm −1156.0251*	+ 34013.9	+103437.5	+ 586.6	− 49009.5	+ 576.7	+ 188.6	+ 955.4	+ 90749.2
	2	233	−1.30	+ 92033.3	+ 91441.4	+ 500.9	− 52503.1	+1088.3	+ 501.9	+1293.9	+134356.6
West part	1	924	mm −1156.0251	− 1000.0	+317389.7	+1665.7	−305658.2	+1743.6	− 91.2	+4428.6	+ 18478.2
	2	924	−0.30	+ 45021.4	+312102.6	+1658.4	−307264.0	+3558.1	+ 724.1	+2880.1	+ 58680.7
V....	1	230	+0.85 mm +1156.0251	− 99036.8	+ 70607.2	+ 262.3	− 74350.8	+1001.5	+ 43.4	+ 938.4	−100534.8
	2	231	+0.10	− 5003.2	+110232.4	+ 665.8	− 55758.1	+2219.5	+ 109.8	+1233.6	+ 53699.8
VI....	1	226	−0.40	+140052.6	+ 98906.5	+ 530.1	− 63979.2	+ 146.5	+ 218.8	+1043.0	+176918.3
	2	225	+0.65	− 27011.5	+ 70535.2	+ 298.7	− 69402.2	+ 379.8	+ 70.3	+ 935.9	− 24193.8
VII....	1	301	−0.45	− 90029.8	+118829.7	+ 681.6	−122664.3	+ 823.4	+ 378.6	+1819.6	− 90161.2
	2	301	−0.50	+ 54023.8	+142115.7	+ 926.7	− 90378.8	+ 556.8	+ 295.2	+1055.6	+108595.0
VIII....	1	196	+0.80	+ 39009.9	+ 53655.8	+ 250.4	− 44521.4	+ 932.5	− 79.5	+1176.9	+ 50424.6
	2	196	+0.80	− 11005.5	+102747.9	+ 692.2	− 43454.5	+ 540.3	− 86.1	+1188.6	+ 50622.9
East part	1	953	+0.80 mm +1156.0251	− 10004.1	+341999.2	+1724.4	−305515.7	+2903.9	+ 561.3	+4977.9	+ 36646.9
	2	953	+1.05	+ 11003.6	+425631.2	+2583.4	−258993.6	+3696.4	+ 389.2	+4413.7	+188723.9
Total..	1	1877	+0.80	− 11004.1	+659388.9	+3390.1	−611173.9	+4647.5	+ 470.1	+9406.5	+ 55125.1
	2	1877	+0.75	+ 56025.0	+737733.8	+4241.8	−566257.6	+7254.5	+1113.3	+7293.8	+247404.6

* Measured with leveling-rod.

The mean value of (Z_1-S_1) for the two measurements of the base was $+570^{\mu}.4$, corresponding to 75°.27 F.

§ **13.** In the following table, derived from the preceding one by expressing the two measurements of a section with the same fraction of a tube, the principal results are given, and also the differences between the two measurements of each section.

To express the sections of the base in terms of entire numbers of S_1^0 and equal fractions of it, the value of the fractions of a tube in metres is needed. The value of S_1 is given in Chapter IX, § 62, and from that its value at 60°.292, for which $Z_1 - S_1 = 0$, may be computed. Transforming it into metres by Clarke's value of the metre (metre=$1^y.093623$) there results—

$$S_1 \text{ at } 60^\circ.292 \text{ F.} = 4^{m}.0009624$$

as an approximation more than sufficient for obtaining the values of the fractions of the tube in metres. It has already been stated that these fractions of tubes are the exact fractions which they purport to be within 10^{μ}, a quantity that is insignificant in comparison with the length of a section, and which enters a section but once.

Principal results of measurement of Chicago Base.

Section.	Measurement.	Number of $(S_1)_{Z_1=S_1}$	Sum of corrections.	Means.	Differences.
			μ	μ	μ
I....	1	227.25 $(S_1)_{S_1=Z_1}$	+ 15890.0	+ 16554.55	−1329.1
	2	227.25 "	+ 17219.1		
II....	1	230.25 "	+ 18262.4	+ 17034.10	+2456.6
	2	230.25 "	+ 15805.8		
III....	1	234.50 "	− 106423.4	− 107562.10	+2277.4
	2	234.50 "	− 108700.8		
IV....	1	232.00 "	−1065275.9	−1065604.00	+ 656.2
	2	232.00 "	−1065932.1		
West part	1	924.00 "	−1137546.9	−1139577.45	+4061.1
	2	924.00 "	−1141608.0		
V....	1	231.00 "	+ 455345.9	+ 454570.95	+1549.9
	2	231.00 "	+ 453796.0		
VI....	1	225.50 "	+ 577014.5	+ 576482.55	+1063.9
	2	225.50 "	+ 575950.6		
VII....	1	300.50 "	+ 109887.3	+ 109241.15	+1292.3
	2	300.50 "	+ 108595.0		
VIII....	1	196.80 "	+ 50424.6	+ 50523.75	− 198.3
	2	196.80 "	+ 50622.9		
East part	1	953.80 "	+1192672.3	+1190818.40	+3707.8
	2	953.80 "	+1188964.5		
Total..	1	1877.80 "	+ 55125.4	+ 51240.95	+7768.9
	2	1877.80 "	+ 47356.5		

Substituting in $924\,(S_1)_{Z_1=S_1} - 1139^{mm}.58$ and in $953.80\,(S_1)_{Z_1=S_1} + 1190^{mm}.82$, which are respectively the measured lengths of the west and east halves of the base, given in the preceding table, the value of $(S_1)_{Z_1=S_1}$, derived from Chapter IX, § 66, namely:

$$(S_1)_{Z_1=S_1} = 4^{y}.37554481 = 13^{ft}.12663443$$

there result $12125^{ft}.2714$ and $12524^{ft}.0909$ as the measured lengths of the west and east halves of the base respectively, and

Chicago Base as measured (mean of two measures) $= 24649^{ft}.3623$

§ **14.** The mean level of Lake Michigan from 1860 to 1875, inclusive, is given in Chapter XXII, § 13, as 581.28 feet above mean tide at New York. The mean height of the tubes during the measurement of the Chicago Base above the mean level of the lake was determined with a leveling-instrument, and is 36.7 feet. The mean heights of the east and west halves of the base differ so little that the difference may be neglected. Hence there results mean height of Chicago Base above mean tide, 618.0 feet, with a probable error not greater than ± 1.0 foot.

The approximate length of the west half of the base is 12,125 feet. Taking 7.321128 as the logarithm of the radius of curvature for the base, there results a correction of $-0^{ft}.3577$ to the west half of the base to reduce it to mean tide at New York. The approximate length of the east half of the base is 12,524 feet, and there results a correction to the east half of the base to reduce it to mean tide of $-0^{ft}.3695$. Applying these corrections, there result at sea-level,

$$\begin{aligned} &\text{West half of Chicago Base} = 924\,(S_1^0) - 1^{m}.24861 \\ &\text{East half of Chicago Base} = 953\,(S_1^0) + 4^{m}.27898 \\ &\text{Chicago Base at sea-level} = 1877\,(S_1^0) + 3^{m}.03037 \end{aligned}$$

From Chapter IX, § 66,

$$S_1^0 = 13^{ft}.12663443$$

and S_1^0 when expressed in terms of $R\,1876$, is

$$S_1^0=4\,R\,1876-58^{\mu}.4+1^{\mu}.326\ (60.29-42.75)$$

Making the proper substitutions there result,

Chicago Base at sea-level=7508 (R1876 at 60°.29 F.)+$2^{m}.96368$.
West half of Chicago Base at sea-level =12124.9137 English feet.
East half of Chicago Base at sea-level=12523.7214 English feet.
Chicago Base at sea-level =24648.6351 English feet.

The value of the base given above in terms of R1876 depends on adjusted expansions of R1876 and S_2. But in Chapter IX, § 66, a value for S_1 in terms of R1876 is given, which depends solely on comparisons of R1876 with S_2 and of S_2 with S_1. This value is—

$$S_1=4\,R\,1876-37^{\mu}.01+1^{\mu}.2744\ (t-59)$$

Expressing the base by this value it becomes—

7508 (R1876 at 60°.29 F.)+$2^{m}.96398$

This value will be of use when the length of R1876 is accurately known.

The Chicago Base was divided into two nearly equal parts by the middle section-stone. The angles of the triangles formed by these halves of the base and station Willow Springs were read with the same care as primary angles. They are given in Chapter XVI, C, and enable either half of the base to be computed from the whole base. The computed values of the east and west halves are, respectively, $12523^{ft}.6833$ and $12124^{ft}.9519$, and their excesses over the measured lengths are $-0^{ft}.038$ and $+0^{ft}.038$, respectively.

§ **15.** In § 6 a value for $\frac{E_{S_1}}{E_{Z_1}-E_{S_1}}$ depending on the temperature has been obtained. Considering this quantity as independent of the temperature, that is, as constant, an approximate value for it might have been obtained from the base-measurements. There were quite large differences of temperature between the measurement and remeasurement of several of the twenty sections into which the Chicago, Sandusky, and Olney Bases are divided. If the measurements have all corrections applied to them save those for temperature of S_1, the remaining differences between the measurement and remeasurement of a section will be attributable to the changes in length of S_1 from temperature and to accidental errors. It will be seen hereafter, in considering the discrepancies between measurement and remeasurement, that the accidental errors are very small, and, therefore, that the differences between two measures of the same section uncorrected for differences of temperature, taken in connection with the values of the metallic temperatures Z_1-S_1 for these measures, should give a value for $\frac{E_{S_1}}{E_{Z_1}-E_{S_1}}$. From twenty sections, with ranges in the mean temperatures of the two measurements of a section which sometimes reached 23°.4 F., a pretty good mean value of $\frac{E_{S_1}}{E_{Z_1}-E_{S_1}}$ should result. For each measure of each section of the three bases, an observation-equation was written in the form $a\frac{E_{S_1}}{E_{Z}-E_{S_1}}+b=v$, in which a is the sum of the observed Z_1-S_1 for the first measure minus that for the second; b is the length of the section uncorrected for temperature resulting from first measure minus that for the second measure; and v is the residual. Solving by least squares the twenty equations of condition thus obtained, there results $\frac{E_{S_1}}{E_{Z_1}-E_{S_1}}=0.658\pm0.002$. This value corresponds nearly to the mean temperature of all the measures of the three bases, which was 74°.47 F. In § 6 the value of $\frac{E_{S_1}}{E_{Z_1}-E_{S_1}}$, derived from office comparisons, is given as $0.64119+0.000389\ (t-32)$, which at 74°.47 F. becomes 0.65771.

SOURCES OF ERROR.

§ **16.** The sources of error in the measurement of a base-line with the Repsold base-apparatus may now be considered.

1. *Errors of alignment.*—The Chicago base-line was divided into eight nearly equal sections by seven marking-stones intermediate to the stones which mark the ends of the base-line. A point at about the middle of the base was first obtained approximately on the line, and the angle at it was then read between the ends of the base-line to an accuracy of one or two seconds. This gave the means of computing the distance by which the point should be moved to make the angle at it 180°, and the middle section-stone was then placed with its mark at this point. The other section-stones were then set in a similar way from the stones at the middle and ends of the base. This made it certain that the different sections were parallel to the line joining the ends of the base, within 5″, a precision much greater than is necessary. The average length of a section was about 940 metres. Between two section-stones the measuring-tube was brought into line by setting its 0^{m} mark on the steel bar over a point already fixed (a section-stone at starting), and then moving the front end of the tube in azimuth till the alignment-telescope, firmly connected with the tube, points to a target at the next section-stone, as distant from the line of the base as the vertical plane through the line of collimation of the alignment-telescope is from the vertical plane through the longitudinal line marked on the steel bar. It is evident, if the longitudinal line on the steel bar is not parallel to the vertical plane of the telescope's motion, that when the telescope points at the distant target the steel bar will not be parallel to the base, its front end each time the tube is set up changing its distance from the base but always remaining on the same side of the base-line, the distance from the base-line of either rear or front end being zero at the beginning and end of the section. The curve drawn through the successive positions of the tube resembles the path of a boat steered toward a fixed point across a river. An investigation by Assistant Engineer J. B. Johnson shows that the curve in polar co-ordinates with the forward section-stone as an origin and with angles counted from the section line may be represented closely enough by

$$\theta = -\text{Nep.}\log\left(\frac{r}{A}\right)\tan\alpha$$

where A is the length of the section, α the deviation from parallelism of the alignment-telescope, and r and θ the co-ordinates. The rectification of this curve gives for its length between two section-stones, $\frac{A}{\cos\alpha_1}$ if α_1 is the actual angular deviation of the steel bar from the telescope plane of motion for this section. Hence the correction to the measured length of the section is $-A'(1-\cos\alpha_1)$ if A' be the measured length. This may be written

$$-\frac{A'}{2}\sin^2 1''\,\alpha_1^2$$

Assuming that the sections are of equal length, as is nearly the case, the sum of the actual corrections for all the sections would be

$$-(\alpha_1^2+\alpha_2^2\ldots.+\alpha_n^2)\frac{A'}{2}\sin^2 1''$$

in which α_n is the actual deviation of the telescope from parallelism for the nth section. Approximately we shall have

$$\frac{[\alpha^2]}{n}=e^2$$

where e is the mean error for the actual errors α_1, α_2, &c. Hence,

$$[\alpha^2]=ne^2$$

A value of e has been determined from numerous repetitions of the adjustment, each followed by a measurement of the distance on a target attached to the tube, between the line of sight of the telescope in its different adjustments and a fixed point of the target. The resulting value for e, or the mean error of adjustment found, was 22″. Hence the total correction to the base for this mean error of adjustment would be

$$-n\frac{A'}{2}\sin^2 1''\,(22)^2$$

As this amounts to less than $0^{mm}.1$ for either the Chicago, Sandusky, or Olney Base, it may be neglected. As the adjustments were only made about once in a week, while a section was measured in two or three days, it has been assumed as an approximation that the adjustment-error was constant for each section. Experience on the Chicago and Sandusky Bases showed that the telescope when once adjusted retained its position with great stability, and that the changes produced by a new adjustment were due to the errors in the adjustment itself and not to change in the position of the telescope. On the Sandusky and Olney Bases, measured in 1878 and 1879, respectively, points were placed accurately on the base-line at intervals of about 300^{m} intermediate to the section-stones and by the method used for placing the section-stones. Then, in measuring the base, whenever one of these points was passed the actual deviation from the base-line was measured. The greatest deviation in any case was $0^{m}.015$ excluding one case of $0^{m}.191$, which arose from an error of adjustment. See Chapter XII, § 2.

On the Olney Base, after adjusting the telescope to parallelism with the steel bar as nearly as possible, by the method given in Chapter VIII, § 11, further changes by computed amounts read on the target were made in the adjustment of the telescope until the tubes would pass within 1^{cm} of the intermediate marks. This once accomplished, no further adjustment was needed during the measurement of the base, although the adjustment was frequently tested. There is of course a slight error in alignment arising from error in pointing telescope at the signal, but as the telescope was a good one with an object-glass about 30^{mm} in diameter, the error from this cause was entirely insignificant in comparison with that already discussed.

§ **17.** 2. *Errors of inclination.*—In § 8, under (6), it has been stated that seven determinations of index-error of the sector, distributed through the measurement, gave a range of but $26''$, and that their mean was taken as the true value of the index-error. We may assume, then, that the probable error in this correction did not exceed $5''$. This would introduce an error into the length of the base which would change sign whenever the inclination changed sign, and so, on a nearly level base, would be largely eliminated. Even if there had been no changes of sign of this correction, since the average inclination was much less than 1°, an error of not more than $\frac{1}{2000000}$ part of the distance would have been introduced. With the changes of sign the error may be neglected. The sector can be read to $30''$, but its vernier was habitually set at exact minutes, and the level, of which one division equals $33''$, was read to tenths of a division. These errors of reading are accidental, and so as often positive as negative, and their probable value does not exceed $5''$. The resulting corrections change sign with the error, so that the total correction to a section or to the base from this error would be very nearly zero. Taking 1° as the inclination, which would give the average error for a tube from error in reading inclination (and this value is too large), this error would be about 2^{μ}. The probable error in the length of the base from this cause would be about $2^{\mu}\sqrt{1878}$ or $\pm 86^{\mu}.6$, a quantity which may be neglected.

§ **18.** 3. *Stability of microscopes.*—It has already been stated that the interval between the position of the 4^{m} mark on the front end of the tube and the position of the 0^{m} mark on the rear end of the tube, when these marks were brought successively under the same microscope, was measured with the microscope. As the distance to be measured was habitually less than $0^{mm}.1$, and as the microscopes can be pointed with a probable error of about $\pm 0^{\mu}.4$ in the comparing-room, which may be increased to $0^{\mu}.7$ or $0^{\mu}.8$ in the field, and since the errors in measuring this quantity are as likely to be positive as negative, their resultant effect on the length of the base does not exceed $0^{mm}.04$, and is entirely insignificant.

But the method of measurement supposes absolute stability in the microscope in the interval between its pointings at the front and rear ends of the tube. Numerous experiments on this point were made on the Sandusky and Olney Bases. The microscope may from temperature-changes in itself and in the stand which carries it, change its pointing. This it undoubtedly does, but experience in open-air comparisons has shown that such changes are very slow, and that in the period of one or two minutes elapsing between successive pointings with the same microscope, the change is entirely insignificant. On the Olney Base, work was suspended at noon for dinner. On twenty-four days the average length of the stop was $1^{h}\ 49^{m}$. The relative movement of the microscopes in this interval, from all causes (excluding one day on which it was $+174^{\mu}.2$), varied between

$+27^{\mu}.5$ and $-18^{\mu}.5$, giving in mean $+1^{\mu}.6$ as the average motion of the microscopes from each other during this stop. The microscope may also change its pointing from disturbances transmitted to it through its stand from the earth, these disturbances arising either from the weight of the men employed in the work, when they change positions in the vicinity of the microscope, or from the disturbance produced by the removal of the front end of the tube from a stand and the placing of the rear end of the tube in the same stand, this disturbance of the tube-stand being transmitted through the ground to the microscope-stand. If by any of these causes the microscope is displaced subsequently to the pointing at the front end of the tube, the displacement will introduce no error, provided the microscope returns to its previous position by the time it is pointed at the rear end of the tube next in order of measurement.

It will be well to restate the position of the microscope-stand with reference to the tube-stand, and the varying positions of the men employed in the work. The vertical axis of the stand supporting the end of the tube is directly under the microscope, so that its feet are not far from the corresponding feet of the microscope-stand. The feet of both rest on the heads of iron pins two inches in diameter and fifteen inches long, shown in Figure 1, Plate VII, driven down to the ground. The axes of the pins supporting corresponding legs of the two stands vary in distance from each other between 7 and 12 inches. When the men approach a microscope to remove the tube from or to place the tube on its stands, they are required to keep 16 inches away from the pins supporting the microscope, or, what is the same thing, 12 inches away from the foot-plate. On the Sandusky and Olney Bases this distance was secured by placing wooden cross-bars on the platform which supports the observer, at the right places to keep the men away from the microscopes. The question arises, and it is a very important one, whether, when a tube-carrier approaches a microscope, keeping at the prescribed distance from its supporting pins, his weight, acting through the ground, displaces the microscope by amounts that need to be considered, and, if so, whether, when he withdraws, the displacement disappears. Two men carry the tube by means of straps passing over their shoulders, one being at the front end and one at the rear. A microscope having been pointed at the front end of a tube, the front carrier then approaches the microscope, lifts the tube from its stand, steps sidewise, and then moves forward. Immediately after the rear carrier approaches the same microscope, places the rear end of the tube on the stand under the microscope, and then withdraws. As at other times the men are two or three feet or more away from the microscope, the danger of disturbance exists only when the tube is being placed on or lifted from its stand. Numerous experiments were made on the Sandusky and the Olney Base to decide the question whether the microscopes were permanently disturbed by the motions of the tube-carriers. The following method was used: The tube being placed under the microscopes as in measuring, pointings were made at both ends of the tube; then a man approached till his foot, on which he bore his weight, was within a known distance, usually six inches or less, of that foot-plate supporting the front or rear microscope to which he approached most nearly in measuring; then both microscopes were read; the man then withdrew and both microscopes were again read. The readings on the tube, corrected for temperature-change, gave the change of interval between the microscopes due to the approach or recession of the man. Assuming that the distant microscope was stable for this short time, about one minute, the change in the interval was the motion of the other microscope in the direction of the base. On the Sandusky Base experiments were only made with the front microscope. The following tables give the results. The first column gives the date; the second gives the distance of the man's foot from the foot-plate of the microscope-stand, and should be increased by three inches to give the distance from the supporting pin; the third column gives the displacement when the man approached the foot-plate; and the fourth, the displacement when he withdrew. A plus sign indicates motion of the microscope in the direction of measurement.

SANDUSKY BASE-LINE.

Apparent movement of microscope at front end of tube.

Date.	Distance of man from rear leg, in inches.	Apparent movement on approaching rear leg.	Apparent movement on receding from rear leg.	Distance of man from front leg, in inches.	Apparent movement on approaching front leg.	Apparent movement on receding from front leg.
1878.		μ	μ		μ	μ
Aug. 12	6	+ 8.4	+3.2	6	−12.2	+ 5.8
15	6	− 6.3	+8.5	6	− 9.3	+ 3.3
16	3	+ 9.3	−0.3	3	−35.7	+ 8.7
19	1	− 2.7	−0.1	1	−19.7	−13.1
20	2	− 4.9	+4.9	2	− 4.2	− 0.5
22	2	+ 1.7	+3.5	2	+10.1	+13.3
23	6	− 2.4	+3.3	6	− 7.0	+ 0.8
27	3	− 4.4	+6.3	3	+ 0.7	− 0.1
28	2	− 3.0	+0.4	2	− 4.9	+ 3.6
29	6	− 4.6	+1.7	6	− 8.9	+ 6.8
Sept. 6	6	− 7.4	+2.0	6	−25.8	+ 0.9
9	6	− 1.7	+1.4	6	−13.2	+ 4.2
16	3	−12.5	+9.9	3	− 3.0	− 4.6
17	6	−11.5	+9.1	6	−16.3	+ 9.3
18	6	− 6.8	+6.1	6	−12.1	− 0.3
30	6	− 4.7	+3.7	6	− 8.7	+ 4.0
Oct. 1	6	+10.2	+3.1	6	− 5.8	− 0.9
2	6	− 0.4	+7.0	6	− 5.2	− 1.9
3	6	− 0.4	+1.7	6	−18.8	+ 1.6
4	6	+ 1.5	+2.1	6	− 4.5	+ 1.4
7	6	− 8.0	+6.1	6	+ 7.5	− 5.0
8	6	− 2.3	+2.7	6	−10.7	+10.2
12	6	+11.4	+5.9	6	+10.1	− 4.2
	Means.......	− 1.8	+4.0	Means.......	− 8.6	+ 1.9

An examination of these tables for cases in which the man's foot was within 6 inches or less of the foot-plate, shows that when the rear foot of the microscope-stand at the front end of the tube was approached, the microscope moved to the rear in seventeen cases out of twenty-three, and in twenty-one cases out of twenty-three moved in the opposite direction when he stepped away. The mean movement in the first case was $-1^{\mu}.8$, and in the second case $+4^{\mu}.0$, so that the reaction would appear from the observations to be greater than the action, giving a displacement of $+2^{\mu}.2$. When the front leg of the front microscope was approached, the microscope moved backward in nineteen cases out of twenty-three, and when the man withdrew it moved forward in fourteen cases out of twenty-three. The mean movement in the first case was $-8^{\mu}.6$, and in the last case $+1^{\mu}.9$, indicating a permanent set of $-6^{\mu}.7$. Since in measurement the front and rear legs are both approached, the mean effect of the approaches will be

$$\frac{-1^{\mu}.8-8^{\mu}.6}{2}=-5^{\mu}.2$$

and of the withdrawals

$$\frac{+4^{\mu}.0+1^{\mu}.9}{2}=+3^{\mu}.0$$

so that the permanent displacement of the microscope would be $-2^{\mu}.2$. If the two tables be compared, it will be noticed that in approaching the rear leg but one displacement greater than $12^{\mu}.0$ was observed out of twenty-three, while in approaching the front leg eight greater displacements were observed, one reaching $35^{\mu}.7$. As these legs are but about two feet apart, it is difficult to suppose there was habitually so great a difference in the character of the soil about them as to give such differing results, and there may be some other unknown source of error in this work about the front leg. The work on the rear leg and all the work on the Olney Base show that the microscope habitually moved toward the man, as would be the case were his weight to slightly depress the soil about the pin nearest him. The work on the front leg, if it were correct, would indicate that the front leg rose under his pressure.

§ **19.** On the Olney Base, similar experiments were made, and with both the front and rear microscopes. Their results are given in the following tables, in which the first column gives the dates; the second gives the displacement produced by man standing within 6 inches of rear foot-plate; the third, the displacement on withdrawing; and the fourth and fifth the like data for man near front foot-plate.

OLNEY BASE-LINE.

Apparent movement of microscope at front end of tube.

[+ indicates forward, — indicates backward movement.]

Date.	On man standing within 6 inches of rear leg.	On man receding from rear leg.	On man standing within 6 inches of front leg.	On man receding from front leg.
1879.	μ	μ	μ	μ
July 25	— 6.2	+ 4.9	+ 8.7	— 5.2
28	— 2.7	+ 5.6	+ 4.5	— 4.0
30	— 8.7	+ 6.7	+11.7	— 7.2
Aug. 1	....	+14.1	+12.9	....
6	— 9.9	+ 8.0	— 3.0	+ 4.0
7	— 9.0	+ 8.7	+16.1	—18.2
8	— 9.5	+12.5	+11.5	— 8.3
11	—11.6	+ 8.6	+ 7.2	— 6.9
14	—20.2	+13.6	+14.7	—16.2
18	— 9.6	+ 6.1	+ 6.8	— 3.1
19	—10.3	+ 9.1	+13.3	—13.1
20	— 7.5	+ 5.7	+13.9	—11.8
21	— 3.5	+ 2.0	+ 9.4	— 9.3
22	— 8.5	+ 4.9	+10.4	— 8.3
28	— 9.8	+ 6.9	+ 9.7	— 9.3
29	—12.0	+ 9.5	+ 4.6	— 5.0
Sept. 1	—12.6	+ 7.5	+11.9	—10.4
2	—14.1	+13.5	+12.4	—13.7
3	—10.2	+ 6.0	+ 9.6	— 4.6
4	—12.1	+12.6	+ 6.5	— 7.4
5	—16.7	+13.0	+ 9.5	— 8.7
8	—17.0	+13.7	+19.8	—12.2
9	—11.5	+ 8.8	+ 4.0	— 3.0
10	—16.1	+15.6	+22.3	—18.6
11	—12.2	+11.5	+15.1	— 9.7
12	—10.6	+14.5	+ 2.2	— 3.5
Means..	—11.1	+ 9.4	+10.2	— 8.5

Apparent movement of microscope at rear end of tube.

Date.	On man standing within 6 inches of rear leg.	On man receding from rear leg.	On man standing within 6 inches of front leg.	On man receding from front leg.
1879.	μ	μ	μ	μ
Aug. 4	—12.9	+ 9.3	+13.3	—11.9
5	—14.5	+ 9.7	+13.8	— 9.0
6	— 9.9	+ 6.7	+ 3.9	— 6.0
7	—18.4	+12.0	+ 6.9	— 7.6
8	— 2.4	— 2.1	+ 8.9	— 6.7
11	—15.4	+11.5	+13.9	7.7
14	—44.3	+32.4	+12.8	—12.3
18	— 9.8	+ 7.7	+ 2.7	— 3.5
19	—10.4	+ 6.5	+ 9.2	— 9.5
20	—11.3	+ 9.0	+11.3	—10.6
21	— 3.9	+ 4.1	+ 5.6	— 5.9
22	—13.8	+ 9.5	+10.3	— 7.4
28	—22.8	+18.7	+10.8	—11.7
29	—13.1	+12.5	+13.1	—10.7
Sept. 1	—14.8	+13.2	+ 6.4	— 4.3
2	—19.6	+13.2	+ 9.7	— 8.7
3	—12.5	+10.7	+10.8	—10.2
4	— 8.6	+10.8	+ 6.8	— 8.6
5	—19.0	+16.6	+ 4.9	— 4.9
8	—15.0	+13.9	+12.7	—10.6
9	— 4.7	+ 7.2	+ 6.7	— 8.8
10	—18.4	+12.5	+11.5	— 7.9
11	— 7.4	+10.3	+10.5	— 7.6
12	+ 2.0	+ 4.3	+ 6.1	— 4.8
Means..	—13.4	+10.8	+ 9.3	— 8.2

From these tables it appears that when a man stood within 6 inches of either front or rear foot-plate of a microscope it moved toward him in 97 cases out of 99; and when the man withdrew the microscope moved in the opposite direction in 97 cases out of 99.

Mean displacements of microscopes.

Position of man.	At front end of tube.	At rear end of tube.
	μ	μ
Within 6 inches of rear foot-plate	−11.1	−13.4
Withdrawn	+ 9.4	+10.8
Within 6 inches of front foot-plate	+10.2	+ 9.3
Withdrawn	− 8.5	− 8.2
Sum	− 0.0	− 1.5

For the front microscope there results then as the permanent displacement resulting from an approach within 6 inches of and a withdrawal from both its front and rear foot-plates (as is the case in measuring, although the approach is not so close) a permanent displacement of the microscope by $0^{\mu}.0$, and for the rear microscope a permanent displacement of $-1^{\mu}.5$, or, in mean, $-0^{\mu}.7$.

The average displacement produced by a man's weight within 6 inches of either front or rear foot-plate on the sandy Sandusky Base was $5^{\mu}.2$, the displacements being irregular in amount and sign. On the Olney Base, much of which was covered with sod, displacements were quite regular in amount and in sign, the average value being $11^{\mu}.0$.

The permanent displacements of the microscope by the approach of men to and their recession from both front and rear foot-plates were $-2^{\mu}.2$ for the Sandusky and $-0^{\mu}.7$ for the Olney base-line. Considering the result on the Sandusky base-line, it is evident that a few additional observations giving displacements no larger than some observed would seriously modify its value, while the Olney permanent displacement is no larger than its probable error.

On the Sandusky Base the recovery exceeded the observed displacement for the rear foot-plate while the mean recovery was slight for the front foot-plate. It is difficult to believe that these two foot-plates should differ widely in their average conduct. The experiments do not then, in average, establish any permanent displacement, though they indicate about $-1^{\mu}.5$. But in all or nearly all of these experiments the man's foot was but 6 inches or less from the foot-plate instead of being a foot or more from it as in actual measurements. We may conclude, then, that in actual measurement the average permanent displacements were so small that the experiments do not enable us to assign a reliable value to them.

§ **20.** Experiments were made on 6 days on the Sandusky Base to see if the pressure on the ground of the tube-carriers between the microscopes and alongside the tube, affected the interval between the microscopes. The tube was first read on with the two microscopes; then the tube-carriers stood at their places beside the tube, and the microscopes were again read, then the tube-carriers stepped away and the microscopes were again read. The observed relative displacements of the microscopes with men beside the tube varied from $+2^{\mu}.1$ to $-9^{\mu}.6$, their mean being $-2^{\mu}.5$. But two measures of displacement when men stepped away were made; their mean was $+1^{\mu}.0$, a plus sign indicating motion of microscopes from each other.

On the Olney Base, such measures were made on twenty-four days. The results are given in the following tables:

OLNEY BASE-LINE.

Relative movement of microscopes when tube-carriers come into and go out of position.

[— indicates that microscopes move towards, + that they move from each other.]

Date.	Relative movement on coming into position.	Relative movement on going out of position.
1879.	μ	μ
July 30	− 5.5	+5.4
Aug. 1	− 7.3	+7.5
4	−11.1	+3.5
5	−10.0	+5.4
7	− 7.6	+6.3
8	11.7	+2.5
11	− 7.6	+8.6
14	− 3.5	+5.7
18	− 3.9	+2.0
19	5.4	+7.7
20	− 2.8	+2.4
21	0.2	+1.0
22	− 1.5	+7.8
28	− 2.9	+3.1
29	− 3.8	+4.5
Sept. 1	+ 0.4	+2.0
2	− 4.2	+2.2
3	1.7	+3.5
5	2.5	+3.7
8	− 3.4	+5.0
9	− 4.2	+3.9
10	− 4.3	+1.8
11	− 4.1	+4.7
12	− 1.3	+1.7
Means..	− 4.6	+4.2

On twenty-three days the microscopes moved toward each other when the men stepped beside the tube, and on all days the microscopes moved apart when the men stepped away, as they always do when measuring is going on. The maximum and minimum movements in the first case were $-11^{\mu}.7$ and $-0^{\mu}.2$; and in the latter case $+8^{\mu}.6$ and $+1^{\mu}.0$, respectively. The mean movements were $-4^{\mu}.6$ and $+4^{\mu}.2$, respectively. The mean permanent set was but $-0^{\mu}.4$, a quantity not greater than its probable error.

It may, then, be said that the experiments of both kinds at both bases indicate in the average no certain permanent displacement of the microscopes by movements of the tube-carriers which need be taken into special account. Such displacements occur, but they seem to be of the nature of accidental errors which eliminate themselves in many observations.

§ **21**. There is another way in which the stability of the microscopes might be affected. In the measurement of a base, immediately after a microscope has been pointed at the front end of a tube, the tube is carried forward and the rear end is placed under the microscope. The question arises whether the removal of the half weight of the tube (about 90 pounds) from the tube-stand, and its replacement, together with the slight shocks incident to the operation, may not transmit some permanent disturbance through the ground to the microscope.

Two kinds of experiments were made to ascertain this. In the first kind the tube, after having been read on by the microscopes, was taken a few steps away, brought back and placed on its stands again, and then read on. Experiments were made in this way on the Sandusky Base on 8 days. The observed relative movement of the microscopes varied from $+6^{\mu}.8$ to $-5^{\mu}.1$, having a mean value of $0^{\mu}.0$. On four days the microscopes appeared to move towards each other, and on four to separate. The experiments do not establish any displacement. In the second kind of experiments to determine the effect of weight on tube-stand in displacing microscopes, the tube was first read on with both microscopes, then iron pins were piled among the braces of a tube-stand symmetrically about its axis, and the microscopes were again read, and finally the pins were removed, and the microscopes were again read in some cases. At the Sandusky base 14 experiments were made on the front microscope on 13 days. The results are given in the following table:

SANDUSKY BASE-LINE.

Apparent movement of microscope at front end of tube when weight was placed on or taken off stand under microscope at front end.

[+ indicates forward, — indicates backward movement.]

Date.	Weight.	Apparent movement on placing weight on stand.	Apparent movement on removing weight from stand.
1878.		μ	μ
Aug. 6	50 pounds	— 3.5	+ 3.3
9	Man's weight...	— 4.5	+ 0.8
9	....do	+ 1.1	+ 0.4
15	... do	— 7.6	+ 3.9
16	... do	+ 7.6	— 2.0
19	... do	—35.2	+11.7
22	... do	+ 1.6	— 0.5
28	... do	— 5.6	....
Sept. 9	... do	— 8.7	....
16	... do	—14.5	....
17	....do	+ 7.4	....
25	....do	+ 4.6	....
Oct. 2	50 pounds......	— 2.6	+ 1.1
12	120 pounds......	— 2.8	....
	Means	— 4.5	+ 2.3

On two days the weight placed on the stand was 50 pounds; on the others it was 120 or 150 pounds. The extreme displacements observed when tube-stand was loaded were $-35^{\mu}.2$ and $+11^{\mu}.0$, the mean displacement being $-4^{\mu}.5$.

If the single observation which gave a displacement $-35^{\mu}.2$ were rejected (and it is the only one exceeding $14^{\mu}.5$), the mean displacement would be reduced to $-2^{\mu}.1$. The displacement resulting from the removal of the weights was observed seven times on six days; its extreme values were $+11^{\mu}.7$ and $-2^{\mu}.0$; its mean value was $+2^{\mu}.3$, essentially the same as the displacement produced by putting on the weights if the one observation referred to, be rejected. The experiments do not establish any permanent displacement on the average produced by adding the weights.

Nine experiments on the same day were made by loading a rear tube-stand in three different positions with 90 pounds, and observing the displacement of the microscope over it on adding and on removing the weights. The mean displacement when weight was added was $+4^{\mu}.1$, and $-0^{\mu}.1$ when weight was removed; but as the microscopes occupied but three positions, the results are not sufficient to justify any conclusions from them.

Similar experiments were made on the Olney Base, the loads being 90 pounds. They were made on but two days. For the front microscope the loading was repeated (the stands remaining in one position each day) four times on the first day, and six times on the second day. They, therefore, show mainly the results of repeating experiments on the same ground. On loading the stand the displacements varied between $+4^{\mu}.5$ and $-3^{\mu}.3$, giving $+0^{\mu}.2$ in mean; on unloading, the displacements varied between $+4^{\mu}.3$ and $-4^{\mu}.2$, giving $0^{\mu}.0$ in the mean.

As the result of the experiments to determine whether a microscope was permanently displaced, either by the removal of the front end of the tube weighing 90 pounds, and replacing it by the rear end having the same weight, or by loading the tube-stand by weights ranging from 50 to 150 pounds, and then removing them, it may be said that no permanent displacement of the microscope is established in either case, and that if such displacement exist it must be very small in comparison with other errors of measurement.

§ **22.** It has already been stated that, to prevent disturbance of the microscope by the observer, he stood at the middle of a plank 8 feet long, shown in Plate XIII, supported at the ends, which were thus about 2½ feet from the microscope foot-plates. At the Olney base-line the experiment was made on 7 days of having a man, in addition to the observer, stand first on the rear and then on the front end of the platform. When the man stepped on the rear end of the front platform, the motion of the microscope varied between $-11^{\mu}.4$ and $+1^{\mu}.9$, being $-1^{\mu}.7$ in mean; when he stepped from the rear to the front end of the front platform, the motions of the microscope varied between $+9^{\mu}.2$ and $+1^{\mu}.7$, being in mean $+6^{\mu}.4$. Experiments on 6 days, in which the man shifted from front to rear end of rear platform, gave motions varying between $-6^{\mu}.2$ and $+1^{\mu}.4$, or in mean $-4^{\mu}.3$. These displacements were the temporary displacements; the permanent ones, as in the case of the tube-carriers, were doubtless much smaller; but the work indicates the necessity for a platform to support the observer.

§ **23.** From the results of these experiments on the stability of the microscopes, it may finally be concluded that while the microscopes may be displaced by small quantities by the motions of the tube-carriers, the average of such displacements must be much less for the tube-carriers, when at their usual distance of 12 inches from the foot-plates, than in the experiments when the distance was 6 inches, and the average displacement for the base on which it was greatest was $11^{\mu}.0$; that these displacements almost entirely disappear when the pressure is removed, the mean permanent displacement for the two bases being but $-1^{\mu}.5$ for a man 6 inches from a foot-plate, a quantity too small to be accurately determined by the experiments made, and which would be still smaller for the distance of 12 inches preserved in measurement. This elimination of permanent displacement might have been anticipated, since the displacement would naturally be toward the man causing it, and the permanent displacement, if any, should be in the same direction. But in a long series there is no reason for expecting a greater permanent displacement from approach to the front foot-plate than to the rear, and, since the number of approaches to each is the same in measuring, elimination of the effect should be expected. It is also concluded that the movement of the tube from microscope to microscope causes in the average no permanent displacement in the microscopes, and that the observer standing on his platform does not permanently affect the microscope by measurable quantities. Slight permanent disturbances of the microscopes undoubtedly occur, but it seems safe to assume that they may be considered as accidental errors, which will show their full effect in remeasurements of the different sections of each base.

As the Chicago Base like the Olney Base was on prairie soil, underlaid within 6 or 12 inches by clay, the conclusions as to the stability of microscopes on the Olney Base can be applied to it. On the Chicago Base, however, the clay was softer, and there were some periods of rain which made the clay quite soft.

§ **24.** 4. *Errors in cut-offs, that is in referring ends of tube to marks on ground.*—The method of making cut-offs has already been described in Chapter VIII, §§ 9, 13. After the microscope has been pointed at the end of a tube, the tube is removed and a horizontal graduated scale, described in Chapter VIII, § 9, and shown in Plate X, Fig. 4, is put in its place, this scale being fixed on top of an axis which can be made vertical, the lower end of this axis having a conical socket which rests on a hemisphere of steel fixed in the ground-plate. The steel axis is made closely vertical with a level of which one division $= 1''.6$, and the small deviations from verticality are read. As the greatest distance of the graduated scale from the ground plate does not exceed $1^{m}.785$, the error in fixing the position of the scale with reference to the ground-mark from this cause cannot exceed $0^{mm}.1$, and as it is as likely to be positive as negative, its cumulated effect in a section or in the base is insignificant. The longitudinal graduation-line of the scale is brought into the direction of the base by the longitudinal microscope-wire. The scale has been compared with a standard and is found to be accurately graduated, so that its errors can be neglected. Its divisions are millimeters at 38° F. In fact, the zero-mark near the axis of revolution was habitually pointed at with scale pointing to rear and front end of section, so that graduation-errors did not usually enter. The error in determining the horizontal distance in the direction of the base between the 4^{m} mark on the end of the tube and the cut-off sphere in the ground-plate being insignificant, the question arises whether the cut-off sphere itself remained immovable in the interval of time between closing work in the evening and beginning it again in the morning, or sometimes for longer intervals.

The cut-off sphere is held in the cut-off plate by stout abutting-screws which give it rectangular horizontal motions. It may be considered as fixed with reference to the cut-off plate. This is a circular disk of iron weighing about 9 kilograms. In using it, three broad-headed wooden stakes were driven about three decimeters into the ground, leaving their heads projecting slightly, and it was placed on the heads of these stakes and snugly confined by nails driven into them.

As three such cut-off plates were placed on stopping work at night (except in a part of the first measurement of the Chicago Base when but two were used) and as their distances apart were again measured on recommencing the work, an idea of their stability can be formed from the results. It has already been stated that in reduction of the base-measurement the assumption is made that the center of gravity of three cut-off spheres does not move and that the measurement is referred to the front cut-off sphere of the three. The motions that we detect, then, are those of this front cut-off sphere with reference to the center of gravity of three. If, as a part of their motion, all three move by equal amounts in the same direction, this part of their motion is undetected.

§ **25.** The following table gives for each cut-off on each measurement of the Chicago, Sandusky, and Olney Bases, the date, the number of the tube at which the cut-off was made, the time-interval between stopping and starting, and the change in the distance between the first and second and the second and third cut-off plates. When the cut-off plates separate from each other the plus sign is given to the change.

Relative movements of cut-off plates.

TWO CUT-OFF PLATES: CHICAGO BASE, FIRST MEASUREMENT.

Dates.	Nos. of tubes.	Intervals of time between cut-off readings.	Relative movement of cut-off plates. 1–2	2–3
1877.		*h.*	*mm.*	*mm.*
June 13–14	78	16	−0.0479	
14–15	139	15	+0.1150	
15–18	210	63	+0.0740	
18–20	228	43	−0.2534	
21–22	280	20	+0.1136	
22–23	367	14	−0.2766	
23–27	458	87	+0.0078	
27–28	513	16	+0.0879	
28–28	513	6	+0.0305	
28–29	536	15	+0.0394	
29–30	636	16	+0.1414	
30–July 2	636	50	+0.0047	
July 2–3	712	15	+0.0885	
3–5	772	42	+0.1242	
5–6	843	18	+0.1687	
6–7	928	15	+0.0473	
7–10	966	68	−0.0117	
10–11	1066	16	−0.0455	
11–12	1154	16	+0.0552	
12–13	1240	16	+0.0528	
23–24	1877	18	+0.1464	

TWO CUT-OFF PLATES: CHICAGO BASE, SECOND MEASUREMENT.

Aug. 22–24	1761	40	−0.1271	

THREE CUT-OFF PLATES: CHICAGO BASE, FIRST MEASUREMENT.

July 13–14	1339–40	16	+0.2417	+0.1816
14–16	1377– 8	45	+0.2133	−0.1203
16–17	1425– 6	17	−0.0701	+0.1766
17–18	1525– 6	17	−0.0981	+0.1316
18–19	1591– 2	17	+0.1018	+0.0981
19–20	1678– 9	19	−0.0106	+0.1792
20–21	1751– 2	16	−0.1248	−0.0026
21–23	1787– 8	45	+0.1799	−0.2573

Relative movements of cut-off plates—Continued.

THREE CUT-OFF PLATES: CHICAGO BASE, SECOND MEASUREMENT.

Dates.	Nos. of tubes.	Intervals of time between cut-off readings.	Relative movement of cut-off plates. 1–2	2–3
1878.		*h.*	*mm.*	*mm.*
July 24–25	42– 3	17	−0.1108	+0.0233
27–28	199–200	17	+0.0700	+0.1318
28–30	233– 4	45	−0.0051	+0.0720
30–31	330– 1	17	−0.3266	+0.1723
31–Aug. 2	441– 2	40	+0.0574	+0.1738
Aug. 2– 3	524– 5	16	−0.1163	+0.1234
3– 4	647– 8	16	−0.1018	−0.0849
4– 6	700– 1	45	−0.1963	+0.0278
6– 7	721– 2	22	+0.0080	+0.2410
8– 9	952– 3	16	−0.0788	+0.1331
9–10	1042– 3	16	+0.0465	+0.1010
10–13	1160– 1	64	−0.0760	+0.6659
13–15	1216– 7	41	−0.2874	+0.0981
15–16	1237– 8	18	−0.0281	−0.0561
16–17	1341– 2	17	−0.0078	+0.0859
17–18	1379– 80	20	+0.0904	+0.0296
18–20	1423– 4	42	−0.0103	+0.3944
20–21	1546– 7	16	+0.1201	−0.1194
21–22	1653– 4	14	−0.1867	+0.1388
22–22	1720– 1	1½	+0.0792	−0.0324
24–25	1806– 7	18	−0.0319	+0.0229

THREE CUT-OFF PLATES: SANDUSKY BASE, FIRST MEASUREMENT.

Aug. 6– 9	42– 3	62	+0.0989	+0.0792
9–12	95– 6	63	+0.0071	−0.0154
12–14	173– 4	38	+0.0954	−0.1875
14–15	242– 3	17	−0.1190	+0.0414
15–16	323– 4	15	+0.0407	+0.1430
16–19	406– 7	65	+0.0720	+0.0355
19–20	489–90	15	−0.0065	+0.0383
20–22	544– 5	40	+0.0476	+0.0681
22–23	647– 8	16	−0.1982	+0.1995
27–28	857– 8	16	+0.0464	+0.0049
28–29	934– 5	17	−0.0705	−0.1107
Sept. 2– 3	1041– 2	17	−0.1229	+0.1557
3– 4	1141– 2	16	−0.1901	+0.1856
4– 6	1195– 6	45	+0.1789	−0.0187
7– 9	1353– 4	46	+0.0906	−0.1009
9–16	1463– 4	161	−0.4603	+0.3222

THREE CUT-OFF PLATES: SANDUSKY BASE, SECOND MEASUREMENT.

Sept. 17–18	93–4	16	+0.0610	−0.1096
18–23	194–5	112	−0.1736	+0.0093
24–25	332–3	16	−0.0462	−0.0141
25–26	484–5	15	−0.2430	+0.1071
30–Oct. 1	640–1	16	+0.0885	+0.0159
Oct. 1– 2	741–2	17	−0.0605	+0.3173
2– 3	841–2	16	+0.1729	−0.1593
3– 4	966–7	16	+0.0295	−0.1290
7– 8	1157–8	15	−0.1112	+0.1528
8–10	1263–4	40	−0.0512	+0.1416
10–11	1352–3	15	−0.0367	+0.0055
11–12	1424–5	20	+0.1434	+0.1112

Relative movements of cut-off plates—Continued.

THREE CUT-OFF PLATES: OLNEY BASE, FIRST MEASUREMENT.

Dates.	Nos. of tubes.	Intervals of time between cut-off readings.	Relative movements of cut-off plates. 1–2	2–3
1879.		*h.*	*mm.*	*mm.*
July 28–29	92– 3	14	−0.0017	+0.0591
29–30	143– 4	17	−0.0846	+0.0578
30–31	243– 4	14	+0.1192	−0.1431
31–Aug. 1	272– 3	22	−0.0619	−0.0770
Aug. 1– 4	362– 3	62	+0.1547	−0.2592
4– 5	463– 4	15	−0.0924	+0.1534
5– 6	547– 8	16	+0.0333	+0.0434
6– 6	593– 4	2½	−0.0334	+0.0039
6– 7	648– 9	15	+0.0478	+0.0786
7– 8	718– 9	15	−0.0065	+0.0100
8–11	821– 2	64	+0.2043	−0.1354
11–12	939–40	15	−0.0340	+0.0827
12–13	994– 5	23	−0.1215	+0.0306
13–14	994– 5	21	−0.0148	−0.0085
13–14	1009–10	19	−0.1984	+0.1111
14–16	1079–80	42	+0.1029	−0.2572
16–18	1079–80	46	+0.1188	−0.1679
18–19	1175– 6	16	−0.0919	+0.0711
19–20	1297– 8	16	+0.0387	−0.0806
20–21	1398– 9	16	−0.0161	+0.0534
21–22	1519–20	16	+0.0545	+0.0382

THREE CUT-OFF PLATES: OLNEY BASE, SECOND MEASUREMENT.

1877.				
Aug. 27–28	70–1	16	+0.0786	+0.0269
28–29	207–8	16	+0.0273	−0.0400
29–Sept. 1	312–3	64	+0.2170	−0.1407
Sept. 1– 2	424–5	16	−0.0300	−0.0605
2– 3	546–7	16	−0.0612	+0.0551
3– 4	641–2	17	+0.1150	−0.0490
4– 5	756–7	16	+0.0442	−0.0332
5– 8	872–3	64	−0.1507	+0.1628
8– 9	1040–1	15	−0.0847	+0.0325
9–10	1204–5	15	+0.0557	+0.0098
10–11	1371–2	15	+0.1019	−0.0433
11–12	1523–4	16	−0.0675	−0.0037

These tables, giving changes in interval between cut-off plates during the suspension of work at night or during storms, give the data for an estimate of the probable error in the assumption that the center of position of the two or three cut-off plates remained unchanged during the suspension.

§ **26.** If Δ_1 represent the change between the first and second cut-off plates, and Δ_2 that between the second and third; and if δx_1, δx_2, δx_3, represent the change in the position of the separate plates taken as positive when they move in the direction of measurement, we shall have

$$\Delta_1 = \delta x_2 - \delta x_1 \qquad \Delta_2 = \delta x_3 - \delta x_2$$

and squaring and summing for the base, including both measurement and remeasurement, with three cut-offs,

$$[\Delta_1^2] + [\Delta_2^2] = [\delta x_1^2 + 2\delta x_2^2 + \delta x_3^2 - 2\delta x_2 \delta x_1 - 2\delta x_3 \delta x_2]$$

Now, as we do not know the laws which control the small disturbances, δx, of the cut-off plates, it must be assumed that they follow the ordinary law of error. The products of the displacements in the second member of the equation just written will then disappear, leaving

$$[\Delta_1^2] + [\Delta_2^2] = [\delta x_1^2] + 2[\delta x_2^2] + [\delta x_3^2]$$

But there is no reason for supposing the sum of the squares of the displacements greater for one cut-off plate than for another. Taking them to be equal to that for the first cut off plate,

$$4[\delta x_1^2]=[\Delta_1^2]+[\Delta_2^2]$$

If there are n cut-offs the square root of the mean square of displacement will be

$$\sqrt{\frac{[\Delta_1^2]+[\Delta_2^2]}{4n}}$$

and the probable displacement of any cut-off plate,

$$\delta x=\pm 0.6745\sqrt{\frac{[\Delta_1^2]+[\Delta_2^2]}{4n}}$$

For the center of position of three cut-off plates the probable displacement is then

$$\delta x_0=\pm 0.6745\sqrt{\frac{[\Delta_1^2]+[\Delta_2^2]}{12n}}$$

or, for two cut-off plates,

$$\delta x_0=\pm 0.6745\sqrt{\frac{[\Delta_1^2]+[\Delta_2^2]}{8n}}$$

These probable displacements include the probable errors of the two measurements of the intervals between the cut-off plates, and hence overestimate the disturbance. But as the errors of measurement are small in comparison with the displacements, it is not necessary to specially consider them. The probable error of a measurement scarcely exceeds 5^{μ}, and this would affect but very slightly the probable value of the displacement. Finding now the values of δx or the probable movement of a single cut-off plate, during the suspension of work, habitually for 15 hours or more, there results

For Chicago Base, $\delta x=\pm 84^{\mu}$
For Sandusky Base, $\delta x=\pm 61^{\mu}$
For Olney Base, $\delta x=\pm 42^{\mu}$

Deriving the values of the δx_0, the probable motion of the center of position of two or three cut-off plates, they are found to be

Chicago Base, 1st measurement, two cut-off plates, $\delta x_0=\pm 60^{\mu}$; number of stops, 21.
Chicago Base, both measurements, three cut-off plates, $\delta x_0=\pm 49^{\mu}$; numbers of stops, 8 and 21.
Sandusky Base, both measurements, three cut-off plates, $\delta x_0=\pm 35^{\mu}$; numbers of stops, 16 and 12.
Olney Base, both measurements, three cut-off plates, $\delta x_0=\pm 24^{\mu}$; numbers of stops, 21 and 12.

The probable error of the cut-off correction, that is, of the relative movement of the last microscope, and the center of position will differ from these only by quantities which may be neglected. Since the observations only gave relative movements of the cut-off plates, they did not detect a common movement of all the plates in the same direction, which without doubt sometimes occurred. Hence the above probable displacements are somewhat too small.

Multiplying each of the δx_0 by the square root of the number of stops, values varying between $\pm 0^{mm}.27$ and $\pm 0^{mm}.08$ are found for the probable errors introduced into a measurement of a base by the instability of the cut-off plates. They are so small that even when increased somewhat on account of the fact that the method of making the cut-off measurements does not detect a common movement of all the cut off plates in the same direction, it will yet be sufficient to assume that these errors develop their full effect in the differences of length of the sections of the base given by the measurement and remeasurement, and therefore do not need a separate estimate.

§ **27.** The following table gives the numbers of times the signs of the change in interval between the first and second, and second and third cut-off plates and microscopes over them were positive and negative for each base and each measurement, a positive sign indicating that the plates or microscopes moved apart.

Number of + and — signs in relative movements of cut-off plates and microscopes.

TWO CUT-OFF PLATES: CHICAGO BASE.

Pair of cut-off plates and microscopes.	Cut-offs.			Microscopes.		
	No. +	No. −	Whole No.	No. +	No. −	Whole No.
1–2	16	6	22	11	7	18

THREE CUT-OFF PLATES: CHICAGO BASE, FIRST MEASUREMENT.

1–2	4	4	8	2	5	7
2–3	5	3	8	5	2	7
Sums	9	7	16	7	7	14

THREE CUT-OFF PLATES: CHICAGO BASE, SECOND MEASUREMENT.

1–2	7	14	21	9	11	20
2–3	17	4	21	11	9	20
Sums	24	18	42	20	20	40

THREE CUT-OFF PLATES: SANDUSKY BASE, FIRST MEASUREMENT.

1–2	9	7	16	5	7	12
2–3	11	5	16	9	3	12
Sums	20	12	32	14	10	24

THREE CUT-OFF PLATES: SANDUSKY BASE, SECOND MEASUREMENT.

1–2	5	7	12	5	6	11
2–3	8	4	12	6	4	10
Sums	13	11	24	11	10	21

THREE CUT-OFF PLATES: OLNEY BASE, FIRST MEASUREMENT.

1–2	9	12	21	9	7	16
2–3	13	8	21	9	7	16
Sums	22	20	42	18	14	32

THREE CUT-OFF PLATES: OLNEY BASE, SECOND MEASUREMENT.

1–2	7	5	12	6	4	10
2–3	5	7	12	9	1	10
Sums	12	12	24	15	5	20

This table shows on the whole a preponderance of cases where there was separation of both cut-off plates and microscopes.

§ **28.** In the measurement of the bases the microscopes usually remained in position over the cut-off plates, and as the measurement of the distances between the cut-off plates gave also the distances between the microscopes, an examination as to their relative motion during suspension of work has also been made. There is a source of error entering this work, however, which does not affect the other. Different parts of the microscope-tripod may have different temperatures at night when the work stops and in the morning when the work begins, due to the changed position of the sun or other causes. This might cause slight motions in the microscopes even when the tripod-feet did not change their position. But this would be partially eliminated from the fact that the sun would change position with reference to both microscopes.

The result of the examination is that the cut-off plates have a somewhat greater stability than the microscopes. Since, however, both cut-off plates and microscopes are most unstable during and after rains, and since at that time the greater weight of the microscope-tripod would tend to make its changes of position the greater, it is probable that in good soil and dry weather the microscopes and the cut-off plates have about the same stability. As a rule, when the change in the interval between cut-off plates was large, the change in the interval between the microscopes was large and usually in the same direction. Thus on the Chicago Base there were sixteen cut-offs in which the change of interval of cut-off plates exceeded $0^{mm}.15$ out of a total of sixty-three. In nine of these the microscope-interval changed by more than $0^{mm}.1$ in the same direction, and in three cases it changed by more than $0^{mm}.1$ in the opposite direction. On the Sandusky Base the interval between cut-off plates changed in thirteen cases out of a total of forty-seven by more than $0^{mm}.15$. In ten of these cases the microscope-interval changed by more then $0^{mm}.1$ in the same direction, and in none of the cases did it change in the opposite direction by $0^{mm}.1$. On the Olney Base the interval between cut-off plates changed by more than $0^{mm}.15$ in four cases out of a total of fifty-one cases. In two of these the microscope-interval changed by more than $0^{mm}.1$ in the same direction, and in none of these cases did it change in the opposite direction by $0^{mm}.1$.

Besides the cut-offs, two or three in number, made when the work was to be suspended for many hours, there was a cut-off made on a single plate on stopping for about 1.9 hours on the average at noon. The measurements made on recommencing the work gave the change in relative position of microscope and cut-off plate from all causes. That change was of course due only in part to change of position of the cut-off plate. The following are the probable changes in position of cut-off plate or microscope for the different bases, derived on the assumption that microscopes and cut-off plates were equally stable from the observed changes in relative position.

Chicago Base, $\pm 16^{\mu}$
Sandusky Base, $\pm 13^{\mu}$
Olney Base, $\pm 10^{\mu}$

The number of cut-offs of this kind for both measurements for all bases varied between twenty-one and twelve, so that the maximum probable error introduced into the measurement of a base from this cause would not exceed $16^{\mu}\sqrt{21} = 0^{mm}.07$. This error is so small that it may be included in the discrepancy of the two measures of each section of a base.

§ **29.** From the discussion of errors of alignment, of inclination, of stability of microscopes and of stability of cut-off plates, §§ 16–28, it is concluded that these errors are either so small that their effect on the whole base can be neglected, or that their effect is sufficiently shown in the discrepancies between the two measurements of each section. The errors which will enter the final value of the Chicago Base will be that in the adopted length of the steel bar S_1 at a chosen temperature, and those due to the determination of the number of times this length of bar is contained in the length of the base, that is, to errors of measurement.

If the full effect of the errors of measurement was shown by the discrepancies between the two measures of each section, a pretty good idea of the resulting probable error of measurement in the mean of two measurements of the base could be obtained in the following way. From § 13 the following table is derived:

Section.	No. of tubes.	Differences of measures.
		mm.
I	227.25	−1.3
II	230.25	+2.5
III	234.50	+2.3
IV	232.00	+0.7
V	231.00	+1.5
VI	225.00	+1.1
VII	300.50	+1.3
VIII	196.80	−0.2

The positive sign prefixed to a difference indicates that the first measurement was the greater. There are but two negative signs to six positive. If the positive signs all corresponded to cases where the temperature of the first measurement differed in the same direction from that of the second, the positive signs might be attributed to an error in the value adopted for the expansions of Z_1 and S_1; but of the six positive differences three, as will be seen by reference to the temperature corrections in § 12, are when the first measurement was hotter and three when it was cooler. Since the first measure of the base was from west to east, the zinc bar was on the north side; in the second measurement it was on the south side. If the position of the sun affected the deduced length of S_1 by slightly heating the nearer bar, the second measurement should have given the greater length. Call d the difference between two measures of the same section and suppose the errors of the different sections independent of each other, then there results for the probable error in the mean of the two measures of the base

$$p.\ e.=0.6745\sqrt{\frac{[d^2]}{4}}=\pm 1^{\mathrm{mm}}.46=\frac{1}{5145800}\text{ part of the base.}$$

Besides the errors of measurement already mentioned, this probable error includes the other accidental errors of measurement, and a part of the errors due to erroneous corrections for temperature. But the discrepancies between the measures of a section do not fully develop the errors arising from temperature.

§ **30.** Neglecting for the present the consideration of the errors arising from the fact that the observed Z_1-S_1 is not always the true metallic temperature of S_1, and leaving this error to be considered later, the errors now to be considered are those in the part of the base expressed by § 6.

$$(1)\qquad n(S_1)_{Z_1=S_1}+0.6522\,[Z_1-S_1]+0.000005106[(Z_1-S_1)^2]$$

and include the error of the standard S_1 at a chosen temperature. The quantity (1) may, so far as probable errors are concerned, be taken as the whole base. Now the sum of the first two terms is identical with

$$(2)\qquad n(S_1)_{Z_1-S_1=m},\text{ or } n(S_1)_m$$

where m is the mean value of Z_1-S_1 for the two measures of the base. Hence the probable error in the sum of the first two terms is the same as that of (2). But the probable error of (2) arises only from the probable error in the length of the standard S_1 for the temperature $Z_1-S_1=m$, and from the error in m. Neglecting the latter for the present, since m for the Chicago Base is $+570^{\mu}$ (§ 12), corresponding to 75°.27 F., and since $n=1877$, there results for the probable error due to the first two terms of (1) from the value of the probable error in S_1, given in Chapter IX, § 63,

$$\pm 1877(3^{\mu}.36)=\pm 6^{\mathrm{mm}}.31$$

The probable error in the third term of (1) cannot be assigned with any precision, since the coefficient of the second power of the temperature in the value for the length of S_1 at any temperature is not precise. A probable error in this coefficient equal to one-fourth of its value seems large enough to attribute to it. Since for the two measures of the Chicago Base the mean value of the third term of (1) (see table in § 12) is $3^{\mathrm{mm}}.82$, this process would assign to the third term a probable error of

$$\pm 0^{\mathrm{mm}}.95$$

Combining it with the $\pm 6^{\mathrm{mm}}.31$ found for the sum of the first two terms of (1), there results as the probable error in the base due to uncertainties in the length of S_1,

$$\pm 6^{\mathrm{mm}}.38$$

§ **31.** It remains to consider the error introduced into the base from the fact that the observed Z_1-S_1 is not always the true metallic temperature of S_1. This subject has been discussed in Chapter IX, §§ 25–53, and it is there seen that owing to differences of temperature of Z_1-S_1, to slight possible bendings of Z_1, but mainly it is believed to the fact that Z_1 does not at the same temperature always have the same length, it is necessary to apply an average correction of $4^{\mu}.6$ (Chapter

IX, § 49) to the length of S_1 computed from the observed Z_1-S_1, during the whole time occupied in the measurement of a base. Whatever error there is in this correction enters the final value of the base. The considerations stated in Chapter IX, § 53, led to the assignment of $\pm 1^{\mu}.5$ as the probable error in this value. For the 1877 (S_1) on the Chicago Base, this gives a probable error of

$$\pm 2^{\mathrm{mm}}.82$$

§ **32.** In § 14 the probable error in the adopted mean height of tube during the measurement of the base above mean tide is given as ± 1 ft. This introduces into the base the probable error of

$$\pm 0^{\mathrm{mm}}.36$$

§ **33.** The probable errors already derived are

	mm.
From discrepancies of measurement, § 29	±1.46
From error of value of S_1, § 30	±6.38
From error in observed $Z_1 - S_1$, § 31	±2.82
From error in mean elevation of tube, § 32	±0.36

The first of these includes in part the errors of the third species. Neglecting this fact and combining all, there results,

$$\text{Probable error in Chicago Base} = \pm 7^{\mathrm{mm}}.4 = \frac{1}{1052200} \text{ part of base.}$$

Hence from § 14 there results for the Chicago Base as the final value at sea-level

CHICAGO BASE = 24648.635 ± 0.023 English feet.

§ **34.** The length of Chicago Base in terms of the metre, R1876, deduced from the length of S_1, depending on intercomparisons of R1876, S_1 and S_2 alone, is, § 14,

$$7508\ (R1876, \text{ at } 60^{\circ}.29 \text{ F.}) + 2^{\mathrm{m}}.96016$$

Its probable error may be derived by observing that this length was obtained from the length of the base in terms of S_1, viz, § 14,

$$1877\ (S_1{}^{0}) + 3^{\mathrm{m}}.03037$$

by substituting for S_1 the value

$$S_1 = 4\ R1876 - 37^{\mu}.01 + 1^{\mu}.2744\ (t-59)$$

The square of the probable error of the length of S_1 just given is, Chapter IX, § 66, the unit being 1^{μ},

$$0.1480 + 0.00048\ (t-59)^2 + 0.00023\ (t-42.75)^2$$

which for $t = 75^{\circ}.27$ F., the mean temperature of Chicago Base, becomes 0.5162. Therefore, by reference to §§ 29–32, the square of the probable error of the length of the base as expressed above, in terms of R1876, is in millimeters,

$$\begin{aligned} &\overline{1877}^2\,(0.000718)^2 \text{ from error in relative length of } S_1, \text{ and } 4\ R1876 \\ &+(0.95)^2 \text{ from error in correction depending on } Z_1 - S_1) \\ &+(1.46)^2 \text{ from residuals of two measures of sections} \\ &+(2.82)^2 \text{ from errors in metallic temperatures} \\ &+(0.36)^2 \text{ from error in elevation above sea.} \end{aligned}$$

Adding and extracting the square root there results $\pm 3^{\mathrm{mm}}.60$, which corresponds to the $\frac{1}{2089000}$ part of the length of the base. The above length of the base may then be written thus:

$$\text{Chicago Base at sea-level} = 7508\ (R1876 \text{ at } 60^{\circ}.29 \text{ F.}) + 2960^{\mathrm{mm}}.16 \pm 3^{\mathrm{mm}}.60$$

When the length of R1876 in metres becomes known, and is substituted in this expression, the probable error of the resulting length of the base will be the square root of the sum of the squares of $3^{\mathrm{mm}}.60$, and 7,508 times the probable error of the length of R1876 at the mean temperature (75°.27 F.) of the base.

COMPARISON OF THE MEASURED LENGTH OF CHICAGO BASE WITH ITS LENGTH COMPUTED FROM FOND DU LAC BASE.

§ **35**. By means of the adjusted values of the angles in the triangulation connecting Chicago and Fond du Lac Bases, given in Chapters XV, C, and XVI, C, the logarithm of the ratio, Chicago Base divided by Fond du Lac Base is found to be 0.0052091. Adding to this the logarithm of Fond du Lac Base expressed in feet, Chapter V, § 10, viz, 4.3865919, there results for the logarithm of Chicago Base computed from Fond du Lac Base, 4.3918010. The logarithm of the measured length of Chicago Base expressed in feet is, § 33, 4.3917929. The discrepancy between these two logarithms is 81 units in the seventh place, a quantity corresponding to 0.46 feet=14.0 centimeters or $\frac{1}{53616}$ part of Chicago Base.

The probable error of this discrepancy, due to uncertainties in the angles of the triangulation, will vary with the triangles used in computing the ratio of the two bases. Using the triangles of the principal chain connecting the bases, given in Chapters XV, D, XVI, D, and XVII, D, and the method given in Chapter IV, § 14, the probable error of the above logarithmic discrepancy dependent on probable errors of observed angles alone is ± 70.03 units in the seventh decimal place. This corresponds to ± 0.398 feet$= \pm 12.1$ centimeters, and is about one-eighth less than the actual discrepancy. Since in computing this probable error, probable errors of observed angles have been used, it is too great. Taking a mean of the approximate ratios of the probable errors of observed to probable errors of adjusted angles given in Chapters XV, C, § 7, and XVI, C, § 11, and multiplying the above probable error by that mean, there results ± 0.275 feet, as the more approximate probable error of the above discrepancy, or of the length of Chicago Base computed from the Fond du Lac Base. The actual discrepancy is not greater, therefore, than may be attributed to errors in the angles of the principal chain connecting the two bases. This chain embraces twenty-nine triangles, and measured along its axis is about 150 miles in length.

CHAPTER XI.

SANDUSKY BASE.

§ **1.** This base-line is situated on a sand-spit which separates Sandusky Bay from Lake Erie. The end of the spit is called Cedar Point. The width of the spit varies from 50 to 500 metres, and as it is concave towards the lake it was necessary to break the length of the base into three nearly equal parts by angles. Looking from the west end of the base toward the east end in both cases the deflection was toward the left hand or north. The angles at these points were measured in the same way as the other primary angles of the triangulation and their values are given in § 8.

The base-line followed the shore closely; the surface was everywhere sand, but in places it was underlaid, at the depth of 3 or 4 feet, by a thin layer of peat or vegetable material. The base was divided into six approximately equal sections by five section marking-stones intermediate to the end-stones. The second and fourth of these stones were at the angles in the base. The marking-stones at the ends of the base and at the ends of the sections were similar to those on the Chicago Base already described in Chapter X, § 1.

The base-line was measured, with the Repsold apparatus, by a party in charge of Assistant Engineer E. S. Wheeler. The measurement was begun on August 5, 1878. The first measurement was completed on September 16, and the remeasurement was completed October 12, 1878. The average number of tubes of four metres each, measured on each day of the first measurement, was 88. On the remeasurement, the average number per day was 100, the maximum being 152, these numbers including the remeasurement of the three cut-off tubes.

In the first measurement of the Chicago Base, for twenty-two out of thirty cut-offs, but two cut-off plates were set on stopping work at night; on the Sandusky Base three cut-off plates were used in both measurements. At the Chicago Base, on reaching a section-stone, cut-off plates were set, and on recommencing the measurement these cut-off plates were measured from, so that the fractional part of the tube measured to reach the section-stone entered the preceding and following sections with opposite signs and was eliminated in the result. On the Sandusky Base this method was impracticable at the angular points of the base, and several fractional parts of tubes enter the base. But as their number is less than the number of sections, their probable errors do not need special consideration. On the Chicago Base the section-stones were placed on the line by setting them nearly in position and then reading the angles at each between the ends of the base-line to an accuracy of a second or two, computing their deviation from the line and then moving them upon it, the tube-telescope being depended on to give the proper alignment at intermediate points. On the Sandusky Base, intermediate points about 300 metres apart were established by the method used for the section-stones. In other respects the methods of measurement and of reduction were the same for the Sandusky as for the Chicago base-line, given in Chapter X.

§ **2.** The west end of the base is approximately in latitude 41° 29′ north, longitude 82° 41′ west of Greenwich, the azimuth of the east end of the base from the west end being 319° 37′ west of south. The length of the first straight portion of the base as measured is approximately 2012.4 metres; of the second, 2055.1 metres; of the third, 2161.5 metres; the angles between the first and second and second and third parts being respectively 177° 45′ 41″.9 and 177° 39′ 52″.2, the opening of the angles being toward the north. The base-line was nearly level, lying on a sand beach.

Referred to the top of the stone, about one metre under ground, which marks the west end of the base-line, the mean elevation of the tube during the measurement of the base above this stone was for the west part, $1^{m}.91$; for the middle part, $1^{m}.01$; for the east part, $1^{m}.03$. By leveling to the surface of the lake and from the gauge-reading at the same time at Cleveland, Ohio, the top of this stone was found to be $1^{m}.085$ above the mean level of Lake Erie between January 1, 1860, and December 30, 1875, inclusive. That mean level is (Chapter XXII, § 13), $572^{ft}.86$ or $174^{m}.605$ above mean tide at New York City. Hence the mean heights of the tubes during base-measurement above tide were for the west, middle, and east parts of the base, respectively, $177^{m}.60$, $176^{m}.70$, and $176^{m}.72$. A probable error of $\pm\ 0^{m}.25$ may be assigned to each of these values.

A line of levels was carried along the base-line by means of the known lengths and inclinations of the steel bar S_1 and by means of the measured heights of its ends above each permanent mark. This was checked by direct measurements of the height of tube above surface of water, which were made at seven different points along the line, the readings of the Cleveland gauge being used to reduce the surface of the water to a common plane. There were two instances in the first measurement of the base where the duplicate records of inclination of tube differed by 1°, namely, at the 120th tube, where the two records were 5° 23′ and 6° 23′, and at the 1162d tube where the records were 4° 36′ and 5° 36′. A determination of the relative elevations of the adjacent section-stones was made on the basis of each of the pair of discrepant records, and the two results so obtained were successively compared with the result obtained from the second measurement of the base which was free from discordant records, and thus it was practicable to decide which was the correct record in the first measurement. In addition to this, a line of spirit-levels was run over the first two sections, the results of which confirmed the decision in regard to the correct record in the case of the 120th tube inclination. A line of spirit-levels was also run over the 5th section, but the work was considered unreliable and was not made use of. Spirit-levels were not run over the remaining portions of the line. The sector level was not as stable as on the Chicago and Olney Bases. Its zero, which was determined every day of measurement, showed a slight steady change in the same direction. It was subsequently found that some of the felt which forms a wrapping to the tube had got between the bottom of the sector-level plate and its seat upon the tube, where it was compressed by the screws fastening the plate to its seat. Possibly this may have been the cause of the change of zero, which, however, only amounted to 2′ 5″ during the double measurement of the base.

§ **3.** In the following table the first column gives the number of the section of the base; the second shows by the numbers 1 and 2 whether the first measurement or the remeasurement in the opposite direction is given on that horizontal line; the third gives the entire number of times that the steel bar S_1^0 at the temperature of 60°.292 F. entered into the measurement of the section; the fourth gives the fractional parts of S_1 which entered the measurement when closing on the permanent mark at the end of a section, this interval being measured by means of the intermediate graduations on S_1; the fifth gives the sums of the parts of the base measured with the cut-off scale in closing on section-stones; (the graduations of this cut-off scale have been examined and found sufficiently accurate to introduce no error that need be considered); the sixth gives for each section the sums of the corrections to S_1^0 for metallic temperature on the assumption that

$$\frac{E_{S_1}}{E_{Z_1}-E_{S_1}}=0.6522$$

its value when $Z_1=S_1$; the seventh gives the sums of corrections to the length of each section on account of the inaccuracy of this last supposition; the eighth gives the sums of the corrections for the sections, on account of inclination of S_1; the ninth gives the sums of the small distances measured with the microscopes in each section; the tenth gives the sums of the movements of the front cut-off plate in each section; the eleventh gives the sums of the corrections for each section on account of the observed Z_1-S_1 not being the correct metallic temperature of S_1; the twelfth gives the sum of the different corrections for each section.

Results of measurement of Sandusky Base.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
Section.	Measurement.	Number of tubes.	Fractional tubes.	Cut-off scale measurements.	Temperature corrections to $(S_1)_{S_1=Z_1}$.	Corrections to S_1 for difference of e at $Z_1-S_1=n$ and $Z_1-S_1=0$.	Correction for inclination.	Sum of intervals between tubes measured with microscopes.	Sum of movements of front cut-off plates.	Corrections for regular residuals.	Sum of the corrections.
			T.	μ	μ	μ	μ	μ	μ	μ	μ
I....	1	243	+0.675	−29011.6	+133730.4	+ 944.3	−117704.7	+ 2474.0	+ 31.0	+1260.5	− 8276.1
	2	243	+0.700	+ 3000.8	− 19924.7	+ 54.1	−100196.0	+ 8674.2	+106.7	+ 281.4	+108003.5
II....	1	259	+0.300	+11004.3	+131750.7	+ 901.5	− 90315.7	+14424.4	+178.9	+1499.4	+ 69443.5
	2	258	+1.325	+51009.0	− 12290.5	+ 103.0	− 78096.4	+ 7818.4	+142.7	+ 604.2	− 30709.6
West part	1	502	+0.975	−18007.3	+265481.1	+1845.8	−208020.4	+16898.4	+209.9	+2759.9	+ 61167.4
	2	503	+0.025	+54009 8	− 32215.2	+ 157.1	−178292.4	+16492.6	+249.4	+ 885.6	−138713.1
III....	1	247	+0.425	+12005.2	+118214.9	+ 726.2	− 28197.2	+10994.0	+128.6	+1498.5	+115370.2
	2	247	+0.450	− 7001.7	+ 45812.9	+ 200.3	− 33575.4	+ 9505.5	−110.3	+ 913.1	+ 15744.4
IV....	1	265	+1.175	+42015.7	+ 91324.0	+ 445.2	− 26467.7	+10760.6	−147.0	+1299.9	+119230.7
	2	265	+1.200	−53019.6	+ 87331.0	+ 404.0	− 27518.8	+ 9370.5	+232.0	+ 545.9	+ 17345.0
Middle part	1	513	+0.600	+54020.9	+209538.9	+1171.4	− 54664.9	+21754.6	− 18.4	+2798.4	+234600.9
	2	513	+0.650	−60021.3	+133143.9	+ 604.3	− 61094.2	+18876.0	+121.7	+1459.0	+ 33089.4
V....	1	277	+0.750	−58025.2	+107851.5	+ 572.3	−138903.1	+14055.6	+166.0	+1360.8	− 72922.1
	2	277	+0.725	+ 8001.6	+ 68288.7	+ 324.3	− 69042.2	+14425.9	− 35.3	+1342.7	+ 23305.7
VI....	1	262	+0.500	+43016.9	+106751.8	+ 639.3	− 65790.1	+11733.8	+ 30.0	+1037.0	+ 97418.7
	2	262	+0.525	− 6001.0	+ 76299.3	+ 436.9	− 85785.7	+11911.5	− 91.0	+1181.9	− 2048.0
East part	1	540	+0.250	−15008.3	+214603.4	+1211.6	−204693.2	+25789.4	+196.0	+2397.8	+ 24496.7
	2	540	+0.250	+ 2000.6	+144588.1	+ 761.2	−154827.9	+26337.4	−126.3	+2524.6	+ 21257.7
Total..	1	1556	+0.825	+21005.3	+689623.4	+4228.8	−467378.5	+64442.4	+387.5	+7956.1	+320265.0
	2	1556	+0.925	− 4010.9	+245516.8	+1522.6	−394214.5	+61706.0	+244.8	+4869.2	− 84366.0

The mean value of Z_1-S_1 for the two measurements of the base was $+\overset{\mu}{460.5}$, corresponding to 72°.38 F.

§ **4.** In the following table, derived from the preceding one by expressing the two measurements of a section with the same fraction of a tube, the principal results are given, and also the differences between the two measurements of each section. To express the section of the base in terms of entire numbers of S_1^0 and equal fractions of it, the values of the fractions of a tube in metres are needed. The value of S_1 is given in Chapter IX, § 66, and from that its value at 60°.292, for which $Z_1-S_1=0$, may be computed. Transforming it into metres by Clarke's value of the metre (metre = 1.093623 yard) there results—

$$S_1 \text{ at } 60^\circ.292 \text{ F.} = 4^m.0009624$$

as an approximation more than sufficient for obtaining the values of the fractions of the tube in metres. It has already been stated that these fractions of tubes are the exact fractions which they purport to be within 10^μ, a quantity that is insignificant in comparison with the length of a section, and which enters a section but once.

Principal results of measurement of Sandusky Base.

Section.	Measurement.	Number of $(S_1)_{Z_1=S_1}$	Sum of corrections.	Mean.	Difference.
			μ	μ	μ
I....	1	243.7	−108300.2	+108151.8	− 296.7
	2	243.7	−108003.5		
II....	1	259.3	+ 69443.5	+ 69379.0	+ 129.0
	2	259.3	+ 69314.5		
West part	1	503.0	− 38856.7	− 38772.8	− 167.7
	2	503.0	− 38689.0		
III....	1	247.4	+215394.3	+215593.4	− 398.3
	2	247.4	+215792.6		
IV....	1	266.2	+ 19206.6	+ 18275.8	+1861.6
	2	266.2	+ 17345.0		
Middle part	1	513.6	+234600.9	+233869.2	+1463.3
	1	513.6	+233137.6		
V....	1	277.7	+127126.1	+125228.0	+3796.3
	2	277.7	+123329.8		
VI....	1	262.5	+ 97418.7	+ 97697.4	− 557.4
	2	262.5	+ 97976.1		
East part	1	540.2	+224544.8	+222925.4	+3238.9
	2	540.2	+221305.9		
Total base	1	1556.8	+420289.0	+418021.8	+4534.5
	2	1556.8	+415754.5		

§ **5.** From the preceding table there result, for the mean lengths as measured of the three parts of the Sandusky Base, the following values, in which $(S_1)_{Z_1=S_1}=S_1^0$ is the length of the steel bar S_1 when at the temperature for which $Z_1=S_1$ or 60°.292 F.

$$\begin{aligned} &&mm. && ft. \\ \text{West part} &=503.0\,(S_1)_{Z_1=S_1}-&38.77&=&6602.5699 \\ \text{Middle part} &=513.6\,(S_1)_{Z_1=S_1}+&233.87&=&6742.6067 \\ \text{East part} &=540.2\,(S_1)_{Z_1=S_1}+&222.93&=&7091.7393 \end{aligned}$$

the values in English feet being derived from the value $(S_1)_{Z_1=S_1}=13^{ft}.12663443$, given in Chapter IX, § 66.

Computing for Clarke's spheroid with the azimuth of the base and the mean heights of the tube for each of the three parts of the base given in § 2 the reduction to sea-level, it is found to be for west part $0^{ft}.1840$; for middle part $0^{ft}.1869$; for east part $0^{ft}.1966$; so that at sea level the parts of the base in terms of $(S_1)_{Z_1=S_1}$ and in feet are

$$\begin{aligned} &&mm. && ft. \\ \text{West part} &=503.0\,(S_1)_{Z_1=S_1}-&94.86&=&6602.3859 \\ \text{Middle part} &=513.6\,(S_1)_{Z_1=S_1}+&176.89&=&6742.4198 \\ \text{East part} &=540.2\,(S_1)_{Z_1=S_1}+&163.01&=&7091.5427 \end{aligned}$$

As stated in § 2, the adjusted angle between the west and middle parts of the base is 177° 45′ 41″.9, and between the middle and east parts of the base is 177° 39′ 52″.2, the opening of each angle being toward the north. Projecting the parts of the base after reduction to sea-level on the line joining the western and eastern ends of the base, also at sea-level, the projections, which will be given both in terms of $(S_1)_{Z_1=S_1}$ and in English feet, are

$$\begin{aligned} &&m. && ft. \\ \text{Projection of west part} &=503.0\,(S_1)_{Z_1=S_1}-&1.75242&=&6596.9477 \\ \text{Projection of middle part} &=513.6\,(S_1)_{Z_1=S_1}+&0.17450&=&6742.4120 \\ \text{Projection of east part} &=540.2\,(S_1)_{Z_1=S_1}-&1.50075&=&7086.0841 \end{aligned}$$

Calling the interval between the west and east ends of the base the Sandusky Base, by summing the projections just given, there results—

Sandusky Base at level of mean tide, New York $=1556.8 (S_1)_{Z_1=S_1} - 3^{m}.07867 = 20425^{ft}.4438$

Substituting for $(S_1)_{Z_1=S_1}$ its value in terms of $R1876$, derived from Chapter IX, § 66, namely $S_1 = 4\ R1876 - 58^{\mu}.40 + 1^{\mu}.326\ (t - 42.75\ \text{F.})$, since $Z_1 = S_1$ at $60^{\circ}.292$ F.,

Sandusky Base at level of mean tide at New York $=6227.2\ (R1876$ at $60^{\circ}.292$ F.$) - 3^{m}.13338$

This last value of the length of the basis, dependent on the adjusted expansion of $R1876$ and S_2. If the length in terms of $R1876$ of S_1 dependent alone on intercomparisons of $R1876$, S_2, and S_1, be used, viz:

$$S_1 = 4\ R1876 - 37^{\mu}.01 + 1^{\mu}.2744\ (t - 59)$$

(see Chapter IX, § 66), the resulting length of the base is

$$6227.2\ (R1876 \text{ at } 60^{\circ}.292 \text{ F.}) - 3^{m}.13372$$

§ **6.** The errors in projecting the parts of the base on the line joining the ends of the base and those in determining the difference of level of the base and of the sea are so small that they may be neglected. Indeed, the corrections from these causes are so small that we may take for the probable error in the final value of the base, the probable error before these corrections are made.

The probable error in the length of the Sandusky Base will be derived in a manner entirely similar to that used in obtaining the probable error in the Chicago Base which is given in Chapter X, §§ 29–32.

From the column of differences, d, in the table given in § 4, there results for the part of the probable error obtained from these differences,

$$0.6745\sqrt{\frac{[d^2]}{4}} = \pm 1^{mm}.45 \text{ or } \frac{1}{4293500} \text{ part of the base.}$$

Following the process in Chapter X, § 30, to find the probable error in the base arising from uncertainty in the length of S_1 (1) of that section becomes $n\ (S_1)_{Z_1-S_1=m} + 0.000005106\ [(Z_1 - S_1)^2]$. For the Sandusky Base, § 3, $m = +460^{\mu}.5$, corresponding to $72^{\circ}.38$ F. Substituting this in the expression for the probable error in the adopted length of S_1 at any temperature, given in Chapter IX, § 63, there results for its probable error at $72^{\circ}.38$ F., $\pm 2^{\mu}.96$. Since $n=1556.8$, there results for the probable error in the base arising from the first term of (1) above, $\pm 4^{mm}.61$. Taking the probable error in the coefficient of the second term at one-fourth of its value, as in Chapter X, § 30, there results as the probable error of the second term $\pm 0^{mm}.72$. Combining it with the $\pm 4^{mm}.61$ already found, there results

$$\pm 4^{mm}.67$$

as the probable error in the value of the base due to uncertainty in the length of S_1.

For the probable error in the adopted lengths of S_1, due to the observed $Z_1 - S_1$, not giving the true temperature of S_1, $\pm 1^{\mu}.5$ will be taken as was done, Chapter X, § 31, for the Chicago Base. Multiplying this by the number of tubes in the base, 1556, there results for this probable error,

$$\pm 2^{mm}.33$$

The probable error in the height of the base, § 2, may be taken as $\pm 0^{m}.25$. This gives a probable error in the reduction to the sea-level of

$$\pm 0^{mm}.24$$

Collecting the various probable errors, there results—

	mm
From discrepancies of measurement	± 1.45
From probable error in value of S_1	± 4.67
From errors in temperatures $Z_1 - S_1$	± 2.33
From errors in mean elevation of tubes	± 0.24

Combining, there results,

$$\text{Probable error in Sandusky Base} = \pm 5^{mm}.42 = \frac{1}{1148600} \text{ part of base.}$$

Hence from § 5 there results,

SANDUSKY BASE AT SEA-LEVEL$=20425^{ft}.4438\pm0^{ft}.0178$

§ **7.** By a process precisely like that followed in Chapter X, § 33, the probable error of Sandusky Base expressed in terms of the metre R1876, § 5, is found to be $\pm3^{mm}.03$ or $\frac{1}{2057000}$ part of the length of the base. Hence

$$\text{Sandusky Base at sea-level}=6227.2(R1876 \text{ at } 60^{\circ}.29 \text{ F.})-3133^{mm}.72\pm3^{mm}.03$$

When the length of R1876 in metres is substituted in this expression, the probable error of the resulting numerical value will be

$$\pm[(6227.2\ \varepsilon)^2+(3.03)^2]^{\frac{1}{2}}$$

in which ε = the probable error of the length of R1876 at the mean temperature ($72^{\circ}.38$ F.) of Sandusky Base.

§ **8.** The Sandusky Base containing two angles differing little from 180°, a new station called Check Base was erected, so as to give triangles of good shape when connected with these angular points and with the ends of the base-line. The angles of the lines joining these points and a few to other stations were read. These angles were adjusted by least squares according to the method given in Chapter XIII, with the conditions introduced needed to prevent change in any of the primary angles previously adjusted. The angles of this subordinate triangulation were read by Lieutenant P. M. Price, Corps of Engineers, in 1878, with the 14-inch Pistor and Martin's theodolite No. 2, the number of combined results obtained for each angle being usually 24. The following tables give the names of the stations and angles, the observed angles, their notation, number of combined results, range in the results, their assigned weights, the local and general corrections for each angle, and the adjusted angles. The normal equations for local adjustment are also given, and in tables following the numerical equations of condition, the general corrections in terms of the correlates, the normal equations for the determination of the correlates, the values of the correlates and their logarithms, the values of the general corrections, the residuals resulting from substitution of the general corrections in the equations of condition, and the derivation of the probable error of an observed angle of weight 1.

Computing with these angles and the value of the whole base given in § 5, the lengths of the three parts of the base, the excesses of the computed over the measured parts are found to be, for the west part, $-0^{ft}.0485$; for the middle part, $+0^{ft}.0151$; for the east part, $+0^{ft}.0333$.

Triangulation about Sandusky Base.

WEST BASE—1.

[Observer, First Lieutenant P. M. Price. Instrument, Pistor & Martin's theodolite, No. 2. Date, September and October, 1878.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	$[v]$	Corrected angle.
	° ′ ″			″		″	″	° ′ ″
East Base and East Angle..	1 11 40.376	1_1	24	6.2	1	+0.172	+0.483	1 11 41.031
East Base and Sandusky..................		$1_{1+2+3+4}$	....	....	....	....	....	74 52 17.009
East Angle and West Angle	1 07 50.394	1_2	24	4.3	1	+0.172	+0.780	1 07 51.346
West Angle and Check Base	35 55 27.588	1_3	24	8.3	1	+0.172	−1.867	35 55 25.893
Check Base and Sandusky..	36 37 18.254	1_4	18	6.3	0.75	+0.229	+0.256	36 37 18.739
Sandusky and Steeple......	2 53 34.229	1_5	11	3.8	0.5	−0.694	0.000	2 53 33.535
Sandusky and East Base...	285 07 40.916	1_{5+6}	7	6.6	0.3	+1.727	+0.348	285 07 42.991
Steeple and East Base......	282 14 09.802	1_6	14	4.5	0.5	−0.694	+0.348	282 14 09.456

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$1.8(1_1)+0.8(1_2)+0.8(1_3)+0.8\ \ (1_4)+0.5(1_5)-0.420=0$$
$$0.8(1_1)+1.8(1_2)+0.8(1_3)+0.8\ \ (1_4)+0.5(1_5)-0.420=0$$
$$0.8(1_1)+0.8(1_2)+1.8(1_3)+0.8\ \ (1_4)+0.5(1_5)-0.420=0$$
$$0.8(1_1)+0.8(1_2)+0.8(1_3)+1.55(1_4)+0.5(1_5)-0.420=0$$
$$0.5(1_1)+0.5(1_2)+0.5(1_3)+0.5\ \ (1_4)+1.0(1_5)+0.322=0$$

NOTE.—The value of $1_{1+2+3+4}$ is taken as exact, being the sum of the adjusted values of the angles 69_3 and 69_4 of Section X of the adjustment of the principal triangulation, Chapter XVII, C, § 4.

Triangulation about Sandusky Base—Continued.

EAST BASE—2.

[Observer, First Lieutenant P. M. Price. Instrument, Pistor & Martin's theodolite, No. 2. Date, September, 1878.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angle.
	° ′ ″			″		″	″	° ′ ″
Sandusky and Check Base..	29 45 32.728	2_1	20	6.2	1	+0.030	+0.039	29 45 32.797
Sandusky and West Base........		$2_{1+2+3+4+5}$						69 32 30.524
Check Base and Steeple....	2 18 30.504	2_2	24	4.1	1	+0.030	−0.706	2 18 29.828
Steeple and East Angle	35 13 35.046	2_3	24	4.1	1	+0.030	−0.707	35 13 34.369
East Angle and West Angle.	1 08 16.291	2_4	24	4.8	1	+0.030	+1.476	1 08 17.797
West Angle and West Base.	1 06 36.904	2_5	24	7.2	1	+0.030	−1.201	1 06 35.733
West Base and Sandusky ..	290 27 28.348	2_6	20	9.1	1	+0.029	+1.099	290 27 29.476

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(2_1)+(2_2)+(2_3)+(2_4)+(2_5)-0.179=0$$
$$(2_1)+2(2_2)+(2_3)+(2_4)+(2_5)-0.179=0$$
$$(2_1)+(2_2)+2(2_3)+(2_4)+(2_5)-0.179=0$$
$$(2_1)+(2_2)+(2_3)+2(2_4)+(2_5)-0.179=0$$
$$(2_1)+(2_2)+(2_3)+(2_4)+2(2_5)-0.179=0$$

NOTE.—The value of $2_{1+2+3+4+5}$ is taken as exact, being the sum of the adjusted values of the angles 68_4 and 68_5 of Section X of the adjustment of the principal triangulation, Chap. XVII, C, § 4.

CHECK BASE—3.

[Observer, First Lieutenant P. M. Price. Instrument, Pistor & Martin's theodolite, No. 2. Date, October, 1878.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angle.
	° ′ ″			″		″	″	° ′ ″
West Base and West Angle.	25 47 54.542	3_1	24	4.6	1	+0.639	−1.048	25 47 54.133
West Angle and East Angle.	45 33 32.923	3_2	24	5.8	1	+0.639	−0.061	45 33 33.501
East Angle and East Base ..	30 36 35.552	3_3	24	6.8	1	+0.639	+0.218	30 36 36.409
East Base and West Base..	258 01 54.425	3_4	24	7.1	1	+0.641	+0.891	258 01 55.957

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(3_1)+(3_2)+(3_3)-2.558=0$$
$$(3_1)+2(3_2)+(3_3)-2.558=0$$
$$(3_1)+(3_2)+2(3_3)-2.558=0$$

WEST ANGLE—4.

[Observer, First Lieutenant P. M. Price. Instrument, Pistor & Martin's theodolite, No. 2. Date, August, 1878.]

Angle as measured between—		Notation.	No. Meas.	Range.	Wt.	(v)	[v]	Corrected angle.
	° ′ ″			″		″	″	° ′ ″
East Base and East Angle..	1 11 51.120	4_1	24	7.8	0.8	+0.093	−1.191	1 11 50.022
East Angle and Check Base.	63 57 37.433	4_2	24	8.6	1	+0.075	+0.592	63 57 38.100
Check Base and West Base.	118 16 40.646	4_3	24	6.2	0.7	+0.107	−0.767	118 16 39.986
West Base and East Base..	176 33 50.402	4_4	24	8.9	0.6	+0.124	+1.366	176 33 51.892

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$1.4(4_1)+0.6(4_2)+0.6(4_3)-0.239=0$$
$$0.6(4_1)+1.6(4_2)+0.6(4_3)-0.239=0$$
$$0.6(4_1)+0.6(4_2)+1.3(4_3)-0.239-0$$

Triangulation about Sandusky Base—Continued.

EAST ANGLE—5.

[Observer, First Lieutenant P. M. Price. Instrument, Pistor & Martin's theodolite No. 2. Date, August, 1878.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angle.
	° ′ ″			″		″	″	° ′ ″
East Base and Check Base	111 51 19.729	5_1	24	10.7	1	−0.474	+0.151	111 51 19.406
Check Base and West Angle	70 28 48.902	5_2	24	11.6	1	−0.474	−0.016	70 28 48.412
West Angle and West Base	1 06 28.158	5_3	24	7.6	0.8	−0.593	−0.825	1 06 26.740
West Base and East Base..	176 33 25.344	5_4	24	13.4	0.8	−0.592	+0.690	176 33 25.442

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$1.8\,(5_1)+0.8\,(5_2)+0.8\,(5_3)+1.706=0$$
$$0.8\,(5^1)+1.8\,(5_2)+0.8\,(5_3)+1.706=0$$
$$0.8\,(5_1)+0.8\,(5_2)+1.6\,(5_3)+1.706=0$$

Numerical equations of condition in the triangulation about the Sandusky Base-line.

SIDE-EQUATIONS.

IX. (10) $+26.7086\,[1_1]\ -1.1768\,[1_2]\ -\ 1.1768\,[1_3]+\ 2.1187\,[2_2]\ +2.1187\,[2_3]\ -25.2863\,[2_4]$
$-25.2863\,[2_5]\ +8.4451\,[5_1]+\ 7.0090\,[5_2]+\ 7.0090\,[5_3]\qquad +\ 0.410=0$

X. (10) $+27.8854\,[1_2]\ -1.1751\,[1_3]\ -10.2873\,[4_2]\ -11.3267\,[4_3]\ +0.4553\,[5_2]\ -\ 7.0090\,[5_3]\ \ -32.336=0$

XI. (10) $+\ 1.0982\,[2_2]\ +1.0982\,[2_3]\ -26.3068\,[2_4]+\ 9.7477\,[4_1]\ -0.5396\,[4_2]\ +\ 8.4451\,[5_1]$
$+\ 7.4643\,[5_2]\qquad +51.151=0$

ANGLE-EQUATIONS.

I. $[1_1]+[1_2]+[1_3]+[1_4]\qquad +\ 0.348=0$

II. $[2_1]+[2_2]+[2_3]+[2_4]+[2_5]\qquad +\ 1.099=0$

III. $[1_1]+[1_2]+[1_3]+[2_2]+[2_3]+[2_4]+[2_5]+[3_1]+[3_2]+[3_3]\ +\ 2.633=0$

IV. $[1_2]+[1_3]+[3_1]+[3_2]+[5_2]+[5_3]\qquad +\ 3.037=0$

V. $[1_3]+[3_1]+[4_3]\qquad +\ 3.682=0$

VI. $[2_2]+[2_3]+[2_4]+[3_2]+[3_3]+[4_1]+[4_2]\qquad +\ 0.379=0$

VII. $[2_2]+[2_3]+[3_3]+[5_1]\qquad +\ 1.043=0$

VIII. $[3_2]+[4_2]+[5_2]\qquad -\ 0.515=0$

NOTE.—In the solution of the equations the side-equations were divided by the numbers in parentheses placed opposite them.

General corrections in terms of the correlates.

$[1_1]=$	+0.2956 I	+0.5123 III	−0.3251 IV	−0.1626 V	+2.2752 IX	−0.4342 X
$[1_2]=$	+0.2956 I	+0.5123 III	+0.6749 IV	−0.1626 V	−0.5138 IX	+2.3550 X
$[1_3]=$	+0.2956 I	+0.5123 III	+0.6749 IV	+0.8374 V	−0.5138 IX	−0.5522 X
$[1_4]=$	+0.3941 I	−0.6502 III	−0.4335 IV	−0.2168 V	−0.5277 IX	−0.5789 X
$[2_1]=$	+0.1667 II	−0.6667 III	−0.5000 VI	−0.3333 VII	+0.7723 IX	+0.4018 XI
$[2_2]=$	+0.1667 II	+0.3333 III	+0.5000 VI	+0.6667 VII	+0.9843 IX	+0.5119 XI
$[2_3]=$	+0.1667 II	+0.3333 III	+0.5000 VI	+0.6667 VII	+0.9843 IX	+0.5119 XI
$[2_4]=$	+0.1667 II	+0.3333 III	+0.5000 VI	−0.3333 VII	−1.7566 IX	−2.2292 XI
$[2_5]=$	+0.1667 II	+0.3333 III	−0.5000 VI	−0.3333 VII	−1.7556 IX	+0.4018 XI
$[3_1]=$	+0.2500 III	+0.5000 IV	+0.7500 V	−0.5000 VI	−0.2500 VII	−0.2500 VIII
$[3_2]=$	+0.2500 III	+0.5000 IV	−0.2500 V	+0.5000 VI	−0.2500 VII	+0.7500 VIII
$[3_3]=$	+0.2500 III	−0.5000 IV	−0.2500 V	+0.5000 VI	+0.7500 VII	−0.2500 VIII
$[4_1]=$	−0.3341 V	+0.7238 VI	−0.2338 VIII	+0.6191 X	+0.9463 XI	
$[4_2]=$	−0.2673 V	+0.5791 VI	+0.8129 VIII	−0.5337 X	−0.2719 XI	
$[4_3]=$	+1 0468 V	−0.6013 VI	−0.2673 VIII	−0.9110 X	−0.3113 XI	
$[5_1]=$	−0.5000 IV	+0.7778 VII	−0.2222 VIII	+0.3067 IX	+0.1845 X	+0.4914 XI
$[5_2]=$	+0.5000 IV	−0.2222 VII	+0.7778 VIII	+0.1627 IX	+0.2305 X	+0.3924 XI
$[5_3]=$	+0.6250 IV	−0.2778 VII	−0.2778 VIII	+0.2034 IX	−0.6456 X	−0.4419 XI

Normal equations for determination of the correlates.

1.	0=+0.3480	+1.2809 I	+0.8867 III	+ 0.5912 IV	+0.2954 V	+0.7197 IX	+0.7897 X
2.	0=+1.0090	+0.8333 II	+0.6667 III	+ 0.5000 VI	+0.3333 VII	−0.7723 IX	−0.4018 XI
3.	0=+2.6330	+0.8867 I	+0.6667 II	+ 3.6201 III	+1.5247 IV	+0.7622 V	+1.5000 VI
		+0.9167 VII	+0.2500 VIII	− 0.2970 IX	+1.3686 X	−0.8036 XI	
4.	0=+3.0370	+0.5912 I	+1.5247 III	+ 3.4748 IV	+1.1748 V	−1.0000 VII	+1.0000 VIII
		−0.6615 IX	+1.3877 X	− 0.0495 XI			
5.	0=+3.6820	+0.2954 I	+0.7622 III	+ 1.1748 IV	+2.6342 V	−1.1013 VI	−0.2500 VII
		−0.5173 VIII	−0.5138 IX	− 1.4632 X	−0.3113 XI		
6.	0=+0.3790	+0.5000 II	+1.5000 III	− 1.1013 V	+3.8029 VI	+1.5000 VII	+1.0791 VIII
		+0.2120 IX	+0.0854 X	− 0.5310 XI			
7.	0=+1.0430	+0.3333 II	+0.9167 III	− 1.0000 IV	−0.2500 V	+1.5000 VI	+2.8612 VII
		−0.4722 VIII	+2.2753 IX	+ 0.1845 X	+1.5152 XI		
8.	0=−0.5150	+0.2500 III	+1.0000 IV	− 0.5173 V	+1.0791 VI	−0.4722 VII	+2.3407 VIII
		+0.1627 IX	−0.3032 X	+ 0.1205 XI			
9.	0=+0.0410	+0.7197 I	−0.7723 II	− 0.2970 III	−0.6615 IV	−0.5138 V	+0.2120 VI
		+2.2753 VII	+0.1627 VIII	+16.0163 IX	−1.5075 X	+5.2191 XI	
10.	0=−3.2336	+0.7897 I	+1.3686 II	+ 1.3877 IV	−1.4632 V	+0.0854 VI	+0.1845 VII
		−0.3032 VIII	−1.5075 IX	+ 8.6779 X	+0.9604 XI		
11.	0=+5.1151	−0.4018 II	−0.8036 III	− 0.0495 IV	−0.3113 V	−0.5310 VI	+1.5152 VII
		+0.1205 VIII	+5.2191 IX	+ 0.9604 X	+7.6228 XI		

VALUES OF THE CORRELATES AND THEIR LOGARITHMS.

I	= +0.0006	log 6.77820
II	= −1.0599	log 0.02527
III	= −0.4958	log 9.69531
IV	= −0.9092	log 9.95866
V	= −0.5335	log 9.72713
VI	= −0.5693	log 9.75534
VII	= +0.3929	log 9.59428
VIII	= +1.0224	log 0.00962
IX	= +0.2943	log 9.46879
X	= +0.7268	log 9.86141
XI	= −1.2338	log 0.09125

VALUES OF THE GENERAL CORRECTIONS.

	″
$[1_1]$	= +0.483
$[1_2]$	= +0.780
$[1_3]$	= −1.867
$[1_4]$	= +0.256
$[2_1]$	= +0.039
$[2_2]$	= −0.706
$[2_3]$	= −0.707
$[2_4]$	= +1.476
$[2_5]$	= −1.201
$[3_1]$	= −1.048
$[3_2]$	= −0.061
$[3_3]$	= +0.218
$[4_1]$	= −1.191
$[4_2]$	= +0.592
$[4_3]$	= −0.767
$[5_1]$	= +0.151
$[5_2]$	= −0.016
$[5_3]$	= −0.825

RESIDUALS RESULTING FROM SUBSTITUTION OF GENERAL CORRECTIONS IN EQUATIONS OF CONDITION.

No. of Eq.	*Residual.*
1	−0.0005
2	−0.0006
3	−0.0011
4	−0.0007
5	−0.0003
6	+0.0003
7	−0.0009
8	−0.0002
9	+0.0120
10	−0.0070
11	+0.0030

$\Sigma p[(v)+[v]]^2=20.44$

Number of conditions local and general $=17$.
Probable error of an observed angle of weight unity$=0.6745\sqrt{\frac{20.44}{17}}$
$=\pm 0''.74$

COMPARISON OF THE MEASURED LENGTH OF SANDUSKY BASE WITH ITS LENGTH COMPUTED FROM CHICAGO BASE.

§ **9.** The adjusted values of the angles of the triangulation connecting Sandusky and Chicago Bases, given in Chapters XVI, C, and XVII, C, furnish for the logarithm of the ratio, Sandusky Base divided by Chicago Base, 9.9183689. The logarithm of Chicago Base expressed in feet is (Chapter X, § 33) 4.3917929. Adding this to the preceding logarithm, there results for the logarithm of Sandusky Base computed from Chicago Base, 4.3101618. From § 6 the logarithm of the measured length of Sandusky Base expressed in feet is found to be 4.3101715. The discrepancy between these two logarithms is 97 units of the seventh place, and is equivalent to 0.456 feet=13.9 centimeters or $\frac{1}{44770}$ part of the Sandusky Base. The probable error of this discrepancy, dependent alone on probable errors of observed angles in the principal chain connecting the two bases (Chapter XVII, D), is found to be by the method explained in Chapter IV, § 14, ±75.23 units in the seventh place of logarithms, and corresponds to ±0.353 feet=±10.8 centimeters. This probable error, which is too large since the adjusted angles are used in computing the ratio of the bases, is about three-fourths the actual discrepancy. If the above probable error be multiplied by the mean of the ratios of probable errors of observed to probable errors of adjusted angles in the triangulation between the two bases, viz., 0.60 (see Chapters XVI, C, § 11, and XVII, C, § 5), there results as the approximate probable error of the above discrepancy, due to errors in the adjusted angles of the principal chain,

$$\pm 0^{ft}.212$$

Measured by this probable error the actual discrepancy is still not greater than can be safely attributed to the small errors in the adjusted angles of the principal chain connecting the bases. This chain embraces forty-five triangles, and, measured along its axis, is about 280 miles in length.

COMPARISON OF THE MEASURED LENGTH OF SANDUSKY BASE WITH ITS LENGTH COMPUTED FROM BUFFALO BASE.

§ **10.** The logarithm of the ratio, Sandusky Base divided by Buffalo Base, given by the adjusted angles (Chapters XVII, C, and XVIII, C), of the triangulation connecting these bases is 9.9628985. The logarithm of the length of Buffalo Base, expressed in feet, is (Chapter VII, § 6) 4.3472757. Adding this to the above logarithm of the ratio, there results for the logarithm of Sandusky Base computed from Buffalo Base 4.3101742. The logarithm of the measured length of Sandusky Base expressed in feet, § 6, is 4.3101715. The difference between the two logarithms gives, in units of the seventh place, 27 ± 60.88, the probable error being computed from the probable errors of observed angles of the principal chain (Chapters XVII, D, XVIII, D, and XIX, D), according to the method given in Chapter IV, § 14. The actual discrepancy is thus about one-half its limiting probable error. The linear equivalent is

$$0^{ft}.127 \pm 0^{ft}.287 = 3^{cm}.9 \pm 8^{cm}.7$$

If in place of the probable errors of observed angles, the approximate probable errors of adjusted angles (Chapters XVII, C, § 5, and XVIII, C, § 5) be used in computing the probable error of the discrepancy, there results,

$$0^{ft}.127 \pm 0^{ft}.171 = 3^{cm}.9 \pm 5^{cm}.2$$

The length of the principal chain of triangles joining Sandusky and Buffalo Bases is about 250 miles. The number of triangles in this chain is thirty-six.

CHAPTER XII.

OLNEY BASE.

§ **1.** This base-line is situated in the southern part of Jasper County, Illinois, about 8 miles from Olney, on a prairie, about one-half the length of the line being on cultivated ground and the other on unbroken prairie sod. The base-line is a straight one, and the greatest difference of elevation of its points is 23 feet. Its length is approximately 6,589 metres; the longitude of its west end is 88° 06′ west from Greenwich; its latitude, 38° 52′ north; and the azimuth of the base-line at the west end is 268° 30′ west of south. The surface soil along the base was underlaid at a depth of about a decimeter by fine clay, which had great stability. Indeed, the base-line as a whole was much firmer than the Chicago Base, which was also on a prairie, with its surface soil underlaid by clay.

The base had marking-stones at its ends similar to those of the Chicago Base, described in Chapter X, § 1. It was divided into six nearly equal sections by marks on stones two feet square and one foot thick, sunk three feet below the surface of the ground, the mark being a small drill-hole in the top of a copper bolt leaded into the stone. The section-stones were placed on the line by first obtaining an approximate position and then reading to within 2″ the angles to the ends of the base. Their deviation was thus known and the stones were then put on the line. Each of these sections was measured in duplicate in opposite directions.

The measurement was made, with the Repsold apparatus, by a party under the charge of Assistant Engineer E. S. Wheeler, between July 9 and September 15, 1879. The methods of measurement and reduction were the same essentially as for the Chicago Base, given in Chapter X. The average number of tubes measured per day, including remeasurements, was 105, and the greatest number measured in one day was 168. Measuring was done on thirty-two days. Guide-points were fixed on the base-line between the section-stones at intervals of 300 metres by first putting them approximately in position and then reading the angles at them subtended by the two ends of the base to an accuracy of about 2″. Their distances from the line were then computed and they were placed upon the line. The adjustment of the tube-telescope and of the sector were tested daily and the other adjustments of the base-apparatus weekly. The adjustments were very stable. Details of the work may be found in the Annual Report of the Chief of Engineers for 1880, p. 2408.

§ **2.** An error of 4′ or 5′ was made in the first adjustment of the tube-telescope to parallelism with the steel bar in beginning the first measurement of the first section at the west end of the base, and as the daily tests showed that the tube-telescope had not changed with reference to the tube, the error was not at first discovered. The marks on the base-line at which the tube-telescope was directed were about 300 metres apart and when the tube approached within about 30^{m} of such a mark it was moved forward 300^{m}. In this first section it was noticed that the tube was off the line when it passed these marks, the maximum deviation being $0^{m}.191$. Its deviation was known at three points of the first section and from these and from the form of the curves produced by a constant deviation of the tube-telescope from parallelism with the steel bar, Chapter X, § 16, the correction to the first measurement of the first section has been obtained. The first section was about $1096^{m}.5$ long, and the resulting correction to its first measurement is $-0^{mm}.85$. This correction has been applied.

§ **3.** As on the Sandusky base-line, the mean elevation of the tube above a bench-mark was determined both by a line of ordinary levels run along the line and by means of the observed inclinations of the tube. There were, however, three cases in which the duplicate note-books gave records discordant by 1° for grade-angles, viz., for tubes 7, 17, and 1618 of the first measurement, and one case, tube 1645 of the second measurement, in which they differed by 50′. The probably correct values in these discordant cases were determined by comparing results for differences of height given by the two measurements and a preliminary measurement of forty-six tubes, all of the discrepancies, except that of tube 1618 of the first measurement, falling within forty-six tube-lengths of West Base. An error of 5° in the recorded grade-angle of tube 9 of the first measurement, both records being erroneous, was also detected by means of the second measurement and the preliminary measurement. The adopted grade-angles gave relative elevations of the section-stones agreeing well with the corresponding relative elevations determined by spirit-level.

The height of the base-tubes during measurement above the sea has been derived by several routes which show considerable discrepancies in their results. The mean heights of the Great Lakes above mean tide at New York are given in Chapter XXII, §13.

1. Starting from Lake Michigan, zenith distances of all stations south to the Olney Base have been observed. This route gives station West Base above mean tide, 485.6 feet.

2. The levels of the Illinois Central Railroad from Chicago to Odin, of the Ohio and Mississippi Railroad from Odin to Olney, and the leveling of the base, give 482.6 feet.

3. Railroad levels from Lake Erie at Cleveland to Olney via Vincennes give 490.6 feet.

4. Levels by the Coast Survey and the Mississippi River Commission from gauge at Carrollton, La., to Cairo, Ill., and then railroad levels from Cairo to Odin, and thence to Olney, give 503.1 feet.

5. The same levels from Carrollton to Cairo, and thence by Vincennes to Olney, give 495.0 feet.

In the absence of data for determining the relative weights of these values, their mean has been taken, and as a probable error of not less than ± 5 feet should be assigned to it, there results,

$$\text{Mean height of agate marking west end of base above mean tide} = 491^{\text{ft}}.38 \pm 5^{\text{ft}}.0$$

The levels over the base-line give mean height of tubes above west end of base during measurement of west half of base, $-1^{\text{ft}}.65$, and during measurement of east half, $-10^{\text{ft}}.90$. There result:

West part of base above mean tide, $489^{\text{ft}}.73 \pm 5^{\text{ft}}.0$
East part of base above mean tide, $480^{\text{ft}}.48 \pm 5^{\text{ft}}.0$

§ **4.** In the following table the first column gives the number of the section of the base; the second shows by the numbers 1 and 2 whether the first measurement or the remeasurement in the opposite direction is given on that horizontal line; the third gives the entire number of times that the steel bar S_1^0 at the temperature of 60°.292 F. entered into the measurement of the section; the fourth gives the fractional parts of S_1 which entered the measurement when closing on the permanent mark at the end of a section, this interval being measured by means of the intermediate graduations on S_1; the fifth gives the sums of the parts of the base measured with the cut-off scale in closing on section-stones (the graduations of this cut-off scale have been examined and found sufficiently accurate to introduce no error that need be considered); the sixth gives for each section the sums of the corrections to S_1^0 for metallic temperature on the assumption that

$$\frac{E_{S_9}}{E_{Z_9} - E_{S_8}} = 0.6522$$

its value when $Z_1 = S_1$; the seventh gives the sums of the corrections to the length of each section on account of the inaccuracy of this last supposition; the eighth gives the sums of the corrections for the sections on account of inclination of S_1; the ninth gives the sums of the small distances measured with the microscopes in each section; the tenth gives the sums of the movements of the front cut-off plate in each section; the eleventh gives the sums of the corrections for each section on account of the observed $Z_1 - S_1$ not being the correct metallic temperature of S_1; the twelfth gives the sum of the different corrections for each section.

Results of measurement of Olney Base.

Section.	Measurement.	Number of tubes.	Fractional tubes.	Cut-off scale measurements.	Temperature corrections. Terms in (Z_1-S_1).	Temperature corrections. Terms in $(Z_1-S_1)^2$.	Correction for inclination.	Intervals between tubes measured with microscopes.	Correction for movements of forward cut-off plate.	Correction for regular residuals.	Sum of the corrections.
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
				μ	μ	μ	μ	μ	μ	μ	μ
I	1	274	+0.150	−50026.6	+131522.1	+ 903.6	−171583.3	− 349.1	− 47.1	+ 667.8	− 847.6* − 88912.6
	2	274	+0.150	+ 7002.1	+ 55964.5	+ 210.0	−153327.6	+ 256.8	− 13.0	+1017.4	− 88889.8
II	1	274	+0.400	+89046.5	+201741.2	+1870.7	− 61850.1	− 2088.7	− 5.5	+1082.3	+229796.4
	2	274	+0.450	+47011.3	+ 41676.0	+ 208.4	− 62609.9	+ 1232.4	+ 29.0	+1712.0	+ 29258.9
III	1	274	+0.275	− 8008.1	+130338.8	+ 894.2	− 28254.5	− 2043.6	+ 40.9	+1055.6	+ 94023.3
	2	274	+0.300	+ 5007.2	+ 20405.6	+ 202.7	− 32931.6	− 1388.7	+ 47.3	+1127.3	− 7530.2
West part	1	822	+0.825	+31011.8	+463602.1	+3668.5	−261687.9	− 4481.4	− 11.7	+2805.7	+234059.5
	2	822	+0.900	+59020.6	+118046.2	+ 621.1	−248869.1	+ 100.1	+ 63.3	+3856.7	− 67161.1
IV	1	273	+0.625	+ 998.4	+128491.2	+ 885.6	−102438.0	− 5569.7	−167.8	+ 906.9	+ 23106.6
	2	273	+0.650	−25010.7	+ 60044.2	+ 266.7	−110782.1	− 1829.6	− 2.7	+1098.6	− 76215.6
V	1	275	+0.950	−33010.7	+127403.8	+ 796.4	− 96114.7	− 318.2	− 23.6	+1455.0	+ 188.0
	2	275	+0.950	+ 8007.9	+ 88031.2	+ 449.9	− 96022.6	− 342.5	− 54.1	+1850.6	+ 1920.4
VI	1	274	+0.825	−19010.2	+162858.9	+1233.4	− 97604.0	+ 347.6	+ 75.2	+1356.9	+ 87278.2
	2	274	+0.850	−30014.1	+116619.9	+ 671.9	−101114.4	+ 926.0	+ 26.6	+1361.8	− 11522.3
East part	1	824	+0.400	−13002.1	+418753.9	+2915.4	−296156.7	− 5540.3	−116.2	+3718.8	+110572.8
	2	824	+0.450	−47016.9	+264695.3	+1388.5	−307919.1	− 1246.1	− 30.2	+4311.0	− 85817.5
Total base	1	1647	+0.225	+18009.7	+882356.0	+6583.9	−557844.6	−10021.7	−127.9	+6524.5	+344632.3
	2	1647	+0.350	+ 12003.7	+382741.5	+2009.6	−556788.2	− 1146.0	+ 33.1	+8167.7	−152978.6

* The correction $847^{\mu}.6$ to the first measurement of the first section is due to errors of alignment, discussed in § 2.

In the following table, derived from the preceding one by expressing the two measurements of a section with the same fraction of a tube, the principal results are given, and also the differences between the two measurements of each section. To express the section of the base in terms of entire numbers of $S_1{}^0$ and equal fractions of it, the values of the fraction of a tube in metres are needed. The value of S_1 is given in Chapter IX, § 66, and from that its value at 60°.292, for which $Z_1 - S_1 = 0$, may be computed. Transforming it into metres by Clarke's value of the metre (metre = 1.093623 yard), there results

$$S_1 \text{ at } 60^\circ.292 \text{ F.} = 4^{\text{m}}.0009624,$$

as an approximation more than sufficient for obtaining the values of the fractions of the tube in metres. It has already been stated that these fractions of tubes are the exact fractions which they purport to be within 10^{μ}, a quantity that is insignificant in comparison with the length of a section, and which enters a section but once.

Principal results of measurement of Olney Base.

Section.	Measurement.	No. of $(S_1)_{Z_1=S_1}$	Sum of corrections.	Mean.	Difference.
			μ	μ	μ
I	1	274.150	− 89760.2	− 89325.00	− 870.4
	2	274.150	− 88889.8		
II	1	274.400	+229796.4	+229551.70	+ 489.4
	2	274.400	+229307.0		
III	1	274.300	− 6000.7	− 6765.45	+1529.5
	2	274 300	− 7530.2		
West part	1	822.850	+134035.5	+133461.25	+1148.5
	2	822.850	+132887.0		
IV	1	273.650	− 76917.4	− 76566.50	− 701.8
	2	273.650	− 76215.6		
V	1	275.950	+ 188.0	+ 1054.20	−1732.4
	2	275.950	+ 1920.4		
VI	1	274.850	− 12745.8	− 12134.05	−1223.5
	2	274.850	− 11522.3		
East part	1	824.450	− 89475.2	− 87646.35	−3657.7
	2	824.450	− 85817.5		
Total base	1	1647.300	+ 44560.3	+ 45814.90	−2509.2
	2	1647.300	+ 47069.5		

§ **5.** From the preceding table the mean measured lengths of the parts of the Olney Base are,

$$\text{West part}=822.9\ (S_1)_{Z_1=S_1}-0^{m}.06659=10801^{ft}.6890$$
$$\text{East part}=824.4\ (S_1)_{Z_1=S_1}+0^{m}.11240=10821^{ft}.9662$$

the values in English feet being derived from $(S_1)_{Z_1=S_1}=13^{ft}.1266344$, given in Chapter IX, § 66.

Computing for Clarke's spheroid for the azimuth of the base and for the mean heights of the tubes during measurement of the west and east parts respectively, given in § 3, the reductions to mean tide at New York, they are found to be, for the west part, $0^{ft}.2525$, and for the east part $0^{ft}.2482$. Applying these corrections, there result,

$$\text{West part at sea-level}=\ 822.9\ (S_1)_{Z_1=S_1}-0^{m}.14354=10801^{ft}.4365$$
$$\text{East part at sea-level}=\ 824.4\ (S_1)_{Z_1=S_1}+0^{m}.03676=10821^{ft}.7180$$
$$\text{Olney Base at sea-level}=1647.3\ (S_1)_{Z_1=S_1}-0^{m}.10678=21623^{ft}.1545$$

If the length of S_1 in terms of R 1876 and its adjusted expansion be used, viz., $S_1=4(R\,1876)-58^{\mu}.40+1^{\mu}.326\,(t-42.75)$, Chapter IX, § 62, there results for the length of Olney Base at sea-level,

$$6589.2\,(R\,1876)-0^{m}.16467$$

The base may also be expressed in terms of R1876 by substituting for $(S_1)_{Z_1=S_1}$ its value in terms of R1876 dependent alone on intercomparisons of R1876, S_2, and S_1, derived from Chapter IX, § 24, namely, $(S_1)_{Z_1=S_1}=4(R\,1876)-37^{\mu}.01+1^{\mu}.2744(t-59)$. This gives

$$\text{Olney Base at sea-level}=6589.2\ (R\,1876\text{ at }60^{\circ}.292\text{ F.})-0^{m}.16504$$

§ **6.** The probable error in the length of the Olney Base will be derived in the same way as that of the Chicago Base given in Chapter X.

From the differences of measurements of the sections of the Olney Base, § 4, and called d, the probable error in the base due to these differences, considered as the result of accidental errors of measurement, is

$$0.6745\sqrt{\frac{[d^2]}{4}}=\pm 0^{mm}.97=\frac{1}{6772000}.$$

The probable error depending on the error in the length of S_1 is obtained from the expression giving the approximate length of the base,

$$(1) \qquad n(S_1)_{Z_1-S_1}=m+0.000005106[(Z_1-S_1)^2]$$

For the Olney Base the mean metallic temperature of both measures was $m=588^{\mu}.7$, corresponding to 75°.75 F. Substituting this value in degrees F. in the expression for the probable error in the adopted length of S_1 for any temperature, given in Chapter IX, § 63, there results for the probable error in S_1 at 75°.75 F., $\pm 3^{\mu}.42$. Since $n=1647$ for the Olney Base there results for the probable error due to the first term of (1) above, $\pm 5^{mm}.63$. Taking the probable error of the coefficient of the second term as one-fourth of its value, as was done in Chapter X, § 30, there results for the probable error of the second term, $\pm 1^{mm}.07$. Combining the last two probable errors, there results, $\pm 5^{mm}.73$ as the probable error in the value of the base due to uncertainty in the length of S_1.

For the probable error in the adopted lengths of S_1 due to the fact that the observed values of Z_1-S_1 are not the true temperatures of S_1, $\pm 1^{\mu}.5$ will be taken as was done for the Chicago Base, Chapter X, § 31. Multiplying this by 1647, the number of S_1 in the base, there results a probable error in the base of $\pm 2^{mm}.47$.

The probable error in the mean height of the tubes during measurement above sea-level has been taken as $\pm 5^{ft}$. This introduces a probable error of $\pm 1^{mm}.57$ into the correction for reduction to sea-level. Collecting now the various probable errors in the length of the base, there results,

	mm.
From discrepancies of measurements	± 0.97
From error in value of length of S_1	± 5.73
From errors in temperature Z_1-S_1	± 2.47
From error in mean elevation of tubes above sea	± 1.57

Combining these, there results,

$$\pm 6^{mm}.51 \text{ or } \tfrac{1}{1013000} \text{ part of the base.}$$

Hence, from § 5,

OLNEY BASE AT SEA-LEVEL $=21623^{ft}.1545 \pm 0\ .0214$

§ **7.** The length of Olney Base in terms of the metre R1876 deduced from the length of S_1 dependent alone on intercomparisons of R1876, S_2, and S_1, is, § 5,

$$6589.2\ (R\,1876 \text{ at } 60°.29 \text{ F.})-0^{m}.16504$$

By a process precisely like that followed in Chapter X, § 33, the probable error of this length is found to be $\pm 3^{mm}.48$, or $\frac{1}{1895000}$ part of the base. Hence,

$$\text{Olney Base at sea-level}=6589.2\ (R\,1876 \text{ at } 60°.29 \text{ F.})-165^{mm}.04\pm 3^{mm}.48$$

When the length of R1876 in metres is substituted in this expression, the probable error of the resulting numerical value will be

$$\pm[(6589.2\ \epsilon)^2+(3.48)^2]^{\frac{1}{2}}$$

in which $\epsilon=$the probable error of the length of R1876 at the mean temperature (75°.75 F.) of Olney Base.

§ **8.** The middle section-stone divided the Olney Base into two nearly equal parts. Station Mound, with these parts, formed two triangles, whose angles were read with the same care as primary angles, and they were adjusted with the primary angles. Their adjusted values are given in Chapter XX, C, § 4. With the value for the whole base given above and these angles, the parts of the base have been computed. The excess of the computed over the measured length was

For the west part of the base, $+0^{ft}.0327$
For the east part of the base, $-0^{ft}.0327$

COMPARISON OF THE MEASURED LENGTH OF OLNEY BASE WITH ITS LENGTH COMPUTED FROM CHICAGO BASE.

§ **9.** The logarithm of the ratio, Olney Base divided by Chicago Base, derived from the adjusted angles in the triangulation (see Chapters XVI, C, and XX, C) connecting them, is 9.9431302. The logarithm of Chicago Base expressed in feet (Chapter X, § 14) is 4.3917929. Hence, by addition, the logarithm of Olney Base computed from Chicago Base is 4.3349231. The logarithm of the measured length of Olney Base expressed in feet is (§ 5) 4.3349191. The discrepancy between the two logarithms is 40, which corresponds to $0^{ft}.199 = 6^{cm}.1$ or to $\frac{1}{108570}$ part of Olney Base.

The probable error of this discrepancy, computed from the probable errors of the observed angles of the principal chain of triangles joining the bases (Chapters XVII, D, and XX, D), according to the method given in Chapter IV, § 14, is ± 55.40 units in the seventh place of logarithms or $\pm 0^{ft}.276 = \pm 8^{cm}.4$. The actual discrepancy is then about two-thirds of its limiting probable error. If the approximate probable errors of adjusted angles in the principal chain connecting the bases be used instead of probable errors of observed angles, the probable error of the discrepancy is $\pm 0^{ft}.157 = \pm 4^{cm}.8$. It would appear quite safe, therefore, to attribute the actual discrepancy between the measured and computed lengths entirely to remaining small errors in the adjusted angles in the principal chain of triangles joining the two bases. This chain is about 200 miles long and embraces thirty-five triangles.

PART III.

PRIMARY TRIANGULATION.

CHAPTER XIII.

ADJUSTMENT OF A TRIANGULATION HAVING BUT ONE MEASURED BASE WHICH IS TAKEN AS EXACT.

§ **1.** In a triangulation-net of p points connected by l lines, if we start from the measured base, two angles observed there will fix a third point. At any two of these three points, two more observed, independent angles will fix a fourth point, and so on. Hence to fix p points of the triangulation, $2p-4$ independent angles will be needed. If m angles are observed, $m-2p+4$ of them will be superfluous.

Independent angles are selected measured angles on which all other measured angles can be made to depend by means of the conditions that the net must geometrically fulfill. Thus, if at a station two angles were measured, and afterward their sum, this last would be a dependent angle if the others were taken as independent.

In triangulations there are more angles measured than the $2p-4$ which are necessary to completely determine all the parts of the net. Hence, when one more angle is measured its value will be known in two ways, (1) by direct observation, (2) by computation from the other $2p-4$ angles. As no measurement is perfectly exact, these two values will differ by a small amount. An equation which expresses a relation, which of geometrical necessity must exist between the different parts of a triangulation-net in order that the net may be a series of points connected by lines, is called an equation of condition. In the case just supposed, the equation of condition is that equation which expresses by symbols the necessary relations between the $2p-4$ angles and the one new angle. When the observed $2p-4$ angles and the observed new angle are substituted in it for their symbols, the resulting equation generally will not be perfectly satisfied, showing that the observed angles do not have precisely the values which could give a possible geometrical net. In order that the triangulation may give a geometrical net, the values of the observed angles must be so corrected as to satisfy all the equations of condition that can be formed in the net; and these corrections, in accordance with the principle of least squares, must also have such values that the sum of their weighted squares shall be a minimum.

The adjustment of a triangulation consists in determining and applying these corrections.

EQUATIONS OF CONDITION.

§ **2.** Equations of condition are usually divided into two classes, namely, angle-equations and side-equations. Into the first, angles enter directly; into the second, the sides of triangles or the sines of their opposite angles.

1st. *Angle-equations.*—Let V', V'', &c., represent the most probable values of the angles, these being the final values which we have to find; let M', M'', &c., represent the values of these angles which result directly from observation, being the means of the observed angles, corrected for instrumental errors, errors of centering, and errors of signals; and v', v'' the corrections to be found, which, applied to M', M'', &c., will give V', V'', &c., so that $V' = M' + v'$, &c.

Angle-equations may arise—

(*a*) At a station from the observation of separate angles, and of their sum or difference. Thus, if $V''' = V'' + V'$, when we substitute $M + v$ for V we have

$$M''' + v''' = M'' + v'' + M' + v'$$

which may be written in the form

$$n' + a'v' + a''v'' + a'''v''' = 0$$

where n', a', a'', a''', are known quantities, and the equation expresses a relation between the corrections v, which must be satisfied. It is called a numerical equation of condition to distinguish it from the geometrical one from which it is derived. Again,

(*b*) If all the angles which make up the circumference at a station be measured, we shall have

$$V' + V'' + V''' + V^{iv} + \text{\&c.}, = 360^\circ$$

or substituting the observed values, with their corrections $M + v$, for V,

$$M' + v' + M'' + v'' + M''' + v''' + M^{iv} + v^{iv} + \text{\&c.}, = 360^\circ$$

This may also be written in the form

$$n' + a'v' + a''v'' + a'''v''' + \text{\&c.}, = 0$$

where n' and a', a'', a''', are known. Again,

(*c*) In an uncrossed polygon of m interior angles, $V', V'' \ldots V^{(m)}$, we have

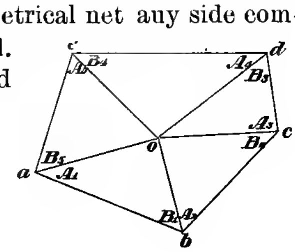
$$V' + V'' \ldots + V^{(m)} = (m-2)\,180^\circ - e$$

where e is the spherical excess of the polygon. Spherical excesses in the adjustment of triangulation result from the formula

$$e = \frac{AB \sin c}{2a^2 (1 - \frac{1}{2} e^2 \cos 2L)^2 \sin 1''}$$

in which the constants are those of Bessel's spheroid. Substituting for V, $M + v$, we again have an equation of condition of the form $n' + a'v' + a''v'' + \text{\&c.}, = 0$.

2d. *Side-equations.*—These arise from the necessity that in any geometrical net any side computed from another side shall be the same by whatever route computed. Thus, in the figure, the same value must be found for cd when computed by either of the following routes:



$$cd = ab \frac{bo}{ab} \frac{co}{bo} \frac{cd}{co} \qquad\qquad cd = ab \frac{ao}{ab} \frac{eo}{ao} \frac{do}{eo} \frac{cd}{do}$$

Hence, dividing one equation by the other,

$$1 = \frac{bo}{ao} \cdot \frac{eo}{bo} \cdot \frac{do}{co} \cdot \frac{eo}{do} \cdot \frac{ao}{eo}$$

or, from the proportionality of sines of the angles to the opposite sides,

$$1 = \frac{\sin A_1}{\sin B_1} \cdot \frac{\sin A_2}{\sin B_2} \cdot \frac{\sin A_3}{\sin B_3} \cdot \frac{\sin A_4}{\sin B_4} \cdot \frac{\sin A_5}{\sin B_5}$$

Substituting for these angles, which are the yet-to-be-found most probable angles, their values in terms of the observed angles M and their needed corrections v, there results—

$$1 = \frac{\sin (M' + v')}{\sin (M'' + v'')} \cdot \frac{\sin (M''' + v''')}{\sin (M^{iv} + v^{iv})} \cdot \frac{\sin (M^{v} + v^{v})}{\sin (M^{vi} + v^{vi})} \cdot \frac{\sin (M^{vii} + v^{vii})}{\sin (M^{viii} + v^{viii})} \cdot \frac{\sin (M^{ix} + v^{ix})}{\sin (M^{x} + v^{x})}$$

which is the equation of condition that the corrections v must rigidly satisfy. For application of the method of least squares this equation must be given a linear form, which can be effected by taking its logarithm, giving,

$$\begin{aligned}(1)\quad & \log \sin (M' + v') + \log \sin (M''' + v''') + \log \sin (M^{v} + v^{v}) + \log \sin (M^{vii} + v^{vii}) + \log \sin (M^{ix} + v^{ix}) = \\ & \log \sin (M'' + v'') + \log \sin (M^{iv} + v^{iv}) + \log \sin (M^{vi} + v^{vi}) + \log \sin (M^{viii} + v^{viii}) + \log \sin (M^{x} + v^{x})\end{aligned}$$

Since v', v'', &c., are only one or two seconds, each term may be developed by Taylor's theorem, stopping at the first power of v and obtaining thus,

$$\log \sin (M' + v') = \log \sin M' + \left(\frac{d \log \sin M'}{d M'}\right) v'$$

$$\log \sin (M'' + v'') = \log \sin M'' + \left(\frac{d \log \sin M''}{d M''}\right) v''$$

and so on, in which, if v is expressed in seconds, $\frac{d \log \sin M}{dM}$ may be replaced by the difference in a table of logarithmic sines for a change of one second in the angle M. Calling these tabular differences δ', δ'', &c., and substituting them in the developments above, and the latter in equation (1) there results,

$$\log \sin M'+\delta' v'+\log \sin M'''+\delta''' v'''+ \cdot \cdot \cdot \cdot =\log \sin M''+\delta'' v''+\log \sin M^{iv}+\delta^{iv} v^{iv}+ \cdot \cdot \cdot \cdot$$

which may be written in the form

$$n'+a' v'+a'' v''+a''' v'''+\text{\&c.}, =0^*$$

Suppose there are in the triangulation-net p points connected by l lines over which observations have been made; l' of these, however, having been observed over in but one direction. Since the ends of the base are known points, there will be $p-2$ points to be determined, requiring $2(p-2)$ observed angles. If m angles are observed, $m-2p+4$ will be superfluous, and will give (including all kinds) as many equations of condition.

In order that the proper number of equations of condition may be obtained, it is desirable to know the number of each kind, and for this purpose they may be divided into local and general equations of condition. The local equations of condition are those which arise at a station when more angles are measured at the station than are necessary to determine all the angles at that station. If there are n distant stations pointed at from a given station, $n-1$ measured angles will determine all angles at this station. If s angles are measured, there will be, at this station, $s-n+1$ equations of condition. The general equations of condition are those involving angles at three or more stations, and are either side-equations or polygon-equations, triangle-equations being included under the last head. Starting from the base, three lines observed over fix the first three points, and to fix the remaining $p-3$ points, two lines are sufficient for each. Hence, $2p-3$ lines will fix the whole net, and if another line is observed over, its value can be obtained by two routes, whence results a side-equation. If there are l lines observed over in one or both directions, there will be $l-2p+3$ side-equations. To obtain the number of polygon-equations, suppose the p points to be connected by p lines, all observed over in both directions. In the resulting polygon, the sum of the angles is known in advance in terms of the angles between its sides. This is a first polygon-equation of condition. If an additional line between two of the p points be observed over in both directions, it will cut the preceding polygon into two new ones and will give a new polygon-equation of condition. It will give but one independent equation of condition, for the sum of the angles of the second new polygon can be derived from that of the first new one and of the primitive polygon. Each new dividing-line giving thus a new polygon-equation of condition, and l' of the whole number l of lines having been observed over in but one direction, there results for the whole number of polygon equations of condition,

$$1+l-l'-p$$

To recapitulate, the whole number of equations of condition in the net is

$$m-2p+4$$

The number of local equations of condition at each station is

$$s-n+1$$

The total number of side-equations in the net is

$$l-2p+3$$

and of polygon equations

$$l-l'-p+1$$

GENERAL METHOD.

§ **3.** Having shown how to obtain the numerical equations of condition in the form

$$n'+a'v'+a''v''+a'''v'''+ \text{\&c.}, =0,$$

* Side-equations are computed with Vlacq's 10-place logarithms.

the method of determining the corrections v', v'', &c., to the mean observed angles M', M'', &c., having weights p', p'', &c., may be given.

Suppose m angles are measured in all, and that there are in the whole net n equations of condition. As already stated, we must have

$$(2) \qquad p'v'^2+p''v''^2+p'''v'''^2+ \qquad (m) = \text{a minimum},$$

the m quantities v', v'', &c., being subject to the n rigid conditions

$$(3) \qquad \left\{\begin{array}{l} n' \;+a'v'+a''v''+\ .\ . \\ n'' \;+b'v'+b''v''+\ .\ . \\ n'''+c'v'+c''v''+\ .\ . \\ \ .\ .\ .\ (n)\ .\ .\ .\ .\ .\ . \end{array}\right\} \begin{array}{l} =0 \\ (m)=0 \\ =0 \\ \end{array}$$

Differentiating (2) and (3), there result—

$$(2') \qquad 2p'v'dv'+2p''v''dv''+\ .\ .\ .\ .\ (m)=0$$

and

$$(3') \qquad \left\{\begin{array}{l} a'dv'+a''dv''+a'''dv'''+\ .\ . \\ b'dv'+b''dv''+b'''dv'''+\ .\ . \\ c'dv'+c''dv''+c'''dv'''+\ .\ . \\ \ .\ .\ .\ .\ (n)\ .\ .\ .\ .\ .\ .\ .\ .\ . \end{array}\right\} \begin{array}{l} =0 \\ (m)=0 \\ =0 \\ \end{array}$$

There are two methods of finding from these equations the values of the unknowns, v.

First. From (3′) the values of n of the dv may be found in terms of the others. These values substituted in (2′) eliminate n of the dv in that equation and leave $m-n$, all independent of each other. The minimum condition of (2) will now be satisfied by placing the coefficients of the $(m-n)$ dv in (2′), which have not been eliminated, separately equal to zero, since these are the partial differential coefficients of (2) with reference to the remaining variables. The resulting $m-n$ equations will give the values of $m-n$ corrections v; the values of the n others have already been obtained in terms of these, so that the values of the corrections for both independent and dependent angles will then be known.

Second. The elimination may be effected by undetermined multipliers or correlates, as follows: Multiply each of the n equations (3′) in order by the correlates $-\mathrm{I}, -\mathrm{II}, -\mathrm{III}$, &c., and add the products to (2′), arranging with reference to dv. Assign such values to the n correlates as to make n of the coefficients of dv zero, leaving $m-n$ coefficients, which, as $m-n$ of the dv are independent of each other, can be separately placed equal to zero. This process then amounts to multiplying each differential equation of condition (3′) by an undetermined multiplier, adding the products to (2′), and then placing the coefficients of all dv separately equal to zero, giving m equations containing as unknowns m corrections and n correlates. From these equations the values of the m corrections can be found in terms of the n correlates. These values of the corrections substituted in the n equations of condition (3) will give n equations containing n correlates as unknowns, from which the values of the correlates can be found. Substituting their resulting values in the expressions for the values of the corrections in terms of the correlates, the corrections themselves become known.

In the adjustment of the Lake-Survey triangulation the two methods have been combined. The labor of adjustment lies largely in the solution of the equations which give the values of the correlates, and increases very rapidly with their number, that is, with the number of equations of condition. Now the local equations of condition, or those which exist between the angles at any one station, usually contain but few variables, so that with little labor, by means of each equation of condition, one correction can be eliminated by the first method from the differential equations (2′) for a minimum. In this way as many corrections can be eliminated from (2′) as there are local equations of condition, and the second method can then be applied to the new form of (2′), and the remaining differential equations of condition in (3′).

If in the n equations of condition there are n' local equations of condition, the number of correlates will thus be reduced from n to $n-n'$.

Going now somewhat more into detail, if the mth angle $V^{(m)}$ entered a local equation of condition, these equations being all of the same form,

$$V^{(m)}=V''+V'''+V^{iv}+C$$

where C is a constant, there results on substituting for the angles their observed values plus their most probable corrections,

$$M^{(m)}+v^{(m)}=M''+v''+M'''+v'''+M^{iv}+v^{iv}+C$$

whence,

$$v^{(m)}=v''+v'''+v^{iv}+C'$$

and

$$dv^{(m)}=dv''+dv'''+dv^{iv}$$

Substituting the value of $dv^{(m)}$ in (2′), there results,

$$(2'')\quad 2p'v'dv'+2p''v''dv''+\ \ldots\ +2p^{(m)}(v''+v'''+v^{iv}+C')(dv''+dv'''+dv^{iv})+\ \ldots\ =0$$

Each additional local equation of condition will insert a similar term in (2″). Eliminating in this way from (2′) as many of the corrections v as there are independent local equations of condition, (2″) may then be written in the following form, in which α, β, γ, &c., are known quantities.

$$(2''')\begin{cases}(\alpha'v'+\alpha''v''+\alpha'''v'''\ +\ \ldots\ +\lambda')\,dv'+(\beta'v'+\beta''v''+\beta'''v'''+\ \ldots\ +\lambda'')dv''\\ +(\gamma'v'+\gamma''v''+\gamma'''v'''+\ \ldots\ +\lambda''')dv'''+\ \ldots\ (m-n')\ \ldots\ =0\end{cases}$$

This is the differential equation for the minimum.

The n' local equations of condition having been already used in eliminating corrections from (2′) there remain to be satisfied in (3′) only the $n-n'$ side- and polygon-equations. Multiplying each of them in order by $-$I, $-$II, $-$III, &c., there results,

$$(4)\qquad\begin{cases}-\mathrm{I}\ \ (a'dv'+a''dv''+a'''dv'''+\ \ldots\ )=0\\ -\mathrm{II}\ \ (b'dv'+b''dv''+b'''dv'''+\ \ldots\ )=0\\ -\mathrm{III}\,(c'dv'+c''dv''+c'''dv'''+\ \ldots\ )=0\\ \ldots\ (n-n')\ \ldots\end{cases}$$

Adding (4) to (2‴), arranging with reference to dv and then placing the coefficients of all dv equal to zero, the following equations result:

$$(5)\qquad\begin{cases}\alpha'v'+\alpha''v''+\alpha'''v'''+\ \ldots\ +\lambda'=a'\,\mathrm{I}+b'\,\mathrm{II}+c'\,\mathrm{III}+\ \ldots\\ \beta'v'+\beta''v''+\beta'''v'''+\ \ldots\ +\lambda''=a''\mathrm{I}+b''\mathrm{II}+c''\mathrm{III}+\ \ldots\\ \ldots\ (m-n')\ \ldots\end{cases}$$

There are as many of these equations as there are corrections remaining to be found. Solving, they give the values of the corrections in terms of the correlates, and these values substituted in the side- and polygon-equations of condition in (3) give as many equations as correlates, from which the latter can be found as already explained.

It will be noticed that thus far all numerical equations of condition have been computed from the geometrical ones by using the means of the observed angles.

MODIFICATION OF GENERAL METHOD.

§ **4.** The foregoing exhibits the general method followed in the adjustment of the Lake-Survey triangulation, but it has been modified in practice. Instead of determining the total corrections v from equations (5) and (3) these corrections have been determined in two parts such that $v=(v)+[v]$, where the (v) are the corrections that result from making a first partial adjustment in which the side- and polygon-equations of condition are neglected and only the local equations are included. The $[v]$ are quantities to be added to the corresponding (v) in order to give the total corrections when all equations of condition are considered, and are obtained by a second adjustment.

From the theory of the first method it is seen that the values of the (v) will at once result by placing the coefficients of the dv in the $(2'')$ or $(2''')$ separately equal to zero, which will give as many equations as there are v, and hence the value of the (v). The resulting values of all (v) which enter no local equation of condition will be zero. The values of the (v) might just as well be found by treating separately each station and its local equations of condition by the first method, and in practice this has been done.

Having found the (v) it remains to find the corrections $[v]$, which added to the (v) shall give the total corrections to the measured angles. It has already been seen that equations (5) and (3) contain the whole solution of the problem of adjustment. Substituting in them for v its value $(v)+[v]$, they become

$$(5')\qquad \begin{cases} \alpha'\{(v')+[v']\}+\alpha''\{(v'')+[v'']\}+\ \ldots\ +\lambda' = a'\,I+b'\,II+c'\,III+\ \&c. \\ \beta'\{(v')+[v']\}+\beta''\{(v'')+[v'']\}+\ \ldots\ +\lambda''=a''I+b''\,II+c''\,III+\ \&c. \\ \ldots\ldots\ldots\ldots\ldots\ldots\ (m-n')\ \ldots\ldots\ldots\ldots\ldots\ldots \end{cases}$$

and

$$(3'')\qquad \begin{cases} n'+a'\{(v')+[v']\}+a''\{(v'')+[v'']\}+\ \ldots\ldots\ldots\ldots\ldots = 0 \\ n''+b'\{(v')+[v']\}+b''\{(v'')+[v'']\}+\ \ldots\ldots\ldots\ldots\ldots = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ (n-n')\ \ldots\ldots\ldots\ldots\ldots\ldots \end{cases}$$

Now the equations by which the (v) were determined are

$$\begin{aligned} &\alpha'(v')+\alpha''(v'')+\ \ldots\ +\lambda' = 0 \\ &\beta'(v')+\beta''(v'')+\ \ldots\ +\lambda''=0 \\ &\ldots\ldots\ldots\ (n')\ \ldots\ldots\ldots \end{aligned}$$

hence, substituting the known values of the (v) in $(5')$ it becomes

$$(5'')\qquad \begin{cases} \alpha'[v']+\alpha''[v'']+\alpha'''[v''']+\ \ldots = a'\,I+b'\,II+c'\,III+\ \ldots \\ \beta'[v']+\beta''[v'']+\beta'''[v''']+\ \ldots = a''I+b''II+c''III+\ \ldots \\ \ldots\ldots\ldots\ldots\ (n-n')\ \ldots\ldots\ldots\ldots\ldots \end{cases}$$

The numerical equations of condition $(3'')$ or (3) were obtained by substituting in the geometrical equations of condition for each most probable angle V its observed value M plus its most probable correction, or $M+v$ for V, or, what is the same, $M+(v)+[v]$ for V. But since the local corrections (v) are known, instead of computing the numerical side- and polygon-equations of condition $(3'')$ with the observed values M of the angles, the locally corrected values of $M+(v)$ may be used. The only change it makes in $(3'')$ is that it combines all the known terms in each equation $(3'')$ into a single term n_0, such that

$$\begin{aligned} n_0' &= n'+a'(v')+a''(v'')+\ \ldots \\ n_0'' &= n''+b'(v')+b''(v'')+\ \ldots \\ &\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}$$

and $(3'')$ then becomes

$$(3''')\qquad \begin{cases} n_0'+a'[v']+a''[v'']+\ \ldots = 0 \\ n_0''+b'[v']+b''[v'']+\ \ldots = 0 \\ \ldots\ldots\ (n-n')\ \ldots\ldots \end{cases}$$

$(5'')$ and $(3''')$ are of the same form as (5) and (3). If the same process that was prescribed to find the values of the v from (5) and (3) be applied to $(5'')$ and $(3''')$, the values of the $[v]$ will result, and $(v)+[v]=v$ gives the total corrections v which, applied to the observed angles, will make them satisfy all the geometrical conditions of the net, and also make the sum of the weighted squares of the corrections to all measured angles a minimum.

A synopsis of the different steps actually followed in the adjustment can now be given.

1. The means of the observed angles are tabulated with their weights. To each angle is assigned a symbol, which is a number with a subscript number. The main number is that of the

station, the stations being numbered consecutively through the portion of the triangulation under consideration. The subscript is the number of the angle at the station, these being numbered from the south round by the west. Thus, 12_3 is the to-be-found most probable value of the third angle at the twelfth station, $12_3'$ is the observed angle, (12_3) is its local correction, and $[12_3]$ the further correction needed to give the total correction $(12_3)+[12_3]$.

2. For each station the observed angles are locally adjusted by the first method. The minimum expression for each station will only include the angles observed there.

3. The numerical side- and polygon-equations of condition are then computed with the locally corrected fundamental angles, and their correlates are written beside them.

4. Equations (5″) are written out, and from them the values of the general corrections are determined in terms of the correlates. The coefficients $\alpha', \alpha'', \ldots \beta', \beta'', \ldots$ &c., are derived from the local adjustment.

5. These values of the general corrections, substituted in the general equations of condition, give the equations for determining the correlates. These equations are solved. The correlates thus found are substituted in the values of the general corrections previously determined, whence the values of the general corrections $[v]$ result. Adding these to the local corrections for the same angles, the total corrections are found. The corrections for angles at a station eliminated in the local adjustment, are derived from those of the angles on which they depend. The corrections are checked by substitution in the equations of condition, the residuals rarely exceeding 0″.02. In the computation of the side-equations the eighth place of logarithms has been retained.

CHAPTER XIV.

TRIANGULATION FROM MINNESOTA POINT BASE TO KEWEENAW BASE.

A.—DESCRIPTIONS OF STATIONS.

NOTE RELATIVE TO ELEVATIONS.

§ **1.** The heights of ground at stations in the triangulation of Lake Superior were determined partly by spirit and partly by trigonometrical leveling, but on the whole with no high degree of precision. They are referred to the surface of the lake at the times of determination. As these times differed in some cases by several years, relative heights are subject to uncertainties arising from fluctuations in elevation of the lake surface. Except for the stations of Minnesota and Keweenaw Bases, a probable error of ± 5 feet may not be too great to assign to these heights.

DESCRIPTIONS OF STATIONS.

§ **2.** NORTH BASE, 1870–'71.*—This station is situated on Minnesota Point, about 2 miles southeast from Duluth, and marks the north end of Minnesota base-line. The height of station used was 50 feet. The geodetic point is marked by the intersection of two lines cut in the surface of a brass frustum, which is leaded into the end of a stone post 1 ft. × 1 ft. × 3 ft. This post is set so that its upper end is about 1 foot below the surface of the ground. Three reference-stones, similar to the above, are set, each 15 feet from the geodetic point, one being in the prolongation of the base-line and one on either side in the line passing through the geodetic point at right-angles to the base-line. A large pine post, used in 1871 as a latitude post, bears north 63° 53′ east, and is 389.17 feet from the geodetic point. Height of ground at station, 2 feet (estimated).

SOUTH BASE, 1870–'71.—This station is situated on Minnesota Point, near its southern extremity, and marks the south end of Minnesota Point base-line. The height of station used was 20 feet. The geodetic point is marked by the intersection of two lines cut in the surface of a brass frustum, which is secured by lead in the end of a stone post 1 ft. × 1 ft. × 3 ft. This post is set so that its upper end is about 2 feet below the surface of the ground. Three reference-stones, similar to the above, are set, each 15 feet from the geodetic point, one being in the prolongation of the base-line and one on either side in the line passing through the geodetic point at right-angles to the base-line. A pine latitude post, used in 1871, was set on line to North Base and 30.8 feet from the geodetic point. The approximate bearing of North Base from South Base is north 36° 06′ west. Height of ground at station, 3.5 feet.

MIDDLE BASE, 1870–'71.—This station is situated on Minnesota Point, near the middle point of Minnesota Point base-line. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 2 feet below the surface of the ground. The height of station used was 10 feet.

ONEOTA, 1870–'71.—This station is situated on the north shore of Saint Louis Bay, about three-quarters of a mile east of the village of Oneota, and on the east bank of a small stream flowing into the bay. The height of station used was 15 feet. The geodetic point is marked by a stone post of the usual form, set so that its surface is about 3 feet below the surface of the ground. A second post, projecting about 6 inches above the ground, is set directly over the first. The geodetic point was 61 feet south of the south rail of the Lake Superior and Mississippi Railway track, about 60 feet east of the stream above named, and about 30 feet from the shore of the bay. Height of ground at station, 15 feet (estimated).

*Dates indicate years when stations were occupied.

LESTER RIVER, 1871.—This station is situated on the north shore of Lake Superior, about 7 miles northeast of Duluth, about 2 miles northeast of the mouth of Lester River, and about 1 mile back from the lake shore. The height of station used was 15 feet. The geodetic point is marked by a brass frustum leaded into the solid surface rock. Three reference pine stumps are blazed on the sides facing the geodetic point, and the initials N., S., W., respectively, formed on them by means of nails driven into the wood. Below each initial is driven another nail, and from the heads of the latter the distances to the geodetic point are: To the one marked N., 16 feet 5 inches; to the one marked S., 15 feet 5 inches; and to the one marked W., 17 feet 4 inches. The bearings of the stumps are approximate. Height of hill at station, 555 feet.

AMINICON RIVER, 1871.—This station is situated on the south shore of Lake Superior, about one-fourth of a mile west of the Aminicon River. The height of station used was 10 feet. The geodetic point is marked by a stone post of the usual form, set so that its upper end is about 3 feet below the surface of the ground. A second stone rising nearly to the ground surface is set directly over the first. Three reference birch stumps are blazed and marked N., S., W., respectively, by means of nails driven into the wood. The stump marked N. is distant from the geodetic point 14 feet 1 inch; the one marked S., 4 feet 3 inches; and the one marked W., 9 feet 7 inches. A large pine post, used in 1871 as a latitude and azimuth post, bears south 50° 54′ east, and is distant 30.43 feet from the geodetic point. Height of ground at station, 50 feet (estimated).

BUCHANAN, 1871.—This station is situated on the north shore of Lake Superior, about 1 mile southwest of Granite Point. It is near the site of the now deserted village of Buchanan, and is not more than 100 feet back from the lake shore. The height of station used was 90 feet. The geodetic point is referred to a brass frustum and an iron bolt leaded into the solid surface rock. The frustum bears south 36° 30′ east and is 2 feet distant from the geodetic point. The iron bolt bears north 36° 30′ west and is 3 feet from the geodetic point. A large bowlder, used as a latitude post in 1871, bears south 78° 44′ west and is 1,194.1 feet distant from the geodetic point. Height of rock at station, 10 feet (estimated).

BRULÉ RIVER, 1871.—This station is situated on the south shore of Lake Superior, about 1 mile east of the mouth of Bois Brulé River. The height of station used was 35 feet. The geodetic point is referred to a latitude post used in 1871, this post bearing south 80° 00′ west and being 69 feet distant from the geodetic point; 69.3 feet directly south of the latitude post is set another post, under which is placed a bowlder, having a cross cut on its upper surface. Height of ground at station, 19 feet.

BURLINGTON, 1871.—This station is situated on the north shore of Lake Superior on the point of land bounding the northeast side of Burlington Bay. The height of station used was 50 feet. The geodetic point is marked by a brass frustum leaded into the solid surface rock. On the north, south, and west, respectively, are set three similar frusta, each being 50 feet distant from the geodetic point. Height of rock at station, 141.5 feet.

CLAY BANKS, 1871.—This station is situated on the south shore of Lake Superior, about 6 miles west of Bark Point and about one-half mile back from the lake shore. The height of station used was 60 feet. The geodetic point is marked by a wrought-iron spike driven into the sandstone surface rock. Centrally over the spike is set a stone marking-post of the usual form. To the west of the geodetic point 23 feet 9 inches is a pine stump marked W., to the south 37 feet is a poplar stump marked S., and to the north 27 feet is a granite bowlder. Height of ground at station, 250 feet (estimated).

SPLIT ROCK, 1870, '71.—This station is situated on the north shore of Lake Superior on Split Rock Point. The height of station used was 58 feet. The geodetic point is marked by a cross cut in the solid surface rock. Three reference-marks of the same kind as the above are cut in the rock—one east, one south, and one west, each being 15 feet from the geodetic point. Height of rock at station, 163 feet.

DETOUR, 1870, '71.—This station is situated on the south shore of Lake Superior, on the point of land nearly due south of Sand Island. The height of station used was 30 feet. The geodetic point is marked by a stone post of the usual form. Three reference-stones are set—one east, one south, and one west, each 15 feet from the geodetic point. Height of ground at station, 12 feet.

WEST SAWTEETH, 1870.—This station is situated on the north shore of Lake Superior, on the highest of the Sawteeth Mountains. It is nearly due north from the mouth of Baptism River and about 2 miles back from the lake shore. The height of station used was 25 feet. The geodetic point is marked by a brass frustum leaded into the solid surface rock. Three pine trees were blazed for references, one being northeast 67.5 feet from geodetic point, one northwest 25 feet, and one south 95 feet. Height of rock at station, 930 feet.

EAST SAWTEETH, 1870.—This station is situated on one of the Sawteeth Mountains, about 1½ miles northeast of West Sawteeth station, and about 2 miles back from the lake shore. The height of station used was 30 feet. The geodetic point is marked by a stone post of the usual form. An astronomical wooden post, used in 1870–'71, bears north 23° 25′ west, and is 50.13 feet distant from the geodetic point. Height of ground at station, 903 feet.

OUTER ISLAND, 1870.—This station is situated on the northwest side of Outer Island, Lake Superior. The height of station used was 51 feet. The geodetic point is marked by a stone post of the usual form, set so that its upper surface is about 3 feet below the surface of the ground. Three similar stones are set for references—one north, one south, and one west, each 9 feet distant from the geodetic point. A stone post, rising about even with the ground-surface, is set directly over the stone marking the geodetic point. An astronomical wooden post, used in 1870, bears north 17° 17′ 50″ east, and is 37 feet distant from the geodetic point. Height of ground at station, 50 feet.

FARQUHAR'S KNOB, 1869, '70.—This station is situated on the north shore of Lake Superior, and is about 4 miles northwest of Sand Bay, whence the trail leading to the station starts. The height of station used was 24 feet. The geodetic point is marked by a cross cut on the solid rock, which is about 3 feet below the surface of the ground. Above this cross is set a stone post of the usual form. An astronomical wooden post, used in 1869, '70, is south 5.9 feet and east 24.9 feet from the geodetic point. Height of ground at station, 1,113.3 feet.

PORCUPINE MOUNTAINS, 1869, '70.—This station is situated on the south shore of Lake Superior, on one of the highest ridges of the Porcupine Mountains. It is about 2 miles south of Carp Lake and about 4 miles south of the "Carp Lake Mining Company's" landing. The height of station used was 25 feet. The geodetic point is marked by a single stone of the usual form. An astronomical wooden post, used in 1869, '70, is 45 feet north and 14.5 feet east from the geodetic point. Height of ground at station, 1,421 feet.

ISLE ROYALE, 1869.—This station is situated near the east end of Isle Royale, about 1¼ miles north from the old mining company's landing in Rock Harbor. The height of station used was 34 feet. The geodetic point is marked by a stone post of the usual form. An astronomical wooden post, used in 1866 and 1869, bears north 88° 3′ 30″ east and is 88 feet distant from the geodetic point. Height of ground at station, 460 feet.

WHEAL KATE, 1869, '70, '71.—This station is situated on Keweenaw Point, about 7 miles southwest of the town of Houghton, about one-quarter of a mile west of the old Ontonagon road and 1½ miles southwest of Wheal Kate mine. The height of station used was 34 feet. The geodetic point is marked by a stone post of the usual form. An astronomical wooden post, used in 1866, '69, '70, bears north 70° 51′ east, and is 98.5 feet distant from the geodetic point. Height of ground at station, 906.5 feet.

ISLE SAINT IGNACE, 1867, '69, '71, '72.—This station is situated near the east end and on the most elevated part of Isle Saint Ignace. It is approached from Saint Ignace Harbor, which lies on the southeast side of the island, and affords safe anchorage in any weather. The height of station used was 9 feet. The geodetic point is marked by a cross cut in the solid surface rock. No reference marks were made. The corners of the station pyramids, however, were surrounded by piles of stones, and as the rock in the immediate vicinity of the station is destitute of vegetation the geodetic point can be easily identified. Height of rock at station, 1,263 feet.

TIP TOP, 1867, '69, '71.—This station is situated on the northeast shore of Lake Superior, about 8 miles northeast of "Simmons' Harbor." This harbor is about 10 miles north of Otter Head. The geodetic point is marked by a brass frustum leaded into the solid surface rock. Over this frustum is placed a tripod about 3 feet high and loaded with stones to keep it in position. Three large bowlders, used to support astronomical instruments, occupy the following positions, respectively: One north 46° 24′ west and 94.6 feet from geodetic point, one north 85° west and 68.5 feet distant, and one south 71° 28′ west and 274.0 feet distant. The latter stone is at the highest point of the hill, and there is some soil in its vicinity. Near the geodetic point the rock is slightly lower and destitute of soil. Height of rock at station, 1,520 feet.

MICHIPICOTEN, 1869, '71.—This station is situated on the northwest side of Michipicoten Island. It is on the highest part of the island and is approached from a mining company's landing on the northwest coast. The height of station used was 9 feet. The geodetic point is marked by a cross cut in the solid surface rock. An astronomical stone post, used in 1869, bears south 44° 04′ west, and is 120.25 feet distant from the geodetic point. Height of rock at station, 937 feet.

VULCAN, 1869, '71, '72.—This station is on Keweenaw Point, about 6 miles southeast of the village of Copper Harbor. It is approached by way of an old mining road leading off to the southeast of the east end of Lake Fanny Hooe. The height of station used was 75 feet. The geodetic point is marked by a single stone of the usual form. An astronomical wooden post bears north 83° 16′ west, and is 48.3 feet distant from the geodetic point. A second astronomical post is set directly south of and 156.16 feet from the former. Height of ground at station, 726 feet.

TRAVERSE POINT, 1871.—This station is situated on the extreme end of Traverse Point. The height of station used was 43 feet. The geodetic point is marked by a stone of the usual form. Height of ground at station, 20.5 feet.

CREBASSA, 1871.—This station is situated on the east side of Keweenaw Point, about one-half mile northeast of the Portage Entry light-house and not more than 10 feet back from the edge of a bluff rising nearly vertically 44 feet above the water of the lake. The height of station used was 24 feet. The geodetic point is marked by a stone post of the usual form. The head of a nail driven into a blazed hemlock tree bears north 43° 53′ west, and is 59.75 feet from the geodetic point. Height of ground at station, 44 feet.

NORTH BASE, 1871.—This station is situated on Keweenaw Point, about 1½ miles south of Portage Entry. It marks the north end of Keweenaw base-line, this line being on the State road connecting Houghton and L'Anse. The height of station used was 30 feet. The geodetic point is marked by the intersection of two lines cut on the end of a copper bolt three-fourths of an inch in diameter and 10 inches long, leaded into the solid sandstone rock, which is about 4 feet below the surface of the overlying soil. The end of the bolt, which projects slightly above the rock, is covered by a telegraph-wire insulator. Directly above is set a marking-stone of the usual form, which rises nearly to the ground surface and is covered by a flat stone about 1 foot square. Another stone post of the usual form, rising about even with the ground surface, is set directly in line to South Base, the distance between the geodetic point and the center mark of the latter stone being 28.37 inches. Height of ground at station, 67.8 feet.

SOUTH BASE, 1871.—This station is situated on Keweenaw Point, about 6½ miles south of Portage Entry, and about one-half mile back from the lake shore. It marks the south end of Keweenaw base-line, and is about 50 feet east of the State road running north along the base-line. The height of the station used was 30 feet. The geodetic point is marked by the intersection of two lines cut on the surface of a brass frustum leaded into the end of a stone post 6 feet long, set so that its upper end is about even with the ground surface. A second stone, 2½ feet long and similarly marked, is set one yard south of the first and exactly in the prolongation of the base-line. The top of this stone is 6 feet below the surface of the ground. Two reference-stones are set, one to the east and one to the west of the line, each about 100 feet distant, and the line joining them passes through the geodetic point at right-angles to the base-line. The east reference is 6 feet long and projects about 1 foot above the ground. The west reference was originally the same as the east one, but in 1873 was found to have been broken off at its upper end. Height of ground at station, 70.2 feet.

MIDDLE BASE, 1871.—This station is situated on Keweenaw Point, about midway between North Base and South Base. The height of station used was 15 feet. The geodetic point is marked by a single stone of the usual form. Height of ground at station, 55.6 feet.

QUAQUAMING, 1871.—This station is situated on the northeast extremity of Pe-qua-qua-wa-ming Point, on the east side of Keweenaw Bay. The height of station used was 25 feet. The geodetic point is marked by a stone of the usual form. Height of ground at station, 7.2 feet.

MIDDLE, 1871.—This station is situated on the east shore of Keweenaw Bay, about midway between Pe-qua-qua-wa-ming Point and Point Abbayé. The height of station used was 25 feet. The geodetic point is marked by a stone of the usual form. Height of ground at station, 10.5 feet.

HURON MOUNTAINS, 1871, '72.—This station is situated on one of the Huron Mountains, about three miles southwest of the mouth of Pine River, and near the north end of Mountain Lake. The height of station used was 15 feet. The geodetic point is marked by a stone post of the usual form. An astronomical stone post, used in 1866, bears south 36° 30′ west, and is 97.5 feet distant from the geodetic point. Height of ground at station, 930 feet.

B.—HISTORICAL NOTE, STATIONS, SIGNALS, INSTRUMENTS, AND METHODS OF OBSERVATION.

HISTORICAL NOTE.

§ **3.** The general triangulation of Lake Superior was begun in 1861 by Captain (afterward General) G. G. Meade, Corps of Topographical Engineers. A preliminary measurement of a base on Minnesota Point at the west end of the lake was made with wooden rods on a stretched rope by Assistant W. H. Hearding. Stations were built and angles read east from the base as far as the line Brulé–Buchanan, shown on Plate I.

In 1865 Colonel W. F. Raynolds, Corps of Engineers, began the triangulation of Keweenaw Bay. Stations were built in the bay, and also station Wheal Kate, by Assistant D. F. Henry.

In 1866 the building of stations was continued by Mr. Henry. The work during the year was principally astronomical, under Lieutenant M. R. Brown, with Assistants O. B. Wheeler and S. W. Robinson. Stations Vulcan, Northeast, Saint Ignace, Isle Royale, Wheal Kate, and Huron Mountains were occupied. Colonel F. U. Farquhar and Lieutenant J. F. Gregory read the angles in Keweenaw Bay from South Base station to the line Traverse Island–Huron Island.

In 1867 the triangulation of Keweenaw Bay was carried on by Assistant O. N. Chaffee. Keweenaw Base was measured with the Bache-Würdemann apparatus by Assistant Henry. The angles of the triangle, Vulcan–Tip Top–Saint Ignace, were read by Lieutenant J. Mercur, Corps of Engineers, Assistant O. B. Wheeler, and Assistant G. Y. Wisner, respectively.

In 1869 three triangulation and astronomical parties, under Lieutenant E. H. Ruffner, Assistant E. S. Wheeler, and Assistant G. Y. Wisner, worked westwardly from the line Vulcan–Saint Ignace to the line Porcupine Mountains–Farquhar's Knob. Three other triangulation and astronomical parties, under Assistants O. B. Wheeler, G. A. Marr, and A. R. Flint, worked from the line Vulcan–Saint Ignace eastwardly to station Mamainse.

In 1870 the work in Lake Superior was continued with General C. B. Comstock, Corps of Engineers, in charge of the survey. As several of the triangles previously measured had too large errors in closure to be of geodetic value, those lying in the chain between Keweenaw and Minnesota Point Bases, whose values were thought defective, were remeasured. The Minnesota Point Base was measured by General Comstock, Lieutenant J. H. Weeden, and Assistant E. S. Wheeler. Triangulation and astronomical work were carried by Lieutenant E. H. Ruffner and Assistants O. B. Wheeler, G. Y. Wisner, and A. R. Flint from the line Wheal Kate–Farquhar's Knob to the line Detour–Split Rock.

In 1871 the triangulation was completed from the line Detour–Split Rock to the Minnesota Point Base. The triangles from the Keweenaw Base to the line Wheal Kate–Vulcan were reread, as also the angles of the triangle Vulcan–Tip Top–Saint Ignace, thus completing the triangulation between the Keweenaw and Minnesota Point Bases. Angles were read by Assistants Wisner, Marr, Flint, Jones, and General Comstock. The telegraphic longitude of North Base station, Minnesota Point, was determined by General Comstock and Assistants Wisner and O. B. Wheeler. The latitude of the same station, of South Base station, Minnesota Point, and the azimuths of Minnesota Point base-line and of Keweenaw base-line were determined by Assistant Engineer Wisner. The final adjustment of the triangulation was made by Assistant Engineers T. W. Wright and C. H. Kummell.

STATIONS AND SIGNALS.

§ 4. In consequence of the mountainous character of much of the shore of Lake Superior, it was sometimes practicable to place the instrument and the signal to be observed on very low structures resting on the rock, thus giving great stability. But in most cases, to avoid cutting lanes for sights through forests, it became necessary to raise the instrument far above the ground. This was usually effected by building a triangular pyramid, the edges of this pyramid being the trunks of stout trees, 12 or 18 inches in diameter at the ground, where they were set far enough apart to give the pyramid stability, and meeting at the vertex of the pyramid, which was cut off to form a stand for the instrument. The legs of this pyramid were strongly braced to each other to give stiffness. Immediately outside of this pyramid, and far enough from it never to come in contact with it, was a second pyramid of quadrangular form, carrying a small platform at a height 2 or 3 feet less than that of the top of the inner pyramid. This gave the observer a place to stand on and move about on without in any way disturbing the instrument. Some of these structures were 75 feet in height, and in one case, where the instrument was supported by a central post, it was 90 feet above the ground. They were built by a steamer's crew.

In the early part of the work the instrument was in the open air; but after 1870, tents were used in all cases to shield it.

As many of the lines to be observed over were long, a form of heliotrope was ordinarily used as a signal. For lines of moderate length Gauss' heliotropes gave beams of light of sufficient size, but on the longer lines they were insufficient. The triangle Vulcan–Tiptop–Saint Ignace had sides whose lengths in order were in round numbers 100, 92, and 93 miles, the first being the length of the line Vulcan–Tiptop. Over long lines a beam of sunlight was flashed to the distant station by means of ordinary mirrors 12 inches high by 9 wide, mounted on wooden stands which permitted their rotation about both a horizontal and a vertical axis. A pole with a wooden screen at its end was run out horizontally from the platform. Through the screen there was a circular hole whose diameter, sometimes reaching 8 or 10 inches, varied with the length of the lines over which the flash was to be sent. The mirror was then placed in such a position that a line through its center and the center of the hole in the screen, 15 or 20 feet distant, pointed at the distant station. The direction of this line was determined with a theodolite when necessary. As the sun's diameter is about 32′ the cone of rays diverging from the screen will have that angle, and great precision in establishing the direction of the cone of light is not necessary. The mirror was constantly turned by an attendant, called a flasher, so as to keep the reflection of the sun on the screen concentric with the hole in it. The sun's rays then reached the distant station. When the air was steady and the opening in the screen was of such size as to give neither too much nor too little light, these flashes appeared at the distant station as bright and perfectly steady points of light, at which a theodolite could be pointed with great precision. The steadiest air was usually between one and three hours before sunset. When the air was unsteady, these points of light frequently expanded, as seen in the telescope to pale disks of a minute or more in diameter, of varying position and frequently changing outlines. Sometimes they would be small, but vibrating through many seconds of arc, and sometimes two or three would be superposed. As some of the lines could only be seen over when refraction had a value greater than its ordinary value, on these the unsteadiness and fluctuations of the heliotrope lights was great, and a station had to be occupied for many weeks to get a few good hours for work. It was only on rare occasions that station Vulcan could be seen from Saint Ignace. Observers used the flashes obtained by cutting off the light sent to a distant station for a shorter or longer interval in place of the dots and dashes of the Morse telegraph code to send messages to that station. This method of sending messages was introduced on the Lake Survey by Assistant Engineers O. B. Wheeler and S. W. Robinson, in 1865, and has since been much used.

INSTRUMENTS AND METHODS OF OBSERVATION OF ANGLES.

§ 5. In the measurement of the angles of the primary triangulation lying between the Keweenaw Point base-line and the Minnesota Point base-line six different theodolites were used. Three of these theodolites, numbered 2561, 2562, and 2563, were made by Oertling, of Berlin, in 1868, and were used in 1869 and 1870 in the absence of better ones. They are all alike in dimensions and form, are non-repeating, and although of large size are very poor instruments. Their horizontal circles are 20 inches in diameter, are graduated to five minutes, and are read by three micrometer-microscopes, one division of the micrometer-head being one second. Their telescopes have a focal length of 30 inches, with object-glasses of 2¼ inches diameter. They have bad axes, bad microscopes, and accidental errors of graduation amounting to 20 seconds. From a comparison of the individual results of the measurements of angles with their means, the following mean errors for one measurement of an angle with these theodolites were found: No. 2561, 4″.48; No. 2562, 3″.10; No. 2563, 3″.91. Of the twenty-eight stations in this section of the triangulation, nine were occupied in whole or in part by these instruments. Other and better ones were obtained as soon as was practicable. None of the Oertling theodolites were used after 1871, and only one, No. 2562, after 1870. In the spring of 1871 two new theodolites were received, one from Troughton & Simms, of London, and one from Pistor & Martins, of Berlin. Troughton & Simms' theodolite No. 1, shown in Plate XVI, is a non-repeating instrument, with a horizontal limb 14 inches in diameter, divided to five minutes, and read by three micrometer-microscopes, the divisions of the micrometer-heads giving single seconds. Its telescope has an object-glass 2¼ inches in diameter, with a focal length of 24 inches, and an eye-piece micrometer. Its 12-inch vertical circle is graduated to ten minutes, and reads by two verniers to ten seconds. It is an excellent instrument. Pistor & Martins' theodolite No. 2, shown in Plate XIV, is a non-repeating theodolite, whose horizontal limb is 14 inches in diameter, is graduated to five minutes, and is read by two micrometer-microscopes, the divisions of whose heads equal two seconds. Its object-glass has a diameter of 2¼ inches and a focal length of 25 inches. It has a watch-telescope which has not been used. Its vertical circle is 10 inches in diameter, is graduated to five minutes, and is read to five seconds by four verniers. It is a good instrument. The sixth theodolite used is a repeating instrument by Gambey, of Paris, with a 10-inch horizontal limb, graduated to five minutes, and reading by two verniers to five seconds. Its telescope has an object-glass 1½ inches in diameter and 19 inches focal length. There is no vertical circle, but it has a watch-telescope of 16 inches focal length, which, however, has not been used. It has been on the Survey since 1851, and in 1871 was a good instrument. In all primary work this instrument has been used as a repeating instrument, five repetitions being obtained in a set, and their result constituting one measurement. An examinat on of more than twenty-five angles, observed prior to 1872 with each of the last-named theodolites, gives the following values for the mean error of one measure of an angle derived from one pointing at each station for the non-repeating theodolites, and from the result of five repetitions for the repeating theodolite: Troughton & Simms' No. 1, 1″.75; Pistor & Martins' No. 2, 2″.24; 10-inch Gambey, 1″.77.

These values, compared with those previously given for the 20-inch Oertling instruments, show how decidedly inferior the latter were. In these values, any twisting in azimuth of instrument or station has not been eliminated. From the table of observed angles in C, § 7, it will be seen that in this triangulation the instruments were used as follows in measurement:

Theodolite used.	Total number of stations.	Total number of angles read.
Gambey No. 1	9	50
Troughton & Simms' No. 1	8	44
Pistor & Martins' No. 2	5	25
Oertling No. 2562	6	24
Oertling No. 2563	3	12
Oertling No. 2561	2	4

For the measurement of the angles given in C, § 7, four stations were occupied in 1869, eight in 1870, twenty in 1871, and one in 1872.

Prior to 1872, in the measurement of angles, much had been left to the discretion of the individual observers. A part of the angles had been read previous to 1870, but not all with the precision deemed necessary. In selecting those to be re-read the tests of closure of triangles and of sum-angles were used. As the shore of Lake Superior is largely a wilderness, the observing parties were moved by a steamer from station to station, the steamer being notified in advance when it was supposed the readings at a given station would be completed. If the intervening weather was unfavorable a small number of measures of the angles would be obtained. If very favorable a very large number might be got. Thus, at Outer Island, one hundred and twenty measures of the angle Detour–Sawteeth West were obtained. As the observing parties were also heliotrope parties for other observing parties, and as there were never more than four, and frequently not more than three, observing parties, a given observing party could only read to two or three stations when the heliotropes were necessary. One of the instruments was a repeating instrument; with non-repeating instruments angles were usually separately read, the method of reiteration not being systematically used.

The aim was to get many measurements, especially with the inferior Oertling instruments, on the principal angles of the triangulation. When this was accomplished sum-angles were freely read if there was spare time. When, on short lines, heliotropes were not used, a pole formed the signal to which readings were made.

In the few angles read in 1869 there were some cases in which the number of measures of the angle obtained by turning the microscopes in the direction of positive graduation differed much from the number obtained by moving the microscopes in the negative direction. Subsequently to 1869 nearly equal numbers of these positive and negative measures were obtained on each angle. The object in taking both positive and negative measures is to eliminate any motion of the horizontal limb of the instrument in azimuth, arising usually from a steady twisting of its support. When the instrument is on a high wooden structure this twisting may exceed 1″ per minute of time.

An examination of this question at station Brulé River, in 1871, and at station Vulcan, in 1872, gave the following results. The twist is called + when in the direction of the sun's daily motion.

STATION BRULÉ RIVER.

[Instrument on post 12 inches square and 35 feet high.]

1871.	Mean twist in 1ᵐ.	Time.	Sky.
	″	*h. m.* *h. m.*	
July 22	+1.10	5 07 to 7 16 p. m.	Clear.
July 22	−1.34	7 16 to 10 27 p. m.	Clear.
July 23	+1.51	3 17 to 5 51 p. m.	Clear.
July 23	+0.14	5 51 to 7 35 p. m.	

STATION VULCAN.

[Instrument on wooden tripod 75 feet high.]

1872.	Mean twist in 1ᵐ.	Time.	Sky.
	″	*h. m.* *h. m.*	
July 24	+0.98	10 40 a. m. to 11 23 a. m.	Clear.
July 24	+0.57	11 23 a. m. to 2 55 p. m.	Clear.
August 27 ..	+0.72	9 00 a. m. to 9 45 a. m.	Bright sunshine.
August 27 ..	+0.30	9 45 a. m. to 11 42 a. m.	Cloudy.
August 27 ..	+0.55	11 42 a. m. to 1 56 p. m.	Faint sunshine.

This twisting is doubtless mainly due to the sun's action on the wooden support of the instrument. During the day the twist is usually in the direction of the sun's motion, while during the night it is in the reverse direction.

Thirty angles, each having at least sixteen measures in both positive and negative directions, were examined to find the difference in the results of the positive and negative measures. It was found that when the microscopes were moved in the direction of the sun's motion, the resulting measure of the angle was on the average 0″.08 smaller than when the microscopes were moved in the opposite direction, indicating that as double the average amount of twist in the interval between pointings at consecutive stations.

While prior to 1872 the readings of angles were not so arranged as to systematically eliminate periodic errors of graduation, yet angles were read on so many parts of the horizontal limb as to approximate to the same result. It was usually the practice after about four measures of an angle had been obtained to turn the horizontal limb in azimuth. An examination of thirteen angles read by different observers showed that each angle was read on an average on thirteen different parts of the limb. Readings on so many portions of the limb with each of two or three microscopes give for a good instrument a satisfactory elimination of accidental, and a partial elimination of periodic, errors of graduation.

NOTE ON PERIODIC ERRORS AND ERRORS OF MICROSCOPE- AND TELESCOPE-POINTINGS.

§ **6.** A single measurement of an angle between two objects is made with a theodolite by pointing the telescope first to one object and noting the readings of the equidistant microscopes on the circle, and then pointing to the other object and again noting the readings of the equidistant microscopes. The difference between the means of the microscope-readings in the two positions is the observed value of the angle. Every such observed value is subject to a correction for periodic error. Denoting this correction by c, it may be expressed by the equation

$$(1) \qquad c=2\,u_1 \cos\left(qz+\tfrac{1}{2}\,qa+\beta_1\right) \sin \tfrac{1}{2}\,qa+2\,u_2 \cos\left(2\,qz+qa+\beta_2\right) \sin qa \\ +2\,u_3 \cos\left(3\,qz+\tfrac{3}{2}\,qa+\beta_3\right) \sin \tfrac{3}{2}\,qa+,\ \&c.$$

In this equation q is the number of equidistant microscopes by which the circle is read, z is the reading of either microscope on the left-hand object (supposing the graduation to increase from left to right), a the angle observed, and u_1, u_2, &c., β_1, β_2, &c., are constants. z and a need be taken only to the nearest degree.

For a circle read by two opposite microscopes, equation (1) becomes

$$(2) \qquad c=2\,u_1 \cos\left(2\,z+a+\beta_1\right) \sin a+2\,u_2 \cos\left(4\,z+2\,a+\beta_2\right) \sin 2\,a+,\ \&c.$$

For a circle read by three equidistant microscopes

$$(3) \qquad c=2\,u_1 \cos\left(3\,z+\tfrac{3}{2}\,a+\beta_1\right) \sin \tfrac{3}{2}\,a+2\,u_2 \cos\left(6\,z+3\,a+\beta_2\right) \sin 3\,a+,\ \&c.$$

For the Pistor & Martins theodolite No. 2, the following weighted mean values of the constants u_1, u_2, β_1, and β_2 result from computations made in 1876 and 1880. u_1 and β_1 are derived from the observations on ten different angles, and u_2 and β_2 from the observations on six different angles.

$$u_1=0''.998\pm0''.034 \qquad \beta_1=\ 49^\circ\pm6^\circ$$
$$u_2=0''.424\pm0''.033 \qquad \beta_2=152^\circ\pm25^\circ$$

These values, substituted in equation (2), give

$$c=2''.00 \cos\left(2\,z+a+49^\circ\right) \sin a+0''.85 \cos\left(4\,z+2a+152^\circ\right) \sin 2a$$

The numerical maximum of c resulting from this expression is $\pm\ 2''.35$.

$$c=+2''.35 \text{ for } z=116^\circ \text{ and } a=\ 62^\circ \text{ or } 240^\circ$$
$$\text{“} \quad z=296^\circ \text{ and } a=\ 62^\circ \text{ or } 240^\circ$$
$$c=-2''.35 \text{ for } z=178^\circ \text{ and } a=118^\circ \text{ or } 298^\circ$$
$$\text{“} \quad z=358^\circ \text{ and } a=118^\circ \text{ or } 298^\circ$$

For the Troughton & Simms theodolite No. 1, the following weighted mean values of the constants u_1, u_2, β_1, and β_2, are the results of a computation made in 1880. u_1 and β_1 were computed from the observations on five different angles and u_2 and β_2 from the observations on six different angles:

$$u_1=0''.172\pm0''.031 \qquad \beta_1=88^\circ\pm24^\circ$$
$$u_2=0''.436\pm0''.053 \qquad \beta_2=18^\circ\pm13^\circ$$

These values, substituted in equation (3), give

$$e=0''.34 \cos (3z+\tfrac{3}{2}a+88^\circ) \sin \tfrac{3}{2}a+0''.87 \cos (6z+3a+18^\circ) \sin 3a$$

This gives, for a numerical maximum of e, $\pm 1''.12$.

$$e=+1''.12 \text{ for } z=44^\circ, 164^\circ, \text{ or } 284^\circ; \text{ and } a=87^\circ, 207^\circ, \text{ or } 327^\circ$$
$$e=-1''.12 \text{ for } z=11^\circ, 131^\circ, \text{ or } 251^\circ; \text{ and } a=33^\circ, 153^\circ, \text{ or } 273^\circ$$

For the details of such a computation reference may be made to the Lake-Survey reports in the Reports of the Chief of Engineers for 1876 and 1879.

A determination of the probable error of pointing a microscope to a division-line of Troughton & Simms' theodolite No. 1 gave

For microscope	A,	$\pm 0''.21$
	B,	$\pm 0''.23$
	C,	$\pm 0''.18$

giving for probable error of a pointing in the mean of the three microscopes $\pm 0''.12$. Numerous pointings with the telescope at a time of steady atmosphere were made to a distant object, and the probable error of one such pointing, the three microscopes being read, was $\pm 0''.39$.

Freeing this from the probable error due to microscope-pointings, namely, $\pm 0''.12$, there results for probable error of telescope-pointing, due to lack of optical power, unsteadiness of instrument and object, &c., $\pm 0''.37$.

For Pistor & Martins' theodolite No. 2, with two microscopes, there was found in the same way,

Probable error of mean pointing on limb with two microscopes, $\pm 0''.14$.

Probable error of one telescope-pointing, when freed from error of microscope-pointings, $\pm 0''.38$.

C.—MEASURED AND ADJUSTED ANGLES BETWEEN MINNESOTA POINT AND KEWEENAW BASES.

§ **7**. In the adjustment of the triangulation between Minnesota and Keweenaw Bases it happened that it could be divided into two entirely independent sections by the line Split Rock–Detour. Hence those two sections were adjusted independently. A sketch of the triangulation will be found in Plate I. The notation is explained in Chapter XIII.

In the following pages the adjustment of the western section is given first.

For each station the name of the observer, of the instrument used, and the date of observation are given. Following this, for each station, a table is given, of which the first column shows the names of the stations between which the angle is observed and the means of observed angles corrected for errors of centering of signal pointed at and of instrument, and for instrumental errors, if there are any known; the second gives the notation for the angles; the third gives the number of measures of the angle, a measure being derived from a single pointing at each of the two stations; the fourth gives the difference in seconds between the least and greatest measure, or the range; the fifth gives the weight, the weights being approximately as the reciprocals of the squares of the mean errors of a single measure, this mean error being derived from the discrepancies between the individual measures and their means. This method gave in some cases exaggerated weights to angles, but with instruments which varied very widely in their excellence, some having such accidental errors of graduation as to make angles read with the same instrument, and having the same number of measures differ widely in accuracy, it was difficult to adopt any other method. The sixth column gives the local correction to each angle; the seventh the additional or general correction, and the eighth column gives the most probable angle.

Below each table are given the normal equations for the local corrections at each station. In the first section there are eleven stations and eleven of the tables.

They are followed by tables giving successively (*a*) the numerical side- and polygon-equations of condition; (*b*) the general corrections in terms of the correlates; (*c*) the normal equations for the correlates; (*d*) the values of the correlates; (*e*) the values of the general corrections; (*f*) the residuals of the general equations of condition.

This completes the first section of the adjustment. The second section follows, arranged in the same way.

SECTION I.—*Triangulation from Minnesota Point base-line to line Split Rock–Detour.*

NORTH BASE, MINNESOTA POINT—1.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms' 14-inch theodolite, No. 1. Date, June and July, 1871.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Oneota and Lester	124 09 40.69	1_1	7	3.0	2.0	−1.055	+0.165	124 09 39.800
Lester and Aminicon	83 26 13.14	1_2	21	7.1	5.0	−0.353	−1.207	83 26 11.580
Aminicon and South Base	30 12 52.43	1_3	9	3.5	5.7	−0.308	+1.056	30 12 53.178
Lester and South Base	113 39 05.07	1_{2+3}	13	7.6	2.2	−0.161	−0.151	113 39 04.758
South Base and Oneota	122 11 15.61	1_{-1-2-3}	30	5.9	13.7	−0.154	−0.014	122 11 15.442

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$15.7(1_1)+13.7(1_2)+13.7(1_3)+25.619=0$$
$$13.7(1_1)+20.9(1_2)+15.9(1_3)+26.719=0$$
$$13.7(1_1)+15.9(1_2)+21.6(1_3)+26.719=0$$

SOUTH BASE, MINNESOTA POINT—2.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms' 14-inch theodolite, No. 1. Date, July, 1871.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Oneota and North Base	23 08 05.26	2_1	29	5.1	23	−0.132	−0.170	23 08 04.958
North Base and Lester	47 31 20.41	2_2	22	7.4	6	−0.505	−0.298	47 31 19.607
Lester and Aminicon	88 55 19.31	2_3	34	7.8	6	−0.000	+0.270	88 55 19.580
Oneota and Lester	70 39 24.60	2_{1+2}	17	5.7	7	+0.433	−0.468	70 39 24.565

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$30(2_1)+7(2_2)+7.49=0$$
$$7(2_1)+13(2_2)+7.49=0$$

ONEOTA—3.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms' 14-inch theodolite, No. 1. Date, June and July, 1871.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
South Base and North Base	34 40 39.66	3_2	19	2.3	31	−0.025	+0.019	34 40 39.654
South Base and Lester	78 27 05.17	3_{1+2}	29	7.1	8	+0.096	−0.639	78 27 04.627
Lester and North Base	43 46 26.40	3_1	6	6.5	1	−0.769	−0.658	43 46 24.973

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$9(3_1)+8(3_2)+7.12=0$$
$$8(3_1)+39(3_2)+7.12=0$$

SECTION I.—*Triangulation from Minnesota Point base-line, &c.*—Continued.

LESTER—4.

[Observer, J. C. Jones. Instrument, Oertling 20-inch theodolite, No. 2562. Date, July, 1871.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Oneota and South Base	30 53 30.81	4_{1+2}	22	6.9	8	+0.507	−0.141	30 53 31.180
South Base and Aminicon ..	37 56 06.78	4_5	35	8.0	7	+0.579	−0.505	37 56 06.854
Aminicon and Brulé	41 43 08.97	4_4	39	10.3	6	+0.464	+0.629	41 43 10.063
Brulé and Buchanan........	51 00 39.77	4_3	35	12.2	3	+0.927	−0.562	51 00 40.135
Aminicon and Buchanan....	92 43 49.42	4_{3+4}	21	13.1	2	+0.711	+0.067	92 43 50.198
North Base and Aminicon ..	56 45 45.12	4_{5+1}	5	5.8	1	+0.150	−2.585	56 45 42.685
Buchanan and Oneota.......	198 26 27.14	$4-1-2-3-4-5$	14	13.1	1	+4.049	+0.579	198 26 31.768
North Base and Buchanan ..	149 29 35.55	$4_{1+3+4+5}$	4	5.2	1	−0.149	−2.518	149 29 32.883

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$9(4_{1+2}) \quad + (4_5) + (4_4) + (4_3) - 6.53 = 0$$
$$2(4_{5+1}) + \quad + (4_4) + (4_3) - 1.69 = 0$$
$$(4_{1+2}) \quad + 8(4_5) + (4_4) + (4_3) - 6.53 = 0$$
$$(4_{1+2}) + (4_{5+1}) + (4_5) + 10(4_4) + 4(4_3) - 9.58 = 0$$
$$(4_{1+2}) + (4_{5+1}) + (4_5) + 4(4_4) + 7(4_3) - 9.58 = 0$$

AMINICON—5.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms' 14-inch theodolite, No. 1. Date, July, 1871.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
South Base and North Base.	13 20 30.11	5_1	11	7.7	2	−0.940	−1.389	13 20 27.781
North Base and Lester......	39 48 06.39	5_2	10	6.8	2	−0.940	+0.912	39 48 06.362
South Base and Lester......	53 08 34.44	5_{1+2}	34	7.6	9	+0.180	−0.477	53 08 34.143
Lester and Buchanan.....	37 47 05.51	5_3	44	8.7	8	−0.032	+0.209	37 47 05.687
Buchanan and Brulé	60 03 58.62	5_4	57	6.1	31	−0.008	−0.251	60 03 58.361
Brulé and South Base.......	209 00 21.42	5_5	17	10.4	2	−0.130	+0.519	209 00 21.809

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$13(5_1) + 11(5_2) + 2(5_3) + 2(5_4) + 22.64 = 0$$
$$11(5_1) + 13(5_2) + 2(5_3) + 2(5_4) + 22.64 = 0$$
$$2(5_1) + 2(5_2) + 10(5_3) + 2(5_4) + 4.10 = 0$$
$$2(5_1) + 2(5_2) + 2(5_3) + 33(5_4) + 4.10 = 0$$

BUCHANAN—6.

[Observer, A. R. Flint. Instrument, Gambey 10-inch repeating theodolite. Date, July, 1871.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Brulé and Burlington.......	95 03 59.79	6_2	37	5.5	14	+0.150	+0.204	95 04 00.144
Brulé and Aminicon........	47 57 36.25	6_3	35	3.2	37	+0.019	−0.260	47 57 36.009
Aminicon and Lester	49 29 05.15	6_1	34	5.7	12	+0.075	−0.101	49 29 05.124
Aminicon and Burlington ..	143 01 36.20	6_{2+3}	27	5.0	23	+0.009	−0.056	143 01 36.153
Brulé and Lester....	97 26 41.29	6_{3+1}	16	6.0	7	+0.204	−0.361	97 26 41.133
Burlington and Lester......	192 30 41.90	6_{1+2+3}	15	4.9	5	−0.466	−0.157	192 30 41.277

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$24(6_1) + 5(6_2) + 12(6_3) - 2.78 = 0$$
$$5(6_1) + 42(6_2) + 28(6_3) - 7.23 = 0$$
$$12(6_1) + 28(6_2) + 72(6_3) - 6.46 = 0$$

SECTION 1.—*Triangulation from Minnesota Point base-line, &c.*—Continued.

BRULÉ—7.

[Observer, G. A. Marr. Instrument, Pistor & Martius' 14-inch theodolite. Date, July, 1871.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Aminicon and Lester	40 25 47.49	7_1	14	5.4	3	−0.317	−0.091	40 25 47.082
Lester and Buchanan.......	31 32 40.22	7_2	26	3.2	21	−0.045	−0.256	31 32 39.919
Aminicon and Buchanan ...	71 58 27.31	7_{1+2}	39	5.3	25	+0.038	−0.347	71 58 27.001
Buchanan and Burlington ..	28 52 27.22	7_3	39	5.2	21	−0.336	+0.013	28 52 26.897
Burlington and Clay Banks.	79 00 44.40	7_4	31	6.7	16	−0.441	+0.152	79 00 44.111
Buchanan and Clay Banks ..	107 53 08.49	7_{3+4}	14	8.2	3	+2.353	+0.165	107 53 11.008

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$28(7_1)+25(7_2) \qquad +10.00=0$$
$$25(7_1)+46(7_2) \qquad +10.00=0$$
$$24(7_3)+\ 3(7_4)+\ 9.39=0$$
$$3(7_3)+19(7_4)+\ 9.39=0$$

BURLINGTON—8.

[Observer, J. C. Jones. Instrument, Oertling 20-inch theodolite, No. 2562. Date, July, 1871.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Brulé and Buchanan..........	56 03 33.85	8_1	32	6.3	9	0.000	+0.085	56 03 33.935
Brulé and Clay Banks	39 25 53.51	8_3	54	9.8	7	0.000	+0.419	39 25 53.929
Clay Banks and Split Rock ..	85 14 19.15	8_2	67	10.1	8	0.000	−0.593	85 14 18.557

CLAY BANKS—9.

[Observer, A. R. Flint. Instrument, Gambey 10-inch repeating theodolite. Date, June and July, 1871.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Burlington and Brulé, ...	61 33 23.42	9_1	76	5.7	31	+0.170	+0.082	61 33 23.672
Split Rock and Burlington..	38 49 38.70	9_2	72	5.2	44	+0.074	−0.116	38 49 38.658
Split Rock and Detour......	69 38 60.47	9_3	72	6.3	39	+0.077	+0.114	69 39 00.661
Detour and Burlington	108 28 38.98	9_{2+3}	14	5.0	6	+0.341	−0.002	108 28 39.319
Split Rock and Brulé	100 23 02.38	9_{1+2}	26	4.6	15	−0.016	−0.034	100 23 02.330
Detour and Brulé...........	170 02 03.37	9_{1+2+3}	23	4.5	11	−0.459	+0.080	170 02 02.991

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$57(9_1)+26(9_2)+11(9_3)-12.48=0$$
$$26(9_1)+76(9_2)+17(9_3)-11.34=0$$
$$11(9_1)+17(9_2)+56(9_3)-\ 7.44=0$$

SPLIT ROCK—10.

[Observer, G. A. Marr. Instrument, Pistor & Martins' 14-inch theodolite, No. 2. Date, June and July, 1871.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Detour and Clay Banks......	41 36 52.11	10_3	54	5.0	28	0.000	+0.190	41 36 52.300
Clay Banks and Burlington..	55 56 05.46	10_1	64	9.4	12	0.000	−0.395	55 56 05.065

SECTION 1.—*Triangulation from Minnesota Point base-line, &c.*—Continued.

DETOUR—11.

[Observer, J. C. Jones. Instrument, Oertling 20-inch theodolite, No. 2562. Date, June, 1871.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angle.
	° ′ ″			″		″	″	° ′ ″
Clay Banks and Split Rock ..	68 44 09.43	11_1	52	9.7	10	0.000	+0.532	68 44 09.962

Numerical equations of condition in the triangulation from Minnesota Point base-line to the line Split Rock–Detour.

SIDE-EQUATIONS.

I. $+14.288\,[1_1] -9.221\,[1_2] - 9.221\,[1_3] +7.391\,[2_1] -11.888\,[2_2] +17.674\,[3_1] -4.302\,[3_2] + 5.666=0$

II. $-11.643\,[1_2] -9.221\,[1_3] -19.279\,[2_2] +0.396\,[2_3] -15.784\,[5_1] + 9.486\,[5_2] \quad -40.762=0$

III. $-30.062\,[5_3] -2.903\,[5_4] +20.744\,[6_1] +2.751\,[6_3] -24.714\,[7_1] +34.299\,[7_2] \quad +14.863=0$

ANGLE-EQUATIONS.

IV. $+[2_1]+[2_2]+[3_1]+[3_2]+[4_{1+2}]+1.248=0$

V. $+[1_2]+[1_3]+[2_2]+[4_{1+5}]-[4_5]+2.530=0$

VI. $-[1_1]-[1_2]-[1_3]+[2_1]+[3_2]+0.164=0$

VII. $+[1_3]+[2_2]+[2_3]+[5_1]+0.362=0$

VIII. $+[2_3]+[4_5]+[5_1]+[5_2]+0.712=0$

IX. $+[4_3]+[6_1]+[6_3]+[7_2]+1.179=0$

X. $+[4_4]+[5_3]+[5_4]+[7_1]-0.496=0$

XI. $+[5_4]+[6_3]+[7_1]+[7_2]+0.858=0$

XII. $+[6_2]+[7_3]+[8_1]-0.301=0$

XIII. $+[7_4]+[8_3]+[9_1]-0.653=0$

XIV. $+[8_2]+[9_2]+[10_1]+1.104=0$

XV. $+[9_3]+[10_3]+[11_1]-0.836=0$

General corrections in terms of correlates.

$[1_1]$	=+0.37738 I	+0.13879 II	−0.13204 V	−0.04687 VI	−0.06170 VII		
$[1_2]$	=−0.17483 I	−0.10737 II	+0.08061 V	−0.01027 VI	−0.05579 VII		
$[1_3]$	=−0.15337 I	−0.05169 II	+0.07071 V	−0.00901 VI	+0.12650 VII		
$[2_1]$	=+0.05258 I	+0.03958 II	+0.01759 IV	−0.02053 V	+0.03812 VI	−0.02053 VII	
$[2_2]$	=−0.11976 I	−0.16962 II	+0.06745 IV	+0.08798 V	−0.02053 VI	+0.08798 VII	
$[2_3]$	=	+0.00660 II				+0.16667 VII	+0.16667 VIII
$[3_1]$	=+0.25216 I		+0.10801 IV	−0.02788 VI			
$[3_2]$	=−0.06276 I		+0.00348 IV	+0.03136 VI			
$[4_3]$	=−0.01243 IV	−0.04877 V	−0.01420 VIII	+0.19514 IX	−0.06910 X		
$[4_4]$	=−0.00621 IV	−0.02437 V	−0.00710 VIII	−0.06910 IX	+0.13212 X		
$[4_5]$	=−0.01198 IV	−0.11853 V	+0.12918 VIII	−0.01420 IX	−0.00710 X		
$[4_{1+5}]$	=+0.00932 IV	+0.53662 V	+0.01065 VIII	−0.06297 IX	−0.03147 X		
$[4_{1+2}]$	=+0.11451 IV	+0.02130 V	−0.01198 VIII	−0.01243 IX	−0.00621 X		
$[5_1]$	=−0.64542 II	+0.02543 III	+0.27170 VII	+0.04340 VIII	−0.01038 X	−0.00213 XI	
$[5_2]$	=+0.61808 II	+0.02543 III	−0.22830 VII	+0.04340 VIII	−0.01038 X	−0.00213 XI	
$[5_3]$	=+0.00520 II	−0.31220 III	−0.00825 VII	−0.01651 VIII	+0.09904 X	−0.00532 XI	
$[5_4]$	=+0.00134 II	+0.00704 III	−0.00213 VII	−0.00426 VIII	+0.02556 X	+0.03088 XI	

General corrections in terms of correlates—Continued.

$[6_1]$	=	+0.09227 III	+0.03807 IX	—0.00739 XI	—0.00049 XII	
$[6_2]$	=	—0.00443 III	—0.01291 IX	—0.01242 XI	+0.03215 XII	
$[6_3]$	=	—0.00984 III	+0.01256 IX	+0.01995 XI	—0.01242 XII	
$[7_1]$	=—0.30080 III	—0.03771 IX	+0.06938 X	+0.03167 XI		
$[7_2]$	=+0.23805 III	+0.04223 IX	—0.03771 X	+0.00453 XI		
$[7_3]$	=				+0.04251 XII	—0.00671 XIII
$[7_4]$	=				—0.00671 XII	+0.05369 XIII
$[8_1]$	=+0.11111 XII					
$[8_2]$	=		+0.12500 XIV			
$[8_3]$	=	+0.14286 XIII				
$[9_1]$	=	+0.02101 XIII	—0.00672 XIV	—0.00209 XV		
$[9_2]$	=	—0.00672 XIII	+0.01627 XIV	—0.00362 XV		
$[9_3]$	=	—0.00209 XIII	—0.00362 XIV	+0.01937 XV		
$[10_1]$	=		+0.08333 XIV			
$[10_3]$	=			+0.03571 XV		
$[11_1]$	=			+0.10000 XV		

Normal equations for determining the correlates.

No. of equation.							
1.	0*=+0.5666	+1.49573 I	+0.57586 II	+0.12222 IV	—0.44796 V	—0.05936 VI	—0.27313 VII
2.	0 =—4.0762	+0.57586 I	+2.10496 II	—0.01602 III	—0.13004 IV	—0.32868 V	+0.05985 VI
		—0.86013 VII	—0.02074 VIII	+0.00654 X	+0.00134 XI		
3.	0 =+1.4863	—0.01602 II	+2.68507 III	+0.02543 VII	+0.05086 VIII	+0.32048 IX	—0.60596 X
		—0.06555 XI	—0.00443 XII				
4.	0 =+1.2480	+0.12222 I	—0.13004 II	+0.31104 IV	+0.08875 V	+0.02107 VI	+0.06745 VII
		—0.01198 VIII	—0.01243 IX	—0.00621 X			
5.	0 =+2.5300	—0.44796 I	—0.32868 II	+0.08875 IV	+0.89445 V	—0.03981 VI	+0.15869 VII
		—0.11853 VIII	—0.04877 IX	—0.02437 X			
6.	0 =+0.1640	—0.05936 I	+0.05985 II	+0.02107 IV	—0.03981 V	+0.13563 VI	—0.02954 VII
7.	0 =+0.3620	—0.27313 I	—0.86013 II	+0.02543 III	+0.06745 IV	+0.15869 V	—0.02954 VI
		+0.65285 VII	+0.21007 VIII	—0.01038 X	—0.00213 XI		
8.	0 =+0.7120	—0.02074 II	+0.05086 III	—0.01198 IV	—0.11853 V	+0.21007 VII	+0.38265 VIII
		—0.01420 IX	—0.02787 X	—0.00426 XI			
9.	0 =+1.1790	+0.32048 III	—0.01243 IV	—0.04877 V	—0.01420 VIII	+0.28800 IX	—0.10681 X
		+0.01709 XI	—0.01291 XII				
10.	0 =—0.4960	+0.00654 II	—0.60596 III	—0.00621 IV	—0.02437 V	—0.01038 VII	—0.02787 VIII
		—0.10681 IX	+0.32610 X	+0.05723 XI			
11.	0 =+0.8580	+0.00134 II	—0.06555 III	—0.00213 VII	—0.00426 VIII	+0.01709 IX	+0.05723 X
		+0.08703 XI	—0.01242 XII				
12.	0 =—0.3010	—0.00443 III	—0.01291 IX	—0.01242 XI	+0.18577 XII	—0.00671 XIII	
13.	0 =—0.6530	—0.00671 XII	+0.21756 XIII	—0.00672 XIV	—0.00209 XV		
14.	0 =+1.1040		—0.00672 XIII	+0.22460 XIV	—0.00362 XV		
15.	0 =—0.8360		—0.00209 XIII	—0.00362 XIV	+0.15508 XV		

* Each of the *side-equations* was divided by 10 for the purpose of avoiding large numbers in the solution.

Values of the correlates and their logarithms.

I =− 2.2987	log 0.3614823−	IX =− 4.6470	log 0.6671727−
II =+ 6.0147	log 0.7792140+	X =+ 0.8166	log 9.9120094+
III =+ 0.0620	log 8.7923917+	XI =− 9.6120	log 0.9828138−
IV =− 1.7040	log 0.2314696−	XII =+ 0.7620	log 9.8819550+
V =− 5.1014	log 0.7076894−	XIII =+ 2.9300	log 0.4668676+
VI =− 3.7945	log 0.5791546−	XIV =− 4.7420	log 0.6759615−
VII =+10.5966	log 1.0251665+	XV =+ 5.3200	log 0.7259116+
VIII =− 9.2137	log 0.9644341−		

NOTE.—The subscripts + and − attached to the last figure of the logarithms indicate the signs of the corresponding numbers.

Values of the general corrections.

″	″	″	″
$[1_1]$ =+0.165	$[4_4]$ =+0.629	$[6_1]$ =−0.101	$[8_2]$ =−0.593
$[1_2]$ =−1.207	$[4_5]$ =−0.505	$[6_2]$ =+0.204	$[8_3]$ =+0.419
$[1_3]$ =+1.056	$[4_{1+2}]$ =−0.141	$[6_3]$ =−0.260	$[9_1]$ =+0.082
$[2_1]$ =−0.170	$[4_{1+5}]$ =−2.585	$[7_1]$ =−0.091	$[9_2]$ =−0.116
$[2_2]$ =−0.298	$[5_1]$ =−1.389	$[7_2]$ =−0.256	$[9_3]$ =+0.114
$[2_3]$ =+0.270	$[5_2]$ =+0.912	$[7_3]$ =+0.013	$[10_1]$ =−0.395
$[3_1]$ =−0.658	$[5_3]$ =+0.209	$[7_4]$ =+0.152	$[10_3]$ =+0.190
$[3_2]$ =+0.019	$[5_4]$ =−0.251	$[8_1]$ =+0.085	$[11_1]$ =+0.532
$[4_3]$ =−0.562			

Residuals resulting from substitution of corrections in the numerical equations of condition.

No. of equation.	Residual.	No. of equation.	Residual.
1	+0.0010	9	+0.0000
2	+0.0022	10	+0.0000
3	−0.0034	11	+0.0001
4	−0.0000	12	−0.0000
5	+0.0001	13	+0.0001
6	−0.0001	14	−0.0000
7	+0.0001	15	+0.0001
8	+0.0001		

SECTION II.—*Triangulation from line Split Rock-Detour to Keweenaw Base-line.*

SPLIT ROCK—10.

[Observers, O. B. Wheeler and J. C. Jones. Instrument, Oertling 20-inch theodolite, No. 2563. Date, September and October, 1870.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angle.
	° ′ ″			″		″	″	° ′ ″
Sawteeth West and Detour........	95 18 31.11	10_2	111	27.4	4	0.000	−0.598	95 18 30.512

SECTION II.—*Triangulation from line Split Rock, &c.*—Continued.

DETOUR—11.

[Observer, Lieut. E. H. Ruffner. Instrument, Oertling 20-inch theodolite, No. 2561. Dates, September and October, 1870.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Split Rock and Sawteeth West ...	29 31 46.20	11_2	60	14.0	4.5	+0.390	−0.427	29 31 46.163
Sawteeth West and Outer Island .	90 02 33.58	11_3	87	23.5	4.0	+0.438	+0.091	90 02 34.109
Split Rock and Outer Island......	119 34 21.86	11_{2+3}	20	12.4	1.4	−1.252	−0.336	119 34 20.272

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$5.9(11_2)+1.4(11_3)-2.912=0$$
$$1.4(11_2)+5.4(11_3)-2.912=0$$

SAWTEETH WEST—12.

[Observer, A. R. Flint. Instrument, Gambey 10-inch repeating theodolite. Dates, August, September, and October, 1870.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Detour and Split Rock............	55 09 45.69	12_1	46	9.0	10	+0.380	−0.107	55 09 45.963
Sawteeth East and Outer Island ..	75 10 43.97	12_{2+3}	12	5.3	27	0.000	−0.007	75 10 43.963
Farquhar's Knob and Outer Island	62 18 23.81	12_3	72	10.5	17	−0.148	+0.590	62 18 24.252
Outer Island and Detour.........	40 04 34.73	12_4	71	12.2	16	+0.080	+0.037	40 04 34.847
Outer Island and Split Rock.......	95 14 21.08	12_{1+4}	81	10.0	19	−0.200	−0.070	95 14 20.810
Farquhar's Knob and Detour.....	102 22 55.95	12_{3+4}	6	6.3	1	+2.522	+0.627	102 22 59.099

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$29(12_1) \qquad +19(12_4)-12.54=0$$
$$+18(12_3)+\ (12_4)+\ 2.59=0$$
$$19(12_1)+\ (12_3)+36(12_4)-\ 9.95=0$$

OUTER ISLAND—13.

[Observer, G. Y. Wisner. Instrument, Oertling 20-inch theodolite, No. 2562. Dates, August, September, and October, 1870.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Detour and Sawteeth West	49 52 56.52	13_1	120	12.1	17	0.000	−0.006	49 52 56.514
Sawteeth West and Farquhar's Knob	80 00 33.86	13_{2+3}	67	14.7	5	0.000	+1.868	80 00 35.728
Sawteeth East and Farquhar's Knob	77 48 12.97	13_3	77	12.4	9	0.000	−0.329	77 48 12.641

SAWTEETH EAST—14.

[Observer, A. R. Flint. Instrument, Gambey 10-inch repeating theodolite. Dates, July and August, 1870.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Farquhar's Knob and Porcupine Mountains	62 59 40.33	14_2	51	13.5	5	−0.046	+0.609	62 59 40.893
Farquhar's Knob and Outer Island.	64 11 34.92	14_{2+3}	83	13.5	7	−0.356	−0.367	64 11 34.197
Outer Island and Bayfield.	36 08 55.86	14_4	30	9.5	4	−0.623	+0.383	36 08 55.620
Porcupine Mts. and Bayfield	37 20 49.55	14_{3+4}	35	9.9	7	−0.033	−0.593	37 20 48.924
Farquhar's Knob and Bayfield	100 20 29.12	14_{2+3+4}	66	16.0	4	+0.681	+0.016	100 20 29.817

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$12(14_2)-\ 7(14_{2+3})-\ 7(14_4)-\ 6.30=0$$
$$-\ 7(14_2)+18(14_{2+3})+11(14_4)+12.94=0$$
$$-\ 7(14_2)+11(14_{2+3})+15(14_4)+12.94=0$$

SECTION II.—*Triangulation from line Split Rock, &c.*—Continued.

FARQUHAR'S KNOB—15.

[Observers, Lieut. E. H. Ruffner and O. B. Wheeler. Instruments, Oertling 20-inch theodolites, Nos. 2561 and 2563. Dates, September, 1869, and July and August, 1870.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Outer Island and Sawteeth West	37 41 15.42	15_{1+2}	78	16.2	6	0.000	+0.445	37 41 15.865
Outer Island and Sawteeth East	38 00 28.22	15_{1+2+3}	91	17.2	7	0.000	+0.520	38 00 28.740
Bayfield and Sawteeth East	29 19 38.66	15_{2+3}	37	17.2	2	−1.762	−0.211	29 19 36.687
Isle Royale East and Wheal Kate	56 31 44.44	15_4	70	28.8	2	+0.000	+1.388	56 31 45.828
Wheal Kate and Porcupine Mts	39 25 02.18	15_5	39	18.0	2	+0.000	+1.383	39 25 03.563
Porcupine Mts. and Bayfield	38 51 19.20	15_{1+6}	36	9.9	6	−0.587	+0.792	38 51 19.405
Porcupine Mts. and Sawteeth East	68 10 54.63	$15_{1+2+3+6}$	49	15.8	4	+0.881	+0.581	68 10 56.092

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$6(15_{2+3})+4(15_{1+6})+12.92=0$$
$$4(15_{2+3})+10(15_{1+6})+12.92=0$$

NOTE.—The angle 15_4 was read by Lieutenant Ruffner with Oertling, No. 2561. All the other angles were read by O. B. Wheeler with Oertling, No. 2563.

PORCUPINE MOUNTAINS—16.

[Observer, G. Y. Wisner. Instrument, Oertling 20-inch theodolite, No. 2562. Dates, October, 1869, and August, 1870.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Bayfield and Sawteeth East	27 56 54.54	16_1	60	18.1	3	+0.173	+0.258	27 56 54.971
Sawteeth East and Farquhar's Knob	48 49 52.86	16_2	72	11.7	9	+0.058	+0.278	48 49 53.196
Farquhar's Knob and Wheal Kate	77 08 08.64	16_3	66	16.5	4	+0.030	+0.446	77 08 09.116
Bayfield and Farquhar's Knob	76 46 47.83	16_{1+2}	57	19.4	2	−0.199	+0.536	76 46 48.167
Bayfield and Wheal Kate	153 54 56.42	16_{1+2+3}	14	13.8	1	−0.119	+0.982	153 54 57.283

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$6(16_1)+3(16_2)+(16_3)-1.24=0$$
$$3(16_1)+12(16_2)+(16_3)-1.24=0$$
$$(16_1)+(16_2)+5(16_3)-0.38=0$$

WHEAL KATE—17.

[Observers, E. S. Wheeler and A. R. Flint. Instruments, Oertling 20-inch theodolite, No. 2563, and Gambey 10-inch repeating theodolite. Dates, July, August, September, and October, 1869; June, 1870; and August, 1871.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Porcupine Mts. and Farquhar's K'b.	63 27 12.34	17_1	30	12.7	2	+0.408	+1.199	63 27 13.947
Farquhar's K'b and Isle Royale East	51 22 53.64	17_2	53	18.0	3	+0.342	+0.823	51 22 54.805
Isle Royale East and Vulcan	53 40 16.84	17_3	41	14.0	6	+0.171	+0.788	53 40 17.799
Huron Island and Porcupine Mts	214 24 18.40	$17_{1+2+3+4}$	6	8.5	0.5	+0.511	+0.200	214 24 19.111
Farquhar's Knob and Huron Island	150 57 08.72	17_{2+3+4}	6	18.5	0.1	−2.557	−0.999	150 57 05.164
Vulcan and Huron Mts	53 16 36.98	17_{4+5}	33	6.3	13	+0.000	+0.487	53 16 37.467
Vulcan and Quaquaming	77 45 17.39	17_{4+5+6}	28	13.0	2	+0.513	−1.478	77 45 16.425
Farquhar's Knob and Quaquaming	182 48 28.75	$17_{2+3+4+5+6}$	4	7.0	0.3	+0.146	+0.133	182 48 29.029
Porcupine Mts. and Quaquaming	246 15 47.00	$17_{1+2+3+4+5+6}$	5	14.0	0.2	−5.356	+1.332	246 15 42.976

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.7(17_1)+0.2(17_2)+0.2(17_3)+0.5(17_{2+3+4})+0.2(17_{4+5+6})-0.028=0$$
$$0.2(17_1)+3.5(17_2)+0.5(17_3)+0.5(17_{4+5+6})-1.622=0$$
$$0.2(17_1)+0.5(17_2)+6.5(17_3)+0.5(17_{4+5+6})-1.622=0$$
$$0.5(17_1)+0.6(17_{2+3+4})+1.330=0$$
$$0.2(17_1)+0.5(17_2)+0.5(17_3)+2.5(17_{4+5+6})-1.622=0$$

NOTE.—The angles 17_2, 17_3, 17_{4+5+6}, $17_{1+2+3+4+5+6}$, and $17_{2+3+4+5+6}$ were read by E. S. Wheeler with the Oertling theodolite, No. 2563

SECTION II.—*Triangulation from line Split Rock, &c.*—Continued.

ISLE ROYALE EAST—18.

[Observer, G. Y. Wisner. Instrument, Oertling 20-inch theodolite, No. 2562. Dates, June, July, August, and September, 1869.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Wheal Kate and Farquhar's Knob.	72 05 50.11	18_1	39	12.2	5	0.000	+0.555	72 05 50.665
Isle St. Ignace and Vulcan........	107 19 06.78	18_4	41	14.0	4	−0.018	+1.689	107 19 08.451
Vulcan and Wheal Kate..........	41 11 08.52	18_5	14	10.4	2	−0.035	−0.287	41 11 08.198
Isle St. Ignace and Wheal Kate ..	148 30 15.23	18_{4+5}	33	12.2	4	+0.017	+1.402	148 30 16.649

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$8(18_4)+4(18_5)+0.28=0$$

$$4(18_4)+6(18_5)+0.28=0$$

VULCAN—19.

[Observers, G. Y. Wisner and A. R. Flint. Instruments, Troughton & Simms' 14-inch theodolite, No. 1, and Gambey 10-inch repeating theodolite. Dates, August and September, 1871.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Huron Mts. and Crebassa	33 26 31.73	19_1	19	5.0	19	0.000	−0.848	33 26 30.882
Huron Mts. and Traverse Point ..	36 10 31.56	19_{1+2}	9	4.3	4	0.000	−1.203	36 10 30.357
Huron Mts. and Wheal Kate......	49 14 53.72	19_{1+2+3}	41	6.7	19	−0.181	−0.235	49 14 53.304
Huron Mts. and Mt. Houghton....	59 59 15.77	$19_{1+2+3+4}$	16	7.8	3	+1.448	+0.129	59 59 17.347
Huron Mts. and Isle Royale East..	134 23 46.08	$19_{1+2+3+4+5}$	18	4.2	11	+0.160	−0.237	134 23 46.003
Isle St. Ignace and Huron Mts	169 35 59.61	$19_{1+2+3+4+5+6}$	5	4.0	6	−0.328	+0.662	169 35 59.944
Wheal Kate and Mt Houghton ...	10 44 22.46	19_4	15	8.5	4	+1.219	+0.364	10 44 24.043
Mt. Houghton and Isle Royale East	74 24 27.08	19_5	20	7.7	5	+1.942	−0.366	74 24 28.656
Isle Royale East and Isle St. Ignace	35 12 14.14	19_6	5	4.5	2	−1.098	+0.899	35 12 13.941
Wheal Kate and Isle Royale East.	85 08 53.63	19_{4+5}	18	6.7	7	−0.929	−0.002	85 08 52.699
Mt. Houghton and Isle St. Ignace .	109 36 41.54	19_{5+6}	16	5.3	7	+0.524	+0.533	109 36 42.597
Wheal Kate and Isle St. Ignace...	120 21 06.65	19_{4+5+6}	6	5.2	2	−0.907	+0.897	120 21 06.640
Isle St. Ignace and Tip Top.......	56 30 26.98	19_7	6	4.4	2	−1.161	−0.537	56 30 25.282
Isle Royale East and Tip Top	91 42 36.47	19_{6+7}	12	6.3	3	+2.391	+0.362	91 42 39.223
Mt. Houghton and Tip Top	166 07 08.26	19_{5+6+7}	23	5.0	11	−0.377	−0.004	166 07 07.879
Huron Mts. and Tip Top..........	226 06 27.40	$19_{1+2+3+4+5+6+7}$	4	8.8	0.3	−2.299	+0.125	226 06 25.226

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$39.3(19_{1+2+3})+20.3(19_4)+17.3(19_5)+6.3(19_6)+0.3(19_7)-43.956=0$$

$$20.3(19_{1+2+2})+33.3(19_4)+26.3(19_5)+8.3(19_6)+0.3(19_7)-78\ 526=0$$

$$17.3(19_{1+2+3})+26.3(19_4)+49.3(19_5)+26.3(19_6)+11.3(19_7)-82.656=0$$

$$6.3(19_{1+2+3})+8.3(19_4)+26.3(19_5)+31.3[19_6)+14.3(19_7)-9.056=0$$

$$0.3(19_{1+2+3})+0.3(19_4)+11.3(19_5)+14.3)19_6)+16.3(19_7)+12.384=0$$

NOTE.—Angles 19_{1+2+3}, $19_{1+2+3+4}$, 19_4, 19_{4+5}, 19_{5+6}, 19_{4+5+6}, 19_7, 19_{5+6+7}, and $19_{1+2+3+4+5+6+7}$ were read by Wisner with the Troughton & Simms' instrument. Angles 19_1 and 19_{1+2} were read by G. Y. Wisner with the Gambey instrument. Angles $19_{1+2+3+4+5}$, $19_{1+2+3+4+5+6}$, 19_5, 19_6, and 19_{6+7} were read by A. R. Flint with the Gambey instrument.

ISLE ST. IGNACE—20.

[Observer, G. A. Marr. Instrument, Pistor & Martins' 14-inch theodolite. Date, July, 1872.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angle.
	° ′ ″			″		″	″	° ′ ″
Vulcan and Isle Royal East	37 28 57.72	20	94	8.6	19	0.000	+0.651	37 28 58.368

SECTION II.—*Triangulation from line Split Rock, &c.*—Continued.

HURON MOUNTAINS—21.

[Observer, A. R. Flint. Instrument, Troughton & Simms' 14-inch theodolite, No. 1. Date, September, 1871.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			″		″	″	° ′ ″
Wheal Kate and Traverse Island .. 17 40 43.39	21_2	20	8.1	6	−0.120	+0.473	17 40 43.743
Traverse Island and Traverse Point. 12 35 27.79	21_3	32	6.8	16	−0.045	−0.284	12 35 27.461
Traverse Point and Vulcan........ 47 12 28.20	21_4	41	6.3	15	+0.094	−0.729	47 12 27.565
Crebassa and Traverse Point 33 41 03.16	21_{1+2+3}	46	7.3	15	+0.142	+0.257	33 41 03.559
Crebassa and Vulcan.............. 80 53 31.81	$21_{1+2+3+4}$	30	7.9	10	−0.214	−0.472	80 53 31.124
Wheal Kate and Vulcan........... 77 28 39.26	21_{2+3+4}	35	8.2	15	+0.049	−0.540	77 28 38.769

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$25(21_{1+2+3}) \qquad +10(21_4)-4.50=0$$
$$+21(21_2)+15(21_3)+15(21_4)+1.80=0$$
$$+15(21_2)+31(21_3)+15(21_4)+1.80=0$$
$$10(21_{1+2+3})+15(21_2)+15(21_3)+40(21_4)-2.70=0$$

TRAVERSE POINT—22.

[Observer, G. A. Marr. Instrument, Pistor & Martins' 14-inch theodolite, No. 2. Date, October, 1871.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			″		″	″	° ′ ″
Vulcan and Huron Mts..... .... 96 37 07.44	22_1	44	4.8	23	−0.093	−0.760	96 37 06.587
Huron Mts. and Traverse Island .. 56 49 44.50	22_{2+3}	8	4.4	2	−0.216	+0.079	56 49 44.363
Huron Mts. and Crebassa 74 49 51.96	22_{2+3+4}	29	4.9	16	−0.109	−0.092	74 49 51.759
Middle and Crebassa 34 53 31.37	22_{3+4}	33	11.8	4	+0.000	+0.026	34 53 31.396
Crebassa and Vulcan 171 26 58.96	$22_{1+2+3+4}$	44	7.3	9	+0.238	−0.852	171 26 58.346
Traverse Island and Crebassa 18 00 07.61	22_4	14	3.5	10	−0.043	−0.171	18 00 07.396

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$32(22_1)+\ 9(22_{2+3})+\ 9(22_4)+5.31=0$$
$$9(22_1)+27(22_{2+3})+25(22_4)+7.71=0$$
$$9(22_1)+25(22_{2+3})+35(22_4)+7.71=0$$

MIDDLE—23.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms' 14-inch theodolite, No. 1. Date, October, 1871.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			″		″	″	° ′ ″
Quaquaming and South Base 13 42 48.28	23_1	24	7.7	7	+0.030	+0.694	13 42 49.004
South Base and North Base 25 20 34.44	23_2	15	6.2	4	+0.486	+1.383	25 20 36.309
North Base and Crebassa 18 36 46.34	23_3	17	6.9	4	+0.486	−1.679	18 36 45.147
Crebassa and Quaquaming 57 40 10.08	23_{1+2+3}	24	5.9	12	−0.018	+0.398	57 40 10.460
South Base and Crebassa.......... 43 57 21.91	23_{2+3}	25	6.3	11	−0.158	−0.296	43 57 21.456
Crebassa and Traverse Island 57 04 51.37	23_4	48	8.6	11	0.000	−0.282	57 04 51.088
Crebassa and Traverse Point 66 38 51.13	23_{4+5}	43	8.3	9	0.000	+0.112	66 38 51.242

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$19(23_1)+12(23_2)+12(23_3)-12.24=0$$
$$12(23_1)+27(23_2)+23(23_3)-24.67=0$$
$$12(23_1)+23)22_2)+27(23_3)-24.67=0$$

SECTION II.—*Triangulation from line Split Rock, &c.*—Continued.

QUAQUAMING—25.

[Observer, A. R. Flint. Instrument, Gambey 10-inch repeating theodolite. Date, October, 1871.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
South Base and Middle Base	29 13 58. 02	25_1	20	3. 5	19	+0. 118	−0. 330	29 13 57. 808
Middle Base and North Base......	25 51 07. 32	25_2	20	4. 6	10	+0. 224	−0. 627	25 51 06. 917
South Base and North Base.	55 05 05. 93	25_{1+2}	24	7. 0	0	−0. 248	−0. 957	55 05 04. 725
South Base and Crebassa	79 12 09. 33	$25_{1+2+3+4}$	23	6. 3	10	0. 000	+1. 191	79 12 10. 521
Wheal Kate and Traverse Island .	64 21 56. 78	25_{4+5}	8	4. 0	3	−0. 112	+0. 675	64 21 57. 343
Wheal Kate and Middle	101 24 50. 72	25_{4+5+6}	8	4. 5	4	+0. 083	+0. 067	101 24 50. 870
Crebassa and Middle	68 00 15. 85	25_{5+6}	25	4. 2	19	−0. 018	−0. 231	68 00 15. 601
Crebassa and Traverse Island	30 57 21. 53	25_5	4	2. 8	2	+0. 167	+0. 377	30 57 22. 074

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$7(25_{4+5})+\ 4(25_{5+6})-4(25_6)+1.52=0$$
$$4(25_{4+5})+23(25_{5+6})-4(25_6)+1.52=0$$
$$-4(25_{4+5})-\ 4(25_{5+6})+6(25_6)-1.52=0$$

CREBASSA—26.

[Observers, G. A. Marr, A. R. Flint, and G. Y. Wisner. Instruments, Pistor & Martins' 14-inch theodolite No. 2. and Gambey 10-inch repeating theodolite. Dates, September and October, 1871.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Traverse Point and Vulcan	5 49 02. 08	26_1	22	6. 5	9	+1. 201	−0. 708	5 49 02. 573
Vulcan and Traverse Island	5 22 56. 43	26_2	29	7. 3	8	+0. 343	+0. 310	5 22 57. 083
Traverse Island and Huron Mts ..	60 17 07. 29	26_3	54	8. 3	12	+0. 193	−0. 396	60 17 07. 087
Traverse Island and Middle	67 15 39. 15	26_{3+4}	64	9. 5	12	−0. 575	−0. 107	67 15 38. 468
Middle and Quaquaming..........	54 19 34. 59	26_5	49	7. 9	15	−0. 570	+0. 278	54 19 34. 298
Qoaquaming and South Base	38 13 52. 80	26_6	47	10. 6	10	0. 000	+0. 798	38 13 53. 598
Traverse Point and Traverse I'd..	11 11 59. 81	26_{1+2}	23	6. 8	5	+0. 244	−0. 398	11 11 59. 656
Traverse Island and Quaquaming	121 35 12. 06	26_{3+4+5}	42	5. 7	16	+0. 535	+0. 171	121 35 12. 766
Quaqusming and North Base	51 05 40. 71	26_{6+7}	63	11. 6	7	0. 000	+0. 371	51 05 41. 081
Traverse Point and Huron Mts ...	71 29 09. 02	26_{1+2+3}	66	11. 7	7	−1. 483	−0. 794	71 29 06. 743
Vulcan and Huron Mts	65 40 03. 36	26_{2+3}	36	6. 9	9	+0. 896	−0. 086	65 40 04. 170
Traverse Point and Middle	78 27 39. 04	$26_{1+2+3+4}$	44	13. 5	4	−0. 411	−0. 505	78 27 38. 124

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$25(26_1)+16(26_2)+\ 7(26_3)+\ 4(26_{3+4})\qquad -34.56=0$$
$$16(26_1)+33(26_2)+16(26_3)+\ 4(26_{3+4})\qquad -31.32=0$$
$$7(26_1)+16(26_2)+28(26_3)\qquad -19.30=0$$
$$4(26_1)+\ 4(26_2)\qquad +32(26_{3+4})+16(26_5)+21.36=0$$
$$+16(26_{3+4})+31(26_5)+26.28=0$$

NOTE.—Angles 26_1, 26_{2+3}, 26_3, 26_{3+4}, $26_{1+2+3+4}$, 26_5, 26_{3+4+5}, and 26_{6+7} were read by G. A. Marr with the Pistor & Martins theodolite. Angles 26_2, 26_{1+2}, and 26_6 were read by G. A. Marr with the Pistor & Martins and Gambey theodolites. Angle 26_{1+2+3} was read partly by G. A. Marr with the Pistor & Martins theodolite, and partly by A. R. Flint and G. Y. Wisner with Gambey theodolite.

SOUTH BASE, KEWEENAW POINT—27.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms 14-inch theodolite No. 1. Date, October, 1871.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
North Base and Crebassa.... ..	7 42 42. 94	27_1	36	8 3	10	0. 000	+0. 964	7 42 43. 904
Crebassa and Middle............	43 29 10. 65	27_2	37	5. 6	15	+0. 277	+0. 210	43 29 11. 137
Middle and Quaquaming.........	19 04 45. 35	27_3	39	8. 1	15	+0. 277	−0. 607	19 04 45. 020
Crebassa and Quaquaming	62 33 59. 33	27_{2+3}	4	3. 6	1. 5	−2. 776	−0. 397	62 33 56. 157

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$16.5(27_2)+\ 1.5(27_3)-4.995=0$$
$$1.5(27_2)+16.5(27_3)-4.995=0$$

SECTION II.—*Triangulation from line Split Rock, &c.*—Continued.

NORTH BASE, KEWEENAW POINT—28.

[Observer, A. R. Flint. Instrument, Gambey 10-inch repeating theodolite. Date, October, 1871.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	$[v]$	Corrected angles.
° ′ ″			″		″	″	° ′ ″
Crebassa and Traverse Island.. 5 18 34.38	28_1	4	2 4	3	0.000	−0.293	5 18 34.087
Crebassa and Middle...... ... 55 57 58.68	28_{1+2}	4	2.2	3	+0.285	+0.689	55 57 59.654
Middle and Quaquaming...... 48 49 13.64	28_3	12	2.5	19	+0.005	−0.044	48 49 13.601
Crebassa and Quaquaming.....104 47 12.66	28_{1+2+3}	8	1.8	17	−0.050	+0.645	104 47 13.255
Quaquaming and South Base... 54 38 15.53	28_4	24	4.2	13	−0.059	−0.072	54 38 15.399
Middle and Seuth Base .... .103 27 28.99	28_{3+4}	20	6.1	6	+0.126	−0.116	103 27 29.000

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$20(28_{1+2})+17(28_3) \qquad -5.78=0$$
$$17(28_{1+2})+42(28_3)+6\ (28_4)-4.70=0$$
$$+6(28_3)+19(28_4)+1.08=0$$

Numerical equations of condition in the triangulation from the line Split Rock–Detour to Keweenaw Base-line.

SIDE-EQUATIONS.

$$\text{I.}\ (10)\ +5.5578\,[28_{1+2}]\ +5.5578\,[28_3]\ +14.9423\,[28_4]\ -7.5481\,[27_1]$$
$$+3.3819\,[27_2]\ +3.3819\,[27_3]\ +16.9927\,[26_{6+7}]\ -26.7263\,[26_6]\ +21.135=0$$
$$\text{II.}\ (30)\ -18.4192\,[28_3]\ +14.9423\,[28_4]\ -7.5481\,[27_1]\ -7.5481\,[27_2]$$
$$+53.3266\,[27_3]\ -60.3353\,[23_1]\ +25.9487\,[23_2]\ +47.475=0$$
$$\text{III.}\ (10)\ +5.5578\,[28_{1+2}]\ +23.9770\,[28_3]\ -15.1151\,[26_5]\ +16.9927\,[26_{6+7}]$$
$$-12.6225\,[23_1]\ -12.6225\,[23_2]\ +13.3262\,[23_3]\ +43.726=0$$
$$\text{IV.}\ (20)\ -21.7711\,[26_{3+4}]\ -12.9466\,[26_5]\ +27.8922\,[25_{5+6}]\ -62.9949\,[25_5]$$
$$+9.7068\,[23_1]\ +9.7068\,[23_2]\ +9.7068\,[23_3]\ +23.3380\,[23_4]\ +34.179=0$$
$$\text{V.}\ (100)\ -243.8175\,[28_1]\ +17.2600\,[28_{1+2}]\ -172.7654\,[26_{3+4}]\ -163.9409\,[26_5]$$
$$-163.9409\,[26_{6+7}]\ -5.3693\,[23_3]\ +8.2619\,[23_4]\ -2.147=0$$
$$\text{VI.}\ (100)\ -106.3363\,[26_1]\ -106.3363\,[26_2]\ +8.8245\,[26_{3+4}]\ -138.5598\,[23_4]$$
$$+124.9286\,[23_{4+5}]\ -69.3446\,[22_{3+4}]\ +134.1377\,[22_4]\ -69.737=0$$
$$\text{VII.}\ (100)\ -106.3363\,[26_1]\ -106.3363\,[26_2]\ +12.0168\,[26_3]\ -13.7629\,[22_{2+3}]$$
$$+64.7931\,[22_4]\ -54.5851\,[21_{1+2+3}]\ +148.8498\,[21_3]\ +30.913=0$$
$$\text{VIII.}\ (10)\ -7.0509\,[26_1]\ +2.4701\,[26_2]\ +2.4701\,[26_3]\ +2.4431\,[22_1]$$
$$+5.7083\,[22_{2+3}]\ +5.7083\,[22_4]\ +28.7942\,[19_{1+2}]\ -31.8808\,[19_1]\ +5.198=0$$
$$\text{IX.}\ (100)\ -24.3099\,[26_2]\ -137.8826\,[26_3]\ +103.5727\,[26_{3+4}]\ +103.5727\,[26_5]$$
$$-71.6531\,[25_{4+5}]\ -71.6531\,[25_5]\ -8.5050\,[25_{5+6}]\ +13.3262\,[23_1]$$
$$+13.3262\,[23_2]\ +13.3262\,[23_3]\ -9.0907\,[23_{4+5}]\ -5.7083\,[22_{2+3}]$$
$$-5.7083\,[22_4]\ +30.1908\,[22_{3+4}]\ +31.5897\,[21_{1+2+3}]\ +114.8858\,[19_1]$$
$$-65.3818\,[19_{1+2+3}]\ +223.2634\,[17_{4+5}]\ -166.6608\,[17_{4+5+6}]\ -328.971=0$$
$$\text{X.}\ (30)\ -34.7388\,[16_1]\ +4.9462\,[16_2]\ +37.4786\,[15_{2+3}]\ -26.1358\,[15_{1+6}]$$
$$+31.4340\,[14_{2+3}]\ -27.5918\,[14_2]\ +31.4340\,[14_4]\ +52.485=0$$
$$\text{XI.}\ (10)\ +27.2544\,[15_{1+2}]\ -26.9418\,[15_{1+2+3}]\ +10.1817\,[14_{2+3}]\ -4.7123\,[13_{2+3}]$$
$$+4.7123\,[13_3]\ -11.0511\,[12_3]\ +0.8590\,[12_{2+3}]\ +22.502=0$$

ANGLE-EQUATIONS.

$$\text{XII.}\ [28_4]\ +[27_1]\ +[27_2]\ +[27_3]\ +[25_1]\ +[25_2]\ +0.462=0$$
$$\text{XIII.}\ [27_2]\ +[27_3]\ +[26_6]\ +[25_{1+2+3+4}]\ -1.592=0$$
$$\text{XIV.}\ [28_{1+2}]\ +[28_3]\ +[26_{6+7}]\ +[25_{1+2+\ +4}]\ -[25_1]\ -[25_2]\ -3.164=0$$
$$\text{XV.}\ [28_{1+2}]\ +[26_5]\ +[26_{6+7}]\ +[23_3]\ +0.341=0$$
$$\text{XVI.}\ [28_3]\ +[25_{1+2+3+4}]\ -[25_1]\ -[25_2]\ +[25_{5+6}]\ +[23_1]\ +[23_2]\ -3.950=0$$
$$\text{XVII.}\ [28_3]\ +[28_4]\ +[27_1]\ +[27_2]\ +[23_2]\ -2.441=0$$

NOTE.—In the solution for determining the general corrections, each of the side-equations was divided by the number inclosed in parenthesis and placed opposite it.

Numerical equations of condition, &c.—Continued.

ANGLE-EQUATIONS—Continued.

XVIII.	$[26_1]$	$+[26_2]$	$+[26_{3+4}]$	$+[23_{4+5}]$	$+[22_{3+4}]$				$+0.367=0$
XIX.	$[26_1]$	$+[26_2]$	$+[26_3]$	$+[22_{2+3}]$	$+[22_4]$	$+[21_{1+2+3}]$			$+0.629=0$
XX.	$[26_2]$	$+[26_3]$	$+[21_{1+2+3}]$	$+[21_4]$	$-[19_1]$				$+1.407=0$
XXI.	$[22_1]$	$+[21_4]$	$+[19_{1+2}]$						$+2.692=0$
XXII.	$[21_2]$	$+[21_3]$	$+[21_4]$	$+[19_{1+2+3}]$	$+[17_{4+5}]$				$+0.288=0$
XXIII.	$[17_{4+5+6}]$	$-[17_{4+5}]$	$+[25_{4+5}]$	$-[25_5]$	$+[21_{1+2+3}]$	$-[21_2]$	$-[21_3]$	$-[26_{3+4}]$	
		$+[26_3]$	$-[26_5]$						$+2.166=0$
XXIV.	$[19_4]$	$+[19_5]$	$+[18_5]$	$+[17_3]$					$-0.499=0$
XXV.	$[20]$	$+[19_6]$	$+[18_4]$						$-3.238=0$
XXVI.	$[18_1]$	$+[17_2]$	$+[15_4]$						$-2.766=0$
XXVII.	$[17_1]$	$+[16_3]$	$+[15_5]$						$-3.028=0$
XXVIII.	$[16_2]$	$+[15_{1+6}]$	$+[15_{2+3}]$	$+[14_2]$					$-1.468=0$
XXIX.	$[15_{1+2+3}]$	$+[14_{2+3}]$	$+[13_3]$						$+0.177=0$
XXX.	$[15_{1+2}]$	$+[13_{2+3}]$	$+[12_3]$						$-2.903=0$
XXXI.	$[13_1]$	$+[12_4]$	$+[11_3]$						$-0.121=0$
XXXII.	$[12_1]$	$+[11_2]$	$+[10_2]$						$+1.131=0$

General corrections in terms of the correlates.

$[28_1]$	=			−0.81273 V		
$[28_{1+2}]$	=+0.04066 I	+0.02555 II	−0.03600 III	+0.01349 V	+0.01046 XII	+0.04503 XIV
	+0.07816 XV	−0.03313 XVI	−0.02267 XVII			
$[28_3]$	=−0.01514 I	−0.03006 II	+0.07504 III	−0.00572 V	−0.01231 XII	+0.00585 XIV
	−0.03313 XV	+0.03898 XVI	+0.02267 XVII			
$[28_4]$	=+0.08342 I	+0.03571 II	−0.02370 III	+0.00181 V	+0.05652 XII	−0.00185 XIV
	+0.01046 XV	−0.01231 XVI	+0.04421 XVII			
$[27_1]$	=−0.07548 I	−0.02516 II	+0.10000 XII		+0.10000 XVII	
$[27_2]$	=+0.01879 I	−0.02525 II	+0.05556 XII	+0.05556 XIII	+0.06111 XVII	
$[27_3]$	=+0.01879 I	+0.11003 II	+0.05556 XII	+0.05556 XIII	−0.00556 XVII	
$[26_1]$	=−0.00394 III	+0.00381 IV	+0.00446 V	−0.03236 VI	−0.03171 VII	−0.04803 VIII
	+0.00495 IX	+0.00261 XV	+0.02495 XVIII	+0.03172 XIX	−0.02695 XX	+0.00416 XXIII
$[26_2]$	=−0.00374 III	+0.00362 IV	+0.00423 V	−0.03070 VI	−0.03334 VII	+0.02803 VIII
	+0.01313 IX	+0.00248 XV	+0.02368 XVIII	+0.00299 XIX	+0.03166 XX	−0.02316 XXIII
$[26_3]$	=+0.00313 III	−0.00302 IV	−0.00353 V	+0.02563 VI	+0.03127 VII	+0.00481 VIII
	−0.05798 IX	−0.00207 XV	−0.01977 XVIII	+0.02607 XIX	+0.02436 XX	+0.04791 XXIII
$[26_{3+4}]$	=+0.03415 III	−0.03303 IV	−0.03859 V	+0.01434 VI	+0.01096 VII	+0.00337 VIII
	+0.01806 IX	−0.02260 XV	+0.03393 XVIII	−0.00585 XIX	−0.00079 XX	−0.01718 XXIII
$[26_5]$	=−0.06639 III	−0.00383 IV	−0.03296 V	−0.00740 VI	−0.00566 VII	−0.00174 VIII
	+0.02409 IX	+0.04392 XV	−0.01751 XVIII	+0.00302 XIX	+0.00041 XX	−0.02339 XXIII
$[26_6]$	=−0.26726 I			+0.10000 XIII		
$[26_{6+7}]$	=+0.24275 I	+0.24275 III	−0.23425 V		+0.14285 XIV	+0.14286 XV
$[25_1]$	=+0.02217 XII	−0.02217 XIV	−0.02217 XVI			
$[25_2]$	=+0.04213 XII	−0.04213 XIV	−0.04213 XVI			
$[25_{1+2+3+4}]$	=+0.10000 XIII	+0.10000 XIV	+0.10000 XVI			
$[25_{4+5}]$	=−0.48366 IV	−0.06232 IX	−0.01544 XVI	+0.08880 XXIII		
$[25_5]$	=−0.84938 IV	+0.09348 IX	+0.02317 XVI	−0.13321 XXIII		
$[25_{5+6}]$	=−0.00297 IV	+0.02340 IX	+0.05019 XVI	−0.03861 XXIII		
$[23_1]$	=−0.16758 II	−0.09661 III	+0.01906 IV	+0.00097 V	+0.00523 IX	−0.01813 XV
	+0.05740 XVI	−0.01813 XVII				
$[23_2]$	=+0.15699 II	−0.30047 III	+0.00513 IV	+0.00594 V	+0.00140 IX	−0.11065 XV
	+0.12122 XVI	+0.13935 XVII				

General corrections in terms of the correlates—Continued.

$[23_3]$	=−0.05925 II	+0.34825 III	+0.00513 IV	−0.00748 V	+0.00140 IX	+0.13935 XV
	−0.12878 XVI	−0.11065 XVII				
$[23_4]$	=+0.10608 IV	+0.00751 V	−0.12596 VI			
$[23_{4+5}]$	=		+0.13881 VI	−0.01010 IX	+0.11111 XVIII	
$[22_1]$	=−0.00261 VI	+0.00008 VII	+0.00179 VIII	+0.00067 IX	−0.01165 XIX	+0.03453 XXI
$[22_{2+3}]$	=−0.10406 VI	−0.06570 VII	+0.01734 VIII	−0.00197 IX	+0.03453 XIX	−0.00971 XXI
$[22_4]$	=+0.11333 VI	+0.06542 VII	+0.00347 VIII	−0.00039 IX	+0.00691 XIX	−0.00194 XXI
$[22_{3+4}]$	=−0.17336 VI			+0.07548 IX	+0.25000 XVIII	
$[21_{1+2+3}]$	=−0.02026 VII	+0.01471 IX	+0.04656 XIX	+0.03016 XX	−0.01641 XXI	−0.00370 XXII
	+0.03385 XXIII					
$[21_2]$	=−0.05019 VII	+0.00292 IX	+0.00924 XIX	−0.01386 XX	−0.02311 XXI	+0.03235 XXII
	−0.04622 XXIII					
$[21_3]$	=+0.07421 VII	+0.00110 IX	+0.00347 XIX	−0.00520 XX	−0.00867 XXI	+0.01213 XXII
	−0.01733 XXIII					
$[21_4]$	=−0.00394 VII	−0.00518 IX	−0.01641 XIX	+0.02461 XX	+0.04102 XXI	+0.00924 XXII
	+0.01537 XXIII					
$[20]$	=+0.05263 XXV					
$[19_1]$	=−0.16779 VIII	+0.06047 IX	+0.05263 XX			
$[19_{1+2}]$	=+0.71986 VIII			+0.25000 XXI		
$[19_{1+2+3}]$	=	−0.02437 IX	+0.03727 XXII	−0.02256 XXIV	−0.00177 XXV	
$[19_4]$	=	+0.01393 IX	−0.02131 XXII	+0.03098 XXIV	+0.01227 XXV	
$[19_5]$	=	+0.00082 IX	−0.00126 XXII	+0.02489 XXIV	−0.04003 XXV	
$[19_6]$	=	+0.00116 IX	−0.00177 XXII	−0.02776 XXIV	+0.08361 XXV	
$[19_7]$	=	−0.00139 IX	+0.00213 XXII	+0.00694 XXIV	−0.04579 XXV	
$[18_4]$	=			−0.12500 XXIV	+0.18750 XXV	
$[18_5]$	=			+0.25000 XXIV	−0.12500 XXV	
$[17_1]$	=+0.04924 IX	−0.02954 XXIII	−0.00985 XXIV	−0.01970 XXVI	+0.44313 XXVII	
$[17_2]$	=+0.09040 IX	−0.05424 XXIII	−0.01808 XXIV	+0.29717 XXVI	−0.01970 XXVII	
$[17_3]$	=+0.04520 IX	−0.02712 XXIII	+0.15763 XXIV	−0.01808 XXVI	−0.00985 XXVII	
$[17_{2+3+4}]$	=−0.04103 IX	+0.02462 XXIII	+0.00821 XXIV	+0.01641 XXVI	−0.36928 XXVII	
$[17_{4+5+6}]$	=−0.69770 IX	+0.41864 XXIII	−0.02712 XXIV	−0.05424 XXVI	−0.02954 XXVII	
$[17_{4+5}]$	=+0.17174 IX	+0.07692 XXII	−0.07692 XXIII			
$[16_1]$	=−0.23310 X	−0.02970 XXVII	−0.04620 XXVIII			
$[16_2]$	=+0.06928 X	−0.00990 XXVII	+0.09571 XXVIII			
$[16_3]$	=+0.03276 X	+0.20792 XXVII	−0.00990 XXVIII			
$[15_{1+2}]$	=+0.45424 XI	+0.16667 XXX				
$[15_{1+2+3}]$	=−0.38488 XI	+0.14286 XXIX				
$[15_{2+3}]$	=+0.36313 X	+0.13636 XXVIII				
$[12_{6+1}]$	=−0.23237 X	+0.04545 XXVIII				
$[15_4]$	=+0.50000 XXVI					
$[15_6]$	=+0.50000 XXVII					
$[14_2]$	=−0.04512 X	+0.02283 XI	+0.11930 XXVIII	+0.02242 XXIX		
$[14_{2+3}]$	=+0.01965 X	+0.10679 XI	+0.02242 XXVIII	+0.10488 XXIX		
$[14_4]$	=+0.03439 X	−0.06766 XI	+0.03923 XXVIII	−0.06645 XXIX		
$[13_1]$	=+0.05882 XXXI					
$[13_{2+3}]$	=−0.09425 XI	+0.20000 XXX				
$[13_3]$	=+0.05236 XI	+0.11111 XXIX				
$[12_1]$	=−0.00171 XI	+0.00155 XXX	−0.02788 XXXI	+0.05275 XXXII		
$[12_3]$	=−0.06154 XI	+0.05569 XXX	−0.00236 XXXI	+0.00155 XXXII		
$[12_4]$	=+0.00261 XI	−0.00236 XXX	+0.04256 XXXI	−0.02788 XXXII		
$[12_{2+3}]$	=+0.00318 XI					
$[11_2]$	=		−0.04682 XXXI	+0.18060 XXXII		
$[11_3]$	=		+0.19732 XXXI	−0.04682 XXXII		
$[10_2]$	=			+0.25000 XXXII		

Normal equations for determining the correlates.

No. of equation.						
1.	0 =+2.11350	+1.33531 I	+0.09851 II	+0.39879 III	−0.39095 V	+0.04552 XII
		−0.22968 XIII	+0.26827 XIV	+0.28341 XV	−0.01514 XVI	+0.01159 XVII
2.	0 =+1.58250	+0.09851 I	+0.71733 II	−0.12347 III	−0.03390 IV	+0.00760 V
		−0.00931 IX	+0.09533 XII	+0.08478 XIII	−0.00451 XIV	−0.03370 XV
		−0.04065 XVI	+0.11223 XVII			
3.	0 =+4.37260	+0.39879 I	−0.12347 II	+1.63806 III	−0.01790 IV	−0.37305 V
		+0.01118 VI	+0.00855 VII	+0.00263 VIII	−0.04292 IX	−0.02370 XII
		+0.28179 XIV	+0.48861 XV	−0.32204 XVI	−0.24913 XVII	+0.02647 XVIII
		−0.00455 XIX	−0.00061 XX	+0.03537 XXIII		
4.	0 =+1.70985	−0.03390 II	−0.01790 III	+2.84763 IV	+0.07183 V	−0.15780 VI
		−0.00826 VII	−0.00254 VIII	−0.29315 IX	+0.00130 XV	+0.02122 XVI
		+0.00513 XVII	−0.02560 XVIII	+0.00441 XIX	+0.00060 XX	+0.39956 XXIII
5.	0 =−0.02147	−0.39095 I	+0.00760 II	−0.37305 III	+0.07183 IV	+2.48959 V
		−0.02305 VI	−0.00966 VII	−0.00297 VIII	−0.07077 IX	+0.00181 XII
		−0.22643 XIV	−0.26115 XV	+0.00119 XVI	+0.00203 XVII	+0.02990 XVIII
		+0.00516 XIX	+0.00070 XX	+0.06802 XXIII		
6.	0 =−0.69737	+0.01118 III	−0.15780 IV	−0.02305 V	+0.68850 VI	+0.15789 VII
		+0.02622 VIII	−0.08311 IX	−0.00740 XV	−0.08327 XVIII	−0.02816 XIX
		−0.00507 XX	−0.00261 XXI	+0.01869 XXIII		
7.	0 =+0.30913	+0.00855 III	−0.00826 IV	−0.00966 V	+0.15789 VI	+0.24588 VII
		+0.02171 VIII	−0.03257 IX	−0.00566 XV	−0.05409 XVIII	−0.05432 XIX
		−0.02627 XX	−0.00386 XXI	+0.02008 XXII	−0.01831 XXIII	
8.	0 =+0.51980	+0.00263 III	−0.00254 IV	−0.00297 V	+0.02622 VI	+0.02171 VII
		+2.66200 VIII	−0.20853 IX	−0.00174 XV	−0.01663 XVIII	+0.00562 XIX
		−0.13495 XX	+0.72165 XXI	+0.00318 XXIII		
9.	0 =−3.28971	−0.00931 II	−0.04292 III	−0.29315 IV	−0.07077 V	−0.08311 VI
		−0.03257 VII	−0.20853 VIII	+1.88993 IX	+0.02549 XV	+0.03003 XVI
		+0.00140 XVII	+0.10152 XVIII	−0.02755 XIX	+0.02515 XX	−0.00451 XXI
		+0.14621 XXII	−1.11468 XXIII	+0.05995 XXIV	+0.00116 XXV	+0.09040 XXVI
		+0.04924 XXVII				
10.	0 =+1.74950	+1.03556 X	+0.02001 XI	+0.03276 XXVII	+0.15492 XXVIII	+0.01965 XXIX
11.	0 =+2.25020	+0.02001 X	+2.52104 XI	+0.02283 XXVIII	−0.22573 XXIX	+0.29845 XXX
		+0.00261 XXXI	−0.00171 XXXII			
12.	0 =+0.46200	+0.04552 I	+0.09533 II	−0.02370 III	+0.00181 V	+0.33194 XII
		+0.11111 XIII	−0.06615 XIV	+0.01046 XV	−0.07661 XVI	+0.19976 XVII
13.	0 =−1.59200	−0.22968 I	+0.08478 II	+0.11111 XII	+0.31111 XIII	+0.10000 XIV
		+0.10000 XVI	+0.05556 XVII			
14.	0 =−3.16400	+0.26827 I	−0.00451 II	+0.28179 III	−0.22643 V	−0.06615 XII
		+0.10000 XIII	+0.35804 XIV	+0.18789 XV	+0.17015 XVI	+0.00400 XVII
15.	0 =+0.34100	+0.28341 I	−0.03370 II	+0.48831 III	+0.00130 IV	−0.26115 V
		−0.00740 VI	−0.00566 VII	−0.00174 VIII	+0.02549 IX	+0.01046 XII
		+0.18789 XIV	+0.40429 XV	−0.16191 XVI	−0.13332 XVII	−0.01751 XVIII
		+0.00302 XIX	+0.00041 XX	−0.02339 XXIII		
16.	0 =−3.95000	−0.01514 I	−0.04065 II	−0.32204 III	+0.02122 IV	+0.00119 V
		+0.03003 IX	−0.07661 XII	+0.10000 XIII	+0.17015 XIV	−0.16191 XV
		+0.43209 XVI	+0.14789 XVII	−0.03861 XXIII		
17.	0 =−2.44100	+0.01159 I	+0.11223 II	−0.24913 III	+0.00513 IV	+0.00203 V
		+0.00140 IX	+0.19976 XII	+0.05556 XIII	+0.00400 XIV	−0.13332 XV
		+0.14789 XVI	+0.37134 XVII			
18.	0 =+0.36700	+0.02647 III	−0.02560 IV	−0.02990 V	−0.08327 VI	−0.05409 VII
		−0.01663 VIII	+0.10152 IX	−0.01751 XV	+0.44367 XVIII	+0.02886 XIX
		+0.00392 XX	−0.03618 XXIII			

Normal equations for determining the correlates—Continued.

No. of equation.						
19.	0 =+0.62900	−0.00455 III	+0.00441 IV	+0.00516 V	−0.02816 VI	−0.05432 VII
		+0.00562 VIII	−0.02755 IX	+0.00302 XV	+0.02886 XVIII	+0.14878 XIX
		+0.05923 XX	−0.02806 XXI	−0.00370 XXII	+0.06276 XXIII	
20.	0 =+1.40700	−0.00061 III	+0.00060 IV	+0.00070 V	−0.00507 VI	−0.02627 VII
		−0.13495 VIII	+0.02515 IX	+0.00041 XV	+0.00392 XVIII	+0.05923 XIX
		+0.16342 XX	+0.02461 XXI	+0.00554 XXII	+0.07397 XXIII	
21.	0 =+2.69200	−0.00261 VI	−0.00386 VII	+0.72165 VIII	−0.00451 IX	−0.02806 XIX
		+0.02461 XX	+0.32555 XXI	+0.00924 XXII	+0.01537 XXIII	
22.	0 =+0.28800	+0.02008 VII	+0.14621 IX	−0.00370 XIX	+0.00554 XX	+0.00924 XXI
		+0.16791 XXII	−0.12510 XXIII	−0.02256 XXIV	−0.00177 XXV	
23.	0 =+2.16600	+0.03537 III	+0.39956 IV	+0.06802 V	+0.01869 VI	−0.01831 VII
		+0.00318 VIII	−1.11468 IX	−0.02339 XV	−0.03861 XVI	−0.03618 XVIII
		+0.06276 XIX	+0.07397 XX	+0.01537 XXI	−0.12510 XXII	+0.90345 XXIII
		−0.02712 XXIV	−0.05424 XXVI	−0.02954 XXVII		
24.	0 =−0.49900	+0.05995 IX	−0.02256 XXII	−0.02712 XXIII	+0.46350 XXIV	−0.15276 XXV
		−0.01808 XXVI	−0.00985 XXVII			
25.	0 =−3.23800	+0.00116 IX	−0.00177 XXII	−0.15276 XXIV	+0.32374 XXV	
26.	0 =−2.76600	+0.09040 IX	−0.05424 XXIII	−0.01808 XXIV	+0.99717 XXVI	−0.01970 XXVII
27.	0 =−3.02800	+0.04924 IX	+0.03276 X	−0.02954 XXIII	−0.00985 XXIV	−0.01970 XXVI
		+1.15105 XXVII	−0.00990 XXVIII			
28.	0 =−1.46800	+0.15492 X	+0.02283 XI	−0.00990 XXVII	+0.39682 XXVIII	+0.02242 XXIX
29.	0 =+0.17700	+0.01965 X	−0.22573 XI	+0.02242 XXVIII	+0.35885 XIX	
30.	0 =−2.90300	+0.29845 XI	+0.42236 XXX	−0.00236 XXXI	+0.00155 XXXII	
31.	0 =−0.12100	+0.00261 XI	−0.00236 XXX	+0.29870 XXXI	−0.07470 XXXII	
32.	0 =+1.13100	−0.00171 XI	+0.00155 XXX	−0.07470 XXXI	+0.48335 XXXII	

Values of the correlates and their logarithms.

I =− 4.002	log 0.6022771−	XVII =+ 6.117	log 0.7865385+
II =− 3.023	log 0.4804381−	XVIII =− 0.414	log 9.6170003−
III =− 4.562	log 0.6591553−	XIX =−13.759	log 1.1385869−
IV =− 0.883	log 9.9459607−	XX =+ 2.512	log 0.4000196+
V =+ 0.360	log 9.5563025+	XXI =−27.030	log 1.4318460−
VI =+ 1.515	log 0.1804126+	XXII =+ 0.742	log 9.8704039+
VII =− 4.963	log 0.6957443−	XXIII =+ 6.019	log 0.7795243+
VIII =+ 7.716	log 0.8874204+	XXIV =+ 5.034	log 0.7019132+
IX =+ 5.201	log 0.7160869+	XXV =+12.363	log 1.0921239+
X =− 2.440	log 0.3873898−	XXVI =+ 2.776	log 0.4434195+
XI =− 2.086	log 0.3193143−	XXVII =+ 2.765	log 0.4416951+
XII =− 0.259	log 9.4132998−	XXVIII =+ 4.953	log 0.6948683+
XIII =− 2.714	log 0.4336098−	XXIX =− 1.981	log 0.2968845−
XIV =+17.084	log 1.2325896+	XXX =+ 8.355	log 0.9219465+
XV =+ 0.652	log 9.8142476+	XXXI =− 0.108	log 9.0334238−
XVI =− 2.459	log 0.3907585−	XXXII =− 2.391	log 0.3785796−

Values of the general corrections.

$[28_1]$	$=-0.293$	$[23_{4+5}]$	$=+0.112$	$[17_{4+5}]$	$=+0.487$
$[28_{1+2}]$	$=+0.689$	$[22_1]$	$=-0.760$	$[16_1]$	$=+0.258$
$[28_3]$	$=-0.044$	$[22_{2+3}]$	$=+0.079$	$[16_2]$	$=+0.278$
$[28_4]$	$=-0.072$	$[22_4]$	$=-0.171$	$[16_3]$	$=+0.446$
$[27_1]$	$=+0.954$	$[22_{3+4}]$	$=+0.026$	$[15_{1+2}]$	$=+0.445$
$[27_2]$	$=+0.210$	$[21_{1+2+3}]$	$=+0.257$	$[15_{1+2+3}]$	$=+0.520$
$[27_3]$	$=-0.607$	$[21_2]$	$=+0.473$	$[15_{2+3}]$	$=-0.211$
$[26_1]$	$=-0.708$	$[21_3]$	$=-0.284$	$[15_4]$	$=+1.388$
$[26_2]$	$=+0.310$	$[21_4]$	$=-0.729$	$[15_5]$	$=+1.383$
$[26_3]$	$=-0.396$	$[20]$	$=+0.651$	$[15_{6+1}]$	$=+0.792$
$[26_{3+4}]$	$=-0.107$	$[19_1]$	$=-0.848$	$[14_2]$	$=+0.609$
$[26_5]$	$=+0.278$	$[19_{1+2}]$	$=-1.203$	$[14_{2+3}]$	$=-0.367$
$[26_6]$	$=+0.798$	$[19_{1+2+3}]$	$=-0.235$	$[14_4]$	$=+0.383$
$[26_{6+7}]$	$=+0.371$	$[19_4]$	$=+0.364$	$[13_1]$	$=-0.006$
$[25_1]$	$=-0.330$	$[19_5]$	$=-0.366$	$[13_{2+3}]$	$=+1.868$
$[25_2]$	$=-0.627$	$[19_6]$	$=+0.899$	$[13_3]$	$=-0.329$
$[25_{1+2+3+4}]$	$=+1.191$	$[19_7]$	$=-0.537$	$[12_1]$	$=-0.107$
$[25_{4+5}]$	$=+0.675$	$[18_4]$	$=+1.689$	$[12_3]$	$=+0.590$
$[25_5]$	$=+0.377$	$[18_5]$	$=-0.287$	$[12_4]$	$=+0.037$
$[25_{5+6}]$	$=-0.231$	$[17_1]$	$=+1.199$	$[12_{2+3}]$	$=-0.007$
$[23_1]$	$=+0.694$	$[17_2]$	$=+0.823$	$[11_2]$	$=-0.427$
$[23_2]$	$=+1.383$	$[17_3]$	$=+0.788$	$[11_3]$	$=+0.091$
$[23_3]$	$=-1.679$	$[17_{2+3+4}]$	$=-0.999$	$[10_2]$	$=-0.598$
$[23_4]$	$=-0.282$	$[17_{4+5+6}]$	$=-1.478$		

Residuals resulting from substitution of general corrections in numerical equations of condition.

No. equation.	Residual.	No. equation.	Residual.
1	−0.0069	17	0.0000
2	−0.0063	18	−0.0001
3	+0.0035	19	−0.0002
4	−0.0108	20	+0.0002
5	−0.0180	21	0.0000
6	−0.0630	22	0.0000
7	+0.0160	23	+0.0004
8	+0.0015	24	+0.0001
9	+0.0220	25	+0.0001
10	0.0000	26	+0.0004
11	+0.0031	27	−0.0003
12	−0.0002	28	0.0000
13	+0.0001	29	+0.0001
14	+0.0001	30	−0.0002
15	−0.0002	31	+0.0001
16	0.0000	32	−0.0001

§ **8.** It is of much interest to know, at least approximately, the probable errors of the observed angles in this triangulation, and the corrections to them which have been deduced afford the means of finding those errors. In a series of m observations giving values l for a linear function of n unknowns, if v is the most probable correction to l we have the equations

$$(1)\qquad \begin{cases} v' = -l' + a\,x + b\,y + c\,z + \ldots \text{ weight } p' \\ v'' = -l'' + a'x + b'y + c'z + \ldots \text{ weight } p'' \\ \ldots\ldots (m) \ldots\ldots \end{cases}$$

If the variables are independent it is well known that, ρ being the probable error of an observation whose weight is unity,

$$(2)\qquad \rho = 0.6745\sqrt{\frac{[pvv]}{m-n}}$$

On the other hand, if the variables are connected by r equations of condition, these equations can be used to eliminate r unknowns from (1), leaving m equations with $n-r$ independent unknowns, to which (2) will apply, becoming then,

$$\rho=0.6745\sqrt{\frac{[pvv]}{m-(n-r)}}=0.6745\sqrt{\frac{[pvv]}{r}} \tag{3}$$

This last equation follows from the fact that in a triangulation each observed angle may be used to give an equation of the form (1), containing only the most probable angle as an unknown, thus giving $m=n$. If there are r equations of condition, (3) gives at once the probable error of an observed angle whose weight equals unity.

It has already been stated that in adjustment the chain of triangles connecting the Keweenaw and Minnesota Point Bases was broken into two sections at the line Split Rock - Detour, the sections being adjusted separately. The first section includes the stations North Base (Minnesota Point), Oneota, South Base (Minnesota Point), Lester, Aminicon, Buchanan, Brulé, Burlington, Clay Banks, Sawteeth West, Detour, and Split Rock. For this section r in equation (3) $=32$. The average weight of an angle in this section is 12.01, the probable error of an angle of weight unity is $1''.36$, and hence the probable error of an angle of average weight is $\pm 0''.39$. Selecting the chain joining the two bases, whose triangles have the best form and their angles the greatest weight, the average weight of an angle in those triangles of this section which enter this principal chain is 17.35, and hence the probable error of an observed angle of average weight in this part of the principal chain is $\pm 0''.33$. The second section includes the following stations: Split Rock, Detour, Outer Island, Sawteeth East, Porcupine Mountains, Farquhar's Knob, Isle Royale, Wheal Kate, Vulcan, Huron Mountains, Traverse Point, Middle, Crebassa, Quaquaming, South Base (Keweenaw Point) and North Base (Keweenaw Point). For this section r in equation (3) is 69, and there results $\rho=\pm 1''.72$ as the probable error for this section of the angle whose weight is unity. It will be remembered that the weight unity is that of an angle for which the mean error for one measure of it as derived from observations at the station alone is $1''$. The average weight of the angles in this sectionis 7.96, and hence the probable error of an angle of average weight is $\pm 0''.61$. The average weight of an angle in those triangles of this section which enter the principal chain is 8.84, and hence the probable error of an observed angle of average weight in this part of the principal chain is $\pm 0''.58$.

The probable error of an observed angle of the whole principal chain may be derived from the excess of the sum of the observed angles of a triangle over 180° plus the spherical excess by using to obtain the probable error of an observed angle, the expression $0.6745\sqrt{\frac{[vv]}{3n}}$ in which the v are the excesses just referred to, and n is the number of triangles. In this way the value $\pm 0''.58$ is found as the probable error of any angle in the principal chain connecting Keweenaw and Minnesota Point Bases.

Colonel Walker, in Survey of India, Vol. II, page 195, shows, for a triangulation whose angles have nearly equal weight, that the probable error of any adjusted angle may be approximately derived from the probable error of an observed angle by multiplying it by $\sqrt{\frac{m-r}{m}}$, in which m is the number of observed angles and r the number of rigid equations of condition which they must satisfy. In this chain, for the section extending from Keweenaw Base to the line Split Rock - Detour, $m=109$ and $r=69$, hence $\sqrt{\frac{m-r}{m}}=0.61$. For the section between Split Rock - Detour and Minnesota Base $m=50$, $r=32$, and $\sqrt{\frac{m-r}{m}}=0.60$. Multiplying the probable error of an observed angle of the main chain in each of the two sections by 0.61 and 0.60, respectively, there result for the probable error of an adjusted angle in the selected chain between Keweenaw Base and the line Detour - Split Rock, $\pm 0''.58\times 0.61=\pm 0''.35$, and between this line and the Minnesota Base $\pm 0''.33 \times 0.60=\pm 0''.20$. Since the weights of angles between the Keweenaw and Minnesota Point Bases differ widely, the values of the ratio $\frac{m-r}{m}$ can only be considered as rough approximations. In the remainder of the triangulation the angles in each section have nearly equal weights.

PROBABLE ERRORS OF OBSERVED AND ADJUSTED ANGLES.

§ **9.** The values just deduced are collected in tabular form, as follows:

Let

m=whole number of observed angles in a section (one adjustment).
r=whole number of rigid conditions in a section.
n=number of triangles in principal chain.
$[pvv]$=sum of weighted squares of corrections to observed angles.
ρ_1=probable error of an observed angle of weight unity.
ρ_s=probable error of an observed angle of average weight in whole section.
ρ_s'=probable error of an adjusted angle of average weight in whole section.
p_s=average weight of an observed angle in whole section.
p_c=average weight of an observed angle in principal chain.
ρ_c=probable error of an observed angle of average weight in principal chain.
ρ_c'=probable error of an adjusted angle of average weight in principal chain.
$[vv]$=sum of squares of closing errors of triangles in principal chain.
ρ_t=probable error of an observed angle in principal chain as derived from the closing errors of triangles.

FOR THE ENTIRE SECTIONS IN THIS CHAPTER.

Section.	Extent of section.	m	r	$[pvv]$	ρ_1	p_s	ρ_s	$\sqrt{\frac{m-r}{m}}$	ρ_s'
					"		"		"
I	Minnesota Point Base to Split Rock-Detour	50	32	130.56	1.36	12.01	0.39	0.60	0.24
II	Split Rock-Detour to Keweenaw Base	109	69	447.68	1.72	7.96	0.61	0.61	0.37

FOR THE PRINCIPAL CHAIN CONNECTING THE MINNESOTA POINT AND KEWEENAW BASES.*

Section.	Divisions of principal chain.	p_c	ρ_c	ρ_c'	From closing errors of triangles.				
					$[vv]$	n	ρ_t	Average error.	Greatest error.
				"			"	"	"
I	Minnesota Point Base to Split Rock-Detour	17.35	0.33	0.20	3.99	9	0.26	0.55	1.03
II	Split Rock-Detour to Keweenaw Bsse	8.84	0.58	0.35	49.13	15	0.70	1.38	3.47
	Entire principal chain				53.12	24	0.58	1.07	3.47

*Given in D, § 10 following.

D.—PRINCIPAL CHAIN OF TRIANGLES BETWEEN KEWEENAW BASE AND MINNESOTA POINT BASE.

§ **10.** In a triangulation lying between two bases and adjusted without reference to them, the values of any side will differ when computed first from one base and then from the other. If proper weights can be assigned to the two values, and their weighted mean be taken, that mean will have a greater accuracy than either of the separate values. But as these separate values differ but slightly, great accuracy in determining their weights is not necessary. As an approximation we may compute from each base, with the probable errors of the observed angles, the weights of a triangle side. The ratio of these two weights will approximate to the ratio of the true weights of this side and may be used in combining the two values of the side computed with the adjusted angles from the two bases. The adoption of such mean values for the sides of the triangles will carry with it new values for the angles, no longer perfectly satisfying their equations of condition, but these changes in the angles will usually be within the probable errors of the angles and so may be neglected.

To obtain approximate weights for the two computed values of a triangle side, the method given in Chapter IV, § 14, may be adopted. In a triangle $A\,B\,C$, if the side $B\,C$ be computed from

$A\,C$, the square of the probable error in $B\,C$, expressed in units of the seventh place of logarithms arising from the errors of the angles, will be $\rho^2(\alpha^2+\beta^2)$, where ρ is the probable error of any observed angle in this part of the triangulation and α and β are changes for 1″ in the seventh place of the logarithmic sines of A and B. For a succession of transverse sides with the same value for ρ this probable error squared will be $\rho^2[\alpha^2+\beta^2]$ for the last side. For the side $A\,B$ the probable error of the side $B\,C$ may be taken as a sufficient approximation. If ρ differs for different parts of the triangulation, these sums will be formed for each value of ρ and their sum will be taken. If to this sum there be added the square of the probable error in the logarithm of the base after the error of the standard of length is excluded, the reciprocal of the sum will be the weight to assign to the logarithm of the side computed from the first base. Finding in the same way the weight of the logarithm of the same side when computed from the second base, the weighted mean of the two values will be the value to be adopted. The reciprocals of the weights are derived from the $\alpha^2+\beta^2$ for each triangle, given in the following table, by adding to $\rho^2[\alpha^2+\beta^2]$, 19.01 for the computations depending on the Keweenaw Base and 58.68 for the Minnesota Point Base, those being the probable errors squared in the seventh place of logarithms of the bases after the error of the standard is excluded, corresponding to the probable errors $0^{\text{in}}.349$ and $0^{\text{in}}.421$ respectively. The bases will be left unchanged, their probable errors being used only in computing the weights of intermediate sides. Calling the weight of the lesser logarithmic value of the side p, where $\sqrt{p}$ is the reciprocal of the probable error, and of the greater, p', the correction to the lesser value to give the weighted mean value will be $+\dfrac{p'}{p+p'}d$ if d be the difference between the two values. The weight of any such mean value will be $p+p'$. When p and p' are equal the corresponding mean side will be that with the greatest probable error, and the probable error will decrease toward each base.

By this method unequal corrections are applied to the logarithms of the different sides of a triangle, but the deviation of the correction to any side from the mean of all the corrections to the sides of that triangle rarely equals 2 in the seventh place of logarithms. This quantity is small compared with the probable error in the logarithms of the sides and if its consideration be neglected we may then suppose that the mean value is applied to the logarithms of all the sides of this triangle.

Within the degree of approximation stated, this method of making the two bases agree amounts then in effect to the following: Starting from one base the logarithms of all the sides of each triangle receive the same correction. This correction varies from triangle to triangle, increasing from one base toward the other. It leaves all angles unchanged, but it effects the agreement of the bases by using values differing by small quantities, sometimes by 6 units in the seventh place of logarithms for the same transverse side of the triangulation, as it forms a part of the preceding or following triangle. It gives a very nearly correct value for the distance between the bases and affects the azimuths of long lines only by insignificant quantities. Instead of distributing the error gradually through the triangulation, as a strict adjustment would do, it does it by small abrupt steps in passing from one triangle to the next and leaves the triangulation a series of slightly disconnected triangles. The method actually followed of giving unequal corrections to the different sides of the same triangle, and of leaving the previously adjusted angles unchanged, gives the same results as those just stated within the degree of approximation previously mentioned (2 units in the seventh place of the logarithms of the sides).

In the following table, giving the principal chain of triangles between Keweenaw and Minnesota Point Bases, the first column gives the names of the stations; the second gives the adjusted angles taken from C, § 7; the third gives the triangle-error or the excess of the sum of the observed angles over 180° plus the spherical excess; the fourth gives the logarithm of the side, expressed in feet, opposite the station in the same line, the value being computed from the Keweenaw Base with the adjusted angles and the length of the base given in Chapter III, § 31; the fifth gives for each triangle α^2 and β^2; the sixth gives the sum of $\alpha^2+\beta^2$ from Keweenaw Base up to the opposite triangle, inclusive; the seventh gives the quantities $\dfrac{1}{p}$; and the eighth the weighted mean logarithms of the sides as depending on both bases and derived by the method just explained.

Both bases depend on the mean of Clarke yards A and B.

From Keweenaw Base to Split Rock-Detour $\rho=\pm 0''.58$, and from Split Rock-Detour to Minnesota Point Base $\rho=\pm 0''.33$. These values are taken from *C*, § 9. The logarithm of the measured value of the length of Minnesota Point Base, expressed in feet, is 4.2982176, and the logarithm of the same, as computed from Keweenaw Base, is 4.2982130; hence, $d=+0.0000046$ The constant for the system $\left(\frac{1}{p}+\frac{1}{p'}\right)=3034$.

Chain of principal triangles between Minnesota Point and Keweenaw Bases.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	α^2 and β^2	$\Sigma(\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Quaquaming	55 05 04.725		4.4622750	216.09			4.4622750
North Base	54 38 15.399	−0.215	4.4598901				4.4598903
South Base	70 16 40.061		4.5222084	57.76	273.85	111	4.5222086
Crebassa	51 05 41.081		4.5222084	289.00			4.5222086
Quaquaming	24 07 05.796	+2.772	4.2424465				4.2424468
North Base	104 47 13.265		4.6164985	30.25	593.10	219	4.6164988
Middle	57 40 10.460		4.6164985	176.89			4.6164988
Crebassa	54 19 34.298	−0.161	4.5993964				4.5993969
Quaquaming	68 00 15.601		4.6568323	72.25	842.24	302	4.6568328
Traverse Point	34 53 31.396		4.6568323	912.04			4.6558328
Crebassa	78 27 38.124	−0.778	4.8905444				4.8905454
Middle	66 38 51.242		4.8622949	82.81	1837.09	637	4.8622959
Huron Mountains	33 41 03.559		4.8622949	998.56			4.8622959
Crebassa	71 29 06.743	−2.079	5.0952326				5.0952241
Traverse Point	74 49 51.759		5.1029023	32.49	2868.14	984	5.1029038
Vulcan	33 26 30.882		5.1029023	1017.61			5.1029038
Huron Mountains	80 53 31.124	−0.724	5.3561741				5.3561762
Crebassa	65 40 04.170		5.3212837	90.25	3976.00	1357	5.3212858
Wheal Kate	53 16 37.467		5.3212837	249.64			5.3212858
Vulcan	49 14 53.304	−0.420	5.2967674				5.2967796
Huron Mountains	77 28 38.769		5.4069075	22.09	4247.73	1449	5.4069097
Isle Royale	41 11 08.198		5.4069075	576.00			5.4069097
Wheal Kate	53 40 17.799	−0.294	5.4944950				5.4944975
Vulcan	85 08 52.699		5.5868062	4.00	4827.73	1643	5.5868087
Farquhar's Knob	56 31 45.828		5.5868062	193.21			5.5868087
Wheal Kate	51 22 54.805	−3.108	5.5583799				5.5583825
Isle Royale	72 05 50.665		5.6440052	46.24	5067.18	1724	5.6440078
Porcupine Mountains	77 08 09.116		5.6440052	23.04			5.6440078
Farquhar's Knob	39 25 03.563	+3.466	5.4577785				5.4577812
Wheal Kate	63 27 13.947		5.6065564	112.36	5202.58	1769	5.6066591
Sawteeth East	62 59 40.893		5.6066564	114.49			5.6066591
Farquhar's Knob	68 10 56.092	+2.361	5.6245197				5.6245226
Porcupine Mountains	48 49 53.196		5.5334542	342.25	5659.32	1923	5.5334571
Outer Island	77 48 12.641		5.5334542	21.16			5.5334571
Farquhar's Knob	38 00 28.740	−0.532	5.3327768				5.3327798
Sawteeth East	64 11 34.197		5.4977361	102.01	5782.49	1964	5.4977391
Sawteeth West	62 18 24.252		5.4977361	121.00			5.4977391
Outer Island	80 00 35.728	+2.755	5.5439415				5.5439449
Farquhar's Knob	37 41 15.865		5.3368598	745.29	6648.78	2256	5.3368632

Chain of principal triangles between Minnesota Point and Keweenaw Bases—Continued.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	α^2 and β^2	$\Sigma(\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Detour	90 02 34. 109		5. 3368598	0. 00	.....	.....	5. 3368632
Sawteeth West	40 04 34. 847	+0. 640	5. 1456116	.....	.....	.....	5. 1456152
Outer Island	49 52 56. 514		5. 2203609	313. 29	6962. 07	2361	5. 2203645
Split Rock	95 18 30. 512		5. 2203609	4. 00	.....	.....	5. 2203645
Detour	29 31 46. 163	−0. 362	5. 9149578	.....	.....	.....	4. 9149615
Sawteeth West	55 09 45. 963		5. 1364519	213. 16	7179. 23	2434	5. 1364556
Clay Banks	69 39 00. 661		5. 1364519	60. 84	.....	.....	5. 1364556
Split Rock	41 36 52. 300	+0. 913	4. 9866826	.....	.....	.....	4. 9866863
Detour	68 44 09. 962		5. 1338189	67. 24	7307. 31	2448	5. 1338226
Burlington	85 14 18. 557		5. 1338189	4. 00	.....	.....	5. 1338226
Clay Banks	38 49 38. 658	−1. 030	4. 9325697	.....	.....	.....	4. 9325734
Split Rock	55 56 05. 065		5. 0535595	204. 49	7515. 80	2471	5. 0535632
Brulé	79 00 44. 111		5. 0535595	16. 81	.....	.....	5. 0535632
Burlington	39 25 53. 929	+0. 382	4. 8644749	.....	.....	.....	4. 8644787
Clay Banks	61 33 23. 672		5. 0057256	129. 96	7662. 57	2487	5. 0057294
Buchanan	95 04 00. 144		5. 0057256	4. 00	.....	.....	5. 0017294
Brulé	28 52 26. 897	+0. 116	4. 6912702	.....	.....	.....	5. 6912740
Burlington	56 03 33. 935		4. 9263031	201. 64	7868. 21	2509	4. 9263069
Aminicon	60 03 58. 361		4. 9263031	146. 41	.....	.....	4. 9263069
Brulé	71 58 27. 001	−0. 809	4. 9666259	.....	.....	.....	4. 9666398
Buchanan	47 57 36. 009		4. 8592830	357. 21	8371. 83	2564	4. 8592869
Lester	41 43 10. 063		4. 8592830	556. 96	.....	.....	4. 8592869
Aminicon	97 51 04. 048	+1. 003	5. 0320564	.....	.....	.....	5. 0320605
Brulé	40 25 47. 082		4. 8480655	610. 09	9538. 88	2691	4. 8480696
South Base	88 55 19. 580		4. 84807[illegible]5	0. 00	.....	.....	4. 8480696
Lester	37 56 06. 854	+0. 047	4. 6368548	.....	.....	.....	4. 6368589
Aminicon	53 08 34. 143		4. 7513044	249. 64	9788. 52	2718	4. 7513085
Oneota	78 27 04. 627		4. 7513044	18. 49	.....	.....	4. 7513085
South Base	70 39 24. 565	−0. 212	4. 7349526	.....	.....	.....	4. 7349569
Lester	30 53 31. 180		4. 4706605	1239. 04	11046. 05	2855	4. 4706648
North Base	122 11 15. 442		4. 4706605	176. 89	.....	.....	4. 4706648
South Base	23 08 04. 958	−0. 476	4. 1374075	.....	.....	.....	4. 1374120
Oneota	34 40 39. 654		4. 2982130	924. 16	12147. 10	2975	4. 2982176

For the side Vulcan - Wheal Kate p becomes nearly equal to p', giving for the probable error squared of the weighted mean logarithm of this side 756.97. Adding to this the probable error squared of the logarithm of the length of the standard, namely, 8.39 (Chapter II, § 14), there results for the probable error of the logarithm of this side $\pm$ 27.66, corresponding to $\pm \frac{1}{156980}$ part of its length. From this side the probable errors decrease toward the Minnesota Point and Keweenaw Bases, where they are respectively $\pm \frac{1}{542500}$ and $\pm \frac{1}{834400}$.

The probable error $\pm$ 27.66, found for the logarithm of the side Vulcan - Wheal Kate, since it has been derived from the probable error of an observed angle, is too great and should be diminished in a ratio somewhat less than that of the probable error of an observed to that of an adjusted angle.

CHAPTER XV.

TRIANGULATION FROM LINE VULCAN-HURON MOUNTAINS TO FOND DU LAC BASE.

A.—DESCRIPTIONS OF STATIONS.

NOTE RELATIVE TO ELEVATIONS.

§ **1.** The heights of ground at stations described in this chapter were determined chiefly by trigonometrical leveling. The heights at stations about Lake Superior and between it and Green Bay are referred to the mean surface of Lake Superior from 1871 to 1875. (See Chapter XXII, § 13.) Heights along Green Bay are referred to its surface at the times of determination and are therefore subject relatively to some uncertainties arising from fluctuations in elevation of the lake-surface. For any of the group of heights referred to Lake Superior the probable error may be estimated as not exceeding ± 5 feet. For the group referred to Green Bay ± 2 feet would appear to be a sufficiently large probable error, since these heights do not depend on trigonometrical levels over long lines. Heights in the vicinity of and south of Fond du Lac Base are referred to the mean surface of Lake Michigan for 1860 to 1875. (See Chapter XXII, § 13.) They were computed from zenith-distances observed in the triangulation and connecting with a point of known height on Milwaukee court-house. The height of East Base given herein is 8 feet greater than that derived by spirit-leveling, as stated in Chapter V, § 6. As no special precision was aimed at in the trigonometrical leveling of this part of the triangulation, a probable error of not less than ± 5 feet may be assigned to the heights.

DESCRIPTIONS OF STATIONS.

§ **2.** IVES HILL, 1872.—This station is situated on one of the Huron Mountains, about 4 miles southeast of the mouth of Pine River, and about one-half mile east of Ives Lake. The height of station used was 38 feet. The geodetic point is marked by a nail leaded into the solid surface-rock. The geodetic point is 11 feet 2 inches distant from each of three small triangles cut in the rock—one south, one west, and one east—the bearings being approximate. Height of rock at station, 1,030 feet (estimated).

GRANITE ISLAND, 1872, '73.—This station is situated on the north side of Granite Island, Lake Superior. The height of station used was 36 feet. The geodetic point is marked by a cross cut in the solid surface rock. The center of Granite Island light-house tower bears south 73° 33′ 46″ west and is 171.32 feet distant from the geodetic point. Height of rock at station, 38.4 feet.

TRILOBA, 1873.—This station is situated on a bald granite knob, about 3 miles southwest of Granite Point. The height of station used was 17 feet. The geodetic point is marked by a brass frustum leaded into the solid rock. No reference marks are made. The hill, however, can be easily identified as it is the highest one in the immediate vicinity. Height of rock at station, 635 feet.

MESNARD, 1873.—This station is situated about 2 miles south of Marquette, on a granite knob called Mount Mesnard. The height of station used was about 4 feet. The geodetic point is marked by a cross cut in the solid surface rock, which is about 3 feet below the surface soil. Around and above this mark a pile of small stones was made. Three reference-crosses are cut in the solid surface-rock in the following positions: one north 45 feet distant, one east 27 feet distant, and one west 17.3 feet distant from the geodetic point. Height of ground at station, 522.1 feet.

SHELTER BAY, 1873.—This station is situated on the south shore of Lake Superior, about 2 miles southwest of Shelter Bay and about one-fourth of a mile south of the west end of Deer Lake.

The height of station used was 30 feet. The geodetic point is marked by a single stone of the usual form. Height of ground at station, 451.7 feet.

GRAND ISLAND, 1872, '73.—This station is situated on Grand Island, about 2 miles south of the light-house, and on the highest part of the island. The height of station used was 50 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 2 feet below the ground surface. Three reference-stones are set, one north, one east, and one south, each 10 feet distant from the geodetic point, the bearings being approximate. Height of ground at station, 388.2 feet.

DIVIDE, 1873, '74.—This station is situated on the dividing ridge of land between Lake Superior and Green Bay, about 6½ miles south of Munising, and about 1,000 feet east of the old Bay de Noquette and Lake Superior road. The height of station used was 110 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is 3½ feet below the ground surface. A second stone of the same form, rising flush with the surface of the ground, is set directly above the first. The geodetic point is south 67° west and 760 feet distant from the northeast corner of section 27, township 46 north, range 19 west. Height of ground at station, 426.7 feet.

MUD LAKE, 1874.—This station is situated in the Northern Peninsula of Michigan, in the southeast corner of section 4, township 45 north, range 20 west. The height of station used was 85 feet. The geodetic point is referred to a stone which bears south 29° 19′ east and is 31.02 feet distant. This stone is set so that its upper end is about 3 feet below the ground surface. A surface stone marked with a cross is set directly over the first stone. Two reference-stones are set, one approximately north 5.75 feet distant, and one approximately south 2.83 feet distant. The geodetic point is also marked by a spike driven into the root of the tree used as the station center-post. The southeast corner of section 4 bears south 44° 06′ east and is 1,061.8 feet distant from the geodetic point. Height of ground at station, 403.5 feet.

STURGEON RIVER, 1874.—This station is situated in the Northern Peninsula of Michigan, in section 16, township 42 north, range 19 west, being south 40° east and 705 feet distant from the quarter-stake on the north side of the section. The height of station used was 30 feet. The geodetic point is marked by a stone set so that its upper end is about 2½ feet below the surface of the ground. Above the latter is set a reference-stone rising 4 to 6 inches above ground. Height of ground at station, 345.4 feet.

MONISTIQUE, 1874.—This station is situated in the Northern Peninsula of Michigan, 975 feet southwest of the northeast corner of section 7, township 43 north, range 18 west. The height of station used was 30 feet, the theodolite having been supported on a tree cut off at this height. The geodetic point is the middle point of the line joining two stones set 5 feet apart in an east-and-west line. Height of ground at station, 394.7 feet.

FISHDAM RIVER, 1874.—This station is situated in the Northern Peninsula of Michigan, about 5 miles northeast of the mouth of Fishdam River, or the head of Big Bay de Noquette. The height of the station used was 48 feet. The geodetic point is marked by a single stone, set so that its upper end is about 2 feet below the ground surface. References were made to the three trees supporting the platform. The tree to the north is distant 8 feet 1 inch; the one east, 6 feet 6 inches; and the one northwest, 12 feet 6 inches. Height of ground at station, 253.6 feet.

PINE HILL, 1874.—This station is situated on Peninsula Point. It bears north 65° east and is 5½ miles distant from Squaw Point. A tree cut off 104 feet above ground was used as a post for supporting the theodolite. The geodetic point is marked by a nail leaded into the solid rock which is about 2 feet below the surface of the ground. A marking-stone of the usual form is set directly over the nail. The following references were made: head of wrought spike in center-post tree, southwest 1 foot 4 inches; head of wrought spike in pine tree, southwest 14 feet 4 inches; head of wrought spike in pine tree, north 14 feet 4 inches; head of wrought spike in pine stump, southeast 34 feet. These distances are each from the geodetic point.

BURNT BLUFF,* 1874.—This station is situated on Burnt Bluff, about 3 miles southwest of Fayette, Michigan. It is on the west side and not more than 100 feet back from the edge of the

*Topographical sketches of nearly all stations hereafter described in this chapter and in Chapters XVI–XX are on file at the Survey Office. They show, on a scale of $\frac{1}{5000}$, the features of the country in detail within a radius of 500 metres from the geodetic point.

bluff. The height of station used was 10 feet. The geodetic point is marked by a single stone of the usual form, set so that its upper end is about 1 foot below the ground surface, and is covered by a cairn. Three reference-stones are set, one east 10 feet distant, one south 10 feet distant, and one south 20 feet distant from the geodetic point, the bearings being approximate. Height of ground at station, 215 feet above Lake Superior, or 235.5 feet above Green Bay.

FORD RIVER, 1874.—This station is situated on the west shore of Green Bay, about 5 miles southwest of Escanaba light-house, about 2 miles northeast of the mouth of Ford River, and about 100 feet back from the shore. The height of station used was 75 feet. The geodetic point is marked by a cross, cut in the solid rock, which is about 3 feet below the ground surface. Above this cross is set a marking-stone of the usual form, rising about to the level of the ground. Two reference-stones are set, one north (approximately) 55.1 feet and one west (approximately) 64.5 feet from the geodetic point. A large bowlder, marked U. S., lies 20.2 feet to the south of the geodetic point. The southeast corner of section 12, township 38 north, range 23 west, bears north 79° 29′ east, and is 435.5 feet distant from the geodetic point. Height of ground at station, 10 feet (estimated).

CEDAR RIVER, 1874.—This station is situated on the west shore of Green Bay, about 2 miles northeast of the mouth of Cedar River, and about 50 feet back from the shore. The height of station used was 48 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 2 feet below the ground surface. Directly above the latter is set another stone, rising about to the level of the ground surface. Three reference-stones are set, one north 13 metres, one south 12.5 metres, and one west 14 metres distant from the geodetic point. Height of ground at station, 3.8 feet.

BOYER'S BLUFF, 1874.—This station is situated on Boyer's Bluff, on the northwest side of Washington Island, and about 15 feet back from the edge of the bluff. The height of station used was 5 feet. The geodetic point is marked by a single stone of the usual form. A reference-stone, 24 feet distant, bears south 43° east from the geodetic point. A second reference is north 33° west, 15 feet 6 inches from the geodetic point. Height of ground at station, 130 feet.

DOOR BLUFF, 1874.—This station is situated on the east shore of Green Bay, on Death's Door Bluff. It is on the highest part and 34 feet back from the edge of the west side of the bluff. The height of station used was 10 feet. The geodetic point is marked by a cross, cut in the solid rock. Above this mark is set a post of the usual form. Cedar reference-stakes were driven on the north, east, and south of geodetic point, each 10 feet distant. Each of these stakes is surrounded by a pile of stones. An astronomical stone post bears south 34° 47′ west, and is 90 feet distant from the geodetic point. Height of ground at station, 100 feet.

ROCHEREAU, 1874.—This station is situated on the west shore of Green Bay, on Point Rochereau. It is about 100 feet back from the low sandy beach and about 200 feet northwest of the extreme point of land. The height of station used was 50 feet. The geodetic point is marked by a stone set inside a wooden box, sunk in the quicksand, the upper end of the stone being about 2½ feet below the ground surface. Another stone, rising 6 inches above ground, is set directly above the former. Two reference stones are set, one north 19° 58′ east, 74.65 feet distant, and one south 79° 53′ east, 65.30 feet distant from the geodetic point. Height of ground at station, 2.5 feet (estimated).

EAGLE BLUFF, 1874.—This station is situated on the east shore of Green Bay, about 40 metres southwest of Eagle Bluff light-house, and about 10 metres back from the edge of the limestone bluff rising perpendicularly from the bay. The height of station used was 12 feet. The geodetic point is marked by a stone of the usual form. A reference-mark cut in the solid rock bears south 72° 10′ west, and is 6.7 metres distant from the geodetic point. The center of the light-house tower bears north 48° 29′ east, and is 39.79 metres distant from the geodetic point. Height of ground at station, 36 feet.

SOUTH EGG, 1874.—This station is situated on the east shore of Green Bay, about 4½ miles southwest of Egg Harbor, and about one-fourth of a mile back from the shore. The height of station used was 20 feet. The geodetic point is marked by a stone of the usual form, rising 3 or 4 inches above ground. Three reference-stones are set, one north 29° east, 53.1 feet distant; one south 55° 60′ east 35.2 feet distant; and one south 25° west, 50.3 feet distant from the geodetic point.

MENOMONEE, 1874.—This station is situated on the southwest corner of Almeda and Water streets, in the village of Menekaunee, Wisconsin. The height of station used was 50 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 3 feet below the ground surface. Directly over the latter is set another stone, rising about even with the surface of the ground. A reference-stone bears north 39° 48′ east, and is 20.5 metres distant. A second reference bears south 59° 22′ east, and is 20.7 metres distant from the geodetic point. Height of ground at station, 3 feet (estimated).

PESHTIGO, 1874.—This station is situated about one-third of a mile northeast of the extremity of Peshtigo Point and about 100 feet back from the shore of the bay. The height of station used was 25 feet. The geodetic point is marked by a stone of the usual form. Three reference-stones are set in the following positions, respectively: one north 2° 48′ east, 75.3 feet distant; one south 2° 50′ west, 25.2 feet distant; and one north 86° 49′ west, 50.1 feet distant from the geodetic point. Height of ground at station, 2 feet (estimated).

DÉBROUX, 1874.—This station is situated on the east shore of Green Bay, about 3½ miles southwest of Little Sturgeon Bay, and about 12 feet back from the edge of a limestone bluff rising perpendicularly about 50 feet from the water. The height of station used was 20 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 1 foot below the surface of the ground. The northeast corner of a frame house bears south 36° 01′ west, and is 232 feet distant from the geodetic point. A reference-stone, set on the east side of the highway, bears south 83° 53′ east, and is 86 feet 10 inches distant. A blazed stump bears north 53° 58′ east, and is 64 feet distant from the geodetic point. Height of ground at station, 53 feet.

GALES, 1873.—This station is situated on the west shore of Green Bay, about 2 miles south of Oconto, Wisconsin, and about 50 feet back from the beach. The height of station used was 50 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 3 feet below the ground surface. Directly above is set another stone, rising about even with the surface of the ground. A reference-stone bears north 49° 29′ west, and is 60 feet 1½ inches distant; a second reference-stone bears north 8° 59′ west, and is 80 feet 3½ inches distant; and a third reference-stone bears south 38° 31′ west, and is 62 feet 7 inches distant from the geodetic point. These bearings are magnetic and were taken in 1873. Height of ground at station, 3 feet (estimated).

RED RIVER, 1873.—This station is situated on the east shore of Green Bay, about 2¼ miles northeast of the mouth of Red River, and about 300 feet back from the beach. The height of station used was 15 feet. The geodetic point is marked by a single stone of the usual form. A reference-stone bears south 8° 42′ west, and is 31 feet 10 inches distant from the geodetic point. A blaze on a hard-maple tree bears south 65° 22′ east, and is 18 feet 7 inches distant from the geodetic point. Height of ground at station, 150 feet.

RED BANKS, 1873.—This station is situated on the east shore of Green Bay, on a red-clay bluff, distant 6.5 miles in a northeasterly direction from Long Tail Point light-house. The height of station used was 15 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 3 feet below the surface of the ground. A reference-stone bears north 26° 36′ east, and is 125 feet 7 inches distant from the geodetic point. A second reference-stone bears north 72° 59′ east, and is 144 feet 4 inches distant from the geodetic point. The bearings here given are magnetic. Height of ground at station, 86 feet.

LITTLE TAIL POINT, 1873.—This station is situated on the west shore of Green Bay, about 0.9 mile south of the mouth of Little Suamico River, on low marshy ground, and about 10 metres back from the shore. The height of station used was 15 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 2½ feet below the surface of the ground. Directly above is set another stone, rising slightly above the ground surface. Two reference-stones are set, one south 00° 27′ west, 16.25 metres distant, and one north 36° 46′ west, 9.25 metres distant from the geodetic point. Height of ground at station, 2 feet.

BRUCE, 1872, '73.—This station is situated in the southeast quarter of the northwest quarter of section 25, Preble Township, Brown County, Wisconsin. The height of station used was 60 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 4½ feet below the surface of the ground. Directly over this stone was set a large oak post, used as an azimuth-post. A reference-stone bears north 14° 50′ east, and is 15.10 metres distant; a second reference bears north 85° 41′ west, and is 182.33 metres distant; a third reference bears south 79° 21′ west, and is 185.81 metres distant from the geodetic point. The latter two stones are set on the east side of a north-and-south road. Height of ground at station, 185 feet.

FORT HOWARD, 1872, '73, '74.—This station is situated in the town of Fort Howard, Wisconsin, on what is called "Private Claim" No. 8, west side Fox River. The height of station used was 75 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is 4 feet below the surface of the ground. Directly over this stone is set another, rising 4 inches above the level of the ground. Two reference-stones are set on the north side of the road to the south of the station. The one farthest east bears south 16° 26′ east, and is 202.94 metres distant from the geodetic point. The westerly stone bears south 57° 05′ west, and is 152.44 metres distant from the geodetic point.

ONEIDA, 1872.—This station is situated in the northeast part of Oneida Reservation, Wisconsin, about one-half mile north of the Green Bay and Lake Pepin Railway, about 2 miles southeast of an Episcopal mission church, and about 1,000 feet southwest of the junction of two roads running to what is called Cook's Mill. The height of station used was 50 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 2½ feet below the surface of the ground. Directly above the latter is set another stone, rising about 6 inches above the ground. A reference-stone 24.7 feet distant from the geodetic point, bears north 3° east. A second reference-stone, 25.2 feet distant, bears south 3° west. These two bearings are magnetic.

EAST DEPERE, 1872.—This station is situated near the northwest corner of Glenmore Township, Brown County, Wisconsin. It is on the line between Glenmore and Rockland Townships, and about 50 feet back from the edge of the limestone ledge running in a northeasterly direction across Brown County. The height of station used was 65 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 2½ feet below the surface of the ground. A second stone is set directly over the latter, and rises nearly to the surface of the ground. Two reference-stones are set, one north 36° 30′ east 24 feet distant and one south 53° 30 east 24.05 feet distant from the geodetic point, the bearings being magnetic.

WEST DEPERE, 1872.—This station is situated near the south side of section 1, Freedom Township, Outagamie County, Wisconsin. It is about one mile north of Sagole post-office, and about 300 metres west of the Green Bay and Appleton road. The height of station used was 75 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 2½ feet below the surface of the ground. Directly above is set another stone, rising slightly above the surface of the ground. A reference-stone bears north 10° 43′ west, and is 11.87 metres distant; a second reference bears south 45° 44′ east, and is 60.85 metres distant from the geodetic point.

FREEDOM, 1872.—This station is situated on a prominent hill in the southwest corner of Freedom Township, Outagamie County, Wisconsin. The height of station used was 35 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 2½ feet below the ground surface. Another stone projecting slightly above ground is set directly over the latter. Two reference-stones are set, one north 5° 30′ east, 17.32 feet distant, and one north 86° 46′ east, 38.50 feet distant from the geodetic point.

CLAYTON, 1872.—This station is situated in the southeast quarter of the southeast quarter of section 14, Clayton Township, Winnebago County, Wisconsin. The height of station used was 50 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 3 feet below the ground surface. A second stone, rising 4 to 6 inches above ground, is set directly over the latter. Two reference-stones are set, one north 2° 48′ east 19.95 feet distant, and one south 86° 27′ east, 35 metres distant from the geodetic point. The corner common to sections 13, 14, 23, and 24 bears south 14° 34′ east, and is 163 metres distant.

CALUMET, 1872.—This station is situated about three-fourths of a mile northeast from the northeast side of Lake Winnebago, and near the line between Harrison and Woodville Townships, Calumet County, Wisconsin. It is about 20 feet back from the edge of the limestone ledge running nearly parallel with the east shore of Lake Winnebago. The height of the station used was 35 feet. The geodetic point is marked by a hole drilled in the solid rock. Above this is placed a marking-stone of the ordinary form, surrounded by a pile of small stones. Three reference-marks were made. The first of these is a hole drilled in the rock between the letters U. S. cut thereon. It bears south 83° 15′ west, and is 25.25 feet distant from the geodetic point. The second reference-mark is a circle and an arrow cut in the rock, the arrow pointing toward the geodetic point; the center of the circle bears north 32° 14′ west, and is 11.93 feet distant. The third mark is a hole drilled in the rock; it bears north 44° 52′ east, and is 46.97 feet distant from the geodetic point.

STOCKBRIDGE, 1872.—This station is situated about 2 miles northwest of the village of Stockbridge, Calumet County, Wisconsin, and about 1,000 feet back from the shore of Lake Winnebago It is nearly due east of a brick manufactory on the lake shore. The height of station used was 3 feet. The geodetic point is marked by a hole drilled in a large fragment of limestone rock which lies about 1½ feet below the surface of the ground. In the bottom of the hole in the rock two copper cents were placed. Directly above was placed a rough stone with a hole drilled in its upper surface, and a pile of small stones was laid around it. A granite bowlder bears north 34° 15′ east, and is 99.85 feet distant from the geodetic point.

OSHKOSH, 1872.—At this point the tower of the Oshkosh high school building was used as a station. The height of the tower is about 80 feet. The geodetic point is marked by an iron screw set in the upper end of the shaft forming the axis of the winding stairway. Two reference-stones are set near the corners of the school-yard on the northeast side. One of these bears north 5° 57′ east, and is 246.65 feet distant from the geodetic point. The second bears north 52° 14′ east, and is 238.02 feet distant from the geodetic point.

ELDORADO, 1872.—This station is situated in section 10, Eldorado Township, Fond du Lac County, Wisconsin. It is about one-eighth of a mile nearly due east of the southwest corner of the northwest quarter of the section, about 175 feet west of the ridge-road between Fond du Lac and Oshkosh, and about 40 feet north of an east and west road on the half-section line. The height of station used was 35 feet. The geodetic point is marked by a single stone of the usual form, set so that its upper end is about 3 feet below the surface of the ground. Directly over the latter is set another stone, rising 4 to 6 inches above ground. Two reference-stones are set in the road on the south side of station. The eastern stone is 48.75 feet from the geodetic point, and the western stone is 35.1 feet distant from the geodetic point.

TAYCHEEDAH, 1872.—This station is situated in the southeast quarter of the southwest quarter of section 20, Taycheedah Township, Fond du Lac County, Wisconsin. The height of station used was 3 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 2½ feet below the surface of the ground. Two reference-stones are set, one north 58° 43′ east, 14.35 feet distant, and one south 47° 42′ east, and 16.15 feet distant from the geodetic point.

SPRINGVALE, 1872.—This station is situated in the west half of the southeast quarter of section 17, Springvale Township, Fond du Lac County, Wisconsin. The height of station used was 50 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 2½ feet below the surface of the ground. Two reference-stones were set, one north (approximately) 13 feet distant, and one southwest (approximately) 14.3 feet distant from the geodetic point. The head of a nail in a black oak nearly west of the geodetic point is 29.55 feet distant. Height of ground at station, 462.6 feet.

EAST BASE, 1872.—This station is situated in the northwest quarter of the southwest quarter of section 4, Byron Township, Fond du Lac County, Wisconsin, and marks the east end of Fond du Lac base-line. The height of station used was 75 feet. The geodetic point is marked by the intersection of two lines cut in the upper surface of a brass frustum leaded in the upper end of a stone post. This post is 8 inches by 8 inches in cross-section, 5 feet long, and is set so that its upper end is 4 feet below the level of the ground surface. Two similar stones are set, one on the north and one on the south of the geodetic point, and in the line passing through it at right-angles to the base-line. Each of these stones is 4.4 metres distant from the geodetic point, and is set about 4 feet below the surface of the ground. Directly over the stone marking the geodetic point is set a stone of the usual form, rising to within a foot of the surface of the ground. A reference-stone set in the road on the north of the station bears north 11° 45′ west, and is 99.59 metres distant. A second reference bears north 78° 02′ east, and is 27.5 metres distant from the geodetic point. Height of ground at station, 257.2 feet.

West Base, 1872.—This station is situated near the center of section 10, Oakfield Township, Fond du Lac County, Wisconsin, and marks the west end of Fond du Lac base-line. The height of station used was 35 feet. The geodetic point is marked in the same manner as East Base, described above, and has likewise two reference-stones placed one on either side, 4.6 metres distant from and in the line passing through the geodetic point at right angles to the base-line. Directly over the stone marking the geodetic point is set a stone of the usual form, rising within a foot of the surface of the ground. Two reference-stones are set in the road south of the station. One of these bears south 5° 53′ west, and is 18.6 metres distant from the geodetic point; the other bears south 52° 35′ east, and is 31.42 metres distant from the geodetic point. Height of ground at station, 264.2 feet.

Middle Base, 1872.—This station is situated in the northeast quarter of the northeast quarter of section 12, Oakfield Township, Fond du Lac County, Wisconsin. It is about 75 feet west of the intersection of the Chicago and Northwestern Railway and the north and south road on the east side of Oakfield Township. The height of station used was 10 feet. The geodetic point is marked by the intersection of two lines cut in the surface of a brass frustum leaded into the upper end of a stone post. This post is 8 inches by 8 inches in cross-section, 3 feet long, and is set so that its upper end is about 3 feet below the surface of the ground. Directly above is set a marking-stone of the ordinary form, rising about even with the surface of the ground.

Oakfield, 1872.—This station is situated in the northwest quarter of the northeast quarter of the southwest quarter of section 34, Oakfield Township, Fond du Lac County, Wisconsin, on Parratt's Hill. The height of station used was 65 feet. The geodetic point is marked by a cross cut in the surface of lead run into a hole drilled in a limestone rock, which is placed so that its upper end is 3 feet below the surface of the ground. Two reference-stones are set, one north 60° 16′ west and 30.8 feet distant, and one south 45° 48′ west and 38.13 feet distant from the geodetic point. Height of ground at station, 586.3 feet.

Waupun, 1872.—At this point the observations were made from the southeast tower of the main building of the Wisconsin State Prison. The height of this tower above ground is about 70 feet, and the position of the instrument is marked by a cross cut in the surface of the limestone block on which it stood. The geodetic point of this station is the center of the trap-door opening into the central tower of the main building. This point is defined by the intersection of the median lines of the door, these lines being indicated by screws set in the door casing. The geodetic point is also marked by a brass screw set in the stairway below the trap-door. The position of the instrument bears south 52° 35′ east, and is 18.311 metres distant from the geodetic point. Two reference-marks are made on the top of the west wall of the prison yard. These marks are ½-inch holes drilled in the flagstones capping the wall, one being near the north, and the other near the south end of the wall. The northerly mark bears north 57° 52′ west, and is 163.34 metres distant from the geodetic point. The southerly mark bears south 60° 09′ west, and is 160 metres distant from the geodetic point. Height of ground at station, 338.5 feet.

Minnesota Junction, 1873.—This station is situated in the southwest quarter of the southeast quarter of section 29, Burnett Township, Dodge County, Wisconsin. The height of station used was 25 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is 4 feet beneath the surface of the ground. Directly over this stone was set a large oak azimuth-post. A reference-stone set in the road south of the station bears south 36° 54′ east, and is 140.57 metres distant from the geodetic point. A stone post marking the southwest corner of the southeast quarter of section 29 bears south 69° 15′ west, and is 345.32 metres distant from the geodetic point. A hole drilled in a large bowlder bears north 87° 23′ east, and is 36.89 metres distant from the geodetic point. Height of ground at station, 416.2 feet.

Horicon, 1873.—This station is situated in section 33, Williamstown Township, Dodge County, Wisconsin. It is about 4 miles northeast of Horicon village, and about 150 metres east of the limestone ledge running in a northerly direction across Dodge County. The height of station used was 50 feet. The geodetic point is marked by a hole drilled in a granite bowlder set so that its upper side is 2 feet below the surface of the ground. A thin limestone slab is placed over the bowlder, and above is set a marking-stone of the usual form, rising about to the ground surface. Two reference-marks were made on the projecting rock of the ledge west of the station.

Each mark consists of a small triangle with a hole in its center, cut in the bare rock. The northerly mark bears north 62° 02′ west, and is 150.27 metres distant from the geodetic point. The southerly mark bears north 73° 41′ west, and is 156.92 metres distant from the geodetic point. Height of ground at station, 536.5 feet.

B.—HISTORICAL NOTE, STATIONS, SIGNALS, INSTRUMENTS, AND METHODS OF OBSERVATION.

HISTORICAL NOTE.

§ **3**. The angles of the primary triangulation between the lines Vulcan – Huron Mountains and Minnesota Junction – Horicon were read in the years 1872, 1873 and 1874. In 1872 the angles were read in Lake Superior from station Huron Mountains to Granite Island, by Assistants G. Y. Wisner, G. A. Marr, and R. S. Woodward, with the following theodolites: Troughton & Simms' No. 1, Pistor & Martins' No. 2, and Gambey No. 1; and in Wisconsin, from Fort Howard to the Fond du Lac Base, by Assistant A. R. Flint, with Repsold Universal Instrument No. 1. In 1873 the Lake Superior triangulation was carried to Mud Lake, by Assistants Wisner and Marr, with theodolites Troughton and Simms' No. 1 and Pistor & Martins' No. 2; and the Green Bay triangulation, from Fort Howard to Gales, by Assistants Flint and Woodward, with Repsold Universal Instrument No. 1 and Gambey No. 1, respectively. In 1874 the triangulation was carried south from Mud Lake by Assistants Wisner and Marr, with the theodolites Troughton & Simms' No. 2 and Pistor & Martins' No. 2, and north from Gales by Assistants Flint and Woodward with the Repsold Universal Instrument and Troughton & Simms' No. 1, till the different parties joined work at Pine Hill.

The angles of many of the triangles in Green Bay had been read in 1864, but not with the highest precision, some of the triangles not closing within 8″. As a necessary part of the general chain they were therefore re-read.

The adjustment of the triangulation was made by Assistants C. H. Kummell, T. W. Wright, T. Russell, and O. B. Wheeler.

The Fond du Lac Base was measured by Assistant E. S. Wheeler in 1872.

STATIONS, SIGNALS, AND INSTRUMENTS.

§ **4**. In this part of the triangulation the instrument was supported on high wooden tripods or stations like those described in Chapter XIV, B, § 4. There were in all forty-nine stations. Ten of these had a height equaling or exceeding 75 feet, while at one, Divide, the instrument was 110 feet above the ground. The longest triangle side was Vulcan – Granite Island, its length being 53.2 miles.

On the longer lines heliotropes of the forms described in Chapter XIV, B, were used as signals; on the shorter ones the heliotrope was replaced by a flat board about 4 feet high and of such width as to subtend an angle of 4″ or 5″, one-half of the height of the board being white and the other black, this difference in color being produced either with white and black paint or by white and black cotton cloths. Whenever the instrument or the signal was not precisely over the point in the underground stone which fixed the trigonometrical point, its horizontal co-ordinates with reference to that stone were carefully measured and corrections were applied in reducing the angles.

The following instruments were used in reading the primary angles between the lines Vulcan – Huron Mountains and Horicon – Minnesota Junction: Troughton & Simms's 14-inch theodolite No. 1, Pistor & Martins' 14-inch theodolite No. 2, Gambey 10-inch theodolite No. 1, Repsold 10-inch Universal Instrument No. 1, and Troughton & Simms' 12-inch theodolite No. 2. The first three of these instruments have already been described in Chapter XIV, B; it remains to describe the last two.

The Repsold Universal Instrument No. 1 has a "broken telescope" whose object-glass has a diameter of $2\frac{1}{4}$ inches and a focal length of 20 inches; its transit axis is 12 inches long; its 10-inch horizontal circle, graduated to 4 minutes, reads by two microscopes to 2 seconds; its 9-inch vertical circle, graduated to 4 minutes, reads by two microscopes to 2 seconds. It is shown in Plate XV, and is a good instrument.

The Troughton & Simms 12-inch theodolite No. 2 is a repeating instrument, but has been used as a reiterating one. Its object-glass is 2 inches in diameter and has 19 inches focal length; its 12-inch horizontal circle is graduated to 5 minutes and reads by two microscopes to 1 second; its 12-inch vertical circle is graduated to 5 minutes and by two verniers reads to 5 seconds. It is a pretty good instrument, but inferior to the 14-inch Troughton & Simms theodolite No. 1. Further details in reference to it and a determination of its periodic errors may be found in the reports of the Chief of Engineers for 1872 and 1876, under the head of Lake Survey.

The correction for periodic error in angles read with the Repsold Universal Instrument No. 1, was determined in 1880, from observations on twenty-one different angles. The correction to a horizontal angle read with this instrument, both microscopes being used, is:

$$c=1''.198 \cos(2z+a+223^\circ) \sin a+0''.826 \cos(4z+2a+323^\circ) \sin 2a+1''.002 \cos(6z+3a+135^\circ) \sin 3a$$

The maximum value of c is $2''.25$ and occurs for

$$\left.\begin{array}{l} z=\ 19^\circ \text{ and } a=96^\circ \text{ or } 276^\circ \\ z=199^\circ \text{ and } a=96^\circ \text{ or } 276^\circ \end{array}\right\} c=+2''.25$$

$$\left.\begin{array}{l} z=115^\circ \text{ and } a=84^\circ \text{ or } 264^\circ \\ z=295^\circ \text{ and } a=84^\circ \text{ or } 264^\circ \end{array}\right\} c=-2''.25$$

In the formula, z is the reading of the finder on the left-hand object (the graduation increasing to the right) and a is the observed angle.

With the Repsold theodolite the independent probable errors of a single microscope-reading and of a single bisection of an object with the cross-wires of the telescope have been found to be $\pm 0''.30$ and $\pm 0''.70$ respectively.

The corresponding probable errors with the Troughton & Simms theodolite No. 2 have been found to be $\pm 0''.25$ and $\pm 0''.51$ respectively.

METHODS OF OBSERVATION.

§ **5.** The instructions for measuring angles, given in 1872, required that, for reiterating instruments, stations should be pointed at successively around the horizon, in the order *A*, *B*, *C*, *D*, *E—E*, *D*, *C*, *B*, *A*; that the telescope should be frequently reversed in such a way as to obtain an equal number of readings with vertical circle right and left, and that the horizontal circle should be turned in azimuth from time to time, so that an equal number of readings of each angle might be obtained on arcs not more than 60° apart uniformly distributed around the limb in order to eliminate periodic error. The instructions as to the elimination of periodic error were only partially carried out in this section of the triangulation in the work first done, but in that done in 1873 and 1874 by Mr. Flint, and that done in 1874 by Mr. Wisner, periodic error was habitually eliminated. The mean of the two values of an angle, obtained by turning the telescope over it, first in the direction of positive graduation and then in the direction of negative graduation, was called a combined result. Thus with the notation above, the mean of *C* to *D* and *D* to *C*, will be a combined result. Gambey theodolite No. 1 was used as a repeating instrument, sets of five repetitions of each successive angle being read, first in the positive, then in the negative order of graduation; the mean of the resulting values was taken as a combined result.

In the spring of 1873 the probable errors of a combined result for each instrument were deduced from the discrepancies between the separate combined results for an angle and their mean, a large number of angles read in 1871 and 1872, with each instrument, being used. As in some of these angles equal numbers of positive and negative results had not been obtained, in such cases the probable error of a single positive measurement was obtained, and also that of a single negative measurement. The mean of these was divided by $\sqrt{2}$ to obtain the probable error of a combined result. The following results for the probable error of a combined result were thus obtained for the different instruments:

	''
Troughton & Simms theodolite No. 1	±0.83
Pistor & Martins theodolite No. 2	±1.17
Repsold theodolite No. 1	±1.18
Gambey theodolite No. 1	±0.66

From this table it was attempted to assign to each instrument the number of combined results to be required from it, in order that the means of the combined results for any angle measured with any instrument might have approximately the same weight. From the above table it is seen that the mean of 20 combined results with Pistor & Martius No. 2, or Repsold No. 1, would have a probable error of $\pm 0''.26$, and 20 was fixed on as the number of combined results to be obtained for each angle with those instruments, to give a mean result whose weight was unity. For Troughton & Simms No. 1 and Gambey No. 1, the number of combined results whose mean would have a probable error of $\pm 0''.26$ would be but about one-half the above number. As the character of those instruments scarcely seemed to justify so marked a superiority, it was decided that sixteen combined results should be obtained with each. Twenty combined results were required from Troughton & Simms theodolite No. 2, to give a mean angle of weight unity. In July, 1873, in order to obtain for the adjustment a greater number of equations of condition, it was decided thereafter to make pointings at stations in the order *A*, *B*, *C*, *D*, *E*, *A*,—*A*, *E*, *D*, *C*, *B*, *A*, with reiterating instruments, and with the repeating instrument, Gambey No. 1, to read the angle needed at each station to close the horizon. An equation of condition for closing the horizon is thus obtained. The assignment of weights to mean angles, by the method above given, is not a satisfactory one, since an instrument which has a large periodic error may, if properly used, give as good results as if it had none, but the discrepancies between its combined results for an angle and their mean may be large, thus giving an apparently large probable error to the mean. Moreover, a given number of results over short lines, or lines over which the distant signals are habitually steady when seen in the telescope, will give a resulting value for the angle of much greater weight than the same number of combined results between two stations which are habitually unsteady. At the time the number of combined results to be obtained with each instrument was fixed on, little was known as to their periodic errors. Subsequent work, however, does not, so far as examined, indicate that the number of results required from each instrument to give a mean angle of weight unity needs any large modification. The following tables show the number of angles read with each instrument, and the number of stations occupied by each observer.

Number of stations occupied and angles read with each instrument for each year from station Vulcan to station Horicon.

Years.	T. & S. No. 1.		P. & M. No. 2.		Gambey No. 1.		Repsold No. 1.		T. & S. No. 2.	
	No. of stations.	No. of angles.	No. of stations.	No. of angles.	No. of stations.	No. of angles.	No. of stations.	No. of angles.	No. of stations.	No. of angles.
1872	1	3	2	18	2	8	19	146	....	....
1873	3	12	3	14	4	14	4	22	....	....
1874	4	18	3	9	....	....	8	35	5	24
Sums	8	33	8	41	6	22	31	203	5	24

Number of stations occupied.

Observers.	1872.	1873.	1874.
A. R. Flint	19	4	8
G. Y. Wisner	2	3	4
G. A. Marr	2	3	2
R. S. Woodward	1	4	5

C.—MEASURED AND ADJUSTED ANGLES BETWEEN THE LINES VULCAN-HURON MOUNTAINS AND MINNESOTA JUNCTION-HORICON.

§ **6.** This chapter, showing the measured and adjusted angles and an abstract of the adjustment of the triangulation between the lines Vulcan-Huron Mountains and Minnesota Junction-Horicon, is arranged in the same form as Chapter XIV, C, to § 7 of which reference may be made for a detailed explanation. For the angles of this chain, however, as stated in Chapter XV, B, § 5, combined results or the means of a positive or negative and an immediately succeeding negative or positive measure are used instead of separate measures as in Chapter XIV, C, § 7, so that the column headed "No. meas." gives the number of combined results. This remark applies also to angles given in Chapters XVI-XX. The scale of weights differs also from that used in Chapter XIV, C. The scale here adopted is that explained in § 5, Chapter XV, B, viz:

Weight 1 to mean of—

Sixteen combined results with Troughton & Simms theodolite No. 1.
Sixteen combined results with Gambey theodolite No. 1.
Twenty combined results with Repsold theodolite No. 1.
Twenty combined results with Pistor & Martins theodolite No. 2.
Twenty combined results with Troughton & Simms theodolite No. 2.

When the number of combined results on any angle differs from the standard number by no more than one-fourth that number, weight unity is assigned to the mean. When the number of combined results differs from the standard number by more than one-fourth that number, a weight equal to the ratio of the given to the standard number is assigned to the mean.

The adjustment of the chain is divided into three sections numbered III, IV, and V, respectively, the lines of division being Pine Hill-Burnt Bluff and Eldorado-Taycheedah. Only two rigorous conditions were neglected by this division of the adjustment, one at Burnt Bluff and one at Eldorado. The locally adjusted angles at these stations with weights resulting from the local adjustment were used in computing the general adjustment.

SECTION III.—*Triangulation from the line Vulcan-Huron Mountains to line Burnt Bluff-Pine Hill.*

VULCAN—1.

[Observer, G. A. Marr. Instrument, Pistor & Martins' 14-inch theodolite No. 2. Dates, July and August, 1872.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Granite Island and Triloba	5 56 02. 632	1_1	3	3. 0	0. 1	−1. 351	+0. 869	5 56 02. 150
Triloba and Ives Hill	16 36 07. 417	1_2	50	9. 4	3	+0. 134	−0. 165	16 36 07. 386
Ives Hill and Huron Mountains	6 10 25. 765	1_3	87	7. 8	4	+0. 090	−0. 142	6 10 25. 713
Granite Island and Huron Mountains	28 42 34. 641	1_{1+2+3}	51	10. 1	3	+0. 046	+0. 562	28 42 35. 249
Huron Mountains and Mount Houghton	59 59 16. 694	1_4	83	7. 4	4	−0. 113	+0. 065	59 59 16. 646
Triloba and Huron Mountains	22 46 33. 879	1_{2+3}	36	7. 5	2	−0. 473	−0. 307	22 46 33. 099
Ives Hill and Mount Houghton	66 09 42. 392	1_{3+4}	28	9. 9	1	+0. 044	−0. 077	66 09 42. 359
Triloba and Mount Houghton	82 45 49. 578	1_{2+3+4}	15	3. 9	1	+0. 409	−0. 242	82 45 49. 745
Granite Island and Mount Houghton	88 41 51. 275	$1_{1+2+3+4}$	2	3. 3	0. 1	−0. 007	+0. 627	88 41 51. 895

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3.2(1_1)+3.1(1_2)+3.1(1_3)+0.1(1_4)+3.6423=0$$
$$3.1(1_1)+9.1(1_2)+6.1(1_3)+1.1(1_4)+2.5463=0$$
$$3.1(1_1)+6.1(1_2)+11.1(1_3)+2.1(1_4)+2.6133=0$$
$$0.1(1_1)+1.1(1_2)+2.1(1_3)+6.1(1_4)+0.4883=0$$

SECTION III.—*Triangulation from the line Vulcan – Huron Mountains to the line Burnt Bluff – Pine Hill*—Continued.

HURON MOUNTAINS—2.

[Observer, R. S. Woodward. Instrument, Gambey 10-inch repeating theodolite. Date, September, 1872.]

Angle as measured between—		Notation.	No. Meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Vulcan and Granite Island	105 20 59.740	2_{1+2}	20	3.1	1	....	+0.391	105 21 00.131
Vulcan and Ives Hill	123 48 10.070	2_{1+2+3}	23	6.1	1	....	+0.379	123 48 10.449
Grand Island and Granite Island	1 59 03.500	2_2	13	4.6	1	....	+0.788	1 59 04.288

GRANITE ISLAND—3.

[Observer, G. A. Marr. Instrument, Pistor and Martins' 14-inch theodolite No. 2. Dates, September, 1872, and July, 1873.]

Angle as measured between—		Notation.	No. Meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Grand Island and Shelter Bay	19 22 43.452	3_1	24	2.4	1	+0.106	+0.149	19 22 43.707
Shelter Bay and Mount Mesnard	47 25 32.458	3_2	24	4.3	1	+0.720	−0.488	47 25 32.690
Mount Mesnard and Triloba	27 04 46.295	3_3	20	3.9	1	+0.720	+0.165	27 04 47.180
Triloba and Ives Hill	84 47 31.673	3_4	36	4.4	2	+0.021	−0.335	84 47 31.359
Ives Hill and Huron Mountains	4 44 35.840	3_5	6	1.1	0.3	+0.240	+0.636	4 44 36.716
Huron Mountains and Vulcan	45 56 31.068	3_6	16	3.6	1	−0.257	+0.475	45 56 31.286
Grand Island and Triloba	93 53 03.740	3_{1+2+3}	31	5.8	2	+0.011	−0.174	93 53 03.577
Grand Island and Ives Hill	178 40 35.438	$3_{1+2+3+4}$	4	3.2	0.2	+0.007	−0.509	178 40 34.936
Shelter Bay and Triloba	74 30 20.515	3_{2+3}	31	4.8	2	−0.322	−0.323	74 30 19.870
Shelter Bay and Ives Hill	159 17 51.613	3_{2+3+4}	2	0.7	0.1	+0.274	−0.658	159 17 51.229
Triloba and Huron Mountains	89 32 07.887	3_{4+5}	35	3.7	2	−0.113	+0.301	89 32 08.075
Triloba and Vulcan	135 28 38.177	3_{4+5+6}	14	3.6	0.7	+0.408	+0.776	135 28 39.361
Huron Mountains and Grand Island	176 34 48.301	3_{6+7}	13	3.6	0.6	+0.174	−0.127	176 34 48.348
Vulcan and Grand Island	130 38 17.100	3_7	1	0.0	0.05	+0.564	−0.602	130 38 17.062

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3.85(3_1)+2.85(3_2)+2.85(3_3)+0.85(3_4)+0.65(3_5)+0.05(3_6)-4.6763=0$$
$$2.85(3_1)+5.95(3_2)+4.95(3_3)+0.95(3_4)+0.65(3_5)+0.05(3_6)-8.3190=0$$
$$2.85(3_1)+4.95(3_2)+5.95(3_3)+0.95(3_4)+0.65(3_5)+0.05(3_6)-8.3190=0$$
$$0.85(3_1)+0.95(3_2)+0.95(3_3)+5.65(3_4)+3.35(3_5)+0.75(3_6)-2.1902=0$$
$$0.65(3_1)+0.65(3_2)+0.65(3_3)+3.35(3_4)+3.65(3_5)+0.75(3_6)-1.7595=0$$
$$0.05(3_1)+0.05(3_2)+0.05(3_3)+0.75(3_4)+0.75(3_5)+1.75(3_6)+0.1771=0$$

IVES HILL—4.

[Observer, G. Y. Wisner. Instrument, Gambey 10-inch repeating theodolite. Date, August, 1872.]

Angle as measured between—		Notation.	No. Meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Huron Mountains and Vulcan	50 01 24.685	4_1	18	4.3	1	+0.080	+0.279	50 01 25.044
Huron Mountains and Granite Island	156 48 12.790	4_{1+2}	15	1.1	1	+0.130	+0.353	156 48 13.273
Granite Island and Triloba	23 11 32.480	4_3	6	1.2	0.4	+0.325	−0.368	23 11 32.437
Triloba and Huron Mountains	179 59 45.936	4_{1+2+3}	18	2.7	1	−0.211	−0.014	179 59 45.711
Vulcan and Triloba	129 58 20.760	4_{2+3}	6	1.9	0.4	+0.200	−0.293	129 58 20.667

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$1.4(4_1)-0.4(4_{1+2})-0.4(4_3)+0.070=0$$
$$-0.4(4_1)+2.4(4_{1+2})+1.4(4_3)-0.736=0$$
$$-0.4(4_1)+1.4(4_{1+2})+1.8(4_3)-0.736=0$$

SECTION III.—*Triangulation from the line Vulcan-Huron Mountains, to the line Burnt Bluff-Pine Hill*—Continued.

TRILOBA—5.

[Observer, G. Y. Wisner. Instrument, Troughton and Simms' 14-inch theodolite No. 1. Dates, June and July, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Ives Hill and Vulcan	33 25 37.206	5_1	16	4.6	1	−0.236	−0.189	33 25 36.781
Vulcan and Granite Island	38 35 20.521	5_2	19	3.8	1	−0.236	+0.368	38 35 20.653
Granite Island and Grand Island	72 49 17.182	5_3	17	4.0	1	+0.203	−0.149	72 49 17.236
Grand Island and Shelter Bay	13 16 32.600	5_4	21	5.1	1	+0.203	−0.715	13 16 32.088
Shelter Bay and Mount Mesnard	35 02 50.513	5_5	15	7.5	1	+0.157	+0.084	35 02 50.754
Ives Hill and Granite Island	72 00 57.018	5_{1+2}	17	4.0	1	+0.237	+0.179	72 00 57.434
Granite Island and Shelter Bay	86 05 50.280	5_{3+4}	8	2.8	0.5	−0.092	−0.864	86 05 49.324
Granite Island and Mount Mesnard	121 08 41.380	5_{3+4+5}	5	3.0	0.3	−0.522	−0.780	121 08 40.078

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(5_1)+1(5_2) \qquad +0.7090=0$$
$$1(5_1)+2(5_2) \qquad +0.7090=0$$
$$1.8(5_3)+0.8(5_4)+0.3(5_5)-0.5745=0$$
$$0.8(5_3)+1.8(5_4)+0.3(5_5)-0.5745=0$$
$$0.3(5_3)+0.3(5_4)+1.3(5_5)-0.3255=0$$

MOUNT MESNARD—6.

[Observer, G. A. Marr. Instrument, Pistor & Martins' 14-inch theodolite. Date, August, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Triloba and Granite Island	31 46 32.993	6_1	27	2.8	1	+0.148	−0.022	31 46 33.119
Granite Island and Shelter Bay	100 25 34.308	6_2	26	5.7	1	+0.148	−0.341	100 25 34.115
Shelter Bay and Triloba	227 47 52.255	6_3	25	5.4	1	+0.148	+0.364	227 47 52.767

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(6_1)+(6_2)-0.444=0$$
$$(6_1)+2(6_2)-0.444=0$$

SHELTER BAY—7.

[Observers, G. Y. Wisner and G. A. Marr. Instruments, Troughton & Simms' 14-inch theodolite No. 1; Pistor & Martins' 14-inch theodolite No. 2. Dates, July, 1873, and June and July, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Mount Mesnard and Triloba	12 45 03.781	7_1	16	4.1	1	....	−1.033	12 45 02.748
Triloba and Granite Island	19 23 53.400	7_2	15	2.1	1	....	−1.113	19 23 52.287
Granite Island and Grand Island	123 34 58.812	7_3	17	2.0	1	....	−0.408	123 34 58.404
Grand Island and Mud Lake	66 00 07.190	7_4	21	2.8	1	+0.155	−0.072	66 00 07.273
Mud Lake and Grand Island	293 59 52.500	7_{-4}	21	3.1	1	+0.155	+0.072	293 59 52.727

NOTE.—The angles 7_1, 7_2, 7_3, were read by G. Y. Wisner with Troughton & Simms' instrument. The angles 7_4, 7_{-4}, were read by G. A. Marr with Pistor & Martins' instrument.

SECTION III.—*Triangulation from the line Vulcan – Huron Mountains to the line Burnt Bluff – Pine Hill*—Continued.

GRAND ISLAND—8.

[Observers, G. Y. Wisner and G. A. Marr. Instruments, Troughton & Simms' 14 inch theodolite No. 1; Pistor & Martins' 14-inch theodolite No. 2. Dates, August and September, 1872, and September and October, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Divide and Mud Lake	28 25 45.526	8_1	22	3.8	1	+1.027	−0.017	28 25 46.536
Mud Lake and Shelter Bay	53 44 02.303	8_{2+3}	15	3.7	1	+1.027	−0.029	53 44 03.301
Wood Island and Shelter Bay	2 31 50.686	8_3	22	4.5	1	−0.590	+0.231	2 31 50.327
Shelter Bay and Triloba	23 44 38.766	8_4	17	3.9	1	−0.085	+0.019	23 44 38.700
Triloba and Granite Island	13 17 41.452	8_5	23	3.1	1	−0.085	−0.060	13 17 41.307
Wood Island and Huron Mountains	41 00 18.690	$8_{3+4+5+6}$	17	2.1	1	....	−0.611	41 00 18.079
Divide and Shelter Bay	82 09 50.753	8_{1+2+3}	12	4.9	0.6	−0.870	−0.046	82 09 49.837
Granite Island and Divide	240 47 49.565	8_{6+7}	23	4.2	1	+0.504	+0.087	240 47 50.156
Wood Island and Granite Island	39 34 09.849	8_{3+4+5}	27	7.3	2	+0.295	+0.190	39 34 10.334

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.6(8_1)+1.6(8_{2+3})\qquad+1(8_4)+1(8_5)-4.1424=0$$
$$1.6(8_1)+2.6(8_{2+3})\qquad+1(8_4)+1(8_5)-4.1424=0$$
$$+3(8_3)+2(8_4)+2(8_5)+2.1100=0$$
$$1.0(8_1)+1.0(8_{2+3})+2(8_3)+4(8_4)+3(8_5)-0.2780=0$$
$$1.0(8_1)+1.0(8_{2+3})+2(8_3)+3(8_4)+4(8_5)-0.2780=0$$

NOTE.—Angles 8_3, $8_{3+4+5+6}$, and 8_{3+4+5} were read by G. Y. Wisner with the Troughton & Simms instrument. All the others were read by G. A. Marr with the Pistor & Martins instrument.

MUD LAKE—9.

[Observer, G. A. Marr. Instrument, Pistor & Martins' 14-inch theodolite No. 2. Date, June, 1874.]

Angles as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Shelter Bay and Grand Island	60 15 50.759	9_1	25	3.4	1	0.000	−0.145	60 15 50.614
Grand Island and Divide	47 20 58.062	9_2	21	2.8	1	+0.119	+0.105	47 20 58.286
Divide and Monistique	72 56 50.257	9_3	21	6.0	1	+0.119	+0.109	72 56 50.485
Monistique and Sturgeon	22 23 22.538	9_4	21	3.3	1	+0.119	−0.453	22 23 22.204
Sturgeon and Grand Island	217 18 48.667	9_{5+1}	21	5.0	1	+0.119	+0.239	217 18 49.025

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(9_2)+1(9_3)+1(9_4)-0.476=0$$
$$1(9_2)+2(9_3)+1(9_4)-0.476=0$$
$$1(9_2)+1(9_3)+2(9_4)-0.476=0$$

DIVIDE—10.

[Observers, G. Y. Wisner and G. A. Marr. Instruments, Troughton & Simms' 14-inch theodolite No. 1; Pistor & Martins' 14-inch theodolite No. 2. Dates, September, 1873, and May and June, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Monistique and Mud Lake	79 11 00.064	10_1	22	2.9	1	+0.413	−0.064	79 11 00.413
Mud Lake and Grand Island	104 13 15.870	10_2	20	2.4	1	....	−0.132	104 13 15.738
Mud Lake and Monistique	280 48 59.110	10_{2+3}	21	3.7	1	+0.413	+0.064	280 48 59.587

NOTE.—The angle 10_2 was read by G. Y. Wisner with the Troughton & Simms instrument. The others were read by G. A. Marr with the Pistor & Martins instrument.

SECTION III.—*Triangulation from the line Vulcan-Huron Mountains to the line Burnt Bluff-Pine Hill*—Continued.

MONISTIQUE—11.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms' 12-inch theodolite No. 2. Date, May, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Fishdam and Sturgeon	53 09 20.379	11_1	16	1.0	1	−0.289	+0.066	53 09 20.156
Sturgeon and Mud Lake	110 01 52.571	11_2	17	7.6	1	−0.245	−0.376	110 01 51.950
Mud Lake and Divide	27 52 09.912	11_3	17	5.9	1	−0.245	+0.186	27 52 09.853
Divide and Fishdam	168 56 38.205	11_4	17	5.9	1	−0.288	+0.124	168 56 38.041
Divide and Sturgeon	222 05 57.963	11_{4+1}	16	6.6	1	+0.044	+0.190	222 05 58.197

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(11_1)+1(11_2)+1(11_3)+1.067=0$$
$$1(11_1)+3(11_2)+2(11_3)+1.513=0$$
$$1(11_1)+2(11_2)+3(11_3)+1.513=0$$

STURGEON—12.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms' 12-inch theodolite No. 2. Date, June, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Mud Lake and Monistique	47 34 47.267	12_1	21	6.4	1	−0.335	−0.286	47 34 46.646
Monistique and Fishdam	89 40 37.148	12_2	21	5.6	1	−0.335	+0.346	89 40 37.159
Fishdam and Burnt Bluff	62 07 43.173	12_3	22	4.1	1	−0.335	−0.464	62 07 42.374
Burnt Bluff and Pine Hill	36 41 19.933	12_4	24	5.1	1	....	+0.738	36 41 20.671
Burnt Bluff and Mud Lake	160 36 53.752	12_{4+5}	20	4.8	1	−0.335	+0.404	160 36 53.821

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(12_1)+1(12_2)+1(12_3)+1.340=0$$
$$1(12_1)+2(12_2)+1(12_3)+1.340=0$$
$$1(12_1)+1(12_2)+2(12_3)+1.340=0$$

FISHDAM—13.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms' 12-inch theodolite No. 2. Date, June, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Burnt Bluff and Sturgeon	91 38 39.171	13_1	20	7.0	1	−0.282	−0.559	91 38 38.330
Sturgeon and Monistique	37 10 03.299	13_2	20	2.5	1	−0.282	+0.250	37 10 03.267
Monistique and Burnt Bluff	231 10 18.376	13_3	17	5.6	1	−0.282	+0.309	231 10 18.403

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(13_1)+1(13_2)+0.846=0$$
$$1(13_1)+2(13_2)+0.846=0$$

SECTION III.—*Triangulation from the line Vulcan-Huron Mountains to the line Burnt Bluff-Pine Hill*—Continued.

BURNT BLUFF—14.

[Observer, R. S. Woodward. Instrument, Troughton & Simms' 14-inch theodolite No. 1. Date, June, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	$[v]$	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Boyer's Bluff and Ford River....	59 24 38.666	14_1	16	6.6	1	+0.491	+0.033	59 24 39.190
Ford River and Pine Hill........	52 46 31.875	14_2	16	4.4	1	+0.491	+0.867	52 46 33.233
Pine Hill and Sturgeon..........	39 43 08.725	14_3	16	8.8	1	+0.491	+0.591	39 43 09.807
Sturgeon and Fishdam	26 13 41.043	14_4	20	6.1	1	+0.489	−0.694	26 13 40.838
Fishdam and Boyer's Bluff......	181 51 57.238	14_5	16	7.4	1	+0.491	......	....

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(14_1)+1(14_2)+1(14_3)+1(14_4)-2.453=0$$
$$1(14_1)+2(14_2)+1(14_3)+1(14_4)-2.453=0$$
$$1(14_1)+1(14_2)+2(14_3)+1(14_4)-2.453=0$$
$$1(14_1)+1(14_2)+1(14_3)+2(14_4)-2.453=0$$

NOTE.—The adjustment of the triangulation was divided at the line Burnt Bluff-Pine Hill. Hence the general corrections to 14_1 and 14_2 are derived from the succeeding section of the adjustment.

PINE HILL—15.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, June, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	$[v]$	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Sturgeon and Burnt Bluff........	103 35 30.334	15_1	20	7.7	1	0.000	+0.738	103 35 31.072
Burnt Bluff and Ford River	76 34 15.669	15_2	19	8.1	1	0.000	+1.083	76 34 16.752

Numerical equations of condition in the triangulation from the line Vulcan-Huron Mountains to the line Burnt Bluff-Pine Hill.

SIDE-EQUATIONS.

$$\text{I. } (10)\ -5.7798\,[2_{1+2}]+14.0967\,[2_{1+2+3}]+17.2425\,[3_5]-3.1316\,[3_6]+11.3037\,[4_1]+6.3490\,[4_{1+2}]-17.961=0$$

$$\text{II. } (30)\ -21.4090\,[3_4]-38.6515\,[3_5]-38.6515\,[3_6]-11.3012\,[4_1]+11.3012\,[4_{1+2}]+17.6502\,[4_3]+31.8992\,[5_1]-26.3858\,[5_2]+57.162=0$$

$$\text{III. } (500)\ +607.7145\,[2_2]+63.0998\,[2_{1+2}]-63.0998\,[2_{1+2+3}]-49.1339\,[4_{1+2}]-49.1433\,[4_3]+6.8348\,[5_1]+6.8348\,[5_2]-6.5090\,[5_3]+929.1941\,[8_5]+840.0884\,[8_3]+840.0884\,[8_4]-840.0884\,[8_{3+4+5+6}]-1149.813=0$$

$$\text{IV. } (20)\ +13.5068\,[3_2]-5.8369\,[3_3]+1.4364\,[5_3]+1.4364\,[5_4]-30.0170\,[5_5]-19.0931\,[6_1]-15.2188\,[6_2]+5.717=0$$

$$\text{V. } (40)\ +61.2899\,[3_1]+1.4296\,[3_2]+1.4296\,[3_3]+6.5090\,[5_3]-89.2381\,[5_4]-27.9221\,[7_2]-13.9421\,[7_3]-108.276=0$$

NOTE.—In the solution for determining the general corrections, each of the side-equations was divided by the number inclosed in parenthesis and placed opposite it.

ANGLE-EQUATIONS.

$$\text{VI. } [1_1]+[1_2]+[1_3]+[2_{1+2}]+[3_6]-1.427=0$$
$$\text{VII. } [1_1]+[1_2]+[3_5]+[3_6]+[4_{1+2}]-[4_1]-1.888=0$$
$$\text{VIII. } [1_3]+[2_{1+2+3}]+[4_1]\quad 0.516=0$$

Numerical equations of condition, &c.—Continued.

ANGLE-EQUATIONS—Continued.

IX.	$[1_1]$	$+[3_4]$	$+[3_5]$	$+[3_6]$	$+[5_2]$						$-2.013=0$
X.	$[1_2]$	$+[4_{1+2}]$	$-[4_1]$	$+[4_3]$	$+[5_1]$						$+0.648=0$
XI.	$[3_1]$	$+[3_2]$	$+[3_3]$	$+[5_3]$	$+[8_5]$						$+0.383=0$
XII.	$[2_2]$	$+[3_1]$	$-[3_2]$	$-[3_3]$	$-[3_4]$	$-[3_5]$	$-[8_3]$	$-[8_4]$	$-[8_5]$	$+[8_{3+4+5+6}]$	$+0.140=0$
XIII.	$[3_3]$	$+[5_3]$	$+[5_4]$	$+[5_5]$	$+[6_1]$						$+0.636=0$
XIV.	$[3_2]$	$+[3_3]$	$+[5_3]$	$+[5_4]$	$+[7_2]$						$+2.300=0$
XV.	$[3_2]$	$+[6_2]$	$+[7_1]$	$+[7_2]$							$+2.976=0$
XVI.	$[3_1]$	$+[7_3]$	$+[8_4]$	$+[8_5]$							$+0.300=0$
XVII.	$[7_4]$	$+[8_{2+3}]$	$+[9_1]$								$+0.246=0$
XVIII.	$[8_1]$	$+[9_2]$	$+[10_2]$								$+0.044=0$
XIX.	$[9_3]$	$+[10_1]$	$+[11_3]$								$-0.231=0$
XX.	$[9_4]$	$+[11_2]$	$+[12_1]$								$+1.115=0$
XXI.	$[11_1]$	$+[12_2]$	$+[13_2]$								$-0.662=0$
XXII.	$[12_3]$	$+[13_1]$	$+[14_4]$								$+1.717=0$
XXIII.	$[12_4]$	$+[14_3]$	$+[15_1]$								$-2.06\ =0$

General corrections in terms of the correlates.

$[1_1]$	=+0.30471 VI	+0.38750 VII	−0.08279 VIII	+0.51132 IX	−0.12382 X		
$[1_2]$	=+0.00428 VI	+0.08016 VII	−0.07588 VIII	−0.12382 IX	+0.20398 X		
$[1_3]$	=+0.00399 VI	−0.15867 VII	+0.16266 VIII	−0.08279 IX	−0.07588 X		
$[1_4]$	=−0.00714 VI	+0.03382 VII	−0.04096 VIII	+0.04245 IX	−0.00863 X		
$[2_{1+2}]$	=−0.57798 I	+0.12620 III	+1.00000 VI				
$[2_2]$	=	+1.21543 III	+1.00000 XII				
$[2_{1+2+3}]$	=+1.40967 I	−0.12620 III	+1.00000 VIII				
$[3_1]$	=−0.05250 I	+0.03062 II	−0.04200 IV	+0.64991 V	+0.01149 VI	−0.01687 VII	−0.02932 IX
	+0.21024 XI	−0.16943 XII	−0.10951 XIII	−0.21902 XIV	−0.10951 XV	+0.42926 XVI	
$[3_2]$	=+0.00144 I	+0.00217 II	+0.51247 IV	−0.16242 V	+0.00252 VI	+0.00383 VII	−0.00612 IX
	+0.04105 XI	−0.03240 XII	−0.42472 XIII	+0.15056 XIV	+0.57528 XV	−0.10951 XVI	
$[3_3]$	=+0.00144 I	+0.00217 II	−0.45473 IV	−0.16242 V	+0.00253 VI	+0.00383 VII	−0.00612 IX
	+0.04105 XI	−0.03240 XII	+0.57528 XIII	+0.15056 XIV	−0.42472 XV	−0.10951 XVI	
$[3_4]$	=−0.60021 I	+0.19394 II	−0.00382 IV	−0.01979 V	−0.01686 VI	−0.36802 VII	+0.02464 IX
	−0.03235 XI	−0.00915 XII	−0.00995 XIII	−0.01990 XIV	−0.00995 XV	−0.01245 XVI	
$[3_5]$	=+1.11381 I	−0.40423 II	+0.00050 IV	−0.04336 V	−0.11654 VI	+0.50826 VII	+0.15710 IX
	−0.02576 XI	−0.24788 XII	+0.00130 XIII	+0.00260 XIV	+0.00130 XV	−0.02836 XVI	
$[3_6]$	=−0.39765 I	−0.64709 II	+0.00097 IV	+0.01779 V	+0.62813 VI	+0.51159 VII	+0.49473 IX
	+0.01655 XI	+0.11685 XII	+0.00253 XIII	+0.00506 XIV	+0.00253 XV	+0.01149 XVI	
$[4_1]$	=+0.89910 I	−0.19266 II	−0.01787 III	−0.71428 VII	+0.76623 VIII	−0.58441 X	
$[4_{1+2}]$	=+0.54520 I	−0.07476 II	−0.01787 III	+0.71428 VII	+0.05195 VIII	+0.12987 X	
$[4_3]$	=−0.22424 I	+0.34218 II	−0.04469 III	−0.71428 VII	+0.12987 VIII	+0.32467 X	
$[5_1]$	=+1.00205 II	+0.00456 III	−0.33333 IX	+0.66667 X			
$[5_2]$	=−0.94079 II	+0.00456 III	+0.66667 IX	−0.33333 X			
$[5_3]$	=−0.00916 III	+0.16988 IV	+0.77673 V	+0.70313 XI	+0.31250 XIII	+0.40625 XIV	
$[5_4]$	=+0.00387 III	+0.16988 IV	−1.61695 V	−0.29688 XI	+0.31250 XIII	+0.40625 XIV	
$[5_5]$	=+0.00122 III	−1.23291 IV	+0.19390 V	−0.09375 XI	+0.62500 XIII	−0.18750 XIV	
$[6_1]$	=−0.38279 IV	+0.66667 XIII	−0.33333 XV				
$[6_2]$	=−0.18907 IV	−0.33333 XIII	+0.66667 XV				
$[7_1]$	=		+1.00000 XV				
$[7_2]$	=−0.69805 V	+1.00000 XIV	+1.00000 XV				

General corrections in terms of the correlates—Continued.

$[7_3]$	$=-0.34855$ V $+1.00000$ XVI
$[7_4]$	$=$ $+0.50000$ XVII
$[8_1]$	$=-0.09143$ III -0.07042 XI $+0.04694$ XII -0.14084 XVI -0.34742 XVII $+0.65258$ XVIII
$[8_{2+3}]$	$=-0.09143$ III -0.07042 XI $+0.04694$ XII -0.14084 XVI $+0.65258$ XVII -0.34742 XVIII
$[8_3]$	$=+0.30405$ III -0.19718 XI -0.20188 XII -0.39436 XVI $+0.09390$ XVII $+0.09390$ XVIII
$[8_4]$	$=+0.10290$ III -0.35212 XI -0.09859 XII $+0.29577$ XVI -0.07042 XVII -0.07042 XVIII
$[8_5]$	$=+0.28111$ III $+0.64789$ XI -0.09859 XII $+0.29577$ XVI -0.07042 XVII -0.07042 XVIII
$[8_{3+4+5+6}]$	$=-1.68018$ III $+1.00000$ XII
$[9_1]$	$=+1.00000$ XVII
$[9_2]$	$=$ $+0.75000$ XVIII -0.25000 XIX -0.25000 XX
$[9_3]$	$=$ -0.25000 XVIII $+0.75000$ XIX -0.25000 XX
$[9_4]$	$=$ -0.25000 XVIII -0.25000 XIX $+0.75000$ XX
$[10_1]$	$=$ $+0.50000$ XIX
$[10_2]$	$=$ $+1.00000$ XVIII
$[11_1]$	$=$ -0.12500 XIX -0.12500 XX $+0.62500$ XXI
$[11_2]$	$=$ -0.37500 XIX $+0.62500$ XX -0.12500 XXI
$[11_3]$	$=$ $+0.62500$ XIX -0.37500 XX -0.12500 XXI
$[12_1]$	$=$ $+0.75000$ XX -0.25000 XXI -0.25000 XXII
$[12_2]$	$=$ -0.25000 XX $+0.75000$ XXI -0.25000 XXII
$[12_3]$	$=$ -0.25000 XX -0.25000 XXI $+0.75000$ XXII
$[12_4]$	$=$ $+1.00000$ XXIII
$[13_1]$	$=$ -0.33333 XXI $+0.66667$ XXII
$[13_2]$	$=$ $+0.66667$ XXI -0.33333 XXII
$[14_3]$	$=$ $+0.80000$ XXIII
$[14_4]$	$=$ $+0.80000$ XXII
$[15_1]$	$=$ $+1.00000$ XXII

Normal equations for determining the correlates.

No. of equation.							
1.	0=−1.79610	+5.72871 I	−0.75959 II	−0.28239 III	+0.00055 IV	−0.08033 V	−0.97563 VI
		+0.36226 VII	+2.30877 VIII	+0.11595 IX	−0.57814 X	−0.04960 XI	−0.46400 XII
		+0.00145 XIII	+0.00289 XIV	+0.00145 XV	−0.05250 XVI		
2.	0=+1.90542	−0.75959 I	+3.35478 II	−0.02545 III	+0.00084 IV	+0.04706 V	−0.64709 VI
		−0.93342 VII	−0.19266 VIII	−1.79817 IX	+1.46213 X	+0.03496 XI	+0.17534 XII
		+0.00217 XIII	+0.00433 XIV	+0.00217 XV	+0.03062 XVI		
3.	0=−2.29963	−0.28239 I	−0.02545 II	+5.54467 III	−0.00221 IV	−0.01011 V	+0.12620 VI
		−0.14407 VIII	+0.00456 IX	−0.04013 X	+0.27195 XI	−1.15281 XII	−0.00407 XIII
		−0.00529 XIV	+0.38401 XVI	−0.09143 XVII	−0.09143 XVIII		
4.	0=+0.28585	+0.00055 I	+0.00084 II	−0.00221 III	+2.86291 IV	−0.41364 V	+0.00097 VI
		+0.00147 VII	−0.00235 IX	+0.18562 XI	−0.01242 XII	−1.73067 XIII	+0.39750 XIV
		+0.32340 XV	−0.04200 XVI				
5.	0=−2.70690	−0.08033 I	+0.04706 II	−0.01011 III	−0.41364 IV	+5.32671 V	+0.01779 VI
		−0.02557 VII	−0.04536 IX	+1.10180 XI	−0.26192 XII	−0.80874 XIII	−1.86311 XIV
		−0.86047 XV	+0.30136 XVI				
6.	0=−1.42700	−0.97563 I	−0.64709 II	+0.12620 III	+0.00097 IV	+0.01779 V	+1.94111 VI
		+0.82058 VII	+0.00399 VIII	+0.79944 IX	+0.00428 X	+0.01655 XI	+0.11685 XII
		+0.00253 XIII	+0.00506 XIV	+0.00253 XV	+0.01149 XVI		
7.	0=−1.88800	+0.36226 I	−0.93342 II	+0.00147 IV	−0.02557 V	+0.82058 VI	+2.91607 VII
		−0.87295 VIII	+1.03933 IX	+0.79444 X	−0.00921 XI	−0.13103 XII	+0.00383 XIII
		+0.00766 XIV	+0.00383 XV	−0.01687 XVI			

Normal equations for determining the correlates—Continued.

No. of equation.							
8.	0=−0.51600	+2.30877 I	−0.19266 II	−0.14407 III	+0.00399 VI	−0.87295 VII	+1.92889 VIII
		−0.08279 IX	−0.66029 X				
9.	0=−2.01300	+0.11595 I	−1.79817 II	+0.00456 III	−0.00235 IV	−0.04536 V	+0.79944 VI
		+1.03933 VII	−0.08279 VIII	+1.85446 IX	−0.45715 X	−0.04156 XI	−0.14018 XII
		−0.00612 XIII	−0.01224 XIV	−0.00612 XV	−0.02932 XVI		
10.	0=+0.64800	−0.57814 I	+1.46213 II	−0.04013 III	+0.00428 VI	+0.79444 VII	−0.66029 VIII
		−0.45715 IX	+1.90960 X				
11.	0=+0.38300	−0.04960 I	+0.03496 II	+0.27195 III	+0.18562 IV	+1.10180 V	+0.01655 VI
		−0.00921 VII	−0.04156 IX	+1.64336 XI	−0.33282 XII	+0.35355 XIII	+0.48835 XIV
		+0.04105 XV	+0.50601 XVI	−0.07042 XVII	−0.07042 XVIII		
12.	0=+0.14000	−0.46400 I	+0.17534 II	−1.15281 III	−0.01242 IV	−0.26192 V	+0.11685 VI
		−0.13103 VII	−0.14018 IX	−0.33282 XI	+2.89032 XII	−0.03240 XIII	−0.06480 XIV
		−0.03240 XV	−0.36661 XVI	+0.04694 XVII	+0.04694 XVIII		
13.	0=+0.63600	+0.00145 I	+0.00217 II	−0.00407 III	−1.73067 IV	−0.80874 V	+0.00253 VI
		+0.00383 VII	−0.00612 IX	+0.35355 XI	−0.03240 XII	+2.49195 XIII	+0.77556 XIV
		−0.75805 XV	−0.10951 XVI				
14.	0=+2.30000	+0.00289 I	+0.00433 II	−0.00529 III	+0.39750 IV	−1.86311 V	+0.00506 VI
		+0.00766 VII	−0.01224 IX	+0.48835 XI	−0.06480 XII	+0.77556 XIII	+2.11362 XIV
		+1.15056 XV	−0.21902 XVI				
15.	0=+2.97600	+0.00145 I	+0.00217 II	+0.32340 IV	−0.86047 V	+0.00253 VI	+0.00383 VII
		−0.00612 IX	+0.04105 XI	−0.03240 XII	−0.75805 XIII	+1.15056 XIV	+3.24195 XV
		−0.10951 XVI					
16.	0=+0.30000	−0.05250 I	+0.03062 II	+0.38401 III	−0.04200 IV	+0.30136 V	+0.01149 VI
		−0.01687 VII	−0.02932 IX	+0.50601 XI	−0.36661 XII	−0.10951 XIII	−0.21902 XIV
		−0.10951 XV	+2.02080 XVI	−0.14084 XVII	−0.14084 XVIII		
17.	0=+0.24600	−0.09143 III	−0.07042 XI	+0.04694 XII	−0.14084 XVI	+2.15258 XVII	−0.34742 XVIII
18.	0=+0.04400	−0.09143 III	−0.07042 XI	+0.04694 XII	−0.14084 XVI	−0.34742 XVII	+2.40258 XVIII
		−0.25000 XIX	−0.25000 XX				
19.	0=−0.23100	−0.25000 XVIII	+1.87500 XIX	−0.62500 XX	−0.12500 XXI		
20.	0=+1.11500	−0.25000 XVIII	−0.62500 XIX	+2.12500 XX	−0.37500 XXI	−0.25000 XXII	
21.	0=−0.66200	−0.12500 XIX	−0.37500 XX	+2.04167 XXI	−0.58333 XXII		
22.	0=+1.71700		−0.25000 XX	−0.58333 XXI	+2.21667 XXII		
23.	0=−2.06700					+2.80000 XXIII	

Values of the correlates and their logarithms.

I	=+0.6352	log 9.8029105$_{+}$	XIII	=−0.7969	log 9.9014038$_{-}$
II	=+0.4550	log 9.6580114$_{+}$	XIV	=+0.1349	log 9.1300119$_{+}$
III	=+0.4831	log 9.6840370$_{+}$	XV	=−1.0330	log 0.0141003$_{-}$
IV	=−0.4314	log 9.6348801$_{-}$	XVI	=−0.3003	log 9.4775553$_{-}$
V	=+0.3081	log 9.4886917$_{+}$	XVII	=−0.1446	log 9.1601683$_{-}$
VI	=+0.6976	log 9.8436065$_{+}$	XVIII	=−0.1324	log 9.1218880$_{-}$
VII	=+0.1938	log 9.2873538$_{+}$	XIX	=−0.1285	log 9.1089031$_{-}$
VIII	=−0.4556	log 9.6585837$_{-}$	XX	=−0.6905	log 9.8391637$_{-}$
IX	=+0.9415	log 9.9738203$_{+}$	XXI	=−0.0584	log 8.7664128$_{-}$
X	=−0.4996	log 9.6986224$_{-}$	XXII	=−0.8678	log 9.9384196$_{-}$
XI	=−0.1651	log 9.2177471$_{-}$	XXIII	=+0.7382	log 9.8681740$_{+}$
XII	=+0.2008	log 9.3027637$_{+}$			

Values of the general corrections.

	″		″		″
$[1_1]$	=+0. 869	$[5_2]$	=+0. 368	$[9_2]$	=+0. 105
$[1_2]$	=−0. 165	$[5_3]$	=−0. 149	$[9_3]$	=−0. 109
$[1_3]$	=−0. 142	$[5_4]$	=−0. 715	$[9_4]$	=−0. 453
$[1_4]$	=+0. 065	$[5_5]$	=+0. 084	$[10_1]$	=−0. 064
$[2_{1+2}]$	=+0. 391	$[6_1]$	=−0. 022	$[10_2]$	=−0. 132
$[2_2]$	=+0. 788	$[6_2]$	=−0. 342	$[11_1]$	=+0. 066
$[2_{1+2+3}]$	=+0. 379	$[7_1]$	=−1. 033	$[11_2]$	=−0. 376
$[3_1]$	=+0. 149	$[7_2]$	=−1. 113	$[11_3]$	=+0. 186
$[3_2]$	=−0. 488	$[7_3]$	=−0. 408	$[12_1]$	=−0. 286
$[3_3]$	=+0. 165	$[7_4]$	=−0. 072	$[12_2]$	=+0. 346
$[3_4]$	=−0. 335	$[8_1]$	=−0. 017	$[12_3]$	=−0. 464
$[3_5]$	=+0. 636	$[8_{2+3}]$	=−0. 029	$[12_4]$	=+0. 738
$[3_6]$	=+0. 475	$[8_3]$	=+0. 231	$[13_1]$	=−0. 559
$[4_1]$	=+0. 279	$[8_4]$	=+0. 019	$[13_2]$	=+0. 250
$[4_{1+2}]$	=+0. 353	$[8_5]$	=−0. 060	$[14_3]$	=+0. 591
$[4_3]$	=−0. 368	$[8_{3+4+5+6}]$	=−0. 611	$[14_4]$	=−0. 694
$[5_1]$	=−0. 189	$[9_1]$	=+0. 145	$[15_1]$	=+0. 738

Residuals resulting from substitution of general corrections in numerical equations of condition.

No. of equation.	Residual.	No. of equation.	Residual.
1	+0. 0010	13	−0. 0001
2	−0. 0060	14	−0. 0001
3	+0. 0500	15	−0. 0001
4	+0. 0020	16	0. 0000
5	−0. 0040	17	−0. 0002
6	+0. 0004	18	−0. 0001
7	+0. 0003	19	−0. 0001
8	+0. 0001	20	−0. 0001
9	0. 0000	21	+0. 0001
10	−0. 0001	22	+0. 0001
11	−0. 0001	23	+0. 0000
12	+0. 0000		

SECTION IV.—*Triangulation from the line Burnt Bluff – Pine Hill to the line Eldorado – Taycheedah.*

FORD RIVER—16.

[Observers, A. R. Flint and G. Y. Wisner. Instruments, Repsold 10-inch theodolite, Troughton & Simms' 14-inch theodolite. Dates, June and July, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Pine Hill and Burnt Bluff........	50 39 10. 853	16_1	16	5. 1	1	−0. 225	+0. 880	50 39 11. 508
Burnt Bluff and Boyer's Bluff....	66 17 33. 377	16_2	15	5. 7	1	+0. 320	+0. 074	66 17 33. 771
Boyer's Bluff and Cedar River...	55 17 15. 508	16_3	16	4. 7	1	+0. 320	−0. 546	55 17 15. 282
Cedar River and Azimuth-Mark.	187 04 00. 321	16_4	16	4 8	1	−0. 225	−0. 204	187 03 59. 892
Cedar River and Burnt Bluff	238 25 09. 385	16_{-2-3}	10	6. 6	0. 5	+1. 090	+0. 472	238 25 10. 947
Azimuth-Mark and Pine Hill	0 41 59. 975	16_5	16	5. 6	1	−0. 224	−0. 204	0 41 59. 547

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(16_1)+1.0(16_2)+1.0(16_3)+1(16_4)+0.034=0$$
$$1(16_1)+2.5(16_2)+1.5(16_3)+1(16_4)-0.831=0$$
$$1(16_1)+1.0(16_2)+2.5(16_3)+1(16_4)-0.831=0$$
$$1(16_1)+1.0(16_2)+1.0(16_3)+2(16_4)+0.034=0$$

NOTE.—Angles 16_2, 16_3, and 16_{-2-3} were read by A. R. Flint with Repsold instrument.

SECTION IV.—*Triangulation from the line Burnt Bluff-Pine Hill to the line Eldorado-Taycheedah*—Continued.

BOYER'S BLUFF—17.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Dates, June and July, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Door Bluff and Eagle Bluff	3 58 07. 984	17_1	12	3. 6	0. 6	0. 000	+1. 429	3 58 09. 413
Door Bluff and Cedar River	55 56 57. 640	17_{1+2}	20	6. 0	1	−0. 126	+0. 166	55 56 57. 680
Cedar River and Ford River	64 11 43. 751	17_3	20	5. 5	1	−0. 126	−0. 393	64 11 43. 232
Burnt Bluff and Door Bluff	185 33 29. 802	17_5	20	7. 0	1	0. 000	−0. 042	185 33 29. 760
Ford River and Door Bluff	239 51 18. 987	17_{4+5}	20	8. 0	1	−0. 126	+0. 227	239 51 19. 088

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(17_{1+2})+1(17_3)+0.378=0$$
$$1(17_{1+2})+2(17_3)+0.378=0$$

CEDAR RIVER—18.

[Observer, R. S. Woodward. Instrument, Troughton & Simms' 12-inch theodolite No. 2. Date, July, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Ford River and Boyer's Bluff....	60 31 03. 300	18_1	20	12. 4	1	+1. 061	−0. 623	60 31 03. 738
Boyer's Bluff and Door Bluff.....	33 57 51. 840	18_2	20	8. 8	1	+1. 061	+0. 081	33 57 52. 982
Door Bluff and Eagle Bluff	40 31 28. 188	18_3	20	6. 3	1	+1. 061	+0. 283	40 31 29. 532
Rochereau and Ford River	181 29 15. 115	18_5	20	9. 7	1	+0. 000	−0. 303	181 29 14. 812
Eagle Bluff and Ford River......	224 59 32. 427	18_{4+5}	20	11. 6	1	+1. 062	+0. 259	224 59 33. 748

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(18_1)+1(18_2)+1(18_3)-4.245=0$$
$$1(18_1)+2(18_2)+1(18_3)-4.245=0$$
$$1(18_1)+1(18_2)+2(18_3)-4.245=0$$

DOOR BLUFF—19.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, July, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Eagle Bluff and Rochereau	46 11 07. 120	19_1	19	6. 8	1	−0 357	−0. 466	46 11 06. 297
Rochereau and Cedar River......	36 18 22. 123	19_2	20	4. 5	1	−0. 357	+0. 565	36 18 22. 331
Cedar River and Boyer's Bluff...	90 05 10. 883	19_3	20	7. 0	1	−0. 357	−0. 094	90 05 10. 432
Boyer's Bluff and Eagle Bluff ...	187 25 20. 888	19_4	19	5. 7	1	+0. 057	−0. 005	187 25 20. 940
Eagle Bluff and Boyer's Bluff....	172 34 38. 230	19_{1+2+3}	10	4. 9	0. 5	+0. 825	+0. 005	172 34 39. 060

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.5(19_1)+1.5(19_2)+1.5(19_3)+1.962=0$$
$$1.5(19_1)+2.5(19_2)+1.5(19_3)+1.962=0$$
$$1.5(19_1)+1.5(19_2)+2.5(19_3)+1.962=0$$

SECTION IV.—*Triangulation from the line Burnt Bluff–Pine Hill to the line Eldorado–Taycheedah*—Continued.

ROCHEREAU—20.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, July, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Cedar River and Door Bluff......	59 39 49.775	20_1	10	5.9	0.5	−0.206	+0.740	59 39 50.309
Door Bluff and Eagle Bluff	42 14 14.191	20_2	11	6.6	0.5	−0.206	−0.442	42 14 13.543
Eagle Bluff and South Egg	37 03 14.150	20_3	12	7.7	0.5	−0.693	+0.922	37 03 14.379
South Egg and Menomonee......	39 12 56.175	20_4	12	7.1	0.5	−0.693	−1.617	39 12 53.865
Cedar River and Eagle Bluff	101 54 03.631	20_{1+2}	10	5.7	0.5	−0.077	+0.298	101 54 03.852
Eagle Bluff and Menomonee.....	76 16 08.529	20_{3+4}	12	7.1	0.5	+0.410	−0.695	76 16 08.244
Menomonee and Cedar River	181 49 47.647	20_5	20	6.5	1.0	−0.140	+0.397	181 49 47.904

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.0(20_1)+1.5(20_2)+1.0(20_3)+1.0(20_4)+2.1055=0$$
$$1.5(20_1)+2.0(20_2)+1.0(20_3)+1.0(20_4)+2.1055=0$$
$$1.0(20_1)+1.0(20_2)+2.0(20_3)+1.5(20_4)+2.8360=0$$
$$1.0(20_1)+1.0(20_2)+1.5(20_3)+2.0(20_4)+2.8360=0$$

EAGLE BLUFF—21.

[Observer, R. S. Woodward. Instrument, Troughton & Simms' 12-inch theodolite. Date, July, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
South Egg and Menomonee	47 48 5.515	21_1	19	6.3	1	+0.212	−0.197	47 48 5.530
Menomonee and Rochereau......	59 43 41.413	21_2	20	11.8	1	+0.211	−0.297	59 43 41.327
Rochereau and Cedar River......	34 35 38.198	21_3	20	6.3	1	+0.212	−0.292	34 35 38.118
Cedar River and Door Bluff......	56 59 02.546	21_{4+5}	20	7.9	1	+0.211	+0.330	56 59 03.087
Boyer's Bluff and Door Bluff.....	3 27 14.093	21_5	9	3.3	0.5	0.000	−2.456	3 27 11.637
Door Bluff and South Egg	160 53 31.270	21_6	25	15.0	1	+0.212	+0.456	160 53 31.938

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(21_1)+1(21_2)+1(21_3)+1(21_{4+5})-1.058=0$$
$$1(21_1)+2(21_2)+1(21_3)+1(21_{4+5})-1.058=0$$
$$1(21_1)+1(21_2)+2(21_3)+1(21_{4+5})-1.058=0$$
$$1(21_1)+1(21_2)+1(21_3)+2(21_{4+5})-1.058=0$$

MENOMONEE—22.

[Observer, R. S. Woodward. Instrument, Troughton & Simms' 14-inch theodolite No. 1. Date, June, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″		° ′ ″
Rochereau and Eagle Bluff.......	44 00 11.966	22_1	16	3.8	1	+0.039	−0.220	44 00 11.785
Eagle Bluff and South Egg	47 44 26.542	22_2	16	5.1	1	+0.039	−0.966	47 44 25.615
South Egg and Débroux	62 48 53.615	22_3	16	4.8	1	+0.039	−0.013	62 48 53.641
Débroux and Peshtigo	11 31 52.860	22_4	16	8.1	1	+0.039	+0.424	11 31 53.323
Peshtigo and Rochereau..........	193 54 34.822	22_5	20	6.8	1	+0.039	+0.775	193 54 35.636

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(22_1)+1(22_2)+1(22_3)+1(22_4)-0.195=0$$
$$1(22_1)+2(22_2)+1(22_3)+1(22_4)-0.195=0$$
$$1(22_1)+1(22_2)+2(22_3)+1(22_4)-0.195=0$$
$$1(22_1)+1(22_2)+1(22_3)+2(22_4)-0.195=0$$

SECTION IV.—*Triangulation from the line Burnt Bluff-Pine Hill to the line Eldorado-Taycheedah*—Continued.

SOUTH EGG—23.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, July, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	$[v]$	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Débroux and Peshtigo	38 24 18.808	23_1	20	6.5	1	+0.033	+0.029	38 24 18.870
Peshtigo and Menomonee........	30 51 58.738	23_2	20	5.6	1	+0.033	−0.035	30 51 58.736
Menomonee and Rochereau	49 02 30.448	23_3	20	7.8	1	+0.018	−0.280	49 02 30.186
Rochereau and Eagle Bluff.......	35 25 00.239	23_4	12	4.6	0.5	+0.037	−0.398	35 24 59.878
Menomonee and Eagle Bluff	84 27 30.713	23_{3+4}	8	5.5	0.5	+0.029	−0.678	84 27 30.064
Eagle Bluff and Débroux	206 16 11.614	23_5	18	7.4	1	+0.032	+0.684	206 16 12.330

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(23_1)+1(23_2)+1\ (23_3)+1\ (23_4)-0.153=0$$
$$1(23_1)+2(23_2)+1\ (23_3)+1\ (23_4)-0.153=0$$
$$1(23_1)+1(23_2)+2.5(23_3)+1.5(23_4)-0.166=0$$
$$1(23_1)+1(23_2)+1.5(23_3)+2\ (23_4)-0.166=0$$

PESHTIGO—24.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, May, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	$[v]$	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Menomonee and South Egg	74 47 15.143	24_1	20	6.4	1	−0.004	−0.216	74 47 14.923
South Egg and Débroux	85 30 00.708	24_2	20	8.3	1	−0.004	−0.048	85 30 00.656
Débroux and Red River	15 56 16.302	24_3	20	4.4	1	−0.311	+0.778	15 56 16.769
Débroux and Gales	56 54 36.578	24_{3+4}	20	4.8	1	+0.307	−0.849	56 54 36.036
Red River and Menomonee	183 46 28.789	24_{-1-2-3}	14	5.2	0.5	−0.623	−0.514	183 46 27.652
Gales and Menomonee	142 48 06.658	24_5	11	4.7	0.5	+0.614	+1.113	142 48 08.385

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2\ (24_1)+1\ (24_2)+0.5(24_3)+0.5(24_{3+4})+0.0145=0$$
$$1\ (24_1)+2\ (24_2)+0.5(24_3)+0.5(24_{3+4})+0.0145=0$$
$$0.5(24_1)+0.5(24_2)+1.5(24_3)\qquad +0.4710=0$$
$$0.5(24_1)+0.5(24_2)+\qquad +1.5(24_{3+4})-0.4565=0$$

DÉBROUX—25.

[Observer, R. S. Woodward. Instrument, Troughton & Simms' 14-inch theodolite No. 1. Date, May, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	$[v]$	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Gales and Peshtigo	79 16 11.664	25_1	16	6.5	1	−0.291	−0.051	79 16 11.322
Peshtigo and Menomonee	8 10 51.244	25_2	15	3.4	1	0.000	+0.017	8 10 51.261
Peshtigo and South Egg	56 05 41.937	25_{2+3}	16	5.1	1	−0.291	−0.268	56 05 41.378
South Egg and Gales	224 38 07.273	25_4	16	3.0	1	−0.292	+0.319	224 38 07.300

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2\ (25_1)+1\ (25_{2+3})+0.874=0$$
$$1\ (25_1)+2\ (25_{2+3})+0.874=0$$

SECTION IV.—*Triangulation from the line Burnt Bluff–Pine Hill to the line Eldorado–Taycheedah*—Continued.

GALES—26.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, September, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Peshtigo and Débroux	43 49 13.840	26_1	20	5.1	1	+0.018	−0.409	43 49 13.449
Débroux and Red River	32 15 59.106	26_2	20	6.1	1	+0.017	−0.039	32 15 59.084
Red River and Red Banks	54 10 34.377	26_3	20	6.7	1	0.000	+0.864	54 10 35.241
Red River and Little Tail	82 26 20.882	26_{3+4}	17	6.2	1	+0.018	−0.547	82 26 20.353
Little Tail and Peshtigo	201 28 26.102	26_5	20	6.7	1	+0.017	+0.995	201 28 27.114

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2\,(26_1)+1\,(26_2)+1\,(26_{3+4})-0.070=0$$
$$1\,(26_1)+2\,(26_2)+1\,(26_{3+4})-0.070=0$$
$$1\,(26_1)+1\,(26_2)+2\,(26_{3+4})-0.070=0$$

RED RIVER—27.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, October, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Red Banks and Little Tail	30 45 05.680	27_1	10	6.8	0.5	+0.463	+0.825	30 45 06.968
Little Tail and Gales	55 51 59.675	27_2	10	4.2	0.5	+0.463	−0.839	55 51 59.299
Gales and Peshtigo	62 56 29.510	27_3	20	7.0	1	+0.177	−0.510	62 56 29.177
Red Banks and Gales	86 37 06.390	27_{1+2}	10	7.2	0.5	−0.109	−0.013	86 37 06.268
Peshtigo and Red Banks	210 26 23.856	27_4	20	6.8	1	+0.176	+0.523	210 26 24.555

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.0(27_1)+1.5(27_2)+1(27_3)-1.7965=0$$
$$1.5(27_1)+2.0(27_2)+1(27_3)-1.7965=0$$
$$1.0(27_1)+1.0(27_2)+2(27_3)-1.2790=0$$

LITTLE TAIL—28.

[Observer, R. S. Woodward. Instrument, Gambey 10-inch repeating theodolite No. 1. Date, September, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Gales and Red River	41 41 41.155	28_1	16	3.6	1	+0.290	−0.183	41 41 41.262
Red River and Red Banks	59 23 47.595	28_2	16	2.9	1	+0.290	−0.140	59 23 47.745
Red Banks and Bruce	27 27 27.537	28_3	16	5.0	1	+0.290	−0.463	27 27 27.364
Bruce and Long Tail P't L't-House	19 00 08.437	28_4	4	2.7	0.2	+0.949	−0.511	19 00 08.875
Long Tail Point Light-House and Fort Howard	14 58 22.308	28_5	4	3.3	0.2	+0.949	+0.516	14 58 23.773
Bruce and Fort Howard	33 58 32.542	28_{4+5}	16	2.7	1	+0.101	+0.005	33 58 32.648
Fort Howard and Gales	197 28 29.911	28_6	16	3.9	1	+0.289	+0.781	197 28 30.981

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(28_1)+1(28_2)+1(28_3)+1\ (28_4)+1\ (28_5)-3.057=0$$
$$1(28_1)+2(28_2)+1(28_3)+1\ (28_4)+1\ (28_5)-3.057=0$$
$$1(28_1)+1(28_2)+2(28_3)+1\ (28_4)+1\ (28_5)-3.057=0$$
$$1(28_1)+1(28_2)+1(28_3)+2.2(28_4)+2\ (28_5)-4.854=0$$
$$1(28_1)+1(28_2)+1(28_3)+2\ (28_4)+2.2(28_5)-4.854=0$$

SECTION IV.—*Triangulation from the line Burnt Bluff-Pine Hill to the line Eldorado-Taycheedah*—Continued.

RED BANKS—29.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, October, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Fort Howard and Little Tail	79 31 42.980	29_{1+2}	10	3.2	0.5	+0.251	−0.441	79 31 42.790
Long Tail Point Light-House and Little Tail	50 09 30.819	29_2	8	4.4	0.5	+0.307	−0.599	50 09 30.527
Little Tail and Gales	50 38 46.750	29_3	10	3.7	0.5	−0.043	−0.137	50 38 46.570
Gales and Red River	39 12 19.686	29_4	11	5.8	0.5	−0.043	−0.202	39 12 19.441
Red River and Court-House	187 05 46.092	29_5	20	5.5	1	+0.275	+0.411	187 05 46.778
Little Tail and Red River	89 51 05.756	29_{3+4}	11	6.8	0.5	+0.594	−0.339	89 51 06.011
Court-House and Long Tail Point Light-House	32 53 35.850	29_{6+1}	8	4.9	0.5	+0.307	+0.527	32 53 36.684
Court-House and Little Tail	83 03 07.290	29_{5+1+2}	12	7.3	0.5	−0.007	−0.072	83 03 07.211
Court-House and Fort Howard	3 31 23.800	29_5	10	4.6	0.5	+0.252	+0.369	3 31 24.421

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$1\ (29_{1+2}) \qquad +0.5(29_3)+0.5(29_4)+0.5(29_5)-0.3460=0$$

$$+1\ (29_2)+0.5(29_3)+0.5(29_4)+0.5(29_5)-0.4015=0$$

$$0.5(29_{1+2})+0.5(29_2)+2.5(29_3)+2.0(29_4)+1.5(29_5)-0.4985=0$$

$$0.5(29_{1+2})+0.5(29_2)+2.0(29_3)+2.5(29_4)+1.5(29_5)-0.4985=0$$

$$0.5(29_{1+2})+0.5(29_2)+1.5(29_3)+1.5(29_4)+2.5(29_5)-0.8385=0$$

BRUCE—31.

[Observers, A. R. Flint and R. S. Woodward. Instruments, Repsold 10-inch theodolite, and Gambey 10-inch repeating theodolite. Dates, July, 1872, and October, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ″ ″
East Depere and West Depere	32 44 08.500	31_1	4	2.7	0.2	−0.646	−0.155	32 44 07.699
West Depere and Fort Howard	14 18 44.480	31_2	5	4.5	0.2	−0.646	+0.001	14 18 43.835
Fort Howard and Oneida	10 24 40.160	31_3	12	7.3	0.5	−0.963	+0.037	10 24 39.234
Oneida and Long Tail Point Light-House	49 24 26.915	31_4	13	7.4	0.5	−0.206	−0.091	49 24 26.618
Long Tail Point Light-House and Little Tail	18 47 35.738	31_5	8	3.4	0.5	−1.071	+0.087	18 47 34.754
Little Tail and Red Banks	38 20 04.017	31_6	6	3.1	0.2	+0.520	+0.893	38 20 05.430
East Depere and Fort Howard	47 02 51.693	31_{1+2}	17	9.3	1	−0.005	−0.154	47 02 51.534
Fort Howard and East Depere	312 57 08.335	31_{-1-2}	8	4.3	0.5	−0.023	+0.154	312 57 08.466
East Depere and Oneida	57 27 31.167	31_{1+2+3}	6	3.7	0.2	−0.282	−0.117	57 27 30.768
Oneida and East Depere	302 32 30.038	31_{-1-2-3}	8	6.5	0.5	−0.923	+0.117	302 32 29.232
Long Tail Point Light-House and East Depere	253 08 01.814	$31_{-1-2-3-4}$	7	4.6	0.5	+0.592	+0.208	253 08 02.614
Red Banks and East Depere	196 00 23.225	31_7	8	2.8	0.5	−0.023	−0.772	196 00 22.430
Fort Howard and Long Tail Point Light-House	59 49 06.140	31_{3+4}	15	8.1	1	−0.234	−0.054	59 49 05.852
Long Tail Point Light-House and Fort Howard	300 10 54.000	31_{-3-4}	10	6.9	0.5	+0.094	+0.054	300 10 54.148
Fort Howard and Little Tail,*	78 36 39.791	31_{3+4+5}	16	4.5	1	+0.782	+0.033	78 36 40.606
Little Tail and Fort Howard	281 23 18.000	31_{-3-4-5}	2	1.4	0.1	+1.427	−0.033	281 23 19.394
Long Tail Point Light-House and Oneida	310 35 33.567	31_{-4}	3	8.6	0.1	−0.276	+0.091	310 35 33.382
Long Tail Point Light-House and Red Banks	57 07 40.367	31_{5+6}	3	2.7	0.1	−1.163	−0.980	57 07 40.184

* All angles except this one were read by A. R. Flint with the Repsold theodolite.

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3.4(31_1)+3.2(31_2)+1.7(31_3)+1.0(31_4)+0.5(31_5)+0.5(31_6)+6.3801=0$$

$$3.2(31_1)+3.4(31_2)+1.7(31_3)+1.0(31_4)+0.5(31_5)+0.5(31_6)+6.3801=0$$

$$1.7(31_1)+1.7(31_2)+4.8(31_3)+3.6(31_4)+1.6(31_5)+0.5(31_6)+9.0114=0$$

$$1.0(31_1)+1.0(31_2)+3.6(31_3)+4.2(31_4)+1.6(31_5)+0.5(31_6)+7.0760=0$$

$$0.5(31_1)+0.5(31_2)+1.6(31_3)+1.6(31_4)+2.2(31_5)+0.6(31_6)+4.5596=0$$

$$0.5(31_1)+0.5(31_2)+0.5(31_3)+0.5(31_4)+0.6(31_5)+0.8(31_6)+1.4563=0$$

SECTION IV.—*Triangulation from the line Burnt Bluff-Pine Hill to the line Eldorado-Taycheedah*—Continued.

FORT HOWARD—32.

[Observers, A. R. Flint and R. S. Woodward. Instruments, Repsold 10-inch theodolite, Gambey 10-inch repeating theodolite. Dates, July, 1872, October, 1873, and June, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Little Tail and Red Banks	39 02 19.064	32_1	23	8.2	1	0.060	−1.275	39 02 17.789
Red Banks and Court-House	23 50 44.025	32_2	4	7.2	0.2	+0.370	+0.209	23 50 44.604
Bruce and Court-House	4 31 44.325	32_3	4	6.4	0.2	+0.370	+0.209	4 31 44.904
Bruce and East Depere	86 53 16.500	32_4	19	7.2	1	+0.307	−0.273	86 53 16.534
Red Banks and Bruce	28 22 28.784	32_{2+3}	19	3.6	1	+0.306	+0.418	28 22 29.508
East Depere and Bruce	273 06 43.338	32_{-4}	13	8.6	0.5	−0.145	+0.273	273 06 43.466
East Depere and Red Banks	244 44 13.343	32_{-2-3-4}	7	4.5	0.5	+0.760	−0.145	244 44 13.958

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$1.7(32_2)+1.5(32_3)+0.5(32_4)-1.3375=0$$
$$1.5(32_2)+1.7(32_3)+0.5(32_4)-1.3375=0$$
$$0.5(32_2)+0.5(32_3)+2.0(32_4)-0.9845=0$$

NOTE.—Angles 32_1 and 32_{2+3} were read partly by A. R. Flint with the Repsold instrument and partly by R. S. Woodward with the Gambey instrument. All the others were read by A. R. Flint with the Repsold instrument.

EAST DEPERE—33.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, July, 1872.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Calumet and Freedom	35 05 43.582	33_1	11	5.1	0.5	+0.076	−0.772	35 05 42.886
Freedom and West Depere	14 38 27.975	33_2	4	2.5	0.2	+0.300	+0.662	14 38 28.937
West Depere and Oneida	46 52 38.681	33_3	16	8.3	1	+0.040	−0.370	46 52 38.351
Oneida and Fort Howard	29 51 48.264	33_4	11	5.1	0.5	+0.539	+0.347	29 51 49.150
Fort Howard and Bruce	46 03 52.047	33_{5+6}	17	5.0	1	+0.270	−0.039	46 03 52.278
Long Tail Point Light-House and Bruce	27 06 16.000	33_6	2	2.0	0.1	0.000	+0.156	27 06 16.156
Freedom and Calumet	324 54 16.760	33_{-1}	10	5.8	0.5	−0.418	+0.772	324 54 17.116
Calumet and West Depere	49 44 11.854	33_{1+2}	10	5.2	0.5	+0.079	−0.110	49 44 11.823
West Depere and Calumet	310 15 47.083	33_{-1-2}	6	4.5	0.3	+0.984	+0.110	310 15 48.177
Bruce and Calumet	187 27 28.256	33_7	7	4.6	0.3	−0.030	+0.172	187 27 28.398
West Depere and Freedom	345 21 31.300	33_{-2}	2	0.2	0.1	+0.425	−0.662	345 21 31.063
Oneida and Freedom	298 28 55.300	33_{-2-3}	2	0.4	0.1	−2.296	−0.290	298 28 52.712
Oneida and West Depere	313 07 20.722	33_{-3}	9	5.0	0.5	+0.557	+0.370	313 07 21.649
Oneida and Bruce	75 55 42.667	33_{4+5+6}	3	3.7	0.1	−1.547	+0.308	75 55 41.428
Bruce and Oneida	284 04 18.467	33_{-4-5-6}	6	4.5	0.3	+0.413	−0.308	284 04 18.572
Bruce and Fort Howard	313 56 07.683	33_{-5-6}	6	4.7	0.3	0.000	+0.039	313 56 07.722

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.1(33_1)+1.1(33_2)+0.3(33_3)+0.3(33_4)+0.3(33_{5+6})-0.7440=0$$
$$1.1(33_1)+1.5(33_2)+0.4(33_3)+0.3(33_4)+0.3(33_{5+6})-0.7919=0$$
$$0.3(33_1)+0.4(33_2)+1.9(33_3)+0.3(33_4)+0.3(33_{5+6})-0.4614=0$$
$$0.3(33_1)+0.3(33_2)+0.3(33_3)+1.2(33_4)+0.7(33_{5+6})-0.9607=0$$
$$0.3(33_1)+0.3(33_2)+0.3(33_3)+0.7(33_4)+2.0(33_{5+6})-1.0417=0$$

SECTION IV.—*Triangulation from the line Burnt Bluff-Pine Hill to the line Eldorado-Taycheedah*—Continued.

ONEIDA—34.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, July, 1872.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Red Banks and Bruce	20 58 35.155	34_1	9	3.6	0.5	+0.123	+0.819	20 58 36.097
Bruce and East Depere	46 36 47.953	34_2	15	5.9	1	+0.061	+0.527	46 36 48.541
East Depere and West Depere	74 20 27.256	34_3	16	6.5	1	+0.061	−0.423	74 20 26.894
West Depere and Long Tail Point Light-House	211 31 45.063	34_4	16	9.7	1	+0.061	+0.135	211 31 45.259
Long Tail Point Light-House and Red Banks	6 32 24.144	34_5	9	6.2	0.5	+0.123	−1.058	6 32 23.209

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$1.0(34_1)+0.5(34_2)+0.5(34_3)+0.5(34_4)-0.2145=0$$
$$0.5(34_1)+1.5(34_2)+0.5(34_3)+0.5(34_4)-0.2145=0$$
$$0.5(34_1)+0.5(34_2)+1.5(34_3)+0.5(34_4)-0.2145=0$$
$$0.5(34_1)+0.5(34_2)+0.5(34_3)+1.5(34_4)-0.2145=0$$

WEST DEPERE—35.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, August, 1872.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Oneida and East Depere	58 46 55.840	35_1	18	6.9	1.0	−0.123	−0.237	58 46 55.480
East Depere and Calumet	92 07 28.216	35_2	14	5.6	0.7	−0.312	−0.624	92 07 27.280
Calumet and Freedom	43 43 20.387	35_3	12	7.0	0.6	−0.548	+0.558	43 43 20.397
East Depere and Oneida	301 13 04.213	35_{-1}	3	3.8	0.1	+0.070	+0.237	301 13 04.520
Freedom and Oneida	165 22 16.669	35_4	18	7.0	1.0	−0.129	+0.303	165 22 16.843
Calumet and East Depere	267 52 31.744	35_{-2}	9	6.2	0.5	+0.352	+0.624	267 52 32.720
East Depere and Freedom	135 50 47.727	35_{2+3}	11	7.7	0.5	+0.016	−0.066	135 50 47.677
Freedom and East Depere	224 09 14.650	35_{-2-3}	2	0.9	0.1	−2.393	+0.066	224 09 12.323
Freedom and Calumet	316 16 39.920	35_{-3}	5	6.0	0.2	+0.241	−0.558	316 16 39.603

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.1(35_1)+1.0(35_2)+1.0(35_3)+1.1173=0$$
$$1.0(35_1)+2.8(35_2)+1.6(35_3)+1.8733=0$$
$$1.0(35_1)+1.6(35_2)+2.4(35^3)+1.9367=0$$

CALUMET—36.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, August, 1872.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″							° ′ ″
Oshkosh and Clayton	47 38 42.917	36_1	12	5.4	0.6	−0.284	−1.005	47 38 41.628
Clayton and Freedom	59 51 21.038	36_2	13	6.2	0.6	−0.286	+0.741	59, 51 21.493
Freedom and West Depere	22 17 56.814	36_3	7	2.8	0.3	−0.027	−0.334	22 17 56.453
West Depere and East Depere	38 08 22.120	36_4	10	3.8	0.5	−0.042	+0.200	38 08 22.278
Clayton and Oshkosh	312 21 16.050	36_{-1}	6	6.1	0.3	+1.317	+1.005	312 21 18.372
Oshkosh and Freedom	107 30 03.225	36_{1+2}	12	7.8	0.6	+0.160	−0.264	107 30 03.121
Freedom and Oshkosh	252 29 56.542	36_{-1-2}	12	6.7	0.6	+0.073	+0.264	252 29 56.879
Freedom and Clayton	300 08 37.933	36_{-2}	6	4.0	0.3	+1.315	−0.741	300 08 38.507
West Depere and Freedom	337 42 03.200	36_{-3}	2	3.6	0.1	+0.013	+0.334	337 42 03.547
Freedom and East Depere	60 26 19.710	36_{3+4}	10	4.9	0.5	−0.845	−0.134	60 26 18.731
East Depere and West Depere	321 51 37.980	36_{-4}	5	1.4	0.2	−0.058	−0.200	321 51 37.722
East Depere and Oshkosh	192 03 40.320	36_5	5	4.9	0.2	−2.570	+0.398	192 03 38.148
East Depere and Freedom	299 33 40.971	36_{-5-4}	9	3.5	0.5	+0.164	+0.134	299 33 41.269

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.3(36_1)+1.4(36_2)+0.2(36_3)+0.2(36_4)+1.0681=0$$
$$1.4(36_1)+2.3(36_2)+0.2(36_3)+0.2(36_4)+1.0693=0$$
$$0.2(36_1)+0.2(36_2)+1.6(36_3)+1.2(36_4)+0.2077=0$$
$$0.2(36_1)+0.2(36_2)+1.2(36_3)+1.9(36_4)+0.2263=0$$

SECTION IV.—*Triangulation from the line Burnt Bluff–Pine Hill to the line Eldorado–Taycheedah*—Continued.

FREEDOM—37.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, August, 1872.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
West Depere and East Depere..	29 30 44.764	37_1	11	7.4	0.5	+0.180	−1.158	29 30 43.786
East Depere and Calumet........	84 27 59.015	37_2	13	7.5	0.6	−0.066	+0.905	84 27 59.854
Calumet and Appleton	38 29 01.382	37_3	13	6.3	0.6	+0.054	+0.113	38 29 01.549
Appleton and Clayton	26 06 56.021	37_4	9	5.0	0.5	−0.255	+0.108	26 06 55.874
Clayton and West Depere	181 25 19.150	37_5	16	8.4	0.8	−0.245	+0.032	181 25 18.937
West Depere and Calumet......	113 58 43.990	37_{1+2}	10	4.3	0.5	−0.097	−0.253	113 58 43.640
Calumet and West Depere	246 01 15.333	37_{-1-2}	3	4.3	0.1	+0.774	+0.253	246 01 16.360
Appleton and West Depere......	207 32 13.077	37_{-1-2-3}	3	1.9	0.1	+1.594	+0.140	207 32 14.811
Calumet and Clayton	64 35 57.150	37_{3+4}	14	6.4	0.7	+0.052	+0.221	64 35 57.423
Clayton and Calumet	295 24 02.537	37_{-3-4}	8	5.9	0.4	+0.261	−0.221	295 24 02.577
Calumet and East Depere........	275 32 02.350	37_{-2}	2	0.9	0.1	−1.299	−0.905	275 32 00.146

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.0(37_1)+1.5(37_2)+0.9(37_3)+0.8(37_4)-0.1049=0$$
$$1.5(37_1)+2.2(37_2)+0.9(37_3)+0.8(37_4)+0.0316=0$$
$$0.9(37_1)+0.9(37_2)+2.6(37_3)+1.9(37_4)+0.2425=0$$
$$0.8(37_1)+0.8(37_2)+1.9(37_3)+2.4(37_4)+0.4187=0$$

CLAYTON—38.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, August, 1872.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Freedom and Appleton	16 34 38.700	38_1	2	1.8	0.1	0.000	0.000	16 34 38.700
Freedom and Calumet	55 32 41.113	38_{1+2}	15	6.2	0.7	+0.265	+0.632	55 32 42.010
Calumet and Stockbridge........	22 05 59.092	38_3	12	3.8	0.6	−0.426	−0.938	22 05 57.728
Stockbridge and Oshkosh........	57 57 40.058	38_4	12	5.4	0.6	−0.432	−0.085	57 57 39.541
Calumet and Freedom	304 27 18.050	38_{-1-2}	2	0.5	0.1	+0.572	−0.632	304 27 17.990
Oshkosh and Freedom	224 23 40.115	38_5	13	6.1	0.6	+0.215	+0.391	224 23 40.721
Stockbridge and Calumet	337 54 01.300	38_{-3}	2	0.8	0.1	+0.034	+0.938	337 54 02.272
Calumet and Oshkosh......	80 03 37.443	38_{3+4}	7	3.4	0.3	+0.849	−1.023	80 03 37.269
Oshkosh and Calumet............	279 56 23.033	38_{-3-4}	3	2.8	0.1	−1.325	+1.023	279 56 22.731
Oshkosh and Stockbridge.... ..	302 02 20.375	38_{-4}	4	3.8	0.2	−0.001	+0.085	302 02 20.459

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$1.4(38_{1+2})+0.6(38_3)+0.6(38_4)+0.1431=0$$
$$0.6(38_{1+2})+1.7(38_3)+1.0(38_4)+0.9964=0$$
$$0.6(38_{1+2})+1.0(38_3)+1.8(38_4)+1.0438=0$$

OSHKOSH—39.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Dates, August and September, 1872.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Clayton and Calumet	52 17 42.953	39_1	15	4.4	0.8	−0.005	−0.769	52 17 42.181
Calumet and Stockbridge...	10 14 20.625	39_2	16	5.5	0.8	−0.102	+0.678	10 14 21.201
Stockbridge and Taycheedah	84 48 34.536	39_3	11	4.7	0.5	+0.399	+0.056	84 48 34.991
Calumet and Clayton	307 42 17.100	39_{-1}	2	1.2	0.1	−0.048	+0.767	307 42 17.819
Stockbridge and Clayton.........	297 27 57.325	39_{-1-2}	4	5.0	0.2	−0.796	+0.089	297 27 56.618
Taycheedah and Clayton	212 39 20.000	38_4	2	1.0	0.1	+1.594	+0.033	212 39 21.627
Stockbridge and Calumet........	349 45 40.300	39_{-2}	2	3.4	0.1	−0.823	−0.678	349 45 38.799
Taycheedah and Stockbridge ..	275 11 24.933	39_{-3}	6	5.3	0.3	+0.132	−0.056	275 11 25.009

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$1.2(39_1)+0.3(39_2)+0.1(39_3)-0.0027=0$$
$$0.3(39_1)+1.2(39_2)+0.1(39_3)+0.0845=0$$
$$0.1(39_1)+0.1(39_2)+0.9(39_3)-0.3479=0$$

SECTION IV.—*Triangulation from the line Burnt Bluff-Pine Hill to the line Eldorado-Tayeheedah*—Continued.

STOCKBRIDGE—40.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, September, 1872.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			″		″	″	° ′ ″
Taycheedah and Oshkosh .. 54 30 03.274	40_{1+2}	19	5.8	1	0.000	+0.215	54 30 03.489
Eldorado and Oshkosh.......... 23 00 55.006	40_2	16	6.5	1	0.000	−0.174	23 00 54.832
Oshkosh and Clayton 59 30 17.380	40_3	15	3.7	1	0.000	+0.589	59 30 17.969

TAYCHEEDAH—41.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, September, 1872.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			″		″	″	° ′ ″
Catholic Church and Oakfield... 17 35 03.586	41_{1+2}	14	5.5	0.7	0.000	−0.022	17 35 03.564
East Base and Oakfield 5 19 00.825	41_2	4	2.9	0.2	0.000	−1.413	5 18 59.412
Oakfield and Eldorado 66 40 21.939	41_3	18	8.0	1.0	0.000	−0.073	66 40 21.866
Eldorado and Oshkosh........... 42 46 57.786	41_4	14	6.8	0.7	+0.074	+0.196	42 46 58.056
Oshkosh and Stockbridge....... 40 41 22.521	41_5	19	5.1	1.0	+0.052	+0.178	40 41 22.751
Eldorado and Stockbridge 83 28 20.950	41_{4+5}	2	3.9	0.1	−0.517	+0.374	83 28 20.807

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$0.8(41_4)+0.1(41_5)-0.0643=0$$
$$0.1(41_4)+1.1(41_5)-0.0643=0$$

NOTE.—The corrections [v] for 41_{1+2}, 41_2, and 41_3 will be found in the adjustment of the triangulation from the line Eldorado-Taycheedah to the line Minnesota Junction-Horicon.

ELDORADO—42.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, September, 1872.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			′		″	″	° ′ ″
Stockbridge and Taycheedah... 65 02 31.683	42_1	18	4.7	1.0	0.000	+0.174	65 02 31.857
Taycheedah and Catholic Church 50 33 26.000	42_2	8	7.6	0.5	+0.688	−0.227	50 33 26.461
Catholic Church and Oakfield .. 24 49 03.680	42_3	10	5.4	0.5	+0.688	+0.287	24 49 04.655
Oakfield and Springvale 47 04 33.855	42_4	20	5.9	1.0	0.000	−0.003	47 04 33.852
Taycheedah and Oakfield....... 75 22 31.744	42_{2+3}	9	4.6	0.5	−0.688	+0.060	75 22 31.116

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$1.0(42_2)+0.5(42_3)-1.032=0$$
$$0.5(42_2)+1.0(42_3)-1.032=0$$

NOTE.—The corrections [v] for 42_2, 42_3, 42_4, 42_{2+3} will be found in the adjustment of the triangulation from the line Eldorado-Taycheedah to the line Minnesota Junction-Horicon.

Numerical equations of condition in the triangulation from the line Pine Hill – Burnt Bluff, to the line Eldorado – Taycheedah.

SIDE-EQUATIONS.

XXX. (7) $-16.4617[17_1]+2.2827[17_{1+2}]-2.7753[19_1]-2.7753[19_2]-0.0317[19_3]-1.8812[21_{4+5}]$
$+15.5629[21_5] \quad +62.282=0$

XXXIII. (20) $+2.7753[19_1]-25.8815[19_2]+16.7589[20_1]+4.4374[20_2]+30.5281[21_3]-13.6817[21_{4+5}]+18.880=0$

XXXVII. (20) $-27.8667[20_3]+25.8020[20_4]-25.7413[21_1]-6.6507[21_2]+0.6411[22_1]+19.7725[22_2]+79.594=0$

XLI. (9) $+4.9140[22_3]-5.9000[22_4]+5.7255[24_1]-1.6570[24_2]+19.0155[25_2]-4.8643[25_{2+3}]+2.106=0$

XLVII. (20) $+1.2441[27_1]-13.0292[27_2]+27.7637[28_1]+4.1276[28_2]+17.2664[29_3]-25.8112[29_4]-9.125=0$

L. (25) $-29.0558[28_3]+11.4638[28_4]+11.4638[28_5]+10.7031[31_3]+10.7031[31_4]+10.7031[31_5]$
$+37.3305[31_6]-25.9652[32_1]+38.9818[32_2]+38.9818[32_3] \quad -96.681=0$

LI. (30) $-9.7614[28_3]+41.1313[28_4]+27.8153[29_2]-48.2672[31_5]+23.8539[31_6] \quad +16.064=0$

LIII. (20) $-2.7470[28_4]+31.2441[28_5]+20.3116[31_1]+20.3116[31_2]+20.3116[31_3]+20.3116[31_4]$
$+27.1486[31_5]-8.7587[32_1]-8.7587[32_2]-8.7587[32_3]+1.1447[32_4]-20.2870[33_{5+6}]$
$+61.4492[33_6] \quad -33.204=0$

LIV. (40) $+22.9867[28_3]+22.9867[29_2]-21.0532[31_4]-28.9025[31_5]-5.9158[31_6]-71.0809[34_1]$
$-35.5278[34_2]-35.5278[34_3]-35.5278[34_4] \quad +97.051=0$

LVI. (30) $+6.3834[31_1]+6.3834[31_2]+6.3834[31_3]+24.4252[31_4]-18.4171[33_4]-18.4171[33_{5+6}]$
$+59.5547[33_6]+40.4182[34_2]+34.4323[34_3]+34.4323[34_4] \quad -12.092=0$

LVIII. (25) $-32.7522[31_1]+45.7288[31_2]+45.7288[31_3]-33.2907[33_3]-13.5720[33_4]-13.5720[33_{5+6}]$
$+12.6284[34_2]+18.5305[34_3] \quad -13.806=0$

LXII. (12.5) $+12.1306[33_1]-17.8330[33_2]-0.7810[35_2]-22.0159[35_3]-9.3651[37_1]-11.4050[37_2]+32.444=0$

NOTE.—In the solution for determining the general corrections, each of the side-equations was divided by the number inclosed in parenthesis and placed opposite it.

ANGLE-EQUATIONS.

XXIV. $[14^2]+[15_2]+[16_1] \quad -2.830=0$

XXV. $[14_1]+[16_2]-[17_{1+2}]-[17_3]-[17_5] \quad -0.376=0$

XXVI. $[16_3]+[17_3]+[18_1] \quad +1.561=0$

XXVII. $[17_{1+2}]+[18_2]+[19_3] \quad -0.153=0$

XXVIII. $[18_3]+[19_1]+[19_2]+[21_{4+5}] \quad -0.712=0$

XXIX. $[17_{1+2}]-[17_1]+[18_2]+[18_3]+[21_{4+5}]-[21_5] \quad -1.887=0$

XXXI. $-[18_1]-[18_2]-[18_5]+[19_2]+[20_1] \quad -2.149=0$

XXXII. $[19_1]+[20_2]+[21_3]+[21_{4+5}] \quad +0.869=0$

XXXIV. $[20_3]+[20_4]+[21_2]+[22_1] \quad +1.212=0$

XXXV. $[21_1]+[22_2]+[23_3]+[23_4] \quad +1.841=0$

XXXVI. $[20_4]+[22_1]+[22_2]+[23_3] \quad +3.082=0$

XXXVIII. $[22_3]+[22_4]+[23_2]+[24_1] \quad -0.160=0$

XXXIX. $[23_1]+[24_2]+[25_{2+3}] \quad +0.287=0$

XL. $[22_3]+[23_1]+[23_2]+[25_{2+3}]-[25_2] \quad +0.304=0$

XLII. $[24_{3+4}]+[25_1]+[26_1] \quad +1.309=0$

XLIII. $[24_{3+4}]-[24_3]+[26_1]+[26_2]+[27_3] \quad +2.585=0$

XLIV. $[26_{3+4}]+[27_2]+[28_1] \quad +1.569=0$

XLV. $[26_3]+[27_1]+[27_2]+[29_4] \quad -0.648=0$

XLVI. $[26_{3+4}]-[26_3]+[28_1]+[28_2]+[29_3] \quad +1.871=0$

XLVIII. $[28_3]+[28_4]+[28_5]+[29_{1+2}]+[32_1] \quad +2.175=0$

XLIX. $[28_4]+[28_5]+[31_3]+[31_4]+[31_5]+[32_1]+[32_2]+[32_3]+0.818=0$

LII. $[31_1]+[31_2]+[32_4]+[33_{5+6}] \quad +0.466=0$

LV. $[31_1]+[31_2]+[31_3]+[33_4]+[33_{5+6}]+[34_2] \quad -0.718=0$

LVII. $[33_3]+[34_3]+[35_1] \quad +1.030=0$

LIX. $[33_1]+[33_2]+[35_2]+[36_4] \quad +0.535=0$

Numerical equations of condition—Continued.

LX.	$[36_3]$	$+[36_4]$	$+[37_2]$	$+[33_1]$		$+0.002=0$
LXI.	$[35_2]$	$+[36_3]$	$+[37_1]$	$+[37_2]$		$+0.029=0$
LXIII.	$[36_2]$	$+[37_3]$	$+[37_4]$	$+[38_{1+2}]$		$-1.593=0$
LXIV.	$[36_1]$	$+[38_3]$	$+[38_4]$	$+[39_1]$		$+2.795=0$
LXV.	$[38_4]$	$+[39_1]$	$+[39_2]$	$+[40_3]$		$-0.415=0$
LXVI.	$[39_3]$	$+[40_{1+2}]$	$+[41_5]$			$-0.449=0$
LXVII.	$[40_{1+2}]$	$-[40_2]$	$+[41_4]$	$+[41_5]$	$+[42_1]$	$-0.938=0$

General corrections in terms of the correlates.

$[14_1]$ =		+0.80000 XXV			
$[14_2]$ =	+0.80000 XXIV				
$[15_2]$ =	+1.00000 XXIV				
$[16_1]$ =	+0.75000 XXIV	−0.12500 XXV	−0.12500 XXVI		
$[16_2]$ =	−0.12500 XXIV	+0.68750 XXV	−0.31250 XXVI		
$[16_3]$ =	−0.12500 XXIV	−0.31250 XXV	+0.68750 XXVI		
$[16_4]$ =	−0.25000 XXIV	−0.12500 XXV	−0.12500 XXVI		
$[17_1]$ =				−1.66667 XXIX	−3.91945 XXX
$[17_{1+2}]$ =	−0.33333 XXV	−0.33333 XXVI	+0.66667 XXVII	+0.66667 XXIX	+0.21264 XXX
$[17_3]$ =	−0.33333 XXV	+0.66667 XXVI	−0.33333 XXVII	−0.33333 XXIX	−0.10632 XXX
$[17_5]$ =	−1.00000 XXV				
$[18_1]$ =	+0.75000 XXVI	−0.25000 XXVII	−0.25000 XXVIII	−0.50000 XXIX	−0.50000 XXXI
$[18_2]$ =	−0.25000 XXVI	+0.75000 XXVII	−0.25000 XXVIII	+0.50000 XXIX	−0.50000 XXXI
$[18_3]$ =	−0.25000 XXVI	−0.25000 XXVII	+0.75000 XXVIII	+0.50000 XXIX	+0.50000 XXXI
$[18_5]$ =	−1.00000 XXXI				
$[19_1]$ =	−0.27273 XXVII	+0.45455 XXVIII	−0.17898 XXX	−0.27273 XXXI	+0.72727 XXXII
	+0.45385 XXXIII				
$[19_2]$ =	−0.27273 XXVII	+0.45455 XXVIII	−0.17898 XXX	+0.72727 XXXI	−0.27273 XXXII
	−0.97900 XXXIII				
$[19_3]$ =	+0.72727 XXVII	−0.54545 XXVIII	+0.21[illegible]96 XXX	−0.27273 XXXI	−0.27273 XXXII
	+0.31508 XXXIII				
$[20_1]$ =	+0.21212 XXXI	−0.78788 XXXII	+0.84089 XXXIII	−0.24242 XXXIV	−0.12121 XXXVI
	+0.01264 XXXVII				
$[20_2]$ =	−0.78788 XXXI	+1.21212 XXXII	−0.39127 XXXIII	−0.24242 XXXIV	−0.12121 XXXVI
	+0.01264 XXXVII				
$[20_3]$ =	−0.12121 XXXI	−0.12121 XXXII	−0.12846 XXXIII	+0.42424 XXXIV	−0.78788 XXXVI
	−2.70655 XXXVII				
$[20_4]$ =	−0.12121 XXXI	−0.12121 XXXII	−0.12846 XXXIII	+0.42424 XXXIV	+1.21212 XXXVI
	+2.66233 XXXVII				
$[21_1]$ =	−0.20000 XXVIII	−0.20000 XXIX	+0.05375 XXX	−0.40000 XXXII	−0.16846 XXXIII
	−0.20000 XXXIV	+0.80000 XXXV	−0.96315 XXXVII		
$[21_2]$ =	−0.20000 XXVIII	−0.20000 XXIX	+0.05375 XXX	−0.40000 XXXII	−0.16846 XXXIII
	+0.80000 XXXIV	−0.20000 XXXV	−0.00862 XXXVII		
$[21_3]$ =	−0.20000 XXVIII	−0.20000 XXIX	+0.05375 XXX	+0.60000 XXXII	+1.35795 XXXIII
	−0.20000 XXXIV	−0.20000 XXXV	+0.32392 XXXVII		
$[21_{4+5}]$ =	+0.80000 XXVII	+0.80000 XXIX	−0.21499 XXX	+0.60000 XXXII	−0.85255 XXXIII
	−0.20000 XXXIV	−0.20000 XXXV	+0.32392 XXXVII		
$[21_5]$ =		−2.00000 XXIX	+4.44654 XXX		

General corrections in terms of the correlates—Continued.

$[22_1]$	=+0.80000 XXXIV	−0.20000 XXXV	+0.60000 XXXVI	−0.17208 XXXVII	−0.40000 XXXVIII
	−0.20000 XL	+0.02191 XLI			
$[22_2]$	=−0.20000 XXXIV	+0.80000 XXXV	+0.60000 XXXVI	+0.78149 XXXVII	−0.40000 XXXVIII
	−0.20000 XL	+0.02191 XLI			
$[22_3]$	=−0.20000 XXXIV	−0.20000 XXXV	−0.40000 XXXVI	−0.20414 XXXVII	+0.60000 XXXVIII
	+0.80000 XL	+0.56791 XLI			
$[22_4]$	=−0.20000 XXXIV	−0.20000 XXXV	−0.40000 XXXVI	−0.20414 XXXVII	+0.60000 XXXVIII
	−0.20000 XL	−0.63365 XLI			
$[23_1]$	=−0.28571 XXXV	−0.09524 XXXVI	−0.23810 XXXVIII	+0.76190 XXXIX	+0.52381 XL
$[23_2]$	=−0.28571 XXXV	−0.09524 XXXVI	+0.76190 XXXVIII	−0.23810 XXXIX	+0.52381 XL
$[23_3]$	=+0.28571 XXXV	+0.76190 XXXVI	−0.09524 XXXVIII	−0.09524 XXXIX	−0.19048 XL
$[23_4]$	=+0.57143 XXXV	−0.47619 XXXVI	−0.19048 XXXVIII	−0.19048 XXXIX	−0.38095 XL
$[24_1]$	=+0.71429 XXXVIII	−0.28571 XXXIX	+0.50701 XLI	−0.14286 XLII	
$[24_2]$	=−0.28571 XXXVIII	+0.71429 XXXIX	−0.31327 XLI	−0.14286 XLII	
$[24_3]$	=−0.14286 XXXVIII	−0.14286 XXXIX	−0.06458 XLI	+0.09524 XLII	−0.66667 XLIII
$[24_{3+4}]$	=−0.14286 XXXVIII	−0.14286 XXXIX	−0.06458 XLI	+0.76190 XLII	+0.66667 XLIII
$[25_1]$	=−0.33333 XXXIX	−0.33333 XL	+0.18016 XLI	+0.66667 XLII	
$[25_2]$	=	−1.00000 XL	+2.11283 XLI		
$[25_{2+3}]$	=+0.66667 XXXIX	+0.66667 XL	−0.36032 XLI	−0.33333 XLII	
$[26_1]$	=+0.75000 XLII	+0.50000 XLIII	−0.25000 XLIV		−0.25000 XLVI
$[26_2]$	=−0.25000 XLII	+0.50000 XLIII	−0.25000 XLIV		−0.25000 XLVI
$[26_3]$	=			+1.00000 XLV	−1.00000 XLVI
$[26_{3+4}]$	=−0.25000 XLII	−0.50000 XLIII	+0.75000 XLIV		+0.75000 XLVI
$[27_1]$	=−0.20000 XLIII	−0.80000 XLIV	+0.40000 XLV	+0.59582 XLVII	
$[27_2]$	=−0.20000 XLIII	+1.20000 XLIV	+0.40000 XLV	−0.83152 XLVII	
$[27_3]$	=+0.70000 XLIII	−0.20000 XLIV	−0.40000 XLV	+0.11785 XLVII	
$[28_1]$	=+0.79630 XLIV	+0.59259 XLVI	+1.06337 XLVII	−0.38889 XLVIII	−0.18519 XLIX
	+0.15183 L	−0.06067 LI	−0.13193 LIII	−0.11706 LIV	
$[28_2]$	=−0.20370 XLIV	+0.59259 XLVI	−0.11844 XLVII	−0.38889 XLVIII	−0.18519 XLIX
	+0.15183 L	−0.06067 LI	−0.13193 LIII	−0.11706 LIV	
$[28_3]$	=−0.20370 XLIV	−0.40741 XLVI	−0.32482 XLVII	+0.61111 XLVIII	−0.18519 XLIX
	−1.01040 L	−0.38605 LI	−0.13193 LIII	+0.45761 LIV	
$[28_4]$	=−0.09259 XLIV	−0.18519 XLVI	−0.14765 XLVII	+0.27778 XLVIII	+0.37037 XLIX
	+0.27745 L	+3.71162 LI	−3.98501 LIII	−0.05321 LIV	
$[28_5]$	=−0.09259 XLIV	−0.18519 XLVI	−0.14765 XLVII	+0.27778 XLVIII	+0.37037 XLIX
	+0.27745 L	−3.14358 LI	+4.51274 LIII	−0.05321 LIV	
$[29_{1+2}]$	=−0.10000 XLV	−0.10000 XLVI	+0.04272 XLVII	+1.17500 XLVIII	+0.16226 LI
	+0.10057 LIV				
$[29_2]$	=−0.10000 XLV	−0.10000 XLVI	+0.04272 XLVII	+0.17500 XLVIII	+1.08944 LI
	+0.67524 LIV				
$[29_3]$	=−0.80000 XLV	+1.20000 XLVI	+2.06843 XLVII	−0.10000 XLVIII	−0.09272 LI
	−0.05747 LIV				
$[29_4]$	=+1.20000 XLV	−0.80000 XLVI	−2.23933 XLVII	−0.10000 XLVIII	−0.09272 LI
	−0.05747 LIV				
$[29_5]$	=−0.20000 XLV	−0.20000 XLVI	+0.08545 XLVII	−0.15000 XLVIII	−0.13908 LI
	−0.08620 LIV				
$[31_1]$	=−0.03246 XLIX	−0.17832 L	−0.13160 LI	+0.20318 LII	+0.18273 LIII
	−0.02409 LIV	+0.10422 LV	+0.05404 LVI	−7.97637 LVIII	

General corrections in terms of the correlates—Continued.

$[31_2]$	=−0.03246 XLIX	−0.17832 L	−0.13160 LI	+ 0.20318 LII	+0.18273 LIII
	−0.02409 LIV	+0.10422 LV	+0.05404 LVI	+ 7.71983 LVIII	
$[31_3]$	=+0.06695 XLIX	+0.17125 L	+0.25148 LI	− 0.19791 LII	−0.17031 LIII
	+0.33586 LIV	+0.49330 LV	−0.31445 LVI	+ 1.21297 LVIII	
$[31_4]$	=+0.05817 XLIX	−0.07360 L	+0.19590 LI	+ 0.07826 LII	+0.08579 LIII
	−0.26171 LIV	−0.43689 LV	+0.49949 LVI	− 0.92198 LVIII	
$[31_5]$	=+0.48848 XLIX	−0.43818 L	−1.55451 LI	+ 0.05474 LII	+0.80874 LIII
	−0.39797 LIV	−0.05437 LV	−0.13725 LVI	− 0.18538 LVIII	
$[31_6]$	=−0.40398 XLIX	+2.35706 L	+2.04467 LI	− 0.22024 LII	−0.78214 LIII
	−0.09736 LIV	−0.12475 LV	−0.08025 LVI	+ 0.11751 LVIII	
$[32_1]$	=+1.00000 XLVIII	+1.00000 XLIX	−1.03861 L		−0.43794 LIII
$[32_2]$	=	+0.33898 XLIX	+0.52857 L	− 0.08475 LII	−0.15331 LIII
$[32_3]$	=	+0.33898 XLIX	+0.52857 L	− 0.08475 LII	−0.15331 LIII
$[32_4]$	=	−0.16949 XLIX	−0.26428 L	+ 0.54237 LII	+0.10527 LIII
$[33_1]$	=−0.01755 LII	+0.01780 LIII	−0.06317 LV	+ 0.03878 LVI	+0.00486 LVII
	+0.02782 LVIII	+0.21856 LIX	+0.77708 LX	+ 1.55093 LXII	
$[33_2]$	=−0.03509 LII	+0.03559 LIII	−0.12633 LV	+ 0.07755 LVI	−0.13113 LVII
	+0.24319 LVIII	+0.57796 LIX	−0.55852 LX	− 2.16336 LXII	
$[33_3]$	=−0.03509 LII	+0.03559 LIII	−0.12633 LV	+ 0.07755 LVI	+0.57310 LVII
	−0.69458 LVIII	−0.12627 LIX	+0.00486 LX	+ 0.19178 LXII	
$[33_4]$	=−0.34858 LII	+0.35358 LIII	+0.74512 LV	− 0.45743 LVI	−0.09124 LVII
	−0.28301 LVIII	−0.13686 LIX	−0.04562 LX	+ 0.08590 LXII	
$[33_{5+6}]$	=+0.63516 LII	−0.64427 LIII	+0.28658 LV	− 0.17593 LVI	−0.03509 LVII
	−0.10885 LVIII	−0.05264 LIX	−0.01755 LX	+ 0.03303 LXII	
$[33_6]$	=	+30.72460 LIII		+19.85160 LVI	
$[34_1]$	=−1.77729 LIV	−0.28571 LV	−1.04079 LVI	− 0.28571 LVII	−0.35610 LVIII
$[34_2]$	=+0.00018 LIV	+0.85714 LV	+0.82688 LVI	− 0.14286 LVII	+0.32709 LVIII
$[34_3]$	=+0.00018 LIV	−0.14286 LV	+0.62735 LVI	+ 0.85714 LVII	−0.56317 LVIII
$[34_4]$	=+0.00018 LIV	−0.14286 LV	+0.62735 LVI	− 0.14286 LVII	−0.17805 LVIII
$[35_1]$	=+0.61758 LVII	−0.11876 LIX	−0.17815 LXI	+ 0.32119 LXII	
$[35_2]$	=−0.11876 LVII	+0.59976 LIX	−0.35036 LXI	+ 0.57961 LXII	
$[35_3]$	=−0.17815 LVII	−0.35036 LIX	+0.72447 LXI	− 1.25409 LXII	
$[36_1]$	=−0.01372 LIX	−0.03773 LX	−0.02401 LXI	− 0.41838 LXIII	+0.69273 LXIV
$[36_2]$	=−0.01372 LIX	−0.03773 LX	−0.02401 LXI	− 0.69273 LXIII	−0.41838 LXIV
$[36_3]$	=−0.74760 LIX	+0.44410 LX	+1.19170 LXI	− 0.02401 LXIII	−0.02401 LXIV
$[36_4]$	=+1.00137 LIX	+0.25377 LX	−0.74760 LXI	− 0.01372 LXIII	−0.01372 LXIV
$[37_1]$	=−0.66687 LX	+0.39951 LXI	−0.19049 LXII	− 0.15342 LXIII	
$[37_2]$	=+0.95224 LX	+0.28537 LXI	−0.36919 LXII	− 0.10959 LXIII	
$[37_3]$	=−0.06944 LX	−0.16666 LXI	+0.13620 LXII	+ 0.25411 LXIII	
$[37_4]$	=−0.04015 LX	−0.09635 LXI	+0.07874 LXII	+ 0.30316 LXIII	
$[38_{1+2}]$	=+0.87884 LXIII	−0.38396 LXIV	−0.17918 LXV		
$[38_3]$	=−0.20478 LXIII	+0.47781 LXIV	−0.44369 LXV		
$[38_4]$	=−0.17918 LXIII	+0.41809 LXIV	+0.86178 LXV		
$[39_1]$	=+0.89390 LXIV	+0.67669 LXV	−0.07519 LXVI		
$[39_2]$	=−0.21721 LXIV	+0.67669 LXV	−0.07519 LXVI		
$[39_3]$	=−0.07519 LXIV	−0.15038 LXV	+1.12782 LXVI		
$[40_{1+2}]$	=	+1.00000 LXVI	+1.00000 LXVII		
$[40_2]$	=		−1.00000 LXVII		
$[40_3]$	=+1.00000 LXV				
$[41_4]$	=	−0.11494 LXVI	+1.14943 LXVII		
$[41_5]$	=	+0.91954 LXVI	+0.80460 LXVII		
$[42_1]$	=		+1.00000 LXVII		

Normal equations for determining the correlates.

No. of equation.					
24.	0=−2.83000	+2.55000 XXIV	− 0.12500 XXV	−0.12500 XXVI	
25.	0=−0.37600	−0.12500 XXIV	+ 3.15417 XXV	−0.64583 XXVI	−0.33333 XXVII
		−0.33333 XXIX	− 0.10632 XXX		
26.	0=+1.56100	−0.12500 XXIV	− 0.64583 XXV	+2.10417 XXVI	−0.58333 XXVII
		−0.25000 XXVIII	− 0.83333 XXIX	−0.10632 XXX	−0.50000 XXXI
27.	0=−0.15300	−0.33333 XXV	− 0.58333 XXVI	+2.14394 XXVII	−0.79545 XXVIII
		+1.16667 XXIX	+ 0.42560 XXX	−0.77273 XXXI	−0.27273 XXXII
		+0.31508 XXXIII			
28.	0=−0.71200	−0.25000 XXVI	− 0.79545 XXVII	−2.45909 XXVIII	+1.30000 XXIX
		−0.57295 XXX	+ 0.95455 XXXI	+1.05455 XXXII	−1.37770 XXXIII
		−0.20000 XXXIV	− 0.20000 XXXV	+0.32392 XXXVII	
29.	0=−1.88700	−0.33333 XXV	− 0.83333 XXVI	+1.16667 XXVII	+1.30000 XXVIII
		+6.13333 XXIX	− 0.52944 XXX	+0.60000 XXXII	−0.85255 XXXIII
		−0.20000 XXXIV	− 0.20000 XXXV	+0.32392 XXXVII	
30.	0=−8.89743	−0.10632 XXV	− 0.10632 XXVI	+0.42560 XXVII	−0.57295 XXVIII
		− 0.52944 XXIX	+19.36967 XXX	−0.17898 XXXI	−0.34022 XXXII
		+0.43589 XXXIII	+ 0.05375 XXXIV	+0.05375 XXXV	−0.08705 XXXVII
31.	0=−2.14900	−0.50000 XXVI	− 0.77273 XXVII	+0.95455 XXVIII	−0.17898 XXX
		+3.93939 XXXI	− 1.06061 XXXII	−0.13811 XXXIII	−0.24242 XXXIV
		−0.12121 XXXVI	+ 0.01264 XXXVII		
32.	0=+0.86900	−0.27273 XXVII	+ 1.05455 XXVIII	+0.60000 XXIX	−0.34022 XXX
		−1.06061 XXXI	+ 3.13939 XXXII	+0.56798 XXXIII	−0.64242 XXXIV
		−0.40000 XXXV	− 0.12121 XXXVI	+0.66048 XXXVII	
33.	0=+0.94400	+0.31508 XXVII	− 1.37770 XXVIII	−0.85255 XXIX	+0.43589 XXX
		−0.13811 XXXI	+ 0.56798 XXXII	+4.60371 XXXIII	−0.42538 XXXIV
		−0.16846 XXXV	− 0.12846 XXXVI	+0.28624 XXXVII	
34.	0=+1.21200	−0.20000 XXVIII	− 0.20000 XXIX	+0.05375 XXX	−0.24242 XXXI
		−0.64242 XXXII	− 0.42538 XXXIII	+2.44848 XXXIV	− 0.40000 XXXV
		+1.02424 XXXVI	− 0.22492 XXXVII	−0.40000 XXXVIII	− 0.20000 XL
		+0.02191 XLI			
35.	0=+1.84100	−0.20000 XXVIII	− 0.20000 XXIX	+0.05375 XXX	−0.40000 XXXII
		−0.16846 XXXIII	− 0.40000 XXXIV	+2.45714 XXXV	+0.88571 XXXVI
		−0.17866 XXXVII	− 0.68571 XXXVIII	−0.28571 XXXIX	−0.77143 XL
		+0.02191 XLI			
36.	0=+3.08200	−0.12121 XXXI	− 0.12121 XXXII	−0.12846 XXXIII	+1.02424 XXXIV
		+0.88571 XXXV	+ 3.17402 XXXVI	+3.27474 XXXVII	−0.89524 XXXVIII
		−0.09524 XXXIX	− 0.59048 XL	+0.04382 XLI	
37.	0=+3.97970	+0.32392 XXVIII	+ 0.32392 XXIX	−0.08705 XXX	+0.01264 XXXI
		+0.66048 XXXII	+ 0.28624 XXXIII	−0.22492 XXXIV	−0.17866 XXXV
		+3.27474 XXXVI	+ 9.22108 XXXVII	−0.40828 XXXVIII	−0.20414 XL
		+0.02236 XLI			
38.	0=−0.16000	−0.40000 XXXIV	− 0.68571 XXXV	−0.89524 XXXVI	−0.40828 XXXVII
		+2.67619 XXXVIII	− 0.52381 XXXIX	+1.12381 XL	+0.44127 XLI
		−0.14286 XLII			
39.	0=+0.28700	−0.28571 XXXV	− 0.09524 XXXVI	−0.52381 XXXVIII	+2.14286 XXXIX
		+1.19048 XL	− 0.67359 XLI	−0.47619 XLII	
40.	0=+0.30400	−0.20000 XXXIV	− 0.77143 XXXV	−0.59048 XXXVI	−0.20414 XXXVII
		+1.12381 XXXVIII	+ 1.19048 XXXIX	+3.51429 XL	−1.90524 XLI
		−0.33333 XLII			
41.	0=+0.23400	+0.02191 XXXIV	+ 0.02191 XXXV	+0.04382 XXXVI	+0.02236 XXXVII
		+0.44127 XXXVIII	− 0.67359 XXXIX	−1.90524 XL	+5.76449 XLI
		+0.11558 XLII			

Normal equations for determining the correlates—Continued.

No. of equation.					
42.	0=+1.30900	— 0.14286 XXXVIII	—0.47619 XXXIX	—0.33333 XL	+ 0.11558 XLI
		+ 2.17857 XLII	+1.16667 XLIII	—0.25000 XLIV	— 0.25000 XLVI
43.	0=+2.58500	+ 1.16667 XLII	+3.03333 XLIII	—0.70000 XLIV	— 0.40000 XLV
		— 0.50000 XLVI	+0.11785 XLVII		
44.	0=+1.56900	— 0.25000 XLII	—0.70000 XLIII	+2.74630 XLIV	+ 0.40000 XLV
		+ 1.34259 XLVI	+0.23185 XLVII	—0.38889 XLVIII	— 0.18519 XLIX
		+ 0.15183 L	—0.06067 LI	—0.13193 LIII	— 0.11706 LIV
45.	0=—0.64800	— 0.40000 XLIII	+ 0.40000 XLIV	+3.00000 XLV	— 1.80000 XLVI
		— 2.47503 XLVII	—0.10000 XLVIII	—0.09272 LI	— 0.05747 LIV
46.	0=+1.87100	— 0.25000 XLII	—0.50000 XLIII	+1.34259 XLIV	— 1.80000 XLV
		+ 4.13518 XLVI	+3.01336 XLVII	—0.87778 XLVIII	— 0.37038 XLIX
		+ 0.30366 L	—0.21406 LI	—0.26387 LIII	— 0.29159 LIV
47.	0=—0.45625	+ 0.11785 XLIII	+0.23185 XLIV	—2.47503 XLV	+ 3.01336 XLVI
		+ 6.70619 XLVII	—0.57740 XLVIII	—0.29530 XLIX	+ 0.24210 L
		— 0.05713 LI	—0.21037 LIII	—0.16211 LIV	
48.	0=+2.17500	— 0.38889 XLIV	—0.10000 XLV	—0.87778 XLVI	— 0.57740 XLVII
		+ 3.34167 XLVIII	+1.55556 XLIX	—1.49411 L	+ 0.34426 LI
		— 0.04214 LIII	+0.45175 LIV		
49.	0=+0.81800	— 0.18519 XLIV	—0.37038 XLVI	—0.29530 XLVII	+ 1.55556 XLIX
		+ 3.03230 XLIX	+0.23289 L	—0.53909 LI	— 0.23441 LII
		+ 0.50739 LIII	—0.43025 LIV	+0.00204 LV	+ 0.04779 LVI
		+ 0.10561 LVIII			
50.	0=—3.86724	+ 0.15183 XLIV	+0.30366 XLVI	+0.24210 XLVII	— 1.49411 XLVIII
		+ 0.23289 XLIX	+7.52967 L	+3.28832 LI	— 0.62093 LII
		— 0.48577 LIII	—0.57390 LIV	—0.18540 LV	— 0.09937 LVI
		+ 0.22068 LVIII			
51.	0=+0.53547	— 0.06067 XLIV	—0.09272 XLV	—0.21406 XLVI	— 0.05713 XLVII
		+ 0.34426 XLVIII	—0.53909 XLIX	+3.28832 L	+ 10.35134 LI
		— 0.26319 LII	—7.34378 LIII	+1.12193 LIV	— 0.01172 LV
		+ 0.15701 LVI	+0.39169 LVIII		
52.	0=+0.46600	— 0.23441 XLIX	—0.62093 L	—0.26319 LI	+ 1.58389 LII
		— 0.17353 LIII	—0.04817 LIV	+0.49502 LV	— 0.06786 LVI
		— 0.03509 LVII	—0.36539 LVIII	—0.05264 LIX	— 0.01755 LX
		+ 0.03303 LXII			
53.	0=—1.66020	— 0.13193 XLIV	—0.26387 XLVI	—0.21037 XLVII	— 0.04214 XLVIII
		+ 0.50739 XLIX	—0.48577 L	—7.34378 LI	— 0.17353 LII
		+104.36597 LIII	—0.58965 LIV	—0.09553 LV	+61.28307 LVI
		+ 0.03559 LVII	—0.10625 LVIII	+0.05340 LIX	+ 0.01780 LX
		— 0.03350 LXII			
54.	0=+2.42628	— 0.11706 XLIV	—0.05747 XLV	—0.29159 XLVI	— 0.16211 XLVII
		+ 0.45175 XLVIII	—0.43025 XLIX	—0.57390 L	+ 1.12193 LI
		— 0.04817 LII	—0.58965 LIII	+4.21972 LIV	+ 0.28786 LV
		— 0.15121 LVI	+0.00018 LVII	+0.60205 LVIII	
55.	0=—0.71800	+ 0.00204 XLIX	—0.18540 L	—0.01172 LI	+ 0.49502 LII
		— 0.09553 LIII	+0.28786 LIV	+2.59058 LV	— 0.01286 LVI
		— 0.26919 LVII	+0.89165 LVIII	—0.18950 LIX	— 0.06317 LX
		+ 0.11893 LXII			
56.	0=—0.40307	+ 0.04779 XLIX	—0.09937 L	+0.15701 LI	— 0.06786 LII
		+ 61.28307 LIII	—0.15121 LIV	—0.01286 LV	+42.71428 LVI
		+ 0.70490 LVII	+0.57613 LVIII	+0.11633 LIX	+ 0.03878 LX
		— 0.07301 LXII			

Normal equations for determining the correlates—Continued.

No. of equation.					
57.	0=+1.03000	−0.03509 LII	+ 0.03559 LIII	+0.00019 LIV	−0.26919 LV
		+0.70490 LVI	+ 2.04782 LVII	−0.13141 LVIII	−0.24503 LIX
		+0.00486 LX	− 0.17815 LXI	+0.51297 LXII	
58.	0=−0.55224	+0.10561 XLIX	+ 0.22068 L	+0.39169 LI	−0.36539 LII
		−0.10625 LIII	+ 0.60205 LIV	+0.89165 LV	+0.57613 LVI
		−0.13141 LVII	+28.50951 LVIII	+0.27102 LIX	+0.02782 LX
		−0.31995 LXII			
59.	0=+0.53500	−0.05264 LII	+ 0.05340 LIII	−0.18950 LV	+0.11633 LVI
		−0.24503 LVII	+ 0.27102 LVIII	+2.39765 LIX	+0.47233 LX
		−1.09796 LXI	− 0.03282 LXII	−0.01372 LXIII	−0.01372 LXIV
60.	0=+0.00200	−0.01755 LII	+ 0.01780 LIII	−0.06317 LV	+0.03878 LVI
		+0.00486 LVII	+ 0.02782 LVIII	+0.47233 LIX	+2.42719 LX
		+0.72947 LXI	+ 1.18174 LXII	−0.14732 LXIII	−0.03773 LXIV
61.	0=+0.02900	−0.17815 LVII	− 1.09796 LIX	+0.72947 LX	+2.60105 LXI
		−1.81377 LXII	− 0.28702 LXIII	−0.02401 LXIV	
62.	0=+2.59552	+0.03303 LII	− 0.03350 LIII	+0.11893 LV	−0.07301 LVI
		+0.51297 LVII	− 0.31995 LVIII	−0.03282 LIX	+1.18174 LX
		−1.81377 LXI	+ 7.24358 LXII	+0.21493 LXIII	
63.	0=−1.59300	−0.01372 LIX	− 0.14732 LX	−0.28702 LXI	+0.21493 LXII
		+2.12884 LXIII	− 0.80234 LXIV	−0.17918 LXV	
64.	0=+2.79500	−0.01372 LIX	− 0.03773 LX	−0.02401 LXI	−0.80234 LXIII
		+2.48253 LXIV	+ 1.09478 LXV	−0.07519 LXVI	
65.	0=−0.41500	−0.17918 LXIII	+ 1.09478 LXIV	+3.21516 LXV	−0.15038 LXVI
66.	0=−0.44900	−0.07519 LXIV	− 0.15038 LXV	+3.04736 LXVI	+1.80460 LXVII
67.	0=−0.93800	+1.80460 LXVI	+ 4.95403 LXVII		

Values of the correlates and of their logarithms.

XXIV	=+1.0835	log 0.0348289+	XLVI	=−1.0081	log 0.0035036−
XXV	=+0.0418	log 8.6211763+	XLVII	=+0.4159	log 9.6189889+
XXVI	=−0.5777	log 9.7618527−	XLVIII	=−0.4045	log 9.6069185−
XXVII	=−0.0886	log 8.9474337−	XLIX	=−0.3550	log 9.5502284−
XXVIII	=−0.1894	log 9.2773800−	L	=+0.5407	log 9.7329564+
XXIX	=+0.2145	log 9.3314273+	LI	=−0.2793	log 9.4460709−
XXX	=−0.4559	log 9.6588696−	LII	=−0.3305	log 9.5191715−
XXXI	=+0.3029	log 9.4812993+	LIII	=−0.1045	log 9.0191163−
XXXII	=−0.4201	log 9.6233527−	LIV	=−0.5284	log 9.7229628−
XXXIII	=−0.2145	log 9.3314273−	LV	=+0.3537	log 9.5486351+
XXXIV	=−0.9001	log 9.9542908−	LVI	=+0.1696	log 9.2294258+
XXXV	=−1.2225	log 0.0872489−	LVII	=−0.5645	log 9.7516639−
XXXVI	=−0.0807	log 8.9068735−	LVIII	=+0.0099	log 7.9956352+
XXXVII	=−0.4427	log 9.6461095−	LIX	=−1.1454	log 0.0589572−
XXXVIII	=−0.4342	log 9.6376898−	LX	=+1.0712	log 0.0298706+
XXXIX	=−0.3705	log 9.5687882−	LXI	=−1.4184	log 0.1517987−
XL	=−0.2850	log 9.4548449−	LXII	=−0.8641	log 9.9365640−
XLI	=−0.1270	log 9.1038037−	LXIII	=+0.2704	log 9.4320067+
XLII	=−0.3702	log 9.5684364−	LXIV	=−1.3007	log 0.1141771−
XLIII	=−1.0346	log 0.0147725−	LXV	=+0.5890	log 9.7701153+
XLIV	=−0.5348	log 9.7281914−	LXVI	=+0.0411	log 8.6138418+
XLV	=−0.1446	log 9.1601683−	LXVII	=+0.1744	log 9.2415665+

Values of the general corrections.

	″		″		″		″		″
[14_1]	=+0.033	[20_4]	=−1.617	[26_1]	=−0.409	[31_5]	=+0.087	[36_3]	=−0.334
[14_2]	=+0.867	[21_1]	=−0.197	[26_2]	=−0.039	[31_6]	=+0.893	[36_4]	=+0.199
[15_2]	=+1.083	[21_2]	=−0.297	[26_3]	=+0.864	[32_1]	=−1.275	[37_1]	=−1.158
[16_1]	=+0.880	[21_3]	=−0.292	[26_{3+4}]	=−0.547	[32_2]	=+0.209	[37_2]	=+0.905
[16_2]	=+0.074	[21_{4+5}]	=+0.330	[27_1]	=+0.825	[32_3]	=+0.209	[37_3]	=+0.113
[16_3]	=−0.546	[21_5]	=−2.456	[27_2]	=−0.839	[32_4]	=−0.273	[37_4]	=+0.108
[16_4]	=−0.204	[22_1]	=−0.220	[27_3]	=−0.510	[33_1]	=−0.772	[38_{1+2}]	=+0.632
[17_1]	=+1.429	[22_2]	=−0.966	[28_1]	=−0.183	[33_2]	=+0.662	[38_3]	=−0.938
[17_{1+2}]	=+0.166	[22_3]	=−0.013	[28_2]	=−0.140	[33_3]	=−0.370	[38_4]	=−0.085
[17_3]	=−0.393	[22_4]	=+0.424	[28_3]	=−0.463	[33_4]	=+0.347	[39_1]	=−0.767
[17_5]	=−0.042	[23_1]	=+0.029	[28_4]	=−0.511	[33_{5+6}]	=−0.039	[39_2]	=+0.678
[18_1]	=−0.623	[23_2]	=−0.035	[28_5]	=+0.516	[33_6]	=+0.156	[39_3]	=+0.056
[18_2]	=+0.081	[23_3]	=−0.280	[29_{1+2}]	=−0.441	[34_1]	=+0.619	[40_{1+2}]	=+0.215
[18_3]	=+0.283	[23_4]	=−0.398	[29_2]	=−0.599	[34_2]	=+0.527	[40_2]	=−0.174
[18_5]	=−0.303	[24_1]	=−0.216	[29_3]	=−0.137	[34_3]	=−0.423	[40_3]	=+0.589
[19_1]	=−0.466	[24_2]	=−0.048	[29_4]	=−0.202	[34_4]	=+0.135	[41_4]	=+0.196
[19_2]	=+0.565	[24_3]	=+0.778	[29_5]	=+0.411	[35_1]	=−0.237	[41_5]	=+0.178
[19_3]	=−0.094	[24_{3+4}]	=−0.849	[31_1]	=−0.155	[35_2]	=−0.624	[42_1]	=+0.174
[20_1]	=+0.740	[25_1]	=−0.051	[31_2]	=+0.001	[35_3]	=+0.558		
[20_2]	=−0.442	[25_2]	=+0.017	[31_3]	=+0.037	[36_1]	=−1.005		
[20_3]	=+0.922	[25_{2+3}]	=−0.268	[31_4]	=−0.091	[36_2]	=+0.741		

Residuals resulting from substitution of general corrections in the numerical equations of condition.

No. of equation.	Residual.	No. of equation.	Residual.	No. of equation.	Residual.
24	−0.00006	39	+0.00004	54	−0.00080
25	+0.00014	40	−0.00012	55	+0.00012
26	−0.00014	41	+0.00126	56	−0.01290
27	+0.00003	42	−0.00003	57	+0.00009
28	+0.00001	43	+0.00009	58	−0.01825
29	−0.00003	44	−0.00011	59	+0.00010
30	+0.00490	45	+0.00004	60	+0.00016
31	+0.00003	46	−0.00016	61	+0.00012
32	−0.00009	47	−0.00080	62	+0.00262
33	−0.00040	48	−0.00003	63	−0.00002
34	+0.00011	49	+0.00005	64	+0.00011
35	+0.00012	50	−0.00775	65	+0.00012
36	−0.00009	51	+0.00930	66	+0.00019
37	−0.00980	52	−0.00012	67	+0.00015
38	−0.00005	53	+0.05080		

SECTION V.—*Triangulation from the line Eldorado-Taycheedah to the line Minnesota Junction-Horicon.*

OAKFIELD—43.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Dates, October and November, 1872.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Horicon and Minnesota Junction ...	32 54 41.589	43_1	9	3.6	0.5	−0.099	−0.259	32 54 41.231
Minnesota Junction and Waupun ..	50 55 03.433	43_2	15	5.2	0.7	−0.175	+0.593	50 55 03.851
Waupun and Springvale	53 57 55.920	43_3	15	5.7	0.7	−0.450	−0.719	53 57 54.751
Springvale and Eldorado	40 02 22.733	43_4	18	6.3	1.0	0.000	+0.256	40 02 22.989
Springvale and West Base	46 49 51.936	43_{4+5}	14	7.0	0.7	−0.345	+0.132	46 49 51.723
Eldorado and Taycheedah	37 57 07.987	43_{5+6}	15	6.3	0.7	0.000	+0.138	37 57 08.125
West Base and East Base	38 49 29.631	43_{6+7}	16	5.8	0.8	−0.109	+0.437	38 49 29.959
Horicon and Waupun	83 49 44.843	43_{1+2}	7	2.8	0.4	−0.095	+0.334	83 49 45.082
East Base and Horicon	136 32 58.844	43_8	9	5.6	0.5	−0.175	−0.184	136 32 58.485
Minnesota Junction and Springvale.	104 52 58.000	43_{2+3}	3	6.3	0.1	+0.728	−0.126	104 52 58.602
Waupun and West Base	100 47 46.675	43_{3+4+5}	8	5.3	0.4	+0.386	−0.587	100 47 46.474

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$1.4(43_1)+0.9(43_2)+0.5(43_3)+0.5(43_{4+5})+0.5(43_{6+7})+0.7481=0$$
$$0.9(43_1)+1.7(43_2)+0.6(43_3)+0.5(43_{4+5})+0.5(43_{6+7})+0.8834=0$$
$$0.5(43_1)+0.6(43_2)+1.7(43_3)+0.9(43_{4+5})+0.5(43_{6+7})+1.2842=0$$
$$0.5(43_1)+0.5(43_2)+0.9(43_3)+1.6(43_{4+5})+0.5(43_{6+7})+1.1489=0$$
$$0.5(43_1)+0.5(43_2)+0.5(43_3)+0.5(43_{4+6})+1.3(43_{6+7})+0.6765=0$$

SPRINGVALE—44.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Dates, September and October, 1872.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Eldorado and East Base	61 07 44.980	44_1	10	5.7	0.5	−0.199	+0.133	61 07 44.914
East Base and Oakfield	31 45 19.178	44_2	13	5.2	0.6	−0.166	+0.039	31 45 19.051
Oakfield and Waupun	40 42 14.262	44_3	16	6.3	0.8	....	−0.825	40 42 13.437
Eldorado and Oakfield	92 53 02.800	44_{1+2}	3	3.9	0.1	+0.993	+0.172	92 53 03.965

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$0.6(44_1)+0.1(44_2)+0.1358=0$$
$$0.1(44_1)+0.7(44_2)+0.1358=0$$

EAST BASE—45.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Dates, October and November, 1872.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Oakfield and West Base	33 28 54.437	45_1	24	6.4	1.0	+0.042	+0.309	33 28 54.788
West Base and Middle Base	0 00 03.797	45_2	13	2.8	0.6	−0.251	−0.081	0 00 03.465
Middle Base and Check Base	24 31 07.875	45_3	17	7.8	1.0	−0.150	−0.046	24 31 07.679
Check Base and Springvale	4 35 14.693	45_4	6	4.5	0.3	−0.851	+0.036	4 35 13.878
Check Base and Taycheedah	109 01 04.340	45_{4+6}	11	9.0	0.5	+0.167	−0.315	109 01 04.192
Taycheedah and Catholic Church...	119 15 57.067	45_6	9	6.0	0.5	+0.168	+0.130	119 15 57.365
Oakfield and Springvale	62 35 19.396	$45_{1+2+3+4}$	16	5.7	0.8	+0.196	+0.218	62 35 19.810
Catholic Church and Oakfield	73 42 50.520	45_7	2	6.1	0.1	+1.988	+0.003	73 42 52.511
West Base and Check Base	24 31 10.800	45_{2+3}	5	8.0	0.2	+0.471	−0.127	24 31 11.144
West Base and Springvale...	29 06 24.620	45_{2+3+4}	4	3.1	0.2	+0.493	−0.091	29 06 25.022
Check Base and Catholic Church ...	228 17 00.590	45_{4+6+6}	2	2.9	0.1	+1.152	−0.185	228 17 01.557

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$1.9(45_1)+0.9(45_2)+0.9(45_3)+0.8(45_4)+0.1(45_{4+6})+0.1(45_6)+0.9284=0$$
$$0.9(45_1)+1.9(45_2)+1.3(45_3)+1.0(45_4)+0.1(45_{4+5})+0.1(45_6)+1.4518=0$$
$$0.9(45_1)+1.3(45_2)+2.3(45_3)+1.0(45_4)+0.1(45_{4+5})+0.1(45_6)+1.4518=0$$
$$0.8(45_1)+1.0(45_2)+1.0(45_3)+1.3(45_4) \qquad +1.4738=0$$
$$0.1(45_1)+0.1(45_2)+0.1(45_3) \qquad +0.7(45_{4+6})+0.2(45_6)-0.1147=0$$
$$0.1(45_1)+0.1(45_2)+0.1(45_3) \qquad +0.2(45_{4+6})+0.7(45_6)-0.1147=0$$

SECTION V.—*Triangulation from the line Eldorado-Taycheedah to the line Minnesota Junction-Horicon*—Continued.

WEST BASE—46.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, October, 1872.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Check Base and Middle Base	25 05 25.357	46_1	7	5.4	0.4	....	+0.162	25 05 25.519
Check Base and East Base	25 05 29.775	46_{1+2}	8	4.7	0.4	−1.005	+0.188	25 05 28.958
East Base and Oakfield	107 41 35.342	46_3	21	8.0	1.0	−0.402	+0.430	107 41 35.370
Check Base and Oakfield	132 46 59.690	46_{1+2+3}	2	1.0	0.1	+4.020	+0.618	132 47 04.328

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$0.5(46_{1+2})+0.1(46_3)+0.5427=0$$
$$0.1(46_{1+2})+1.1(46_3)+0.5427=0$$

MIDDLE BASE—48.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, November, 1872.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
West Base and Check Base	87 55 04.350	48_1	12	5.5	0.6	....	+0.146	87 55 04.496
Check Base and East Base	92 05 02.436	48_2	14	6.1	0.7	....	-0.028	92 05 02.408

CHECK BASE—49.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, November, 1872.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
East Base and Middle Base	63 23 49.835	49_1	17	4.4	1	....	+0.094	63 23 49.929
Middle Base and West Base	66 59 29.800	49_2	17	5.6	1	....	+0.201	66 59 30.001

WAUPUN—50.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, October, 1872.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Springvale and Oakfield	85 19 52.958	50_1	19	6.8	1	....	−0.660	85 19 52.298
Oakfield and Horicon	60 50 13.287	50_2	19	12.1	1	....	+0.307	60 50 13.594
Horicon and Minnesota Junction	34 00 48.328	50_3	16	8.3	1	....	+0.273	34 00 48.601

MINNESOTA JUNCTION—51.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, May, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Waupun and Oakfield	34 13 54.910	51_1	20	5.0	1	....	−0.416	34 13 54.494
Oakfield and Horicon	58 27 05.085	51_2	20	8.0	1	....	+0.089	58 27 05.174
Horicon and Lebanon	60 40 56.071	51_3	20	6.5	1	....	+0.215	60 40 56.286

NOTE.—The correction [v] for 51_3 is derived from the succeeding section of the adjustment. 51_3 above is designated l_1 in the succeeding section.

SECTION V.—*Triangulation from the line Eldorado-Taycheedah to the line Minnesota Junction-Horicon*—Continued.

HORICON—52.

[Observer, R. S. Woodward. Instrument, 10-inch Gambey repeating theodolite. Date, May, 1873.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			″			″	° ′ ″
Minnesota Junction and Oakfield.... 88 38 14.394	52_{1+2}	16	2.3	1	....	−0.228	88 38 14.166
Waupun and Oakfield 35 20 00.900	52_2	16	2.9	1	....	+1.014	35 20 01.914
Woodland and Lebanon 34 19 08.384	52_3	16	3.0	1	....	−0.295	34 19 08.089
Lebanon and Minnesota Junction ... 90 13 47.021	52_4	16	5.0	1	....	+0.215	90 13 47.236

NOTE.—Angles 52_3 and 52_4 are the same as 2_1 and 2_2 of the succeeding section of the adjustment, from which their corrections [v] are derived.

Numerical equations of condition in the triangulation from the line Eldorado-Taycheedah to the line Minnesota Junction-Horicon.

SIDE-EQUATIONS.

LXVIII. (100) $-2.2692[41_{1+2}] - 18.6499[41_2] - 20.9191[41_3] - 32.7460[42_2]$
$+5.4942[42_3] - 156.4680[43_4] + 156.4680[43_{4+5}] - 183.4638[43_{5+6}]$
$+156.4680[43_{6+7}] + 91.3420[45_1] + 91.3420[45_2] + 91.3420[45_3]$
$+91.3420[45_{4+5}] + 6.8508[45_6] - 49.271 = 0$

LXIX. (70) $+226.2427[41_2] - 9.0796[41_3] + 5.4942[42_2] + 5.4942[42_3]$
$-19.5821[42_4] - 1.0608[44_1] - 35.0786[44_2] - 102.2612[45_1]$
$+102.2612[45_2] + 102.2612[45_3] + 10.9192[45_4] + 91.3420[45_{4+5}] + 329.841 = 0$

LXXV. (30) $-46.1591[45_2] + 0.0021[45_3] - 44.9676[46_1] + 44.9657[46_{1+2}]$
$+0.7655[48_1] + 0.7662[48_2] - 5.016 = 0$

LXXXI. (20) $+30.2555[43_1] - 2.2765[43_2] + 11.7495[50_2] - 31.1998[50_3]$
$-0.9868[51_1] - 13.9140[51_2] + 14.934 = 0$

NOTE.—In the solution for determining the general corrections, each of the side-equations was divided by the number inclosed in parenthesis and placed opposite it.

ANGLE-EQUATIONS.

LXX. $[41_2] + [41_3] + [42_2] + [42_3] + [42_4] + [44_1] - [45_4] + [45_{4+5}] + 1.647 = 0$

LXXI. $[41_3] + [42_2] + [42_3] + [43_{5+6}] - 0.125 = 0$

LXXII. $[42_4] + [43_4] + [44_1] + [44_2] - 0.425 = 0$

LXXIII. $[43_{4+5}] + [43_{6+7}] + [44_2] + [45_1] + [45_2] + [45_3] + [45_4] - 0.826 = 0$

LXXIV. $[43_{6+7}] + [45_1] + [46_3] - 1.176 = 0$

LXXVI. $[45_2] + [45_3] + [46_{1+2}] + [49_1] + [49_2] - 0.356 = 0$

LXXVII. $[45_3] + [48_2] + [49_1] - 0.020 = 0$

LXXVIII. $[46_1] + [48_1] + [49_2] - 0.509 = 0$

LXXIX. $[43_3] + [44_3] + [50_1] + 2.204 = 0$

LXXX. $[43_2] + [50_2] + [50_3] + [51_1] - 0.758 = 0$

LXXXII. $[43_1] + [43_2] + [50_2] + [52_2] - 1.655 = 0$

LXXXIII. $[43_1] + [51_2] + [52_{1+2}] + 0.398 = 0$

General corrections in terms of the correlates.

$[41_{1+2}]$	=−0.03241 LXVIII			
$[41_2]$	=−0.93250 LXVIII	+16.16020 LXIX	+5.00000 LXX	
$[41_3]$	=−0.20919 LXVIII	−0.12971 LXIX	+1.00000 LXX	+1.00000 LXXI
$[42_2]$	=−0.47324 LXVIII	+0.05232 LXIX	+0.66667 LXX	+0.66667 LXXI
$[42_3]$	=+0.29156 LXVIII	+0.05232 LXIX	+0.66667 LXX	+0.66667 LXXI
$[42_4]$		−0.27974 LXIX	+1.00000 LXX	+1.00000 LXXII
$[43_1]$	=−0.48773 LXVIII	−0.31171 LXXIII	−0.18948 LXXIV	−0.04283 LXXIX
	−0.51298 LXXX	+1.82936 LXXXI	+0.65770 LXXXII	+1.17067 LXXXIII
$[43_2]$	=−0.20059 LXVIII	−0.12820 LXXIII	−0.10366 LXXIV	−0.13983 LXXIX
	+0.94687 LXXX	−0.88380 LXXXI	+0.43390 LXXXII	−0.51298 LXXXIII
$[43_3]$	=−0.83383 LXVIII	−0.53291 LXXIII	−0.11810 LXXIV	+0.90452 LXXIX
	−0.13983 LXXX	−0.04887 LXXXI	−0.18266 LXXXII	−0.04283 LXXXIII
$[43_4]$	=−1.56468 LXVIII	+1.00000 LXXII		
$[43_{4+5}]$	=+1.25365 LXVIII	+0.80122 LXXIII	−0.14978 LXXIV	−0.41481 LXXIX
	−0.02454 LXXX	−0.18212 LXXXI	−0.14677 LXXXII	−0.12223 LXXXIII
$[43_{5+6}]$	=−2.62091 LXVIII	+1.42857 LXXI		
$[43_{6+7}]$	=+1.30686 LXVIII	+0.83523 LXIII	+0.98501 LXXIV	−0.11810 LXXIX
	−0.10366 LXXX	−0.27484 LXXXI	−0.29314 LXXXII	−0.18948 LXXXIII
$[44_1]$	=+0.09635 LXIX	+1.70732 LXX	+1.46341 LXXII	0.24390 LXXIII
$[44_2]$	=−0.72964 LXIX	−0.24390 LXX	+1.21951 LXXII	+1.46341 LXXIII
$[44_3]$	=			+1.25000 LXXIX
$[45_1]$	=+0.43246 LXVIII	+0.66136 LXIX	+0.23619 LXX	+0.24174 LXXIII
	+0.76479 LXXIV	+0.21840 LXXV	−0.22712 LXXVI	−0.08517 LXXVII
$[45_2]$	=+0.48869 LXVIII	+0.72320 LXIX	+0.42170 LXX	+0.11838 LXXIII
	−0.14195 LXXIV	−1.68246 LXXV	+0.74953 LXXVI	−0.34393 LXXVII
$[45_3]$	=+0.29320 LXVIII	+0.43392 LXIX	+0.25302 LXX	+0.07102 LXXIII
	−0.08517 LXXIV	+0.52924 LXXV	+0.44972 LXXVI	+0.79365 LXXVII
$[45_4]$	=−0.86759 LXVIII	−1.17708 LXIX	−1.43359 LXX	+0.47477 LXXIII
	−0.29593 LXXIV	+0.75270 LXXV	−0.78274 LXXVI	−0.29353 LXXVII
$[45_{4+5}]$	=+1.25549 LXVIII	+1.82779 LXIX	+1.45434 LXX	−0.04791 LXXIII
	−0.05974 LXXIV	+0.10387 LXXV	−0.10802 LXXVI	−0.04051 LXXVII
$[45_6]$	=−0.43433 LXVIII	−0.78199 LXIX	−0.54566 LXX	−0.04791 LXXIII
	−0.05974 LXXIV	+0.10387 LXXV	−0.10802 LXXVI	−0.04051 LXXVII
$[46_1]$	=	−3.74730 LXXV		+2.50000 LXXVIII
$[46_{1+2}]$	=−0.18519 LXXIV	+3.05323 LXXV	+2.03704 LXXVI	
$[46_3]$	=+0.92593 LXXIV	−0.27757 LXXV	−0.18519 LXXVI	
$[48_1]$	=+0.04253 LXXV	+1.66667 LXXVIII		
$[48_2]$	=+0.03649 LXXV	+1.42857 LXXVII		
$[49_1]$	=+1.00000 LXXVI	+1.00000 LXXVII		
$[49_2]$	=+1.00000 LXXVI		+1.00000 LXXVIII	
$[50_1]$	=+1.00000 LXXIX			
$[50_2]$	=	+1.00000 LXXX	+0.58748 LXXXI	+1.00000 LXXXII
$[50_3]$	=	+1.00000 LXXX	−1.55999 LXXXI	
$[51_1]$	=	+1.00000 LXXX	−0.04934 LXXXI	
$[51_2]$	=		−0.69570 LXXXI	+1.00000 LXXXIII
$[52_{1+2}]$	=			+1.00000 LXXXIII
$[52_2]$	=+1.00000 LXXXII			

Normal equations for determining the correlates.

No. of equation.					
68.	0=−0.49271	+13.87867 LXVIII	+0.27597 LXIX	+0.79970 LXX	−3.01178 LXXI
		−1.56468 LXXII	+2.90727 LXXIII	+1.73932 LXXIV	−0.75190 LXXV
		+0.78189 LXXVI	+0.29320 LXXVII	−0.83383 LXXIX	−0.20059 LXXX
		−0.71500 LXXXI	−0.68832 LXXXII	−0.48773 LXXXIII	
69.	0=+4.71201	+0.27597 LXVIII	+57.55592 LXIX	+18.95661 LXX	−0.02507 LXXI
		−0.91303 LXXII	−0.08824 LXXIII	+0.66136 LXXIV	−1.11271 LXXV
		+1.15712 LXXVI	+0.43392 LXXVII		
70.	0=+1.64700	+0.79970 LXVIII	+18.95661 LXIX	+12.92858 LXX	+2.33333 LXXI
		+2.46342 LXXII	−0.76658 LXXIII	+0.23619 LXXIV	−0.64883 LXXV
		+0.67472 LXXVI	+0.25302 LXXVII		
71.	0=−0.12500	−3.01178 LXVIII	−0.02507 LXIX	+2.33333 LXX	+3.76190 LXXI
72.	0=−0.42500	−1.56468 LXVIII	−0.91303 LXIX	+2.46342 LXX	+4.68292 LXXII
		+1.21951 LXXIII			
73.	0=−0.82600	+2.90727 LXVIII	−0.08824 LXIX	−0.76658 LXX	+1.21951 LXXII
		+4.00577 LXXIII	+1.07697 LXXIV	−0.18213 LXXV	+0.18940 LXXVI
		+0.07102 LXXVII	−0.53291 LXXIX	−0.12820 LXXX	−0.45696 LXXXI
		−0.43991 LXXXII	−0.31171 LXXXIII		
74.	0=−1.17600	+1.73932 LXVIII	+0.66136 LXIX	+0.23619 LXX	+1.07697 LXXIII
		+2.67573 LXXIV	−0.05917 LXXV	−0.41231 LXXVI	−0.08517 LXXVII
		−0.11810 LXXIX	−0.10366 LXXX	−0.27484 LXXXI	−0.29314 LXXXII
		−0.18948 LXXXIII			
75.	0=−0.16720	−0.75190 LXVIII	−1.11271 LXIX	−0.64883 LXX	−0.18213 LXXIII
		−0.05917 LXXIV	+12.78402 LXXV	+1.90001 LXXVI	+0.56573 LXXVII
		−3.70477 LXXVIII			
76.	0=−0.35600	+0.78189 LXVIII	+1.15712 LXIX	+0.67472 LXX	+0.18940 LXXIII
		−0.41231 LXXIV	+1.90001 LXXV	+5.23629 LXXVI	+1.44972 LXXVII
		+1.00000 LXXVIII			
77.	0=−0.02000	+0.29320 LXVIII	+0.43392 LXIX	+0.25302 LXX	+0.07102 LXXIII
		−0.08517 LXXIV	+0.56573 LXXV	+1.44972 LXXVI	+3.22222 LXXVII
78.	0=−0.50900	+3.70477 LXXV	+1.00000 LXXVI	+5.16667 LXXVIII	
79.	0=+2.20400	−0.83383 LXVIII	−0.53291 LXXIII	−0.11810 LXXIV	+3.15452 LXXIX
		−0.13983 LXXX	−0.04887 LXXXI	−0.18266 LXXXII	−0.04283 LXXXIII
80.	0=−0.75800	−0.20059 LXVIII	−0.12820 LXXIII	−0.10366 LXXIV	−0.13983 LXXIX
		+3.94687 LXXX	−1.90565 LXXXI	+1.43390 LXXXII	−0.51298 LXXXIII
81.	0=+0.74670	−0.71500 LXVIII	−0.45696 LXXIII	−0.27484 LXXIV	−0.04887 LXXIX
		−1.90565 LXXX	+6.13315 LXXXI	+1.53304 LXXXII	+1.13366 LXXXIII
82.	0=−1.65500	−0.68832 LXVIII	−0.43991 LXXIII	−0.29314 LXXIV	−0.18266 LXXIX
		+1.43390 LXXX	+1.53304 LXXXI	+3.09160 LXXXII	+0.65769 LXXXIII
83.	0=+0.39800	−0.48773 LXVIII	−0.31171 LXXIII	−0.18948 LXXIV	−0.04283 LXXIX
		−0.51298 LXXX	+1.13366 LXXXI	+0.65769 LXXXII	+3.17067 LXXXIII

Values of the correlates and their logarithms.

LXVIII	=+0.6715	log 9.8270201+	LXXVI	=+0.1142	log 9.0575900+
LXIX	=+0.3282	log 9.5160989+	LXXVII	=−0.0202	log 8.3049212−
LXX	=−1.2180	log 0.0856330−	LXXVIII	=+0.0872	log 8.9407654+
LXXI	=+1.3284	log 0.1233387+	LXXIX	=−0.6598	log 9.8193860−
LXXII	=+1.3067	log 0.1161726+	LXXX	=−0.4382	log 9.6416922−
LXXIII	=−1.1017	log 0.0420673−	LXXXI	=−0.4561	log 9.6590886−
LXXIV	=+0.4921	log 9.6920445+	LXXXII	=+1.0137	log 0.0059094+
LXXV	=+0.0151	log 8.1789769+	LXXXIII	=−0.2281	log 9.3581778−

Values of the general corrections.

″	″	″
$[41_{1+2}]=-0.022$	$[43_{6+7}]=+0.437$	$[46_3]=+0.430$
$[41_2]=-1.413$	$[44_1]=+0.133$	$[48_1]=+0.146$
$[41_3]=-0.073$	$[44_2]=+0.039$	$[48_2]=-0.028$
$[42_2]=-0.227$	$[44_3]=-0.825$	$[49_1]=+0.094$
$[42_3]=+0.287$	$[45_1]=+0.309$	$[49_2]=+0.201$
$[42_4]=-0.003$	$[45_2]=-0.081$	$[50_1]=-0.660$
$[43_1]=-0.259$	$[45_3]=-0.046$	$[50_2]=+0.308$
$[43_2]=+0.593$	$[45_4]=+0.036$	$[50_3]=+0.273$
$[43_3]=-0.719$	$[45_{4+5}]=-0.315$	$[51_1]=-0.416$
$[43_4]=+0.256$	$[45_6]=+0.130$	$[51_2]=+0.089$
$[43_{4+5}]=+0.132$	$[46_1]=+0.162$	$[52_{1+2}]=-0.228$
$[43_{5+6}]=+0.138$	$[46_{1+2}]=+0.188$	$[52_2]=+1.014$

Residuals resulting from substitution of general corrections in the numerical equations of condition.

No. of equation.	Residual.	No. of equation.	Residual.
68	−0.00600	76	−0.00004
69	−0.00049	77	−0.00001
70	−0.00005	78	+0.00004
71	+0.00001	79	+0.00006
72	−0.00002	80	−0.00002
73	−0.00005	81	+0.00100
74	−0.00002	82	+0.00001
75	−0.00360	83	+0.00003

PROBABLE ERRORS OF OBSERVED AND ADJUSTED ANGLES.

§ **7.** Let—

m = whole number of observed angles in a section (one adjustment).
r = whole number of rigid conditions in a section.
n = number of triangles in principal chain.
$[pvv]$ = sum of weighted squares of corrections to observed angles.
ρ_1 = probable error of an observed angle of weight unity.
ρ_s = probable error of an observed angle of average weight in whole section.
ρ_s' = probable error of an adjusted angle of average weight in whole section.
p_s = average weight of an observed angle in whole section.
p_c = average weight of an observed angle in principal chain.
ρ_c = probable error of an observed angle of average weight in principal chain.
ρ_c' = probable error of an adjusted angle of average weight in principal chain.
$[vv]$ = sum of squares of closing errors of triangles in principal chain.
ρ_t = probable error of an observed angle in principal chain as derived from the closing errors of triangles.

Proceeding as in Chapter XIV, C, § 8, there are found the following values:

FOR THE ENTIRE SECTIONS.

Section.	Extent of section.	m	r	$[pvv]$	ρ_1	p_s	ρ_s	$\sqrt{\frac{m-r}{m}}$	ρ_s'
					″		″		″
III	Vulcan-Huron Mountains to Pine Hill-Burnt Bluff	80	52	21.12	0.43	1.09	0.41	0.59	0.24
IV	Pine Hill-Burnt Bluff to Eldorado-Taycheedah	188	129	55.80	0.44	0.64	0.55	0.56	0.31
V	Eldorado-Taycheedah to Minnesota Junction-Horicon	48	28	8.88	0.38	0.63	0.48	0.65	0.31

FOR THE PRINCIPAL CHAIN CONNECTING THE LINE VULCAN - HURON MOUNTAINS WITH FOND DU LAC BASE, GIVEN IN D, § 8, FOLLOWING.

Section.	Extent of principal chain in each section.	p_c	ρ_c	ρ_c'	From closing errors of triangles. $[vv]$	n	ρ_t	Average error.	Greatest error.
			"	"			"	"	"
III	Vulcan - Huron Mountains to Pine Hill-Burnt Bluff..	1.03	0.42	0.25	24.51	13	0.54	1.21	2.56
IV	Pine Hill - Burnt Bluff to Eldorado-Taycheedah......	0.79	0.50	0.28	40.02	26	0.48	1.04	3.10
V	Eldorado - Taycheedah to Fond du Lac Base	0.76	0.44	0.28	0.98	4	0.19	0.43	0.71
	Entire principal chain		....	......	65.51	43	0.48	1.03	3.10

D.—CHAIN OF PRINCIPAL TRIANGLES BETWEEN THE LINE VULCAN-HURON MOUNTAINS AND THE FOND DU LAC BASE.

§ 8. In Chapter XIV, D, a method of computing the weighted mean logarithms of the sides in a chain of triangles depending on two bases has been given, and this method is there applied to the system of triangles between Keweenaw and Minnesota Point Bases. The two systems connecting the Keweenaw with the Minnesota and the Fond du Lac Bases have a common part, embracing six triangles lying between Keweenaw Base and the line Vulcan – Huron Mountains. Computing the common line Vulcan – Huron Mountains from each of the three bases and taking the weighted mean, it is found that the effect of the Fond du Lac Base on the logarithm of this line is less than unity in the seventh place. The probable error of the logarithm of this weighted mean line is (probable error of standard being excluded) 26.24 in the seventh place, while the corresponding value obtained from Chapter XIV, D, where Keweenaw and Minnesota Point Bases are alone considered, is 27.38. The weighted mean logarithms of the sides of the common triangles given in Chapter XIV, D, therefore need no modification on account of the Fond du Lac Base, and for computing the weighted mean logarithms of the triangle-sides between the Fond du Lac Base and the line Vulcan – Huron Mountains, the latter line may be taken as a base, using the weighted mean logarithm and probable error of its length derived from Chapter XIV, D.

The adjustment of the triangulation between the line Vulcan–Huron Mountains and the Fond du Lac Base gives the following values for the probable error, ρ, of an observed angle of average weight in the respective sections of the principal chain. See Chapter XV, C, § 7.

From the line Vulcan – Huron Mountains to the line Pine Hill – Burnt Bluff.......... $\rho=\pm 0''.42$
From the line Pine Hill – Burnt Bluff to the line Eldorado – Taycheedah .... $\rho=\pm 0''.50$
From the line Eldorado – Taycheedah to the line Minnesota Junction – Horicon. ..... $\rho=\pm 0''.43$

Using the notation of Chapter XIV, D,

For the first section, $\Sigma(\alpha^2+\beta^2)\rho^2=$ 4021
For the second section, $\Sigma(\alpha^2+\beta^2)\rho^2=$ 3874
For the third section, $\Sigma(\alpha^2+\beta^2)\rho^2=$ 586

Sum.... ... 8481

The square of the probable error in the logarithm of the weighted mean value of the side Vulcan – Huron Mountains, obtained from Chapter XIV, D, is 750, the error of standard bar being excluded, and the corresponding quantity for the Fond du Lac Base is 36, both being expressed, as are the above sums, in units of the seventh decimal place. Hence, the constant

$$\frac{1}{p}+\frac{1}{p'}=8481+750+36=9267$$

The logarithms of the Fond du Lac Base resulting by computation from the weighted mean length of the line Vulcan – Huron Mountains, given in Chapter XIV, D, and from the direct measurement, are—

From computation (feet) 4.3865922
From measurement (feet) 4.3865918

The difference between these logarithms gives $d=-4$, in units of the seventh decimal place.

From these values of $\left(\frac{1}{p}+\frac{1}{p'}\right)$ and d, and from the values of $\frac{1}{p}$ given in the table, the weighted mean logarithms result.

The line in the system having the maximum probable error is Burnt Bluff-Sturgeon. For this line $\frac{1}{p}=4622$, and $\frac{1}{p'}=4645$, giving for the square of the probable error of the weighted mean logarithm of this side, 2317. Adding to this the square of the probable error of the logarithm of the length of the standard 15-feet bar, viz, 8.39 (Chapter II, § 14), there results for the probable error of the logarithm of the above line, ± 48.22, corresponding to the $\frac{1}{90066}$ part of its length.

In the table which follows, the first column gives the names of the stations; the second gives the adjusted values of the angles, taken from Chapter XV, C, § 6; the third gives the error of closure; the fourth gives the logarithm of the side in feet opposite the station on the same line, this value being computed from the weighted mean value of the logarithm of the line Vulcan - Huron Mountains, and the angles in the second column; the fifth gives for each triangle α^2 and β^2; the sixth gives the sum of $(\alpha^2+\beta^2)$ from the first triangle in a group having the same probable error for an observed angle, up to the opposite triangle, inclusive, in the direction of the Fond du Lac Base; the seventh gives the quantities $\frac{1}{p}$; and the eighth gives the weighted mean logarithms of the sides.

Chain of principal triangles between Vulcan - Huron Mountains and Fond du Lac Base.

Stations.	Angles. ° ′ ″	Errors of closure. ″	Logarithms of sides in feet.	α^2 and β^2	$\Sigma(\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
Granite Island	45 56 31.286		5.3212858	412.09			5.3212858
Vulcan	28 42 35.249	+1.217	5.1463514				5.1463514
Huron Mountains	105 21 00.131		5.4490065	33.64	445.73	830	5.4490065
Ives Hill	106 46 48.229		5.4490065	39.69			5.4490065
Granite Island	50 41 08.002	+1.504	5.3564611				5.3564610
Vulcan	22 32 09.536		5.0513906	2580.64	3066.06	1301	5.0513905
Triloba	72 00 57.434		5.0513906	46.24			5.0513905
Granite Island	84 47 31.359	−0.606	5.0713486				5.0713485
Ives Hill	23 11 32.437		4.6684400	2410.81	5523.11	1743	4.6684399
Mount Mesnard	31 46 33.119		4.6684400	1156.00			4.6684399
Granite Island	27 04 47.180	+0.794	4.6051924				4.6051923
Triloba	121 08 40.078		4.8793674	161.29	6840.40	1980	4.8793673
Shelter Bay	32 08 55.035		4.8793674	1122.25			4.8793673
Granite Island	47 25 32.690	−2.107	5.0204754				5.0204753
Mount Mesnard	100 25 34.115		5.1461318	15.21	7977.86	2184	5.1461317
Grand Island	37 02 20.007		5.1461318	778.41			5.1461317
Shelter Bay	123 34 58.404	+0.691	5.2869708				5.2869707
Granite Island	19 22 43.707		4.8871679	3588.01	12344.28	2969	4.8871678
Mud Lake	60 15 50.614		4.8871679	146.41			4.8871678
Grand Island	53 44 03.301	+0.936	4.8549747				4.8549746
Shelter Bay	66 00 07.273		4.9092250	86.49	12577.18	3011	4.9092249
Divide	104 13 15.738		4.9092250	28.09			4.9092249
Mud Lake	47 20 58.286	+1.102	4.7893245				4.7893244
Grand Island	28 25 46.536		4.6004197	1513.21	14118.48	3288	4.6004196

Chain of principal triangles between Vulcan-Huron Mountains and Fond du Lac Base—Continued.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	α^2 and β^2	$\Sigma(\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Monistique	27 52 09.853	+0.518	4.6004197	1584.04			4.6004196
Mud Lake	72 56 50.485		4.9111523				4.9111521
Divide	79 11 00.413		4.9228928	16.00	15718.52	3576	4.9228926
Sturgeon	47 34 46.646	−1.576	4.9228928	368.64			4.9228926
Monistique	110 01 51.950		5.0276103				5.0276101
Mud Lake	22 23 22.204		4.6355208	2611.21	18698.37	4111	4.6355206
Fishdam	37 10 03.267	−0.244	4.6355208	767.29			4.6355206
Sturgeon	89 40 37.159		4.8543709				4.8543707
Monistique	53 09 20.156		4.7576124	249.64	19715.30	4294	4.7576122
Burnt Bluff	26 13 40.838	−1.845	4.7576124	1823.29			4.7576122
Sturgeon	62 07 42.374		5.0586975				5.0586973
Fishdam	91 38 38.330		5.1120682	1.00	21539.59	4622	5.1120680
Pine Hill	103 35 31.072	+2.558	5.1120682	26.01			5.1120680
Burnt Bluff	39 43 09.807		4.9299230				4.9299228
Sturgeon	36 41 20.671		4.9007206	800.89	22366.49	4771	4.9007204
Ford River	50 39 11.508	+3.096	4.9007206	295.84			4.9007204
Burnt Bluff	52 46 33.233		4.9134234				4.9134232
Pine Hill	76 34 16.752		5.0003215	25.00	320.84	4851	5.0003213
Boyer's Bluff	54 17 49.328	+1.061	5.0003215	231.04			5.0003213
Ford River	66 17 33.771		5.0524486				5.0524484
Burnt Bluff	59 24 39.190		5.0256589	153.76	705.64	4947	5.0256587
Cedar River	60 31 03.738	−0.307	5.0256589	141.61			5.0256587
Boyer's Bluff	64 11 43.232		5.0402656				5.0402654
Ford River	55 17 15.282		5.0007688	213.16	1060.41	5036	5.0007686
Door Bluff	90 05 10.432	+0.731	5.0007688	0.00			5.0007686
Cedar River	33 57 52.982		4.7479330				4.7479328
Boyer's Bluff	55 56 57.680		4.9190838	204.49	1264.90	5087	4.9190836
Eagle Bluff	56 59 03.087	+1.270	4.9190838	187.69			4.9190836
Door Bluff	82 29 28.628		4.9918305				4.9918303
Cedar River	40 31 29.532		4.8083348	605.16	2057.75	5285	4.8083346
Rochereau	42 14 13.543	−1.010	4.8083348	538.24			4.8083346
Eagle Bluff	91 34 41.205		4.9806723				4.9806721
Door Bluff	46 11 06.297		4.8391208	408.04	3004.03	5522	4.8391206
Menomonee	44 00 11.785	−0.552	4.8391208	475.24			4.8391206
Eagle Bluff	59 43 41.327		4.9336587				4.9336585
Rochereau	76 16 08.244		4.9847319	26.01	3505.28	5648	4.9847317
South Egg	84 27 30.064	−1.561	4.9847319	4.00			4.9847317
Menomonee	47 44 25.615		4.8560597				4.8560595
Eagle Bluff	47 48 05.530		4.8564800	364.81	3874.09	5739	4.8564798
Peshtigo	74 47 14.923	+0.267	4.8564800	32.49			4.8564798
South Egg	30 51 58.736		4.5821189				4.5821187
Menomonee	74 20 46.964		4.8555570	34.81	3941.39	5756	4.8555568
Débroux	56 05 41.378	−0.549	4.8555570	201.64			4.8555568
Peshtigo	85 30 00.656		4.9351584				4.9351581
South Egg	38 24 18.870		4.7297435	707.56	4850.59	5984	4.7297432

Chain of principal triangles between Vulcan-Huron Mountains and Fond du Lac Base—Continued.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	α^2 and β^2	$\Sigma\ (\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Gales	43 49 13.449		4.7297435	479.61			4.7297432
Peshtigo	56 54 36.036	−1.275	4.8125343				4.8125340
Débroux	79 16 11.322		4.8817260	16.00	5346.20	6107	4.8817257
Red River	62 56 29.177		4.8817260	114.49			4.8817257
Gales	76 05 12.533	−1.755	4.9191394				4.9191391
Peshtigo	40 58 19.267		4.7487699	590.49	6051.18	6284	4.7487696
Little Tail	41 41 41.262		4.7487699	556.96			4.7487696
Red River	55 51 59.300	−0.797	4.8437322				4.8437319
Gales	82 26 20.353		4.9220502	7.84	6615.98	6425	4.9220499
Red Banks	89 51 06.011		4.9220502	0.00			4.9220499
Little Tail	59 23 47.745	+1.693	4.8569091				4.8569088
Red River	30 45 06.968		4.6307454	1253.16	7869.14	6738	4.6307451
Fort Howard	39 02 17.789		4.6307454	670.81			4.6307451
Little Tail	61 26 00.012	−1.532	4.7751396				4.7751393
Red Banks	79 31 42.790		4.8242222	15.21	8555.16	6910	4.8242219
Bruce	78 36 40.606		4.8242222	17.64			4.8242219
Fort Howard	67 24 47.297	+0.370	4.7982007				4.7982004
Little Tail	33 58 32.648		4.5801471	973.44	9546.24	7158	4.5801468
East Depere	46 03 52.278		4.5801471	412.09			4.5801468
Bruce	47 02 51.534	+0.106	4.5872052				4.5872049
Fort Howard	86 53 16.534		4.7221005	1.00	9959.33	7261	4.7221002
Oneida	46 36 48.541		4.7221005	396.01			4.7221002
East Depere	75 55 41.428	+0.620	4.8474920				4.8474917
Bruce	57 27 30.768		4.7865526	182.25	10537.59	7405	4.7865523
West Depere	58 46 55.480		4.7865526	163.84			4.7865523
East Depere	46 52 38.351	−1.052	4.7177421				4.7177418
Oneida	74 20 26.894		4.8380581	34.81	10736.24	7455	4.8380578
Calumet	38 08 22.278		4.8380581	718.24			4.8380578
West Depere	92 07 27.280	−0.809	5.0470686				5.0470683
East Depere	49 44 11.823		4.9299372	316.84	11771.32	7713	4.9299369
Freedom	113 58 43.640		4.9299372	86.49			4.9299369
Calumet	22 17 56.453	−0.701	4.5482779				4.5482776
West Depere	43 43 20.397		4.8087162	484.00	12341.81	7857	4.8087159
Clayton	55 32 42.010		4.8087162	207.36			4.8087159
Calumet	59 51 21.493	+1.625	4.8293868				4.8293865
Freedom	64 35 57.423		4.8483348	100.00	12649.17	7933	4.8483345
Oshkosh	52 17 42.181		4.8483348	265.69			4.8483345
Clayton	80 03 37.269	−2.235	4.9434969				4.9434966
Calumet	47 38 41.628		4.8186992	368.64	13283.50	8092	4.8186989
Stockbridge	59 30 17.969		4.8186992	153.76			4.8186989
Oshkosh	62 32 03.382	−0.124	4.8314207				4.8314203
Clayton	57 57 39.541		4.8115921	174.24	13611.50	8174	4.8115917
Taycheedah	40 41 22.751		4.8115921	600.25			4.8115917
Stockbridge	54 30 03.489	+0.900	4.9080618				4.9080614
Oshkosh	84 48 34.991		4.9955867	4.00	14215.75	8325	4.9955863

Chain of principal triangles between Vulcan - Huron Mountains and Fond du Lac Base—Continued.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	α^2 and β^2	$(\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Eldorado	65 02 31.857	+1.063	4.9955867	96.04			4.9955863
Taycheedah	83 28 20.807		5.0353378				5.0353374
Stockbridge	31 29 08.657		4.7560696	1183.36	15495.15	8645	4.7560692
Oakfield	37 57 08.125	−0.563	4.7560696	723.61			4.7560692
Eldorado	75 22 31.116		4.9528881				4.9528877
Taycheedah	66 40 21.866		4.9301566	81.90	805.51	8793	4.9301562
Springvale	92 53 03.965	+0.060	4.9301566	1.00			4.9301562
Oakfield	40 02 22.989		4.7391325				4.7391321
Eldorado	47 04 33.852		4.7953711	384.16	1190.67	8863	4.7953707
East Base	62 35 19.810	+0.402	4.7953711	118.81			4.7953707
Oakfield	85 39 21.682		4.8458431				4.8458427
Springvale	31 45 19.051		4.5683190	1156.00	2465.48	9097	4.5683186
West Base	107 41 35.370	+0.707	4.5683190	46.24			4.5683186
East Base	33 28 54.788		4.3310457				4.3310453
Oakfield	38 49 29.959		4.3865922	686.44	3198.16	9231	4.3865918

CHAPTER XVI.

TRIANGULATION FROM FOND DU LAC BASE TO CHICAGO BASE.

A.—DESCRIPTIONS OF STATIONS.

NOTE RELATIVE TO ELEVATIONS.

§ **1.** The heights of ground at stations described in this chapter are referred to the mean surface of Lake Michigan for the years 1860 to 1875. (See Chapter XXII, § 13.) From the line Minnesota Junction - Horicon to the line Warren - Fremont, heights were determined by trigonometrical leveling from a point of known height on Milwaukee court-house. In this group, as stated in Chapter XV, A, heights may be regarded as having a probable error of at least ± 5 feet. Heights of Chicago Base stations were determined by spirit-leveling and may be considered precise within 1 foot. Heights of Willow Springs and Morgan Park depend on zenith distances to and from the stations of Chicago Base, and cannot have a greater probable error than ± 1 foot. The same probable error may be assigned to the heights of stations Millers, Michigan City, and Bald Tom, which were determined by zenith distances over short lines to the surface of Lake Michigan.

DESCRIPTIONS OF STATIONS.

§ **2.** WOODLAND,* 1873.—This station is situated in the southwest quarter of section 33, Herman Township, Dodge County, Wisconsin. It is about 1⅛ miles east of Woodland railway station, and about 10 metres north of the middle of the road on the line between Herman and Rubicon Townships. The height of station used was 47 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is 2½ feet below the surface of the ground. A reference-stone set on the south side of the road south of the station, bears south 1° 38′ east, and is 17.07 metres distant from the geodetic point. A stone post set at the southwest corner of section 33 bears south 87° 46′ 26″ west, and is 320.59 metres distant from the geodetic point. A similar stone post at the southeast corner of the southwest quarter of section 33 bears south 88° 41′ 24″ east, and is 484.49 metres distant from the geodetic point. Height of ground at station, 607.2 feet.

LEBANON, 1873.—This station is situated in the northwest quarter of the southwest quarter of section 3, Lebanon Township, Dodge County, Wisconsin. The height of station used was 58 feet. The geodetic point is marked by a hole drilled in a granite bowlder having the letters "U. S." cut thereon, and set so that its upper surface is 3 feet below the surface of the ground. Above the bowlder is placed a limestone slab, and above the latter is set a stone of the usual form rising to the surface of the ground. A stone post set at the intersection of the west section and east-and-west half-section lines bears north 85° 05′ west, and is 318.33 metres distant from the geodetic point. Height of ground at station, 436.8 feet.

ERIN, 1873.—This station is situated on a prominent hill in the southwest quarter of the northeast quarter of section 14, Erin Township, Washington County, Wisconsin. The height of station used was 3 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is 2 feet beneath the surface of the ground. Above the latter is set a granite rock having an inch hole drilled in its upper surface and rising 4 inches above ground. A hole drilled in a large granite bowlder bears north 78° 09′ east, and is 20.83 metres distant from the geodetic point. The

* See note relative to topographical sketches of stations under Burnt Bluff, Chapter XV, A, § 2.

southwest corner of a Catholic church bears north 10° 32′ east, and is 21.7 metres distant from the geodetic point. The southeast corner of the same church bears north 22° 04′ east, and is 19.33 metres distant from the geodetic point. Height of hill, 775.8 feet.

LISBON, 1873.—This station is situated in the southwest quarter of section 24, Lisbon Township, Waukesha County, Wisconsin. The height of station used was 74 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is 2 feet 8 inches beneath the surface of the ground. Directly above the latter is set another stone rising about 4 inches above ground. A reference-stone bears south 53° 25′ west, and is 14.75 metres distant from the geodetic point. A second reference-stone bears north 41° 29′ west, and is 15 metres distant from the geodetic point. A hole drilled in a large granite bowlder, marked "U. S.", bears south 38° 39′ west, and is 14.4 metres distant from the geodetic point. Height of ground at station, 471.6 feet.

DELAFIELD, 1873.—This station is situated in section 29, Delafield Township, Waukesha County, Wisconsin, on "Government Hill." The height of station used was 4 feet. The geodetic point is marked by a stone of the ordinary form, set so that its upper end is 3 feet below the surface of the ground. Directly above the latter is set another stone rising about even with the ground surface. Nearly due south, 10.75 metres from the geodetic point, is set a reference-stone. Another reference-stone 6.62 metres distant from the geodetic point bears (approximately) south 35° east. Height of hill, 659.7 feet.

NEW BERLIN, 1873.—This station is situated in the northwest quarter of section 19, New Berlin Township, Waukesha County, Wisconsin. The height of station used was 45 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is 2½ feet below the surface of the ground. Directly above is set another stone of the usual form rising 2 inches above the ground surface. Two reference-stones are set, one north 59° 57′ east, and 118.4 metres distant; and one south 7° 11′ east, and 46.32 metres distant from the geodetic point. Height of ground at station, 491.1 feet.

MILWAUKEE COURT-HOUSE, 1873.—The geodetic point of this station is determined by the axis of the dome on the court-house building. Height of top of statue, 256 feet.

WATERFORD, 1873.—This station is situated in the east half of the northeast quarter of section 22, Waterford Township, Racine County, Wisconsin. The height of station used was 54 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is 3 feet below the surface of the ground. Two reference-stones are set on the north side of the east-and-west road south of the station. The easterly stone bears south 21° 27′ east, and is 77.65 metres distant from the geodetic point. The westerly stone bears south 31° 24′ west, and is 86.95 metres distant from the geodetic point. Height of ground at station, 366.9 feet.

CALEDONIA, 1873.—This station is situated in the southwest quarter of section 31, Caledonia Township, Racine County, Wisconsin. The height of station used was 4 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 2½ feet below the surface of the ground. Two reference-stones are set, one south 69° 11′ west, and 268.6 metres distant from the geodetic point; and one north 73° 47′ west, and 284.2 metres distant from the geodetic point. Height of ground at station, 208.8 feet.

DOVER, 1873.—This station is situated in the southwest quarter of section 6, Dover Township, Racine County, Wisconsin. The height of station used was 72 feet. The geodetic point is marked by a stone set so that its upper end is 3 feet below the surface of the ground. Two reference-stones are set in the road south of the station: one of these bears south 14° 13′ east, and is 318.58 metres distant from the geodetic point; the other bears south 35° 08′ west, and is 190.42 metres distant from the geodetic point. Height of ground at station, 278.7 feet.

SOMERS, 1873.—This station is situated in the southeast quarter of the northeast quarter of section 22, Somers Township, Kenosha County, Wisconsin. The height of station used was 30 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is 2½ feet below the surface of the ground. Three reference-stones are set on the east boundary-line of section 22. The most northerly stone bears north 13° 13′ east, and is 117.73 metres distant from the geodetic point. The most southerly stone, which is at the half-section corner, bears south 7° 56′ east, and is 189.23 metres distant from the geodetic point. The third stone bears north 76° 07′ east, and is 27.37 metres distant from the geodetic point. Height of ground at station, 182.3 feet.

BRISTOL, 1873.—This station is situated in section 17, Bristol Township, Kenosha County, Wisconsin, about 100 metres north of a farm residence owned in 1873 by Mr. O. C. Stonebreaker. The height of station used was 54 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is 3½ feet beneath the surface of the ground. Directly over the latter is set another stone rising 3 or 4 inches above the ground surface. Two reference-stones are set, one north 28° 52′ east, and 18.07 metres distant from the geodetic point; and one south 70° 48′ east, and 10.79 metres distant from the geodetic point.

BENTON, 1873.—This station is situated in the northwest quarter of the northwest quarter of section 7, Benton Township, Lake County, Illinois. The height of station used was 65 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 2½ feet below the surface of the ground. Directly above the latter is set another stone rising 3 or 4 inches above the ground. A reference-stone bears south 13° 04′ west, and is 565.9 metres distant from the geodetic point. Another reference-stone bears north 68° 59′ east, and is 19.65 metres distant from the geodetic point. Height of ground at station, 212.6 feet.

ANTIOCH, 1873.—This station is situated in section 27, Antioch Township, Lake County, Illinois. The height of station used was 54 feet. The geodetic point is marked by a stone of the ordinary form, set so that its upper end is 3 feet below the surface of the ground. Directly above the latter is set another stone rising 6 inches above the ground. Two reference-stones are set, one south 46° 13′ east, and 32.12 metres distant from the geodetic point; and one south 60° 32′ west, and 23.59 metres distant from the geodetic point. Height of ground at station, 279.1 feet.

WARREN, 1873.—This station is situated in the southeast quarter of section 24, Warren Township, Lake County, Illinois. The height of station used was 60 feet. The geodetic point is marked by a stone of the usual form, set so that its top is 2 feet 8 inches below the surface of the ground. Another stone, rising 4 to 6 inches above ground, is set directly over the latter. A reference-stone is set nearly due west, 20.49 metres distant from the geodetic point. Another reference-stone, set at the southeast corner of the southeast quarter of section 24, is slightly west of south, and is 94.04 metres distant from the geodetic point. Height of ground at station, 148.4 feet.

FREMONT, 1873.—This station is situated in section 22, Fremont Township, Lake County, Illinois, near Dean's Corners. The height of station used was 41 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is 3 feet beneath the surface of the ground. Three reference-stones are set in the road north of the station. One stone bears north 47° 02′ west, and is 33.62 metres distant; one bears north 20° 27′ west, and is 6.16 metres distant; and one bears north 36° 01′ east, and is 30.61 metres distant from the geodetic point.

DEERFIELD, 1873, '74.—This station is situated in the southwest quarter of section 28, Deerfield Township, Lake County, Illinois, about ¼ mile north of Deerfield Corners. The height of station used was 75 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is 3 feet below the surface of the ground. Another stone rising 6 inches above ground is set directly over the stone marking the geodetic point. A reference-stone bears north 13° 34′ east, and is 110.87 metres distant from the geodetic point. Another reference-stone, set on the east side of the road west of the station, bears south 57° 02′ west, and is 123.09 metres distant from the geodetic point.

PALATINE, 1873, '74.—This station is situated on the northeast corner of Sherman and Benton streets, Palatine, Illinois. The height of station used was 75 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is 3 feet below the surface of the ground. Directly above the latter is set another stone rising about 4 inches above ground. Two reference-stones are set on the east side of Benton street. The northerly stone bears north 81° 33′ west, and is 47.15 metres distant from the geodetic point. The southerly stone bears south 26° 25′ west, and is 104.08 metres distant from the geodetic point.

PARK RIDGE, 1874.—This station is situated in Meacham's Park, in the village of Park Ridge, Cook County, Illinois. The height of station used was 40 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is about 3 feet below the surface of the ground. Directly above is placed another stone, rising about even with the surface of the ground. A reference-stone, set at the southwest corner of Washington and Elm streets, bears south 44° 25′ east, and is 201.25 metres distant from the geodetic point. A second reference-stone, set at the southwest corner of Prospect avenue and Elm street, bears south 44° 48′ west, and is 214.69 metres distant from the geodetic point.

LOMBARD, 1874.—This station is situated in the northwest quarter of section 4, York Township, Du Page County, Illinois. The height of station used was 75 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is 3 feet below the surface of the ground. A reference-stone bears north 8° 52′ west, and is 117.93 metres distant from the geodetic point. Another reference-stone, set in the road east of the station, bears north 85° 11′ east, and 150.05 metres distant from the geodetic point. A third reference-stone, set at the center of section 4, bears south 13° 05′ east, and is 575.32 metres distant from the geodetic point.

SHOT TOWER, 1874, '77.—This station is the tower of the Chicago Shot Tower Company, situated on the west side of Clinton street, between Lake and Fulton streets. The height of the tower is 190 feet. The geodetic point is indicated by the intersection of two lines, each of which is determined by two screws set in opposite sides of the casing to the door opening to the top of the tower.

WILLOW SPRINGS, 1874, '77, '79.—This station is situated in the northwest quarter of section 3, Palos Township, Cook County, Illinois. It is about 1 mile southeast of Mount Forest station, on the Chicago and Alton Railway. The height of station used was 75 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is 4 feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Two reference-stones are set, one south 49° 46′ west, and 38.20 metres distant from the geodetic point, and one north 51° 53′ west, and 38.78 metres distant from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 147.3 feet.

MILITARY ACADEMY, 1874.—This station is situated in the northwest quarter of section 19, Calumet Township, Cook County, Illinois, being the dome of the Mount Vernon Military Academy. The geodetic point is marked by a screw set in the floor of the dome in the prolongation of the axis of the flagstaff.

WEST BASE, 1877.—This station, marking the west end of the Chicago base-line, is situated near the center of section 13, Lyons Township, Cook County, Illinois. The height of station used was 65 feet. The geodetic point is marked by an agate hemisphere, set in a brass cylinder, which is leaded into the top of a granite block 1 foot square and 3 feet long, set 4 feet below the surface of the ground, surrounded by a prism of brickwork 3 feet square and 3 feet deep. The brickwork is extended 8 inches above the top of the stone block, and a large, flat stone is laid on top of the brick. Two supplementary stone posts of a similar description were set without brickwork, one on the line of the base, bearing south 69° 01′ east, distant 3.982 metres from the geodetic point, and one on the prolongation of the base line, bearing north 69° 01′ west, distant 4.002 metres. Two stone reference-posts are set on the west side of the road west of the station, one bearing south 82° 53′ west, distant 53.12 metres; and one bearing north 50° 44′ west, distant 54.51 metres from the geodetic point. The quarter-section stone at the center of section 13 bears north 80° 44′ west, and is distant 37 metres. The corner of sections 7, 12, 13, and 18 bears north 43° 14′ east, and is distant 1,162 metres from the geodetic point. The height of ground at the station above Lake Michigan is 37.6 feet.

EAST BASE, 1877.—This station, marking the east end of the Chicago base-line, is situated in the northeast quarter of section 27, Lake Township, Cook County, Illinois. The height of station used was 6 feet. The geodetic point was marked in the same manner as at West Base. Two supplementary stones, similar to those at West Base, were set on a line passing through the geodetic point at right angles to the direction of the base-line, one bearing north 21° 03′ east, distant 4.48 metres, and one bearing south 21° 03′ west, distant 4.03 metres from the geodetic point. Two stone reference-posts were set under the nearest road-fences to the east and north, projecting 6 inches above the surface of the ground, as follows: one bearing south 88° 13′ east, distant 367.4 metres; and one bearing north 3° 48′ east, distant 439.4 metres from the geodetic point. The corner of sections 22, 23, 26, and 27, bears north 38° 41′ east, and is distant 580.6 metres from the geodetic point. The height of ground at the station above Lake Michigan is 35.0 feet.

MIDDLE BASE, 1877.—This station, near the middle of the Chicago base-line, is located near the end of the 924th tube of the measurement of the base from West Base. The height of station used was 6 feet. The geodetic point is marked by a brass cylinder leaded into the top of a limestone block 2 feet square and 1 foot thick, placed 3 feet below the surface of the ground. The height of ground at the station above Lake Michigan is 32.9 feet.

MORGAN PARK, 1877, '79.—This station is situated in Calumet Township, Cook County, Illinois, about 13 miles south of Chicago, and about 300 metres westerly from Morgan Park railway station on the Chicago, Rock Island and Pacific Railroad, on lot 2, section 1, of the Blue Island Land Company's subdivision, near the corner of Morgan and Armida avenues. The height of station used was 66 feet. The geodetic point is marked by a small hole drilled in a stone post of the usual form, lettered U. S. on top, set 3 feet below the surface of the ground. A similar stone post is set directly over the first as a surface-mark. Three stone reference-posts are set as follows: two on the east side of Armida avenue, one bearing south 13° 46′ west, distant 36.08 metres, and one bearing south 81° 42′ west, distant 13.38 metres, and one on the south side of Morgan avenue, bearing north 3° 11′ west, distant 58.53 metres from the geodetic point. The Military Academy bears south 41° 35′ west, and is distant 277.3 metres. The southeast corner of section 13, Worth Township, Cook County, Illinois, bears north 84° 46′ west and is distant 637.8 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 83.7 feet.

MILLERS, 1874.—This station is situated on a sand dune in section 33, township 37 north, range 7 west, Lake County, Indiana, about 2 miles northeast of Millers Station on the Lake Shore and Michigan Southern Railway. The height of station used was 73 feet. The geodetic point is marked by a stone post of the usual form, set so that its upper end is about 2 feet below the surface of the ground. Three stone reference-posts are set as follows: one north 3.77 metres, one south 3.74 metres, and one west 4.9 metres from the geodetic point, the bearings being magnetic. The height of ground at the station above Lake Michigan is 128 feet.

MICHIGAN CITY, 1874, '77.—This station is situated in section 22, township 38 north, range 4 west, Michigan Township, La Porte County, Indiana, about 2 miles northeast of Michigan City, Indiana, on the last of the range of high sand hills in that direction, about one-half mile northwesterly from the track of the Michigan Central Railroad. The height of station used was about 70 feet. The geodetic point was marked by a stone post of the usual form, set 4 feet below the surface of the ground, with a wooden post set directly over it as a surface-mark. Three stone reference-posts were set as follows: one bearing south 81° 43′ east, distant 3.36 metres; one bearing south 13° 51′ west, distant 13.19 metres; and one bearing north 53° 43′ west, distant 13.78 metres from the geodetic point. A "mile post" on the boundary-line of Michigan and Indiana, an oak post 4½ inches square and about 3 feet high, standing in a marsh, bears north 66° 46′ 17″ east, and is distant 6935.6 metres. The light-house at Michigan City bears south 62° 46′ west, and is distant 3,145 metres from the geodetic point. The height of ground at the station above Lake Michigan is 148.4 feet.

OTIS, 1874.—This station is situated in the northwest quarter of section 10, township 36 north, range 4 west, New Durham Township, La Porte County, Indiana. The height of station used was 75 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is 3 feet below the surface of the ground. Directly above the latter is set another stone, rising to within 8 inches of the ground surface. Two reference-stones are set in the road north of the station. One of these bears north 45° west, and is 49.36 metres distant from the geodetic point. The other bears north 38° 30′ east, and is 47.32 metres distant from the geodetic point. The corner of sections 3, 4, 9, and 10 bears north 9° 08′ 25″ west, and is distant 441.7 metres from the geodetic point.

SPRINGVILLE, 1874.—This station is situated in the southeast quarter of section 10, township 37 north, range 3 west, Center Township, La Porte County, Indiana. The height of station used was 74 feet. The geodetic point is marked by a stone of the usual form, set so that its upper end is 2½ feet below the surface of the ground. Two reference-stones are set in the road east of the station. One bears north 63° 43′ east, and is 273.81 metres distant from the geodetic point; the other bears south 86° 14′ east, and is 245.25 metres distant from the geodetic point. The southeast corner of section 10 bears south 86° 00′ east, and is 252.46 metres distant from the geodetic point. A third reference-stone bears south 6° 15′ east (magnetic), and is 13 metres distant from the geodetic point.

BALD TOM, 1874, '77.—This station is situated in the southwest quarter of section 35, range 20 west, township 6 south, Lake Township, Berrien County, Michigan, about three-fourths of a mile northwest of Brown's railway station, on the Chicago and Michigan Lake Shore Railroad, on the highest hill in the vicinity. A small tripod, about 4 feet high, was used to support the instrument during observations. The geodetic point was marked by a stone post of the usual form, set 3 feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Three stone reference-posts were set as follows: One bearing south 43° 34′ west, distant 19.37 metres; one bearing north 46° 34′ west, distant 7.56 metres; and one bearing north 74° 12′ east, distant 22.25 metres from the geodetic point. The southeast corner of section 35 bears south 52° 01½′ east, and is distant 1038.9 metres from the geodetic point. The height of ground at the station above Lake Michigan is 240.1 feet.

B.—HISTORICAL NOTES, RECONNAISSANCE, STATIONS, SIGNALS, INSTRUMENTS, AND METHODS OF OBSERVATION.

HISTORICAL NOTES.

§ **3.** In the development of the triangulation south of the Fond du Lac Base and east of the Chicago Base, the methods of procedure varied only in the details. In the present chapter, therefore, it is proposed to give such explanations of those methods as are applicable to the whole triangulation, and specify such differences as are applicable to the various sections thereof. By reference to Plates III, IV, V, and XXIII it will be seen that the triangulation here considered may be divided into five chains having a base-line near either extremity of each. These chains are, that lying between Fond du Lac and Chicago Bases, that between Chicago and Sandusky Bases, that between Sandusky and Buffalo Bases, that between Buffalo and Sandy Creek Bases, and that between Chicago and Olney Bases. The lengths of these chains, measured along their axes, are in order, approximately, 150, 280, 250, 210, and 200 miles, giving an aggregate length of 1,090 miles. The whole number of stations in this triangulation is 171. The whole number of lines observed over in both directions is 412. The average length of line is 15 miles, the longest line being from Station Houghton to Station Edinboro, across Lake Erie, 54 miles, and the shortest, excluding parts of bases, the Sandy Creek base-line, 3 miles.

The angles of the triangles between Fond du Lac and Chicago Bases were measured by A. R. Flint and R. S. Woodward during 1873, 1874, and 1877. The adjustments of this system were made by T. W. Wright, C. H. Kummell, and T. Russell. Fond du Lac Base was measured by E. S. Wheeler, with the Bache-Würdemann apparatus, in 1872.

Between Chicago and Sandusky Bases the angles of the triangulation were measured by G. Y. Wisner, A. R. Flint, R. S. Woodward, and J. H. Darling during 1874, 1877, 1878, and 1879. The adjustments were made by T. W. Wright, C. H. Kummell, O. B. Wheeler, T. Russell, and W. Voigt. Chicago Base was measured in 1877 and Sandusky Base in 1878 by E. S. Wheeler, with the Repsold metallic-thermometer apparatus.

The angles of the triangles joining Sandusky and Buffalo Bases were measured by G. Y. Wisner, A. R. Flint, J. H. Darling. R. S. Woodward, G. A. Marr, T. Russell, and W. A. Metcalf during 1875, 1876, and 1877. The adjustments were made by T. W. Wright and C. H. Kummell.

Between Buffalo and Sandy Creek Bases the angles of the triangles were observed by G. Y. Wisner, R. S. Woodward, J. H. Darling, T. Russell, and W. A. Metcalf in 1874, 1875, 1877, and 1878, and the adjustment was made by T. W. Wright, C. H. Kummell, T. Russell, and J. H. Darling. Sandy Creek Base was measured in 1874 and Buffalo Base in 1875 by E. S. Wheeler, with the Bache-Würdemann apparatus.

The angles of the triangulation in Illinois, connecting the Chicago and Olney Bases, were measured in 1879 by G. Y. Wisner, A. R. Flint, R. S. Woodward, and J. H. Darling. Olney Base was measured by E. S. Wheeler, with the Repsold apparatus, in 1879. The adjustments in this system were made by G. Y. Wisner, O. B. Wheeler, C. H. Kummell, and W. Voigt.

RECONNAISSANCE.

§ **4.** In the vicinity of the lakes and between Lakes Erie and Michigan the country is generally rolling or hilly, and partly covered with timber. In Illinois the surface is more nearly level and for the most part but little wooded. The whole of this country is quite accessible by roads, and maps of it, based on the original land surveys and giving relative positions of points approximately, are available. The most formidable obstacles encountered in reconnaissance, therefore, were in localities where the surface of the ground was nearly level and heavily wooded. The parties engaged in this work, consisting usually of two persons, were provided with a map of the country, a field-glass, a pocket sextant, and creepers for climbing trees. The most favorable position and requisite height for a station were oftenest determined by observations from trees. In a few cases buildings, public or private, have been used both as aids in reconnaissance and as stations in the subsequent work. The following table gives the names of parties by whom stations were located, the number located by each, and the parts of the triangulation in which they were located:

Name.	No. stations.	Part of triangulation.
Capt. W. R. Livermore	10	East end of Lake Ontario.
A. R. Flint	65	Wisconsin, Illinois, Indiana, and Michigan.
G. A. Marr	68	New York, Canada, Pennsylvania, Ohio, and Michigan.
E. S. Wheeler	26	Wisconsin, Illinois, and New York.
G. Y. Wisner	1	Michigan.
R. S. Woodward	1	Illinois.

STATIONS.

§ **5.** With few exceptions the observing-stations like those described in Chapters XIV, B, and XV, B, consisted of two concentric pyramids, the inner one for supporting the instrument, of triangular form, and the outer one supporting the observer's platform, quadrangular. Some built in 1873 had both pyramids triangular, but as they were more readily overturned by gales than those having the outer pyramid quadrangular, the latter form only was built after that year. These pyramids were usually constructed of sawed pine timber. The inner pyramid or tripod was so braced as to give a stable support, even when 100 feet or more in height, for the theodolite. When not set on the surface rock, the corner posts of the outer pyramids of nearly all stations erected after 1873 rested on the ends of four sills of heavy timber laid in a trench 4 or 5 feet deep dug in the form of a square about the station. The sills were halved together at their ends, and to each corner post was spiked a piece of strap iron, passing in the form of a loop around the sill, halved with the one on which the post rested. In this manner, on filling the trench over the sills with earth or stones, a good anchorage for the station was secured. For stations set on surface rock anchorage was secured by means of iron staples leaded into holes drilled 1½ or 2 feet into the rock, and straps passing through the staples and spiked to the posts. The outer pyramid was generally truncated and surmounted by a more obtuse pyramid, whose vertex was about 15 feet above the platform, and which terminated in a pole projecting 8 to 10 feet above the vertex. The tripods of thirty-two stations in this triangulation were 100 feet or more in height, the highest being 125 feet. The average height of tripod was 73 feet.

SIGNALS.

§ **6.** For lines not exceeding 20 miles in length the signals most commonly used were targets made of plane boards 3 to 5 feet long, and covered on the upper half with black and on the lower half with white cloth. They were in most cases nailed to the pyramid pole, care being taken to make them accurately vertical. Whenever possible, also, they were set so as to be illuminated by the sun's rays during the latter part of the day, when the atmosphere is most likely to be steady. The angular width of these signals varied considerably, as one of them frequently served for different lines at the same time, but did not as a rule exceed 5″. For lines greater than 20 miles in length beams of sunlight from heliotrope instruments or from mirrors not mounted on telescopes were used as signals. The angular breadth of mirror used in reflecting a beam varied with the transparency of the air and with the length of line over which the beam was observed. In ordinarily clear air, a surface 0''.1 in diameter was found sufficiently large on lines 40 to 50 miles long. In some cases surfaces of no greater breadth than 0''.05 have been used on lines 25 miles long. During exceptionally clear weather targets have sometimes been read to over lines 30 miles long; while on the other hand it has occasionally been necessary to use sunlight on lines 10 miles or less in length.

The positions of signals with reference to the geodetic point were carefully noted. For this purpose a theodolite was generally used to determine the points of intersection of the vertical lines through the geodetic point and through the center of the signal with a horizontal plane, on which the rectangular or polar coördinates of the signal could be measured. In all cases after the season of 1875, the positions of signals were determined at as short an interval before and as soon after they were observed as possible; and if the second determination showed a movement which would affect any angle by 0''.25, that angle was remeasured.

INSTRUMENTS.

§ **7.** The instruments used in the measurement of angles were the following for the respective systems:

Between Fond du Lac and Chicago Bases, theodolites Repsold No. 1, Gambey No. 1, and Troughton & Simms No. 2.

Between Chicago and Sandusky Bases, theodolites Repsold No. 1, Troughton & Simms Nos. 1, 2, 3, and 4, and Pistor & Martins No. 2.

Between Sandusky and Buffalo Bases, theodolites Repsold No. 1, Troughton & Simms Nos. 1, 2, 3, and 4, Pistor & Martins No. 2, and Gambey No. 1.

Between Buffalo and Sandy Creek Bases, theodolites Repsold No. 1, Troughton & Simms Nos. 1, 2, and 3, Pistor & Martins No. 2, and Gambey No. 1.

Between Chicago and Olney Bases, theodolites Troughton & Simms Nos. 1, 3, and 4, Pistor & Martins No. 2, and Repsold No. 1.

Of these theodolites, the Gambey only has been used as a repeating instrument. Troughton & Simms theodolite No. 1, Pistor & Martins No. 2, and Gambey No. 1, have been described in Chapter XIV, B, and Repsold No. 1 and Troughton & Simms No. 2 in Chapter XV, B.

The Troughton & Simms theodolites Nos. 3 and 4 are of the same form and dimensions, and are of the non-repeating class. They have horizontal circles, 14 inches in diameter, graduated to 5 minutes, and read by three equidistant microscopes, whose micrometer-heads are graduated to 1 second. Their vertical circles are 12 inches in diameter, graduated to 5 minutes, and read by two opposite microscopes whose micrometer-heads are graduated to 1 second. Their telescopes have a focal length of 30 inches and objectives $2\frac{1}{2}$ inches in diameter. The object-end of the telescope may be turned through between the wyes without raising the axis from its supports. The periodic errors affecting horizontal angles measured with these instruments have been determined from the measures of a large number of angles. For No. 3 the correction to the observed value of an angle, a, between two objects is—

$$c=+1''.87\cos\left(3z+\frac{3}{2}a+327^\circ\right)\sin\frac{3}{2}a$$

z being the reading of the finding-microscope on the left-hand object, the graduation increasing from left to right. The probable errors of the coefficient, 1''.87, and of the constant angle, 327°, resulting in their determination are $\pm 0''.02$ and $\pm 1^\circ$, respectively. For No. 4 the corresponding correction is—

$$c=+1''.59\cos\left(3z+\frac{3}{2}a+67^\circ\right)\sin\frac{3}{2}a$$
$$+1''.16\cos(6z+3a+329^\circ)\sin 3a$$
$$+0''.74\cos\left(9z+\frac{9}{2}a+323^\circ\right)\sin\frac{9}{2}a$$

z being here the reading of the finder, which for either instrument is 180° from microscope A.

The probable errors of the first, second, and third coefficients in this series are $\pm 0''.05$, $\pm 0''.08$, and $\pm 0''.16$, respectively; and the probable errors of the corresponding constant angles are $\pm 3^\circ$, $\pm 14^\circ$, and $\pm 16^\circ$. The numerical maximum of c in the last expression is—

$$c = \pm 2''.45$$

$$c = +2''.45 \text{ for } z = \text{either } 65^\circ, 185^\circ, \text{ or } 305^\circ$$
$$\text{and } a = \text{either } 71^\circ, 191^\circ, \text{ or } 311^\circ$$

$$c = -2''.45 \text{ for } z = \text{either } 16^\circ, 136^\circ, \text{ or } 256^\circ$$
$$\text{and } a = \text{either } 49^\circ, 169^\circ, \text{ or } 289^\circ$$

The independent probable errors of a single micrometer-reading on the horizontal circle and a single bisection of an object with the cross-wires of the telescope have been found to be for No. 3 $\pm 0''.22$ and $\pm 0''.34$, respectively.

The corresponding probable errors for No. 4 have been found to be $\pm 0''.27$ and $\pm 0''.36$, respectively.

The theodolites when in use were mounted on trivets or stands, which were securely attached to the station tripod by means of screws or bolts. For those instruments having no provision for shifting their horizontal circles, stands with movable parts were provided, so that the zeroes of the circles could be fixed in any desired azimuth. The position of the center of the instrument, with reference to the vertical line through the geodetic point, was determined with the same care and in the same manner as the position of a signal.

MEASUREMENT OF HORIZONTAL ANGLES.

§ **8.** The main features of the methods followed in the measurement of horizontal angles were in accordance with the requirements stated in Chapter XV, B, § 5, and need be only briefly recapitulated here. In all cases combined results, *i. e.*, means of a positive and an immediately succeeding negative measure of an angle, were obtained. With non-repeating instruments the stations situated about the horizon in the order *A*, *B*, *C*, . . . *E*, of their azimuths were usually pointed to in the order *A*, *B*, *C*, . . . *E*; *E*, . . . *C*, *B*, *A*, when the horizon was not closed. When the horizon was closed, they were pointed to in the order *A*, *B*, *C*, . . . *E*, *A* ; *A*, *E*, . . . *C*, *B*, *A*, in case they were all visible, and not so numerous as to make too long an interval between consecutive pointings to any station. Otherwise, as many consecutive stations were pointed to as could be conveniently at one time, and the remaining angles making up the horizon were measured as opportunity offered. In the later work, in order to avoid an excess of pointings to any one station, the initial station was changed from *A* to *B*, *B* to *C*, &c., whenever the circle was shifted in azimuth. With the only repeating instrument used the angles between consecutive stations were measured separately in sets of five repetitions. The mean of a set obtained by shifting the verniers in one direction and an immediately succeeding set obtained by shifting them in the opposite direction was always taken in order to eliminate as well as possible any effect of twist of station or instrument. Except in that part of the triangulation lying between the lines Minnesota Junction – Horicon and Warren – Fremont the horizons at all stations were closed. Prior to 1877 few angles other than those between consecutive stations about the horizon were observed. During 1877, 1878, and 1879, however, the work was strengthened by the measurement of many sum-angles.

In instrumental manipulation errors due to lack of collimation and inequality in heights of wyes were eliminated by frequently transiting the telescope and obtaining the same number of measures in the two positions of the telescope. To eliminate the effect of accidental and periodic errors of graduation the zero of the circle was frequently changed in azimuth, the number of such changes varying from six to twenty. Instructions given in 1872 required the elimination of periodic error by systematic changes of zero, but they were not fully complied with by all the observers before 1876. On this account the changes in position of the azimuth circle and the number of measures in those different positions were not for all observers such as to secure the best elimination of periodic errors before that year. An idea of the extent to which angles measured previous to 1876 may involve errors of the periodic class can be formed from the following statements relative to the manipulation of each instrument, and from the periodic errors of these instruments already referred to.

Repsold theodolite No. 1, 1873 *and* 1874.—Elimination of periodic error not systematic. Angles were observed on eight to sixteen different parts of circle.

Repsold theodolite No. 1, 1875.—Elimination of periodic error nearly complete except at Stony Point, N. Y., Duck Island, and Scottsville stations. Angles observed on twelve or more different parts of the circle.

Troughton & Simms theodolite No. 1, 1874 *and* 1875.—Elimination of periodic error partial. Angles observed on eight to ten different parts of circle.

Troughton & Simms theodolite No. 2, 1874.—Elimination of periodic error not generally complete. Where elimination was not complete, angles were corrected before using them in adjustment. With same instrument in 1875, elimination was systematic.

Pistor & Martins theodolite No. 2, 1875.—Elimination generally systematic; when not so, corrections were applied to angles before using them in adjustment.

Gambey repeating theodolite No. 1, 1873.—Elimination systematic, except at stations Horicon and Lebanon. Angles observed on eight to sixteen different parts of circle.

Gambey repeating theodolite No. 1, 1874 *and* 1875.—Elimination not generally systematic. Angles observed on ten or more different parts of the circle.

It is thus seen that the angles likely to be vitiated by periodic errors are those measured with the Repsold, Troughton & Simms No. 1, and the Gambey theodolites prior to 1876. With the Repsold and Troughton & Simms theodolites the maximum errors possible from this cause in an angle are $\pm 2''.25$ and $\pm 1''.12$, respectively. (See Chapter XV, B, § 4, and Chapter XIV, B, § 6.) If we suppose only such angles as might produce these maximum errors had been observed with these instruments, the most unfavorable of all suppositions, and that all errors between zero and the limiting values, $\pm 2''.25$ and $\pm 1''.12$, are equally probable (also an unfavorable supposition, since an inspection of the functions expressing the periodic errors shows that small errors are more likely to occur than large errors), the probable error of any such angle would be, if measured with the Repsold theodolite, $\pm \frac{2''.25}{2}$, or if measured with the Troughton & Simms theodolite, $\pm \frac{1''.12}{2}$. These probable errors are applicable, however, only to the measurement or mean of the measurements of the angle on one arc of the circle. In case the angle were measured the same number of times on n different but random arcs, the probable error of the mean angle would be, under the same unfavorable supposition as to the law of error, approximately $\pm \frac{2''.25}{2\sqrt{n}}$ in the first case and $\pm \frac{1''.12}{2\sqrt{n}}$ in the second. For $n=9$, which is about the minimum number of changes of the circle with the Repsold and about the average number with the Troughton & Simms theodolite, the above expressions give $\pm 0''.37$ and $\pm 0''.19$, respectively. The angles which could produce the first of these results, with the Repsold theodolite, are 84°, 96°, 264°, and 276° (Chapter XV, B, § 4); those which could produce the second result, with the Troughton & Simms theodolite, are 33°, 87°, 153°, 207°, 273°, and 327° (Chapter XIV, B, § 6). The angles actually measured with these instruments were of all magnitudes from 0° up to 360°, but the greater portion of them were near 60°. It may be concluded, therefore, that the probable errors, arising from periodic errors, of angles measured with the Repsold and Troughton & Simms No. 1 instruments in this triangulation are on the average considerably less than $\pm 0''.37$ and $\pm 0''.19$, respectively. The periodic errors of the Gambey repeating theodolite have not been determined, but judging from the discrepancies between the mean and individual results for angles so observed as to give means free from such errors, they cannot be large, and the probable error from this cause of an angle measured with this instrument may be safely assumed as not exceeding the larger of the two probable errors derived above, viz, $\pm 0''.37$. The angles measured prior to 1876 and likely to be affected by periodic errors are confined chiefly to the systems of triangles between Fond du Lac and Chicago Bases and between Buffalo and Sandy Creek Bases. About one-third of the stations between the last two bases were reoccupied, however, during 1877 and 1878. Nearly all angles that had been measured previously at these stations were remeasured, and many new angles were also observed, care being taken to make the elimination of periodic errors in the mean angles certain.

The method of eliminating periodic errors which has been carefully adhered to since 1876, is to obtain the same number of measures with the zero of the azimuth circle in each of n successive positions such that n times the angular distance between any two consecutive positions equals the angular distance between consecutive microscopes. The number n is thus determined by the total number of measures or results to be obtained. If, for example, 16 combined measures are to be made, n must be 16, 8, 4, or 2. Usually $n=8$ was used for 16 results.

Errors of run were determined by frequent comparisons of the micrometer-screws with spaces of known values on the circles, and corrections to the observed angles were applied whenever the effect of such errors was appreciable.

The number of combined results required in the measurement of angles varied with the different instruments, the aim being to have these numbers such that the mean values of all angles would have the same weight, and that the probable error of a mean angle as derived from the discrepancies between it and the individual measures should be about $\pm 0''.25$. The following table shows the number of combined results required with each instrument in each of the different years during which it was used. These numbers were derived in the manner explained in Chapter XV, B, § 5, from the probable errors of single combined results. With Troughton & Simms theodolites Nos. 3 and 4, in 1876 and the early part of 1877, while they were under examination, it was deemed best to obtain twenty-four combined results on a fully measured angle. After being thoroughly tested their performance was found equal to that of Troughton & Simms theodolite No. 1, and the number of results required was reduced to 16.

Instruments.	Number of results required for weight 1.						
	1873.	1874.	1875.	1876.	1877.	1878.	1879.
Troughton & Simms No. 1	......	16	16	16	16	16	16
Troughton & Simms No. 2	......	20	24	......	......	......	......
Troughton & Simms No. 3	......	......	......	24	24, 16	16	16
Troughton & Simms No. 4	......	......	......	24	24, 16	......	16
Pistor & Martins No. 2	......	......	16	18	......	......	16
Repsold No. 1	20	20	24	24	......	......	16
Gambey No. 1	16	20	16	......	......	......	......

During the measurement of angles the theodolite was always shielded from the sun by a tent specially designed for the purpose, or by an equivalent covering of canvas. Observations were made only when the atmosphere was steady, so as to give sharply defined images of signals. On high stations not sheltered by timber, angles were measured only when the wind was very light or calm, a condition which in some cases had to be waited for many days. The rapidity attained in observing has varied somewhat with the conditions, distinctness of signals, &c., and with the observers. Using non-repeating instruments, however, under ordinarily favorable circumstances skilled observers have made an average of one pointing per minute, such pointing including the fixing of the cross wires of the telescope on the image of a signal and readings of the two or three micrometers. With the repeating instrument used, the average time required to make a set of five repetitions was four minutes. As a rule, the days when any observations could be made did not furnish suitable conditions for a longer interval than two hours, and this occurred usually between 2 p. m. and sundown. Occasionally, cloudy days furnished an interval of steady atmosphere long enough to enable the observer to make all the requisite measures at a station, but advantage was seldom taken of such opportunities, as it has been deemed best not to have important angles depend wholly or mainly on measures made during one day.

Below are given two tables, the first showing the number of stations occupied and the number of angles measured by the different observers in different years, and the second the number of stations occupied and the number of angles measured with the different instruments in different years.

Table showing number of stations occupied and number of angles measured by different observers in different years.

Observers.	1873.		1874.		1875.		1876.		1877.		1878.		1879.	
	Number of stations.	Number of angles.	Number of stations.	Number of angles.	Number of stations.	Number of angles.	Number of stations.	Number of angles.	Number of stations.	Number of angles.	Number of stations.	Number of angles.	Number of stations.	Number of angles.
G. Y. Wisner			7	29	9	45	4	17	15	76	11	53	16	87
A. R. Flint	10	34	8	38			3	17	13	67			6	47
G. A. Marr							1	7						
E. S. Wheeler													1	3
R. S. Woodward	9	26	5	20	10	33	3	12	11	68	4	35	16	100
J. H. Darling					6	26	2	9	8	46			14	94
T. Russell			5	15	8	29								
W. A. Metcalf					6	34								

Table showing number of stations occupied and number of angles measured with the different instruments in different years.

Instruments.	1873.		1874.		1875.		1876.		1877.		1878.		1879.	
	Number of stations.	Number of angles.	Number of stations.	Number of angles.	Number of stations.	Number of angles.	Number of stations.	Number of angles.	Number of stations.	Number of angles.	Number of stations.	Number of angles.	Number of stations.	Number of angles.
Trou hton & Simms No. 1			7	29	9	45	4	17	15	76	11	53	8	50
Troughton & Simms No. 2			5	20	10	33								
Troughton & Simms No. 3							3	12	11	68	4	35	16	100
Troughton & Simms No. 4							2	9	8	46			14	94
Pistor & Martins No. 2					6	34	1	7					14	84
Repsold No. 1	10	34	8	38	6	26	3	17					1	3
Gambey No. 1	9	26	5	15	8	29								

MEASUREMENT OF ZENITH DISTANCES.

§ **9.** Previous to the year 1876 but little attention was paid to the measurement of zenith distances, although enough were observed to give the elevations of nearly all stations occupied. On nearly all lines of the triangulation measured during 1876 and subsequent years, however, reciprocal zenith distances were observed. These were not generally simultaneous, no attempt having been made to render them such. They were observed usually during the afternoon from 1 to 4 o'clock, or before the atmosphere had become sufficiently steady for the measurement of horizontal angles. Ten separate measures of the zenith distance of each station visible from the one occupied were required, and they were generally distributed over several days, so as to eliminate as much as possible any unusual effects of refraction.

C.—MEASURED AND ADJUSTED ANGLES BETWEEN THE LINES MINNESOTA JUNCTION-HORICON AND MICHIGAN CITY-BALD TOM.

§ **10.** The following tables give an abstract of the adjustment of the triangulation comprised within the above-stated limits. A sketch of this triangulation is given in Plate III.

The adjustment is made in two parts, viz: Section VI, extending from the line Minnesota Junction-Horicon to the line Warren-Fremont, and Section VII, extending from Warren-Fremont to Michigan City-Bald Tom. The scale of weights assigned to observed angles is as follows:

Weight 1 to mean of—

16 combined results by Gambey theodolite No. 1
20 combined results by Repsold theodolite No. 1
20 combined results by Troughton & Simms theodolite No. 2

Weight 1 was also assigned when the number of combined results differed by no more than one-fourth of the above numbers, but when the number of combined results differed from the standard number by more than one-fourth that number, a weight equal to the ratio of the given number to the standard number was assigned.

For a detailed explanation of the tables see Chapter XIV, C, § 7. Here, however, as in Chapter XV, C, the column headed "No. meas." gives the number of combined results, a combined result being the mean of a positive and a negative measure. The quantity designated "Range" is the difference between the greatest and least combined results.

One condition, viz., that of a sum-angle at station Michigan City was violated by the above division of the triangulation. This sum-angle formed the link connecting Section VII and Section VIII of Chapter XVII, C. The locally adjusted angles, with their resulting weights, were used in computing the general adjustments of Sections VII and VIII.

SECTION VI.—*Triangulation from the line Minnesota Junction – Horicon to the line Warren – Fremont.*

NOTE.—For angles at stations 1 and 2 see stations 51 and 52 of the preceding section of the adjustment.

LEBANON—3.

[Observer, R. S. Woodward. Instrument, Gambey 10-inch repeating theodolite. Date, May, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Minnesota Junction and Woodland	63 10 05.353	3_{1+2}	16	3.3	1	....	+0.215	63 10 05.568
Horicon and Woodland	34 04 48.946	3_2	16	3.5	1	....	−0.510	34 04 48.436
Woodland and Erin	62 54 02.743	3_3	16	4.0	1	....	−0.131	62 54 02.612
Woodland and Delafield	112 28 47.435	3_{3+4}	16	3.1	1	....	+0.255	112 28 47.690

WOODLAND—4.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, May, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Erin and Lebanon	80 57 44.418	4_1	20	3.3	1	....	+0.125	80 57 44.543
Lebanon and Horicon	111 36 04.168	4_2	20	5.9	1	....	−0.295	111 36 03.873

ERIN—5.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, May, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Milwaukee Court-House and Lisbon	15 26 29.895	5_1	11	4.5	0.5	....	+0.329	15 26 30.224
Lisbon and Delafield	54 16 34.192	5_2	20	4.8	1	....	+0.309	54 16 34.501
Delafield and Lebanon	86 50 06.367	5_3	20	4.8	1	....	+0.255	86 50 06.622
Lebanon and Woodland	36 08 13.362	5_4	20	6.0	1	....	+0.125	36 08 13.487

DELAFIELD—6.

[Observer, R. S. Woodward. Instrument, Gambey 10-inch repeating theodolite. Date, June, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Lebanon and Lisbon	84 12 49.215	6_{1+2}	16	3.6	1	....	+0.255	84 12 49.470
Erin and Lisbon	40 37 39.671	6_2	17	6.3	1	....	+0.167	40 37 39.838
Lisbon and New Berlin	58 18 36.172	6_3	16	3.4	1	....	+0.871	58 18 37.043
New Berlin and Waterford	41 54 11.647	6_4	16	4.9	1	....	+0.002	41 54 11.640

SECTION VI.—*Triangulation from the line Minnesota Junction–Horicon to the line Warren–Fremont*—Continued.

LISBON—7.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, May and June, 1873.]

Angle as measured between—		Notation.	No meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Milwaukee Court-House and New Berlin	62 14 26. 525	7_1	10	2. 7	0. 5	—0. 062	—0. 371	62 14 26. 092
New Berlin and Delafield	57 18 45. 457	7_2	20	5. 6	1. 0	—0. 031	+0. 624	57 18 46. 050
Delafield and Erin	85 05 46. 407	7_3	20	6. 0	1. 0	—0. 031	+0. 068	85 05 46. 444
Erin and Milwaukee Court-House	155 21 01. 888	7_4	4	6. 6	0. 2	—0. 153	—0. 321	155 21 01. 414

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$0.7(7_1)+0.2(7_2)+0.2(7_3)+0.0554=0$$
$$0.2(7_1)+1.2(7_2)+0.2(7_3)+0.0554=0$$
$$0.2(7_1)+0.2(7_2)+1.2(7_3)+0.0554=0$$

NEW BERLIN—8.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, June, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Caledonia and Waterford	50 25 16. 426	8_1	20	7. 3	1. 0	....	—0. 062	50 25 16. 364
Waterford and Delafield	100 47 03. 312	8_2	15	6. 4	1. 0	......	+0. 002	100 47 03. 314
Delafield and Erin	47 09 47. 945	8_3	9	4. 8	0. 5	....	+0. 511	47 09 48. 456
Delafield and Lisbon	64 22 37. 004	8_{3+4}	18	5. 2	1. 0	....	+0. 681	64 22 37. 685
Lisbon and Milwaukee Court-House	74 44 34. 565	8_5	11	7. 5	0. 5	....	—0. 045	74 44 34. 520

WATERFORD—9.

[Observer, R. S. Woodward. Instrument, Gambey 10-inch repeating theodolite. Date, June, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Delafield and New Berlin	37 18 45. 959	9_1	16	3. 6	1	....	+0. 002	37 18 45. 961
New Berlin and Caledonia	88 10 53. 829	9_2	16	2. 8	1	....	+0. 251	88 10 54. 080
New Berlin and Dover	126 53 43. 625	9_{2+3}	16	3. 2	1	....	—0. 313	126 53 43. 312

CALEDONIA—10.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, July, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Somers and Dover	70 28 13. 293	10_1	20	6. 1	1	....	+0. 855	70 28 14. 148
Dover and Waterford	49 21 10. 503	10_2	20	5. 5	1	....	—0. 313	49 21 10. 190
Waterford and New Berlin	41 23 50. 823	10_3	20	7. 1	1	....	—0. 062	41 23 50. 761

DOVER—11.

[Observer, R. S. Woodward. Instrument, Gambey 10-inch theodolite. Date, July, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Waterford and Somers	151 26 04. 950	11_{1+2}	16	4. 6	1	....	—0. 313	151 26 04. 637
Caledonia and Somers	59 30 02. 222	11_2	16	4. 1	1	....	+1. 168	59 30 03. 390
Somers and Bristol	54 44 08. 372	11_3	16	3. 1	1	....	—1. 124	54 44 07. 248

SECTION VI.—*Triangulation from the line Minnesota Junction - Horicon to the line Warren - Fremont*—Continued.

SOMERS—12.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, July, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Benton and Bristol	57 50 16.650	12_1	20	7.7	1	....	+0.052	57 50 16.702
Bristol and Dover	50 44 36.660	12_2	20	6.2	1	....	−1.124	50 44 35.536
Dover and Caledonia	50 01 42.189	12_3	18	8.5	1	....	+0.855	50 01 43.044

BRISTOL—13.

[Observer, R. S. Woodward. Instrument, Gambey 10-inch repeating theodolite. Date, July, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Dover and Somers	74 31 18.887	13_1	16	3.1	1	....	−1.124	74 31 17.763
Somers and Antioch	127 57 24.116	13_{2+3}	16	5.2	1	....	+0.052	127 57 24.168
Benton and Antioch	67 57 04.784	13_3	16	2.9	1	....	−0.390	67 57 04.394

BENTON—14.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, July, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Warren and Bristol	115 07 28.300	14_{1+2}	20	5.0	1	....	−0.118	115 07 28.182
Antioch and Somers	111 54 54.150	14_{2+3}	20	5.8	1	....	−0.220	111 54 53.930
Bristol and Somers	62 09 23.748	14_3	20	5.0	1	....	+0.272	62 09 24.020

ANTIOCH—15.

[Observer, R. S. Woodward. Instrument, Gambey 10-inch repeating theodolite. Date, July, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Bristol and Warren	115 44 30.809	15_{1+2}	16	4.3	1	....	−0.337	115 44 30.472
Benton and Warren	53 27 04.119	15_2	16	2.9	1	....	+0.220	53 27 04.339
Warren and Fremont	55 31 49.442	15_3	14	4.4	1	....	+0.644	55 31 50.086

WARREN—16.

[Observer, A. R. Flint. Instrument, Repsold 10-inch theodolite. Date, August, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Fremont and Antioch	67 11 46.258	16_2	20	5.1	1	....	+0.644	67 11 46.902
Antioch and Benton	61 10 58.008	16_3	20	6.5	1	....	−0.118	61 10 57.890

FREMONT—17.

[Observer, R. S. Woodward. Instrument, Gambey 10-inch repeating theodolite. Date, August, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angle.
	° ′ ″			″			″	° ′ ″
Antioch and Warren	57 16 22.953	17_1	16	3.1	1	....	+0.644	57 16 23.597

Numerical equations of condition in the triangulation from the line Minnesota Junction – Horicon to the line Warren – Fremont.

SIDE-EQUATIONS.

VII. (40) $-27.8529\,[6_2] - 3.3114\,[6_3] + 27.3494\,[7_2] + 29.1559\,[7_3] - 87.4835\,[8_3] + 67.9611\,[8_{3+4}] - 13.061 = 0$

VIII. (100) $+206.1077\,[5_1] - 15.1430\,[5_2] + 24.5415\,[6_2] - 12.9988\,[6_3] - 152.4494\,[7_1] - 129.8837\,[7_2]$
$-129.8837\,[7_3] + 10.0984\,[8_{3+4}] - 28.3088\,[8_5] - 30.831 = 0$

NOTE.—In the solution for determining the general corrections each of the side-equations was divided by the number inclosed in parenthesis and placed opposite it.

ANGLE-EQUATIONS.

I.	$[1_1]$	$+[2_2]$	$+[3_{1+2}]$	$-[3_2]$		$-1.154=0$
II.	$[2_1]$	$+[3_2]$	$+[4_2]$			$+1.100=0$
III.	$[3_3]$	$+[4_1]$	$+[5_4]$			$-0.119=0$
IV.	$-[3_3]$	$+[3_{3+4}]$	$+[5_3]$	$+[6_{1+2}]$	$-[6_2]$	$-0.730=0$
V.	$[5_2]$	$+[6_2]$	$+[7_3]$			$-0.543=0$
VI.	$[6_3]$	$+[7_2]$	$+[8_{3+4}]$			$-2.175=0$
IX.	$[6_4]$	$+[8_2]$	$+[9_1]$			$-0.005=0$
X.	$[8_1]$	$+[9_2]$	$+[10_3]$			$-0.128=0$
XI.	$-[9_2]$	$+[9_{2+3}]$	$+[10_2]$	$+[11_{1+2}]$	$-[11_2]$	$+2.359=0$
XII.	$[10_1]$	$+[11_2]$	$+[12_3]$			$-2.879=0$
XIII.	$[11_3]$	$+[12_2]$	$+[13_1]$			$+3.373=0$
XIV.	$[12_1]$	$+[13_{2+3}]$	$-[13_3]$	$+[14_3]$		$-0.766=0$
XV.	$[13_3]$	$-[14_3]$	$+[14_{2+3}]$	$+[15_{1+2}]$	$-[15_2]$	$+1.438=0$
XVI.	$[14_3]$	$-[14_{2+3}]$	$+[14_{1+2}]$	$+[15_2]$	$+[16_3]$	$-0.476=0$
XVII.	$[15_3]$	$+[16_2]$	$+[17_1]$			$-1.931=0$

Values of the general corrections in terms of the correlates.

$[1_1]$	$=+1.00000$ I			
$[2_1]$	$=$	$+1.00000$ II		
$[2_2]$	$=+1.00000$ I			
$[3_{1+2}]$	$=+1.00000$ I			
$[3_2]$	$=-1.00000$ I	$+1.00000$ II		
$[3_3]$	$=+1.00000$ III	-1.00000 IV		
$[3_{3+4}]$	$=$	$+1.00000$ IV		
$[4_1]$	$=+1.00000$ III			
$[4_2]$	$=+1.00000$ II			
$[5_1]$	$=+4.12216$ VIII			
$[5_2]$	$=-0.15143$ VIII	$+1.00000$ V		
$[5_3]$	$=+1.00000$ IV			
$[5_4]$	$=+1.00000$ III			
$[6_{1+2}]$	$=+1.00000$ IV			
$[6_2]$	$=-1.00000$ IV	$+1.00000$ V	-0.69632 VII	$+0.24542$ VIII
$[6_3]$	$=$	$+1.00000$ VI	-0.08279 VII	-0.12999 VIII
$[6_4]$	$=+1.00000$ IX			
$[7_1]$	$=-0.22222$ V	-0.22222 VI	-0.31392 VII	-1.79417 VIII
$[7_2]$	$=-0.11111$ V	$+0.88888$ VI	$+0.52678$ VII	-0.67142 VIII
$[7_3]$	$=+0.88888$ V	-0.11111 VI	$+0.57194$ VII	-0.67142 VIII
$[8_1]$	$=+1.00000$ X			
$[8_2]$	$=+1.00000$ IX			

Values of the general corrections in terms of the correlates—Continued.

[8_3]	=−4.37418 VII		
[8_{3+4}]	=+1.00000 VI	+1.69903 VII	+0.10098 VIII
[8_5]	=−0.56618 VIII		
[9_1]	=+1.00000 IX		
[9_2]	=+1.00000 X	−1.00000 XI	
[9_{2+3}]	=+1.00000 XI		
[10_1]	=+1.00000 XII		
[10_2]	=+1.00000 XI		
[10_3]	=+1.00000 X		
[11_{1+2}]	=+1.00000 XI		
[11_2]	=−1.00000 XI	+1.00000 XII	
[11_3]	=+1.00000 XII		
[12_1]	=+1.00000 XIV		
[12_2]	=+1.00000 XIII		
[12_3]	=+1.00000 XII		
[13_1]	=+1.00000 XIII		
[13_{2+3}]	=+1.00000 XIV		
[13_3]	=−1.00000 XIV	+1.00000 XV	
[14_{1+2}]	=+1.00000 XVI		
[14_{2+3}]	=+1.00000 XV	−1.00000 XVI	
[14_3]	=+1.00000 XIV	−1.00000 XV	+1.00000 XVI
[15_{1+2}]	=+1.00000 XV		
[15_2]	=−1.00000 XV	+1.00000 XVI	
[15_3]	=+1.00000 XVII		
[16_2]	=+1.00000 XVII		
[16_3]	=+1.00000 XVI		
[17_1]	=+1.00000 XVII		

Normal equations for determining the correlates.

No. of equation.							
1.	0 =−1.15400	+4.00000 I	−1.00000 II				
2.	0 =+1.10000	−1.00000 I	+3.00000 II				
3.	0 =−0.11900	+3.00000 III	−1.00000 IV				
4.	0 =−0.73000	−1.00000 III	+5.00000 IV	−1.00000 V	+ 0.69632 VII	− 0.24542 VIII	
5.	0 =−0.54300	−1.00000 IV	+2.88889 V	−0.11111 VI	− 0.12438 VII	− 0.57743 VIII	
6.	0 =−2.17500	−0.11111 V	+2.88889 VI	+2.14302 VII	− 0.70043 VIII		
7.	0 =−0.32653	+0.69632 IV	−0.12438 V	+2.14302 VI	+13.72220 VII	− 0.93704 VIII	
8.	0 =−0.30831	−0.24542 IV	−0.57743 V	−0.70043 VI	− 0.93704 VII	+13.24598 VIII	
9.	0 =−0.00500	+3.00000 IX					
10.	0 =−0.12800	+3.00000 X	−1.00000 XI				
11.	0 =+2.35900	−1.00000 X	+5.00000 XI	−1.00000 XII			
12.	0 =−2.87900	−1.00000 XI	+3.00000 XII				
13.	0 =+3.37300	+3.00000 XIII					
14.	0 =−0.76600	+4.00000 XIV	−2.00000 XV	+1.00000 XVI			
15.	0 =+1.43800	−2.00000 XIV	+5.00000 XV	−3.00000 XVI			
16.	0 =−0.47600	+1.00000 XIV	−3.00000 XV	+5.00000 XVI			
17.	0 =−1.93100	+3.00000 XVII					

Values of the correlates.

I	=+0.2147	X	=−0.0617
II	=−0.2951	XI	=−0.3131
III	=+0.1248	XII	=+0.8553
IV	=+0.2553	XIII	=−1.1243
V	=+0.3208	XIV	=+0.0523
VI	=+0.8713	XV	=−0.3372
VII	=−0.1169	XVI	=−0.1176
VIII	=+0.0798	XVII	=+0.6437
IX	=+0.0017		

Values of the general corrections.

	″		″		″		″
$[1_1]$	=+0.215	$[6_{1+2}]$	=+0.255	$[9_1]$	=+0.002	$[13_1]$	=−1.124
$[2_1]$	=−0.295	$[6_2]$	=+0.166	$[9_2]$	=+0.251	$[13_{2+3}]$	=+0.052
$[2_2]$	=+0.215	$[6_3]$	=+0.871	$[9_{2+3}]$	=−0.313	$[13_3]$	=−0.389
$[3_{1+2}]$	=+0.215	$[6_4]$	=+0.002	$[10_1]$	=+0.855	$[14_{1+2}]$	=−0.118
$[3_2]$	=−0.510	$[7_1]$	=−0.371	$[10_2]$	=−0.313	$[14_{2+3}]$	=−0.220
$[3_3]$	=−0.131	$[7_2]$	=+0.624	$[10_3]$	=−0.062	$[14_3]$	=+0.272
$[3_{3+4}]$	=+0.255	$[7_3]$	=+0.068	$[11_{1+2}]$	=−0.313	$[15_{1+2}]$	=−0.337
$[4_1]$	=+0.125	$[8_1]$	=−0.062	$[11_2]$	=+1.168	$[15_2]$	=+0.220
$[4_2]$	=−0.295	$[8_2]$	=+0.002	$[11_3]$	=−1.124	$[15_3]$	=+0.644
$[5_1]$	=+0.329	$[8_3]$	=+0.511	$[12_1]$	=+0.052	$[16_2]$	=+0.644
$[5_2]$	=+0.309	$[8_{3+4}]$	=+0.681	$[12_2]$	=−1.124	$[16_3]$	=−0.118
$[5_3]$	=+0.255	$[8_5]$	=−0.045	$[12_3]$	=+0.855	$[17_1]$	=+0.664
$[5_4]$	=+0.125						

SECTION VII.—*Triangulation from the line Warren–Fremont to the line Michigan City–Bald Tom*

WARREN—16.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Date, August, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angle.
	° ′ ″			″		″	″	° ′ ″
Deerfield and Fremont	65 12 13.589	16_1	20	5	1	0.000	−0.352	65 12 13.237

FREMONT—17.

[Observer, R. S. Woodward. Instrument, Gambey theodolite No. 1. Date, August, 1873.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Warren and Deerfield	70 00 09.894	17_2	16	4.9	1	0.000	−0.972	70 00 08.922
Warren and Palatine	123 34 05.391	17_{2+3}	16	2.9	1	0.000	+0.620	123 34 06.011

SECTION VII.—*Triangulation from the line Warren–Fremont to the line Michigan City–Bald Tom*—Continued.

DEERFIELD—18.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Dates, August, 1873, and July, 1874.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			″		″	″	° ′ ″
Park Ridge (new) and Palatine 74 16 19.722	18_{1+2}	20	5.8	1	−0.217	−0.868	74 16 18.637
Palatine and Park Ridge (new) 285 43 40.711	18_{-1-2}	20	6.0	1	−0.216	+0.868	285 43 41.363
Park Ridge (old) and Palatine 74 15 03.666	18_2	15	7.9	0.75	−0.025	−0.120	74 15 03.521
Park Ridge (old) and Fremont...... 130 12 54.805	18_{2+3}	5	2.7	0.25	+0.750	+0.462	130 12 56.017
Palatine and Fremont 55 57 51.939	18_3	15	7.6	0.75	−0.025	+0.582	55 57 52.496
Fremont and Warren 44 47 38.845	18_4	20	6.6	1	+0.169	−0.420	44 47 38.594
Warren and Evanston University .. 143 29 19.733	18_5	20	6.3	1	+0.169	−0.068	143 29 19.834
Evanston University and Park Ridge (old) 41 30 05.360	18_{6+1}	20	5.4	1	+0.169	+0.026	41 30 05.555

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(18_{1+2}) \qquad +0.433=0$$
$$2.00(18_2)+1.25(18_3)+(18_4)+(18_5)-0.257=0$$
$$1.25(18_2)+2.00(18_3)+(18_4)+(18_5)-0.257=0$$
$$(18_2)+(18_3)+2(18_4)+(18_5)-0.457=0$$
$$(18_2)+(18_3)+(18_4)+2(18_5)-0.457=0$$

NOTE.—18_{1+2} and 18_{-1-2} were read in 1874, the remainder in 1873.

PALATINE—19.

[Observer, R. S. Woodward. Instruments, Gambey repeating theodolite No. 1, and Troughton & Simms theodolite No. 2. Dates, September and October, 1873, and July and August, 1874.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			″		″	″	° ′ ″
Fremont and Deerfield............... 70 28 10.622	19_1	39	7.6	2	−0.047	+0.569	70 28 11.144
Deerfield and Park Ridge (new) 53 19 05.178	19_2	21	8.8	1	−0.094	−1.067	53 19 04.017
Park Ridge (new) and Lombard 47 34 27.447	19_{3+4}	20	8.3	1	−0.094	−0.019	47 34 27.334
Lombard and Fremont 188 38 17.081	19_5	20	4.1	1	−0.093	+0.517	188 38 17.505

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3(19_1)+(19_2)+(19_{3+4})+0.328=0$$
$$(19_1)+2(19_2)+(19_{3+4})+0.328=0$$
$$(19_1)+(19_2)+2(19_{3+4})+0.328=0$$

NOTE.—19_1 was partly measured with the Gambey instrument in 1873; the remainder of the angles were read with the Troughton & Simms instrument in 1874.

PARK RIDGE (new)—20.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Dates, July and August, 1874.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			″		″	″	° ′ ″
Shot Tower and Lombard........... 93 50 42.697	20_1	20	6.7	1	+0.082	+0.921	93 50 43.700
Lombard and Palatine 75 50 49.080	20_2	10	5.8	0.5	−0.695	+0.530	75 50 48.915
Lombard and Deerfield 128 15 27.356	20_{2+3}	10	5.5	0.5	+0.859	−1.222	128 15 26.993
Palatine and Deerfield 52 24 40.525	20_3	10	11.8	0.5	−0.695	−1.752	52 24 38.078
Deerfield and Evanston University . 77 24 56.159	20_4	17	6.7	1	+0.082	+0.170	77 24 56.411
Evanston University and Shot Tower 60 28 52.682	20_5	18	6.9	1	+0.083	+0.131	60 28 52.896

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(20_1)+(20_2)+(20_3)+(20_4)+1.143=0$$
$$(20_1)+2.0(20_2)+1.5(20_3)+(20_4)+2.267=0$$
$$(20_1)+1.5(20_2)+2.0(20_3)+(20_4)+2.267=0$$
$$(20_1)+(20_2)+(20_3)+2(20_4)+1.143=0$$

SECTION VII.—*Triangulation from the line Warren–Fremont to the line Michigan City–Bald Tom*—Continued.

LOMBARD—21.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 2. Date, August, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Palatine and Park Ridge (new)	56 34 44.857	21_1	20	8.1	1	+0.492	−0.648	56 34 44.701
Park Ridge (new) and Shot Tower	46 37 58.092	21_2	25	11.9	1.25	0.000	−0.140	46 37 57.952
Park Ridge (new) and Willow Springs	102 39 56.945	21_{2+3}	20	7.2	1	+0.492	+0.849	102 39 58.286
Willow Springs and Palatine	200 45 16.722	21_4	20	8.0	1	+0.492	−0.201	200 45 17.013

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(21_1)+(21_{2+3})-1.476=0$$

$$(21_1)+2(21_{2+3})-1.476=0$$

SHOT TOWER—22.

[Observer, A. R. Flint. Instrument, Repsold theodolite, No. 1. Dates, August and September, 1874, and September, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Michigan City and Millers	27 20 26.443	22_1	16	5.5	1	+0.337	−0.547	27 20 26.233
Millers and Willow Springs	92 50 52.692	$22_{2+3+4+5+6}$	9	6.7	0.5	+1.688	−0.047	92 50 54.333
Morgan Park and East Base	19 46 29.144	22_{3+4}	20	8.0	1	−0.036	−0 216	19 46 28.892
West Base and Morgan Park	315 48 37.587	$22_{-3-4-5-6-7}$	20	5.7	1	−0.034	−0.149	315 48 37.404
Military Acad. and Willow Springs	37 14 29.470	22_{4+5+6}	10	4.0	0.5	−0.030	−0.758	37 14 28.682
East Base and Middle Base	13 16 16.074	22_5	21	6.9	1	−0.036	−0.592	13 16 15.446
Middle Base and Willow Springs ...	4 36 20.391	22_6	19	6.0	1	−0.036	+0.181	4 36 20.536
Willow Springs and West Base	6 32 17.031	22_7	20	4.8	1	−0.085	+0.776	6 32 17.722
Willow Springs and Lombard	48 55 01.519	22_{7+8}	28	7.7	1.5	−0.072	+0.451	48 55 01.898
Willow Springs and Park Ridge (new)	88 26 19 537	22_{7+8+9}	10	4.9	0.5	+1.054	+0.644	88 26 21.235
West Base and Lombard	42 22 44.601	22_8	8	10.7	0.5	−0.100	−0.325	42 22 44.176
Lombard and Park Ridge (new)	39 31 19.301	22_9	20	6.0	1	−0.157	+0.193	39 31 19.337
Park Ridge (new) and Water Works.	97 12 21.930	22_{10+11}	20	9.2	1	+0.371	−0.268	97 12 22.033
Evanston University and Water Works	59 06 46.639	22_{11}	19	9.0	1	0.000	+0.079	59 06 46.718
Water Works and Michigan City ...	54 09 55.612	22_{12}	20	7.5	1	+0.336	+0.218	54 09 56.166
Water Works and Millers	81 30 22.322	22_{12+1}	25	6.5	1.25	+0.406	−0.329	81 30 22.399
Water Works and Military Academy	137 06 47.682	$22_{12+1+2+3}$	20	7.7	1	−0.014	+0.382	137 06 48.050
Water Works and Willow Springs ..	174 21 17.566	$22_{12+1+2+3+4+5+6}$	24	5.9	1	−0.458	−0.376	174 21 16.732

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(22_1)+(22_{2+3+4+5+6})+(22_7)+(22_8)+(22_9)+(22_{10+11})-2.390=0$$

$$(22_1)+2.75(22_{2+3+4+5+6})+2.25(22_7)+2.25(22_8)+2.25(22_9)+2.25(22_{10+11})-5.044=0$$

$$+2(22_{3+4})+(22_5)+(22_6)+(22_7)+0.227=0$$

$$+1.5(22_{4+5+6})+(22_7)+(22_8)+(22_9)+(22_{10+11})+0.015=0$$

$$+(22_{3+4})+2(22_5)+(22_6)+(22_7)+0.227=0$$

$$+(22_{3+4})+(22_5)+2(22_6)+(22_7)+0.227=0$$

$$(22_1)+2.25(22_{2+3+4+5+6})+(22_{3+4})+(22_{4+5+6})+(22_5)+(22_6)+8.25(22_7)+6.25(22_8)+4.75(22_9)+4.25(22_{10+11})-3.505=0$$

$$(22_1)+2.25(22_{2+3+4+5+6})+(22_{4+5+6})+6.25(22_7)+6.75(22_8)+4.75(22_9)+4.25(22_{10+11})-3.732=0$$

$$(22_1)+2.25(22_{2+3+4+5+6})+(22_{4+5+6})+4.75(22_7)+4.75(22_8)+5.75(22_9)+4.25(22_{10+11})-3.902=0$$

$$(22_1)+2.25(22_{2+3+4+5+6})+(22_{4+5+6})+4.25(22_7)+4.25(22_8)+4.25(22_9)+5.25(22_{10+11})-4.600=0$$

NOTE.—22_{3+4}, 22_5, 22_6, 22_7, $22_{-3-4-5-6-7}$, 22_8, and part of 22_{7+8} were measured in 1877; the remainder in 1874.

SECTION VII.—*Triangulation from the line Warren–Fremont to the line Michigan City–Bald Tom*—Continued.

WILLOW SPRINGS—23.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Dates, August, September, and October, 1874, and August, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	$[v]$	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Lombard and Park Ridge (new)	34 04 25.366	23_1	8	6.9	0.5	+0.415	+0.439	34 04 26.220
Lombard and West Base............	58 26 51.887	23_{1+2}	24	6.3	1	−0.019	+0.394	58 26 52.262
Lombard and Military Academy....	136 22 54.882	$23_{1+2+3+4+5+6+7}$	12	4.0	0.5	−0.149	+0.228	136 22 54.961
Park Ridge (new) and Military Acad.	102 18 28.538	$23_{2+3+4+5+6+7}$	8	4.7	0.5	+0.414	−0.211	102 18 28.741
West Base and Shot Tower	16 36 06.564	23_3	20	4.3	1	−0.124	+0.445	16 36 06.885
West Base and Middle Base	25 36 29.405	23_{3+4}	12	5.1	0.5	+0.211	−0.010	25 36 29.606
Shot Tower and Middle Base	9 00 23.250	23_4	20	5.3	1	−0.074	−0.455	9 00 22.721
Shot Tower and Military Academy.	61 19 57.750	$23_{4+5+6+7}$	6	6.2	0.25	−1.325	−0.611	61 19 55.814
Shot Tower and Azimuth Mark.....	70 32 47.323	$23_{4+5+6+7+8}$	20	5.5	1	+0.352	−0.362	70 32 47.313
Azimuth Mark and Shot Tower.....	289 27 12.253	$23_{-4-5-6-7-8}$	20	6.4	1	+0.072	+0.362	289 27 12.687
Middle Base and East Base	16 58 34.415	23_5	24	6.4	1	+0.031	+0.518	16 58 34.964
East Base and Morgan Park........	34 24 49.432	23_6	24	10.2	1	+0.032	−0.668	34 24 48.796
Morgan Park and Military Academy	0 56 09.308	23_7	24	4.7	1	+0.031	−0.006	0 56 09.333
Military Acad. and Azimuth Mark..	9 12 51.416	23_8	22	4.7	1	−0.166	+0.249	9 12 51.497
Azimuth Mark and Lombard	214 24 13.902	23_9	20	6.0	1	+0.115	−0.477	214 24 13.540

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$(23_1)-0.5(23_{1+2})-0.5(23_3)-0.50(23_4)-0.50(23_5)-0.50(23_6)-0.50(23_7) \qquad -0.476=0$$
$$-0.5(23_1)+3.0(23_{1+2})+2.0(23_3)+2.00(23_4)+2.00(23_5)+2.00(23_6)+2.00(23_7)+(23_8)+0.637=0$$
$$-0.5(23_1)+2.0(23_{1+2})+3.5(23_3)+2.50(23_4)+2.00(23_5)+2.00(23_6)+2.00(23_7)+(23_8)+0.841=0$$
$$-0.5(23_1)+2.0(23_{1+2})+2.5(23_3)+5.75(23_4)+4.25(23_5)+4.25(23_6)+4.25(23_7)+3(23_8)+1.077=0$$
$$-0.5(23_1)+2.0(23_{1+2})+2.0(23_3)+4.25(23_4)+5.25(23_5)+4.25(23_6)+4.25(23_7)+3(23_8)+0.873=0$$
$$-0.5(23_1)+2.0(23_{1+2})+2.0(23_3)+4.25(23_4)+4.25(23_5)+5.25(23_6)+4.25(23_7)+3(23_8)+0.873=0$$
$$-0.5(23_1)+2.0(23_{1+2})+2.0(23_3)+4.25(23_4)+4.25(23_5)+4.25(23_6)+5.25(23_7)+3(23_8)+0.873=0$$
$$+(23_{1+2})+(23_3)+3.00(23_4)+3.00(23_5)+3.00(23_6)+3.00(23_7)+4(23_8)+0.746=0$$

NOTE.—23_{1+2}, 23_3, 23_4, 23_{3+4}, 23_6, 23_6, 23_7, and $23_{4+5+6+7}$ were measured in 1877; the remainder in 1874.

MILITARY ACADEMY—24.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Date, October, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	$[v]$	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Willow Springs and Shot Tower....	81 25 36.653	24_1	20	7.2	1	+0.420	−0.725	81 25 36.348
Willow Springs and Millers	176 37 34.549	24_{1+2}	12	4.5	0.5	−0.310	−0.096	176 37 34.143
Shot Tower and Millers	95 11 56.327	24_2	11	5.3	0.5	+0.839	+0.629	95 11 57.795
Millers and Willow Springs	183 22 25.497	24_3	21	5.0	1	+0.264	+0.096	183 22 25.857

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.5(24_1)+1.5(24_2)-2.307=0$$
$$1.5(24_1)+2.0(24_2)-2.307=0$$

SECTION VII.—*Triangulation from the line Warren-Fremont to the line Michigan City-Bald Tom*—Continued.

WEST BASE—25.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Dates, June and July, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Shot Tower and East Base..........	59 55 44.873	25_{1+2}	24	8.1	1	−0.160	+0.219	59 55 44.932
Middle Base and East Base..........	00 00 00.815	25_2	24	3.0	1	+0.161	+0.290	00 00 01.266
Middle Base and Morgan Park.......	21 01 39.159	25_{2+3}	24	7.2	1	−0.160	+0.636	21 01 39.635
Morgan Park and Military Academy	1 02 00.014	25_4	24	5.5	1	−0.161	−1.102	1 01 58.751
Military Academy and Willow Springs..............................	74 52 13.298	25_5	24	6.6	1	−0.160	+0.333	74 52 13.471
Willow Springs and Shot Tower....	203 08 24.434	25_6	24	6.7	1	−0.161	−0.204	203 08 24.069

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$+2(25_{1+2})-(25_2)+(25_{2+3})+(25_4)+(25_5)+0.963=0$$
$$-(25_{1+2})+2(25_2)-(25_{2+3})-(25_4)-(25_5)-0.963=0$$
$$+(25_{1+2})-(25_2)+2(25_{2+3})+(25_4)+(25_5)+0.963=0$$
$$+(25_{1+2})-(25_2)+(25_{2+3})+2(25_4)+(25_5)+0.963=0$$
$$+(25_{1+2})-(25_2)+(25_{2+3})+(25_4)+2(25_5)+0.963=0$$

EAST BASE—26.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Date, July, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Willow Springs and West Base.....	40 29 05.139	26_1	24	7.7	1	+0.064	−0.227	40 29 04.976
West Base and Middle Base........	00 00 01.185	26_2	24	2.4	1	+0.065	−0.023	00 00 01.227
Middle Base and Shot Tower	95 39 20.289	26_3	24	5.6	1	+0.064	+0.082	95 39 20.435
Shot Tower and Morgan Park.......	123 09 05.901	26_4	24	5.4	1	+0.065	+0.540	123 09 06.506
Morgan Park and Military Academy	1 41 46.055	26_5	24	5.1	1	+0.064	+0.158	1 41 46.277
Military Academy and Willow Springs............................	99 00 41.045	26_6	24	6.2	1	+0.064	−0.530	99 00 40.579

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(26_1)+(26_2)+(26_3)+(26_4)+(26_5)-0.386=0$$
$$(26_1)+2(26_2)+(26_3)+(26_4)+(26_5)-0.386=0$$
$$(26_1)+(26_2)+2(26_3)+(26_4)+(26_5)-0.386=0$$
$$(26_1)+(26_2)+(26_3)+2(26_4)+(26_5)-0.386=0$$
$$(26_1)+(26_2)+(26_3)+(26_4)+2(26_5)-0.386=0$$

MIDDLE BASE—27.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Date, July, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Willow Springs and West Base.....	57 27 38.801	27_1	24	6.4	1	−0.091	−0.106	57 27 38.604
West Base and Shot Tower.	108 55 38.692	27_2	24	6.5	1	−0.091	−0.378	108 55 38.223
Shot Tower and East Base...........	71 04 24.575	27_3	25	7.2	1.25	−0.073	−0.230	71 04 24.272
East Base and Morgan Park........	27 20 56.669	27_4	22	8.8	1	−0.091	+0.283	27 20 56.861
Morgan Park and Willow Springs..	95 11 21.700	27_5	21	8.3	1	−0.091	+0.431	95 11 22.040

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(27_1)+(27_2)+(27_3)+(27_4)+0.437=0$$
$$(27_1)+2(27_2)+(27_3)+(27_4)+0.437=0$$
$$(27_1)+(27_2)+2.25(27_3)+(27_4)+0.437=0$$
$$(27_1)+(27_2)+(27_3)+2(27_4)+0.437=0$$

SECTION VII.—*Triangulation from the line Warren–Fremont to the line Michigan City–Bald Tom*—Continued.

MORGAN PARK—28.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Date, August, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Willow Springs and West Base.....	27 05 54.324	28_1	24	6.2	1	+0.040	+0.321	27 05 54.685
West Base and East Base...........	17 46 51.014	28_2	24	7.5	1	+0.041	−1.150	17 46 49.905
East Base and Shot Tower..... ...	37 04 24.114	28_3	24	9.4	1	+0.040	+0.742	37 04 24.896
Shot Tower and Willow Springs....	278 02 50.386	28_4	24	7.4	1	+0.041	+0.087	278 02 50.514

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(28_1)+(28_2)+(28_3)-0.162=0$$
$$(28_1)+2(28_2)+(28_3)-0.162=0$$
$$(28_1)+(28_2)+2(28_3)-0.162=0$$

MILLERS—29.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 2. Date, August and September, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Military Academy and Shot Tower	29 11 38.326	29_1	21	10.0	1	−0.006	+0.317	29 11 38.637
Shot Tower and Michigan City.....	114 11 26.524	29_2	18	12.3	1	−0.006	−0.618	114 11 25.900
Michigan City and Otis....... ...	29 17 11.870	29_3	20	10.4	1	−0.006	+0.154	29 17 12.018
Otis and Military Academy.	187 19 43.303	29_4	20	6.8	1	−0.005	+0.147	187 19 43.445

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(29_1)+(29_2)+(29_3)+0.023=0$$
$$(29_1)+2(29_2)+(29_3)+0.023=0$$
$$(29_1)+(29_2)+2(29_3)+0.023=0$$

MICHIGAN CITY—30.

[Observers, R. S. Woodward and A. R. Flint. Instruments, Troughton and Simms theodolite No. 2, and Repsold theodolite No. 1. Dates, October and November, 1874, and May, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Bald Tom and Galena...............	54 47 01.324	30_1	16	5.2	1	+0.496	....	....
Bald Tom and Springville	75 43 51.528	30_{1+2}	20	5.7	1	+0.003	+0.117	75 43 51.648
Galena and Millers	141 35 29.122	30_{2+3+4}	16	4.6	1	+0.496	....	....
Springville and Otis	55 23 13.333	30_3	20	8.9	1	+0.285	−0.181	55 23 13.437
Otis and Millers	65 15 26.004	30_4	20	10.9	1	+0.285	+0.172	65 15 26.461
Millers and Shot Tower	38 28 11.265	30_5	28	6.1	1.5	+0.333	−0.260	38 28 11.338
Millers and Bald Tom (W.)	161 01 12.248	30_{5+6}	20	8.7	1	+0.283	+0.004	161 01 12.535
Shot Tower and Bald Tom..........	125 09 16.465	30_{6+7}	18	5.9	1	+0.499	+0.152	125 09 17.116
Bald Tom (W.) and Springville.....	78 20 07.278	30_{7+1+2}	20	12.9	1	+0.284	+0.005	78 20 07.567

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(30_1)-(30_{1+2})-(30_3)-(30_4)\qquad -0.419=0$$
$$-(30_1)+3(30_{1+2})+2(30_3)+2(30_4)+(30_5)\qquad -0.986=0$$
$$-(30_1)+2(30_{1+2})+4(30_3)+3(30_4)+(30_5)+(30_{5+6})-2.123=0$$
$$-(30_1)+2(30_{1+2})+3(30_3)+4(30_4)+(30_5)+(30_{5+6})-2.123=0$$
$$(30_{1+2})+(30_3)+(30_4)+2.5(30_5)\qquad -1.405=0$$
$$(30_3)+(30_4)\qquad +2(30_{5+6})-1.137=0$$

NOTE.—30_{1+2} was measured in 1874; 30_1, 30_{2+3+4}, 30_5, and 30_{6+7}, were measured in 1877, by A. R. Flint, with the Repsold instrument. The remainder were read in 1874, by R. S. Woodward, with the Troughton & Simms instrument.

SECTION VII.—*Triangulation from the line Warren-Fremont to the line Michigan City-Bald Tom*—Continued.

OTIS—31.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Dates, October and November, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Millers and Michigan City....... ...	85 27 22.727	31_1	20	6.7	1	+0.042	+0.004	85 27 22.773
Michigan City and Springville.......	51 45 07.262	31_2	21	5.4	1	0.000	−0.524	51 45 06.738
Michigan City and Millers	274 32 37.189	31_{2+3}	20	6.1	1	+0.042	−0.004	274 32 37.227

SPRINGVILLE—32.

[Observer, R. S. Woodward. Instrument, Troughton and Simms theodolite No. 2. Date, November, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Otis and Michigan City.............	72 51 40.470	32_1	20	12.7	1	+0.053	−0.242	72 51 40.281
Michigan City and Bald Tom.......	77 50 41.254	32_2	22	14.1	1	0.000	−0.067	77 50 41.187
Michigan City and Bald Tom (W.)..	77 59 02.240	32_{2+3}	10	8.6	0.5	+0.150	+0.080	77 59 02.470
Michigan City and Otis......	287 08 19.500	32_{2+3+4}	16	13.2	1	−0.023	+0.242	287 08 19.719
Bald Tom (W.) and Otis............	209 09 16.787	32_4	4	1.8	0.25	+0.300	+0.162	209 09 17.249

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.25(32_1)+0.25(32_{2+3})-0.156=0$$
$$0.25(32_1)+0.75(32_{2+3})-0.126=0$$

BALD TOM—33.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Date, November, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angle.
	° ′ ″			″		″	″	° ′ ″
Springville and Michigan City.......	26 25 28.206	33_4	20	5.0	1	0.000	−0.067	26 25 28.139

Numerical equations of condition in the triangulation from the line Warren-Fremont to the line Michigan City-Bald Tom.

SIDE-EQUATIONS.

XXIII. (20)	$-41.3655\,[18_{1+2}]$	$-35.4361\,[18_3]$	$-35.4361\,[18_4]$	$-35.4361\,[18_5]$	$+15.6837\,[19_2]$	
	$-19.2434\,[19_{3+4}]$	$+3.1714\,[20_1]$	$+3.1714\,[20_2]$	$+3.1714\,[20_3]$	$+14.7958\,[20_4]$	
	$+13.8943\,[21_1]$	$-19.8881\,[21_2]$	$+25.5220\,[22_9]$	$-30.0339\,[22_{10+11}]$	$+30.0339\,[22_{11}]$	$-26.859=0$
XXV. (20)	$-19.8881\,[21_2]$	$-4.7318\,[21_{2+3}]$	$-0.5740\,[22_7]$	$-0.5740\,[22_8]$	$+24.9479\,[22_{10}]$	
	$-55.3710\,[23_1]$	$+24.2420\,[23_{1+2}]$	$+24.2420\,[23_3]$			$+0.677=0$
XXIX. (7)	$+5.6306\,[22_{3+4}]$	$+5.6306\,[22_5]$	$+5.6306\,[22_6]$	$-21.6594\,[22_7]$	$+4.8617\,[23_3]$	
	$-7.1010\,[23_4]$	$-7.1010\,[23_5]$	$-7.1010\,[23_6]$	$+3.3512\,[25_{1+2}]$	$-3.3512\,[25_2]$	
	$+3.3512\,[25_{2+3}]$	$-5.2873\,[25_4]$	$-5.2873\,[25_5]$			$+7.922=0$
XXXIII. (20)	$-31.2400\,[23_4]$	$-31.2409\,[23_5]$	$+11.9527\,[23_6]$	$-21.9108\,[26_1]$	$-21.9108\,[26_2]$	
	$-21.9108\,[26_3]$	$+13.7527\,[26_4]$	$-2.9768\,[28_1]$	$-2.9768\,[28_2]$	$+24.8899\,[28_3]$	$-22.093=0$
XXXIV. (8)	$-8.8397\,[25_{1+2}]$	$-3.3512\,[25_2]$	$+3.3512\,[25_{2+3}]$	$-2.0853\,[26_2]$	$-2.0853\,[26_3]$	
	$+13.7528\,[26_4]$	$-14.8230\,[28_2]$	$+13.0438\,[28_3]$			$-33.266=0$
XXXV. (25)	$-4.5007\,[23_3]$	$-4.5007\,[23_4]$	$-4.5007\,[23_5]$	$+25.1821\,[23_6]$	$+25.1821\,[23_7]$	
	$+51.9554\,[25_2]$	$-51.9554\,[25_{2+3}]$	$-51.9554\,[25_4]$	$+5.6928\,[25_5]$	$-3.3391\,[26_1]$	
	$+21.3101\,[26_2]$	$+21.3101\,[26_3]$	$+21.3101\,[26_4]$	$+21.3101\,[26_5]$		$-38.707=0$

Numerical equations of condition, &c.—Continued.

SIDE-EQUATIONS—Continued.

XXXIX. (20)	$-29.3470\,[25_{2+3}]$	$+\ 7.8469\,[25_4]$	$+\ 7.8469\,[25_5]$	$+11.5277\,[27_1]$	$+29.9943\,[27_2]$	
	$+29.9943\,[27_3]$	$+29.9943\,[27_4]$	$-\ 9.2415\,[28_1]$			$+38.697=0$
XL. (10)	$-\ 8.8399\,[25_{1+2}]$	$+\ 8.8399\,[25_2]$	$+\ 3.3512\,[25_3]$	$+18.6073\,[26_3]$	$+18.6073\,[26_4]$	
	$-\ 7.2200\,[27_2]$	$+\ 3.1176\,[27_3]$	$-15.4897\,[27_4]$	$-14.8230\,[28_2]$	$+\ 3.7813\,[28_3]$	$-31.816=0$
XLI. (20)	$+31.9066\,[25_{2+3}]$	$+20.6841\,[26_1]$	$+20.6841\,[26_2]$	$-\ 3.9813\,[26_3]$	$-\ 3.9813\,[26_4]$	
	$-11.5211\,[27_1]$	$-43.4277\,[27_2]$	$-43.4277\,[27_3]$	$-29.9943\,[27_4]$	$-10.7626\,[28_1]$	
	$+21.1440\,[28_2]$					$-$ [illegible]$.052=0$
XLIII. (20)	$-32.2095\,[22_{4+5+6}]$	$+59.9069\,[22_5]$	$+59.9069\,[22_6]$	$-11.5119\,[23_4]$	$-11.5119\,[23_5]$	
	$+18.1708\,[23_6]$	$+18.1708\,[23_7]$	$-\ 3.3391\,[26_1]$	$-\ 3.3391\,[26_2]$	$-\ 3.3391\,[26_3]$	
	$-17.9988\,[26_4]$	$-17.9988\,[26_5]$				$+25.163=0$

NOTE.—In the solution for determining the general corrections, each of the side-equations was divided by the number inclosed in parenthesis and placed opposite it.

ANGLE-EQUATIONS.

XVIII.	$+[16_1]+[17_2]+[18_4]$	$+1.745=0$
XIX.	$-[17_2]+[17_{2+3}]+[18_3]+[19_1]$	$-2.743=0$
XX.	$+[18_{1+2}]+[19_2]+[20_3]$	$+3.687=0$
XXI.	$+[19_{3+4}]+[20_2]+[21_1]$	$+0.137=0$
XXII.	$+[20_1]+[21_2]+[22_9]$	$-0.973=0$
XXIV.	$-[21_2]+[21_{2+3}]+[22_7]+[22_8]+[23_{1+2}]+[23_3]$	$-2.279=0$
XXVI.	$+[22_{3+4}]+[22_5]+[22_6]+[23_4]+[23_5]+[23_6]+[28_1]+[28_2]+[28_3]$	$+1.318=0$
XXVII.	$+[22_{3+4}]+[22_5]+[22_6]+[22_7]+[25_{1+2}]-[25_2]+[25_{2+3}]+[28_2]+[28_3]$	$-0.307=0$
XXVIII.	$+[23_3]+[23_4]+[23_5]+[23_6]+[25_4]+[25_5]+[28_1]$	$+0.608=0$
XXX.	$+[22_5]+[22_6]+[22_7]+[25_{1+2}]+[26_2]+[26_3]$	$-0.643=0$
XXXI.	$+[22_5]+[22_6]+[23_4]+[23_5]+[26_1]+[26_2]+[26_3]$	$+0.516=0$
XXXII.	$+[23_6]-[26_1]-[26_2]-[26_3]-[26_4]+[28_1]+[28_2]$	$+1.868=0$
XXXVI.	$+[22_6]+[22_7]+[25_{1+2}]-[25_2]+[27_2]$	$-0.508=0$
XXXVII.	$+[22_5]+[26_3]+[27_3]$	$+0.741=0$
XXXVIII.	$+[23_3]+[23_4]+[25_{2+3}]+[25_4]+[25_5]+[27_1]$	$+0.249=0$
XLII.	$+[22_{4+5+6}]+[23_4]+[23_5]+[23_6]+[23_7]+[24_1]$	$+2.095=0$
XLIV.	$-[22_{4+5+6}]+[22_{2+3+4+5+6}]+[24_2]+[29_1]$	$-1.657=0$
XLV.	$+[22_1]+[29_2]+[30_5]$	$+1.426=0$
XLVI.	$+[29_3]+[30_4]+[31_1]$	$-0.330=0$
XLVII.	$+[30_3]+[31_2]+[32_1]$	$+0.946=0$
XLVIII.	$+[30_{1+2}]+[32_2]+[33_4]$	$+0.018=0$

General corrections in terms of the correlates.

$[16_1]$	$=+1.00000$ XVIII			
$[17_2]$	$=+1.00000$ XVIII	-1.00000 XIX		
$[17_{2+3}]$	$=+1.00000$ XIX			
$[18_{1+2}]$	$=+0.50000$ XX	-1.03414 XXIII		
$[18_2]$	$=-0.17391$ XVIII	-0.40580 XIX	$+1.33526$ XXIII	
$[18_3]$	$=-0.17391$ XVIII	$+0.92753$ XIX	$+1.02714$ XXIII	
$[18_{3+4+5}]$	$=+0.39131$ XVIII	$+0.57971$ XIX	-2.41382 XXIII	
$[18_4]$	$=+0.78261$ XVIII	-0.17391 XIX	-0.69334 XXIII	
$[18_5]$	$=-0.21739$ XVIII	-0.17391 XIX	-0.69334 XXIII	
$[19_1]$	$=+0.42857$ XIX	-0.14286 XX	-0.14286 XXI	$+0.02542$ XXIII
$[19_2]$	$=-0.14286$ XIX	$+0.71429$ XX	-0.28571 XXI	$+0.33501$ XXIII

General corrections in terms of the correlates—Continued.

$[19_{3+4}]$	=−0.14286 XIX	−0.28571 XX	+0.71429 XXI	−0.91132 XXIII	
$[20_1]$	=−0.15385 XX	−0.15385 XXI	+0.76923 XXII	−0.09745 XXIII	
$[20_{1+2+3}]$	=+0.30769 XX	+0.30769 XXI	+0.46153 XXII	−0.22739 XXIII	
$[20_2]$	=−0.76923 XX	+1.23077 XXI	−0.15385 XXII	−0.06497 XXIII	
$[20_3]$	=+1.23077 XX	−0.76923 XXI	−0.15385 XXII	−0.06497 XXIII	
$[20_4]$	=−0.15385 XX	−0.15385 XXI	−0.23077 XXII	+0.48360 XXIII	
$[21_1]$	=+0.66667 XXI	+0.46315 XXIII	−0.33333 XXIV	+0.07886 XXV	
$[21_2]$	=+0.80000 XXII	−0.79552 XXIII	−0.80000 XXIV	−0.79552 XXV	
$[21_{2+3}]$	=−0.33333 XXI	−0.23157 XXIII	+0.66667 XXIV	−0.15773 XXV	
$[22_1]$	=−0.01488 XXII	+0.02055 XXIII	−0.00801 XXIV	−0.01833 XXV	+0.00171 XXVI
	−0.00058 XXVII	+0.00847 XXIX	−0.00115 XXX	+0.00114 XXXI	−0.00172 XXXVI
	+0.00057 XXXVII	+0.03282 XLII	+0.04944 XLIII	−0.21676 XLIV	+0.61658 XLV
$[22_{2+3+4+5+6}]$	=−0.10418 XXII	+0.14385 XXIII	−0.05610 XXIV	−0.12834 XXV	+0.01203 XXVI
	−0.00400 XXVII	+0.05928 XXIX	−0.00801 XXX	+0.00802 XXXI	−0.01202 XXXVI
	+0.00401 XXXVII	+0.22973 XLII	+0.34596 XLIII	+0.48273 XLIV	−0.18394 XLV
$[22_{3+4}]$	=+0.01588 XXII	+0.00394 XXIII	−0.02991 XXIV	+0.02067 XXV	+0.36355 XXVI
	+0.21215 XXVII	+0.76088 XXIX	−0.57570 XXX	−0.42430 XXXI	−0.36355 XXXVI
	−0.21215 XXXVII	+0.00343 XLII	+1.27644 XLIII	+0.00058 XLIV	+0.00057 XLV
$[22_{3+4+5+6}]$	=+0.04761 XXII	+0.02082 XXIII	−0.08973 XXIV	+0.06201 XXV	+1.09065 XXVI
	+0.63645 XXVII	+2.28264 XXIX	+0.27290 XXX	+0.72710 XXXI	−0.09065 XXXVI
	+0.36355 XXXVII	+0.01029 XLII	−2.16138 XLIII	+0.00174 XLIV	+0.00171 XLV
$[22_{4+5+6}]$	=−0.08930 XXII	+0.12329 XXIII	−0.04809 XXIV	−0.11001 XXV	+0.01029 XXVI
	−0.00345 XXVII	+0.05079 XXIX	−0.00688 XXX	+0.00686 XXXI	−0.01031 XXXVI
	+0.00343 XXXVII	+0.86358 XLII	+1.37024 XLIII	−0.63385 XLIV	+0.03282 XLV
$[22_5]$	=+0.01588 XXII	+0.00694 XXIII	−0.02991 XXIV	+0.02067 XXV	+0.36355 XXVI
	+0.21215 XXVII	+0.76088 XXIX	+0.42430 XXX	+0.57570 XXXI	−0.36355 XXXVI
	+0.78785 XXXVII	+0.00343 XLII	−1.71891 XLIII	+0.00058 XLIV	+0.00057 XLV
$[22_{5+6}]$	=+0.03176 XXII	+0.01388 XXIII	−0.05982 XXIV	+0.04134 XXV	+0.72710 XXVI
	+0.42430 XXVII	+1.52176 XXIX	+0.84860 XXX	+1.15140 XXXI	+0.27290 XXXVI
	+0.57570 XXXVII	+0.00686 XLII	−3.43782 XLIII	+0.00116 XLIV	+0.00114 XLV
$[22_6]$	=+0.01588 XXII	+0.00694 XXIII	−0.02991 XXIV	+0.02067 XXV	+0.36355 XXVI
	+0.21215 XXVII	+0.76088 XXIX	+0.42430 XXX	+0.57570 XXXI	+0.63645 XXXVI
	−0.21215 XXXVII	+0.00343 XLII	−1.71891 XLIII	+0.00058 XLIV	+0.00057 XLV
$[22_7]$	=−0.06354 XXII	−0.02778 XXIII	+0.11963 XXIV	−0.08269 XXV	−0.45420 XXVI
	+0.15141 XXVII	−2.23922 XXIX	+0.30281 XXX	−0.30280 XXXI	+0.45421 XXXVI
	−0.15140 XXXVII	−0.01374 XLII	+0.88487 XLIII	−0.00229 XLIV	−0.00229 XLV
$[22_{7+8}]$	=−0.22238 XXII	−0.09725 XXIII	+0.41871 XXIV	−0.28941 XXV	−0.08973 XXVI
	+0.02990 XXVII	−0.44234 XXIX	+0.05981 XXX	−0.05982 XXXI	+0.08972 XXXVI
	−0.02991 XXXVII	−0.04809 XLII	+0.10174 XLIII	−0.00801 XLIV	−0.00801 XLV
$[22_8]$	=−0.15884 XXII	−0.06947 XXIII	+0.29908 XXIV	−0.20672 XXV	+0.36447 XXVI
	−0.12151 XXVII	+1.79688 XXIX	−0.24300 XXX	+0.24298 XXXI	−0.36449 XXXVI
	+0.12149 XXXVII	−0.03135 XLII	−0.78313 XLIII	−0.00572 XLIV	−0.00572 XLV
$[22_9]$	=+0.58701 XXII	+1.09549 XXIII	−0.22238 XXIV	+0.73862 XXV	+0.04764 XXVI
	−0.01590 XXVII	+0.23492 XXIX	−0.03178 XXX	+0.03176 XXXI	−0.04766 XXXVI
	+0.01588 XXXVII	−0.08930 XLII	−0.23896 XLIII	−0.01488 XLIV	−0.01488 XLV
$[22_{10}]$	=−0.23068 XXII	−2.68488 XXIII	−0.12421 XXIV	−0.28418 XXV	+0.02661 XXVI
	−0.00888 XXVII	+0.13121 XXIX	−0.01775 XXX	+0.01774 XXXI	−0.02662 XXXVI
	+0.00887 XXXVII	−0.15799 XLII	−0.30758 XLIII	−0.02633 XLIV	−0.02633 XLV
$[22_{10+11}]$	=−0.23068 XXII	−1.18318 XXIII	−0.12121 XXIV	−0.28418 XXV	+0.02661 XXVI
	−0.00888 XXVII	+0.13121 XXIX	−0.01775 XXX	+0.01774 XXXI	−0.02662 XXXVI
	+0.00887 XXXVII	−0.15799 XLII	−0.30758 XLIII	−0.02633 XLIV	−0.02633 XLV

General corrections in terms of the correlates—Continued.

[22_{11}]	=+1. 50170 XXIII				
[23_{1}]	=+0. 15741 XXIV	−3. 02637 XXV	+0. 11575 XXVI	+0. 17594 XXVIII	−0. 07562 XXIX
	+0. 06482 XXXI	+0. 05093 XXXII	−0. 07079 XXXIII	+0. 08009 XXXV	+0. 07408 XXXVIII
	+0. 16668 XLII	−0. 05522 XLIII			
[23_{1+2}]	=+0. 36111 XXIV	+0. 16854 XXV	−0. 09721 XXVI	−0. 36110 XXVIII	−0. 08467 XXIX
	−0. 02777 XXXI	−0. 06944 XXXII	+0. 00185 XXXIII	−0. 08737 XXXV	−0. 22222 XXXVIII
	−0. 16665 XLII	+0. 11020 XLIII			
[23_{1+2+3}]	=+0. 64815 XXIV	+0. 34982 XXV	−0. 26851 XXVI	+0. 01853 XXVIII	+0. 47174 XXIX
	−0. 20370 XXXI	−0. 06481 XXXII	+0. 27942 XXXIII	−0. 14557 XXXV	+0. 14815 XXXVIII
	−0. 33332 XLII	+0. 00052 XLIII			
[23_{3}]	=+0. 28704 XXIV	+0. 18128 XXV	−0. 17130 XXVI	+0. 37963 XXVIII	+0. 55641 XXIX
	−0. 17583 XXXI	+0. 00463 XXXII	+0. 27757 XXXIII	−0. 05818 XXXV	+0. 37037 XXXVIII
	−0. 16667 XLII	−0. 10968 XLIII			
[23_{4}]	=−0. 13889 XXIV	−0. 20681 XXV	+0. 31944 XXVI	+0. 13890 XXVIII	−0. 44946 XXIX
	+0. 47223 XXXI	−0. 15277 XXXII	−0. 82900 XXXIII	−0. 36027 XXXV	+0. 44444 XXXVIII
	+0. 16669 XLII	+0. 54942 XLIII			
[23_{4+5}]	=−0. 20370 XXIV	−0. 42637 XXV	+0. 71296 XXVI	+0. 53703 XXVIII	−0. 84541 XXIX
	+1. 09259 XXXI	−0. 37963 XXXII	−1. 93370 XXXIII	−0. 92979 XXXV	+0. 29629 XXXVIII
	+0. 33333 XLII	+1. 31872 XLIII			
[23_{4+5+6}]	=−0. 26851 XXIV	−0. 64593 XXV	+1. 10648 XXVI	+0. 93518 XXVIII	−1. 24138 XXIX
	+0. 71296 XXXI	+0. 39352 XXXII	−0. 87827 XXXIII	−0. 31200 XXXV	+0. 14814 XXXVIII
	+0. 50000 XLII	+0. 60387 XLIII			
[23_{5}]	=−0. 06481 XXIV	−0. 21956 XXV	+0. 39352 XXVI	+0. 39815 XXVIII	−0. 39597 XXIX
	+0. 62037 XXXI	−0. 22685 XXXII	−1. 10471 XXXIII	−0. 56952 XXXV	−0. 14815 XXXVIII
	+0. 16667 XLII	+0. 76929 XLIII			
[23_{6}]	=−0. 06481 XXIV	−0. 21956 XXV	+0. 39352 XXVI	+0. 39815 XXVIII	−0. 39597 XXIX
	−0. 37963 XXXI	+0. 77315 XXXII	+1. 05543 XXXIII	+0. 61779 XXXV	−0. 14815 XXXVIII
	+0. 16667 XLII	−0. 71485 XLIII			
[23_{6+7}]	=−0. 12962 XXIV	−0. 43912 XXV	−0. 21296 XXVI	−0. 20370 XXVIII	+0. 22250 XXIX
	−0. 75926 XXXI	+0. 54630 XXXII	+1. 51272 XXXIII	+1. 23558 XXXV	−0. 29630 XXXVIII
	+0. 33334 XLII	−1. 42970 XLIII			
[23_{7}]	=−0. 06481 XXIV	−0. 21956 XXV	−0. 60648 XXVI	−0. 60185 XXVIII	+0. 61847 XXIX
	−0. 37963 XXXI	−0. 22685 XXXII	+0. 45729 XXXIII	−0. 61779 XXXV	−0. 14815 XXXVIII
	+0. 16667 XLII	−0. 71485 XLIII			
[23_{8}]	=+0. 08796 XXIV	+0. 56162 XXV	−0. 30788 XXVI	−0. 25464 XXVIII	+0. 34930 XXIX
	−0. 19908 XXXI	−0. 10880 XXXII	+0. 24589 XXXIII	−0. 19293 XXXV	−0. 03704 XXXVIII
	−0. 41668 XLII	+0. 08310 XLIII			
[24_{1}]	=+0. 72727 XLII	−0. 54545 XLIV			
[24_{2}]	=−0. 54545 XLII	+0. 90909 XLIV			
[25_{1+2}]	=+0. 50000 XXVII	−0. 33333 XXVIII	+0. 49115 XXIX	+0. 83333 XXX	−1. 06044 XXXIV
	+1. 00116 XXXV	+0. 66667 XXXVI	−0. 50000 XXXVIII	+0. 11378 XXXIX	−0. 64518 XL
	−0. 26589 XLI				
[25_{1+2+3}]	=+1. 50000 XXVII	−1. 00000 XXVIII	+1. 47345 XXIX	+0. 50000 XXX	−0. 13360 XXXIV
	−1. 15294 XXXV	+1. 00000 XXXVI	−0. 50000 XXXVIII	−1. 12601 XXXIX	−0. 71643 XL
	+0. 79766 XLI				
[25_{2}]	=−0. 50000 XXVII	+0. 33333 XXVIII	−0. 49115 XXIX	+0. 16667 XXX	−0. 46342 XXXIV
	+1. 07705 XXXV	−0. 66667 XXXVI	+0. 50000 XXXVIII	−0. 11378 XXXIX	+0. 64518 XL
	+0. 26589 XLI				
[25_{2+3}]	=+0. 50000 XXVII	−0. 33333 XXVIII	+0. 49115 XXIX	−0. 16667 XXX	+0. 46342 XXXIV
	−1. 07705 XXXV	−0. 33333 XXXVI	+0. 50000 XXXVIII	−1. 35357 XXXIX	+0. 573[illegible]3 XL
	+1, 32944 XLI				

General corrections in terms of the correlates—Continued.

$[25_{2+3+4+5}]$	= −0.50000 XXVII	+1.00000 XXVIII	−0.99469 XXIX	−0.50000 XXX	+0.55246 XXXIV
	−0.92523 XXXV	−1.00000 XXXVI	+1.50000 XXXVIII	−0.34133 XXXIX	+1.05155 XL
	+0.79766 XLI				
$[25_1]$	=−0.50000 XXVII	+0.66667 XXVIII	−0.74292 XXIX	−0.16667 XXX	+0.04452 XXXIV
	−1.07705 XXXV	−0.33333 XXXVI	+0.50000 XXXVIII	+0.50612 XXXIX	+0.23881 XL
	−0.26589 XLI				
$[25_{1+5}]$	=−1.00000 XXVII	+1.33333 XXVIII	−1.48584 XXIX	−0.33333 XXX	+0.08904 XXXIV
	+0.15182 XXXV	−0.66667 XXXVI	+1.00000 XXXVIII	+1.01224 XXXIX	+0.47762 XL
	−0.53178 XLI				
$[25_5]$	=−0.50000 XXVII	+0.66667 XXVIII	−0.74292 XXIX	−0.16667 XXX	+0.04452 XXXIV
	+1.22887 XXXV	−0.33333 XXXVI	+0.50000 XXXVIII	+0.50612 XXXIX	+0.23881 XL
	−0.26589 XLI				
$[26_1]$	=−0.33333 XXX	+0.50000 XXXI	−0.33333 XXXII	−0.66238 XXXIII	−0.19964 XXXIV
	−0.67959 XXXV	−0.16667 XXXVII	−0.62024 XL	+0.75581 XLI	−0.21651 XLIII
$[26_{1+2+3}]$	=+1.00000 XXX	+1.50000 XXXI	−1.00000 XXXII	−1.98714 XXXIII	−1.12024 XXXIV
	−0.06683 XXXV	+0.50000 XXXVII	+0.00001 XL	+1.03418 XLI	−0.64953 XLIII
$[26_2]$	=+0.66667 XXX	+0.50000 XXXI	−0.33333 XXXII	−0.66238 XXXIII	−0.46030 XXXIV
	+0.30638 XXXV	−0.16667 XXXVII	−0.62024 XL	+0.75581 XLI	−0.21651 XLIII
$[26_{2+3}]$	=+1.33333 XXX	+1.00000 XXXI	−0.66667 XXXII	−1.32476 XXXIII	−0.92060 XXXIV
	+0.61276 XXXV	+0.66667 XXXVII	+0.62025 XL	+0.27837 XLI	−0.43302 XLIII
$[26_{2+3+4}]$	=+1.00000 XXX	+0.50000 XXXI	−1.00000 XXXII	−0.20396 XXXIII	+0.59886 XXXIV
	+0.91914 XXXV	+0.50000 XXXVII	+1.86074 XL	−0.19907 XLI	+0.08345 XLIII
$[26_3]$	=+0.66677 XXX	+0.50000 XXXI	−0.33333 XXXII	−0.66238 XXXIII	−0.46030 XXXIV
	+0.30638 XXXV	+0.83333 XXXVII	+1.24049 XL	−0.47744 XLI	−0.21651 XLIII
$[26_{3+4}]$	=+0.33333 XXX	−0.66667 XXXII	+0.45842 XXXIII	+1.05916 XXXIV	+0.61276 XXXV
	+0.66667 XXXVII	+2.48098 XL	−0.95488 XLI	+0.29996 XLIII	
$[26_4]$	=−0.33333 XXX	−0.50000 XXXI	−0.33333 XXXII	+1.12080 XXXIII	+1.51946 XXXIV
	+0.30638 XXXV	−0.16667 XXXVII	+1.24049 XL	−0.47744 XLI	+0.51647 XLIII
$[26_5]$	=−0.33333 XXX	−0.50000 XXXI	+0.66667 XXXII	+0.43316 XXXIII	−0.19964 XXXIV
	+0.30638 XXXV	−0.16667 XXXVII	−0.62024 XL	−0.27838 XLI	+0.51647 XLIII
$[27_1]$	=−0.20833 XXXVI	−0.16667 XXXVII	+0.79167 XXXVIII	−0.41876 XXXIX	+0.42115 XL
	+0.67066 XLI				
$[27_2]$	=+0.79167 XXXVI	−0.16667 XXXVII	−0.20833 XXXVIII	+0.50488 XXXIX	−0.30085 XL
	−0.92467 XLI				
$[27_{2+3}]$	=+0.62500 XXXVI	+0.50000 XXXVII	−0.37500 XXXVIII	+0.90878 XXXIX	+0.28548 XL
	−1.66440 XLI				
$[27_3]$	=−0.16667 XXXVI	+0.66667 XXXVII	−0.16667 XXXVIII	+0.40390 XXXIX	+0.58633 XL
	−0.73973 XLI				
$[27_4]$	=−0.20833 XXXVI	−0.16667 XXXVII	−0.20833 XXXVIII	+0.50488 XXXIX	−1.12782 XL
	−0.25300 XLI				
$[28_1]$	=+0.25000 XXVI	−0.50000 XXVII	+0.75000 XXVIII	+0.50000 XXXII	−0.38555 XXXIII
	+0.05560 XXXIV	−0.34656 XXXIX	+0.27597 XL	−0.66790 XLI	
$[28_{1+2}]$	=+0.50000 XXVI	+0.50000 XXVIII	+1.00000 XXXII	−0.77110 XXXIII	−1.74168 XXXIV
	−0.23104 XXXIX	−0.93036 XL	+0.25953 XLI		
$[28_2]$	=+0.25000 XXVI	+0.50000 XXVII	−0.25000 XXVIII	+0.50000 XXXII	−0.38555 XXXIII
	−1.79728 XXXIV	+0.11552 XXXIX	−1.20633 XL	+0.92743 XLI	
$[28_3]$	=+0.25000 XXVI	+0.50000 XXVII	−0.25000 XXVIII	−0.50000 XXXII	+1.00780 XXXIII
	+1.68608 XXXIV	+0.11552 XXXIX	+0.65440 XL	−0.12977 XLI	
$[29_1]$	=+0.75000 XLIV	−0.25000 XLV	−0.25000 XLVI		
$[29_2]$	=−0.25000 XLIV	+0.75000 XLV	−0.25000 XLVI		
$[29_3]$	=−0.25000 XLIV	−0.25000 XLV	+0.75000 XLVI		

General corrections in terms of the correlates—Continued.

$[30_{1+2}]$	=—0. 13418 XLV	—0. 08911 XLVI	—0. 08911 XLVII	+0. 48013 XLVIII
$[30_3]$	=—0. 06789 XLV	—0. 21109 XLVI	+0. 45296 XLVII	—0. 08911 XLVIII
$[30_4]$	=—0. 06789 XLV	+0. 45296 XLVI	—0. 21109 XLVII	—0. 08911 XLVIII
$[30_5]$	=+0. 39777 XLV	—0. 06789 XLVI	—0. 06789 XLVII	—0. 13418 XLVIII
$[30_{5+6}]$	=+0. 06789 XLV	—0. 12094 XLVI	—0. 12094 XLVII	+0. 08911 XLVIII
$[31_1]$	=+0. 50000 XLVI			
$[31_2]$	=+1. 00000 XLVII			
$[32_1]$	=+0. 46154 XLVII			
$[32_2]$	=+1. 00000 XLVIII			
$[32_{2+3}]$	=—0. 15385 XLVII			
$[33_4]$	=+1. 00000 XLVIII			

Normal equations for determining the correlates.

No. of equation.						
18.	0=+1. 74500	+ 2. 78261 XVIII	—1. 17391 XIX	—0. 69334 XXIII		
19.	0=—2. 74300	— 1. 17391 XVIII	+3. 35610 XIX	—0. 14286 XX	—0. 14286 XXI	— 1. 00172 XXIII
20.	0=+3. 68700	— 0. 14286 XIX	+2. 44506 XX	—1. 05494 XXI	—0. 15385 XXII	— 0. 26407 XXIII
21.	0=+0. 13700	— 0. 14286 XIX	—1. 05494 XX	+2. 61173 XXI	—0. 15385 XXII	— 0. 51314 XXIII
		— 0. 33333 XXIV	+0. 07886 XXV			
22.	0=—0. 97300	— 0. 15385 XX	—0. 15385 XXI	+2. 15624 XXII	+0. 20252 XXIII	— 1. 02238 XXIV
		— 0. 05690 XXV	+0. 04764 XXVI	—0. 01590 XXVII	+0. 23492 XXIX	— 0. 03178 XXX
		+ 0. 03176 XXXI	—0. 04766 XXXVI	+0. 01588 XXXVII	—0. 08930 XLII	— 0. 23896 XLIII
		— 0. 01488 XLIV	—0. 01488 XLV			
23.	0=—1. 34290	— 0. 69334 XVIII	—1. 00172 XIX	—0. 26407 XX	—0. 51314 XXI	+ 0. 20252 XXII
		+14. 81168 XXIII	+0. 46670 XXIV	+2. 21516 XXV	+0. 02082 XXVI	— 0. 00696 XXVII
		+ 0. 10274 XXIX	—0. 01390 XXX	+0. 01388 XXXI	—0. 02084 XXXVI	+ 0. 00694 XXXVII
		+ 0. 12329 XLII	+0. 15695 XLIII	+0. 02056 XLIV	+0. 02056 XLV	
24.	0=—2. 27900	— 0. 33333 XXI	—1. 02238 XXII	+0. 46670 XXIII	+2. 53353 XXIV	+ 0. 69820 XXV
		— 0. 35824 XXVI	+0. 02990 XXVII	+0. 01853 XXVIII	+0. 02940 XXIX	+ 0. 05981 XXX
		— 0. 26352 XXXI	—0. 06481 XXXII	+0. 27942 XXXIII	—0. 14555 XXXV	+ 0. 08972 XXXVI
		— 0. 02991 XXXVII	+0. 14815 XXXVIII	—0. 38141 XLII	+0. 10226 XLIII	— 0. 00801 XLIV
		— 0. 00801 XLV				
25.	0=+0. 03395	+ 0. 07886 XXI	—0. 05690 XXII	+2. 21516 XXIII	+0. 69820 XXIV	+10. 56072 XXV
		— 0. 58392 XXVI	—0. 02068 XXVII	—0. 46465 XXVIII	+1. 08690 XXIX	— 0. 04135 XXX
		— 0. 38503 XXXI	—0. 21956 XXXII	+0. 53467 XXXIII	—0. 39814 XXXV	— 0. 06202 XXXVI
		+ 0. 02067 XXXVII	—0. 02553 XXXVIII	—0. 97550 XLII	—0. 14749 XLIII	— 0. 01833 XLIV
		— 0. 01833 XLV				
26.	0=+1. 31800	+0. 04764 XXII	+0. 02082 XXIII	—0. 35824 XXIV	—0. 58392 XXV	+2. 94713 XXVI
		+1. 13645 XXVII	+1. 18518 XXVIII	+1. 04126 XXIX	+0. 27290 XXX	+1. 44006 XXXI
		+0. 89352 XXXII	—0. 64157 XXXIII	—0. 05560 XXXIV	—0. 31200 XXXV	—0. 09065 XXXVI
		+0 36355 XXXVII	+0. 14814 XXXVIII	—0. 11552 XXXIX	—0. 27596 XL	+0. 12976 XLI
		+0. 51029 XLII	—1. 55752 XLIII	+0. 00174 XLIV	+0. 00171 XLV	
27.	0=—0. 30700	— 0. 01590 XXII	—0. 00696 XXIII	+0. 02990 XXIV	—0. 02068 XXV	+1. 13645 XXVI
		+3. 28786 XXVII	—1. 50000 XXVIII	+1. 51687 XXIX	+1. 07571 XXX	+0. 42430 XXXI
		+ . 62225 XXXIII	—0. 24480 XXXIV	—1. 15294 XXXV	+1. 36356 XXXVI	+0. 21215 XXXVII
		—0. 50000 XXXVIII	—0. 89497 XXXIX	—1. 26836 XL	+1. 59532 XLI	—0. 00345 XLII
		—1. 27651 XLIII	—0. 00055 XLIV	—0. 00058 XLV		
28.	0=+0. 60800	+0. 01853 XXIV	—0. 46465 XXV	+1. 18518 XXVI	—1. 50000 XXVII	+3. 39814 XXVIII
		—2. 17081 XXIX	—0. 33333 XXX	+0. 53703 XXXI	+0. 89815 XXXII	—0. 98625 XXXIII
		+0. 14464 XXXIV	—0. 21836 XXXV	—0. 66667 XXXVI	+1. 51851 XXXVIII	+0. 66568 XXXIX
		+0. 75359 XL	—1. 19968 XLI	+0. 33333 XLII	+0. 49418 XLIII	

Normal equations for determining the correlates—Continued.

No. of equation.						
29.	0=+1.13171	+0.23492 XXII	+0.10274 XXIII	+0.02940 XXIV	+1.08690 XXV	+1.04126 XXVI
		+1.51687 XXVII	−2.17081 XXVIII	+12.23814 XXIX	−0.22631 XXX	+0.67634 XXXI
		−0.39597 XXXII	+1.08373 XXXIII	−0.13121 XXXIV	−0.39053 XXXV	−0.49604 XXXVI
		+0.76088 XXXVII	−0.88773 XXXVIII	−1.30365 XXXIX	−0.70374 XL	+0.78354 XLI
		−0.57212 XLII	−5.16528 XLIII	+0.00849 XLIV	+0.00847 XLV	
30.	0=−0.64300	−0.03178 XXII	−0.01390 XXIII	+0.05981 XXIV	−0.04135 XXV	+0.27290 XXVI
		+1.07571 XXVII	−0.33333 XXVIII	−0.22631 XXIX	+3.31807 XXX	+1.84860 XXXI
		−0.66667 XXXII	−1.32476 XXXIII	−1.98104 XXXIV	+1.61392 XXXV	+1.39378 XXXVI
		+1.09097 XXXVII	−0.50000 XXXVIII	+0.11378 XXXIX	−0.02493 XL	+0.01248 XLI
		−0.00688 XLII	−2.98597 XLIII	−0.00113 XLIV	−0.00115 XLV	
31.	0=+0.51600	+0.03176 XXII	+0.01388 XXIII	−0.26352 XXIV	−0.38503 XXV	+1.44006 XXVI
		+0.42430 XXVII	+0.53703 XXVIII	+0.67634 XXIX	+1.84860 XXX	+3.74400 XXXI
		−1.37963 XXXII	−3.92084 XXXIII	−1.12024 XXXIV	−0.99662 XXXV	+0.27290 XXXVI
		+1.07570 XXXVII	+0.29629 XXXVIII	+1.03418 XLI	+0.34019 XLII	−2.76864 XLIII
		+0.00116 XLIV	+0.00114 XLV			
32.	0=+1.86800	−0.06481 XXIV	−0.21956 XXV	+0.89352 XXVI	+0.89815 XXVIII	−0.39597 XXIX
		−0.66667 XXX	−1.37963 XXXI	+3.10648 XXXII	+1.15067 XXXIII	−2.14090 XXXIV
		+0.37824 XXXV	−0.33333 XXXVII	−0.14815 XXXVIII	−0.23104 XXXIX	−2.17086 XL
		−0.29721 XLI	+0.16667 XLII	−0.58179 XLIII		
33.	0=−1.10165	+0.27942 XXIV	+0.53467 XXV	−0.64157 XXVI	+0.62225 XXVII	−0.98625 XXVIII
		+1.08373 XXIX	−1.32476 XXX	−3.92084 XXXI	+1.15067 XXXII	+7.96842 XXXIII
		+4.62968 XXXIV	+2.10574 XXXV	−0.66238 XXXVII	−0.55142 XXXVIII	+0.17816 XXXIX
		+1.80588 XL	−1.66143 XLI	−0.42098 XLII	−1.42067 XLIII	
34.	0=−4.15812	−0.05560 XXVI	−0.24480 XXVII	+0.14464 XXVIII	−0.13121 XXIX	−1.98104 XXX
		−1.12024 XXXI	−2.14090 XXXII	+4.62968 XXXIII	+10.49132 XXXIV	−1.64162 XXXV
		−0.59702 XXXVI	−0.46030 XXXVII	+0.55246 XXXVIII	−0.67076 XXXIX	+5.95608 XL
		−2.08402 XLI	+1.00073 XLIII			
35.	0=−1.54828	−0.14555 XXIV	−0.39814 XXV	−0.31200 XXVI	−1.15294 XXVII	−0.21836 XXVIII
		−0.39053 XXIX	+1.61392 XXX	−0.99662 XXXI	+0.37824 XXXII	+2.10574 XXXIII
		−1.64162 XXXIV	+9.55267 XXXV	−0.07589 XXXVI	+0.30638 XXXVII	−1.34368 XXXVIII
		+1.63996 XXXIX	+0.84641 XL	−2.22618 XLI	+0.30579 XLII	−1.11747 XLIII
36.	0=−0.50800	−0.04766 XXII	−0.02084 XXIII	+0.08972 XXIV	−0.06202 XXV	−0.09065 XXVI
		+1.36356 XXVII	−0.66667 XXVIII	−0.49604 XXIX	+1.39378 XXX	+0.27290 XXXI
		−0.59702 XXXIV	−0.07589 XXXV	+3.21567 XXXVI	−0.53022 XXXVII	−1.20833 XXXVIII
		+0.73244 XXXIX	−1.59121 XL	−1.45645 XLI	−0.01031 XLII	−0.83404 XLIII
		−0.00171 XLIV	−0.00172 XLV			
37.	0=+0.74100	+0.01588 XXII	+0.00694 XXIII	−0.02991 XXIV	+0.02067 XXV	+0.36355 XXVI
		+0.21215 XXVII	+0.76088 XXIX	+1.09097 XXX	+1.07570 XXXI	−0.33333 XXXII
		−0.66238 XXXIII	−0.46030 XXXIV	+0.30638 XXXV	−0.53022 XXXVI	+2.28785 XXXVII
		−0.16667 XXXVIII	+0.40390 XXXIX	+1.82682 XL	−1.21717 XLI	+0.00343 XLII
		−1.93542 XLIII	+0.00058 XLIV	+0.00057 XLV		
38.	0=+0.24900	+0.14815 XXIV	−0.02553 XXV	+0.14814 XXVI	−0.50000 XXVII	+1.51851 XXVIII
		−0.88773 XXIX	−0.50000 XXX	+0.29629 XXXI	−0.14815 XXXII	−0.55142 XXXIII
		+0.55246 XXXIV	−1.34368 XXXV	−1.20833 XXXVI	−0.16667 XXXVII	+3.10648 XXXVIII
		−0.76009 XXXIX	+1.47270 XL	+1.46832 XLI	+0.43974 XLIII	
39.	0=+1.93178	−0.11552 XXVI	−0.89497 XXVII	+0.66568 XXVIII	−1.30365 XXIX	+0.11378 XXX
		−0.23104 XXXII	+0.17816 XXXIII	−0.67076 XXXIV	+1.63996 XXXV	+0.73244 XXXVI
		+0.40390 XXXVII	−0.76009 XXXVIII	+4.42229 XXXIX	−1.80294 XL	−4.33998 XLI
40.	0=−3.18150	−0.27596 XXVI	−1.26836 XXVII	+0.75359 XXVIII	−0.70374 XXIX	−0.02493 XXX
		−2.17086 XXXII	+1.80588 XXXIII	+5.95608 XXXIV	+0.84611 XXXV	−1.59121 XXXVI
		+1.82682 XXXVII	+1.47270 XXXVIII	−1.80294 XXXIX	+10.13219 XL	−1.45613 XLI
		+0.55814 XLIII				

Normal equations for determining the correlates—Continued.

No. of equation.						
41.	0=−0.20260	+0.12976 XXVI	+1.59532 XXVII	−1.19968 XXVIII	+0.78354 XXIX	+0.01248 XXX
		+1.03418 XXXI	−0.29721 XXXII	−1.66143 XXXIII	−2.08402 XXXIV	−2.22618 XXXV
		−1.45645 XXXVI	−1.21717 XXXVII	+1.46832 XXXVIII	−4.33998 XXXIX	−1.45613 XL
		+8.82134 XLI	−0.50754 XLIII			
42.	0=+2.09500	−0.08930 XXII	+0.12329 XXIII	−0.38141 XXIV	−0.97550 XXV	+0.51029 XXVI
		−0.00345 XXVII	+0.33333 XXVIII	−0.57212 XXIX	−0.00688 XXX	+0.34019 XXXI
		+0.16667 XXXII	−0.42098 XXXIII	+0.30579 XXXV	−0.01031 XXXVI	+0.00343 XXXVII
		+2.25752 XLII	+1.25926 XLIII	−1.17930 XLIV	+0.03282 XLV	
43.	0=−1.25810	−0.23896 XXII	+0.15695 XXIII	+0.10226 XXIV	−0.14749 XXV	−1.55752 XXVI
		−1.27651 XXVII	+0.49418 XXVIII	−5.16528 XXIX	−2.98597 XXX	−2.76864 XXXI
		−0.58179 XXXII	−1.42067 XXXIII	+1.00073 XXXIV	−1.11747 XXXV	−0.83404 XXXVI
		−1.93542 XXXVII	+0.43974 XXXVIII	+0.55814 XL	−0.50754 XLI	+1.25926 XLII
		+15.38335 XLIII	−1.02428 XLIV	+0.04944 XLV		
44.	0=−1.65700	−0.01438 XXII	+0.02056 XXIII	−0.00801 XXIV	−0.01833 XXV	+0.00174 XXVI
		−0.00055 XXVII	+0.00849 XXIX	−0.00113 XXX	+0.00116 XXXI	−0.00171 XXXVI
		+0.00058 XXXVII	−1.17930 XLII	−1.02428 XLIII	+2.77567 XLIV	−0.46676 XLV
		−0.25000 XLVI				
45.	0=+1.42600	−0.01488 XXII	+0.02056 XXIII	−0.00801 XXIV	−0.01833 XXV	+0.00171 XXVI
		−0.00058 XXVII	+0.00847 XXIX	−0.00115 XXX	+0.00114 XXXI	−0.00172 XXXVI
		+0.00057 XXXVII	+0.03282 XLII	+0.04944 XLIII	−0.46676 XLIV	+1.76435 XLV
		−0.31789 XLVI	−0.06789 XLVII	−0.13418 XLVIII		
46.	0=−0.33000	−0.25000 XLIV	−0.31789 XLV	+1.70296 XLVI	−0.21109 XLVII	−0.08911 XLVIII
47.	0=+0.94600	−0.06789 XLV	−0.21109 XLVI	+1.91450 XLVII	−0.08911 XLVIII	
48.	0=+0.01800	−0.13418 XLV	−0.08911 XLVI	−0.08911 XLVII	+2.48013 XLVIII	

Values of the correlates and their logarithms.

XVIII	=−0.3524	log 9.5469989_{-}	XXXIV	=+0.2969	log 9.4725809_{+}
XIX	=+0.6200	log 9.7923847_{+}	XXXV	=+0.6223	log 9.7939858_{+}
XX	=−1.6261	log 0.2111552_{-}	XXXVI	=−2.1601	log 0.3344759_{-}
XXI	=−0.4851	log 9.6858134_{-}	XXXVII	=−2.9333	log 0.4673580_{-}
XXII	=+0.7814	log 9.8928901_{+}	XXXVIII	=−0.9778	log 9.9902456_{-}
XXIII	=+0.0529	log 8.7234600_{+}	XXXIX	=−0.4391	log 9.6425931_{-}
XXIV	=+1.0210	log 0.0090300_{+}	XL	=+0.8354	log 9.9218841_{+}
XXV	=−0.1176	log 9.0704812_{-}	XLI	=−1.2026	log 0.0801284_{-}
XXVI	=−3.6391	log 0.5610000_{-}	XLII	=−0.8698	log 9.9394444_{-}
XXVII	=+3.5744	log 0.5532032_{+}	XLIII	=+0.1732	log 9.2384476_{+}
XXVIII	=+1.1362	log 0.0554433_{+}	XLIV	=+0.1700	log 9.2303978_{+}
XXIX	=−0.0018	log 7.2480000_{-}	XLV	=−0.7653	log 9.8838260_{-}
XXX	=−0.3317	log 9.5207324_{-}	XLVI	=+0.0075	log 7.8733000_{+}
XXXI	=+3.4073	log 0.5324155_{+}	XLVII	=−0.5236	log 9.7189748_{-}
XXXII	=+1.8781	log 0.2737233_{+}	XLVIII	=−0.0672	log 8.8273900_{-}
XXXIII	=−0.0641	log 8.8067200_{-}			

Values of the general corrections.

″	″	″	″	″
$[16_1] = -0.352$	$[20_4] = +0.170$	$[22_{11}] = +0.079$	$[25_4] = -1.102$	$[29_1] = +0.317$
$[17_2] = 0.972$	$[21_1] = -0.648$	$[23_1] = +0.439$	$[25_5] = -0.333$	$[29_2] = -0.618$
$[17_{2+3}] = +0.620$	$[21_2] = -0.140$	$[23_{1+2}] = +0.394$	$[26_1] = -0.227$	$[29_3] = +0.154$
$[18_{1+2}] = -0.868$	$[21_{2+3}] = +0.849$	$[23_3] = +0.445$	$[26_2] = -0.023$	$[30_{1+2}] = +0.117$
$[18_2] = -0.120$	$[22_1] = -0.547$	$[23_4] = -0.455$	$[26_3] = +0.082$	$[30_3] = -0.181$
$[18_3] = +0.582$	$[22_{2+3+4+5+6}] = -0.047$	$[23_5] = +0.518$	$[26_4] = +0.540$	$[30_4] = +0.172$
$[18_4] = -0.420$	$[22_{3+4}] = -0.216$	$[23_6] = -0.668$	$[26_5] = +0.158$	$[30_5] = -0.260$
$[18_5] = -0.068$	$[22_{4+5+6}] = -0.758$	$[23_7] = -0.006$	$[27_1] = -0.106$	$[30_{5+6}] = +0.004$
$[19_1] = +0.569$	$[22_5] = -0.592$	$[23_8] = +0.249$	$[27_2] = -0.378$	$[31_1] = +0.004$
$[19_2] = -1.067$	$[22_6] = +0.181$	$[24_1] = -0.725$	$[27_3] = -0.230$	$[31_2] = -0.524$
$[19_{3+4}] = -0.019$	$[22_7] = +0.776$	$[24_2] = +0.629$	$[27_4] = +0.283$	$[32_1] = -0.242$
$[20_1] = +0.921$	$[22_8] = -0.325$	$[25_{1+2}] = +0.219$	$[28_1] = +0.321$	$[32_2] = -0.067$
$[20_2] = +0.530$	$[22_9] = +0.193$	$[25_2] = +0.290$	$[28_2] = -1.150$	$[32_{2+3}] = +0.080$
$[20_3] = -1.752$	$[22_{10+11}] = -0.268$	$[25_{2+3}] = +0.636$	$[28_3] = +0.742$	$[33_4] = -0.067$

Residuals resulting from substitution of general corrections in numerical equations of condition.

No. of equation.	Residual.	No. of equation.	Residual.
18	−0.0001	34	−0.0112
19	+0.0002	35	+0.0525
20	+0.0001	36	−0.0001
21	−0.0002	37	+0.0003
22	+0.0001	38	−0.0001
23	+0.0060	39	+0.0240
24	+0.0004	40	−0.0080
25	+0.0100	41	−0.0160
26	+0.0004	42	+0.0002
27	+0.0007	43	−0.0260
28	−0.0001	44	−0.0001
29	+0.0021	45	+0.0003
30	+0.0006	46	0.0000
31	+0.0005	47	0.0000
32	−0.0005	48	+0.0009
33	−0.0040		

PROBABLE ERRORS OF OBSERVED AND ADJUSTED ANGLES.

§ **11.** Let

m=whole number of observed angles in a section.

r=whole number of rigid conditions in a section.

n=number of triangles in principal chain.

$[prr]$=sum of weighted squares of corrections to observed angles.

ρ_1=probable error of an observed angle of weight unity.

ρ_s=probable error of an observed angle of average weight in whole section.

ρ_s'=probable error of an adjusted angle of average weight in whole section.

p_s=average weight of an observed angle in whole section.

p_c=average weight of an observed angle in principal chain.

ρ_c=probable error of an observed angle of average weight in principal chain.

ρ_c'=probable error of an adjusted angle of average weight in principal chain.

ρ_t=probable error of an observed angle in the principal chain as derived from the errors in closing of triangles.

$[ee]$=sum of squares of closing errors in triangles.

Proceeding as in § 8, Chapter XIV, C, there are found the following values:

FOR THE ENTIRE SECTIONS OF THIS CHAPTER.

Section.	Extent of section.	m	r	$[pvv]$	ρ_1	p_s	ρ_s	$\sqrt{\frac{m-r}{m}}$	ρ_s'
					$''$		$''$		$''$
VI	Minnesota Junction-Horicon to Warren-Fremont...	50	18	11.53	0.54	0.94	0.56	0.80	0.45
VII	Warren-Fremont to Michigan City-Bald Tom.......	104	64	29.06	0.45	0.93	0.49	0.62	0.31

FOR THE PRINCIPAL CHAIN CONNECTING THE FOND DU LAC AND CHICAGO BASES, GIVEN IN D, § 12, FOLLOWING.

Section.	Extent of principal chain in each section.	p_c	ρ_c	ρ_c'	From closing errors of triangles.				
					$[vv]$	n	ρ_t	Average error.	Greatest error.
				$''$			$''$	$''$	$''$
V	Fond du Lac Base to Minnesota Junction-Horicon...	0.77	0.43	0.26	11.81	5	0.60	1.32	2.65
VI	Minnesota Junction-Horicon to Warren-Fremont...	0.90	0.57	0.46	39.80	15	0.63	1.28	3.37
VII	Warren-Fremont to Chicago Base	0.84	0.48	0.30	43.61	9	0.86	1.80	4.69
	Entire principal chain	......	......	......	95.22	29	0.71	1.45	4.69

D.—PRINCIPAL CHAIN OF TRIANGLES BETWEEN FOND DU LAC AND CHICAGO BASES.

§ **12.** The principal chain of triangles between Fond du Lac and Chicago Bases has two triangles which enter the principal chain between Fond du Lac and Keweenaw Bases, and three triangles which enter the principal chain between Chicago and Sandusky Bases. The complication involved in making the sides of these common triangles depend on three or more bases has been avoided without material loss of accuracy in the following manner: The chains running north and south from Fond du Lac Base diverge from the line Oakfield-Springvale. The chains running north and east from Chicago Base diverge from the line Willow Springs-Shot Tower. The adjusted value of the length of the first line has been taken from the principal chain between Keweenaw and Fond du Lac Bases, Chapter XV, D, § 8; the adjusted value of the second has been taken from the principal chain between Chicago and Sandusky Bases, Chapter XVII, D, § 6; and these two lines have been used as bases in adjusting the intervening triangle-sides. The logarithms of these adjusted lines and their probable errors expressed in units of the seventh place of decimals, as derived from Chapter XV, D, and Chapter XVII, D, are, the unit of length being one foot—

Oakfield-Springvale...... .... 4.7953707±19.65
Willow Springs-Shot Tower.............................. 4.9117967±19.65

These sides will not have their values changed in obtaining weighted mean sides for the intervening triangulation. The logarithm of the second of these lines computed from the first through the intervening triangles is 4.9118042, giving a discrepancy of 75 to be distributed among the logarithms of the intermediate sides. The probable errors of observed angles of average weight in this principal chain of triangles are, between the lines Fond du Lac Base and Minnesota Junction-Horicon, $\pm 0''.43$; between the lines Minnesota Junction-Horicon and Warren-Fremont, $\pm 0''.57$; and between the lines Warren-Fremont and Willow Springs-Shot Tower, $\pm 0''.48$. See Chapters XV, C, and XVI, C. Using the notation of Chapter XIV, D, we have—

$$\frac{1}{p}+\frac{1}{p'}=\Sigma\,(\alpha^2+\beta^2)\,\rho^2+386+386 \qquad d=-75$$

With the above values of ρ and the corresponding sums of $(\alpha^2+\beta^2)$ for the parts of the chain,

$\Sigma (\alpha^2+\beta^2) \rho^2$ is found to be 4101 in units of the seventh decimal place of logarithms. Therefore the constant

$$\frac{1}{p}+\frac{1}{p'}=4873.$$

The additional data required in computing the changes in the logarithms of the lines are given in the following tables, which are arranged in the same manner as those in Chapter XIV, D, and show the principal triangles, &c., between the lines Oakfield–Springvale and Willow Springs–Shot Tower. The logarithms of the sides in the fourth column are computed from the logarithm of the line Oakfield–Springvale and the angles of the triangles given in the second column. The line in the chain having the maximum probable error is Delafield–New Berlin, for which $\frac{1}{p}=2390$ and $\frac{1}{p'}=2483$, giving for the probable error of the logarithm of this line, neglecting error of standard, which is here insignificant, $\pm$ 34.9, corresponding to $\frac{1}{124450}$ part of its length.

Principal chain of triangles between Fond du Lac and Chicago Bases.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	α^2 and β^2	$\Sigma(\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Waupun	85 19 52.298		4.7953707	4.00			4.7953707
Oakfield	53 57 54.751	−2.654	4.7045798				4.7045790
Springvale	40 42 13.437		4.6111598	600.25	604.25	499	4.6111590
Horicon	35 20 01.914		4.6111598	882.09			4.6111590
Waupun	60 50 13.594	+1.560	4.7901096				4.7901086
Oakfield	83 49 45.082		4.8464535	4.84	1491.18	666	4.8464525
Minnesota Junction	92 40 59.668		4.8464535	1.00			4.8464525
Horicon	53 18 12.252	−1.296	4.7510017				4.7510004
Waupun	34 00 48.601		4.5946427	973.44	2465.62	848	4.5946414
Lebanon	29 05 17.132		4.5946427	1428.84			4.5946414
Horicon	90 13 47.236	+1.155	4.9078664				4.9078643
Minnesota Junction	60 40 56.286		4.8483453	139.24	1568.08	1356	4.8483432
Woodland	111 36 03.873		4.8483453	68.89			4.8483432
Lebanon	34 04 48.436	−1.100	4.6284301				4.6284275
Horicon	34 19 08.089		4.6310935	954.81	2591.78	1687	4.6310909
Erin	36 08 13.487		4.6310935	829.44			4.6310909
Lebanon	62 54 02.612	+0.119	4.8099453				4.8099423
Woodland	80 57 44.543		4.8550235	11.56	3432.78	1959	4.8550205
Delafield	43 35 09.632		4.8550235	488.41			4.8550205
Erin	86 50 06.622	+0.729	5.0158634				5.0158600
Lebanon	49 34 45.078		4.8980829	324.00	4245.19	2222	4.8980795
New Lisbon	85 05 46.444		4.8980829	4.00			4.8980795
Delafield	40 37 39.838	+0.513	4.7133504				4.7133469
Erin	54 16 34.501		4.8091465	228.01	4477.20	2298	4.8091430
New Berlin	64 22 37.685		4.8091465	102.01			4.8091430
Delafield	58 18 37.043	+2.145	4.7839849				4.7839812
New Lisbon	57 18 46.050		4.7792255	184.96	4764.17	2390	4.7792218
Waterford	37 18 45.961		4.7792255	767.29			4.7792218
New Berlin	100 47 03.314	+0.006	4.9888964				4.9888921
Delafield	41 54 11.649		4.8213292	547.56	6079.02	2816	4.8213249
Caledonia	41 23 50.761		4.8213292	571.21			4.8213249
Waterford	88 10 54.080	+0.127	5.0007272				5.0007224
New Berlin	50 25 16.364		4.8878583	302.76	6952.99	3099	4.8878535

Principal chain of triangles between Fond du Lac and Chicago Bases—Continued.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	α^2 and β^2	$\Sigma(\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Dover	91 56 01.247		4.8878583	1.00			4.8878535
Caledonia	49 21 10.190	−2.358	4.7681955				4.7681904
Waterford	38 42 49.232		4.6842830	686.44	7640.43	3322	4.6842779
Somers	50 01 43.044		4.6842830	309.76			4.6842779
Dover	59 30 03.390	+2.878	4.7351718				4.7351665
Caledonia	70 28 14.148		4.7741149	54.76	8004.95	3440	4.7741096
Bristol	74 31 17.763		4.7741149	33.64			4.7741096
Somers	50 44 35.536	−3.372	4.6790779				4.6790725
Dover	54 44 07.248		4.7021118	219.04	8257.63	3521	4.7021064
Benton	62 09 24.020		4.7021118	123.21			4.7021064
Bristol	60 00 19.774	+0.766	4.6931021				4.6930965
Somers	57 50 16.702		4.6831981	174.24	8555.08	3618	4.6831925
Antioch	62 17 26.133		4.6831981	123.21			4.6831925
Benton	49 45 29.910	−1.439	4.6188092				4.6188035
Bristol	67 57 04.394		4.7031155	72.25	8750.54	3681	4.7031098
Warren	61 10 57.890		4.7031155	134.56			4.7031098
Antioch	53 27 04.339	+0.476	4.6654360				4.6654302
Benton	65 21 58.272		4.7190905	94.09	8979.19	3755	4.7190847
Fremont	57 16 23.597		4.7190905	184.96			4.7190847
Warren	67 11 46.902	+1.932	4.7588162				4.7588102
Antioch	55 31 50.086		4.7103141	210.25	9374.40	3883	4.7103081
Deerfield	44 47 38.594		4.7103141	449.44			4.7103081
Fremont	70 00 08.922	−1.576	4.8353888				4.8353826
Warren	65 12 13.237		4.8203884	96.04	545.48	4010	4.8203822
Palatine	70 28 11.144		4.8203884	54.76			4.8203822
Deerfield	55 57 52.496	+2.672	4.7645159				4.7645096
Fremont	53 33 57.089		4.7516705	240.25	840.49	4079	4.7516642
Park Ridge (new)	52 24 38.078		4.7516705	262.44			4.7516642
Palatine	53 19 04.017	−4.692	4.7568781				4.7568717
Deerfield	74 16 18.637		4.8361523	34.81	1137.74	4148	4.8361459
Lombard	56 34 44.701		4.8361523	193.21			4.8361459
Park Ridge (new)	75 50 48.915	−0.434	4.9012630				4.9012564
Palatine	47 34 27.334		4.7827953	372.49	1703.44	4280	4.7827887
Shot Tower	39 31 19.337		4.7827953	650.25			4.7827887
Lombard	46 37 57.952	+0.898	4.8405974				4.8405906
Park Ridge (new)	93 50 43.700		4.9781042	1.00	2354.69	4432	4.9780974
Willow Springs	75 02 59.147		4.9781042	31.36			4.9780974
Shot Tower	48 55 01.898	+2.558	4.8702924				4.8702855
Lombard	56 02 00.334		4.9118042	201.64	2587.69	4487	4.9117967

CHAPTER XVII.

TRIANGULATION FROM CHICAGO BASE TO SANDUSKY BASE.

A.—DESCRIPTIONS OF STATIONS.

NOTE RELATIVE TO ELEVATIONS.

§ **1.** The elevations of the surface of the ground at the stations between Lakes Michigan and Erie depend upon the heights of the ground at stations Cedar Point and Stony Point, near the shore of Lake Erie, and at stations Bald Tom and Michigan City, near Lake Michigan. The heights of stations Cedar Point and Stony Point above Lake Erie were accurately determined by spirit leveling. The heights of stations Bald Tom and Michigan City above Lake Michigan were determined by single zenith distances over short lines. The relative heights of the intermediate stations were derived by trigonometrical leveling, reciprocal, non-simultaneous zenith distances having been observed over fifty-one lines, and single zenith distances over thirteen lines of the triangulation. The coefficient of refraction used in computations depending on single zenith distances was 0.06, a mean of the values deduced from the trigonometrical leveling in Illinois.

The total discrepancy shown by comparing the relative elevation of Lakes Michigan and Erie, computed by way of the triangulation, with their relative elevation determined by precise leveling (Chapter XXII, § 13), was 2.75 feet. This discrepancy was distributed amongst the heights of stations between the lakes proportionally to their distances from either lake, the heights of stations Cedar Point, Stony Point, Bald Tom, and Michigan City, derived in the manner stated above, being taken as exact. Excepting stations Cedar Point and Stony Point, the heights are all referred to the mean level of Lake Michigan, which is 8.4 feet higher than the mean level of Lake Erie. (See Chapter XXII, § 13.) An estimate of the probable errors of these heights may be formed thus: First. Considering errors of closure, found by summing relative heights around triangles, and disregarding distance as in this case unimportant, since the lines do not vary greatly in length, the probable error of the relative height of two stations is found to be about $\pm$ 1 foot. The height of any station in the chain can in general be determined by at least four independent routes, and as the number of lines along either flank may be taken as 14 (the actual numbers are 15 and 13) the probable error of any height, *i. e.*, of the mean of the independent values, cannot exceed $\pm 1 \text{ foot} \times \sqrt{\frac{7}{4}} = \pm 1.3$ feet. The heights having the greatest probable errors are those of stations near the middle of the chain between the lakes, and the probable errors of heights in either direction from the middle part of the chain decrease from station to station, becoming zero for the heights of stations on the lake shore. Second. Using the above discrepancy in the relative heights of Lakes Michigan and Erie, viz, 2.75 feet, as a basis, the maximum probable error of a height would be about $\frac{1}{3} \times 2.75$ feet or $\pm$ 1 foot, and the probable errors at nearly all stations would be less than $\pm$ 1 foot. It may be concluded, therefore, that the probable errors of the heights of these stations do not on the average exceed $\pm$ 1 foot.

The elevations of stations east of the line Cedar Point–Stony Point described in this chapter are referred to the mean surface of Lake Erie, and were determined either by direct connection with the surface of the lake or by trigonometric leveling, as stated in greater detail in Chapter XVIII, A, § 1. The precision of these elevations may be taken as equal to that of the heights of stations in Southern Michigan.

DESCRIPTIONS OF STATIONS.

§ **2.** GALENA, 1877.*—This station is situated in the southwest quarter of section 32, township 38 north, range 2 west, Galena Township, La Porte County, Indiana, about 4 miles northwest of the village of Rolling Prairie, a station on the Lake Shore and Michigan Southern Railway, ¼ mile northeast of a school-house and cemetery at the corner of sections 31 and 32, and ¼ mile east and ⅛ mile north of the western and southern section-lines. The height of station used was 65 feet. The geodetic point is marked by a stone post of the usual form set 2½ feet below the surface of the ground. Three stone reference-posts are set as follows: One bearing south 32° 24′ west, distant 60.0 metres; one bearing south 84° 19′ west, distant 62.1 metres; and one bearing south 83° 48′ east, distant 36.6 metres from the geodetic point; a land-survey stone on the south line of section 32, ¼ mile east of the southwest corner of that section, bears south 8° 06′ west, and is distant 214.0 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 375.9 feet.

BERTRAND, 1877.—This station is situated in section 15, Bertrand Township, Berrien County, Michigan, about 4 miles south of Buchanan, a station on the Michigan Central Railroad. The height of station used was 72 feet. The geodetic point is marked by a stone post of the usual form set 3 feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Three stone reference-posts are set as follows: One bearing north 2° 13′ west, distant 13.11 metres; one on the section-line between sections 14 and 15, bearing south 84° 13′ east, distant 9.69 metres; and one at the corner of sections 14, 15, 22, and 23 of township 8 south, range 18 west, bearing south 2° 46′ 45″ east, distant 309.7 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 356.9 feet.

CARLISLE, 1877.—This station is situated in the southeast quarter of section 11, township 37 north, range 1 west, Olive Township, Saint Joseph County, Indiana, about 2 miles southwest of Carlisle village, on the Lake Shore and Michigan Southern Railroad, on what is known as Jarrett's Hill. The height of station used was 75 feet. The geodetic point is marked by a stone post of the usual form, set 2½ feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Three stone reference-posts are set as follows: One bearing north 32° 16′ west, distant 16.1 metres; one bearing south 86° 16′ east, distant 14.4 metres; and one bearing south 18° 24′ west, distant 8.7 metres from the geodetic point. The corner of sections 2, 3, 10, and 11 bears north 39° 09′ 30″ west, and is distant 1640.9 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 332.2 feet.

MILTON, 1877.—This station is situated in the southeast quarter of section 4, township 8 south, range 16 west, Milton Township, Cass County, Michigan, about 4 miles east of the town of Niles, and 230 metres southeasterly from the center of the section, on the highest ground in the vicinity. The height of station used was 66 feet. The geodetic point is marked by stone posts in the usual manner. Three stone reference-posts are set as follows: Two on the east side of the road just west of the station, one bearing south 62° 30′ west, distant 131.8 metres, and one bearing north 62° 03′ west, distant 130.9 metres; and one at the center of section 4, bearing north 32° 06′ west, distant 230.4 metres from the geodetic point. The corner of sections 4, 5, 8, and 9 bears south 57° 12′ 45″ west, and is distant 1113.7 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 314.1 feet.

PENN, 1877.—This station is situated in the northwest quarter of section 20, township 37 north, range 3 east, Penn Township, Saint Joseph County, Indiana, about 3 miles southeast of the city of South Bend. The height of station used was 40 feet. The geodetic point is marked by stone posts in the usual manner. Three stone reference-posts are set as follows: One bearing north 86° 30′ east, distant 99.7 metres; one bearing south 7° 37′ east, distant 41.9 metres; and one bearing south 51° 56′ west, distant 69.6 metres from the geodetic point. The corner of sections 17, 18, 19, and 20 bears north 63° 51′ west, and is distant 788.4 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 309.3 feet.

CALVIN, 1878.—This station is situated in the northeast quarter of section 27, township 7 south, range 14 west, Calvin Township, Cass County, Michigan, about 7 miles south of Vandalia, a station

* See note relative to topographical sketches of stations, under Burnt Bluff, Chap. XV, A, § 2.

on the Michigan Central Railroad, and about 7 miles southeast of Cassopolis, a station on the same road. The height of station used was 110 feet. The geodetic point is marked by stone posts in the usual manner. Three stone reference-posts are set as follows: One bearing north 59° 28′ east, distant 44.6 metres; one bearing south 21° 36′ east, distant 87.5 metres; and one bearing south 9° 47′ west, distant 82.1 metres from the geodetic point. The quarter-section corner on the west side of section 27 bears south 67° 07′ 30″ west, and is distant 1004.5 metres from the geodetic point. The height of ground at the station above the mean level of Lake Michigan is 429.3 feet.

JEFFERSON, 1878.—This station is situated in the southwest quarter of section 2, township 37 north, range 6 east, Jefferson Township, Elkhart County, Indiana, about 3 miles south of Bristol, a station on the Lake Shore and Michigan Southern Railway. The height of station used was 64 feet. The geodetic point is marked by stone posts in the usual manner. Three stone reference-posts are set as follows: One bearing south 27° 28′ west, distant 13.5 metres; one bearing north 15° 01′ west, distant 23.6 metres; and one bearing north 25° 44′ east, distant 95.0 metres from the geodetic point. The corner of sections 2, 3, 10, and 11 bears south 62° 43′ west, and is distant 350 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 388.3 feet.

PORTER, 1878.—This station is situated in section 36, township 7 south, range 13 west, Porter Township, Cass County, Michigan, about 8 miles west of White Pigeon village. The height of station used was 73 feet. The geodetic point is marked by stone posts in the usual manner. Three stone reference-posts were set as follows: One bearing south 48° 02′ east, distant 156.5 metres; one bearing north 73° 29′ east, distant 120.6 metres; and one bearing north 73° 24′ west, distant 108.3 metres from the geodetic point. The quarter-section corner on the west side of section 36 bears south 56° 55′ 03″ west, and is distant 1304.0 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 368.4 feet.

VAN BUREN, 1878.—This station is situated in the northwest quarter of section 33, township 38 north, range 8 east, Van Buren Township, LaGrange County, Indiana. The height of station used was 64 feet. The geodetic point is marked by stone posts in the usual manner. Three stone reference-posts are set as follows: One bearing south 3° 20′ west, distant 130.0 metres; one bearing north 22° 59′ west, distant 27.6 metres; and one bearing south 89° 11′ east, distant 175.9 metres from the geodetic point. The quarter-section corner on the west side of section 33 bears south 55° 44′ west, and is distant 752.4 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 369.7 feet.

SHERMAN, 1878.—This station is situated in the northeast quarter of section 22, township 7 south, range 10 west, Sherman Township, Saint Joseph County, Michigan, about 4 miles northwesterly from Sturgis, Michigan. The height of station used was 64 feet. The geodetic point is marked by stone posts in the usual manner. Three stone reference-posts are set as follows: One bearing south 28° 58′ east, distant 45.9 metres; one bearing south 21° 16′ west, distant 41.7 metres; one bearing north 70° 17′ west, distant 26.8 metres from the geodetic point. The quarter-section corner on the south side of section 22 bears south 24° 12′ 30″ west, and is distant 921.0 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 455.7 feet.

MONGO, 1878.—This station is situated in section 4, township 37 north, range 11 east, Springfield Township, LaGrange County, Indiana, about ¾ mile northeast of Mongo Mills. The height of station used was 66 feet. The geodetic point is marked by stone posts in the usual manner. Three stone reference-posts are set as follows: One bearing south 59° 13′ east, distant 28.6 metres; one bearing south 14° 19′ east, distant 16.7 metres; and one bearing south 57° 03′ west, distant 30.1 metres. The corner of sections 4, 5, 8, and 9 bears south 22° 44′ 00″ west, distant 873.5 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 445.5 feet.

BRONSON, 1878.—This station is situated in the southeast quarter of the southwest quarter of section 16, township 7 south, range 8 west, Bronson Township, Branch County, Michigan, about three miles in a westerly direction from Bronson railway-station, on the Lake Shore and Michigan Southern Railway. The height of station used was 74 feet. The geodetic point is marked by stone posts in the usual manner. Three stone reference-posts are set as follows: One bearing north 40° 13′ east, distant 106.5 metres; one bearing north 85° 08′ east, distant 68.6 metres; and one bearing south 36° 12′ east, distant 116.4 metres from the geodetic point. The quarter-section corner on the south side of section 16 bears south 19° 07′ east, and is distant 208.9 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 416.4 feet.

FREMONT, 1878.—This station is situated in section 31, township 38 north, range 14 east, Fremont Township, Steuben County, Indiana, about 3 miles southwest of the village of Fremont, a station on the Fort Wayne, Jackson and Saginaw Railroad. The height of station used was 62 feet. The geodetic point is marked by stone posts in the usual manner. Three stone reference-posts are set as follows: One bearing north 22° 00′ west, distant 46.5 metres; one bearing north 42° 29′ east, distant 22.9 metres; and one bearing south 5° 15′ west, distant 55.7 metres from the geodetic point. The northwest corner of section 31 bears north 28° 10′ west, and is distant 834.6 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 560.2 feet.

QUINCY, 1878.—This station is situated in the southwest quarter of section 8, township 6 south, range 5 west, Quincy Township, Branch County, Michigan, about 2 miles northwesterly from Quincy railroad-station, on the Lake Shore and Michigan Southern Railway. The height of station used was 104 feet. The geodetic point is marked by stone posts in the usual manner. Three stone reference-posts are set as follows: Two approximately on the section line between sections 7 and 8, one bearing south 18° 02′ west, distant 41.5 metres, and one bearing north 16° 53′ west, distant 46.25 metres; and one bearing north 35° 13′ east, distant 181.5 metres from the geodetic point. The quarter-section corner at the middle of the south side of section 7 bears south 56° 55′ 10″ west, and is distant 980.5 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 478.9 feet.

READING, 1878, 1879.—This station is situated in section 26, township 7 south, range 4 west, Reading Township, Hillsdale County, Michigan, on the grounds of the factory of the Colby Wringer Company, in the village of Reading, on the Fort Wayne, Jackson and Saginaw Railroad. The height of station used was 115 feet. The geodetic point is marked by two stone posts in the usual manner. Three stone reference-posts are set as follows: One bearing north 48° 28′ east, distant 135.8 metres; one bearing south 83° 12′ east, distant 102.9 metres; and one bearing north 86° 20′ west, distant 147.8 metres from the geodetic point. The southeast corner of the factory of the Colby Wringer Company bears north 51° 05′ west, and is distant 50.25 metres. The northwest corner of section 26, being the corner of sections 22, 23, 26, and 27, bears north 44° 11′ west, and is distant 715.32 metres. The northeast corner of section 26, being the corner of sections 23, 24, 25, and 26, bears north 65° 56′ 50″ east, and is distant 1235.9 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 627.2 feet.

BUNDAY, 1879.—This station is situated in section 7, township 5 south, range 1 west, Somerset Township, Hillsdale County, Michigan, about 2 miles west of Somerset Center, on a hill known as Bunday's Hill. The height of station used was 65 feet. The geodetic point is marked by a stone post set 2½ feet below the ground-surface. Three stone reference-posts are set as follows: Two approximately on the section-line on the east side of section 7, one bearing north 85° 56′ east, distant 132.0 metres, and one bearing south 54° 50′ east, distant 161.7 metres; and one on the north side of the road south of the station, bearing south 14° 47′ east, distant 210.9 metres from the geodetic point. The southeast corner of section 8 bears south 56° 15′ 45″ east, and is distant 2166.0 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 702.2 feet.

HILLSDALE, 1878, 1879.—This station is situated in the northwest quarter of section 27, township 6 south, range 3 west, Hillsdale Township, Hillsdale County, Michigan, about one mile west of the town of Hillsdale. The height of station used was 65 feet. The geodetic point is marked by a stone post buried in the ground, with another stone post set directly over it as a surface-mark. Three stone reference-posts are set, as follows: Two on the section-line on the west side of section 27; one bearing north 35° 03′ west, distant 163.3 metres, and one bearing north 74° 02′ west, distant 98.3 metres; and one on the north side of the road south of the station, bearing south 1° 33′ west, distant 64.4 metres from the geodetic point. The clock-tower of Hillsdale College bears north 45° 52′ 28″ east, and is distant 1898.5 metres from the geodetic point. The northwest corner of section 27 bears north 6° 59′ 18″ west, and is distant 742.5 metres. The quarter-section corner on the west side of section 27 bears south 51° 51′ west, and is distant 118.8 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 657.1 feet.

WHEATLAND, 1879.—This station is situated in section 29, township 6 south, range 1 west, Wheatland Township, Hillsdale County, Michigan, about 4 miles in a north-easterly direction from Pittsford, a station on the Lake Shore and Michigan Southern Railway. The height of station used was 110 feet. The geodetic point is marked by a stone post buried in the ground, with another stone post set directly over it as a surface-mark. Three stone reference-posts are set as follows: Two in the road on the west line of section 29, one bearing north 79° 13′ west, distant 351.6 metres, and one bearing south 54° 04′ west, distant 411.2 metres; and one in the road on the south line of section 29, bearing south 24° 35′ east, distant 662.2 metres from the geodetic point. The corner of sections 19, 20, 29, and 30 bears north 18° 50′ 20″ west, and is distant 1053.9 metres; the quarter-section corner on the west side of section 29 bears north 59° 46′ 20″ west, and is distant 392.3 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 665.8 feet.

PITTSFORD, 1879.—This station is situated about 2½ miles, in a southerly direction, from Pittsford, a station on the Lake Shore and Michigan Southern Railway, in section 25, township 7 south, range 2 west, Jefferson Township, Hillsdale County, Michigan, about one-third mile west of the east line and a few metres north of the south line of the section. The height of station used was 74 feet. The geodetic point is marked by a stone post set three feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Three stone reference-posts are set as follows: One bearing south 81° 35′ east, distant 39.97 metres; one bearing south 5° 45′ west, distant 16.09 metres; and one bearing north 77° 47′ west, distant 18.70 metres from the geodetic point. The southeast corner of section 36, Jefferson Township, being the common corner of Jefferson, Pittsford, Wright, and Ransom Townships, bears south 20° 12′ 39″ east, and is distant 1719.0 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 567.1 feet.

WOODSTOCK, 1879.—This station is situated near the south line of section 36, township 5 south, range 1 east, Woodstock Township, Lenawee County, Michigan. The height of station used was 60 feet. The geodetic point is marked by stone posts in the usual manner. Three stone reference-posts are set on the south line of section 36 as follows: One bearing south 54° 41′ east, distant 59.7 metres; one bearing south 0° 16′ west, distant 35.2 metres; and one bearing south 35° 00′ west, distant 43.5 metres from the geodetic point. The northeast corner of section 1, township 6 south, range 1 east, Rollin Township, bears south 88° 57′ 35″ east, and is distant 1132.9 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 608.6 feet.

FAIRFIELD, 1879.—This station is situated about one-fourth mile north of the village of Fairfield, about 1¼ miles north of the railway station Fairfield, on the Chicago and Canada Southern Railroad, and about 6 miles south of Adrian, on a slight rise of ground near the northeast corner of section 3, township 8 south, range 3 east, Fairfield Township, Lenawee County, Michigan. The height of station used was 100 feet. The geodetic point is marked by a small hole drilled in the top of a stone post set 2½ feet below the surface of the ground, with another stone post set directly over it as a surface-mark. Two stone reference-posts, with the letters U. S. cut on their tops, are set as follows: One bearing north 81° 01′ east, distant 21.5 metres, and one bearing north 86° 25′ west, distant 36.0 metres from the geodetic point. A stone post of the same description, set 3 inches underground at the corner of sections 34 and 35, township 7 south, range 3 east, and sections 2 and 3, township 8 south, range 3 east, bears north 51° 43′ 58″ east, and is distant 203.0 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 216.9 feet.

RAISIN, 1879.—This station is situated about 6 miles northeasterly from Adrian, and 4 miles south of Tecumseh, in section 14, township 6 south, range 4 east, Raisin Township, Lenawee County, Michigan. The height of station used was 41 feet. The geodetic point is marked by a small hole drilled in the top of a stone post set 2½ feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Two stone reference-posts, marked U. S. on their tops, are set along the road fence west of the station as follows: One bearing north 35° 41′ west, distant 172.9 metres; and one bearing south 49° 54′ west, distant 127.7 metres from the geodetic point. A stone post of the same description, set 3 inches below the surface, at the corner of sections 14, 15, 22, and 23, of township 6 south, range 4 east, bears south 12° 04′ 18″ west, and is distant 454.8 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 269.3 feet.

BLISSFIELD, 1879.—This station is situated in section 22, township 7 south, range 5 east, Blissfield Township, Lenawee County, Michigan, about 3 miles southwest of the village of Deerfield, and about 3 miles in a northeasterly direction from the village of Blissfield. The height of station used was 117 feet. The geodetic point is marked by a stone post set 3 feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Three stone reference-posts are set along the road east of the station, as follows: One on the east side of the road, bearing north 45° 11′ east, distant 39.0 metres; and two on the west side of the road, one bearing north 83° 25′ east, distant 10.86 metres, and one bearing south 21° 01′ east, distant 37.0 metres from the geodetic point. The corner of sections 22, 23, 26, and 27 bears south 3° 35′ 15″ east, and is distant 351.4 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 114.7 feet.

DUNDEE, 1879.—This station is situated within a few metres of the line between sections 25 and 36, in township 6 south, range 6 east, this being also the line between Dundee and Summerfield Townships, Monroe County, Michigan, on the northwest side of the road between Petersburg and Dundee, and about 2½ miles from each place. The height of station used was 106 feet. The geodetic point is marked by stone posts in the usual manner. Three stone reference-posts are set as follows: Two on the northwest side of the road above mentioned, one bearing north 76° 51′ east, distant 22.4 metres, and one bearing south 4° 49′ east, distant 17.55 metres; and one on the southeast side of the road, bearing south 18° 53′ east, distant 34.05 metres from the geodetic point. The corner of sections 25, 26, 35, and 36, bears north 87° 32′ west, and is distant 110.25 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 99.5 feet.

BEDFORD, 1879.—This station is situated in section 5, township 8 south, range 7 east, Bedford Township, Monroe County, Michigan, about 250 metres in a southerly direction from the quarter-section corner on the north side of the section. The height of station used was 94 feet. The geodetic point is marked by a small hole drilled in the top of a stone post set 2½ feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Three stone reference-posts are set as follows: One bearing south 30° 17′ east, distant 32.8 metres; one bearing south 49° 44′ west, distant 33.0 metres; and one bearing north 39° 12′ west, distant 41.0 metres from the geodetic point. The corner of sections 4 and 5, township 8 south, range 7 east, and sections 32 and 33, township 7 south, range 7 east, bears north 70° 59′ 40″ east, and is distant 823.2 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 96.6 feet.

MONROE, 1879.—This station is situated in the cemetery of the Monroe County Poor-farm, in Monroe Township, Monroe County, Michigan, about 4 miles west of the city of Monroe. The height of station used was 115 feet. The geodetic point is marked by a small hole drilled in the top of a stone post set 3 feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Three stone reference-posts are set in the corners of the cemetery lot, as follows: One bearing north 13° 32′ west, distant 42.0 metres; one bearing south 5° 22′ east, distant 20.0 metres; and one bearing south 73° 36′ west, distant 31.4 metres from the geodetic point. The station is 34.6 metres from the center of the road which runs on the line between Raisinville and Monroe Townships. The southwest corner of private claim No. 432 bears south 25° 07′ west, and is distant 745.3 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 43.9 feet.

STONY POINT, 1877, 1879.—This station is situated on the end of Stony Point, a point of land projecting into Lake Erie at its western end, in Monroe County, Michigan. The height of station used was 55 feet. The geodetic point is marked by a hole drilled in the top of a stone buried beneath the surface of the ground. Another stone post is set directly over the geodetic point for a surface-mark. Three stone reference-posts are set as follows: One bearing south 15° 03′ west, distant 22.5 metres; one bearing south 84° 14′ west, distant 5.7 metres; and one bearing north 25° 03′ west, distant 16.7 metres from the geodetic point. The height of ground at the station above mean level of Lake Erie is 5.2 feet.

CEDAR POINT, 1877, 1879.—This station is situated near the end of Cedar Point, Lucas County, Ohio, on the east side of Maumee Bay. The height of station used was 70 feet. The geodetic point is marked by a stone post of the usual form, set about 3 feet below the surface of the ground. A stone post, projecting about 6 inches above the surface, is set directly over the geodetic point as a surface-mark. Three stone reference-posts are set as follows: One bearing south 61° 05′ east, distant 28.8 metres; one bearing north 61° 00′ east, distant 30.2 metres; and one bearing north 13° 28′ east, distant 35.5 metres from the geodetic point. The height of ground at the station above mean level of Lake Erie is 4.5 feet.

MIDDLE SISTER, 1877.—This station is situated on the west side of Middle Sister Island, near the western end of Lake Erie, about 150 metres north of the south end of the island and 15 metres from the lake shore. The height of station used was 55 feet. The geodetic point is marked by a nail leaded into the solid rock, which is one or two feet below the surface of the ground. Three stone reference-posts are set as follows: One bearing south 39° 57′ west, distant 21.9 metres; one bearing north 33° 49′ east, distant 32.8 metres; and one bearing south 43° 23′ east, distant 21.9 metres from the geodetic point. The height of ground at the station above Lake Erie is about 10 feet.

LOCUST POINT, 1877.—This station is situated on the south shore of Lake Erie, on Locust Point, in Ottawa County, Ohio, about 10 metres back from the lake. The height of station used was 41 feet. The geodetic point is marked by a stone post of the usual form, set 3 feet below the surface of the ground. Three stone reference-posts are set as follows: One bearing south 64° 41′ east, distant 45.0 metres; one bearing south 18° 26′ west, distant 24.3 metres; and one bearing north 86° 30′ west, distant 24.3 metres from the geodetic point. The height of ground at the station above Lake Erie is about 2 feet.

POINTE PELÉE, 1877.—This station is situated on the northwest point of Pointe Pelée Island, an island in the west end of Lake Erie, in the Province of Ontario. It stands in a grove about 210 metres in a southerly direction from the lake shore. The height of station used was 59 feet. The geodetic point is marked by a hole drilled in a brass bolt leaded into the solid limestone rock between the letters U. S. cut in the rock, 19 inches below the surface of the ground. A stone post is set directly over the geodetic point for a surface-mark. Three stone reference-posts are set as follows: One bearing north 22° 44′ east, distant 61.7 metres; one bearing south 80° 12′ east, distant 35.9 metres; and one bearing south 44° 47′ west, distant 31.5 metres from the geodetic point. The east corner of a stone house bears south 53° 36′ west, and is distant 194.8 metres. The above bearings are magnetic. The height of ground at the station above mean level of Lake Erie is 28.4 feet.

MIDDLE BASS, 1877.—This station is situated on Middle Bass Island, in the western end of Lake Erie. It is on the north side of the island, about 650 metres from the eastern end. The height of station used was 75 feet. The geodetic point is marked by a nail leaded into the solid rock about 2½ feet below the surface of the ground. Two stone reference-posts are set as follows: One bearing north 79° 26′ east, distant 17.5 metres; and one bearing south 81° 06′ west, distant 23.05 metres from the geodetic point. The height of ground at the station above the mean level of Lake Erie is 7.8 feet.

KELLEY'S ISLAND, 1877.—This station is situated on Kelley's Island, Erie County, Ohio, somewhat to the north and east of the center of the island, in a piece of open woods. It is about 670 metres south of the bay on the north shore of the island, 243 metres south of a road running east and west, and about 1080 metres east of the road running north from the landing on the south side of the island. The height of station used was 76 feet. The geodetic point is marked by a brass wire leaded into the solid rock between the letters U. S. cut in the rock, 16½ inches below the surface. A stone post is set for a surface reference-mark directly over the geodetic point. Three stone reference-posts, marked U. S. on top, are set as follows: One bearing north 20° 07′

east, distant 32.7 metres; one bearing south 31° 32′ east, distant 30.3 metres; and one bearing north 87° 11′ west, distant 45.9 metres from the geodetic point. The Island House, a hotel at the landing on the south shore of the island, bears south 45° 55′ west, and is distant 1720 metres from the geodetic point. The height of ground at the station above mean level of Lake Erie is 41.1 feet.

DANBURY, 1877.—This station is situated in Danbury Township, Ottawa County, Ohio, about two miles northeast of Danbury railway station, on the Lake Shore and Michigan Southern Railway, and about two-thirds of a mile southwest of the corner of lots 3, 4, 9, and 10 in said township. The height of station used was 116 feet. The geodetic point is marked by a stone post of the usual form, set so that its top is 3 feet below the surface of the ground. Another stone post is set directly above the geodetic point as a surface-mark. Three stone reference-posts are set as follows: One in a fence corner, bearing south 82° 20′ east, distant 61.57 metres; one by a fence on the east of the station, bearing north 57° 24′ east, distant 67.57 metres; and one by a fence on the south of the station, bearing south 3° 36′ west, distant 9.84 metres from the geodetic point. The height of ground at the station above mean level of Lake Erie is 21.4 feet.

WEST BASE, 1877, 1878.—This station, marking the northwest end of the Sandusky base-line, is situated on the northeast side of Cedar Point, about 40 metres from the lake shore and 1016 metres from the light-house at the end of the Point. The height of station used was 70 feet. The geodetic point is marked by a stone post of the usual form, set in a bed of brickwork 3 feet square by 4 feet deep. Three stone reference-posts are set as follows: One bearing south 8° 02′ east, distant 26.3 metres; one bearing south 55° 11′ west, distant 19.5 metres; and one bearing north 64° 21′ west, distant 38.8 metres from the geodetic point. An azimuth post on the line to East Base bears south 40° 23′ east, and is distant 46.91 metres from the geodetic point. The height of ground at the station above mean level of Lake Erie is 6.6 feet.

EAST BASE, 1877, 1878.—This station, marking the southeast end of the Sandusky base-line, is situated on the northeast side of Cedar Point, northeasterly from Sandusky, Ohio, about 4½ miles from the Cedar Point light and 32 metres back from the lake shore. The height of station used was 65 feet. The geodetic point is marked by a stone post of the usual form, set in a bed of brickwork 3 feet square by about 3 feet deep. Three stone reference-posts are set as follows: One bearing south 13° 52′ east, distant 29.87 metres; one bearing south 69° 36′ west, distant 24.78 metres; and one bearing north 50° 57′ west, distant 47.7 metres from the geodetic point. The height of ground at the station above mean level of Lake Erie is 5.8 feet.

SANDUSKY, 1877.—This station is situated about 3½ miles southwest of Sandusky, near the eastern border of Margaretta Township, Erie County, Ohio, about 1⅓ miles south of the west corner of Perkins and Portland Townships. The height of station used was 115 feet. The geodetic point is marked by a brass wire leaded into the solid rock, between the letters U. S. cut in the rock, about 3 feet below the surface of the ground. A stone post is set directly over the geodetic point for a surface mark. Three stone reference-posts are set as follows: One bearing north 3° 34′ west, distant 69.0 metres; one bearing south 69° 12′ east, distant 50.4 metres; and one bearing south 71° 40′ west, distant 43.3 metres from the geodetic point. The Sandusky court-house bears north 33° 37′ east, and is distant 5907.0 metres from the geodetic point. The height of ground at the station above mean level of Lake Erie is 59.3 feet.

TOWNSEND, 1877.—This station is situated in lot 56, Townsend Township, Huron County, Ohio, about half a mile northeast of the village of Collins, and one-eighth of a mile south of the corner of lots 41, 42, 55, and 56. The height of station used was 105 feet. The geodetic point is marked by a brass bolt leaded into the solid rock between the letters U. S. cut therein, about 2½ feet below the surface of the ground. Three stone reference-posts are set as follows: One bearing south 14° 10′ west, distant 42.1 metres; one bearing north 9° 10′ west, distant 64.0 metres; and one bearing north 82° 52′ east, distant 44.1 metres. The height of ground at the station above mean level of Lake Erie is 349.6 feet.

BROWNHELM, 1877.—This station is situated near the center of Brownhelm Township, Lorain County, Ohio, about 1½ miles in a southerly direction from Brownhelm railway depot, and one-half mile northwesterly from the corner of lots 44, 45, 52, and 53 in Brownhelm Township. The height of station used was 124 feet. The geodetic point is marked in the usual manner. Three stone reference-posts are set as follows: One bearing south 87° 31′ east, distant 57.5 metres; one bearing south 10° 06′ west, distant 14.65 metres; and one bearing south 53° 41′ west, distant 42.35 metres. The height of ground at the station above mean level of Lake Erie is 180.8 feet.

CAMDEN, 1877.—This station is situated near the north side of Camden Township, Lorain County, Ohio, about one-half mile northeast of the village of Kipton and one-third mile southeasterly from the corner of lots 5 and 6 in Camden Township and 15 and 16 in Henrietta Township. The height of station used was 106 feet. The geodetic point is marked by a brass wire leaded into the solid sandstone rock, between the letters U. S. cut therein, 2½ feet below the surface of the ground. A stone post is set directly over the geodetic point as a surface-mark. Three stone reference-posts are set as follows: One bearing north 83° 18′ east, distant 76.8 metres; one bearing south 8° 19′ west, distant 99.8 metres; and one bearing north 37° 35′ west, distant 44.4 metres from the geodetic point. The height of ground at the station above mean level of Lake Erie is 318.5 feet.

ELYRIA, 1877.—This station is situated in the northwest corner of Elyria Township, Lorain County, Ohio, about 4½ miles west of north of the village of Elyria, about 1 mile west of the Cleveland, Tuscarawas Valley and Wheeling Railroad, and within the fork of two public roads leading to Charleston and North Amherst. The height of station used was 116 feet. The geodetic point is marked in the usual manner. Three stone reference-posts are set along the fence on the east of the station as follows: One bearing north 19° 44′ east, distant 27.1 metres; one bearing north 86° 16′ east, distant 9.1 metres; and one bearing south 12° 35′ east, distant 32.3 metres from the geodetic point. The height of ground at the station above mean level of Lake Erie is 182.4 feet.

GRAFTON, 1877.—This station is situated in the northwest part of section 89, in the southeast part of Grafton Township, Lorain County, Ohio, about 1 mile northwest of the village of Erhart, and 184.5 metres southwest of the track of the Cleveland, Tuscarawas Valley and Wheeling Railroad. The height of station used was 110 feet. The geodetic point is marked by a hole drilled in the top of a stone post set 27 inches below the surface, a stone post being set directly over it for a surface-mark. Three stone reference-posts are set as follows: One bearing north 46° 18′ west, distant 52.0 metres; one bearing north 9° 04′ east, distant 71.8 metres; and one bearing south 72° 03′ east, distant 43.6 metres from the geodetic point. The first and last reference-stones mentioned are set along a fence running northwest and southeast, 10 metres northeast of the station. The second is set by a fence running in a northeasterly direction from the station. The height of ground at the station above mean level of Lake Erie is 380.7 feet.

OLMSTED, 1877.—This station is situated in section 13, Olmsted Township, Cuyahoga County, Ohio, about 2 miles west of the village of Olmsted Falls, and one-third of a mile south of the corner of sections 8, 14, and 13. The height of station used was 119 feet. The geodetic point is marked by a stone post of the usual form set 2.3 feet below the surface, with a stone post set directly above for a surface-mark. Three stone reference-posts are set as follows: One bearing north 8° 10′ west, distant 90.9 metres; one bearing south 80° 26′ east, distant 67.7 metres; and one bearing south 13° 07′ west, distant 71.7 metres from the geodetic point. An east-and-west rail fence is south 17.1 metres; a north-and-south line fence is 15 metres west; an east-and-west highway is north 321 metres; and the Lake Shore and Michigan Southern Railway is south 653 metres. The first and last mentioned reference-stones are set by the north-and-south line fence, and the second is set by the east-and-west rail fence named above. The height of ground at the station above mean level of Lake Erie is 221.5 feet.

ROYALTON, 1877.—This station is situated in section 12, Royalton Township, Cuyahoga County, Ohio, about half a mile north of the village of North Royalton, on the east side of a north-and-south highway. The height of station used was 35 feet. The geodetic point is marked by a hole drilled in the top of a stone post set 28 inches below the surface of the ground, a stone post being placed directly over it as a surface-mark. Three stone reference-posts are set as follows: One on the east side of the north-and-south highway, bearing north 4° 14′ west, distant 50.6 metres; one on the north side of a northeast-and-southwest highway, bearing north 80° 57′ east, distant 73.2 metres; and one on the west side of the north-and-south highway, bearing south 35° 28′ west, distant 41.4 metres from the geodetic point. The northwest corner of a church with a stone foundation, at the junction of the two roads, bears south 11° 57′ east, and is distant 17.7 metres from the geodetic point. The road fence on the west of the station is distant 5.5 metres. The height of ground at the station above mean level of Lake Erie is 699.1 feet.

ROCKPORT, 1877.—This station is situated in section 19, in Rockport Township, Cuyahoga County, Ohio, about 6 miles westerly from Cleveland, ½ mile northwest of the village of Rockport, and ¼ mile northwest of the Lake Shore and Michigan Southern Railway. The intersection of the Lake Shore and Michigan Southern Railway and the Rocky River Railroad is north 2004.11 metres and east 3124.98 metres from the station. The height of station used was 95 feet. The geodetic point is marked by a stone post of the usual form set 27 inches below the surface. Three stone reference-posts are set as follows: One bearing south 52° 32′ west, distant 364.1 metres; one bearing north 19° 19′ west, distant 22.7 metres; and one bearing north 59° 47′ east, distant 194.3 metres from the geodetic point. The first and last named stones are set on the southeast side of the road just northwest of the station, the first being set in the corner of the road fence and a line fence south of the station, and the last in the corner of the road fence and a line fence northeast of the station. The second stone is set by the fence on the northwest side of the road. The height of ground at the station above mean level of Lake Erie is 208.5 feet.

WARRENSVILLE, 1877.—This station is situated in Warrensville Township, Cuyahoga County, Ohio, about 3 miles northeast of Randall railway station on the Mahoning Division of the New York, Pennsylvania and Ohio Railway. The height of station used was 92 feet. The geodetic point is marked by a stone post of the usual form set 20 inches below the surface. Three stone reference-posts are set as follows: One bearing south 73° 46′ east, distant 56.7 metres; one bearing south 89° 13′ west, distant 75.5 metres; and one bearing north 55° 54′ east, distant 66.4 metres from the geodetic point. The first and last mentioned reference-stones are set by a line fence east of the station, and the second by a line fence west of the station. The height of ground at the station above mean level of Lake Erie is 641.5 feet.

WILLOUGHBY, 1877.—This station is situated on the shore of Lake Erie near the mouth of Chagrin River on the west side, and about 3 miles west of north of Willoughby railway station, in Willoughby Township, Lake County, Ohio. The height of station used was 107 feet. The geodetic point is marked in the usual manner. Three stone reference-posts are set as follows: One bearing south 86° 40′ east, distant 10.1 metres; one bearing north 74° 44′ east, distant 45.6 metres; and one bearing north 13° 25′ west, distant 27.0 metres from the geodetic point. The lake shore on the northwest is distant about 80 metres. The height of ground at the station above mean level of Lake Erie is 39.7 feet.

CHESTER, 1877.—This station is situated in Chester Township, Geauga County, Ohio, about ¾ mile west of Chester Cross Roads, and about 9 metres north of the east-and-west road from that place. The height of station used was 89 feet. The geodetic point is marked in the usual manner. Three stone reference-posts are set along the road fence south of the station as follows: One bearing south 63° 03′ west, distant 19.25 metres; one bearing south 6° 23′ west, distant 9.0 metres; and one bearing south 59° 30′ east, distant 17.75 metres from the geodetic point. The height of ground at the station above mean level of Lake Erie is 719.2 feet.

B.—STATIONS, SIGNALS, INSTRUMENTS, AND METHODS OF OBSERVATION.

§ **3.** See Chapter XVI, B.

C.—MEASURED AND ADJUSTED ANGLES BETWEEN THE LINES MICHIGAN CITY-BALD TOM AND WILLOUGHBY-CHESTER.

§ **4.** The following tables contain an abstract of the adjustment of the triangulation comprised within the above-stated limits. A sketch of this triangulation is given in Plate IV. The adjustment is made in three parts or sections, namely:

Section VIII, extending from line Michigan City–Bald Tom to line Fremont–Quincy.

Section IX, extending from line Fremont–Quincy to line Cedar Point–Stony Point.

Section X, extending from line Cedar Point–Stony Point to line Willoughby–Chester.

The scale of weights assigned to observed angles is as follows: On the same line with the name of each instrument here given is the number of combined results to the mean of which was assigned weight unity.

Troughton & Simms theodolite No. 1, 16.
Troughton & Simms theodolite No. 3, 16.
Troughton & Simms theodolite No. 4, 24 at Olmsted, Royalton, Grafton, and Camden.
Troughton & Simms theodolite No. 4, 20 at Townsend.
Troughton & Simms theodolite No. 4, 16 at remaining stations where it was used.
Repsold theodolite No. 1, 20.
Pistor & Martins theodolite No. 2, 16.

When the number of results obtained for an angle differed from the standard number by more than one-fourth of the latter, the weight assigned was the ratio of the former to the latter, rounded to the nearest one-fourth.

For a detailed explanation of the tables see Chapter XIV, C, § 7, and the remark in Chapter XV, C, § 6, relating to "No. meas." By the above division of the triangulation, a sum-angle condition was disregarded in the general adjustment at each of stations Michigan City, Fremont, Quincy, Cedar Point, Willoughby, and Chester. The locally adjusted angles at these stations, with resulting weights, were used in computing the general adjustment.

SECTION VIII.—*Triangulation from the line Michigan City - Bald Tom to the line Fremont - Quincy.*

MICHIGAN CITY—30.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Date, May, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angle.
	° ′ ″			″		″	″	° ′ ″
Bald Tom and Galena	54 47 01.324	30_1	16	5.2	1	+0.496	−0.262	54 47 01.558

NOTE.—The weight and local correction of 30_1 are taken from the previous section of the adjustment.

BALD TOM—33.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Date, June, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Bertrand and Carlisle	34 20 43.604	33_1	20	7.9	1	+0.048	+0.549	34 20 44.201
Carlisle and Galena	33 39 32.466	33_2	20	9.3	1	+0.048	−1.052	33 39 31.462
Galena and Michigan City	35 06 57.523	33_3	20	7.5	1	+0.048	+0.052	35 06 57.623
Michigan City and Bertrand	256 52 46.214	33_5	20	13.3	1	+0.049	+0.451	256 52 46.714

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(33_1)+(33_2)+(33_3)-0.193=0$$
$$(33_1)+2(33_2)+(33_3)-0.193=0$$
$$(33_1)+(33_2)+2(33_3)-0.193=0$$

GALENA—34.

[Observer, A. R. Flint. Instrument, Pistor & Martins theodolite No. 2. Date, September, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Michigan City and Bald Tom	90 06 01.579	34_1	16	7.1	1	+0.087	+0.188	90 06 01.854
Bald Tom and Bertrand	54 20 26.885	34_2	16	5.7	1	+0.087	−0.337	54 20 26.635
Bertrand and Carlisle	32 05 30.872	34_3	16	6.6	1	+0.087	−0.439	32 05 30.520
Carlisle and Michigan City	183 28 00.316	34_4	16	6.6	1	+0.087	+0.588	183 28 00.991

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(34_1)+(34_2)+(34_3)-0.348=0$$
$$(34_1)+2(34_2)+(34_3)-0.348=0$$
$$(34_1)+(34_2)+2(34_3)-0.348=0$$

Section VIII.—*Triangulation from the line Michigan City–Bald Tom to the line Fremont–Quincy*—Continued.

BERTRAND—35.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Date, June, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Milton and Penn	54 31 31.269	35_1	16	8.6	1	−0.029	+0.258	54 31 31.498
Penn and Carlisle	82 27 30.229	35_2	16	11.3	1	−0.030	−0.904	82 27 29.295
Carlisle and Galena	31 31 31.206	35_3	16	5.3	1	−0.029	+0.794	31 31 31.971
Galena and Bald Tom	57 39 19.343	35_4	16	8.0	1	−0.030	−0.299	57 39 19.014
Bald Tom and Milton	133 50 08.101	35_5	16	6.3	1	−0.030	+0.151	133 50 08.222

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(35_1)+(35_2)+(35_3)+(35_4)+0.148=0$$
$$(35_1)+2(35_2)+(35_3)+(35_4)+0.148=0$$
$$(35_1)+(35_2)+2(35_3)+(35_4)+0.148=0$$
$$(35_1)+(35_2)+(35_3)+2(35_4)+0.148=0$$

CARLISLE—36.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Date, September, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Galena and Bald Tom	59 54 32.221	36_1	16	5.8	1	+0.046	+0.057	59 54 32.324
Bald Tom and Bertrand	56 28 25.129	36_2	16	4.6	1	+0.047	+0.559	56 28 25.735
Bertrand and Milton	22 28 11.313	36_3	16	9.8	1	+0.046	−0.918	22 28 10.441
Milton and Penn	34 41 13.601	36_4	16	6.9	1	+0.047	−0.212	34 41 13.436
Penn and Galena	186 27 37.503	36_5	16	3.4	1	+0.047	+0.514	186 27 38.064

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(36_1)+(36_2)+(36_3)+(36_4)-0.233=0$$
$$(36_1)+2(36_2)+(36_3)+(36_4)-0.233=0$$
$$(36_1)+(36_2)+2(36_3)+(36_4)-0.233=0$$
$$(36_1)+(36_2)+(36_3)+2(36_4)-0.233=0$$

MILTON—37.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Date, October, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Calvin and Jefferson	33 36 05.626	37_1	16	6.2	1	−0.096	−0.375	33 36 05.155
Jefferson and Penn	75 36 29.914	37_2	16	6.4	1	−0.096	−0.315	75 36 29.503
Penn and Carlisle	51 46 54.881	37_3	16	9.0	1	−0.096	+0.179	51 46 54.964
Carlisle and Bertrand	20 32 48.765	37_4	16	7.2	1	−0.096	+0.560	20 32 49.229
Bertrand and Calvin	178 27 41.293	37_5	16	7.5	1	−0.095	−0.049	178 27 41.149

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(37_1)+(37_2)+(37_3)+(37_4)+0.479=0$$
$$(37_1)+2(37_2)+(37_3)+(37_4)+0.479=0$$
$$(37_1)+(37_2)+2(37_3)+(37_4)+0.479=0$$
$$(37_1)+(37_2)+(37_3)+2(37_4)+0.479=0$$

SECTION VIII.—*Triangulation from the line Michigan City–Bald Tom to the line* ***Fremont–Quincy***—Continued.

PENN—38.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Date, October, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Carlisle and Bertrand	40 23 08.566	38_1	16	6.8	1	−0.058	−0.879	40 23 07.629
Bertrand and Milton	53 08 44.389	38_2	18	8.5	1	−0.058	+0.695	53 08 45.026
Milton and Calvin	39 23 36.605	38_3	17	8.2	1	−0.058	+0.029	39 23 36.576
Calvin and Jefferson	34 48 45.446	38_4	17	5.7	1	−0.058	−0.083	34 48 45.305
Jefferson and Carlisle	192 15 45.283	38_5	17	8.5	1	−0.057	+0.238	192 15 45.464

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(38_1)+(38_2)+(38_3)+(38_4)+0.289=0$$
$$(38_1)+2(38_2)+(38_3)+(38_4)+0.289=0$$
$$(38_1)+(38_2)+2(38_3)+(38_4)+0.289=0$$
$$(38_1)+(38_2)+(38_3)+2(38_4)+0.289=0$$

CALVIN—39.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, May, 1878.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Sherman and Porter	8 16 26.722	39_1	16	5.7	1	+0.016	−1.112	8 16 25.626
Porter and Van Buren	22 16 39.841	39_2	18	3.9	1	+0.017	+0.933	22 16 40.791
Van Buren and Jefferson	31 43 07.476	39_3	18	3.1	1	+0.016	+0.049	31 43 07.541
Jefferson and Penn	78 08 16.569	39_4	16	4.4	1	+0.017	−0.064	78 08 16.522
Penn and Milton	31 23 49.758	39_5	16	3.7	1	+0.016	−0.127	31 23 49.647
Milton and Sherman	188 11 39.536	39_6	16	4.4	1	+0.016	+0.321	188 11 39.873

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(39_1)+(39_2)+(39_3)+(39_4)+(39_5)-0.098=0$$
$$(39_1)+2(39_2)+(39_3)+(39_4)+(39_5)-0.098=0$$
$$(39_1)+(39_2)+2(39_3)+(39_4)+(39_5)-0.098=0$$
$$(39_1)+(39_2)+(39_3)+2(39_4)+(39_5)-0.098=0$$
$$(39_1)+(39_2)+(39_3)+(39_4)+2(39_5)-0.098=0$$

JEFFERSON—40.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Dates, April and May, 1878.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Penn and Milton	30 11 09.818	40_1	16	3.1	1	+0.065	+0.152	30 11 10.035
Milton and Calvin	36 51 49.694	40_2	16	9.1	1	+0.066	−0.096	36 51 49.664
Calvin and Porter	42 07 22.333	40_3	17	6.2	1	+0.065	+0.308	42 07 22.706
Porter and Sherman	45 41 34.091	40_4	16	5.6	1	+0.066	+0.083	45 41 34.240
Sherman and Van Buren	21 59 39.920	40_5	16	4.6	1	+0.066	−0.797	21 59 39.189
Van Buren and Penn	183 08 23.750	40_6	16	4.2	1	+0.066	+0.350	183 08 24.166

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(40_1)+(40_2)+(40_3)+(40_4)+(40_5)-0.394=0$$
$$(40_1)+2(40_2)+(40_3)+(40_4)+(40_5)-0.394=0$$
$$(40_1)+(40_2)+2(40_3)+(40_4)+(40_5)-0.394=0$$
$$(40_1)+(40_2)+(40_3)+2(40_4)+(40_5)-0.394=0$$
$$(40_1)+(40_2)+(40_3)+(40_4)+2(40_5)-0.394=0$$

SECTION VIII.—*Triangulation from the line Michigan City–Bald Tom to the line Fremont–Quincy*—Continued.

PORTER—41.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, May, 1878.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Sherman and Van Buren	50 46 16.654	41_1	18	4.3	1	−0.242	−0.077	50 46 16.335
Van Buren and Jefferson	57 46 38.243	41_2	16	4.4	1	−0.242	−0.023	57 46 37.978
Jefferson and Calvin	83 52 49.259	41_3	16	3.3	1	−0.242	+0.473	83 52 49.490
Calvin and Sherman	167 34 16.812	41_4	18	4.3	1	−0.242	−0.373	167 34 16.197

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(41_1)+(41_2)+(41_3)+0.968=0$$
$$(41_1)+2(41_2)+(41_3)+0.968=0$$
$$(41_1)+(41_2)+2(41_3)+0.968=0$$

VAN BUREN—42.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, May, 1878.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Jefferson and Calvin	38 28 16.908	42_1	16	3.1	1	−0.058	+0.243	38 28 17.093
Calvin and Porter	16 03 52.416	42_2	16	2.6	1	−0.058	−0.243	16 03 52.115
Porter and Sherman	85 45 49.792	42_3	16	6.1	1	−0.058	−0.216	85 45 49.518
Sherman and Mongo	52 35 17.153	42_4	16	6.0	1	−0.058	−0.116	52 35 16.979
Mongo and Jefferson	167 06 44.021	42_5	16	6.8	1	−0.058	+0.332	167 06 44.295

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(42_1)+(42_2)+(42_3)+(42_4)+0.290=0$$
$$(42_1)+2(42_2)+(42_3)+(42_4)+0.290=0$$
$$(42_1)+(42_2)+2(42_3)+(42_4)+0.290=0$$
$$(42_1)+(42_2)+(42_3)+2(42_4)+0.290=0$$

SHERMAN—43.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite, No. 1. Date, September, 1878.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Bronson and Mongo	51 33 31.608	43_1	16	2.2	1	−0.041	+0.068	51 33 31.635
Mongo and Van Buren	82 45 41.030	43_2	16	6.4	1	−0.041	−0.301	82 45 40.688
Van Buren and Jefferson	17 42 22.268	43_3	16	4.2	1	−0.041	+0.401	17 42 22.628
Jefferson and Porter	25 45 33.693	43_4	16	3.6	1	−0.041	−1.206	25 45 32.446
Porter and Calvin	4 09 17.514	43_5	16	3.9	1	−0.041	+0.892	4 09 18.365
Calvin and Bronson	178 03 34.133	43_6	16	4.5	1	−0.041	+0.146	178 03 34.238

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(43_1)+(43_2)+(43_3)+(43_4)+(43_5)+0.246=0$$
$$(43_1)+2(43_2)+(43_3)+(43_4)+(43_5)+0.246=0$$
$$(43_1)+(43_2)+2(43_3)+(43_4)+(43_5)+0.246=0$$
$$(43_1)+(43_2)+(43_3)+2(43_4)+(43_5)+0.246=0$$
$$(43_1)+(43_2)+(43_3)+(43_4)+2(43_5)+0.246=0$$

SECTION VIII.—*Triangulation from the line Michigan City–Bald Tom to the line Fremont–Quincy*—Continued.

MONGO—44.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, September, 1878.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Van Buren and Sherman............	44 39 03.939	44_1	18	3.9	1	−0.155	−0.277	44 39 03.507
Sherman and Bronson	48 20 14.027	44_2	18	5.1	1	−0.156	+0.092	48 20 13.963
Bronson and Fremont	79 21 42.987	44_3	18	8.1	1	−0.155	+0.016	79 21 42.848
Fremont and Van Buren............	187 38 59.669	44_4	18	3.0	1	−0.156	+0.169	187 38 59.682

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(44_1)+(44_2)+(44_3)+0.622=0$$
$$(44_1)+2(44_2)+(44_3)+0.622=0$$
$$(44_1)+(44_2)+2(44_3)+0.622=0$$

BRONSON—45.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, September, 1878.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Quincy and Fremont.................	58 53 49.490	45_1	16	6.0	1	−0.076	+0.042	58 53 49.456
Fremont and Mongo	61 17 52.291	45_2	16	4.1	1	−0.077	−0.091	61 17 52.123
Mongo and Sherman............. ...	80 06 15.286	45_3	16	3.9	1	−0.076	0.014	80 06 15.196
Sherman and Quincy................	159 42 03.239	45_4	16	5.6	1	−0.077	+0.063	159 42 03.225

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(45_1)+(45_2)+(45_3)+0.306=0$$
$$(45_1)+2(45_2)+(45_3)+0.306=0$$
$$(45_1)+(45_2)+2(45_3)+0.306=0$$

FREMONT—46.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, October, 1878.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Mongo and Bronson	39 20 26.133	46_1	16	4.6	1	+0.161	−0.115	39 20 26.179
Bronson and Quincy	62 01 52.306	46_2	16	3.6	1	+0.161	−0.016	62 01 52.451
Quincy and Reading.................	45 34 24.583	46_3	16	4.6	1	+0.161	....	....
Reading and Mongo	213 03 16.334	46_4	16	6.0	1	+0.161	....	....

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(46_1)+(46_2)+(46_3)-0.644=0$$
$$(46_1)+2(46_2)+(46_3)-0.644=0$$
$$(46_1)+(46_2)+2(46_3)-0.644=0$$

QUINCY—47.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, October, 1878.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Hillsdale and Reading	31 53 15.287	47_1	17	3.5	1	−0.103	....	....
Reading and Fremont	54 23 55.762	47_2	17	4.4	1	−0.103	....	....
Fremont and Bronson	59 04 20.005	47_3	17	5.9	1	−0.103	−0.016	59 04 19.886
Bronson and Hillsdale	214 38 29.359	47_4	17	8.5	1	−0.104	....	....

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(47_1)+(47_2)+(47_3)+0.413=0$$
$$(47_1)+2(47_2)+(47_3)+0.413=0$$
$$(47_1)+(47_2)+2(47_3)+0.413=0$$

Numerical equations of condition in the triangulation from the line Michigan City–Bald Tom to the line Fremont–Quincy.

SIDE-EQUATIONS.

LIII.	(10)	$-13.7944\,[34_2]+1.3126\,[34_3]-0.3011[35_3]+13.0324[35_4]-12.2009[36_1]+13.9501[36_2]$	$-7.043=0$
LIV.	(10)	$-15.0045\,[35_1]+2.7875\,[35_2]-13.5915[36_3]+16.8307[36_4]-9.8717[37_3]+6.7079[37_4]$	$-4.501=0$
LVIII.	($14\frac{2}{7}$)	$-31.6888\,[37_1]+5.4028\,[37_2]-5.9556[38_3]+24.3246[38_4]-11.8931[39_4]-7.4706[39_5]$	$-9.694=0$
LXIII.	(10)	$+11.0680\,[39_1]-4.2315\,[39_2]-4.2315[39_3]+7.0648[41_1]+7.0648[41_2]+2.2575[41_3]$	
		$+7.0386\,[43_4]-36.5951\,[43_5]$	$+57.247=0$
LXV.	($12\frac{1}{2}$)	$-15.2995\,[39_2]+18.7668\,[39_3]-13.2709[41_2]+2.2575[41_3]-11.4986[42_1]+14.9986[42_2]$	$+18.427=0$
LXVII.	($12\frac{1}{2}$)	$-8.3876\,[40_3]-8.3876\,[40_4]-7.5845[40_5]-26.4972[42_1]-4.4095[42_2]-4.4095[42_3]$	
		$-19.2123\,[43_3]+17.3828\,[43_4]+17.3828[43_5]$	$+14.805=0$

NOTE.—In the solution for determining the general corrections, each of the side-equations was divided by the number inclosed in parenthesis and placed opposite it.

ANGLE-EQUATIONS.

XLIX.	$[30_1]+[33_3]+[34_1]$	$+0.021=0$
L.	$[33_1]+[33_2]+[34_2]+[35_4]$	$+1.139=0$
LI.	$[33_1]+[35_3]+[35_4]+[36_2]$	$-1.603=0$
LII.	$[33_2]+[34_2]+[34_3]+[36_1]$	$+1.771=0$
LV.	$[35_1]+[37_3]+[37_4]+[38_2]$	$-1.692=0$
LVI.	$[35_2]+[36_3]+[36_4]+[38_1]$	$+2.913=0$
LVII.	$[36_4]+[37_3]+[38_1]+[38_2]$	$+0.217=0$
LIX.	$[37_1]+[39_4]+[39_5]+[40_2]$	$+0.662=0$
LX.	$[37_2]+[38_3]+[38_4]+[40_1]$	$+0.217=0$
LXI.	$[38_4]+[39_4]+[40_1]+[40_2]$	$+0.091=0$
LXII.	$[39_1]+[39_2]+[39_3]+[40_3]+[40_4]+[43_4]+[43_5]$	$+0.053=0$
LXIV.	$[39_2]+[39_3]+[40_3]+[41_3]$	$-1.763=0$
LXVI.	$[39_3]+[40_3]+[40_4]+[40_5]+[42_1]$	$+0.114=0$
LXVIII.	$[40_4]+[40_5]+[41_2]+[42_1]+[42_2]$	$+0.737=0$
LXIX.	$[40_4]+[41_1]+[41_2]+[43_4]$	$\pm 1.223=0$
LXX.	$[41_1]+[42_3]+[43_3]+[43_4]$	$+1.098=0$
LXXI.	$[42_4]+[43_2]+[44_1]$	$+0.694=0$
LXXII.	$[43_1]+[44_2]+[45_3]$	$-0.146=0$
LXXIII.	$[44_3]+[45_2]+[46_1]$	$+0.190=0$
LXXIV.	$[45_1]+[46_2]+[47_3]$	$-0.009=0$

General corrections in terms of the correlates.

$[30_1]$	$=+0.65625$ XLIX					
$[33_1]$	$=-0.25000$ XLIX	$+0.50000$ L	$+0.75000$ LI	-0.25000 LII		
$[33_2]$	$=-0.25000$ XLIX	$+0.50000$ L	-0.25000 LI	$+0.75000$ LII		
$[33_3]$	$=+0.75000$ XLIX	-0.50000 L	-0.25000 LI	-0.25000 LII		
$[34_1]$	$=+0.75000$ XLIX	-0.25000 L	-0.50000 LII	$+0.31204$ LIII		
$[34_2]$	$=-0.25000$ XLIX	$+0.75000$ L	$+0.50000$ LII	-1.06740 LIII		
$[34_3]$	$=-0.25000$ XLIX	-0.25000 L	$+0.50000$ LII	$+0.44331$ LIII		
$[35_1]$	$=-0.20000$ L	-0.40000 LI	-0.25463 LIII	-1.25611 LIV	$+0.80000$ LV	-0.20000 LVI
$[35_2]$	$=-0.20000$ L	-0.40000 LI	-0.25463 LIII	$+0.52309$ LIV	-0.20000 LV	$+0.80000$ LVI
$[35_3]$	$=-0.20000$ L	$+0.60000$ LI	-0.28473 LIII	$+0.24434$ LIV	-0.20000 LV	-0.20000 LVI

General corrections in terms of the correlates—Continued.

[35_4]	=+0.80000 L	+0.60000 LI	+1.04861 LIII	+0.24434 LIV	—0.20000 LV
	—0.20000 LVI				
[36_1]	=—0.20000 LI	+0.80000 LII	—1.25508 LIII	—0.06478 LIV	—0.40000 LVI
	—0.20000 LVII				
[36_2]	=+0.80000 LI	—0.20000 LII	+1.36003 LIII	—0.06478 LIV	—0.40000 LVI
	—0.20000 LVII				
[36_3]	=—0.20000 LI	—0.20000 LII	- 0.03498 LIII	—1.42393 LIV	+0.60000 LVI
	—0.20000 LVII				
[36_4]	=—0.20000 LI	—0.20000 LII	– 0.03498 LIII	+1.61829 LIV	+0.60000 LVI
	+0.80000 LVII				
[37_1]	=+0.06327 LIV	—0.40000 LV	—0.20000 LVII	—1.85022 LVIII	+0.80000 LIX
	—0.20000 LX				
[37_2]	=+0.06327 LIV	—0.40000 LV	—0.20000 LVII	+0.74620 LVIII	—0.20000 LIX
	+0.80000 LX				
[37_3]	=—0.92390 LIV	+0.60000 LV	+0.80000 LVII	+0.36800 LVIII	—0.20000 LIX
	—0.20000 LX				
[37_4]	=+0.73406 LIV	+0.60000 LV	—0.20000 LVII	+0.36800 LVIII	—0.20000 LIX
	—0.20000 LX				
[38_1]	=—0.20000 LV	+0.80000 LVI	+0.60000 LVII	—0.25716 LVIII	—0.40000 LX
	—0.20000 LXI				
[38_2]	=+0.80000 LV	—0.20000 LVI	+0.60000 LVII	—0.25716 LVIII	—0.40000 LX
	—0.20000 LXI				
[38_3]	=—0.20000 LV	—0.20000 LVI	—0.40000 LVII	—0.67405 LVIII	+0.60000 LX
	—0.20000 LXI				
[38_4]	=—0.20000 LV	—0.20000 LVI	—0.40000 LVII	+1.44556 LVIII	+0.60000 LX
	+0.80000 LXI				
[39_1]	=+0.22591 LVIII	—0.33333 LIX	—0.16667 LXI	+0.50000 LXII	+1.06339 LXIII
	—0.33333 LXIV	—0.04623 LXV	—0.16667 LXVI		
[39_2]	=+0.22591 LVIII	—0.33333 LIX	—0.16667 LXI	+0.50000 LXII	—0.46657 LXIII
	+0.66667 LXIV	—1.27019 LXV	—0.16667 LXVI		
[39_3]	=+0.22591 LVIII	—0.33333 LIX	—0.16667 LXI	+0.50000 LXII	—0.46657 LXIII
	+0.66667 LXIV	+1.45511 LXV	+0.83333 LXVI		
[39_4]	=—0.60661 LVIII	+0.66667 LIX	+0.83333 LXI	—0.50000 LXII	—0.04341 LXIII
	—0.33333 LXIV	—0.04623 LXV	—0.16667 LXVI		
[39_5]	=—0.29703 LVIII	+0.66667 LIX	—0.16667 LXI	—0.50000 LXII	—0.04341 LXIII
	—0.33333 LXIV	—0.04623 LXV	—0.16667 LXVI		
[40_1]	=—0.16667 LIX	+0.83333 LX	+0.66667 LXI	—0.33333 LXII	—0.16667 LXIV
	—0.50000 LXVI	+0.32480 LXVII	—0.33333 LXVIII	—0.16667 LXIX	
[40_2]	=+0.83333 LIX	—0.16667 LX	+0.66667 LXI	—0.33333 LXII	—0.16667 LXIV
	—0.50000 LXVI	+0.32480 LXVII	—0.33333 LXVIII	—0.16667 LXIX	
[40_3]	=—0.16667 LIX	—0.16667 LX	—0.33333 LXI	+0.66667 LXII	+0.83333 LXIV
	+0.50000 LXVI	—0.34621 LXVII	—0.33333 LXVIII	—0.16667 LXIX	
[40_4]	=—0.16667 LIX	—0.16667 LX	—0.33333 LXI	+0.66667 LXII	—0.16667 LXIV
	+0.50000 LXVI	—0.34621 LXVII	+0.66667 LXVIII	+0.83333 LXIX	
[40_5]	=—0.16667 LIX	—0.16667 LX	—0.33333 LXI	—0.33333 LXII	—0.16667 LXIV
	+0.50000 LXVI	—0.28196 LXVII	+0.66667 LXVIII	—0.16667 LXIX	
[41_1]	=+0.29680 LXIII	—0.25000 LXIV	+0.22027 LXV	—0.25000 LXVIII	+0.50000 LXIX
	+0.75000 LXX				
[41_2]	=+0.29680 LXIII	—0.25000 LXIV	—0.84140 LXV	+0.75000 LXVIII	+0.50000 LXIX
	—0.25000 LXX				
[41_3]	=—0.18393 LXIII	+0.75000 LXIV	+0.40087 LXV	—0.25000 LXVIII	—0.50000 LXIX
	—0.25000 LXX				

General corrections in terms of the correlates—Continued.

$[42_1]$	=−0.97589 LXV	+0.80000 LXVI	−1.55472 LXVII	+0.60000 LXVIII	−0.20000 LXX
	−0.20000 LXXI				
$[42_2]$	=+1.14389 LXV	−0.20000 LXVI	+0.21230 LXVII	+0.60000 LXVIII	−0.20000 LXX
	−0.20000 LXXI				
$[42_3]$	=−0.05600 LXV	−0.20000 LXVI	+0.21230 LXVII	−0.40000 LXVIII	+0.80000 LXX
	−0.20000 LXXI				
$[42_4]$	=−0.05600 LXV	−0.20000 LXVI	+0.56506 LXVII	−0.40000 LXVIII	−0.20000 LXX
	+0.80000 LXXI				
$[43_1]$	=−0.33333 LXII	+0.49261 LXIII	−0.20738 LXVII	−0.16667 LXIX	−0.33333 LXX
	−0.16667 LXXI	+0.83333 LXXII			
$[43_2]$	=−0.33333 LXII	+0.49261 LXIII	−0.20738 LXVII	−0.16667 LXIX	−0.33333 LXX
	+0.83333 LXXI	−0.16667 LXXII			
$[43_3]$	=−0.33333 LXII	+0.49261 LXIII	−1.74436 LXVII	−0.16667 LXIX	+0.66667 LXX
	−0.16667 LXXI	−0.16667 LXXII			
$[43_4]$	=+0.66667 LXII	+1.19647 LXIII	+1.18324 LXVII	+0.83333 LXIX	+0.66667 LXX
	−0.16667 LXXI	−0.16667 LXXII			
$[43_5]$	=+0.66667 LXII	−3.16690 LXIII	+1.18324 LXVII	−0.16667 LXIX	−0.33333 LXX
	−0.16667 LXXI	−0.16667 LXXII			
$[44_1]$	=+0.75000 LXXI	−0.25000 LXXII	−0.25000 LXXIII		
$[44_2]$	=−0.25000 LXXI	+0.75000 LXXII	−0.25000 LXXIII		
$[44_3]$	=−0.25000 LXXI	−0.25000 LXXII	+0.75000 LXXIII		
$[45_1]$	=−0.25000 LXXII	−0.25000 LXXIII	+0.75000 LXXIV		
$[45_2]$	=−0.25000 LXXII	+0.75000 LXXIII	−0.25000 LXXIV		
$[45_3]$	=+0.75000 LXXII	−0.25000 LXXIII	−0.25000 LXXIV		
$[46_1]$	=+0.75000 LXXIII				
$[46_2]$	=+0.75000 LXXIV				
$[47_3]$	=+0.75000 LXXIV				

Normal equations for determining the correlates.

No. of equation.						
49.	0=+0.02100	+2.15625 XLIX	−0.75000 L	−0.25000 LI	−0.75000 LII	+0.31204 LIII
50.	0=+1.13900	−0.75000 XLIX	+2.55000 L	+1.10000 LI	+1.00000 LII	−0.01879 LIII
		+0.24434 LIV	−0.20000 LV	−0.20000 LVI		
51.	0=−1.60300	−0.25000 XLIX	+1.10000 L	+2.75000 LI	−0.45000 LII	+2.12391 LIII
		+0.42390 LIV	−0.40000 LV	−0.80000 LVI	−0.20000 LVII	
52.	0=+1.77100	−0.75000 XLIX	+1.00000 L	−0.45000 LI	+2.55000 LII	−1.87917 LIII
		−0.06478 LIV	−0.40000 LVI	−0.20000 LVII		
53.	0=−0.70430	+0.31204 XLIX	−0.01879 L	+2.12391 LI	−1.87917 LII	+6.33433 LIII
		+0.29974 LIV	−0.25463 LV	−0.32459 LVI	−0.03498 LVII	
54.	0=−0.45010	+0.24434 L	+0.42390 LI	−0.06478 LII	+0.29974 LIII	+8.09402 LIV
		−1.44595 LV	+0.71745 LVI	+0.69439 LVII	−0.11643 LVIII	+0.06327 LIX
		+0.06327 LX				
55.	0=−1.69200	−0.20000 L	−0.40000 LI	−0.25463 LIII	−1.44595 LIV	+2.80000 LV
		−0.40000 LVI	+1.20000 LVII	+0.47884 LVIII	−0.40000 LIX	−0.80000 LX
		−0.20000 LXI				
56.	0=+2.91300	−0.20000 L	−0.80000 LI	−0.40000 LII	−0.32459 LIII	+0.71745 LIV
		−0.40000 LV	+2.80000 LVI	+1.20000 LVII	−0.25716 LVIII	−0.40000 LX
		−0.20000 LXI				
57.	0=+0.21700	−0.20000 LI	−0.20000 LII	−0.03498 LIII	+0.69439 LIV	+1.20000 LV
		+1.20000 LVI	+2.80000 LVII	−0.14632 LVIII	−0.20000 LIX	−1.00000 LX
		−0.40000 LXI				

Normal equations for determining the correlates—Continued.

No. of equation.						
58.	0=−0.67858	−0.11643 LIV	+0.47884 LV	−0.25716 LVI	−0.14632 LVII	+7.78913 LVIII
		−2.75386 LIX	+1.51771 LX	+0.83895 LXI	+0.67773 LXII	+0.05884 LXIII
		+0.45182 LXIV	+0.06267 LXV	+0.22591 LXVI		
59.	0=+0.66200	+0.06327 LIV	−0.40000 LV	−0.20000 LVII	−2.75386 LVIII	+ 2.96667 LIX
		−0.36667 LX	+1.33333 LXI	−1.33333 LXII	−0.08682 LXIII	− 0.83333 LXIV
		−0.09246 LXV	−0.83333 LXVI	+0.32480 LXVII	−0.33333 LXVIII	− 0.16667 LXIX
60.	0=+0.21700	+0.06327 LIV	−0.80000 LV	−0.40000 LVI	−1.00000 LVII	+ 1.51771 LVIII
		−0.36667 LIX	+2.83333 LX	+1.26667 LXI	−0.33333 LXII	− 0.16667 LXIV
		−0.50000 LXVI	+0.32480 LXVII	−0.33333 LXVIII	−0.16667 LXIX	
61.	0=+0.09100	−0.20000 LV	−0.20000 LVI	−0.40000 LVII	+0.83895 LVIII	+ 1.33333 LIX
		+1.26667 LX	+2.96667 LXI	−1.16667 LXII	−0.04341 LXIII	− 0.66667 LXIV
		−0.04623 LXV	−1.16667 LXVI	+0.64960 LXVII	−0.66667 LXVIII	− 0.33333 LXIX
62.	0=+0.05300	+0.67773 LVIII	−1.33333 LIX	−0.33333 LX	−1.16667 LXI	+ 4.16667 LXII
		−1.84018 LXIII	+1.66667 LXIV	+0.13869 LXV	+1.50000 LXVI	+ 1.67406 LXVII
		+0.33333 LXVIII	+1.33333 LXIX	+0.33333 LXX	−0.33333 LXXI	− 0.33333 LXXII
63.	0=+5.72470	+0.05884 LVIII	−0.08682 LIX	−0.04341 LXI	−1.84018 LXII	+14.38111 LXIII
		−1.11707 LXIV	−0.47773 LXV	−0.46657 LXVI	−3.49724 LXVII	+ 0.29680 LXVIII
		+1.79007 LXIX	+1.98588 LXX	+0.49261 LXXI	+0.49261 LXXII	
64.	0=−1.76300	+0.45182 LVIII	−0.83333 LIX	−0.16667 LX	−0.66667 LXI	+ 1.66667 LXII
		−1.11707 LXIII	+2.91667 LXIV	+0.58579 LXV	+1.16667 LXVI	− 0.34621 LXVII
		−0.58333 LXVIII	−0.66667 LXIX	−0.25000 LXX		
65.	0=+1.47416	+0.06267 LVIII	−0.09246 LIX	−0.04623 LXI	+0.13869 LXII	− 0.47773 LXIII
		+0.58579 LXIV	+6.97522 LXV	+0.47922 LXVI	+1.68491 LXVII	− 0.67340 LXVIII
		−0.62113 LXIX	+0.16427 LXX	−0.05600 LXXI		
66.	0=+0.11400	+0.22591 LVIII	−0.83333 LIX	−0.50000 LX	−1.16667 LXI	+ 1.50000 LXII
		−0.46657 LXIII	+1.16667 LXIV	+0.47922 LXV	+3.13333 LXVI	− 2.52910 LXVII
		+1.60000 LXVIII	+0.50000 LXIX	−0.20000 LXX	−0.20000 LXXI	
67.	0=+1.18440	+0.32480 LIX	+0.32480 LX	+0.64960 LXI	+1.67406 LXII	−3.49724 LXIII
		−0.34621 LXIV	+1.68491 LXV	−2.52910 LXVI	+9.75351 LXVII	−1.97059 LXVIII
		+0.83703 LXIX	−0.34882 LXX	+0.35768 LXXI	−0.20738 LXXII	
68.	0=+0.73700	−0.33333 LIX	−0.33333 LX	−0.66667 LXI	+0.33333 LXII	+0.29680 LXIII
		−0.58333 LXIV	−0.67340 LXV	+1.60000 LXVI	−1.97059 LXVII	+3.28333 LXVIII
		+1.16667 LXIX	−0.65000 LXX	−0.40000 LXXI		
69.	0=+1.22300	−0.16667 LIX	−0.16667 LX	−0.33333 LXI	+1.33333 LXII	+1.79007 LXIII
		−0.66667 LXIV	−0.62113 LXV	+0.50000 LXVI	+0.83703 LXVII	+1.16667 LXVIII
		+2.66667 LXIX	+1.16667 LXX	−0.16667 LXXI	−0.16667 LXXII	
70.	0=+1.09800	+0.33333 LXII	+1.98588 LXIII	−0.25000 LXIV	+0.16427 LXV	−0.20000 LXVI
		−0.34882 LXVII	−0.65000 LXVIII	+1.16667 LXIX	+2.88333 LXX	−0.53333 LXXI
		−0.33333 LXXII				
71.	0=+0.69400	−0.33333 LXII	+0.49261 LXIII	−0.05600 LXV	−0.20000 LXVI	+0.35768 LXVII
		−0.40000 LXVIII	−0.16667 LXIX	−0.53333 LXX	+2.38333 LXXI	−0.41667 LXXII
		−0.25000 LXXIII				
72.	0=−0.14600	−0.33333 LXII	+0.49261 LXIII	−0.20738 LXVII	−0.16667 LXIX	−0.33333 LXX
		−0.41667 LXXI	+2.33333 LXXII	−0.50000 LXXIII	−0.25000 LXXIV	
73.	0=+0.19000	−0.25000 LXXI	−0.50000 LXXII	+2.25000 LXXIII	−0.25000 LXXIV	
74.	0=−0.00900	−0.25000 LXXII	−0.25000 LXXIII	+2.25000 LXXIV		

Values of the correlates and their logarithms.

XLIX =−0.3987 log 9.6006898−	LXII =−0.7087 log 9.8504379−
L =−0.5326 log 9.7264175−	LXIII =−0.6548 log 9.8161418−
LI =+0.6304 log 9.7996438+	LXIV =+0.9998 log 9.9999131+
LII =−0.9707 log 9.9870806−	LXV =−0.0358 log 8.5543680−
LIII =−0.4201 log 9.6233630−	LXVI =−0.7868 log 9.8958644−
LIV =+0.2312 log 9.3639315+	LXVII =−0.6771 log 9.8306721−
LV =+0.4542 log 9.6572376+	LXVIII =−0.7703 log 9.8866768−
LVI =−1.1189 log 0.0487991−	LXIX =+1.5449 log 0.1889088+
LVII =+0.0026 log 7.4216039+	LXX =−0.7860 log 9.8954502−
LVIII = −0.0724 log 8.8596186−	LXXI =−0.4470 log 9.6503561−
LIX =−0.4863 log 9.6868953−	LXXII =−0.0776 log 8.8900855−
LX =−0.2385 log 9.3775248−	LXXIII =−0.1538 log 9.1869281−
LXI =+0.0409 log 8.6118295+	LXXIV =−0.0217 log 8.3367176−

Values of the general corrections.

″	″	″	″	″
$[30_1]$ =−0.262	$[36_1]$ =+0.057	$[38_4]$ =−0.083	$[41_1]$ =−0.077	$[43_5]$ =+0.892
$[33_1]$ =+0.549	$[36_2]$ =+0.559	$[39_1]$ =−1.112	$[41_2]$ =−0.023	$[44_1]$ =−0.277
$[33_2]$ =−1.052	$[36_3]$ =−0.918	$[39_2]$ =+0.933	$[41_3]$ =+0.473	$[44_2]$ =+0.092
$[33_3]$ =+0.052	$[36_4]$ =−0.212	$[39_3]$ =+0.049	$[42_1]$ =+0.243	$[44_3]$ =+0.016
$[34_1]$ =+0.188	$[37_1]$ =−0.375	$[39_4]$ =−0.064	$[42_2]$ =−0.243	$[45_1]$ =+0.042
$[34_2]$ =−0.337	$[37_2]$ =−0.315	$[39_5]$ =−0.127	$[42_3]$ =−0.216	$[45_2]$ =−0.091
$[34_3]$ =−0.439	$[37_3]$ =+0.179	$[40_1]$ =+0.152	$[42_4]$ =−0.116	$[45_3]$ =−0.014
$[35_1]$ =+0.258	$[37_4]$ =+0.560	$[40_2]$ =−0.096	$[43_1]$ =+0.068	$[46_1]$ =−0.115
$[35_2]$ =−0.904	$[38_1]$ =−0.879	$[40_3]$ =+0.308	$[43_2]$ =−0.301	$[46_2]$ =−0.016
$[35_3]$ =+0.794	$[38_2]$ =+0.695	$[40_4]$ =+0.083	$[43_3]$ =+0.401	$[47_3]$ =−0.016
$[35_4]$ =−0.299	$[38_3]$ =+0.029	$[40_5]$ =−0.797	$[43_4]$ =−1.206	

Residuals resulting from substitution of general corrections in numerical equations of condition.

No. of equation.	Residual.	No. of equation.	Residual.
49	0.0000	62	0.0000
50	0.0000	63	+0.0040
51	−0.0001	64	−0.0001
52	0.0000	65	+0.0002
53	−0.0002	66	0.0000
54	−0.0001	67	−0.0012
55	0.0000	68	+0.0001
56	0.0000	69	+0.0001
57	0.0000	70	+0.0001
58	−0.0003	71	0.0000
59	0.0000	72	0.0000
60	0.0000	73	0.0000
61	0.0000	74	0.0000

SECTION IX.—*Triangulation from the line Fremont-Quincy to the line Cedar Point-Stony Point.*

FREMONT—46.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, October, 1878.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angle.
	° ′ ″			″		″	″	° ′ ″
Quincy and Reading	45 34 24.583	46_3	16	4.6	1	+0.161	−0.588	45 34 24.156

NOTE.—The weight and local correction of 46_3 are taken from the previous section of the adjustment.

SECTION IX.—*Triangulation from the line Fremont - Quincy to the line Cedar Point - Stony Point*—Continued.

QUINCY—47.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, October, 1878.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Hillsdale and Reading	31 53 15. 287	47_1	17	3. 5	1	−0. 103	−0. 216	31 53 14. 968
Reading and Fremont	54 23 55. 762	47_2	17	4. 4	1	−0. 103	−0. 588	54 23 55. 071

NOTE.—The weights and local corrections of 47_1 and 47_2 are taken from the previous section of the adjustment.

READING—48.

[Observers, G. Y. Wisner and R. S. Woodward. Instruments, Troughton & Simms theodolites Nos. 1 and 3. Dates, October, 1878, and July, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Fremont and Quincy	80 01 42. 379	48_1	16	4. 4	1	+0. 018	−0. 430	80 01 41. 967
Pittsford and Quincy	220 35 01. 027	48_{1+5}	4	1. 3	0. 25	−0. 199	−0. 078	220 35 00. 750
Quincy and Hillsdale	85 29 02. 121	48_2	21	5. 7	1. 25	−0. 016	+0. 021	85 29 02. 126
Quincy and Pittsford	139 24 59. 448	48_{2+3+4}	4	3. 9	0. 25	−0. 276	+0. 078	139 24 59. 250
Hillsdale and Wheatland	29 57 24. 894	48_3	16	5. 0	1	+0. 106	+0. 004	29 57 25. 004
Hillsdale and Pittsford	53 55 57. 168	48_{3+4}	21	7. 6	1. 25	−0. 101	+0. 057	53 55 57. 124
Wheatland and Pittsford	23 58 31. 961	48_4	16	4. 0	1	+0. 106	+0. 053	23 58 32. 120
Pittsford and Fremont	140 33 18. 413	48_5	16	4. 0	1	+0. 018	+0. 352	140 33 18. 783

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(48_1)+\quad(48_2)+\quad(48_3)+\quad(48_4)-0.232=0$$
$$(48_1)+2.75(48_2)+1.50(48_3)+1.50(48_4)-0.291=0$$
$$(48_1)+1.50(48_2)+3.75(48_3)+2.75(48_4)-0.682=0$$
$$(48_1)+1.50(48_2)+2.75(48_3)+3.75(48_4)-0.682=0$$

NOTE.—48_3 and 48_4 were measured by Mr. Woodward with the Troughton & Simms No. 3, in July, 1879. The remainder were measured by Mr. Wisner with the Troughton & Simms No. 1, in October, 1878.

BUNDAY—49.

[Observer, G. Y. Wisner. Instruments, Pistor & Martins theodolite No. 1, and Troughton & Simms theodolite No 1. Dates, May and June, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Woodstock and Wheatland	63 53 06. 544	49_1	20	7. 2	1	−0. 018	−0. 535	63 53 05. 991
Wheatland and Hillsdale	48 28 35. 971	49_2	20	4. 9	1	−0. 018	+0. 046	48 28 35. 999
Hillsdale and Woodstock	247 38 17. 534	49_3	32	8. 7	1. 5	−0. 013	+0. 489	247 38 18. 010

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.5(49_1)+1.5(49_2)+0.073=0$$
$$1.5(49_1)+2.5(49_2)+0.073=0$$

NOTE.—49_3 was partly measured with the Troughton & Simms instrument, the remainder of the angles with the Pistor & Martins instrument.

SECTION IX.—*Triangulation from the line Fremont - Quincy to the line Cedar Point - Stony Point*—Continued.

HILLSDALE—50.

[Observer, G. Y. Wisner. Instruments, Pistor & Martins theodolite No. 2, and Troughton & Simms theodolite No. 1. Dates, November 1878, and June, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	$[v]$	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Bunday and Wheatland	44 25 40.613	50_1	16	5.5	1	−0.053	−0.607	44 25 39.953
Bunday and Pittsford	80 47 32.819	50_{1+2}	18	6.8	1	−0.315	−0.140	80 47 32.364
Wheatland and Pittsford	36 21 51.996	50_2	16	5.8	1	−0.052	+0.467	36 21 52.411
Pittsford and Reading	91 34 24.758	50_3	18	5.8	1	+0.175	−0.064	91 34 24.869
Pittsford and Bunday	279 12 27.619	50_{3+4+5}	16	6.3	1	−0.123	+0.140	279 12 27.636
Reading and Quincy	62 37 43.405	50_4	18	4.3	1	+0.175	−0.043	62 37 43.537
Quincy and Bunday	125 00 18.808	50_5	17	4.2	1	+0.175	+0.247	125 00 19.230

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$4(50_1)+3(50_2)+(50_3)+(50_4)+0.018=0$$
$$3(50_1)+4(50_2)+(50_3)+(50_4)+0.018=0$$
$$(50_1)+(50_2)+2(50_3)+(50_4)-0.420=0$$
$$(50_1)+(50_2)+(50_3)+2(50_4)-0.420=0$$

NOTE.—50_1, 50_2, and 50_{3+4+5} were measured with the Pistor & Martins instrument, in 1879; the remainder with the Troughton & Simms instrument, in 1878.

WHEATLAND—51.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Date, June, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	$[v]$	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Woodstock and Fairfield	43 30 48.485	51_1	16	3.8	1	−0.031	−0.069	43 30 48.385
Woodstock and Pittsford	127 57 45.129	51_{1+2}	8	4.5	0.5	+0.143	+0.010	127 57 45.282
Fairfield and Pittsford	84 26 56.849	51_2	16	6.9	1	−0.031	+0.079	84 26 56.897
Pittsford and Reading	56 02 33.220	51_3	16	8.3	1	−0.007	+0.319	56 02 33.532
Pittsford and Hillsdale	78 08 51.025	51_{3+4}	8	3.9	0.5	+0.317	+0.301	78 08 51.643
Pittsford and Bunday	165 14 36.313	51_{3+4+5}	8	4.8	0.5	−0.224	+0.221	165 14 36.310
Reading and Hillsdale	22 06 18.136	51_4	16	4.7	1	−0.007	−0.018	22 06 18.111
Hillsdale and Bunday	87 05 44.569	51_5	16	4.9	1	+0.178	−0.080	87 05 44.667
Hillsdale and Woodstock	153 53 23.272	51_{5+6}	8	4.0	0.5	+0.114	−0.311	153 53 23.075
Bunday and Woodstock	66 47 38.573	51_6	16	5.7	1	+0.066	−0.231	66 47 38.408

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3.0(51_1)+2.0(51_2)+1.5(51_3)+1.5(51_4)+(51_5)-0.000=0$$
$$2.0(51_1)+3.0(51_2)+1.5(51_3)+1.5(51_4)+(51_5)-0.000=0$$
$$1.5(51_1)+1.5(51_2)+3.5(51_3)+2.5(51_4)+1.5(51_5)-0.131=0$$
$$1.5(51_1)+1.5(51_2)+2.5(51_3)+3.5(51_4)+1.5(51_5)-0.131=0$$
$$(51_1)+(51_2)+1.5(51_3)+1.5(51_4)+2.5(51_5)-0.362=0$$

SECTION IX.—*Triangulation from the line Fremont–Quincy to the line Cedar Point–Stony Point*—Continued.

PITTSFORD—52.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Dates, May, and June, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Reading and Hillsdale	34 29 38.427	52_1	22	4.4	1.25	+0.059	+0.059	34 29 38.545
Reading and Wheatland	99 58 54.846	52_{1+2}	16	4.4	1	+0.037	+0.031	99 58 54.914
Hillsdale and Wheatland	65 29 16.120	52_2	16	5.4	1	−0.277	+0.028	65 29 16.369
Hillsdale and Woodstock	98 42 36.930	52_{2+3}	16	4.0	1	−0.203	−0.200	98 42 36.527
Wheatland and Woodstock	33 13 20.268	52_3	16	4.2	1	+0.062	−0.172	33 13 20.158
Wheatland and Fairfield	78 52 58.851	52_{3+4}	16	4.3	1	+0.253	−0.179	78 52 58.925
Woodstock and Fairfield	45 39 38.915	52_4	16	4.5	1	−0.141	−0.007	45 39 38.767
Fairfield and Reading	181 08 05.901	52_5	16	3.6	1	+0.112	+0.148	181 08 06.161

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3.25(52_1)+2(52_2)+(52_3)+(52_4)-0.668=0$$
$$2.00(52_1)+4(52_2)+2(52_3)+(52_4)-1.210=0$$
$$(52_1)+2(52_2)+4(52_3)+2(52_4)-0.579=0$$
$$(52_1)+(52_2)+2(52_3)+3(52_4)-0.037=0$$

WOODSTOCK—53.

[Observer, G. Y. Wisner. Instrument, Pistor & Martins theodolite No. 2. Date, June, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Raisin and Blissfield	14 00 39.893	53_1	16	5.5	1	−0.414	+0.009	14 00 39.488
Raisin and Fairfield	39 01 55.187	53_{1+2}	8	4.7	0.5	−0.662	−0.148	39 01 54.376
Bunday and Pittsford	291 51 49.097	$53_{1+2+3+6}$	16	7.1	1	−0.511	+0.246	291 51 48.832
Blissfield and Fairfield	25 01 15.459	53_2	16	6.5	1	−0.414	−0.157	25 01 14.888
Fairfield and Pittsford	89 33 18.150	53_3	16	4.9	1	−0.745	−0.025	89 33 17.380
Pittsford and Wheatland	18 48 54.536	53_4	16	6.0	1	+0.354	+0.046	18 48 54.936
Wheatland and Bunday	49 19 16.170	53_5	16	5.0	1	+0.354	−0.292	49 19 16.232
Bunday and Raisin	163 16 37.401	53_6	16	9.0	1	−0.744	+0.419	163 16 37.076

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.5(53_1)+1.5(53_2)+(53_3)+(53_4)+(53_5)+1.691=0$$
$$1.5(53_1)+2.5(53_2)+(53_3)+(53_4)+(53_5)+1.691=0$$
$$(53_1)+(53_2)+2(53_3)+(53_4)+(53_5)+1.609=0$$
$$(53_1)+(53_2)+(53_3)+3(53_4)+2(53_5)-0.197=0$$
$$(53_1)+(53_2)+(53_3)+2(53_4)+3(53_5)-0.197=0$$

FAIRFIELD—54.

[Observer. J. H. Darling. Instrument, Troughton & Simms theodolite No. 4. Dates, May, June, and July, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Pittsford and Wheatland	16 40 05.321	54_1	20	5.9	1	+0.367	−0.552	16 40 05.136
Pittsford and Woodstock	44 47 05.503	54_{1+2}	16	5.7	1	+0.192	−0.133	44 47 05.562
Wheatland and Woodstock	28 06 59.798	54_2	28	7.3	1.75	+0.209	+0.419	28 07 00.426
Woodstock and Raisin	76 00 48.800	54_3	16	5.5	1	−0.257	−0.336	76 00 48.207
Woodstock and Pittsford	315 12 53.685	54_{3+4+5}	20	7.2	1	+0.620	+0.133	315 12 54.438
Raisin and Blissfield	45 24 59.650	54_4	16	4.1	1	−0.258	−0.018	45 24 59.374
Blissfield and Pittsford	193 47 06.627	54_5	16	3.3	1	−0.257	+0.487	193 47 06.857

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$4(54_1)+3.00(54_2)+(54_3)+(54_4)-1.580=0$$
$$3(54_1)+4.75(54_2)+(54_3)+(54_4)-1.580=0$$
$$(54_1)+(54_2)+2(54_3)+(54_4)+0.196=0$$
$$(54_1)+(54_2)+(54_3)+2(54_4)+0.196=0$$

SECTION IX.—*Triangulation from the line Fremont - Quincy to the line Cedar Point - Stony Point*—Continued.

RAISIN—55.

[Observer, J. H. Darling. Instrument, Troughton & Simms theodolite No. 4. Date, June, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Dundee and Bedford	21 35 46.551	55_1	19	6.5	1	−0.005	−0.454	21 35 46.092
Dundee and Blissfield	40 27 53.449	55_{1+2}	17	5.2	1	−0.158	+0.330	40 27 53.621
Bedford and Blissfield	18 52 06.748	55_2	23	6.8	1.25	−0.004	+0.784	18 52 07.528
Blissfield and Fairfield	73 50 35.936	55_3	20	8.2	1	+0.123	+0.006	73 50 36.065
Blissfield and Woodstock	138 47 55.099	55_{3+4}	16	5.5	1	−0.285	−0.161	138 47 54.653
Fairfield and Woodstock	64 57 18.632	55_4	20	7.9	1	+0.123	−0.167	64 57 18.588
Woodstock and Dundee	180 44 12.059	55_5	20	12.5	1	−0.163	−0.169	180 44 11.727

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3(55_1)+2.00(55_2)+(55_3)+(55_4)-0.224=0$$
$$2(55_1)+3.25(55_2)+(55_3)+(55_4)-0.224=0$$
$$(55_1)+(55_2)+3(55_3)+2(55_4)-0.605=0$$
$$(55_1)+(55_2)+2(55_3)+3(55_4)-0.605=0$$

BLISSFIELD—56.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Date, June, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Fairfield and Woodstock	33 32 59.957	56_1	8	5.7	0.5	−0.646	−0.647	33 32 58.664
Fairfield and Raisin	60 44 25.267	56_{1+2}	18	9.5	1	+0.325	−0.374	60 44 25.218
Fairfield and Dundee	156 30 34.879	56_{1+2+3}	8	3.8	0.5	+0.046	−0.109	156 30 34.816
Woodstock and Raisin	27 11 26.972	56_2	12	4.6	1	−0.691	+0.273	27 11 26.554
Woodstock and Dundee	122 57 34.879	56_{2+3}	8	4.0	0.5	+0.735	+0.538	122 57 36.152
Raisin and Dundee	95 46 09.169	56_3	18	9.2	1	+0.164	+0.265	95 46 09.598
Raisin and Bedford	143 49 56.766	56_{3+4}	8	3.4	0.5	−1.059	+0.439	143 49 56.146
Dundee and Bedford	48 03 45.820	56_4	18	4.7	1	+0.554	+0.174	48 03 46.548
Bedford and Fairfield	155 25 38.676	56_5	18	7.6	1	+0.025	−0.065	155 25 38.636

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3.0(56_1)+2.5(56_2)+1.5(56_3)+(56_4)+2.865=0$$
$$2.5(56_1)+4.0(56_2)+2.0(56_3)+(56_4)+3.496=0$$
$$1.5(56_1)+2.0(56_2)+3.5(56_3)+1.5(56_4)+0.946=0$$
$$(56_1)+(56_2)+1.5(56_3)+2.5(56_4)-0.294=0$$

DUNDEE—57.

[Observer, G. Y. Wisner. Instrument, Pistor & Martins theodolite No. 2. Date, June, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Monroe and Bedford	67 06 25.495	57_1	20	8.7	1	−0.134	−0.749	67 06 24.612
Bedford and Blissfield	75 21 26.527	57_2	20	4.6	1	−0.134	+0.007	75 21 26.400
Blissfield and Raisin	43 45 56.591	57_3	20	7.3	1	−0.135	+0.847	43 45 57.303
Raisin and Monroe	173 46 11.925	57_4	20	8.1	1	−0.155	−0.105	173 46 11.685

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(57_1)+(57_2)+(57_3)+0.538=0$$
$$(57_1)+2(57_2)+(57_3)+0.538=0$$
$$(57_1)+(57_2)+2(57_3)+0.538=0$$

SECTION IX.—*Triangulation from the line Fremont-Quincy to the line Cedar Point-Stony Point*—Continued.

BEDFORD—58.

[Observer, J. H. Darling. Instrument, Troughton & Simms theodolite No. 4. Date, June, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Blissfield and Raisin	17 17 56.801	58_1	28	10.5	1.75	+0.165	−0.281	17 17 56.685
Blissfield and Dundee	56 34 47.822	58_{1+2}	16	4.1	1	−0.397	+0.051	56 34 47.476
Cedar Point and Dundee	222 25 18.581	58_{1+2+5}	16	4.3	1	+0.237	+0.246	222 25 19.064
Raisin and Dundee	39 16 50.294	58_2	28	7.9	1.75	+0.165	+0.332	39 16 50.791
Dundee and Monroe	68 37 31.705	58_3	16	4.8	1	−0.016	+0.044	68 37 31.733
Dundee and Cedar Point	137 34 41.229	58_{3+4}	16	4.9	1	0.047	−0.246	137 34 40.936
Dundee and Blissfield	303 25 12.384	58_{3+4+5}	20	7.3	1	+0.191	−0.051	303 25 12.524
Monroe and Cedar Point	68 57 09.509	58_4	16	7.1	1	−0.016	−0.290	68 57 09.203
Cedar Point and Blissfield	165 50 31.180	58_5	16	10.9	1	+0.213	+0.195	165 50 31.588

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$4.75(58_1)+3.00(58_2)+(58_3)+(58_4)-1.248=0$$
$$3.00(58_1)+4.75(58_2)+(58_3)+(58_4)-1.248=0$$
$$(58_1)+(58_2)+4(58_3)+3(58_4)-0.220=0$$
$$(58_1)+(58_2)+3(58_3)+4(58_4)-0.220=0$$

MONROE—59.

[Observer, J. H. Darling. Instrument, Troughton & Simms theodolite No. 4. Dates, May and June, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Stony Point and Cedar Point	68 58 06.371	59_1	20	10.6	1	+0.114	−0.556	68 58 05.929
Stony Point and Bedford	143 27 17.610	59_{1+2}	20	5.2	1	−0.416	−0.485	143 27 16.709
Cedar Point and Bedford	74 29 10.595	59_2	20	4.2	1	+0.114	+0.071	74 29 10.780
Bedford and Dundee	44 16 04.515	59_3	20	7.9	1	−0.301	−0.079	44 16 04.135
Dundee and Stony Point	172 16 38.893	59_4	20	7.0	1	−0.301	+0.564	172 16 39.156

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3(59_1)+2(59_2)+(59_3)-0.270=0$$
$$2(59_1)+3(59_2)+(59_3)-0.270=0$$
$$(59_1)+(59_2)+2(59_3)+0.374=0$$

CEDAR POINT—60.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Dates, May and June, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Bedford and Monroe	36 33 41.076	60_1	16	5.6	1	+0.084	−0.117	36 33 41.043
Monroe and Stony Point	39 32 51.317	60_2	16	5.4	1	+0.084	−0.744	39 32 50.657
Stony Point and Bedford	283 53 27.356	60_3	16	5.7	1	+0.083	+0.861	283 53 28.300

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(60_1)+(60_2)-0.251=0$$
$$(60_1)+2(60_2)-0.251=0$$

STONY POINT—61.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Dates, May and June, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Cedar Point and Monroe	71 29 05.221	61_1	20	4.2	1	+0.042	−0.802	71 29 04.461
Monroe and Cedar Point	288 30 54.695	61_2	18	5.6	1	+0.042	+0.802	288 30 55.539

Numerical equations of condition in the triangulation from the line Fremont–Quincy to the line Stony Point–Cedar Point.

SIDE-EQUATIONS.

LXXIX. (25) $-10.3216\,[49_1]+18.6434\,[49_2]-21.4799\,[50_1]+28.5957\,[50_2]-9.6008\,[52_2]$
$+32.1484\,[52_3]-61.7953\,[53_4]+18.0967\,[53_5]\qquad -19.385=0$

LXXXIII. (20) $+16.4135\,[50_2]+15.8351\,[50_3]-14.1792\,[51_3]+51.8396\,[51_4]-34.3480\,[52_1]$
$-3.7057\,[52_2]\qquad +0.749=0$

LXXXV. (10) $-22.1771\,[51_1]+2.0463\,[51_2]-4.1373\,[52_3]+16.4378\,[52_4]-7.1555\,[53_3]$
$-6.9919\,[53_4]\qquad -2.143=0$

XC. (25) $-18.1117\,[54_3]-12.8673\,[54_4]+24.0499\,[55_3]+33.8881\,[55_4]-31.7509\,[56_1]$
$+40.9855\,[56_2]\qquad -32.515=0$

XCIV. (30) $-26.6749\,[56_3]-28.8022\,[56_4]+11.7302\,[57_2]+33.7127\,[57_3]-67.6040\,[58_1]$
$+25.7421\,[58_2]\qquad -44.124=0$

NOTE.—In the solution for determining the general corrections each of the side-equations was divided by the number inclosed in parenthesis and placed opposite it.

ANGLE-EQUATIONS.

LXXV.	$[46_3]+[47_2]+[48_1]$	$+1.606=0$
LXXVI.	$[47_1]+[48_2]+[50_4]$	$+0.238=0$
LXXVII.	$[48_3]+[48_4]+[50_3]+[52_1]$	$-0.052=0$
LXXVIII.	$[48_4]+[51_3]+[52_1]+[52_2]$	$-0.403=0$
LXXX.	$[49_1]-[51_1]-[51_2]-[51_3]-[51_4]-[51_5]+[53_5]$	$+1.058=0$
LXXXI.	$[49_2]+[50_1]+[51_5]$	$+0.641=0$
LXXXII.	$[50_2]+[51_3]+[51_4]+[52_2]$	$-0.740=0$
LXXXIV.	$[51_1]+[51_2]+[52_3]+[53_4]$	$+0.115=0$
LXXXVI.	$[51_1]+[53_3]+[53_4]+[54_2]$	$-0.371=0$
LXXXVII.	$[52_4]+[53_3]+[54_1]+[54_2]$	$+0.165=0$
LXXXVIII.	$[53_1]+[53_2]+[54_3]+[55_4]$	$+0.652=0$
LXXXIX.	$[53_2]+[54_3]+[54_4]+[56_1]$	$+1.158=0$
XCI.	$[54_4]+[55_3]+[56_1]+[56_2]$	$+0.386=0$
XCII.	$[55_1]+[57_2]+[57_3]+[58_2]$	$-0.733=0$
XCIII.	$[55_1]+[55_2]+[56_3]+[57_3]$	$-1.443=0$
XCV.	$[56_4]+[57_2]+[58_1]+[58_2]$	$-0.232=0$
XCVI.	$[57_1]+[58_3]+[59_3]$	$+0.784=0$
XCVII.	$[58_4]+[59_2]+[60_1]$	$+0.335=0$
XCVIII.	$[59_1]+[60_2]+[61_1]$	$+2.102=0$

General corrections in terms of the correlates.

$[46_3]$	$=+0.75000$ LXXV				
$[47_1]$	$=+0.75000$ LXXVI				
$[47_2]$	$=+0.75000$ LXXV				
$[48_1]$	$=+0.64458$ LXXV	-0.16868 LXXVI	-0.12048 LXXVII	-0.06024 LXXVIII	
$[48_2]$	$=-0.16868$ LXXV	$+0.53012$ LXXVI	-0.19276 LXXVII	-0.09638 LXXVIII	
$[48_3]$	$=-0.06024$ LXXV	-0.09638 LXXVI	$+0.21686$ LXXVII	-0.39157 LXXVIII	
$[48_4]$	$=-0.06024$ LXXV	-0.09638 LXXVI	$+0.21686$ LXXVII	$+0.60843$ LXXVIII	
$[49_1]$	$=-0.53769$ LXXIX	$+0.62500$ LXXX	-0.37500 LXXXI		
$[49_2]$	$=+0.62091$ LXXIX	-0.37500 LXXX	$+0.62500$ LXXXI		
$[50_1]$	$=-0.05882$ LXXVI	-0.05882 LXXVII	-0.97639 LXXIX	$+0.58823$ LXXXI	-0.41176 LXXXII
	-0.38450 LXXXIII				
$[50_2]$	$=-0.05882$ LXXVI	-0.05882 LXXVII	$+1.02662$ LXXIX	-0.41176 LXXXI	$+0.58823$ LXXXII
	$+0.43617$ LXXXIII				
$[50_3]$	$=-0.29412$ LXXVI	$+0.70588$ LXXVII	-0.01673 LXXIX	-0.05882 LXXXI	-0.05882 LXXXII
	$+0.51061$ LXXXIII				

General corrections in terms of the correlates—Continued.

$[50_4]$ =+0. 70588 LXXVI —0. 29412 LXXVII —0. 01673 LXXIX —0. 05882 LXXXI —0. 05882 LXXXII
—0. 28115 LXXXIII

$[51_1]$ =—0. 06250 LXXVIII —0. 12500 LXXX —0. 04167 LXXXI —0. 12500 LXXXII —0. 11769 LXXXIII
+0. 29167 LXXXIV —1. 50473 LXXXV +0. 64583 LXXVI

$[51_2]$ =—0. 06250 LXXVIII —0. 12500 LXXX —0. 04167 LXXXI —0. 12500 LXXXII —0. 11769 LXXXIII
+0. 29167 LXXXIV +0. 91761 LXXXV —0. 35416 LXXXVI

$[51_3]$ =+0. 64584 LXXVIII —0. 04167 LXXX —0. 12500 LXXXI +0. 29167 LXXXII —1. 37587 LXXXIII
—0. 12500 LXXXIV +0. 12582 LXXXV —0. 06250 LXXXVI

$[51_4]$ =—0. 35416 LXXVIII —0. 04167 LXXX —0. 12500 LXXXI +0. 29167 LXXXII +1. 92507 LXXXIII
—0. 12500 LXXXIV +0. 12582 LXXXV —0. 06250 LXXXVI

$[51_5]$ =—0. 12500 LXXVIII —0. 25000 LXXX +0. 58333 LXXXI —0. 25000 LXXXII —0. 23537 LXXXIII
—0. 08334 LXXXIV +0. 08388 LXXXV —0. 04167 LXXXVI

$[52_1]$ =+0. 47058 LXXVII +0. 23528 LXXVIII +0. 16600 LXXIX —0. 23530 LXXXII —0. 76457 LXXXIII
+0. 05882 LXXXIV —0. 21773 LXXXV —0. 11765 LXXXVII

$[52_2]$ =—0. 23530 LXXVII +0. 21568 LXXVIII —0. 42532 LXXIX +0. 45098 LXXXII +0. 32054 LXXXIII
—0. 19608 LXXXIV +0. 17781 LXXXV +0. 05882 LXXXVII

$[52_3]$ =+0. 05882 LXXVII —0. 13726 LXXVIII +0. 67414 LXXIX —0. 19608 LXXXII —0. 06469 LXXXIII
+0. 46568 LXXXIV —0. 62779 LXXXV —0. 26471 LXXXVII

$[52_4]$ =—0. 11765 LXXVII —0. 05882 LXXVIII —0. 36299 LXXIX +0. 05882 LXXXII +0. 19115 LXXXIII
—0. 26471 LXXXIV +0. 97976 LXXXV +0. 52941 LXXXVII

$[53_1]$ =+0. 07947 LXXIX —0. 04545 LXXX —0. 04545 LXXXIV +0. 12937 LXXXV —0. 18182 LXXXVI
—0. 13636 LXXXVII +0. 36364 LXXXVIII —0. 31818 LXXXIX

$[53_2]$ =+0. 07947 LXXIX —0. 04545 LXXX —0. 04545 LXXXIV +0. 12937 LXXXV —0. 18182 LXXXVI
—0. 13636 LXXXVII +0. 36364 LXXXVIII +0. 68182 LXXXIX

$[53_3]$ =+0. 15890 LXXIX —0. 09091 LXXX —0. 09091 LXXXIV —0. 45683 LXXXV +0. 63636 LXXXVI
+0. 72727 LXXXVII —0. 27273 LXXXVIII —0. 13636 LXXXIX

$[53_4]$ =—1. 83622 LXXIX +0. 36364 LXXX +0. 63636 LXXXIV —0. 37989 LXXXV +0. 54545 LXXXVI
—0. 09091 LXXXVII —0. 09091 LXXXVIII —0. 04545 LXXXIX

$[53_5]$ =+1. 35947 LXXIX +0. 63636 LXXX —0. 36364 LXXXIV +0. 31930 LXXXV —0. 45455 LXXXVI
—0. 09091 LXXXVII —0. 09091 LXXXVIII —0. 04545 LXXXIX

$[54_1]$ =—0. 28572 LXXXVI +0. 21428 LXXXVII —0. 07143 LXXXVIII —0. 14286 LXXXIX +0. 08851 XC
—0. 07143 XCI

$[54_2]$ =+0. 40816 LXXXVI +0. 12244 LXXXVII —0.04082 LXXXVIII —0. 08164 LXXXIX +0. 05058 XC
—0. 04082 XCI

$[54_3]$ =—0. 04082 LXXXVI —0. 11225 LXXXVII +0. 70408 LXXXVIII +0. 40816 LXXXIX —0. 35778 XC
—0. 29592 XCI

$[54_4]$ =—0.04082 LXXXVI —0. 11225 LXXXVII —0. 29592 LXXXVIII +0. 40816 LXXXIX —0. 14800 XC
+0. 70408 XCI

$[55_1]$ =—0. 05154 LXXXVIII —0. 11946 XC —0. 05154 XCI +0. 58762 XCII +0. 25772 XCIII

$[55_2]$ =—0. 04124 LXXXVIII —0. 09557 XC —0. 04124 XCI —0. 32990 XCII +0. 20618 XCIII

$[55_3]$ =—0. 38144 LXXXVIII +0. 07799 XC +0. 61855 XCI —0. 05154 XCII —0. 09278 XCIII

$[55_4]$ =+0. 61855 LXXXVIII +0. 47151 XC —0. 38144 XCI —0. 05154 XCII —0. 09278 XCIII

$[56_1]$ =+0. 72906 LXXXIX —1. 60432 XC +0. 31527 XCI —0. 02956 XCIII —0. 13033 XCIV
—0. 10837 XCV

$[56_2]$ =—0. 41379 LXXXIX +1. 48656 XC +0. 17241 XCI —0. 17241 XCIII +0. 12020 XCIV
+0. 03448 XCV

$[56_3]$ =—0. 02956 LXXXIX —0. 24511 XC —0. 20197 XCI +0. 48768 XCIII —0. 23027 XCIV
—0. 21182 XCV

$[56_4]$ =—0. 10837 LXXXIX +0. 19417 XC —0. 07389 XCI —0. 21182 XCIII —0. 34608 XCIV
+0. 55665 XCV

$[57_1]$ =—0. 50000 XCII —0. 25000 XCIII —0. 37869 XCIV —0. 25000 XCV +0. 75000 XCVI

$[57_2]$ =+0. 50000 XCII —0. 25000 XCIII +0. 01232 XCIV +0. 75000 XCV —0. 25000 XCVI

General corrections in terms of the correlates—Continued.

[57_3]	=+0.50000 XCII	+0.75000 XCIII	+0.74507 XCIV	−0.25000 XCV	−0.25000 XCVI
[58_1]	=−0.21606 XCII	−0.98621 XCIV	+0.13931 XCV	−0.01990 XCVI	−0.01990 XCVII
[58_2]	=+0.35537 XCII	+0.79181 XCIV	+0.13931 XCV	−0.01990 XCVI	−0.01990 XCVII
[58_3]	=−0.01990 XCII	+0.02777 XCIV	−0.03980 XCV	+0.57712 XCVI	−0.42289 XCVII
[58_4]	=−0.01990 XCII	+0.02777 XCIV	−0.03980 XCV	−0.42289 XCVI	+0.57712 XCVII
[59_1]	=−0.12500 XCVI	−0.37500 XCVII	+0.62500 XCVIII		
[59_2]	=−0.12500 XCVI	+0.62500 XCVII	−0.37500 XCVIII		
[59_3]	=+0.62500 XCVI	−0.12500 XCVII	−0.12500 XCVIII		
[60_1]	=+0.66667 XCVII	+0.33333 XCVIII			
[60_2]	=−0.33333 XCVII	+0.66667 XCVIII			
[61_1]	=+0.50000 XCVIII				

Normal equations for determining the correlates.

No. of equation.					
75.	0=+1.60600	+2.14458 LXXV	−0.16868 LXXVI	−0.12048 LXXVII	−0.06024 LXXVIII
76.	0=+0.23800	−0.16868 LXXV	+1.98600 LXXVI	−0.48688 LXXVII	−0.09638 LXXVIII
		−0.01673 LXXIX	−0.05882 LXXXI	−0.05882 LXXXII	−0.28115 LXXXIII
77.	0=−0.05200	−0.12048 LXXV	−0.48688 LXXVI	+1.61018 LXXVII	+0.45214 LXXVIII
		+0.14927 LXXIX	−0.05882 LXXXI	−0.29412 LXXXII	−0.25396 LXXXIII
		+0.05882 LXXXIV	−0.21773 LXXXV	−0.11765 LXXXVII	
78.	0=−0.40300	−0.06024 LXXV	−0.09638 LXXVI	+0.45214 LXXVII	+1.70523 LXXVIII
		−0.25932 LXXIX	−0.04167 LXXX	−0.12500 LXXXI	+0.50735 LXXXII
		−1.81990 LXXXIII	−0.26226 LXXXIV	+0.08590 LXXXV	−0.06250 LXXXVI
		−0.05882 LXXXVII			
79.	0=−0.77540	−0.01673 LXXVI	+0.14927 LXXVII	−0.25932 LXXVIII	+9.25133 LXXIX
		+0.82178 LXXX	−0.35548 LXXXI	+0.60130 LXXXII	+0.62297 LXXXIII
		−1.16208 LXXXIV	+0.29457 LXXXV	−1.67732 LXXXVI	−0.20409 LXXXVII
		+0.15894 LXXXVIII	+0.07947 LXXXIX		
80.	0=+1.05800	−0.04167 LXXVIII	+0.82178 LXXIX	+1.84470 LXXX	−0.62500 LXXXI
		−0.08334 LXXXII	−0.07845 LXXXIII	−0.61364 LXXXIV	+0.57090 LXXXV
		−0.57955 LXXXVI	−0.09091 LXXXVII	−0.09091 LXXXVIII	−0.04545 LXXXIX
81.	0=+0.64100	−0.05882 LXXVI	−0.05882 LXXVII	−0.12500 LXXVIII	−0.35548 LXXIX
		−0.62500 LXXX	+1.79656 LXXXI	−0.66176 LXXXII	−0.61987 LXXXIII
		−0.08334 LXXXIV	+0.08388 LXXXV	−0.04167 LXXXVI	
82.	0=−0.74000	−0.05882 LXXVI	−0.29412 LXXVII	+0.50735 LXXVIII	+0.60130 LXXIX
		−0.08334 LXXX	−0.66176 LXXXI	+1.62255 LXXXII	+1.30591 LXXXIII
		−0.44608 LXXXIV	+0.42045 LXXXV	−0.12500 LXXXVI	+0.05882 LXXXVII
83.	0=+0.03745	−0.28115 LXXVI	−0.25396 LXXVII	−1.81990 LXXVIII	+0.62297 LXXIX
		−0.07845 LXXX	−0.61987 LXXXI	+1.30591 LXXXII	+7.98109 LXXXIII
		−0.30007 LXXXIV	+0.57791 LXXXV	−0.11769 LXXXVI	+0.19115 LXXXVII
84.	0=+0.11500	+0.05882 LXXVII	−0.26226 LXXVIII	−1.16208 LXXIX	−0.61364 LXXX
		−0.08334 LXXXI	−0.44608 LXXXII	−0.30007 LXXXIII	+1.68538 LXXXIV
		−1.59480 LXXXV	+0.83712 LXXXVI	−0.35562 LXXXVII	−0.09091 LXXXVIII
		−0.04545 LXXXIX			
85.	0=−0.21430	−0.21773 LXXVII	+0.08590 LXXVIII	+0.29457 LXXIX	+0.57090 LXXX
		+0.08388 LXXXI	+0.42945 LXXXII	+0.57791 LXXXIII	−1.59480 LXXXIV
		+5.98758 LXXXV	−2.34145 LXXXVI	+0.52293 LXXXVII	+0.25874 LXXXVIII
		+0.12937 LXXXIX			
86.	0=−0.37100	−0.06250 LXXVIII	−1.67732 LXXIX	−0.57955 LXXX	−0.04167 LXXXI
		−0.12500 LXXXII	−0.11769 LXXXIII	+0.83712 LXXXIV	−2.34145 LXXXV
		+2.23580 LXXXVI	+0.75880 LXXXVII	−0.40446 LXXXVIII	−0.26345 LXXXIX
		+0.05058 XC	−0.04082 XCI		

Normal equations for determining the correlates—Continued.

No. of equation.					
87.	0=+.16500	−0.011765 LXXVII	−0.05882 LXXVIII	−0.20409 LXXIX	−0.09091 LXXX
		+0.05882 LXXXII	+0.19115 LXXXIII	−0.35562 LXXXIV	+0.52293 LXXXV
		+0.75880 LXXXVI	+1.59340 LXXXVII	−0.38498 LXXXVIII	−0.36086 LXXXIX
		+0.13909 XC	−0.11225 XCI		
88.	0=+0.65200	+0.15894 LXXIX	−0.09091 LXXX	−0.09091 LXXXIV	+0.25874 LXXXV
		−0.40446 LXXXVI	−0.38498 LXXXVII	+2.04991 LXXXVIII	+0.77180 LXXXIX
		+0.11373 XC	−0.67736 XCI	−0.05154 XCII	−0.09278 XCIII
89.	0=+1.15800	+0.07947 LXXIX	−0.04545 LXXX	−0.04545 LXXXIV	+0.12937 LXXXV
		−0.26345 LXXXVI	−0.36086 LXXXVII	+0.77180 LXXXVIII	+2.22720 LXXXIX
		−2.11010 XC	+0.72343 XCI	−0.02956 XCIII	+0.13033 XCIV
		−0.10837 XCV			
90.	0=−1.30060	+0.05058 LXXXVI	+0.13909 LXXXVII	+0.11373 LXXXVIII	−2.11010 LXXXIX
		+5.52419 XC	−0.18777 XCI	−0.11946 XCII	−0.46014 XCIII
		+0.03153 XCIV	+0.19417 XCV		
91.	0=+0.38600	−0.04082 LXXXVI	−0.11225 LXXXVII	−0.67736 LXXXVIII	+0.72343 LXXXIX
		−0.18777 XC	+1.81031 XCI	−0.05154 XCII	−0.29475 XCIII
		+0.25053 XCIV	−0.07389 XCV		
92.	0=−0.73300	−0.05154 LXXXVIII	−0.11946 XC	−0.05154 XCI	+1.94299 XCII
		+0.75772 XCIII	+1.54920 XCIV	+0.63931 XCV	−0.51990 XCVI
		−0.01990 XCVII			
93.	0=−1.44300	−0.09278 LXXXVIII	−0.02956 LXXXIX	−0.46014 XC	−0.29475 XCI
		+0.75772 XCII	+1.70158 XCIII	+0.51480 XCIV	−0.46182 XCV
		−0.25000 XCVI			
94.	0=−1.47080	+0.13033 LXXXIX	+0.03153 XC	+0.25053 XCI	+1.54920 XCII
		+0.51480 XCIII	+4.28093 XCIV	−0.52816 XCV	−0.35092 XCVI
		+0.02777 XCVII			
95.	0=−0.23200	−0.10837 LXXXIX	+0.19417 XC	−0.07389 XCI	+0.63931 XCII
		−0.46182 XCIII	−0.52816 XCIV	+1.58527 XCV	−0.28980 XCVI
		−0.03980 XCVII			
96.	0=+0.78400	−0.51990 XCII	−0.25000 XCIII	−0.35092 XCIV	−0.28980 XCV
		+1.95212 XCVI	−0.54789 XCVII	−0.12500 XCVIII	
97.	0=+0.33500	−0.01990 XCII	+0.02777 XCIV	−0.03980 XCV	−0.54789 XCVI
		+1.86879 XCVII	−0.70833 XCVIII		
98.	0=+2.10200	−0.10200 XCVI	−0.70833 XCVII	+1.79167 XCVIII	

Values of the correlates and their logarithms.

LXXV = −0.7837	log 9.8941338$_-$	LXXXVII = −1.3050	log 0.1156172$_-$
LXXVI = −0.2884	log 9.4600103$_-$	LXXXVIII = −0.4845	log 9.6852669$_-$
LXXVII = −0.2403	log 9.3807538$_-$	LXXXIX = −0.1653	log 9.2182729$_-$
LXXVIII = +0.0489	log 8.6890423$_+$	XC = +0.2404	log 9.3810248$_+$
LXXIX = +0.2760	log 9.4408933$_+$	XCI = −0.2165	log 9.3353977$_-$
LXXX = −1.1560	log 0.0629391$_-$	XCII = −1.4338	log 0.1564916$_-$
LXXXI = −0.8947	log 9.9516677$_-$	XCIII = +1.4807	log 0.1704671$_+$
LXXXII = −0.3015	log 9.4792873$_-$	XCIV = +0.8059	log 9.9062596$_+$
LXXXIII = −0.0874	log 8.9413623$_-$	XCV = +1.2315	log 0.0904379$_+$
LXXXIV = −0.8066	log 9.9066690$_-$	XCVI = −0.6431	log 9.8082515$_-$
LXXXV = +0.5915	log 9.7719621$_+$	XCVII = −0.9769	log 9.9898457$_-$
LXXXVI = +1.2845	log 0.1087273$_+$	XCVIII = −1.6043	log 0.2052821$_-$

Values of the general corrections.

	"		"		"		"
[46_3]	=−0. 588	[51_1]	=−0. 069	[53_5]	=−0. 292	[57_1]	=−0. 749
[47_1]	=−0. 216	[51_2]	=+0. 079	[54_1]	=−0. 552	[57_2]	=+0. 007
[47_2]	=−0. 588	[51_3]	=+0. 319	[54_2]	=+0. 419	[57_3]	=+0. 847
[48_1]	=−0. 430	[51_4]	=−0. 018	[54_3]	=−0. 336	[58_1]	=−0. 281
[48_2]	=+0. 021	[51_5]	=−0. 080	[54_4]	=−0. 018	[58_2]	=+0. 332
[48_3]	=+0. 004	[52_1]	=+0. 059	[55_1]	=−0. 454	[58_3]	=+0. 044
[48_4]	=+0. 053	[52_2]	=−0. 028	[55_2]	=+0. 784	[58_4]	=−0. 290
[49_1]	=−0. 535	[52_3]	=−0. 172	[55_3]	=+0. 006	[59_1]	=−0. 556
[49_2]	=+0. 046	[52_4]	=−0. 007	[55_4]	=−0. 167	[59_2]	=+0. 071
[50_1]	=−0. 607	[53_1]	=+0. 009	[56_1]	=−0. 647	[59_3]	=−0. 079
[50_2]	=+0. 467	[53_2]	=−0. 157	[56_2]	=+0. 273	[60_1]	=−0. 117
[50_3]	=−0. 064	[53_3]	=−0. 025	[56_3]	=+0. 265	[60_2]	=−0. 744
[50_4]	=−0. 043	[53_4]	=+0. 046	[56_4]	=+0. 174	[61_1]	=−0. 802

Residuals resulting from substitution of general corrections in numerical equations of condition.

No. of equation.	Residual.	No. of equation.	Residual.
75	0. 0000	87	0. 0000
76	0. 0000	88	0. 0000
77	0. 0000	89	−0. 0001
78	0. 0000	90	−0. 0017
79	−0. 0025	91	0. 0000
80	0. 0000	92	0. 0000
81	0. 0000	93	0. 0000
82	0. 0000	94	−0. 0009
83	+0. 0006	95	0. 0000
84	0. 0000	96	0. 0000
85	−0. 0002	97	0. 0000
86	+0. 0001	98	0. 0000

SECTION X.—*Triangulation from the line Willoughby - Chester to the line Stony Point - Cedar Point.*

CHESTER—56.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, June, 1877.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angle.
° ′ ″			″		″	″	° ′ ″
Warrensville and Willoughby 92 12 11. 653	56_1	18	3. 5	1	+0. 052	+0. 104	92 12 11. 809

NOTE.—The weight and local correction of 56_1 are taken from Section XI of the adjustment.

WILLOUGHBY—57.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, June, 1877.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			″		″	″	° ′ ″
Chester and Warrensville............. 38 10 03. 094	57_2	20	2. 7	1. 25	−0. 155	+0. 085	38 10 03. 024
Warrensville and Rockport........... 37 51 09. 507	57_3	21	4. 3	1. 25	−0. 156	+0. 262	37 51 09. 613

NOTE.—The weights and local corrections of 57_2 and 57_3 are taken from Section XI of the adjustment.

SECTION X.—*Triangulation from the line Willoughby - Chester to the line Stony Point - Cedar Point*—Continued.

WARRENSVILLE—58.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, July, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Royalton and Olmsted	25 19 42. 976	58_1	16	5. 6	1	+0. 500	−0. 088	25 19 43. 388
Royalton and Rockport	37 10 08. 566	58_{1+2}	16	4. 1	1	−0. 262	+0. 139	37 10 08. 443
Olmsted and Rockport	11 50 24. 328	58_2	16	5. 5	1	+0. 500	+0. 227	11 50 25. 055
Rockport and Willoughby	107 17 58. 683	58_3	16	3. 8	1	+0. 239	+0. 193	107 17 59. 115
Willoughby and Chester	49 37 45. 632	58_4	16	4. 6	1	+0. 239	−0. 097	49 37 45. 774
Chester and Royalton	165 54 06. 663	58_5	16	5. 6	1	+0. 240	−0. 235	165 54 06. 668

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3(58_1)+2(58_2)+(58_3)+(58_4)-2.980=0$$
$$2(58_1)+3(58_2)+(58_3)+(58_4)-2.980=0$$
$$(58_1)+(58_2)+2(58_3)+(58_4)-1.718=0$$
$$(58_1)+(58_2)+(58_3)+2(58_4)-1.718=0$$

ROCKPORT—59.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, July 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Willoughby and Warrensville	34 50 52. 431	59_1	16	3. 4	1	+0. 030	+0. 130	34 50 52. 591
Warrensville and Royalton	76 57 22. 951	59_2	16	5. 6	1	+0. 030	+0. 088	76 57 23. 069
Royalton and Olmsted	73 38 29. 469	59_3	16	4. 7	1	+0. 030	+0. 081	73 38 29. 580
Olmsted and Willoughby	174 33 15. 028	59_4	16	3. 3	1	+0. 031	−0. 299	174 33 14. 760

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(59_1)+(59_2)+(59_3)-0.121=0$$
$$(59_1)+2(59_2)+(59_3)-0.121=0$$
$$(59_1)+(59_2)+2(59_3)-0.121=0$$

ROYALTON—60.

[Observer, J. H. Darling. Instrument, Troughton and Simms theodolite No. 4. Date, July, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Grafton and Camden	23 18 50. 804	60_1	24	4. 9	1	−0. 011	−0. 079	23 18 50. 714
Camden and Elyria	23 11 22. 884	60_2	24	6. 0	1	−0. 011	−0 316	23 11 22. 557
Elyria and Olmsted	2 18 39. 079	60_3	24	3. 1	1	−0. 011	−0. 046	2 18 39. 022
Olmsted and Rockport	54 12 25. 161	60_4	24	5. 3	1	−0. 011	+0. 226	54 12 25. 376
Rockport and Warrensville	65 52 29. 150	60_5	24	6. 6	1	−0. 011	+0. 306	65 52 29. 445
Warrensville and Grafton	191 06 12. 989	60_6	26	9. 9	1	−0. 012	−0. 091	191 06 12. 886

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(60_1)+(60_2)+(60_3)+(60_4)+(60_5)+0.067=0$$
$$(60_1)+2(60_2)+(60_3)+(60_4)+(60_5)+0.067=0$$
$$(60_1)+(60_2)+2(60_3)+(60_4)+(60_5)+0.067=0$$
$$(60_1)+(60_2)+(60_3)+2(60_4)+(60_5)+0.067=0$$
$$(60_1)+(60_2)+(60_3)+(60_4)+2(60_5)+0.067=0$$

SECTION X.—*Triangulation from the line Willoughby – Chester to the line Stony Point – Cedar Point*—Continued.

OLMSTED—61.

[Observer, J. H. Darling. Instrument, Troughton and Simms theodolite No. 4. Date, July, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Rockport and Warrensville	17 33 42.514	61_1	22	6.4	1	+0.141	+0.136	17 33 42.791
Rockport and Royalton	52 09 05.508	61_{1+2}	4	2.5	0.2	−0.273	+0.450	52 09 05.685
Warrensville and Royalton	34 35 22.439	61_2	22	5.3	1	+0.141	+0.314	34 35 22.894
Royalton and Grafton	80 52 00.469	61_3	24	5.5	1	+0.087	+0.084	80 52 00.640
Grafton and Camden	57 53 14.881	61_4	24	5.9	1	+0.087	−0.465	57 53 14.503
Camden and Elyria	36 10 22.838	61_5	24	5.6	1	+0.087	−0.064	36 10 22.861
Elyria and Rockport	132 55 16.230	61_6	25	5.0	1	+0.086	−0.005	132 55 16.311

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.2(61_1)+1.2(61_2)+(61_3)+(61_4)+(61_5)-0.740=0$$
$$1.2(61_1)+2.2(61_2)+(61_3)+(61_4)+(61_5)-0.740=0$$
$$(61_1)+(61_2)+2(61_3)+(61_4)+(61_5)-0.629=0$$
$$(61_1)+(61_2)+(61_3)+2(61_4)+(61_5)-0.629=0$$
$$(61_1)+(61_2)+(61_3)+(61_4)+2(61_5)-0.629=0$$

GRAFTON—62.

[Observer, J. H. Darling. Instrument, Troughton and Simms theodolite No. 4. Date, July, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Camden and Elyria	45 27 23.543	62_1	24	6.3	1	+0.099	+0.179	45 27 23.821
Elyria and Olmsted	38 49 36.563	62_2	24	7.5	1	+0.099	−0.261	38 49 36.401
Olmsted and Royalton	50 19 07.951	62_3	24	5.4	1	+0.099	−0.056	50 19 07.994
Royalton and Camden	225 23 51.548	62_4	24	7.2	1	+0.098	+0.138	225 23 51.784

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(62_1)+(62_2)+(62_3)-0.395=0$$
$$(62_1)+2(62_2)+(62_3)-0.395=0$$
$$(62_1)+(62_2)+2(62_3)-0.395=0$$

ELYRIA—63.

[Observer, G. Y. Wisner. Instrument, Troughton and Simms theodolite No. 1. Dates, July and August, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Olmsted and Royalton	2 45 43.415	63_1	16	4.8	1	−0.044	−0.326	2 45 43.045
Royalton and Grafton	44 21 04.409	63_2	16	5.8	1	−0.044	−0.392	44 21 03.973
Grafton and Camden	68 15 03.279	63_3	18	3.8	1	−0.044	+0.490	68 15 03.725
Camden and Brownhelm	38 49 27.559	63_4	18	3.6	1	−0.044	+0.443	38 49 27.958
Brownhelm and East Base	16 32 52.678	63_5	16	5.3	1	−0.044	+0.102	16 32 52.736
East Base and West Base	5 37 21.814	63_6	16	7.1	1	−0.044	−0.429	5 37 21.341
West Base and Olmsted	183 38 27.153	63_7	16	5.0	1	−0.043	+0.112	183 38 27.222

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(63_1)+(63_2)+(63_3)+(63_4)+(63_5)+(63_6)+0.307=0$$
$$(63_1)+2(63_2)+(63_3)+(63_4)+(63_5)+(63_6)+0.307=0$$
$$(63_1)+(63_2)+2(63_3)+(63_4)+(63_5)+(63_6)+0.307=0$$
$$(63_1)+(63_2)+(63_3)+2(63_4)+(63_5)+(63_6)+0.307=0$$
$$(63_1)+(63_2)+(63_3)+(63_4)+2(63_5)+(63_6)+0.307=0$$
$$(63_1)+(63_2)+(63_3)+(63_4)+(63_5)+2(63_6)+0.307=0$$

SECTION X.—*Triangulation from the line Willoughby - Chester to the line Stony Point - Cedar Point*—Continued.

CAMDEN—64.

[Observer, J. H. Darling. Instrument, Troughton and Simms theodolite No. 4. Date, August, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Townsend and Brownhelm	90 19 61. 029	64_1	24	4. 9	1	−0.022	+0. 425	90 20 01. 432
Brownhelm and Elyria	43 06 02. 093	64_2	24	6. 9	1	−0. 022	−0. 407	43 06 01. 664
Elyria and Olmsted	28 27 47. 300	64_3	25	7. 5	1	−0. 022	−0. 130	28 27 47. 148
Olmsted and Royalton	15 44 43. 903	64_4	24	7. 0	1	−0. 022	+0. 401	15 44 44. 282
Royalton and Grafton	22 05 02. 342	64_5	24	7. 5	1	−0. 022	−0. 066	22 05 02. 254
Grafton and Townsend	160 16 23. 466	64_6	22	4. 7	1	−0. 023	−0. 223	160 16 23. 220

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(64_1)+(64_2)+(64_3)+(64_4)+(64_5)+0.133=0$$
$$(64_1)+2(64_2)+(64_3)+(64_4)+(64_5)+0.133=0$$
$$(64_1)+(64_2)+2(64_3)+(64_4)+(64_5)+0.133=0$$
$$(64_1)+(64_2)+(64_3)+2(64_4)+(64_5)+0.133=0$$
$$(64_1)+(64_2)+(64_3)+(64_4)+2(64_5)+0.133=0$$

BROWNHELM—65.

[Observer, G. Y. Wisner. Instrument, Troughton and Simms theodolite No. 1. Date, August, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Elyria and Camden	98 04 30. 919	65_1	16	5. 2	1	+0. 158	−0. 254	98 04 30. 823
Camden and Townsend	50 46 04. 864	65_2	16	4. 3	1	+0. 158	+0. 341	50 46 05. 363
Townsend and Sandusky	47 23 45. 986	65_3	16	3. 9	1	+0. 158	+0. 290	47 23 46. 434
Sandusky and East Base	8 27 39. 686	65_4	16	6. 3	1	+0. 158	−0. 356	8 27 39. 488
East Base and West Base	6 35 53. 970	65_5	16	10. 5	1	+0. 158	−1. 206	6 35 52. 922
West Base and Kelley's	17 53 03. 552	65_6	18	7. 9	1	+0. 158	+0. 925	17 53 04. 635
Kelley's and Elyria	130 48 59. 916	65_7	16	9. 9	1	+0. 159	+0. 260	130 49 00. 335

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(65_1)+(65_2)+(65_3)+(65_4)+(65_5)+(65_6)-1.107=0$$
$$(65_1)+2(65_2)+(65_3)+(65_4)+(65_5)+(65_6)-1.107=0$$
$$(65_1)+(65_2)+2(65_3)+(65_4)+(65_5)+(65_6)-1.107=0$$
$$(65_1)+(65_2)+(65_3)+2(65_4)+(65_5)+(65_6)-1.107=0$$
$$(65_1)+(65_2)+(65_3)+(65_4)+2(65_5)+(65_6)-1.107=0$$
$$(65_1)+(65_2)+(65_3)+(65_4)+(65_5)+2(65_6)-1.107=0$$

TOWNSEND—66.

[Observer, J. H. Darling. Instrument, Troughton & Simms theodolite No. 4. Date, August, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Sandusky and West Base	19 40 28. 184	66_1	20	5. 0	1	−0. 333	+0. 296	19 40 28. 147
Sandusky and East Base	21 18 58. 049	66_{1+2}	10	5. 2	0. 5	+0. 371	−0. 164	21 18 58. 256
West Base and East Base	1 38 30. 902	66_2	20	6. 6	1	−0. 333	−0. 460	1 38 30. 109
East Base and Kelley's	7 27 32. 553	66_3	20	4. 9	1	−0. 148	+0. 178	7 27 32. 583
Kelley's and Brownhelm	71 10 56. 529	66_4	20	6. 7	1	−0. 148	+0. 206	71 10 56. 587
Brownhelm and Camden	38 53 53. 787	66_5	20	5. 7	1	−0. 148	+0. 077	38 53 53. 716
Camden and Sandusky	221 08 39. 304	66_6	20	5. 6	1	−0. 149	−0. 297	221 08 38. 858

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.5(66_1)+1.5(66_2)+(66_3)+(66_4)+(66_5)+1.777=0$$
$$1.5(66_1)+2.5(66_2)+(66_3)+(66_4)+(66_5)+1.777=0$$
$$(66_1)+(66_2)+2(66_3)+(66_4)+(66_5)+1.259=0$$
$$(66_1)+(66_2)+(66_3)+2(66_4)+(66_5)+1.259=0$$
$$(66_1)+(66_2)+(66_3)+(66_4)+2(66_5)+1.259=0$$

SECTION X.—*Triangulation from the line Willoughby – Chester to the line Stony Point – Cedar Point*—Continued.

SANDUSKY—67.

[Observer, J. H. Darling. Instrument, Troughton & Simms theodolite No. 4. Date, September, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Danbury and Kelley's	42 52 18. 222	67_1	20	6. 1	1	+0. 079	−0. 118	42 52 18. 183
Kelley's and West Base	22 32 18. 600	67_2	20	4. 6	1	+0. 079	+0. 260	22 32 18. 939
West Base and East Base	35 35 11. 917	67_3	20	4. 9	1	+0. 079	+0. 624	35 35 12. 620
East Base and Brownhelm	23 27 01. 315	67_4	20	4. 0	1	+0. 079	−0. 381	23 27 01. 013
Brownhelm and Townsend	32 38 47. 519	67_5	20	4. 3	1	+0. 079	−0. 050	32 38 47. 548
Townsend and Danbury	202 54 21. 952	67_6	20	6. 0	1	+0. 080	−0. 335	202 54 21. 697

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(67_1)+(67_2)+(67_3)+(67_4)+(67_5)-0.475=0$$
$$(67_1)+2(67_2)+(67_3)+(67_4)+(67_5)-0.475=0$$
$$(67_1)+(67_2)+2(67_3)+(67_4)+(67_5)-0.475=0$$
$$(67_1)+(67_2)+(67_3)+2(67_4)+(67_5)-0.475=0$$
$$(67_1)+(67_2)+(67_3)+(67_4)+2(67_5)-0.475=0$$

EAST BASE—68.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, August, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Elyria and Brownhelm	8 09 09. 213	68_1	16	3. 4	1	+0. 106	+0. 465	8 09 09. 784
Brownhelm and Townsend	45 30 06. 026	68_2	16	3. 0	1	+0. 106	−0. 033	45 30 06. 099
Townsend and Sandusky	102 35 14. 334	68_3	18	6. 5	1	+0. 107	−0. 655	102 35 13. 786
Sandusky and Danbury	47 05 16. 167	68_4	18	6. 5	1	+0. 106	−0. 108	47 05 16. 165
Danbury and West Base	22 27 13. 678	68_5	18	4. 0	1	+0. 107	+0. 574	22 27 14. 359
West Base and Kelley's	24 39 32. 309	68_6	18	5. 2	1	+0. 106	+0. 156	24 39 32. 571
Kelley's and Elyria	109 33 27. 528	68_7	16	5. 5	1	+0. 107	−0. 399	109 33 27. 236

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(68_1)+(68_2)+(68_3)+(68_4)+(68_5)+(68_6)-0.745=0$$
$$(68_1)+2(68_2)+(68_3)+(68_4)+(68_5)+(68_6)-0.745=0$$
$$(68_1)+(68_2)+2(68_3)+(68_4)+(68_5)+(68_6)-0.745=0$$
$$(68_1)+(68_2)+(68_3)+2(68_4)+(68_5)+(68_6)-0.745=0$$
$$(68_1)+(68_2)+(68_3)+(68_4)+2(68_5)+(68_6)-0.745=0$$
$$(68_1)+(68_2)+(63_3)+(68_4)+(68_5)+2(68_6)-0.745=0$$

WEST BASE—69.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Dates, August and September, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Elyria and Brownhelm	9 07 40. 373	69_1	17	4. 9	1	+0. 075	+1. 113	9 07 41. 561
Brownhelm and East Base	31 01 58. 603	69_2	16	4. 1	1	+0. 076	−0. 924	31 01 57. 755
East Base and Townsend	6 13 46. 424	69_3	16	3. 6	1	+0. 075	−0. 865	6 13 45. 634
Townsend and Sandusky	68 38 30. 724	69_4	16	6. 3	1	+0. 076	+0. 575	68 38 31. 375
Sandusky and Danbury	72 37 42. 849	69_5	16	5. 7	1	+0. 075	+0. 848	72 37 43. 772
Danbury and Middle Bass	52 38 51. 957	69_6	16	5. 2	1	+0. 076	+0. 362	52 38 52. 395
Middle Bass and Kelley's	15 37 21. 499	69_7	16	6. 3	1	+0. 075	−1. 108	15 37 20. 466
Kelley's and Elyria	104 04 06. 983	69_8	21	9. 3	1. 25	+0. 060	−0. 001	104 04 07. 042

SECTION X.—*Triangulation from the line Willoughby - Chester to the line Stony Point - Cedar Point*—Continued.

WEST BASE—69—Continued.

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.25(69_1)+1.25(69_2)+1.25(69_3)+1.25(69_4)+1.25(69_5)+1.25(69_6)+1.25(69_7)-0.735=0$$
$$1.25(69_1)+2.25(69_2)+1.25(69_3)+1.25(69_4)+1.25(69_5)+1.25(69_6)+1.25(69_7)-0.735=0$$
$$1.25(69_1)+1.25(69_2)+2.25(69_3)+1.25(69_4)+1.25(69_5)+1.25(69_6)+1.25(69_7)-0.735=0$$
$$1.25(69_1)+1.25(69_2)+1.25(69_3)+2.25(69_4)+1.25(69_5)+1.25(69_6)+1.25(69_7)-0.735=0$$
$$1.25(69_1)+1.25(69_2)+1.25(69_3)+1.25(69_4)+2.25(69_5)+1.25(69_6)+1.25(69_7)-0.735=0$$
$$1.25(69_1)+1.25(69_2)+1.25(69_3)+1.25(69_4)+1.25(69_5)+2.25(69_6)+1.25(69_7)-0.735=0$$
$$1.25(69_1)+1.25(69_2)+1.25(69_3)+1.25(69_4)+1.25(69_5)+1.25(69_6)+2.25(69_7)-0.735=0$$

DANBURY—70.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Date, September, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Locust Point and Middle Bass	76 48 48.084	70_1	16	8.2	1	−0.016	−0.421	76 48 47.647
Middle Bass and Kelley's	39 18 37.729	70_2	16	5.5	1	+0.321	+0.171	39 18 38.221
Middle Bass and Sandusky	136 39 28.718	$70_{2+3+4+5}$	6	7.4	0.25	−1.356	+0.527	136 39 27.889
Kelley's and West Base	55 23 09.948	70_3	16	5.8	1	+0.321	−0.038	55 23 10.231
West Base and East Base	10 02 44.696	70_4	16	3.7	1	+0.171	+0.110	10 02 44.977
West Base and Locust Point	188 29 23.052	70_{4+5+6}	6	5.4	0.25	+0.561	+0.288	188 29 23.901
East Base and Sandusky	31 54 54.055	70_5	16	4.8	1	+0.121	+0.284	31 54 54.460
East Base and Locust Point	178 26 38.498	70_{5+6}	6	5.6	0.25	+0.248	+0.178	178 26 38.924
Sandusky and Locust Point	146 31 44.775	70_6	16	6.3	1	−0.205	−0.106	146 31 44.464

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.50(70_1)+1.50(70_2)+1.50(70_3)+1.25(70_4)+\quad(70_5)-0.914=0$$
$$1.50(70_1)+2.75(70_2)+1.75(70_3)+1.50(70_4)+1.25(70_5)-1.487=0$$
$$1.50(70_1)+1.75(70_2)+2.75(70_3)+1.50(70_4)+1.25(70_5)-1.487=0$$
$$1.25(70_1)+1.50(70_2)+1.50(70_3)+2.50(70_4)+1.25(70_5)-1.368=0$$
$$(70_1)+1.25(70_2)+1.25(70_3)+1.25(70_4)+2.25(70_5)-1.285=0$$

KELLEY'S—71.

[Observer, J. H. Darling. Instrument, Troughton & Simms theodolite No. 4. Date, September, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Brownhelm and Townsend	28 28 40.951	71_1	18	4.4	1	+0.190	+0.843	28 28 41.984
Townsend and East Base	9 19 44.669	71_2	20	3.3	1	+0.190	−0.236	9 19 44.623
East Base and West Base	11 06 40.660	71_3	20	4.2	1	+0.189	+0.347	11 06 41.196
West Base and Sandusky	16 33 44.363	71_4	16	4.0	1	+0.190	+0.091	16 33 44.644
Sandusky and Danbury	39 46 53.135	71_5	18	6.8	1	+0.189	−0.627	39 46 52.697
Danbury and Middle Bass	91 27 49.334	71_6	16	4.8	1	+0.190	−0.695	91 27 48.829
Middle Bass and Brownhelm	163 16 25.561	71_7	18	5.9	1	+0.189	+0.277	163 16 26.027

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(71_1)+(71_2)+(71_3)+(71_4)+(71_5)+(71_6)-1.327=0$$
$$(71_1)+2(71_2)+(71_3)+(71_4)+(71_5)+(71_6)-1.327=0$$
$$(71_1)+(71_2)+2(71_3)+(71_4)+(71_5)+(71_6)-1.327=0$$
$$(71_1)+(71_2)+(71_3)+2(71_4)+(71_5)+(71_6)-1.327=0$$
$$(71_1)+(71_2)+(71_3)+(71_4)+2(71_5)+(71_6)-1.327=0$$
$$(71_1)+(71_2)+(71_3)+(71_4)+(71_5)+2(71_6)-1.327=0$$

SECTION X.—*Triangulation from the line Willoughby – Chester to the line Stony Point – Cedar Point*—Continued.

MIDDLE BASS—72.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Dates, September and October, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Kelley's and West Base	16 34 14.051	72_1	16	3.4	1	−0.018	−0.437	16 34 13.596
Kelley's and Danbury	49 13 32.816	72_{1+2}	4	4.7	0.25	+0.641	−0.016	49 13 33.441
Kelley's and Locust Point	108 09 49.039	72_{1+2+3}	10	3.2	0.5	−0.048	−0.182	108 09 48.809
Pelée and Danbury	158 57 48.385	72_{1+2+5}	8	3.8	0.5	+0.868	−0.033	158 57 49.220
West Base and Danbury	32 39 19.442	72_2	15	6.9	1	−0.018	+0.421	32 39 19.845
Danbury and Locust Point	58 56 14.751	72_3	9	6.0	0.5	+0.783	−0.166	58 56 15.368
Danbury and Middle Sister	121 01 44.126	72_{3+4}	4	1.4	0.25	+0.736	−0.223	121 01 44.639
Locust Point and Middle Sister	62 05 30.426	72_4	9	8.1	0.5	−1.098	−0.057	62 05 29.271
Locust Point and Pelée	142 05 53.819	72_{4+5}	8	4.1	0.5	+1.394	+0.199	142 05 55.412
Pelée and Locust Point	217 54 05.668	72_{-4-5}	6	3.5	0.25	−0.881	−0.199	217 54 04.588
Middle Sister and Pelée	80 00 26.058	73_5	16	4.6	1	−0.173	+0.256	80 00 26.141
Middle Sister and Kelley's	189 44 41.877	72_{5+6}	8	6.0	0.5	−0.196	+0.239	189 44 41.920
Kelley's and Middle Sister	170 15 18.129	72_{-5-6}	8	3.7	0.5	+0.190	−0.239	170 15 18.080
Pelée and Kelley's	109 44 15.488	72_5	16	2.9	1	+0.308	−0.017	109 44 15.779

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3.75(72_1)+2.75(72_2)+2.50(72_3)+2.00(72_4)+(72_5)+0.532=0$$
$$2.75(72_1)+3.75(72_2)+2.50(72_3)+2.00(72_4)+(72_5)+0.532=0$$
$$2.50(72_1)+2.50(72_2)+3.75(72_3)+2.75(72_4)+1.50(72_5)+0.435=0$$
$$2.00(72_1)+2.00(72_2)+2.75(72_3)+4.00(72_4)+2.25(72_5)+2.703=0$$
$$(72_1)+(72_2)+1.50(72_3)+2.25(72_4)+3.25(72_5)+1.896=0$$

PELÉE—73.

[Observer, J. H. Darling. Instrument, Troughton & Simms theodolite No. 4. Date, October, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Middle Bass and Middle Sister	64 46 53.046	73_1	24	7.7	1.25	+0.051	+0.252	64 46 53.348
Middle Sister and Kingsville	70 48 10.143	73_2	16	4.5	1	+0.064	−0.126	70 48 10.081
Kingsville and Middle Bass	224 24 56.632	73_3	16	3.5	1	+0.065	−0.126	224 24 56.571

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.25(73_1)+(73_2)-0.180=0$$
$$(73_1)+2(73_2)-0.180=0$$

LOCUST POINT—74.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Date, October, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Cedar Point and Middle Sister	79 21 34.373	74_1	16	7.4	1	+0.207	+1.001	79 21 35.581
Middle Sister and Middle Bass	52 38 36.770	74_2	16	5.9	1	+0.207	−0.282	52 38 36.695
Middle Bass and Danbury	44 14 58.577	74_3	16	5.1	1	+0.207	−0.576	44 14 58.208
Danbury and Cedar Point	183 44 49.452	74_4	16	5.7	1	+0.207	−0.143	183 44 49.516

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(74_1)+(74_2)+(74_3)-0.828=0$$
$$(74_1)+2(74_2)+(74_3)-0.828=0$$
$$(74_1)+(74_2)+2(74_3)-0.828=0$$

SECTION X.—*Triangulation from the line Willoughby-Chester to the line Stony Point-Cedar Point*—Continued.

MIDDLE SISTER—75.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, October, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Kingsville and Middle Bass	85 37 23.710	75_{1+2}	18	5.1	1	0.000	0.000	85 37 23.710
Stony Point and Pelée	164 49 32.788	75_{1+6}	24	7.4	1.5	+0.080	−0.267	164 49 32.601
Pelée and Middle Bass	35 12 41.307	75_2	18	6.2	1	+0.122	+0.040	35 12 41.469
Middle Base and Locust Point	65 15 55.992	75_3	16	5.5	1	+0.122	−0.539	65 15 55.575
Locust Point and Cedar Point	42 43 13.288	75_4	16	6.2	1	+0.122	+0.744	42 43 14.154
Cedar Point and Stony Point	51 58 36.057	76_5	16	7.5	1	+0.122	+0.022	51 58 36.201

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.5(75_2)+1.5(75_3)+1.5(75_4)+1.5(75_5)-0.852=0$$
$$1.5(75_2)+2.5(75_3)+1.5(75_4)+1.5(75_5)-0.852=0$$
$$1.5(75_2)+1.5(75_3)+2.5(75_4)+1.5(75_5)-0.852=0$$
$$1.5(75_2)+1.5(75_3)+1.5(75_4)+2.5(75_5)-0.852=0$$

CEDAR POINT—76.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Date, October, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Stony Point and Middle Sister	47 31 39.268	76_1	16	4.2	1	+0.010	−0.100	47 31 39.178
Locust Point and Stony Point	254 33 09.586	76_{-1-2}	16	3.4	1	+0.009	−0.522	254 33 09.073
Middle Sister and Locust Point	57 55 11.117	76_2	16	5.3	1	+0.010	+0.622	57 55 11.749

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(76_1)+(76_2)-0.029=0$$
$$(76_1)+2(76_2)-0.029=0$$

STONY POINT—77.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, October, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Middle Sister and Cedar Point	80 29 45.811	77_1	20	4.6	1.25	+0.118	+0.187	80 29 46.116
Cedar Point and Middle Sister	279 30 13.924	77_{-1}	16	5.8	1	+0.147	−0.187	279 30 13.884

NORMAL EQUATION FOR LOCAL ADJUSTMENT.

$$2.25(77_1)-0.265=0$$

Numerical equations of condition in the triangulation from the line Willoughby-Chester to the line Cedar Point-Stony Point.

SIDE-EQUATIONS.

$$\text{VI.}\quad (40)\quad -42.2417\,[59_2]-37.3638\,[59_3]+12.1963\,[60_4]+21.6259\,[60_5]-66.5280\,[61_1]+30.5331\,[61_2]-3.114=0$$

$$\text{IX.}\quad (10)\quad +1.5551\,[60_1]+1.5551\,[60_2]-18.4229\,[60_3]+3.3850\,[61_3]+1.4947\,[61_4]+1.4947\,[61_5]+19.5566\,[63_1]-1.9809\,[63_2]+5.876=0$$

$$\text{XII.}\quad (20)\quad +30.4336\,[60_1]-18.4229\,[60_2]-18.4229\,[60_3]+3.3850\,[61_3]-13.2143\,[61_4]+27.1154\,[64_4]-24.7789\,[64_5]-23.209=0$$

Numerical equations of condition—Continued.

SIDE-EQUATIONS—Continued.

XV. $(12\frac{1}{2})$ $+14.7000\,[61_4]+1.4947\,[61_5]+19.5566\,[63_1]+19.5566\,[63_2]-8.3998\,[63_3]$
$+9.2458\,[64_3]-17.8696\,[64_4]-17.8696\,[64_5]$ $-32.277=0$

XX. (60) $-26.1646\,[63_4]+70.8633\,[63_5]+0.1226\,[64_1]+22.4996\,[64_2]-4.2297\,[66_3]$
$-4.2297\,[66_4]+26.0956\,[66_5]-146.9743\,[68_1]+20.6897\,[68_2]$ $+82.140=0$

XXII. (50) $-11.1450\,[65_1]-11.1450\,[65_2]-11.1450\,[65_3]-11.1450\,[65_4]+34.6311\,[65_5]$
$+126.4871\,[68_1]-20.4872\,[68_2]-20.4872\,[68_3]-20.4872\,[68_4]-20.4872\,[68_5]$
$-106.0933\,[69_1]+24.9500\,[69_2]$ $+119.678=0$

XXV. (30) $+3.6965\,[66_1]+3.6965\,[66_2]+7.9262\,[66_3]+7.9262\,[66_4]-48.5385\,[67_4]$
$+32.8642\,[67_5]-54.5019\,[68_2]-33.8122\,[68_3]$ $-43.201=0$

XXVIII. (20) $+3.6965\,[66_1]+7.3019\,[66_2]+7.3019\,[66_3]+7.3019\,[66_4]-12.6327\,[67_3]$
$-12.6327\,[67_4]+32.8642\,[67_5]-33.6759\,[69_2]-33.6759\,[69_3]-5.9995\,[69_4]$ $-52.598=0$

XXX. $(33\frac{1}{3})$ $+12.6327\,[67_3]-35.9058\,[67_4]-61.1226\,[68_2]-61.1226\,[68_3]-27.3104\,[68_4]$
$-27.3104\,[68_5]+40.9956\,[69_2]+5.9995\,[69_3]+5.9995\,[69_4]$ $-11.281=0$

XXXIII. $(16\frac{2}{3})$ $+3.6965\,[66_1]+3.6965\,[66_2]+3.6965\,[66_3]+10.8715\,[66_4]-3.1183\,[67_2]$
$-3.1183\,[67_3]-3.1183\,[67_4]+32.8642\,[67_5]-29.2103\,[71_1]+9.6039\,[71_2]$
$+9.6039\,[71_3]+9.6039\,[71_4]$ $+23.598=0$

XXXIV. $(14\frac{2}{7})$ $-3.6054\,[66_2]-3.6054\,[66_3]+3.5696\,[66_4]+18.6536\,[69_2]+18.6536\,[69_3]$
$-9.0228\,[69_4]-9.0228\,[69_5]-9.0228\,[69_6]-9.0228\,[69_7]-20.4586\,[71_1]$
$+18.3556\,[71_2]+18.3556\,[71_3]$ $+52.921=0$

XXXV. $(16\frac{2}{3})$ $-16.2514\,[68_2]-16.2514\,[68_3]-16.2514\,[68_4]-16.2514\,[68_5]+11.0590\,[68_6]$
$+25.9733\,[69_2]-9.0228\,[69_3]-9.0228\,[69_4]-9.0228\,[69_5]-9.0228\,[69_6]$
$-9.0228\,[69_7]-8.7815\,[71_1]-8.7815\,[71_2]+18.3556\,[71_3]$ $+15.914=0$

XXXVIII. $(14\frac{2}{7})$ $+22.6805\,[67_1]-13.0929\,[67_2]-13.0929\,[67_3]-1.5465\,[68_4]-21.1034\,[68_5]$
$-21.1031\,[68_6]+12.3404\,[70_3]+12.3404\,[70_4]+2.7148\,[70_5]$ $+27.821=0$

XLII. (30) $+22.6805\,[67_1]-50.7352\,[67_2]-25.9075\,[69_5]-34.2990\,[69_6]-34.2990\,[69_7]$
$+17.2473\,[70_3]+2.7148\,[70_4]+2.7148\,[70_5]$ $+11.839=0$

XLV. (40) $-16.0701\,[69_6]+75.2962\,[69_7]-27.4446\,[70_2]-1.7299\,[70_3]+33.4449\,[71_4]$
$+33.4449\,[71_5]+32.9069\,[71_6]$ $+134.686=0$

NOTE.—In the solution for determining the general corrections, each of the side-equations was divided by the number inclosed in parenthesis and placed opposite it.

ANGLE-EQUATIONS.

I. $[56_1]+[57_2]+[58_2]$ $-0.091=0$

II. $[57_3]+[58_3]+[59_1]$ $-0.585=0$

III. $[58_1]+[58_2]+[59_2]+[60_5]$ $-0.533=0$

IV. $[58_1]+[60_4]+[60_5]+[61_2]$ $-0.757=0$

V. $[59_3]+[60_4]+[61_1]+[61_2]$ $-0.757=0$

VII. $[60_1]+[60_2]+[60_3]+[61_3]+[62_3]$ $+0.413=0$

VIII. $[60_1]+[60_2]+[62_2]+[62_3]+[63_2]$ $+1.104=0$

X. $[60_1]+[62_1]+[62_2]+[62_3]+[64_5]$ $+0.283=0$

XI. $[61_4]+[62_1]+[62_2]+[64_4]+[64_5]$ $+0.212=0$

XIII. $[60_2]+[63_2]+[63_3]+[64_3]+[64_4]$ $-0.053=0$

XIV. $[61_5]+[63_1]+[63_2]+[63_3]+[64_3]$ $+0.422=0$

XVI. $[63_4]+[64_2]+[65_1]$ $+0.217=0$

XVII. $[64_1]+[65_2]+[66_5]$ $-0.842=0$

XVIII. $[63_5]-[65_1]-[65_2]-[65_3]-[65_4]+[68_1]$ $-0.546=0$

XIX. $[65_3]+[65_4]+[66_3]+[66_4]+[68_2]$ $-0.286=0$

XXI. $[63_5]+[63_6]-[65_1]-[65_2]-[65_3]-[65_4]-[65_5]+[69_1]$ $-1.970=0$

XXIII. $[65_3]+[66_1]+[66_2]+[66_3]+[66_4]+[67_5]$ $-0.460=0$

XXIV. $[66_1]+[66_2]+[67_4]+[67_5]+[68_3]$ $+1.250=0$

XXVI. $[65_3]+[65_4]+[65_5]+[66_2]+[66_3]+[66_4]+[69_2]+[69_3]$ $+3.136=0$

XXVII. $[65_4]+[65_5]+[67_3]+[67_4]+[69_2]+[69_3]+[69_4]$ $+2.531=0$

Numerical equations of condition—Continued.

ANGLE-EQUATIONS—Continued.

XXIX.	$[67_3] + [68_4] + [68_5] + [69_3] + [69_4]$	$-0.800=0$
XXXI.	$[65_3] + [65_4] + [65_5] + [65_6] + [66_4] + [71_1]$	$-0.703=0$
XXXII.	$[66_1] + [66_2] + [66_3] + [67_2] + [67_3] + [67_4] + [67_5] + [71_2] + [71_3] + [71_4]$	$-0.670=0$
XXXVI.	$[67_2] + [67_3] + [68_4] + [68_5] + [68_6] + [71_3] + [71_4]$	$-1.944=0$
XXXVII.	$[67_1] + [67_2] + [67_3] + [68_4] + [70_5]$	$-0.943=0$
XXXIX.	$[67_2] + [69_5] + [69_6] + [69_7] + [71_4]$	$-0.453=0$
XL.	$[67_1] + [67_2] + [69_5] + [70_4] + [70_5]$	$-1.384=0$
XLI.	$[69_6] + [69_7] + [70_3] + [71_4] + [71_5]$	$+1.320=0$
XLIII.	$[69_6] + [70_2] + [70_3] + [72_2]$	$-0.916=0$
XLIV.	$[70_2] + [71_6] + [72_1] + [72_2]$	$+0.540=0$
XLVI.	$[70_1] + [72_3] + [74_3]$	$+1.163=0$
XLVII.	$[72_5] + [73_1] + [75_2]$	$-0.548=0$
XLVIII.	$[72_4] + [74_2] + [75_3]$	$+0.878=0$
XLIX.	$[74_1] + [75_4] + [76_2]$	$-2.367=0$
L.	$[75_5] + [76_1] + [77_1]$	$-0.109=0$

General corrections in terms of the correlates.

$[56_1]$	=+0.75000 I				
$[57_2]$	=+0.61176 I				
$[57_3]$	=+0.61176 II				
$[58_1]$	=−0.09091 I	−0.09091 II	+0.27273 III	+0.63636 IV	
$[58_2]$	=−0.09091 I	−0.09091 II	+0.27273 III	−0.36364 IV	
$[58_3]$	=−0.27273 I	+0.72727 II	−0.18182 III	−0.09091 IV	
$[58_4]$	=+0.72727 I	−0.27273 II	−0.18182 III	−0.09091 IV	
$[59_1]$	=+0.75000 II	−0.25000 III	−0.25000 V	+0.49754 VI	
$[59_2]$	=−0.25000 II	+0.75000 III	−0.25000 V	−0.55850 VI	
$[59_3]$	=−0.25000 II	−0.25000 III	+0.75000 V	−0.43656 VI	
$[60_1]$	=−0.16667 III	−0.33333 IV	−0.16667 V	−0.14093 VI	+0.50000 VII
	+0.66667 VIII	+0.41072 IX	+0.83333 X	+1.57512 XII	−0.16667 XIII
$[60_{1+2}]$	=−0.33333 III	−0.66667 IV	−0.33333 V	−0.28186 VI	+1.00000 VII
	+1.33333 VIII	+0.82144 IX	+0.66667 X	+0.70741 XII	+0.66667 XIII
$[60_2]$	=−0.16667 III	−0.33333 IV	−0.16667 V	−0.14093 VI	+0.50000 VII
	+0.66667 VIII	+0.41072 IX	−0.16667 X	−0.86771 XII	+0.83333 XIII
$[60_{2+3}]$	=−0.33333 III	−0.66667 IV	−0.33333 V	−0.28186 VI	+1.00000 VII
	+0.33333 VIII	−1.17636 IX	−0.33333 X	−1.73542 XII	+0.66667 XIII
$[60_3]$	=−0.16667 III	−0.33333 IV	−0.16667 V	−0.14093 VI	+0.50000 VII
	−0.33333 VIII	−1.58708 IX	−0.16667 X	−0.86771 XII	−0.16667 XIII
$[60_4]$	=−0.16667 III	+0.66667 IV	+0.83333 V	+0.16398 VI	−0.50000 VII
	−0.33333 VIII	+0.25521 IX	−0.16667 X	+0.05344 XII	−0.16667 XIII
$[60_5]$	=+0.83333 III	+0.66667 IV	−0.16667 V	+0.39972 VI	−0.50000 VII
	−0.33333 VIII	+0.25521 IX	−0.16667 X	+0.05344 XII	−0.16667 XIII
$[61_1]$	=−0.23684 IV	+0.52632 V	−1.45007 VI	−0.13158 VII	−0.08387 IX
	−0.13158 XI	+0.06467 XII	−0.13158 XIV	−0.17057 XV	
$[61_2]$	=+0.76316 IV	+0.52632 V	+0.97646 VI	−0.13158 VII	−0.08387 IX
	−0.13158 XI	+0.06467 XII	−0.13158 XIV	−0.17057 XV	
$[61_3]$	=−0.13158 IV	−0.26316 V	+0.11840 VI	+0.81579 VII	+0.22108 IX
	−0.18421 XI	+0.25978 XII	−0.18421 XIV	−0.23879 XV	
$[61_4]$	=−0.13158 IV	−0.26316 V	+0.11840 VI	−0.18421 VII	+0.03205 IX
	+0.81579 XI	−0.57019 XII	−0.18421 XIV	+0.93793 XV	
$[61_{4+5}]$	=−0.26316 IV	−0.52632 V	+0.23680 VI	−0.36842 VII	+0.06410 IX
	+0.63158 XI	−0.47966 XII	+0.63158 XIV	+0.81872 XV	

General corrections in terms of the correlates—Continued.

$[61_5]$	=−0.13158 IV	−0.26316 V	+0.11840 VI	−0.18421 VII	+0.03205 IX
	−0.18421 XI	+0.09053 XII	+0.81579 XIV	−0.11921 XV	
$[62_1]$	=−0.25000 VII	−0.50000 VIII	+0.25000 X	+0.50000 XI	
$[62_2]$	=−0.25000 VII	+0.50000 VIII	+0.25000 X	+0.50000 XI	
$[62_3]$	=+0.75000 VII	+0.50000 VIII	+0.25000 X	−0.50000 XI	
$[63_1]$	=−0.14286 VIII	+1.70458 IX	−0.28571 XIII	+0.57143 XIV	+1.21352 XV
	−0.14286 XVI	−0.14286 XVIII	−0.10643 XX	−0.28571 XXI	
$[63_{1+2}]$	=+0.71429 VIII	+1.25541 IX	+0.42857 XIII	+1.14286 XIV	+2.42704 XV
	−0.28571 XVI	−0.28571 XVIII	−0.21286 XX	−0.57143 XXI	
$[63_2]$	=+1.85714 VIII	−0.44917 IX	+0.71429 XIII	+0.57143 XIV	+1.21352 XV
	−0.14286 XVI	−0.14286 XVIII	−0.10643 XX	−0.28571 XXI	
$[63_3]$	=−0.14286 VIII	−0.25108 IX	+0.71429 XIII	+0.57143 XIV	−1.02299 XV
	−0.14286 XVI	−0.14286 XVIII	−0.10643 XX	−0.28571 XXI	
$[63_4]$	=−0.14286 VIII	−0.25108 IX	−0.28571 XIII	−0.42857 XIV	−0.35101 XV
	+0.85714 XVI	−0.14286 XVIII	−0.54251 XX	−0.28571 XXI	
$[63_5]$	=−0.14286 VIII	−0.25108 IX	−0.28571 XIII	−0.42857 XIV	−0.35101 XV
	−0.14286 XVI	+0.85714 XVIII	+1.07463 XX	+0.71429 XXI	
$[63_6]$	=−0.14286 VIII	−0.25108 IX	−0.28571 XIII	−0.42857 XIV	−0.35101 XV
	−0.14286 XVI	−0.14286 XVIII	−0.10643 XX	+0.71429 XXI	
$[64_1]$	=−0.16667 X	−0.33333 XI	−0.01947 XII	−0.33333 XIII	−0.16667 XIV
	+0.35325 XV	−0.16667 XVI	+0.83333 XVII	−0.06080 XX	
$[64_2]$	=−0.16667 X	−0.33333 XI	−0.01947 XII	−0.33333 XIII	−0.16667 XIV
	+0.35325 XV	+0.83333 XVI	−0.16667 XVII	+0.31215 XX	
$[64_3]$	=−0.16667 X	−0.33333 XI	−0.01947 XII	+0.66667 XIII	+0.83333 XIV
	+1.09291 XV	−0.16667 XVI	−0.16667 XVII	−0.06284 XX	
$[64_4]$	=−0.16667 X	+0.66667 XI	+1.33630 XII	+0.66667 XIII	−0.16667 XIV
	−1.07632 XV	−0.16667 XVI	−0.16667 XVII	−0.06284 XX	
$[64_{4+5}]$	=+0.66667 X	+1.33333 XI	+0.07788 XII	+0.33333 XIII	−0.33333 XIV
	−2.15264 XV	−0.33333 XVI	−0.33333 XVII	−0.12568 XX	
$[64_5]$	=+0.83333 X	+0.66667 XI	−1.25842 XII	−0.33333 XIII	−0.16667 XIV
	−1.07632 XV	−0.16667 XVI	−0.16667 XVII	−0.06284 XX	
$[65_1]$	=+0.85714 XVI	−0.14285 XVII	−0.42857 XVIII	−0.28571 XIX	−0.28571 XXI
	−0.19447 XXII	−0.14286 XXIII	−0.42857 XXVI	−0.28571 XXVII	−0.57143 XXXI
$[65_{1+2+3+4}]$	=+0.42857 XVI	+0.42857 XVII	−1.71429 XVIII	+0.85714 XIX	−1.14286 XXI
	+0.77788 XXII	+0.42857 XXIII	+0.28571 XXVI	−0.14285 XXVII	−0.28571 XXXI
$[65_2]$	=−0.14286 XVI	+0.85714 XVII	−0.42857 XVIII	−0.28571 XIX	−0.28571 XXI
	−0.19447 XXII	−0.14286 XXIII	−0.42857 XXVI	−0.28571 XXVII	−0.57143 XXXI
$[65_3]$	=−0.14286 XVI	−0.14286 XVII	−0.42857 XVIII	+0.71429 XIX	−0.28571 XXI
	−0.19447 XXII	+0.85714 XXIII	+0.57143 XXVI	−0.28571 XXVII	+0.42857 XXXI
$[65_4]$	=−0.14286 XVI	−0.14286 XVII	−0.42857 XVIII	+0.71429 XIX	−0.28571 XXI
	−0.19447 XXII	−0.14286 XXIII	+0.57143 XXVI	+0.71429 XXVII	+0.42857 XXXI
$[65_5]$	=−0.14286 XVI	−0.14285 XVII	+0.57143 XVIII	−0.28571 XIX	−0.28571 XXI
	+0.72105 XXII	−0.14286 XXIII	+0.57143 XXVI	+0.71429 XXVII	+0.42857 XXXI
$[65_6]$	=−0.14286 XVI	−0.14285 XVII	+0.57143 XVIII	−0.28571 XIX	+0.71429 XXI
	+0.02843 XXII	−0.14286 XXIII	−0.42857 XXVI	−0.28571 XXVII	+0.42857 XXXI
$[66_1]$	=−0.10000 XVII	−0.20000 XIX	−0.02939 XX	+0.20000 XXIII	+0.40000 XXIV
	−0.00355 XXV	−0.50000 XXVI	−0.05317 XXVIII	−0.10000 XXXI	+0.30000 XXXII
	+0.00131 XXXIII	+0.07597 XXXIV			
$[66_{1+2}]$	=−0.20000 XVII	−0.40000 XIX	−0.05878 XX	+0.40000 XXIII	+0.80000 XXIV
	−0.00710 XXV	+0.07393 XXVIII	−0.20000 XXXI	+0.60000 XXXII	+0.00262 XXXIII
	−0.10045 XXXIV				

General corrections in terms of the correlates—Continued.

$[66_{1+2+3}]$	=−0. 40000 XVII	+0. 20000 XIX	−0. 18807 XX	+0. 80000 XXIII	+0. 60000 XXIV
	+0. 12678 XXV	+0. 50000 XXVI	+0. 23800 XXVIII	−0. 40000 XXXI	+1. 20000 XXXII
	+0. 00524 XXXIII	−0. 32709 XXXIV			
$[66_2]$	=−0. 10000 XVII	−0. 20000 XIX	−0. 02039 XX	+0. 20000 XXIII	+0. 40000 XXIV
	−0. 00355 XXV	+0. 50000 XXVI	+0. 12710 XXVIII	−0. 10000 XXXI	+0. 30000 XXXII
	+0. 00131 XXXIII	−0. 17642 XXXIV			
$[66_{2+3}]$	=−0. 30000 XVII	+0. 40000 XIX	−0. 15868 XX	+0. 60000 XXIII	+0. 20000 XXIV
	+0. 13033 XXV	+1. 00000 XXVI	+0. 29117 XXVIII	−0. 30000 XXXI	+0. 90000 XXXII
	+0. 00393 XXXIII	−0. 40306 XXXIV			
$[66_{2+3+4}]$	=−0. 50000 XVII	+1. 00000 XIX	−0. 28797 XX	+1. 00000 XXIII	+0. 26421 XXV
	+1. 50000 XXVI	+0. 45524 XXVIII	+0. 50000 XXXI	+0. 50000 XXXII	+0. 43705 XXXIII
	−0. 12745 XXXIV				
$[66_3]$	=−0. 20000 XVII	+0. 60000 XIX	−0. 12929 XX	+0. 40000 XXIII	−0. 20000 XXIV
	+0. 13388 XXV	+0. 50000 XXVI	+0. 16407 XXVIII	−0. 20000 XXXI	+0. 60000 XXXII
	+0. 00262 XXXIII	−0. 22664 XXXIV			
$[66_{3+4}]$	=−0. 40000 XVII	+1. 20000 XIX	−0. 25858 XX	+0. 80000 XXIII	−0. 40000 XXIV
	+0. 26776 XXV	+1. 00000 XXVI	+0. 32814 XXVIII	+0. 60000 XXXI	+0. 20000 XXXII
	+0. 43574 XXXIII	+0. 04897 XXXIV			
$[66_4]$	=−0. 20000 XVII	+0. 60000 XIX	−0. 12929 XX	+0. 40000 XXIII	−0. 20000 XXIV
	+0. 13388 XXV	+0. 50000 XXVI	+0. 16107 XXVIII	+0. 80000 XXXI	−0. 40000 XXXII
	+0. 43312 XXXIII	+0. 27561 XXXIV			
$[66_5]$	=+0. 80000 XVII	−0. 40000 XIX	+0. 37614 XX	−0. 60000 XXIII	−0. 20000 XXIV
	−0. 13033 XXV	−0. 50000 XXVI	−0. 20103 XXVIII	−0. 20000 XXXI	−0. 40000 XXXII
	−0. 21917 XXXIII	+0. 02574 XXXIV			
$[67_1]$	=−0. 16667 XXIII	−0. 33333 XXIV	+0. 08708 XXV	−0. 33333 XXVII	−0. 06332 XXVIII
	−0. 16667 XXIX	+0. 11637 XXX	−0. 66667 XXXII	−0. 23509 XXXIII	−0. 33333 XXXVI
	+0. 50000 XXXVII	+1. 62853 XXXVIII	−0. 16667 XXXIX	+0. 66667 XL	+0. 91188 XLII
$[67_2]$	=−0. 16667 XXIII	−0. 33333 XXIV	+0. 08708 XXV	−0. 33333 XXVII	−0. 06332 XXVIII
	−0. 16667 XXIX	+0. 11637 XXX	+0. 33333 XXXII	−0. 42219 XXXIII	+0. 66667 XXXVI
	+0. 50000 XXXVII	−0. 87561 XXXVIII	+0. 83333 XXXIX	+0. 66667 XL	−1. 53531 XLII
$[67_{2+3}]$	=−0. 33333 XXIII	−0. 66667 XXIV	+0. 17416 XXV	+0. 33333 XXVII	−0. 75828 XXVIII
	+0. 66667 XXIX	+0. 61172 XXX	+0. 66667 XXXII	−0. 84438 XXXIII	+1. 33333 XXXVI
	+1. 00000 XXXVII	−1. 75122 XXXVIII	+0. 66667 XXXIX	+0. 33333 XL	−1. 37945 XLII
$[67_{2+3+4}]$	=−0. 50000 XXIII	−1. 35671 XXV	+1. 00000 XXVII	−1. 45324 XXVIII	+0. 50000 XXIX
	−0. 34909 XXX	+1. 00000 XXXII	−1. 26657 XXXIII	+1. 00000 XXXVI	+0. 50000 XXXVII
	−1. 71033 XXXVIII	+0. 50000 XXXIX	−1. 22359 XLII		
$[67_3]$	=−0. 16667 XXIII	−0. 33333 XXIV	+0. 08708 XXV	+0. 66667 XXVII	−0. 69496 XXVIII
	+0. 83333 XXIX	+0. 49535 XXX	+0. 33333 XXXII	−0. 42219 XXXIII	+0. 66667 XXXVI
	+0. 50000 XXXVII	−0. 87561 XXXVIII	−0. 16667 XXXIX	−0. 33333 XL	+0. 15586 XLII
$[67_{3+4}]$	=−0. 33333 XXIII	+0. 33333 XXIV	−1. 44379 XXV	+1. 33333 XXVII	−1. 38992 XXVIII
	+0. 66667 XXIX	−0. 46546 XXX	+0. 66667 XXXII	−0. 84438 XXXIII	+0. 33333 XXXVI
	−0. 83472 XXXVIII	−0. 33333 XXXIX	−0. 66667 XL	+0. 31172 XLII	
$[67_4]$	=−0. 16667 XXIII	+0. 66667 XXIV	−1. 53087 XXV	+0. 66667 XXVII	−0. 69496 XXVIII
	−0. 16667 XXIX	−0. 96081 XXX	+0. 33333 XXXII	−0. 42219 XXXIII	−0. 33333 XXXVI
	−0. 50000 XXXVII	+0. 04089 XXXVIII	−0. 16667 XXXIX	−0. 33333 XL	+0. 15586 XLII
$[67_5]$	=+0. 83333 XXIII	+0. 66667 XXIV	+1. 18255 XXV	−0. 33333 XXVII	+1. 57989 XXVIII
	−0. 16667 XXIX	+0. 11637 XXX	+0. 33333 XXXII	+1. 73676 XXXIII	−0. 33333 XXXVI
	−0. 50000 XXXVII	+0. 04089 XXXVIII	−0. 16667 XXXIX	−0. 33333 XL	+0. 15586 XLII
$[68_1]$	=+0. 85714 XVIII	−0. 14286 XIX	−2. 14889 XX	+2. 40249 XXII	−0. 14286 XXIV
	+0. 42054 XXV	−0. 28571 XXIX	+0. 75800 XXX	+0. 46240 XXXV	−0. 42857 XXXVI
	−0. 14286 XXXVII	+0. 43753 XXXVIII			

General corrections in terms of the correlates—Continued.

[68_2]	=−0.14286 XVIII	+0.85714 XIX	+0.64551 XX	−0.53699 XXII	−0.14286 XXIV
	−1.39619 XXV	−0.28571 XXIX	−1.07568 XXX	−0.51268 XXXV	−0.42857 XXXVI
	−0.14286 XXXVII	+0.43753 XXXVIII			
[68_{2+3}]	=−0.28571 XVIII	+0.71429 XIX	+0.94619 XX	−1.07398 XXII	+0.71429 XXIV
	−2.10272 XXV	−0.57143 XXIX	−2.15136 XXX	−1.02536 XXXV	−0.85714 XXXVI
	−0.28571 XXXVII	+0.87506 XXXVIII			
[$68_{2+3+4+5}$]	=−0.57143 XVIII	+0.42857 XIX	+1.54755 XX	−2.14796 XXII	+0.42857 XXIV
	−1.26164 XXV	+0.85714 XXIX	−2.27398 XXX	−2.05072 XXXV	+0.28571 XXXVI
	+0.42857 XXXVII	+0.16462 XXXVIII			
[68_3]	=−0.14286 XVIII	−0.14286 XIX	+0.30068 XX	−0.53699 XXII	+0.85714 XXIV
	−0.70653 XXV	−0.28571 XXIX	−1.07568 XXX	−0.51268 XXXV	−0.42857 XXXVI
	−0.14286 XXXVII	+0.43753 XXXVIII			
[68_4]	=−0.14286 XVIII	−0.14286 XIX	+0.30068 XX	−0.53699 XXII	−0.14286 XXIV
	+0.42054 XXV	+0.71429 XXIX	−0.06131 XXX	−0.51268 XXXV	+0.57143 XXXVI
	+0.85714 XXXVII	+0.32927 XXXVIII			
[68_{4+5}]	=+0.28571 XVIII	−0.28571 XIX	+0.60136 XX	−1.07398 XXII	−0.28571 XXIV
	+0.84108 XXV	+1.42857 XXIX	−0.12262 XXX	−1.02536 XXXV	+1.14286 XXXVI
	+0.71429 XXXVII	−0.71014 XXXVIII			
[68_5]	=−0.14286 XVIII	−0.14286 XIX	+0.30068 XX	−0.53699 XXII	−0.14286 XXIV
	+0.42054 XXV	+0.71429 XXIX	−0.06131 XXX	−0.51268 XXXV	+0.57143 XXXVI
	−0.14286 XXXVII	−1.03971 XXXVIII			
[68_{5+6}]	=−0.28571 XVIII	−0.28571 XIX	+0.60136 XX	−0.66424 XXII	−0.28571 XXIV
	+0.84108 XXV	+0.42857 XXIX	+0.69559 XXX	+0.61326 XXXV	+1.14286 XXXVI
	−0.28571 XXXVII	−2.07912 XXXVIII			
[68_6]	=−0.14286 XVIII	−0.14286 XIX	+0.30068 XX	−0.12725 XXII	−0.14286 XXIV
	+0.42054 XXV	−0.28571 XXIX	+0.75800 XXX	+1.12591 XXXV	+0.57143 XXXVI
	−0.14286 XXXVII	−1.03971 XXXVIII			
[69_1]	=+0.87179 XXI	−1.91381 XXII	−0.25641 XXVI	−0.38462 XXVII	+0.47020 XXVIII
	−0.25641 XXIX	−0.20383 XXX	−0.01091 XXXIV	+0.14724 XXXV	−0.38462 XXXIX
	−0.12821 XL	−0.25641 XLI	+0.40387 XLII	−0.12821 XLIII	−0.18983 XLV
[69_2]	=−0.12821 XXI	+0.70706 XXII	+0.74359 XXVI	+0.61538 XXVII	−1.21360 XXVIII
	−0.25641 XXIX	+1.02604 XXX	+1.29484 XXXIV	+1.70564 XXXV	−0.38462 XXXIX
	−0.12821 XL	−0.25641 XLI	+0.40387 XLII	−0.12821 XLIII	−0.18983 XLV
[69_{2+3}]	=−0.25642 XXI	+0.91512 XXII	+1.48718 XXVI	+1.23076 XXVII	−2.42720 XXVIII
	+0.48718 XXIX	+1.00220 XXX	+2.58968 XXXIV	+1.31151 XXXV	−0.76924 XXXIX
	−0.25642 XL	−0.51282 XLI	+0.80774 XLII	−0.25642 XLIII	−0.37966 XLV
[69_3]	=−0.12821 XXI	+0.20806 XXII	+0.74359 XXVI	+0.61538 XXVII	−1.21360 XXVIII
	+0.74359 XXIX	−0.02384 XXX	+1.29484 XXXIV	−0.39413 XXXV	−0.38462 XXXIX
	−0.12821 XL	−0.25641 XLI	+0.40387 XLII	−0.12821 XLIII	−0.18983 XLV
[69_{3+4}]	=−0.25642 XXI	+0.41512 XXII	+0.48718 XXVI	+1.23076 XXVII	−1.04338 XXVIII
	+1.48718 XXIX	−0.04768 XXX	+0.65233 XXXIV	−0.78826 XXXV	−0.76924 XXXIX
	−0.25642 XL	−0.51282 XLI	+0.80774 XLII	−0.25642 XLIII	−0.37963 XLV
[$69_{3+4+5+6+7}$]	=−0.64105 XXI	+1.04030 XXII	−0.28205 XXVI	+0.07690 XXVII	+0.36722 XXVIII
	+0.71795 XXIX	−0.65917 XXX	−1.27520 XXXIV	−1.97065 XXXV	+1.07690 XXXIX
	+0.35895 XL	+0.71795 XLI	−1.13083 XLII	+0.35895 XLIII	+0.53151 XLV
[69_4]	=−0.12821 XXI	+0.20803 XXII	−0.25641 XXVI	+0.61538 XXVII	+0.17022 XXVIII
	+0.74359 XXIX	−0.02384 XXX	−0.64251 XXXIV	−0.39413 XXXV	−0.38462 XXXIX
	−0.12821 XL	−0.25641 XLI	+0.40387 XLII	−0.12821 XLIII	−0.18983 XLV
[$69_{4+5+6+7}$]	=−0.51284 XXI	+0.83224 XXII	−1.02564 XXVI	−0.53348 XXVII	+1.58032 XXVIII
	−0.02564 XXIX	−0.63533 XXX	−2.57004 XXXIV	−1.57652 XXXV	+1.46152 XXXIX
	+0.48716 XL	+0.97436 XLI	−1.53470 XLII	+0.48716 XLIII	+0.72134 XLV

General corrections in terms of the correlates—Continued.

[69$_5$]	=−0.12821 XXI	+0.20806 XXII	−0.25641 XXVI	−0.38462 XXVII	+0.47020 XXVIII
	−0.25641 XXIX	−0.20383 XXX	−0.64251 XXXIV	−0.39413 XXXV	+0.61538 XXXIX
	+0.87179 XL	−0.25641 XLI	−0.45971 XLII	−0.12821 XLIII	−0.18983 XLV
[69$_6$]	=−0.12821 XXI	+0.20806 XXII	−0.25641 XXVI	−0.38462 XXVII	+0.47020 XXVIII
	−0.25641 XXIX	−0.20383 XXX	−0.64251 XXXIV	−0.39413 XXXV	+0.61538 XXXIX
	−0.12821 XL	+0.74359 XLI	−0.73943 XLII	+0.87179 XLIII	−0.59158 XLV
[69$_{6+7}$]	=−0.25642 XXI	+0.41612 XXII	−0.51282 XXVI	−0.76924 XXVII	+0.94040 XXVIII
	−0.51282 XXIX	−0.40766 XXX	−1.28502 XXXIV	−0.78826 XXXV	+1.23076 XXXIX
	−0.25642 XL	+1.48718 XLI	−1.47886 XLII	+0.74358 XLIII	+1.10100 XLV
[69$_7$]	=−0.12821 XXI	+0.20806 XXII	−0.25641 XXVI	−0.38462 XXVII	+0.47020 XXVIII
	−0.25641 XXIX	−0.20383 XXX	−0.64251 XXXIV	−0.39413 XXXV	+0.61538 XXXIX
	−0.12821 XL	+0.74359 XLI	−0.73943 XLII	−0.12821 XLIII	+1.69258 XLV
[70$_1$]	=−0.04651 XXXVII	−0.25469 XXXVIII	−0.14839 XL	−0.18272 XLI	−0.11818 XLII
	−0.36544 XLIII	−0.18272 XLIV	+0.13327 XLV	+0.68881 XLVI	
[70$_2$]	=−0.11628 XXXVII	−0.36935 XXXVIII	−0.27575 XL	−0.24252 XLI	−0.16438 XLII
	+0.51496 XLIII	+0.75748 XLIV	−0.50923 XLV	−0.18272 XLVI	
[70$_3$]	=−0.11628 XXXVII	+0.49448 XXXVIII	−0.27575 XL	+0.75748 XLI	+0.41053 XLII
	+0.51496 XLIII	−0.24252 XLIV	+0.13354 XLV	−0.18272 XLVI	
[70$_{3+4}$]	=−0.30233 XXXVII	+0.95657 XXXVIII	+0.27353 XL	+0.59801 XLI	+0.36855 XLII
	+0.19602 XLIII	−0.40199 XLIV	+0.24996 XLV	−0.28460 XLVI	
[70$_4$]	=−0.18605 XXXVII	+0.46209 XXXVIII	+0.54928 XL	−0.15947 XLI	−0.04198 XLII
	−0.31894 XLIII	−0.15947 XLIV	+0.11632 XLV	−0.10188 XLVI	
[70$_{4+5}$]	=+0.51162 XXXVII	+0.33351 XXXVIII	+1.06090 XL	−0.27575 XLI	−0.06254 XLII
	−0.55150 XLIII	−0.27575 XLIV	+0.20113 XLV	−0.14839 XLVI	
[70$_5$]	=+0.69767 XXXVII	−0.12858 XXXVIII	+0.51162 XL	−0.11628 XLI	−0.02056 XLII
	−0.23256 XLIII	−0.11628 XLIV	+0.08481 XLV	−0.04651 XLVI	
[71$_1$]	=+0.85714 XXXI	−0.42857 XXXII	−1.74920 XXXIII	−1.59463 XXXIV	−0.53368 XXXV
	−0.28571 XXXVI	−0.14286 XXXIX	−0.28571 XLI	−0.14286 XLIV	−0.35642 XLV
[71$_{1+2}$]	=+0.71429 XXXI	+0.14286 XXXII	−1.16955 XXXIII	−0.47227 XXXIV	−1.06736 XXXV
	−0.57143 XXXVI	−0.28571 XXXIX	−0.57143 XLI	−0.28571 XLIV	−0.71284 XLV
[71$_2$]	=−0.14286 XXXI	+0.57143 XXXII	+0.57965 XXXIII	+1.12236 XXXIV	−0.53368 XXXV
	−0.28571 XXXVI	−0.14286 XXXIX	−0.28571 XLI	−0.14286 XLIV	−0.35642 XLV
[71$_{2+3}$]	=−0.28571 XXXI	+1.14286 XXXII	+1.15930 XXXIII	+2.24472 XXXIV	+0.56087 XXXV
	+0.42857 XXXVI	−0.28571 XXXIX	−0.57143 XLI	−0.28571 XLIV	−0.71284 XLV
[71$_{2+3+4}$]	=−0.42857 XXXI	+1.71429 XXXII	+1.73895 XXXIII	+2.08219 XXXIV	+0.55408 XXXVI
	+1.14286 XXXVI	+0.57143 XXXIX	+0.14286 XLI	−0.42857 XLIV	−0.23314 XLV
[71$_3$]	=−0.14286 XXXI	+0.57143 XXXII	+0.57965 XXXIII	+1.12236 XXXIV	+1.09455 XXXV
	+0.71429 XXXVI	−0.14286 XXXIX	−0.28571 XLI	−0.14286 XLIV	−0.35642 XLV
[71$_4$]	=−0.14286 XXXI	+0.57143 XXXII	+0.57965 XXXIII	−0.16253 XXXIV	−0.00679 XXXV
	+0.71429 XXXVI	+0.85714 XXXIX	+0.71429 XLI	−0.14286 XLIV	+0.47970 XLV
[71$_{4+5}$]	=−0.28571 XXXI	+0.14286 XXXII	+0.58307 XXXIII	−0.32506 XXXIV	−0.01358 XXXV
	+0.42857 XXXVI	+0.71429 XXXIX	+1.42857 XLI	−0.28571 XLIV	+0.95940 XLV
[71$_5$]	=−0.14286 XXXI	−0.42857 XXXII	+0.00342 XXXIII	−0.16253 XXXIV	−0.00679 XXXV
	−0.28571 XXXVI	−0.14286 XXXIX	+0.71429 XLI	−0.14286 XLIV	+0.47970 XLV
[71$_6$]	=−0.14286 XXXI	−0.42857 XXXII	+0.00342 XXXIII	−0.16253 XXXIV	−0.00679 XXXV
	−0.28571 XXXVI	−0.14286 XXXIX	−0.28571 XLI	+0.85714 XLIV	+0.46625 XLV
[72$_1$]	=−0.34022 XLIII	+0.31956 XLIV	−0.19296 XLVI	+0.01557 XLVII	−0.03588 XLVIII
[72$_2$]	=+0.65978 XLIII	+0.31956 XLIV	−0.19296 XLVI	+0.01557 XLVII	−0.03588 XLVIII
[72$_3$]	=−0.19296 XLIII	−0.38592 XLIV	+0.77116 XLVI	−0.00609 XLVII	−0.33378 XLVIII
[72$_4$]	=−0.03588 XLIII	−0.07176 XLIV	−0.33378 XLVI	−0.29587 XLVII	+0.68179 XLVIII
[72$_5$]	=+0.01557 XLIII	+0.03114 XLIV	−0.00609 XLVI	+0.50575 XLVII	−0.29587 XLVIII
[73$_1$]	=+0.57143 XLVII				

General corrections in terms of the correlates—Continued.

$[73_2]$	=−0.28571 XLVII			
$[74_1]$	=−0.25000 XLVI	−0.25000 XLVIII	+0.75000 XLIX	
$[74_2]$	=−0.25000 XLVI	+0.75000 XLVIII	−0.25000 XLIX	
$[74_3]$	=+0.75000 XLVI	−0.25000 XLVIII	−0.25000 XLIX	
$[75_2]$	=+0.78571 XLVII	−0.21429 XLVIII	−0.21429 XLIX	−0.21429 L
$[75_3]$	=−0.21429 XLVII	+0.78571 XLVIII	−0.21429 XLIX	−0.21429 L
$[75_4]$	=−0.21429 XLVII	−0.21429 XLVIII	+0.78571 XLIX	0.21429 L
$[75_5]$	=−0.21429 XLVII	−0.21429 XLVIII	−0.21429 XLIX	+0.78571 L
$[76_1]$	=−0.33333 XLIX	+0.66667 L		
$[76_2]$	=+0.66667 XLIX	−0.33333 L		
$[77_1]$	=+0.44444 L			

Normal equations for determining the correlates.

No. of equation.					
1.	0=−0.09100	+2.08903 I	−0.27273 II	−0.18182 III	−0.09091 IV
2.	0=−0.58500	−0.27273 I	+2.08903 II	−0.43182 III	−0.09091 IV
		−0.25000 V	+0.49754 VI		
3.	0=−0.53300	−0.18182 I	−0.43182 II	+2.12879 III	+0.93939 IV
		−0.41667 V	−0.15878 VI	−0.50000 VII	−0.33333 VIII
		+0.25521 IX	−0.16667 X	+0.05344 XII	−0.16667 XIII
4.	0=−0.75700	−0.09091 I	−0.09091 II	+0.93939 III	+2.73286 IV
		+1.19298 V	+1.54016 VI	−1.13158 VII	−0.66667 VIII
		+0.42655 IX	−0.33333 X	−0.13158 XI	+0.17155 XII
		−0.33333 XIII	−0.13158 XIV	−0.17057 XV	
5.	0=−0.75700	−0.25000 II	−0.41667 III	+1.19298 IV	+2.63597 V
		−0.74619 VI	−0.76316 VII	−0.33333 VIII	+0.08747 IX
		−0.16667 X	−0.26316 XI	+0.18278 XII	−0.16667 XIII
		−0.26316 XIV	−0.34114 XV		
6.	0=−0.07785	+0.49754 II	−0.15878 III	+1.54016 IV	−0.74619 V
		+4.42082 VI	−0.30439 VII	−0.28186 VIII	+0.29127 IX
		−0.14093 X	+0.11840 XI	−0.01301 XII	−0.14093 XIII
		+0.11840 XIV	+0.15349 XV		
7.	0=+0.41300	−0.50000 III	−1.13158 IV	−0.76316 V	−0.30439 VI
		+3.06579 VII	+1.50000 VIII	−0.54456 IX	+0.75000 X
		−0.68421 XI	+0.09948 XII	+0.50000 XIII	−0.18421 XIV
		−0.23879 XV			
8.	0=+1.10400	−0.33333 III	−0.66667 IV	−0.33333 V	−0.28186 VI
		+1.50000 VII	+3.19048 VIII	+0.37227 IX	+1.16667 X
		+0.70741 XII	+1.38095 XIII	+0.57143 XIV	+1.21352 XV
		−0.14286 XVI	−0.14286 XVIII	−0.10643 XX	−0.28571 XXI
9.	0=+0.58760	+0.25521 III	+0.42655 IV	+0.08747 V	+0.29127 VI
		−0.54456 VII	+0.37227 VIII	+6.55858 IX	+0.41072 X
		+0.03205 XI	+1.72483 XII	−0.28953 XIII	+1.03638 XIV
		+2.17438 XV	−0.25108 XVI	−0.25108 XVIII	−0.18706 XX
		−0.50216 XXI			
10.	0=+0.28300	−0.16667 III	−0.33333 IV	−0.16667 V	−0.14093 VI
		+0.75000 VII	+1.16667 VIII	+0.41072 IX	+2.41667 X
		+1.16667 XI	+0.31670 XII	−0.50000 XIII	−0.16667 XIV
		−1.07632 XV	−0.16667 XVI	−0.16667 XVII	−0.06284 XX
11.	0=+0.21200	−0.13158 IV	−0.26316 V	+0.11840 VI	−0.68421 VII
		+0.03205 IX	+1.16667 X	+3.14912 XI	−0.49231 XII
		+0.33333 XIII	−0.51754 XIV	−1.21471 XV	−0.33333 XVI
		−0.33333 XVII	−0.12568 XX		

Normal equations for determining the correlates—Continued.

No. of equation.					
12.	0=-1.16045	+0.05344 III	+0.17155 IV	+0.18276 V	-0.01301 VI
		+0.09948 VII	+0.70741 VIII	+1.72483 IX	+0.31670 X
		-0.49231 XI	+7.78696 XII	+0.44912 XIII	+0.07106 XIV
		-0.78586 XV	-0.01947 XVI	-0.01947 XVII	-0.00734 XX
13.	0=-0.05300	-0.16667 III	-0.33333 IV	-0.16667 V	-0.14093 VI
		+0.50000 VII	+1.38095 VIII	-0.28953 IX	-0.50000 X
		+0.33333 XI	+0.44912 XII	+3.59525 XIII	+1.80952 XIV
		+0.20712 XV	-0.61906 XVI	-0.33333 XVII	-0.28571 XVIII
		-0.33854 XX	-0.57143 XXI		
14.	0=+0.42200	-0.13158 IV	-0.26316 V	+0.11840 VI	-0.18421 VII
		+0.57143 VIII	+1.03638 IX	-0.16667 X	-0.51754 XI
		+0.07106 XII	+1.80952 XIII	+3.36341 XIV	+2.37775 XV
		-0.59524 XVI	0.16667 XVII	-0.42857 XVIII	-0.38213 XX
		-0.85714 XXI			
15.	0=+2.58216	-0 17057 IV	-0.34114 V	+0.15349 VI	-0.23879 VII
		+1.21352 VIII	+2.17438 IX	-1.07632 X	-1.21471 XI
		-0.78586 XII	+0.20712 XIII	+2.37775 XIV	+9.45976 XV
		+0.00224 XVI	+0.35325 XVII	-0.35101 XVIII	-0.12832 XX
		-0.70202 XXI			
16.	0=+0.21700	-0.14286 VIII	-0.25108 IX	-0.16667 X	-0.33333 XI
		-0.01947 XII	-0.61906 XIII	-0.59524 XIV	+0.00224 XV
		+2.54762 XVI	-0.30953 XVII	-0.57143 XVIII	-0.28571 XIX
		-0.23036 XX	-0.57143 XXI	-0.19447 XXII	-0.14286 XXIII
		-0.42857 XXVI	-0.28571 XXVII	-0.57143 XXXI	
17.	0=-0.84200	-0.16667 X	-0.33333 XI	-0.01947 XII	-0.33333 XIII
		-0.16667 XIV	+0.35325 XV	-0.30953 XVI	+2.49048 XVII
		-0.42857 XVIII	-0.68571 XIX	+0.31534 XX	-0.28571 XXI
		-0.19447 XXII	-0.74286 XXIII	-0.20000 XXIV	-0.13033 XXV
		-0.92857 XXVI	-0.28571 XXVII	-0.20103 XXVIII	-0.77143 XXXI
		-0.40000 XXXII	-0.21917 XXXIII	+0.02574 XXXIV	
18.	0=-0.54600	-0.14286 VIII	-0.25108 IX	-0.28571 XIII	-0.42857 XIV
		-0.35101 XV	-0.57143 XVI	-0.42857 XVII	+3.42857 XVIII
		-1.00000 XIX	-1.07426 XX	+1.85714 XXI	+3.18037 XXII
		-0.42857 XXIII	-0.14286 XXIV	+0.42054 XXV	-0.28571 XXVI
		+0.14286 XXVII	-0.28571 XXIX	+0.75800 XXX	+0.28571 XXXI
		+0.46240 XXXV	-0.42857 XXXVI	-0.14286 XXXVII	+0.43753 XXXVIII
19.	0=-0.28600	-0.28571 XVI	-0.68571 XVII	-1.00000 XVIII	+3.48571 XIX
		+0.38693 XX	-0.57143 XXI	-0.92593 XXII	+1.51429 XXIII
		-0.54286 XXIV	-1.12843 XXV	+2.14286 XXVI	+0.42857 XXVII
		+0.32814 XXVIII	-0.28571 XXIX	-1.07568 XXX	+1.45714 XXXI
		+0.20000 XXXII	+0.43574 XXXIII	+0.04897 XXXIV	-0.51268 XXXV
		-0.42857 XXXVI	-0.14286 XXXVII	+0.43753 XXXVIII	
20.	0=+1.36900	-0.10643 VIII	-0.18706 IX	-0.06284 X	-0.12568 XI
		-0.00734 XII	-0.33854 XIII	-0.38213 XIV	-0.12832 XV
		-0.23036 XVI	+0.31534 XVII	-1.07426 XVIII	+0.38593 XIX
		+7.29098 XX	+0.96820 XXI	-6.07024 XXII	-0.31736 XXIII
		+0.24190 XXIV	-1.58715 XXV	-0.28797 XXVI	-0.11056 XXVIII
		+0.60136 XXIX	-2.22770 XXX	-0.12929 XXXI	-0.18807 XXXII
		-0.12604 XXXIII	+0.00775 XXXIV	-1.30947 XXXV	+0.90204 XXXVI
		+0.30068 XXXVII	-0.92089 XXXVIII		

Normal equations for determining the correlates—Continued.

No. of equation.					
21.	0=−1.97000	−0.28571 VIII	−0.50216 IX	−0.57143 XIII	−0.85711 XIV
		−0.70202 XV	−0.57143 XVI	−0.28571 XVII	+1.85714 XVIII
		−0.57143 XIX	+0.96820 XX	+3.72893 XXI	−1.85698 XXII
		−0.28571 XXIII	−1.11355 XXVI	−0.95605 XXVII	+0.47020 XXVIII
		−0.25641 XXIX	−0.20383 XXX	−0.14286 XXXI	−0.01091 XXXIV
		+0.11724 XXXV	−0.38462 XXXIX	−0.12821 XL	−0.25641 XLI
		+0.40387 XLII	−0.12821 XLIII	−0.18983 XLV	
22.	0=+2.39356	−0.19447 XVI	−0.19447 XVII	+3.18037 XVIII	−0.92593 XIX
		−6.07024 XX	−1.85698 XXI	+12.04427 XXII	−0.19447 XXIII
		−0.53699 XXIV	+1.58080 XXV	+1.24723 XXVI	+1.64976 XXVII
		−1.60329 XXVIII	−0.65786 XXIX	+3.79377 XXX	+0.30054 XXXI
		+0.66928 XXXIV	+2.54870 XXXV	−1.20123 XXXVI	−0.53699 XXXVII
		+1.03939 XXXVIII	+0.62418 XXXIX	+0.20806 XL	+0.41612 XLI
		−0.65543 XLII	+0.20806 XLIII	+0.30807 XLV	
23.	0=−0.46000	−0.14286 XVI	−0.74286 XVII	−0.42857 XVIII	+1.51429 XIX
		−0.31736 XX	−0.28571 XXI	−0.19447 XXII	+2.89048 XXIII
		+1.06667 XXIV	+1.44321 XXV	+1.57143 XXVI	−0.61905 XXVII
		+1.98196 XXVIII	−0.16667 XXIX	+0.11637 XXX	+0.82857 XXXI
		+1.13333 XXXII	+2.17512 XXXIII	−0.05148 XXXIV	−0.33333 XXXVI
		−0.50000 XXXVII	+0.04089 XXXVIII	−0.16667 XXXIX	−0.33333 XL
		+0.15586 XLII			
24.	0=+1.25000	−0.20000 XVII	−0.14286 XVIII	−0.54286 XIX	+0.24190 XX
		−0.53699 XXII	+1.06667 XXIII	+2.99048 XXIV	−1.06195 XXV
		+0.33333 XXVII	+0.95886 XXVIII	−0.61905 XXIX	−1.92012 XXX
		−0.20000 XXXI	+1.26667 XXXII	+1.31719 XXXIII	−0.10045 XXXIV
		−0.51268 XXXV	−1.09524 XXXVI	−1.14286 XXXVII	+0.51931 XXXVIII
		−0.33333 XXXIX	−0.66667 XL	+0.31172 XLII	
25.	0=−1.44003	−0.13033 XVII	+0.42054 XVIII	−1.12843 XIX	−1.58715 XX
		+1.58080 XXII	+1.44321 XXIII	−1.06195 XXIV	+7.17500 XXV
		+0.26421 XXVI	−1.44379 XXVII	+2.95094 XXVIII	+0.92816 XXIX
		+4.84861 XXX	+0.13388 XXXI	−0.04738 XXXII	+2.70110 XXXIII
		+0.60056 XXXIV	+1.50925 XXXV	+1.43578 XXXVI	+0.68178 XXXVII
		−1.30937 XXXVIII	+0.08708 XXXIX	+0.17416 XL	−0.08143 XLII
26.	0=+3.13600	−0.42857 XVI	−0.92857 XVII	−0.28571 XVIII	+2.14286 XIX
		−0.28797 XX	−1.11355 XXI	+1.24723 XXII	+1.57143 XXIII
		+0.26421 XXV	+1.70147 XXVI	+2.37363 XXVII	−1.97196 XXVIII
		+0.48718 XXIX	+1.00220 XXX	+1.78571 XXXI	+0.50000 XXXII
		+0.43705 XXXIII	+2.46223 XXXIV	+1.31151 XXXV	−0.76924 XXXIX
		−0.25642 XL	−0.51282 XLI	+0.80774 XLII	−0.25642 XLIII
		−0.37966 XLV			
27.	0=+2.53100	−0.28571 XVI	−0.28571 XVII	+0.14286 XVIII	+0.42857 XIX
		−0.95605 XXI	+1.64976 XXII	−0.61905 XXIII	+0.33333 XXIV
		−1.44379 XXV	+2.37363 XXVI	+4.60805 XXVII	−3.64690 XXVIII
		+1.89744 XXIX	+0.51290 XXX	+0.85714 XXXI	+0.66667 XXXII
		0.84438 XXXIII	+1.94717 XXXIV	+0.91738 XXXV	+0.33333 XXXVI
		−0.83472 XXXVIII	−1.48719 XXXIX	−1.05130 XL	−0.76923 XLI
		+1.52333 XLII	−0.38463 XLIII	−0.56949 XLV	
28.	0=−2.62990	−0.20103 XVII	+0.32814 XIX	−0.11056 XX	+0.47020 XXI
		−1.60329 XXII	+1.98196 XXIII	+0.95886 XXIV	+2.95094 XXV
		−1.97196 XXVI	−3.64690 XXVII	+7.66626 XXVIII	−1.73834 XXIX
		−1.19513 XXX	+0.16407 XXXI	+0.36465 XXXII	+3.54701 XXXIII
		−4.20025 XXXIV	−2.09009 XXXV	−0.75828 XXXVI	−0.82160 XXXVII
		+0.59443 XXXVIII	+1.34728 XXXIX	+0.34356 XL	+0.94040 XLI
		−1.42200 XLII	+0.47020 XLIII	+0.69622 XLV	

Normal equations for determining the correlates—Continued.

No. of equation.					
29.	0=−0.80000	− 0.28571 XVIII	−0.28571 XIX	+0.60136 XX	−0.25641 XXI
		− 0.65786 XXII	−0.16667 XXIII	−0.61905 XXIV	+0.92816 XXV
		+ 0.48718 XXVI	+1.89744 XXVII	−1.73834 XXVIII	+3.74908 XXIX
		+ 0.32505 XXX	+0.33333 XXXII	−0.42219 XXXIII	+0.65233 XXXIV
		− 1.81362 XXXV	+1.80953 XXXVI	+1.21429 XXXVII	−1.58605 XXXVIII
		− 0.93591 XXXIX	−0.58975 XL	−0.51282 XLI	+0.96360 XLII
		− 0.25642 XLIII	−0.37966 XLV		
30.	0=−0.33843	+ 0.75800 XVIII	−1.07568 XIX	−2.22770 XX	−0.20383 XXI
		+ 3.79377 XXII	+0.11637 XXIII	−1.92012 XXIV	+4.84861 XXV
		+ 1.00220 XXVI	+0.51290 XXVII	−1.19513 XXVIII	+0.32505 XXIX
		+ 6.52138 XXX	−0.23272 XXXII	+0.29477 XXXIII	+1.70989 XXXIV
		+ 4.67611 XXXV	+1.24710 XXXVI	+0.66678 XXXVII	−1.39840 XXXVIII
		− 0.49512 XXXIX	+0.02891 XL	−0.40766 XLI	+0.53328 XLII
		− 0.20383 XLIII	−0.30181 XLV		
31.	0=−0.70300	− 0.57143 XVI	−0.77143 XVII	+0.28571 XVIII	+1.45714 XIX
		− 0.12929 XX	−0.14286 XXI	+0.36054 XXII	+0.82857 XXIII
		− 0.20000 XXIV	+0.13388 XXV	+1.78571 XXVI	+0.85714 XXVII
		+ 0.16407 XXVIII	+3.37143 XXXI	−0.82857 XXXII	−1.31608 XXXIII
		− 1.31902 XXXIV	−0.53368 XXXV	−0.28571 XXXVI	−0.14286 XXXIX
		− 0.28571 XLI	−0.14286 XLIV	−0.35642 XLV	
32.	0=−0.67000	− 0.40000 XVII	+0.20000 XIX	−0.18807 XX	+1.13333 XXIII
		+ 1.26667 XXIV	−0.04738 XXV	+0.50000 XXVI	+0.66667 XXVII
		+ 0.36465 XXVIII	+0.33333 XXIX	−0.23272 XXX	−0.82857 XXXI
		+ 4.24762 XXXII	+2.21438 XXXIII	+1.75510 XXXIV	+0.55408 XXXV
		+ 1.80952 XXXVI	−1.66944 XXXVIII	+0.90476 XXXIX	−0.33333 XL
		+ 0.14286 XLI	−1.06773 XLII	−0.42857 XLIV	−0.23314 XLV
33.	0=+1.41588	− 0.21917 XVII	+0.43574 XIX	−0.12604 XX	+2.17512 XXIII
		+ 1.31719 XXIV	+2.70110 XXV	+0.43705 XXVI	−0.84438 XXVII
		+ 3.54701 XXVIII	−0.42219 XXIX	+0.29477 XXX	−1.31608 XXXI
		+ 2.21438 XXXII	+8.01301 XXXIII	+4.10183 XXXIV	+1.25462 XXXV
		+ 0.31492 XXXVI	−1.07947 XXXVII	+0.40063 XXXVIII	+0.15746 XXXIX
		− 0.65728 XL	+0.58307 XLI	+0.53626 XLII	+0.00342 XLIV
		+ 0.49033 XLV			
34.	0=+3.70447	+ 0.02574 XVII	+0.04897 XIX	+0.00775 XX	−0.01091 XXI
		+ 0.66928 XXII	−0.05148 XXIII	−0.10045 XXIV	+0.00056 XXV
		+ 2.46223 XXVI	+1.94717 XXVII	−4.20025 XXVIII	+0.65233 XXIX
		+ 1.70989 XXX	−1.31902 XXXI	+1.75510 XXXII	+4.10183 XXXIII
		+10.34319 XXXIV	+4.19317 XXXV	+0.95983 XXXVI	−2.09006 XXXIX
		− 0.64251 XL	−1.61008 XLI	+2.02403 XLII	−0.64251 XLIII
		− 0.16253 XLIV	−1.35683 XLV		
35.	0=+0.95484	+ 0.46240 XVIII	−0.51268 XIX	−1.30947 XX	+0.14724 XXI
		+ 2.54870 XXII	−0.51268 XXIV	+1.50923 XXV	+1.31151 XXVI
		+ 0.91738 XXVII	−2.09009 XXVIII	−1.81362 XXIX	+4.67611 XXX
		− 0.53368 XXXI	+0.55408 XXXII	+1.25462 XXXIII	+4.19317 XXXIV
		+ 8.23950 XXXV	+1.18834 XXXVI	−0.51268 XXXVII	−0.85041 XXXVIII
		− 1.18918 XXXIX	−0.39413 XL	−0.80184 XLI	+1.24159 XLII
		− 0.39413 XLIII	−0.00679 XLIV	−0.60052 XLV	
36.	0=−1.94400	− 0.42857 XVIII	−0.42857 XIX	+0.90204 XX	−1.20123 XXII
		− 0.33333 XXIII	−1.09524 XXIV	+1.43578 XXV	+0.33333 XXVII
		− 0.75828 XXVIII	+1.80953 XXIX	+1.24710 XXX	−0.28571 XXXI
		+ 1.80952 XXXII	+0.31492 XXXIII	+0.95983 XXXIV	+1.18834 XXXV
		+ 4.47619 XXXVI	+1.57143 XXXVII	−3.50137 XXXVIII	+1.38095 XXXIX
		+ 0.33333 XL	+0.42857 XLI	−1.37945 XLII	−0.28571 XLIV
		+ 0.12328 XLV			

Normal equations for determining the correlates—Continued.

No. of equation.					
37.	0=−0.94300	−0.14286 XVIII	−0.14286 XIX	+0.30068 XX	−0.53699 XXII
		−0.50000 XXIII	−1.14286 XXIV	+0.68178 XXV	−0.82160 XXVIII
		+1.21429 XXIX	+0.66678 XXX	−1.07947 XXXIII	−0.51268 XXXV
		+1.57143 XXXVI	+3.05481 XXXVII	+0.07800 XXXVIII	+0.50000 XXXIX
		+1.51162 XL	−0.11628 XLI	−0.48813 XLII	−0.23256 XLIII
		−0.11628 XLIV	+0.08481 XLV	−0.04651 XLVI	
38.	0=+1.94747	+0.43753 XVIII	+0.43753 XIX	−0.92089 XX	+1.03939 XXII
		+0.04089 XXIII	+0.51931 XXIV	−1.30937 XXV	−0.83472 XXVII
		+0.59443 XXVIII	−1.58605 XXIX	−1.39840 XXX	−1.66944 XXXII
		+0.40063 XXXIII	−0.85041 XXXV	−3.50137 XXXVI	+0.07800 XXXVII
		+8.02854 XXXVIII	−0.87561 XXXIX	+1.08643 XL	+0.49448 XLI
		+3.02646 XLII	+0.12513 XLIII	−0.36935 XLIV	+0.23204 XLV
		−0.25469 XLVI			
39.	0=−0.45300	−0.38462 XXI	+0.62418 XXII	−0.16667 XXIII	−0.33333 XXIV
		+0.08708 XXV	−0.76924 XXVI	−1.48719 XXVII	+1.34728 XXVIII
		−0.93591 XXIX	−0.49512 XXX	−0.14286 XXXI	+0.90476 XXXII
		+0.15746 XXXIII	−2.09006 XXXIV	−1.18918 XXXV	+1.38095 XXXVI
		+0.50000 XXXVII	−0.87561 XXXVIII	+3.53661 XXXIX	+1.28204 XL
		+1.94506 XLI	−3.47388 XLII	+0.61537 XLIII	−0.14286 XLIV
		+1.39087 XLV			
40.	0=−1.38400	−0.12821 XXI	+0.20806 XXII	−0.33333 XXIII	−0.66667 XXIV
		+0.17416 XXV	−0.25642 XXVI	−1.05130 XXVII	+0.34356 XXVIII
		−0.58975 XXIX	+0.02891 XXX	−0.33333 XXXII	−0.65728 XXXIII
		−0.64251 XXXIV	−0.39413 XXXV	+0.33333 XXXVI	+1.51162 XXXVII
		+1.08643 XXXVIII	+1.28204 XXXIX	+3.26603 XL	−0.53216 XLI
		−1.14568 XLII	−0.67971 XLIII	−0.27575 XLIV	+0.01130 XLV
		−0.14839 XLVI			
41.	0=+1.32000	−0.25641 XXI	+0.41612 XXII	−0.51282 XXVI	−0.76923 XXVII
		+0.94040 XXVIII	−0.51282 XXIX	−0.40766 XXX	−0.28571 XXXI
		+0.14286 XXXII	+0.58307 XXXIII	−1.61008 XXXIV	−0.80184 XXXV
		+0.42857 XXXVI	−0.11628 XXXVII	+0.49448 XXXVIII	+1.94506 XXXIX
		−0.53216 XL	+3.67323 XLI	−1.06833 XLII	+1.25854 XLIII
		−0.52823 XLIV	+2.19404 XLV	−0.18272 XLVI	
42.	0=+0.39463	+0.40387 XXI	−0.65543 XXII	+0.15586 XXIII	+0.31172 XXIV
		−0.08143 XXV	+0.80774 XXVI	+1.52333 XXVII	−1.42200 XXVIII
		+0.96360 XXIX	+0.53328 XXX	−1.06773 XXXII	+0.53626 XXXIII
		+2.02403 XXXIV	+1.24159 XXXV	−1.37945 XXXVI	−0.48813 XXXVII
		+3.02646 XXXVIII	−3.47388 XXXIX	−1.14568 XL	−1.06833 XLI
		+5.60401 XLII	−0.49328 XLIII	−0.16438 XLIV	−0.99981 XLV
		−0.11848 XLVI			
43.	0=−0.91600	−0.12821 XXI	+0.20806 XXII	−0.25642 XXVI	−0.38463 XXVII
		+0.47020 XXVIII	−0.25642 XXIX	−0.20383 XXX	−0.64251 XXXIV
		−0.39413 XXXV	−0.23256 XXXVII	+0.12513 XXXVIII	+0.61537 XXXIX
		−0.67971 XL	+1.25854 XLI	−0.49328 XLII	+2.56149 XLIII
		+0.83452 XLIV	−0.96717 XLV	−0.55840 XLVI	+0.01557 XLVII
		−0.03588 XLVIII			
44.	0=+0.54000	−0.14286 XXXI	−0.42857 XXXII	+0.00342 XXXIII	−0.16253 XXXIV
		−0.00679 XXXV	−0.28571 XXXVI	−0.11628 XXXVII	−0.36935 XXXVIII
		−0.14286 XXXIX	−0.27575 XL	−0.52823 XLI	−0.16438 XLII
		+0.83452 XLIII	+2.25374 XLIV	−0.04298 XLV	−0.56864 XLVI
		+0.03114 XLVII	−0.07176 XLVIII		

Normal equations for determining the correlates—Continued.

No. of equation.					
45.	0=+3.36715	−0.18983 XXI	+0.30807 XXII	−0.37966 XXVI	−0.56949 XXVII
		+0.69622 XXVIII	−0.37966 XXIX	−0.30181 XXX	−0.35642 XXXI
		−0.23314 XXXII	+0.49033 XXXIII	−1.35683 XXXIV	−0.60052 XXXV
		+0.12328 XXXVI	+0.08481 XXXVII	+0.23204 XXXVIII	+1.39087 XXXIX
		+0.01130 XL	+2.19404 XLI	−0.99981 XLII	−0.96717 XLIII
		−0.04298 XLIV	+4.95315 XLV	+0.13327 XLVI	
46.	0=+1.16300	−0.04651 XXXVII	−0.25469 XXXVIII	−0.14839 XL	−0.18272 XLI
		−0.11848 XLII	0.55840 XLIII	−0.56864 XLIV	+0.13327 XLV
		+2.20997 XLVI	−0.00609 XLVII	−0.58378 XLVIII	−0.25000 XLIX
47.	0=−0.54800	+0.01557 XLIII	+0.03114 XLIV	−0.00609 XLVI	+1.86289 XLVII
		−0.51016 XLVIII	−0.21429 XLIX	−0.21429 L	
48.	0=+0.87800	−0.03588 XLIII	−0.07176 XLIV	−0.58378 XLVI	−0.51016 XLVII
		+2.21750 XLVIII	−0.46429 XLIX	−0.21429 L	
49.	0=−2.36700	−0.25000 XLVI	−0.21429 XLVII	−0.46429 XLVIII	+2.20238 XLIX
		−0.54762 L			
50.	0=−0.10900	−0.21429 XLVII	−0.21429 XLVIII	−0.54762 XLIX	+1.89682 L

Values of the correlates and of their logarithms.

I	=+0.1381	log 9.1403195+	XXVI	=−0.8350	log 9.9216813−
II	=+0.4285	log 9.6319711+	XXVII	=−1.2970	log 0.1129433−
III	=+0.6017	log 9.7793872+	XXVIII	=−0.4713	log 9.6732974−
IV	=−0.3155	log 9.4989994−	XXIX	=+1.6223	log 0.2101365+
V	=+0.5697	log 9.7556767+	XXX	=+0.4516	log 9.6547155+
VI	=+0.2031	log 9.3077741+	XXXI	=+0.6668	log 9.8240151+
VII	=+0.0825	log 8.9162960+	XXXII	=+0.4572	log 9.6768582+
VIII	=−0.4406	log 9.6440051−	XXXIII	=+0.3762	log 9.5754188+
IX	=−0.0492	log 8.6918768−	XXXIV	=−0.6489	log 9.8122045−
X	=+0.1644	log 9.2158490+	XXXV	=+0.7044	log 9.8478132+
XI	=−0.1227	log 9.0887030−	XXXVI	=−0.5639	log 9.7512329−
XII	=+0.1398	log 9.1454450+	XXXVII	=−0.0780	log 8.8920946−
XIII	=+0.2689	log 9.4296715+	XXXVIII	=−0.4408	log 9.6442810−
XIV	=−0.0267	log 8.4260230−	XXXIX	=+0.5897	log 9.7706017+
XV	=−0.2016	log 9.3044474−	XL	=+0.6394	log 9.8057998+
XVI	=+0.0545	log 8.7362371+	XLI	=−0.6796	log 9.8322725−
XVII	=+0.6490	log 9.8122179+	XLII	=+0.4727	log 9.6746040+
XVIII	=+1.2818	log 0.1078271+	XLIII	=+0.8581	log 9.9335328+
XIX	=+1.4178	log 0.1516272+	XLIV	=−0.7518	log 9.8761139−
XX	=−0.6352	log 9.8029173−	XLV	=−0.2682	log 9.4283940−
XXI	=−0.5411	log 9.7332775−	XLVI	=−0.4331	log 9.6366082−
XXII	=−0.7801	log 9.8921670−	XLVII	=+0.4402	log 9.6436994+
XXIII	=−0.6511	log 9.8136544−	XLVIII	=−0.1388	log 9.1424833−
XXIV	=+0.7139	log 9.8536495+	XLIX	=+1.1440	log 0.0584374+
XXV	=−0.1996	log 9.3002041−	L	=+0.4218	log 9.6251092+

Values of the general corrections.

	″		″		″		″
$[56_1]$	$=+0.104$	$[63_1]$	$=-0.326$	$[67_2]$	$=+0.260$	$[71_2]$	$=-0.236$
$[57_2]$	$=+0.085$	$[63_2]$	$=-0.392$	$[67_3]$	$=+0.624$	$[71_3]$	$=+0.347$
$[57_3]$	$=+0.262$	$[63_3]$	$=+0.490$	$[67_4]$	$=-0.381$	$[71_4]$	$=+0.091$
$[58_1]$	$=-0.088$	$[63_4]$	$=+0.443$	$[67_5]$	$=-0.050$	$[71_5]$	$=-0.627$
$[58_2]$	$=+0.227$	$[63_5]$	$=+0.102$	$[68_1]$	$=+0.465$	$[71_6]$	$=-0.695$
$[58_3]$	$=+0.193$	$[63_6]$	$=-0.429$	$[68_2]$	$=-0.033$	$[72_1]$	$=-0.437$
$[58_4]$	$=-0.097$	$[64_1]$	$=+0.425$	$[68_3]$	$=-0.655$	$[72_2]$	$=+0.421$
$[59_1]$	$=+0.130$	$[64_2]$	$=-0.407$	$[68_4]$	$=-0.108$	$[72_3]$	$=-0.166$
$[59_2]$	$=+0.088$	$[64_3]$	$=-0.130$	$[68_5]$	$=+0.574$	$[72_4]$	$=-0.057$
$[59_3]$	$=+0.081$	$[64_4]$	$=+0.401$	$[68_6]$	$=+0.156$	$[72_5]$	$=+0.256$
$[60_1]$	$=-0.079$	$[64_5]$	$=-0.066$	$[69_1]$	$=+1.113$	$[73_1]$	$=+0.252$
$[60_2]$	$=-0.316$	$[65_1]$	$=-0.254$	$[69_2]$	$=-0.924$	$[73_2]$	$=-0.126$
$[60_3]$	$=-0.046$	$[65_2]$	$=+0.341$	$[69_3]$	$=-0.865$	$[74_1]$	$=+1.001$
$[60_4]$	$=+0.226$	$[65_3]$	$=+0.290$	$[69_4]$	$=+0.575$	$[74_2]$	$=-0.282$
$[60_5]$	$=+0.306$	$[65_4]$	$=-0.356$	$[69_5]$	$=+0.848$	$[74_3]$	$=-0.576$
$[61_1]$	$=+0.136$	$[65_5]$	$=-1.206$	$[69_6]$	$=+0.362$	$[75_{1+2}]$	$=\ 0.000$
$[61_2]$	$=+0.314$	$[65_6]$	$=+0.925$	$[69_7]$	$=-1.108$	$[75_2]$	$=+0.040$
$[61_3]$	$=+0.084$	$[66_1]$	$=+0.296$	$[70_1]$	$=-0.421$	$[75_3]$	$=-0.539$
$[61_4]$	$=-0.465$	$[66_2]$	$=-0.460$	$[70_2]$	$=+0.171$	$[75_4]$	$=+0.744$
$[61_5]$	$=-0.064$	$[66_3]$	$=+0.178$	$[70_3]$	$=-0.038$	$[75_5]$	$=+0.022$
$[62_1]$	$=+0.179$	$[66_4]$	$=+0.206$	$[70_4]$	$=+0.110$	$[76_1]$	$=-0.100$
$[62_2]$	$=-0.261$	$[66_5]$	$=+0.077$	$[70_5]$	$=+0.284$	$[76_2]$	$=+0.622$
$[62_3]$	$=-0.056$	$[67_1]$	$=-0.118$	$[71_1]$	$=+0.843$	$[77_1]$	$=+0.187$

Residuals resulting from substitution of general corrections in numerical equations of condition.

No. of equation.	Residual.	No. of equation.	Residual.	No. of equation.	Residual.
1	0.0000	18	+0.0001	35	0.0000
2	0.0000	19	−0.0001	36	−0.0001
3	0.0000	20	+0.0024	37	−0.0001
4	0.0000	21	+0.0001	38	−0.0004
5	0.0000	22	+0.0055	39	0.0000
6	0.0000	23	−0.0001	40	0.0000
7	0.0000	24	0.0000	41	−0.0001
8	−0.0001	25	+0.0009	42	0.0000
9	−0.0005	26	−0.0001	43	0.0000
10	0.0000	27	0.0000	44	0.0000
11	0.0000	28	0.0000	45	−0.0016
12	−0.0006	29	0.0000	46	0.0000
13	0.0000	30	+0.0007	47	0.0000
14	0.0000	31	−0.0001	48	0.0000
15	−0.0005	32	−0.0001	49	0.0000
16	0.0000	33	−0.0020	50	0.0000
17	+0.0001	34	-0.0004		

PROBABLE ERRORS OF OBSERVED AND ADJUSTED ANGLES.

§ **5.** Let—

m = whole number of observed angles in a section (one adjustment).
r = whole number of rigid conditions in a section.
n = number of triangles in principal chain.
$[pvv]$ = sum of weighted squares of corrections to observed angles.
ρ_1 = probable error of an observed angle of weight unity.
ρ_s = probable error of an observed angle of average weight in whole section.
ρ_s' = probable error of an adjusted angle of average weight in whole section.
p_s = average weight of an observed angle in whole section.
p_c = average weight of an observed angle in principal chain.
ρ_c = probable error of an observed angle of average weight in principal chain.
ρ_c' = probable error of an adjusted angle of average weight in principal chain.
$[vv]$ = sum of squares of closing errors of triangles in principal chain.
ρ_t = probable error of an observed angle in principal chain as derived from the closing errors of triangles.

Proceeding as in Chapter XIV, C, § 8, there are found the following values:

FOR THE ENTIRE SECTIONS OF THIS CHAPTER.

Section.	Extent of section.	m	r	$[pvv]$	ρ_1	p_s	ρ_s	$\sqrt{\frac{m-r}{m}}$	ρ_s'
					″		″		″
VIII	Michigan City - Bald Tom to Fremont - Quincy	67	39	14.42	0.41	1.02	0.41	0.65	0.26
IX	Fremont - Quincy to Cedar Point - Stony Point	93	65	17.78	0.35	0.99	0.36	0.55	0.19
X	Cedar Point - Stony Point to Chester - Willoughby...	126	84	25.00	0.37	0.95	0.38	0.58	0.22

FOR THE PRINCIPAL CHAIN CONNECTING THE CHICAGO AND SANDUSKY BASES, GIVEN IN D, § 6, FOLLOWING.

Section.	Extent of principal chain in each section.	p_c	ρ_c	ρ_c'	From closing errors of triangles.				
					$[vv]$	n	ρ_t	Average error.	Greatest error.
			″	″			″	″	″
VII	Chicago Base to Michigan City - Bald Tom	0.82	0.50	0.31	11.52	9	0.44	0.96	2.07
VIII	Michigan City - Bald Tom to Fremont - Quincy.......	0.88	0.44	0.28	22.70	14	0.50	1.00	2.91
IX	Fremont - Quincy to Cedar Point - Stony Point	1.00	0.35	0.19	13.73	14	0.39	0.78	1.86
X	Cedar Point - Stony Point to Sandusky Base..........	0.82	0.41	0.23	15.64	8	0.54	1.07	2.71
	Entire principal chain	......	......	......	63.59	45	0.46	0.94	2.91

D.—PRINCIPAL CHAIN OF TRIANGLES BETWEEN CHICAGO AND SANDUSKY BASES.

§ **6.** The principal triangles connecting Chicago and Sandusky Bases are forty-five in number, and form a chain about 280 miles in length. In the adjustment of this chain for the discrepancy between the measured value of either base and its value computed from the other base through the triangulation, the bases are considered exact. Were they not considered exact, their probable errors would not in this chain require corrections to the logarithms of their lengths as great as a unit in the seventh place of decimals. The triangles fall in four different sections of the adjustment, the dividing lines of which, lying between the bases, are Michigan City - Bald Tom, Fremont - Quincy, and Stony Point - Cedar Point. Starting from Chicago Base, the probable errors of observed angles of average weight in the parts of this chain separated by the above lines are (see Chapters XVI, C, § 11, and XVII, C, § 5) $\pm 0''.49$, $\pm 0''.44$, $\pm 0''.35$, $\pm 0''.41$. With these values we find, using the notation of Chapter XIV, D, § 10, and the values of α^2 and β^2, given in the following tables,

$$\frac{1}{p}+\frac{1}{p'}=\Sigma\,(\alpha^2+\beta^2)\,\rho^2=5659$$

The logarithm of the measured value of Sandusky Base expressed in feet is 4.3101715. The logarithm of Sandusky Base, computed from Chicago Base through the triangulation, is 4.3101618. Hence, $d=+97$.

The additional data required in deriving the corrections to the logarithms of the lines, as computed from the Chicago Base, are given in the tables which follow, these being arranged in the same manner as those in Chapter XIV, D, § 10. The line in the system having the greatest probable error is Jefferson - Milton. For this line $\frac{1}{p}=2923$ and $\frac{1}{p'}=2736$, giving for the probable error of its logarithm ± 37.6 in units of the seventh decimal place, which is equivalent to $\frac{1}{115530}$ part of the line's length. This probable error is, however, independent of the probable errors of the bases, and is, therefore, somewhat too small. But since the probable error of either base is only about $\frac{1}{1000000}$ of its length, the above result would not be materially changed by taking the probable errors of the bases into account.

Principal chain of triangles between Chicago and Sandusky Bases.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	α^2 and β^2	$\Sigma\,(\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Willow Springs	42 35 04. 570		4. 3917929	524. 41			4. 3917929
East Base	40 29 04. 976	+0. 484	4. 3738195				4. 3738197
West Base	96 55 50. 591		4. 5582257	6. 25	530. 66	127	4. 5582259
Morgan Park	44 52 44. 590		4. 5582257	445. 21			4. 5582259
Willow Springs	34 24 48. 796	−1. 627	4. 4618324				4. 4618328
East Base	100 42 26. 856		4. 7020314	16. 00	991. 87	237	4. 7020318
Shot Tower	37 39 04. 874		4. 7020314	745. 29			4. 7020318
Willow Springs	60 23 46. 481	−1. 251	4. 8553445				4. 8553452
Morgan Park	81 57 09. 486		4. 9117960	9. 00	1746. 16	417	4. 9117967
Military Academy	81 25 36. 348		4. 9117960	10. 24			4. 9117967
Shot Tower	37 14 28. 682	−1. 187	4. 6985550				4. 6985558
Willow Springs	61 19 55. 814		4. 8598813	132. 25	1888. 65	452	4. 8598821
Millers	29 11 38. 637		4. 8598813	1421. 29			4. 8598821
Shot Tower	55 36 25. 651	+2. 071	5. 0882191				5. 0882205
Military Academy	95 11 57. 795		5. 1698790	3. 61	3313. 55	793	5. 1698804
Michigan City	38 28 11. 338		5. 1698790	702. 25			5. 1698804
Millers	114 11 25. 900	−0. 762	5. 3361056				5. 3361079
Shot Tower	27 20 26. 233		5. 0380925	1656. 49	5672. 29	1358	5. 0380948
Otis	85 27 22. 773		5. 0380925	2. 89			5. 0380948
Michigan City	65 15 26. 461	+0. 650	4. 9976392				4. 9976421
Millers	29 17 12. 018		4. 7289264	1413. 76	7088. 94	1696	4. 7289293
Springville	72 51 40. 281		4. 7289264	42. 25			4. 7289293
Michigan City	55 23 13. 437	−0. 608	4. 6640572				4. 6640602
Otis	51 45 06. 738		4. 6437091	275. 56	7406. 75	1772	4. 6437121
Bald Tom	26 25 28. 139		4. 6437091	1797. 76			4. 6437121
Michigan City	75 43 51. 648	−0. 015	4. 9817236				4. 9817274
Springville	77 50 41. 187		4. 9854855	20. 25	9224. 76	2208	4. 9854893

Principal chain of triangles between Chicago and Sandusky Bases—Continued.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	a^2 and β^2	$\Sigma(a^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Galena	90 06 01.854		4.9854855	0.00			4.9854893
Bald Tom	35 06 57.623	+0.113	4.7453296				4.7453335
Michigan City	54 47 01.558		4.8976979	222.01	222.01	2250	4.8977018
Carlisle	59 54 32.324		4.8976979	148.84			4.8977018
Bald Tom	33 39 31.462	−1.503	4.7042677				4.7042716
Galena	86 25 57.155		4.9597243	1.69	372.54	2270	4.9597282
Bertrand	89 10 50.985		4.9597243	0.09			4.9597282
Carlisle	56 28 25.735	+1.639	4.8807434				4.8807476
Bald Tom	34 20 44.201		4.7111882	948.64	1321.27	2461	4.7111924
Penn	40 23 07.629		4.7111882	610.09			4.7111924
Bertrand	82 27 29.295	−2.908	4.8958896				4.8958941
Carlisle	57 09 23.877		4.8240228	184.96	2116.32	2612	4.8240273
Milton	72 19 44.193		4.8240228	46.24			4.8240273
Penn	53 08 45.026	+1.413	4.7481935				4.7481981
Bertrand	54 31 31.498		4.7558374	225.00	2387.56	2664	4.7558420
Jefferson	30 11 10.035		4.7558374	1310.44			4.7558420
Milton	75 36 29.503	−0.364	5.0405874				5.0405924
Penn	74 12 21.881		5.0377210	34.81	3732.81	2923	5.0377260
Calvin	109 32 06.169		5.0377210	56.25			5.0377260
Jefferson	36 51 40.664	−0.650	4.8415570				4.8415624
Milton	33 36 05.155		4.8065162	1004.89	4703.95	3124	4.8065216
Porter	83 52 49.490		4.8065162	4.84			4.8065216
Jefferson	42 07 22.706	+1.619	4.6355416				4.6355470
Calvin	53 59 48.332		4.7169376	234.09	5032.88	3170	4.7169430
Van Buren	54 32 09.208		4.7169376	225.00			4.7169430
Porter	57 46 37.978	−0.963	4.7334184				4.7334239
Jefferson	67 41 13.429		4.7722577	73.96	5331.84	3227	4.7722632
Sherman	43 27 55.074		4.7722577	492.84			4.7722632
Van Buren	85 45 49.518	−1.480	4.9335354				4.9335412
Porter	50 46 16.335		4.8238155	295.84	6120.52	3377	4.8238213
Mongo	44 39 03.507		4.8238155	453.69			4.8238213
Sherman	82 45 40.688	−0.948	4.9735177				4.9735237
Van Buren	52 35 16.979		4.8769704	259.21	6833.42	3514	4.8769764
Bronson	80 06 15.196		4.8769704	13.69			4.8769764
Mongo	48 20 13.963	−0.127	4.7568414				4.7568475
Sherman	51 33 31.635		4.7773785	278.89	7126.00	3570	4.7773846
Fremont	39 20 26.179		4.7773785	660.49			4.7773846
Bronson	61 17 52.123	−0.261	4.9184012				4.9184075
Mongo	79 21 42.848		4.9678094	15.21	7801.70	3699	4.9678157
Quincy	59 04 19.886		4.9678094	158.76			4.9678157
Fremont	62 01 52.451	−0.008	4.9804764				4.9804828
Bronson	58 53 49.456		4.9670114	161.29	8121.75	3760	4.9670178
Reading	80 01 41.967		4.9670114	13.69			4.9670178
Quincy	54 23 55.071	−1.530	4.8837586				4.8837651
Fremont	45 34 24.156		4.8274094	428.49	442.18	3815	4.8274159

Principal chain of triangles between Chicago and Sandusky Bases—Continued.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	α^2 and β^2	$\Sigma\ (\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Hillsdale	62 37 43.537		4.8274094	118.81			4.8274159
Reading	85 29 02.126	−0.182	4.8776235				4.8776303
Quincy	31 53 14.968		4.6018152	1149.21	1710.20	3973	4.6018220
Pittsford	34 29 38.545		4.6018152	936.36			4.6018220
Hillsdale	91 34 24.869	+0.185	4.8485897				4.8485968
Reading	53 55 57.124		4.7563389	237.16	2883.72	4119	4.7563460
Wheatland	78 08 51.643		4.7563389	19.36			4.7563460
Hillsdale	36 21 52.411	+1.282	4.5386945				4.5387016
Pittsford	65 29 16.369		4.7246790	92.16	2995.24	4133	4.7246861
Bunday	48 28 35.999		4.7246790	349.69			4.7246861
Wheatland	87 05 44.667	−0.534	4.8498216				4.8498289
Hillsdale	44 25 39.953		4.6954833	462.25	3807.18	4234	4.6954906
Woodstock	49 19 16.232		4.6954833	327.61			4.6954906
Wheatland	66 47 38.408	−0.656	4.7789593				4.7789667
Bunday	63 53 05.991		4.7688332	106.09	4240.88	4288	4.7688406
Fairfield	28 07 00.426		4.7688332	1552.36			4.7688406
Woodstock	108 22 12.316	+0.158	5.0728497				5.0728575
Wheatland	43 30 48.385		4.9334833	492.84	6286.08	4543	4.9334911
Raisin	64 57 18.588		4.9334833	98.01			4.9334911
Fairfield	76 00 48.206	−1.448	4.9632958				4.9633038
Woodstock	39 01 54.376		4.7755346	670.81	7054.90	4639	4.7755426
Blissfield	60 44 25.218		4.7755346	139.24			4.7755426
Raisin	73 50 36.065	−0.196	4.8173118				4.8173199
Fairfield	45 24 59.374		4.6874312	428.49	7622.63	4710	4.6874393
Dundee	43 45 57.303		4.6874312	484.00			4.6874393
Blissfield	95 46 09.598	+1.313	4.8452997				4.8453080
Raisin	40 27 53.621		4.6597374	610.09	8716.72	4846	4.6597457
Bedford	56 34 47.476		4.6597374	193.21			4.6597457
Dundee	75 21 26.400	+0.255	4.7238914				4.7238998
Blissfield	48 03 46.548		4.6097332	357.21	9267.14	4915	4.6097416
Monroe	44 16 04.135		4.6097332	466.56			4.6097416
Bedford	68 37 31.733	−1.235	4.7349212				4.7349297
Dundee	67 06 24.612		4.7302387	79.21	9812.91	4983	4.7302472
Cedar Point	36 33 41.044		4.7302387	806.56			4.7302472
Monroe	74 29 10.780	−0.154	4.9391054				4.9391141
Bedford	68 57 09.203		4.9252371	65.61	10685.08	5091	4.9252458
Stony Point	71 29 01.461		4.9252371	49.00			4.9252458
Cedar Point	39 32 50.657	−1.862	4.7522650				4.7522738
Monroe	68 58 05.929		4.9183791	65.61	10799.69	5106	4.9183879
Middle Sister	51 58 36.201		4.9183791	272.25			4.9183879
Cedar Point	47 31 39.178	+0.359	4.8898068				4.8898156
Stony Point	80 29 46.116		5.0159833	12.96	285.21	5153	5.0159921
Locust Point	79 21 35.581		5.0159833	16.00			5.0159921
Middle Sister	42 43 14.154	+2.706	4.8550155				4.8550244
Cedar Point	57 55 11.749		4.9515555	174.24	475.45	5184	4.9515644

Principal chain of triangles between Chicago and Sandusky Bases—Continued.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	α^2 and β^2	$\Sigma\ (\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Middle Bass	62 05 29.271		4.9515555	123.21			4.9515644
Locust Point	52 38 36.695	−1.647	4.9055517				4.9055607
Middle Sister	65 15 55.575		4.9634610	94.09	692.75	5220	4.9634700
Danbury	76 48 47.647		4.9634610	24.01			4.9634700
Middle Bass	58 56 15.368	−0.189	4.9078471				4.9078562
Locust Point	44 14 58.208		4.8187868	466.56	1183.32	5301	4.8187959
Kelley's	91 27 48.829		4.8187868	0.36			4.8187959
Danbury	39 18 38.221	−0.065	4.6206913				4.6207005
Middle Bass	49 13 33.441		4.6981910	331.24	1514.92	5356	4.6982002
West Base	68 16 12.861		4.6981919	70.56			4.6982002
Danbury	55 23 10.231	−0.469	4.6456025				4.6456118
Kelley's	56 20 37.341		4.6505232	198.81	1784.29	5401	4.6505325
Sandusky	65 24 37.122		4.6505232	92.16			4.6505325
West Base	72 37 43.772	+1.909	4.6715369				4.6715463
Danbury	41 57 39.437		4.5169926	547.56	2424.01	5506	4.5170020
East Base	69 32 30.524		4.5169926	62.41			4.5170020
West Base	74 52 17.009	+1.243	4.5299680				4.5299777
Sandusky	35 35 12.620		4.3101618	864.36	3350.78	5659	4.3101715

CHAPTER XVIII.

TRIANGULATION FROM SANDUSKY BASE TO BUFFALO BASE.

A.—DESCRIPTIONS OF STATIONS.

NOTE RELATIVE TO ELEVATIONS.

§ **1.** The elevations of the surface of the ground at stations west of Font Hill, described in this chapter and in Chapter XVII, A, § 2, were determined for the most part by trigonometrical leveling. The heights of those stations near the shore of Lake Erie were, in nearly every case, determined by means of a single zenith distance to some point of known height above the surface of the lake. For stations remote from the lake shore, heights were computed from non-simultaneous, reciprocal zenith distances. The latter data were sufficiently numerous, also, to give a good mean value for the coefficient of refraction. Fifty-one independent values were obtained from observations over as many different lines, which varied in length from 6 to 44 miles, averaging 18 miles. The greatest and least values for the coefficient of refraction were 0.145 and 0.042. Only six values exceeded 0.09, and they were obtained from observations made either early in the spring or late in the fall on lines extending over water. The weighted mean of the remaining coefficients, which were obtained from observations made during the summer season, on 37 lines extending over land and 8 lines extending over water, was 0.061. This value was used in computing heights dependent on single zenith distances. No rigorous adjustment of these heights has been made. The adopted values are quite accordant, however, as shown by the numerous checks resulting both from the connections with the lake surface and from the independent routes along which the computation may be carried. The probable error of any height may be estimated as not exceeding on the average ± 1 foot except for stations Grand River, Silver Creek, and Sturgeon Point, to whose heights may be assigned a probable error of ± 4 feet. The heights of stations referred to Lake Erie are above the mean level of that lake given in Chapter XXII, § 13. The heights of stations east of Font Hill referred to the mean level of Lake Ontario are above the mean level of that lake given in Chapter XXII, § 13, and were determined as stated in Chapter XIX, § 1.

DESCRIPTIONS OF STATIONS.

§ **2.** LITTLE MOUNTAIN, 1877.*—This station is situated on a high hill, called Little Mountain, in Mentor Township, Lake County, Ohio, about 4 miles south of Mentor railway station on the Lake Shore and Michigan Southern Railway. The height of station used was 104 feet. The geodetic point is marked by an iron bolt leaded into the solid rock. Three stone reference-posts are set, as follows: one bearing north 28° 11′ east, distant 95 metres; one bearing south 81° 41′ east, distant 97.3 metres; and one bearing south 5° 53′ east, distant 124.2 metres from the geodetic point. The height of ground at the station above mean level of Lake Erie is 674.7 feet.

CLARIDON, 1877.—This station is situated in lot 1, section 14, Claridon Township, Geauga County, Ohio, about 1½ miles in a southerly direction from the village of East Claridon, and 2 miles from the railway station. The height of station used was 100 feet. The geodetic point is marked in the usual manner. Three stone reference-posts were set as follows: one bearing south 83° 38′ east, distant 87.8 metres; one bearing south 23° 11′ east, distant 65.5 metres; and one bearing south 3° 41′ west, distant 61.2 metres from the geodetic point. The first-mentioned reference-stone is set by the north-and-south line-fence east of the station, and the last two by the east-

* See note relative to topographical sketches of stations under Burnt Bluff, Chapter XV, A, § 2.

and-west line-fence south of the station. The height of ground at the station above mean level of Lake Erie is 793.0 feet.

MESOPOTAMIA, 1877.—This station is situated in section 57, Mesopotamia Township, Trumbull County, Ohio, about one-third of a mile east of the west line, and one-half mile south of the center line of the township, and about 4 miles east of the village of Middlefield. The height of station used was 70 feet. The geodetic point was marked in the usual manner. Three stone reference-posts were set as follows: two along a line-fence to the north of the station, one bearing north 23° 23′ west, distant 102.6 metres, one bearing north 9° 51′ east, distant 96.0 metres; and one by a north-and-south line-fence, bearing south 20° 40′ east, distant 129.4 metres from the geodetic point. The height of ground at the station above mean level of Lake Erie is 599.2 feet.

THOMPSON, 1876, 1877.—This station is situated in Thompson Township, Geauga County, Ohio, about one-half mile southeast of the village of Thompson, on Thompson Ledge, 313 metres south along the ledge from the road leading east from Thompson. The height of station used was 88.3 feet. The geodetic point is marked by a brass rod leaded into the solid rock, about 6 inches below the surface of the coarse, compact, pebbly soil. Five stone reference-posts are set as follows: one bearing north 34° 02′ west, distant 85.4 metres; one bearing north 81° 42′ east, distant 6.2 metres; one bearing south 25° 06′ east, distant 36.0 metres; one bearing south 19° 34′ west, distant 34.2 metres; and one bearing south 77° 02′ west, distant 35.0 metres from the geodetic point. The bluff is distant 12 metres on the east. The height of ground at the station above mean level of Lake Erie is 734.5 feet.

ANDOVER, 1876, 1877.—This station is situated on the highest point of Owen's Mound, in Andover Township, Ashtabula County, Ohio, about 1 mile north of the village of West Andover, and about 2½ miles west of Andover railway station on the Franklin division of the Lake Shore and Michigan Southern Railway. The height of station used was 113 feet. The geodetic point is marked by a stone post of the usual form, set 2½ feet below the surface, with a stone post set directly over it as a surface-mark. Three stone reference-posts are set along the fence south of the station, as follows: One bearing south 73° 59′ east, distant 32.38 metres; one bearing south 4° 04′ east, distant 10.56 metres; and one bearing south 75° 07′ west, distant 39.0 metres from the geodetic point. The height of ground at the station above mean level of Lake Erie is 617.8 feet.

CONNEAUT, 1876, 1877.—This station is situated on the South Ridge, a height of land running in an easterly and westerly direction, in the southwest corner of Conneaut Township, Ashtabula County, Ohio, about 5 miles southwest of the village of Conneaut, 2 miles west of the village of South Ridge, and 3 miles east of the village of Kingsville, on the north side of the Ridge Road. The height of station used was 115 feet. The geodetic point was marked by a stone post of the usual form, set 2.6 feet below the surface of the ground. Three stone reference-posts are set along the road-fence, as follows: One bearing south 75° 04′ west, distant 29.56 metres; one bearing south 14° 05′ east, distant 8.1 metres; and one bearing south 75° 43′ east, distant 33.88 metres from the geodetic point. The height of ground at the station above mean level of Lake Erie is 308.9 feet.

EDINBORO, 1876.—This station is situated in the western part of Washington Township, Erie County, Pennsylvania, about 3 miles west of Conneauttee Lake and the village of Edinboro. The height of station used was 115 feet. The geodetic point is marked by a stone post of the usual form, set 2 feet below the surface of the ground, with a stone post set directly over it for a surface-mark. Three stone reference-posts are set as follows: One bearing west, distant 8.5 metres; one east, distant 5.81 metres; and one south, distant 5.1 metres from the geodetic point. These bearings are approximate. The height of ground at the station above mean level of Lake Erie is 929.6 feet.

HOUGHTON, 1876, 1877.—This station is situated on a large sand hill about 7 miles east of Port Burwell, Province of Ontario, and 1 mile east of Houghton. The height of station used was 53 feet. The geodetic point is marked by a stone post, set 3 feet below the surface of the ground, with the letters U. S. cut thereon, the tops of the letters being north. Three stone reference-posts are set as follows: One in a clump of willows, bearing north 51° 04′ west, distant 117.75 metres; one north, distant 4.4 metres, with letters U. S. cut in top, the tops of the letters being west; and one south, distant 4.5 metres from the geodetic point. The directions of the last two stones mentioned are approximate. The height of ground at the station above mean level of Lake Erie is 187.1 feet.

ERIE, 1876.—This station is situated in the western end of Mill Creek Township, Erie County, Pennsylvania, about 8 miles by the roads southwesterly from the city of Erie, and 2 miles easterly from Swan railway station. The height of station used was 114 feet. The geodetic point is marked by a stone of the usual form, set 2½ feet below the surface of the ground. Three stone reference-posts are set along a fence west of the station, as follows: One bearing north 54° 54′ west, distant 56.39 metres; one bearing south 56° 59′ west, distant 31.29 metres; and one bearing south 2° 38′ west, distant 56.34 metres from the geodetic point. The road southeast of the station is distant about 244 metres. The height of ground at the station above mean level of Lake Erie is 285.9 feet.

LONG POINT, 1876.—This station is situated on the north side of Long Point, in the county of Norfolk, Province of Ontario. The height of station used was 123 feet. The geodetic point is marked by a hole drilled in the top of a stone post, set 2 feet below the sandy surface of the ground. Three stone reference-posts were set as follows: One bearing south 52° 27′ east, distant 49.4 metres; one bearing north 82° 51′ west, distant 72.3 metres; and one bearing north 33° 47′ west, distant 63.7 metres from the geodetic point. An oak tree, 1.7 feet in diameter, bears south 56° 45′ east, distant 45.7 metres; a pine tree, 1.4 feet in diameter, bears south 52° 08′ east, distant 51.7 metres; one, 1.5 feet in diameter, bears north 23° 25′ east, distant 27.4 metres; one, 1 foot in diameter, bears north 81° 25′ east, and is distant 12.5 metres from the geodetic point. The water's edge, northeast, is distant about 44 metres. The light-house at the eastern extremity of the point bears south 76° 31′ east, and is distant 5828 metres. The height of ground at the station above mean level of Lake Erie is 5.1 feet.

WESTFIELD, 1876.—This station is situated in Westfield Township, Chautauqua County, New York, on a hill, at the junction of three wagon roads, about 5 miles southwest of the village of Westfield. The height of station used was 90 feet. The geodetic point is marked by a stone post of the usual form, set 20 inches below the surface of the ground. Three stone reference-posts are set as follows: One by the road-fence on the south of the station, bearing south 1° 40′ west, distant 12.06 metres; and two along the road fence on the northwest of the station, one bearing north 81° 08′ west, distant 9.02 metres, and one bearing north 15° 29′ west, distant 21.18 metres from the geodetic point. The height of ground at the station above mean level of Lake Erie is 886.4 feet.

GRAND RIVER, 1876.—This station is situated in the west part of lot 17 of the 5th concession, Township of Dunn, County of Haldimand, Province of Ontario, 2½ miles west of Port Maitland, and about half a mile back from the lake shore. The height of station used was 100 feet. The geodetic point is marked by a stone post of the usual form buried in the ground, with a stone post set directly over it for a surface-mark. Three stone reference-posts are set as follows: One bearing south 84° 00′ west, distant 98.05 metres; one bearing south 0° 57′ west, distant 702.4 metres; and one bearing south 15° 44′ east, distant 679.4 metres from the geodetic point. Mohawk light-house bears south 79° 52′ east. The height of ground at the station above mean level of Lake Erie is 50.3 feet.

SILVER CREEK, 1876.—This station is situated in Sheridan Township, Chautauqua County, New York, about 3 miles west of the village of Silver Creek, three-fourths of a mile east of Sheridan railway station, and just south of the track of the Lake Shore and Michigan Southern Railway at a deep cut in slate rock. The height of station used was 75 feet. The geodetic point is marked by a stone post of the usual form, set 2½ feet below the surface. Three stone reference-posts are set as follows: One bearing north 7° 12′ east, distant 30.66 metres; one bearing north 34° 30′ east, distant 62.84 metres; and one bearing north 72° 23′ east, distant 89.89 metres from the geodetic point. The first two reference-stones mentioned are set along the fence south of the railroad, the third by a fence east of the station. The height of ground at the station above mean level of Lake Erie is 153.3 feet.

STURGEON POINT, 1876.—This station is situated in Evans Township, Erie County, New York, at the extreme end of Sturgeon Point, about 3 miles northwest of Angola, a village on the Lake Shore and Michigan Southern Railway, and about 3 miles west of Derby, at the end of the road leading from Derby to the lake shore. It is about 10 metres back from the edge of the bluff. The height of station used was 36 feet. The geodetic point is marked by a stone post of the usual form, set 1½ feet below the surface, with a stone post set directly over it for a surface-mark. Two stone reference-posts are set by the fences on the northeast and northwest of the station, as follows: One bearing north 45° 52′ east, distant 32.75 metres; and one bearing north 75° 03′ west, distant 5.5 metres from the geodetic point. The height of ground at the station above mean level of Lake Erie is 37.9 feet.

SUGAR LOAF, 1876.—This station is situated on Sugar Loaf Hill, about $1\frac{3}{4}$ miles southwest of Port Colborne, Province of Ontario, about 600 metres northwesterly from Sugar Loaf Point, and 160 metres north from the lake shore. The height of station used was 50 feet. The geodetic point is marked by a stone post of the usual form, set 2 feet below the surface of the ground. Three stone reference-posts are set as follows: One, marked U. S. on top, on the east side of the town-line road between the towns of Wainfleet and Humberstone, projecting 8 inches above the surface, bearing north 0° 19′ east, distant 252.31 metres; one, marked with a cross on top and L. S. on south face near top, projecting 9 inches above the surface, at the foot of the hill, near the corner of a field, bearing north 64° 04′ east, distant 82.21 metres; and one, marked with a cross on top, projecting 6 inches above the surface on top of the hill, bearing south 43° 11′ east, distant 16.9 metres from the geodetic point. The Port Colborne light-house bears north 84° 49′ east. The height of ground at the station above mean level of Lake Erie is 141.2 feet.

FONT HILL, 1875.—This station is situated in lot 5 of the 7th concession, township of Pelham, county of Welland, Province of Ontario, on a hill about 1 mile northwest of the village of Font Hill. The height of station used was $47\frac{1}{2}$ feet. The geodetic point is marked by a stone post of the usual form, set 3 feet below the surface of the ground, with a stone post set directly over it for a surface-mark. Three stone reference-posts are set as follows: One bearing north 27° 18′ east, distant 303.91 metres; one bearing north 32° 32′ west, distant 330.75 metres; and one bearing north 89° 54′ west, distant 51.75 metres from the geodetic point. The height of ground at the station above the mean level of Lake Ontario from 1860 to 1875 is 618.6 feet.

HAMBURG, 1875.—This station is situated in Hamburg Township, Erie County, New York, on the shore of Lake Erie, about 7 miles south of Buffalo, 2 miles northeast of Hamburg railway station on the Lake Shore and Michigan Southern Railway, and three-fourths of a mile northwest of Bay View railway station on the same railway. It is 295 metres west of the Bay View Hotel, a summer resort, and 15 metres back from the edge of the bluff. The height of station used was 5 feet. The geodetic point is marked by a stone post of the usual form, set 28 inches below the surface of the ground. Three stone reference posts are set as follows: One bearing south 12° 14′ east, distant 18.13 metres; one bearing south 81° 08′ east, distant 18.12 metres; and one bearing north 64° 32′ east, distant 42.5 metres from the geodetic point. The height of ground at the station above Lake Erie is about 20 feet; above mean level of Lake Ontario, 347.6 feet.

MIDDLE BASE, 1875.—This station, near the middle of the Buffalo base-line, is situated in Tonawanda Township, Erie County, New York, on the east side of the track of the Erie Railway, about 1 metre from the fence. The height of station used was 3 feet. The geodetic point is marked by a stone post 5 feet long, marked in the usual manner, set $3\frac{1}{2}$ feet below the surface of the ground. A stone reference-post, projecting 6 inches above the ground, is set on a line through the geodetic point, parallel to the railway, about 1.2 metres in a northerly direction from the point. The height of ground at the station above mean level of Lake Erie is 29.3 feet.

WEST BASE, 1875.—This station, marking the southwestern end of the Buffalo base line, is situated in Buffalo Township, Erie County, New York, about 5 miles nearly north of the Buffalo City Hall, on the north side of the Erie Railway, and the west side of Delaware street. The height of station used was 87 feet. The geodetic point is marked by a small hole drilled in a piece of brass leaded into the top of a stone post, set $3\frac{1}{2}$ feet below the surface of the ground. Two stone posts are set on a line at right angles to the base-line through the geodetic point, at a depth below the surface of the ground about the same as that of the geodetic point, as follows: one bearing south 32° 49′ east, distant 3.68 metres; and one bearing north 32° 49′ west, distant 3.71 metres from the geodetic point. Three stone reference-posts are set as follows: one at the intersection of the base-line and the town-line between Tonawanda and Buffalo on the west of Delaware street, bearing north 57° 11′ east, distant 605.02 metres; one 1.8 metres east of the east side of Delaware street, and 8.02 metres south of the Erie Railway track, bearing south 84° 46′ east, distant 775.54 metres; and one in the edge of a piece of woods, bearing south 25° 36′ east, distant 88.23 metres from the geodetic point. The height of ground at the station above mean level of Lake Erie is 32.3 feet; above mean level of Lake Ontario, 358.4 feet.

EAST BASE, 1875.—This station, marking the northeastern end of the Buffalo base-line, is situated in the southwestern part of Amherst Township, Erie County, New York, about 8 miles northeast of the Buffalo City Hall, $4\frac{1}{2}$ miles southeast of Tonawanda, and about $1\frac{3}{4}$ miles in a northerly direction from the post-office of Eggertsville. The height of station used was 75 feet. The geodetic point is marked by a cross on a piece of brass leaded into a stone post set 3 feet below the surface. Two stone posts are set on a line at right angles to the base-line through the geodetic point, at a depth below the surface of the ground about the same as that of the geodetic point, as follows: one bearing north 32° 46′ west, distant 3.66 metres; and one bearing south 32° 46′ east, distant 3.86 metres from the geodetic point. Three stone reference-posts are set as follows: one on the north side of the road south of the station, at a bend towards the east, bearing south 49° 10′ east, distant 105.0 metres; one on the north side of the same road, 65.0 metres northeast of the first, bearing south 77° 08′ east, distant 135.1 metres from the geodetic point; and one on the west side of the same road 60.96 metres south of the first, bearing south 29° 37′ east, and distant 148.8 metres from the geodetic point. The height of ground at the station above mean level of Lake Ontario is 346.2 feet.

BUFFALO PLAINS, 1875.—This station is situated in the northeast corner of Buffalo Township, Erie County, New York, on the grounds of the Erie County Poor House. The height of station used was 50 feet. The geodetic point is marked by a stone post of the usual form, set 2 feet below the surface of the ground. Three stone reference-posts are set as follows: one bearing south 78° 12′ west, distant 22.10 metres; one by the reservoir of the County House, bearing north 34° 17′ east, distant 127.56 metres; and one bearing south 82° 38′ east, distant 28.19 metres from the geodetic point. The buildings of the County House are distant about 470 metres in a northwesterly direction. The height of ground at the station above mean level of Lake Ontario is 435 feet.

BUFFALO, 1875.—The new City Hall tower was used as a station at this point. The City Hall is a large granite building on the square inclosed by Eagle, Franklin, Church, and Delaware streets in the city of Buffalo, New York. The geodetic point is marked by a cross cut in a piece of brass leaded into the granite shelf at the base of the pedestals of the statues on the top of the tower on the north side. It is 0.175 metres from the inner edge of the stone shelf, 0.744 metres from the outer edge, and 1.212 metres from the base of the pedestal of the statue on the northeast corner of the tower. Its height above the ground is 160.5 feet. The height of ground at the station above the mean level of Lake Ontario is 358.6 feet.

RIDGEWAY, 1875.—This station is situated in an open cultivated field, just southwest of the intersection of two roads, on a slight rise of round about half a mile southwest of the village of Ridgeway, in the township of Bertie, county of Welland, Province of Ontario. The height of station used was 100 feet. The geodetic point is marked by a hole drilled in the top of a stone post set 20 inches below the surface of the ground. A stone post is set directly over the geodetic point as a surface-mark. Three stone reference-posts are set along the highway fence east of the station, as follows: one bearing north 18° 14′ east, distant 24.1 metres; one bearing south 14° 14′ east, distant 33.4 metres; and one bearing south 6° 23′ east, distant 86.2 metres from the geodetic point. The fence north of the station is distant 27 metres, that east of the station 9 metres. The Methodist church, Ridgeway, bears north 47° 44′ east, and is distant about half a mile. The height of ground at the station above mean level of Lake Erie is 89.6 feet; above mean level of Lake Ontario, 416 feet.

DRUMMONDVILLE, 1875.—This station stands on a sand-bank in the village of Drummondville, Province of Ontario, on the north side of Lundy's Lane, about 20 metres east of a frame tower called the General Scott monument. The height of station used was 44 feet. The ground on which the station was built is not permanent, but the reference-stones are so set as not to be disturbed. The geodetic point is marked by a stone post of the usual form set 4 feet below the surface. A stone post is set directly over it as a surface-mark, and three reference-stones are set as follows; one in the southwest corner of the lot, distant 8.07 metres; one by the fence south of the station, distant 10.10 metres from the geodetic point, and 10.3 metres from the first reference-stone; and one by the fence west of the station, distant 6.76 metres from the geodetic point, and 12.44 metres from the first reference-stone. The height of ground at the station above Lake Erie is 136.6 feet; above Lake Ontario, 462.8 feet.

TONAWANDA, 1875.—This station is situated in Tonawanda Township, Erie County, New York, about 2 miles south of the village of Tonawanda. The height of station used was 85 feet. The geodetic point is marked by a hole drilled in the top of a stone post set 2 feet below the surface of the ground. Three stone reference-posts are set as follows: One bearing north 69° 07′ east, distant 25.2 metres; one bearing north 48° 43′ west, distant 10.65 metres; and one bearing south 74° 23′ west, distant 18.7 metres from the geodetic point. An azimuth post, used in 1875, bears north 55° 18′ east, and is distant 9.24 metres. The height of ground at the station above Lake Erie is 52 feet; above mean level of Lake Ontario, 378.1 feet.

PEKIN, 1875.—This station is situated on the western border of Cambria Township, Niagara County, New York, in lot 28, about half a mile east of the village of Pekin, on the north side of the street, just east of a cemetery. The height of station used was 101 feet. The geodetic point is marked by a piece of iron leaded into a stone post of the usual form set 2.2 feet below the surface, with a stone post set directly over it for a surface-mark. Three stone reference-posts are set as follows: one on the north side of the street and on the west boundary of the cemetery, bearing south 59° 22′ west, distant 286.36 metres; one on the same side of the street, and on a farm boundary, bearing south 2° 54′ west, distant 148.25 metres from the geodetic point, and 8.47 metres distant from a stone in the middle of the street marking the corner of the farms; and one on the same farm boundary, bearing north 12° 28′ west, distant 22.58 metres from the geodetic point. The height of ground at the station above mean level of Lake Ontario is 410.6 feet.

FALKIRK, 1875.— This station is situated in Newstead Township, Erie County, New York, about one-fourth of a mile northeast of the village of Falkirk, 1 mile east of the village of Akron, and 80 metres south of the edge of a limestone bluff 60 feet or more in height. The height of station used was 75 feet. The geodetic point is marked by a nail leaded into the solid rock about 2 feet below the surface of the ground. Three stone reference-posts are set as follows: One bearing south 22° 12′ west, distant 25.9 metres; one bearing south 36° 52′ east, distant 29.2 metres; and one bearing south 59° 04′ east, distant 44.75 metres from the geodetic point. The height of ground at the station above mean level of Lake Ontario is 593.2 feet.

B.—STATIONS, SIGNALS, INSTRUMENTS, AND METHODS OF OBSERVATION.

§ **3.** See Chapter XVI, B.

C.—MEASURED AND ADJUSTED ANGLES BETWEEN THE LINES WILLOUGHBY-CHESTER AND FALKIRK-PEKIN.

§ **4.** The following tables give an abstract of the adjustment of the triangulation comprised within the above-stated limits. A sketch to accompany it is given in Plate V. The adjustment is made in two sections, namely:

Section XI, extending from Willoughby-Chester to Grand River-Westfield.

Section XII, extending from Grand River-Westfield to Falkirk-Pekin.

Weight unity was assigned to the following number of combined results for the respective instruments used in measuring the angles:

Troughton & Simms No. 1, 16.
Troughton & Simms No. 2, 24.
Troughton & Simms No. 3, 24.
Troughton & Simms No. 4, 24.
Pistor & Martins No. 2, 24, at Drummondville and Buffalo.
Pistor & Martins No. 2, 20, at Sugar Loaf.
Gambey No. 1, 16.
Repsold No. 1, 20.

For a detailed explanation of the tables, see Chapter XIV, C, § 7, and see the remark in Chapter XV, C, § 6, relating to the column headed "No. meas."

At each of the above-named stations marking the lines of division of the adjustment, a sum-angle condition was disregarded in deriving the general corrections to the angles. The locally-adjusted angles, with their resulting weights, at these stations, were used in computing the general corrections.

SECTION XI.—*Triangulation from the line Grand River – Westfield to the line Willoughby – Chester.*

GRAND RIVER—44.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Date, August, 1876.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angle.
	° ′ ″			″		″	″	° ′ ″
Westfield and Long Point............	52 26 09.982	44_4	14	7.2	1	—0.201	—0.011	52 26 09.770

NOTE.—The weight and local correction of 44_4 are taken from Section XII of the adjustment.

WESTFIELD—45.

[Observers, G. Y. Wisner and R. S. Woodward. Instruments, Troughton & Simms theodolites Nos. 1 and 3. Dates, June and October, 1876.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Erie and Long Point..................	64 51 01.072	45_1	14	5.7	1	—0.170	+0.011	64 51 00.913
Long Point and Grand River	52 02 06.509	45_2	16	4.8	1	—0.171	—0.009	52 02 06.329

NOTES.—The weights and local corrections of 45_1 and 45_2 are taken from Section XII of the adjustment.
45_1 and 45_2 were partly measured by Mr. Wisner with the Troughton & Simms No. 1, in June, 1876, and partly by Mr. Woodward with the Troughton & Simms No. 3, in October, 1876.

LONG POINT—46.

[Observer, J. H. Darling. Instrument, Troughton & Simms 14-inch theodolite No. 4. Date, August, 1876.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Grand River and Westfield	75 31 50.868	46_1	16	7.6	0.67	—0.117	—0.204	75 31 50.547
Westfield and Erie....................	57 28 15.676	46_2	28	7.4	1	—0.078	—0.108	57 28 15.490
Erie and Houghton	88 43 44.666	46_3	24	7.9	1	—0.079	+0.435	88 43 45.022
Houghton and Grand River.........	138 16 09.142	46_4	26	7.3	1	—0.078	—0.123	138 16 08.941

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$1.67(46_1)+(46_2)+(46_3)+0.352=0$$
$$(46_1)+2(46_2)+(46_3)+0.352=0$$
$$(46_1)+(46_2)+2(46_3)+0.352=0$$

ERIE—47.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Dates, September and October, 1876.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Edinboro and Conneant.............	64 35 04.513	47_1	16	2.2	1	—0.190	—0.110	64 35 04.213
Conneaut and Houghton............	84 19 49.098	47_2	17	2.1	1	—0.190	+0.260	84 19 49.168
Houghton and Long Point..........	37 51 30.347	47_3	18	3.7	1	—0.190	+0.316	37 51 30.473
Long Point and Westfield...........	57 40 50.245	47_4	18	4.5	1	—0.190	—0.226	57 40 49.829
Westfield and Edinboro.............	115 32 46.747	47_5	17	2.9	1	—0.190	—0.240	115 32 46.317

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(47_1)+(47_2)+(47_3)+(47_4)+0.950=0$$
$$(47_1)+2(47_2)+(47_3)+(47_4)+0.950=0$$
$$(47_1)+(47_2)+2(47_3)+(47_4)+0.950=0$$
$$(47_1)+(47_2)+(47_3)+2(74_4)+0.950=0$$

SECTION XI.—*Triangulation from the line Grand River - Westfield to the line Willoughby - Chester*—Continued.

HOUGHTON—48.

[Observers, A. R. Flint and G. Y. Wisner. Instruments, Repsold theodolite No. 1, and Troughton & Simms theodolite No. 1. Dates, August and September, 1876, and May, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Long Point and Erie	53 24 50. 557	48_1	26	5. 2	1. 25	−0. 025	+0. 023	53 24 50. 555
Long Point and Edinboro	60 24 52. 304	48_{1+2}	16	4. 7	1	+0. 474	+0. 250	60 24 53. 028
Edinboro and Long Point	299 35 07. 699	48_{-1-2}	16	6. 3	1	−0. 477	−0. 250	299 35 06. 972
Erie and Edinboro	7 00 02. 617	48_2	18	4. 5	1	−0. 371	+0. 277	7 00 02. 473
Edinboro and Erie	352 59 57. 175	48_{-2}	18	4. 2	1	+0. 579	−0. 227	352 59 57. 527
Erie and Conneaut	31 08 13. 648	48_{2+3}	16	4. 0	1	+0. 045	+0. 300	31 08 13. 993
Conneaut and Erie	328 51 46. 263	48_{-2-3}	15	3. 4	1	+0. 044	−0. 300	328 51 46. 007
Erie and Long Point	306 35 08. 732	48_{2+3+4}	26	7. 5	1. 25	+0. 736	−0. 023	306 35 09. 445

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$4.5(48_1)+2(48_2) \quad +0.854=0$$
$$2(48_1)+4(48_2) \quad +1.535=0$$
$$2(48_{2+3})-0.089=0$$

NOTE.—48_{2+3} and 48_{-2-3} were measured by G. Y. Wisner with the Troughton & Simms instrument in 1877. The remainder were read by Mr. Flint with the Repsold instrument in 1876.

EDINBORO—49.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Dates, September and October, 1876.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Andover and Conneaut	42 42 44. 255	49_1	28	7. 3	1	−0. 136	−0. 110	42 42 44. 009
Conneaut and Houghton	60 04 52. 940	49_2	22	7. 1	1	+0. 128	+0. 417	60 04 53. 485
Conneaut and Erie	84 09 59. 681	49_{2+3}	28	5. 5	1	−0. 263	+0. 104	84 09 59. 522
Houghton and Erie	24 05 06. 222	49_3	25	3. 7	1	+0. 128	−0. 313	24 05 06. 037
Erie and Andover	233 07 16. 599	49_4	28	8. 0	1	−0. 136	+0. 006	233 07 16. 469

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(49_1)+(49_2)+(49_3)+0.016=0$$
$$(49_1)+3(49_2)+2(49_3)-0.503=0$$
$$(49_1)+2(49_2)+3(49_3)-0.503=0$$

CONNEAUT—50.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Dates, August and September, 1876, and May, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Houghton and Erie	64 32 03. 469	50_1	16	3. 8	1	−0. 107	+0. 473	64 32 03. 835
Erie and Houghton	295 27 56. 744	50_{-1}	16	2. 2	1	−0. 106	−0. 473	295 27 56. 165
Thompson and Erie	185 49 12. 366	50_{1+5}	16	6. 2	1	+0. 070	−0. 187	185 49 12. 249
Erie and Edinboro	31 14 58. 049	50_2	16	2. 3	1	+0. 069	+0. 043	31 14 58. 161
Edinboro and Andover	85 07 58. 672	50_3	17	7. 0	1	+0. 070	−0. 303	85 07 58. 439
Andover and Thompson	57 47 50. 635	50_4	17	5. 4	1	+0. 069	+0. 447	57 47 51. 151

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(50_1) \quad +0.213=0$$
$$2(50_2)+(50_3)+(50_4)-0.278=0$$
$$(50_2)+2(50_3)+(50_4)-0.278=0$$
$$(50_2)+(50_3)+2(50_4)-0.278=0$$

NOTE.—50_1 and 50_{-1} were measured in 1877, the remainder in 1876.

SECTION XI.—*Triangulation from the line Grand River - Westfield to the line Willoughby - Chester*—Continued.

ANDOVER—51.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1. Dates, October and November, 1876.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Mesopotamia and Thompson........	41 49 59.265	51_{1+2}	24	6.1	1	+0.144	−0.034	41 49 59.375
Thompson and Mesopotamia........	318 10 00.448	51_{-1-2}	24	7.3	1	+0.143	+0.034	318 10 00.625
Edinboro and Claridon	203 15 31.780	51_{1+5}	21	7.1	1	+0.404	−0.151	203 15 32.033
Claridon and Thompson	27 33 58.518	51_2	24	4.3	1	+0.403	−0.065	27 33 58.856
Thompson and Conneaut	77 01 07.833	51_3	24	6.5	1	+0.403	+0.483	77 01 08.719
Conneaut and Edinboro	52 09 20.256	51_4	26	4.7	1	+0.403	−0.267	52 09 20.392

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(51_{1+2}) \qquad -0.287=0$$
$$2(51_2)+(51_3)+(51_4)-1.613=0$$
$$(51_2)+2(51_3)+(51_4)-1.613=0$$
$$(51_2)+(51_3)+2(51_4)-1.613=0$$

THOMPSON—52.

[Observer, J. H. Darling. Instrument, Troughton & Simms theodolite No. 4. Date, November, 1876.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Conneaut and Andover	45 11 02.049	52_1	27	6.3	1	+0.141	+0.787	45 11 02.977
Andover and Mesopotamia	70 00 41.285	52_2	28	6.6	1	+0.141	−0.104	70 00 41.322
Mesopotamia and Claridon...........	25 50 49.951	52_3	28	8.1	1	+0.141	−0.228	25 50 49.864
Claridon and Little Mountain.......	57 19 50.582	52_4	28	6.2	1	+0.141	−0.607	57 19 50.116
Little Mountain and Conneaut......	167 37 35.427	52_5	26	6.8	1	+0.142	+0.152	167 37 35.721

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(52_1)+(52_2)+(52_3)+(52_4)-0.706=0$$
$$(52_1)+2(52_2)+(52_3)+(52_4)-0.706=0$$
$$(52_1)+(52_2)+2(52_3)+(52_4)-0.706=0$$
$$(52_1)+(52_2)+(52_3)+2(52_4)-0.706=0$$

MESOPOTAMIA—53.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, May, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Claridon and Thompson	47 45 15.101	53_1	16	3.2	1	+0.060	−0.231	47 45 14.930
Thompson and Andover	68 09 21.256	53_2	16	3.0	1	+0.060	+0.341	68 09 21.657
Andover and Claridon	244 05 23.462	53_3	16	2.7	1	+0.061	−0.110	244 05 23.413

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(53_1)+(53_2)-0.181=0$$
$$(53_1)+2(53_2)-0.181=0$$

SECTION XI.—*Triangulation from the line Grand River - Westfield to the line Willoughby - Chester*—Continued.

CLARIDON—54.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, June, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Chester and Little Mountain	40 25 02.053	54_1	17	4.8	1	+0.155	+0.388	40 25 02.596
Little Mountain and Thompson	59 41 45.047	54_2	17	3.5	1	+0.155	−0.512	59 41 44.690
Thompson and Andover	56 34 32.185	54_3	16	4.3	1	+0.155	−0.460	56 34 31.880
Andover and Mesopotamia	49 49 23.439	54_4	16	6.4	1	+0.155	+0.336	49 49 23.930
Mesopotamia and Chester	153 29 16.500	54_5	17	6.5	1	+0.156	+0.248	153 29 16.904

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(54_1)+(54_2)+(54_3)+(54_4)-0.776=0$$
$$(54_1)+2(54_2)+(54_3)+(54_4)-0.776=0$$
$$(54_1)+(54_2)+2(54_3)+(54_4)-0.776=0$$
$$(54_1)+(54_2)+(54_3)+2(54_4)-0.776=0$$

LITTLE MOUNTAIN—55.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, June, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Thompson and Claridon	62 58 26.422	55_1	16	2.2	1	+0.059	−0.414	62 58 26.067
Claridon and Chester	71 41 06.338	55_2	16	4.2	1	+0.059	+0.487	71 41 06.884
Chester and Willoughby	75 50 29.709	55_3	16	4.6	1	+0.059	−0.418	75 50 29.350
Willoughby and Thompson	149 29 57.294	55_4	16	2.9	1	+0.060	+0.345	149 29 57.699

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(55_1)+(55_2)+(55_3)-0.237=0$$
$$(55_1)+2(55_2)+(55_3)-0.237=0$$
$$(55_1)+(55_2)+2(55_3)-0.237=0$$

CHESTER—56.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, June, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Warrensville and Willoughby	92 12 11.653	56_1	18	3.5	1	+0.052	....	....
Willoughby and Little Mountain	54 01 00.333	56_2	18	5.8	1	+0.052	−0.573	54 00 59.812
Little Mountain and Claridon	67 53 50.998	56_3	18	4.4	1	+0.052	+0.106	67 53 51.156
Claridon and Warrensville	145 52 56.808	56_4	18	3.2	1	+0.052	....	....

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(56_1)+(56_2)+(56_3)-0.208=0$$
$$(56_1)+2(56_2)+(56_3)-0.208=0$$
$$(56_1)+(56_2)+2(56_3)-0.208=0$$

SECTION XI.—*Triangulation from the line Grand River – Westfield to the line Willoughby – Chester*—Continued.

WILLOUGHBY—57.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Date, June, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angle.
	° ′ ″			″		″	″	° ′ ″
Little Mountain and Chester	50 08 31.940	57_1	20	5.6	1.25	−0.156	−0.467	50 08 31.317
Chester and Warrensville	38 10 03.094	57_2	20	2.7	1.25	−0.155	....	....
Warrensville and Rockport..........	37 51 09.507	57_3	21	4.3	1.25	−0.156	....	....
Rockport and Little Mountain.......	233 50 16.120	57_4	16	3.7	1	−0.194	....	....

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.25(57_1)+\quad(57_2)+\quad(57_3)+0.661=0$$
$$(57_1)+2.25(57_2)+\quad(57_4)+0.661=0$$
$$(57_1)+\quad(57_2)+2.25(57_3)+0.661=0$$

Numerical equations of condition in the triangulation from the line Grand River – Westfield to the line Willoughby – Chester.

SIDE-EQUATIONS.

VII. (30) $+34.9240\,[47_1]+37.0144\,[47_2]-12.1164\,[49_2]+47.1023\,[49_3]-12.1601\,[50_1]-\;2.1327\,[50_2]+19.819=0$

XIII. (15) $-\;9.8192\,[52_2]-\;2.1605\,[52_3]+10.2285\,[53_1]+18.6688\,[53_2]-13.8962\,[54_3]+17.7784\,[54_1]-17.878=0$

NOTE.—In the solution for determining the general corrections, each of the side-equations was divided by the number inclosed in parenthesis and placed opposite it.

ANGLE-EQUATIONS.

I.	$[44_4]$	$+[45_2]$	$+[46_1]$			$+0.224=0$
II.	$[45_1]$	$+[46_2]$	$+[47_4]$			$+0.322=0$
III.	$[46_3]$	$+[47_3]$	$+[48_1]$			$-0.774=0$
IV.	$[47_2]$	$+[48_{2+3}]$	$+[50_1]$			$-1.033=0$
V.	$-[48_2]$	$+[48_{2+3}]$	$+[49_2]$	$+[50_1]$	$+[50_2]$	$-1.005=0$
VI.	$[47_1]$	$+[49_2]$	$+[49_3]$	$+[50_2]$		$-0.037=0$
VIII.	$[49_1]$	$+[50_3]$	$+[51_4]$			$+0.679=0$
IX.	$[50_4]$	$+[51_3]$	$+[52_1]$			$-1.717=0$
X.	$[51_{1+2}]$	$+[52_2]$	$+[53_2]$			$-0.202=0$
XI.	$[52_3]$	$+[53_1]$	$+[54_3]$	$+[54_4]$		$+0.582=0$
XII.	$[51_2]$	$+[52_2]$	$+[52_3]$	$+[54_3]$		$+0.857=0$
XIV.	$[52_4]$	$+[54_2]$	$+[55_1]$			$+1.532=0$
XV.	$[54_1]$	$+[55_2]$	$+[56_3]$			$-0.981=0$
XVI.	$[55_3]$	$+[56_2]$	$+[57_1]$			$+1.458=0$

General corrections in terms of the correlates.

$[44_4]$	$=+0.83333$ I				
$[45_1]$	$=+0.73333$ II				
$[45_2]$	$=+0.73333$ I				
$[46_1]$	$=+1.00000$ I	-0.33333 II	-0.33333 III		
$[46_2]$	$=-0.33333$ I	$+0.77778$ II	-0.22222 III		
$[46_3]$	$=-0.33333$ I	-0.22222 II	$+0.77778$ III		
$[47_1]$	$=-0.20000$ II	-0.20000 III	-0.20000 IV	$+0.80000$ VI	$+0.68454$ VII
$[47_2]$	$=-0.20000$ II	-0.20000 III	$+0.80000$ IV	-0.20000 VI	$+0.75422$ VII
$[47_3]$	$=-0.20000$ II	$+0.80000$ III	-0.20000 IV	-0.20000 VI	-0.47959 VII
$[47_4]$	$=+0.80000$ II	-0.20000 III	-0.20000 IV	-0.20000 VI	-0.47959 VII
$[48_1]$	$=+0.28571$ III	$+0.14286$ V			
$[48_2]$	$=-0.14286$ III	-0.32143 V			
$[48_{2+3}]$	$=+0.50000$ IV	$+0.50000$ V			
$[49_1]$	$=-0.12500$ V	-0.25000 VI	-0.14578 VII	$+0.62500$ VIII	
$[49_2]$	$=+0.62500$ V	$+0.25000$ VI	-0.84121 VII	-0.12500 VIII	

General corrections in terms of the correlates—Continued.

$[49_3]$	=−0.37500 V	+0.25000 VI	+1.13276 VII	−0.12500 VIII		
$[50_1]$	=+0.50000 IV	+0.50000 V	−0.20267 VII			
$[50_2]$	=+0.75000 V	+0.75000 VI	−0.05332 VII	−0.25000 VIII	−0.25000 IX	
$[50_3]$	=−0.25000 V	−0.25000 VI	+0.01777 VII	+0.75000 VIII	−0.25000 IX	
$[50_4]$	=−0.25000 V	−0.25000 VI	+0.01777 VII	−0.25000 VIII	+0.75000 IX	
$[51_{1+2}]$	=+0.50000 X					
$[51_2]$	=−0.25000 VIII	−0.25000 IX	+0.75000 XII			
$[51_3]$	=−0.25000 VIII	+0.75000 IX	−0.25000 XII			
$[51_4]$	=+0.75000 VIII	−0.25000 IX	−0.25000 XII			
$[52_1]$	=+0.80000 IX	−0.20000 X	−0.20000 XI	−0.40000 XII	+0.15973 XIII	−0.20000 XIV
$[52_2]$	=−0.20000 IX	+0.80000 X	−0.20000 XI	+0.60000 XII	−0.49488 XIII	−0.20000 XIV
$[52_3]$	=−0.20000 IX	−0.20000 X	+0.80000 XI	+0.60000 XII	+0.01570 XIII	−0.20000 XIV
$[52_4]$	=−0.20000 IX	−0.20000 X	−0.20000 XI	−0.40000 XII	+0.15973 XIII	+0.80000 XIV
$[53_1]$	=−0.33333 X	+0.66667 XI	+0.03974 XIII			
$[53_2]$	=+0.66667 X	−0.33333 XI	+0.60243 XIII			
$[54_1]$	=−0.40000 XI	−0.20000 XII	−0.05176 XIII	−0.20000 XIV	+0.80000 XV	
$[54_2]$	=−0.40000 XI	−0.20000 XII	−0.05176 XIII	+0.80000 XIV	−0.20000 XV	
$[54_3]$	=+0.60000 XI	+0.80000 XII	−0.97817 XIII	−0.20000 XIV	−0.20000 XV	
$[54_4]$	=+0.60000 XI	−0.20000 XII	+1.13347 XIII	−0.20000 XIV	−0.20000 XV	
$[55_1]$	=+0.75000 XIV	−0.25000 XV	−0.25000 XVI			
$[55_2]$	=−0.25000 XIV	+0.75000 XV	−0.25000 XVI			
$[55_3]$	=−0.25000 XIV	−0.25000 XV	+0.75000 XVI			
$[56_2]$	=+0.75000 XVI					
$[56_3]$	=+0.75000 XV					
$[57_1]$	=+0.61176 XVI					

Normal equations for determining the correlates.

No. of equation.							
1.	0=+0.22400	+2.56667 I	−0.33333 II	−0.33333 III			
2.	0=+0.32200	−0.33333 I	+2.31111 II	−0.42222 III	−0.20000 IV	−0.20000 VI	−0.47959 VII
3.	0=−0.77400	−0.33333 I	−0.42222 II	+1.86349 III	−0.20000 IV	+0.14286 V	−0.20000 VI
		−0.47959 VII					
4.	0=−1.03300	−0.20000 II	−0.20000 III	+1.80000 IV	+1.00000 V	−0.20000 VI	+0.55155 VII
5.	0=−1.00500	+0.14286 III	+1.00000 IV	+2.69643 V	+1.00000 VI	−1.09720 VII	−0.37500 VIII
		−0.25000 IX					
6.	0=−0.03700	−0.20000 II	−0.20000 III	−0.20000 IV	+1.00000 V	+2.05000 VI	+0.92277 VII
		−0.50000 VIII	−0.25000 IX				
7.	0=+0.66063	−0.47959 II	−0.47959 III	+0.55155 IV	−1.09720 V	+0.92277 VI	+3.93167 VII
		−0.12800 VIII	+0.01777 IX				
8.	0=+0.67900	−0.37500 V	−0.50000 VI	−0.12800 VII	+2.12500 VIII	−0.50000 IX	−0.25000 XII
9.	0=−1.71700	−0.25000 V	−0.25000 VI	+0.01777 VII	−0.50000 VIII	+2.30000 IX	−0.20000 X
		−0.20000 XI	−0.65000 XII	+0.15973 XIII	−0.20000 XIV		
10.	0=−0.20200	−0.20000 IX	+1.96667 X	−0.53333 XI	+0.60000 XII	+0.10755 XIII	−0.20000 XIV
11.	0=+0.58200	−0.20000 IX	−0.53333 X	+2.66667 XI	+1.20000 XII	+0.21074 XIII	−0.60000 XIV
		−0.40000 XV					
12.	0=+0.85700	−0.25000 VIII	−0.65000 IX	+0.60000 X	+1.20000 XI	+2.75000 XII	−1.45735 XIII
		−0.60000 XIV	−0.20000 XV				
13.	0=−1.19187	+0.15973 IX	+0.10755 X	+0.21074 XI	−1.45735 XII	+3.34818 XIII	+0.10796 XIV
		−0.05176 XV					
14.	0=+1.53200	−0.20000 IX	−0.20000 X	−0.60000 XI	−0.60000 XII	+0.10796 XIII	+2.35000 XIV
		−0.45000 XV	−0.25000 XVI				
15.	0=−0.98100	−0.40000 XI	−0.20000 XII	−0.05176 XIII	−0.45000 XIV	+2.30000 XV	−0.25000 XVI
16.	0=+1.45800	−0.25000 XIV	−0.25000 XV	+2.11176 XVI			

Values of the correlates and their logarithms.

I =−0.0128	log 8.1082267−	IX =+0.6345	log 9.8024590+
II =+0.0155	log 8.1906118+	X =−0.0689	log 8.8384083−
III =+0.5577	log 9.7464006+	XI =−0.4056	log 9.6080765−
IV =+1.5544	log 0.1915600+	XII =+0.0858	log 8.9337403+
V =−0.9548	log 9.9798942−	XIII =+0.4175	log 9.6206253+
VI =+1.1246	log 0.0510135+	XIV =−0.7587	log 9.8800873−
VII =−0.8532	log 9.9310458−	XV =+0.1414	log 9.1504801+
VIII =−0.1154	log 9.0621305−	XVI =−0.7635	log 9.8828106−

Values of the general corrections.

	″		″		″		″
$[44_4]$	=−0.011	$[48_1]$	=+0.023	$[51_{1+2}]$	=−0.034	$[54_1]$	=+0.388
$[45_1]$	=+0.011	$[48_2]$	=+0.227	$[51_2]$	=−0.065	$[54_2]$	=−0.512
$[45_2]$	=−0.009	$[48_{2+3}]$	=+0.300	$[51_3]$	=+0.483	$[54_3]$	=−0.460
$[46_1]$	=−0.204	$[49_1]$	=−0.110	$[51_4]$	=−0.267	$[54_4]$	=+0.336
$[46_2]$	=−0.108	$[49_2]$	=+0.417	$[52_1]$	=+0.787	$[55_1]$	=−0.414
$[46_3]$	=+0.435	$[49_3]$	=−0.313	$[52_2]$	=−0.104	$[55_2]$	=+0.487
$[47_1]$	=−0.110	$[50_1]$	=+0.473	$[52_3]$	=−0.228	$[55_3]$	=−0.418
$[47_2]$	=+0.260	$[50_2]$	=+0.043	$[52_4]$	=−0.607	$[56_2]$	=−0.573
$[47_3]$	=+0.316	$[50_3]$	=−0.303	$[53_1]$	=−0.231	$[56_3]$	=+0.106
$[47_4]$	=−0.226	$[50_4]$	=+0.447	$[53_2]$	=+0.341	$[57_1]$	=−0.467

Residuals resulting from substitution of general corrections in numerical equations of condition.

No. of equation.	Residual.	No. of equation.	Residual.
1	0.00000	9	+0.00001
2	0.00000	10	0.00000
3	0.00000	11	0.00000
4	0.00000	12	0.00000
5	0.00000	13	+0.00003
6	0.00000	14	0.00000
7	0.00000	15	-0.00000
8	0.00000	16	0.00000

SECTION XII.—*Triangulation from the line Falkirk-Pekin to the line Grand River-Westfield.*

FALKIRK—29.

[Observer, T. Russell. Instrument, Gambey theodolite No. 1. Dates, July and August, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Tonawanda and Pekin	31 23 10.691	29_1	17	3.1	1	+0.082	+0.457	31 23 11.230

NOTE.—The weight and local correction of 29_1 are taken from Section XIII of the adjustment.

SECTION XIII.—*Triangulation from the line Falkirk-Pekin to the line Grand River-Westfield*—Continued.

PEKIN—30.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms 14-inch theodolite No. 1. Dates, July and August, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Falkirk and Tonawanda	66 25 49.294	30_2	18	5.9	1	+0.065	+0.471	66 25 49.830
Tonawanda and Drummondville	59 58 19.872	30_3	18	5.6	1	+0.065	+0.121	59 58 20.058

NOTE.—The weights and local corrections of 30_2 and 30_3 are taken from Section XIII of the adjustment.

TONAWANDA—31.

[Observers, T. Russell and G. Y. Wisner. Instruments, Gambey theodolite No. 1, and Troughton & Simms 14-inch theodolite No. 1. Dates, August, September, and October, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Buffalo and Ridgeway	49 07 09.914	31_1	16	4.3	1	+0.202	+0.963	49 07 11.079
Ridgeway and Drummondville	75 12 24.354	31_2	16	2.6	1	+0.019	−0.833	75 12 23.540
Ridgeway and Falkirk	220 32 32.861	31_{2+3+4}	16	5.4	1	−0.507	+0.019	220 32 32.373
Falkirk and Ridgeway	139 27 28.337	31_{-2-3-4}	16	3.0	1	−0.691	−0.019	139 27 27.627
Drummondville and Pekin	63 09 08.104	31_3	16	2.9	1	+0.018	+0.193	63 09 08.315
Pekin and Falkirk	82 10 59.840	31_4	17	2.5	1	+0.019	+0.659	82 11 00.518
Falkirk and East Base	16 06 31.787	31_5	16	5.2	1	+0.202	+0.067	16 06 32.056
East Base and Buffalo Plains	32 36 50.065	31_6	28	4.8	2	+0.101	+0.348	32 36 50.514
Buffalo Plains and West Base	36 04 44.909	31_7	16	4.6	1	+0.202	−1.027	36 04 44.084
West Base and Buffalo	5 32 10.063	31_8	16	4.8	1	+0.201	−0.370	5 32 09.894

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(31_1)+(31_2)+(31_3)+(31_4)+(31_5)+(31_6)+(31_7)-0.964=0$$
$$(31_1)+4(31_2)+3(31_3)+3(31_4)+(31_5)+(31_6)+(31_7)-0.892=0$$
$$(31_1)+3(31_2)+4(31_3)+3(31_4)+(31_5)+(31_6)+(31_7)-0.892=0$$
$$(31_1)+3(31_2)+3(31_3)+4(31_4)+(31_5)+(31_6)+(31_7)-0.892=0$$
$$(31_1)+(31_2)+(31_3)+(31_4)+2(31_5)+(31_6)+(31_7)-0.964=0$$
$$(31_1)+(31_2)+(31_3)+(31_4)+(31_5)+3(31_6)+(31_7)-0.964=0$$
$$(31_1)+(31_2)+(31_3)+(31_4)+(31_5)+(31_6)+2(31_7)-0.964=0$$

NOTE.—31_1, 31_{2+3+4}, 31_5, part of 31_6, 31_7, and 31_8, were read by Mr. Wisner with the Troughton & Simms instrument, in September, 1875 The remainder were read by Mr. Russell with the Gambey instrument, in August and October, 1875.

DRUMMONDVILLE—32.

[Observer, W. A. Metcalf. Instrument, Pistor & Martins 14-inch theodolite No. 2. Dates, August and September, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Pekin and Tonawanda	56 52 32.523	32_1	24	3.9	1	+0.041	−0.103	56 52 32.461
Tonawanda and Buffalo	21 48 17.006	32_2	16	6.0	1	−0.059	+0.541	21 48 17.488
Tonawanda and Ridgeway	52 38 14.004	32_{2+3}	10	3.3	0.5	+0.200	+0.597	52 38 14.801
Buffalo and Ridgeway	30 49 57.316	32_3	22	5.2	1	−0.059	+0.056	30 49 57.313
Ridgeway and Sugar Loaf	39 16 00.424	32_4	24	6.4	1	+0.042	−0.531	39 15 59.935
Sugar Loaf and Font Hill	43 43 09.498	32_5	24	4.4	1	+0.041	+0.302	43 43 09.841
Font Hill and Pekin	167 30 03.185	32_6	17	3.0	1	+0.042	−0.265	167 30 02.962

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(32_1)+(32_2)+(32_3)+(32_4)+(32_5)-0.048=0$$
$$(32_1)+2.5(32_2)+1.5(32_3)+(32_4)+(32_5)+0.111=0$$
$$(32_1)+1.5(32_2)+2.5(32_3)+(32_4)+(32_5)+0.111=0$$
$$(32_1)+(32_2)+(32_3)+2(32_4)+(32_5)-0.048=0$$
$$(32_1)+(32_2)+(32_3)+(32_4)+2(32_5)-0.048=0$$

SECTION XIII.—*Triangulation from the line Falkirk-Pekin to the line Grand River-Westfield*—Continued.

RIDGEWAY—33.

[Observer, J. H. Darling. Instrument, Repsold theodolite No. 1. Dates, August, September, and October, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Drummondville and Tonawanda	52 09 22.166	33_1	22	4.3	1	−0.089	+0.523	52 09 22.600
Tonawanda and West Base	14 00 57.972	33_2	24	6.1	1	−0.089	+0.261	14 00 58.144
West Base and Buffalo	27 27 02.939	33_3	24	5.7	1	−0.088	−1.096	27 27 01.755
Buffalo and Hamburg	36 17 07.977	33_4	40	7.6	1.5	−0.059	+0.058	36 17 07.976
Hamburg and Sturgeon Point	54 38 21.840	33_5	24	7.4	1	−0.089	+0.422	54 38 22.173
Sturgeon Point and Silver Creek	21 31 10.234	33_6	26	4.9	1	−0.088	+0.205	21 31 10.351
Silver Creek and Sugar Loaf	70 07 44.262	33_7	26	4.7	1	−0.089	−0.169	70 07 44.004
Sugar Loaf and Font Hill	44 39 44.577	33_8	24	5.7	1	−0.089	−0.845	44 39 43.643
Font Hill and Drummondville	39 08 28.801	33_9	24	5.6	1	−0.088	+0.641	39 08 29.354

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(33_1)+(33_2)+(33_3)+(33_4)+(33_5)+(33_6)+(33_7)+(33_8)+0.768=0$$
$$(33_1)+2(33_2)+(33_3)+(33_4)+(33_5)+(33_6)+(33_7)+(33_8)+0.768=0$$
$$(33_1)+(33_2)+2(33_3)+(33_4)+(33_5)+(33_6)+(33_7)+(33_8)+0.768=0$$
$$(33_1)+(33_2)+(33_3)+2.5(33_4)+(33_5)+(33_6)+(33_7)+(33_8)+0.768=0$$
$$(33_1)+(33_2)+(33_3)+(33_4)+2(33_5)+(33_6)+(33_7)+(33_8)+0.768=0$$
$$(33_1)+(33_2)+(33_3)+(33_4)+(33_5)+2(33_6)+(33_7)+(33_8)+0.768=0$$
$$(33_1)+(33_2)+(33_3)+(33_4)+(33_5)+(33_6)+2(33_7)+(33_8)+0.768=0$$
$$(33_1)+(33_2)+(33_3)+(33_4)+(33_5)+(33_6)+(33_7)+2(33_8)+0.768=0$$

BUFFALO—34.

[Observers, W. A. Metcalf and J. H. Darling. Instruments, Pistor & Martius 14-inch theodolite No. 2, and Repsold theodolite No. 1. Dates, September, October, and November, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Hamburg and Sturgeon Point	40 46 37.873	34_1	24	5.8	1	+0.067	−0.871	40 46 37.069
Hamburg and Ridgeway	94 27 25.537	34_{1+2}	24	5.2	1	−0.178	+0.229	94 27 25.588
Ridgeway and Hamburg	265 32 34.751	34_{-1-2}	24	7.4	1	−0.110	−0.229	265 32 34.412
Sturgeon Point and Ridgeway	53 40 47.352	34_2	24	6.8	1	+0.067	+1.100	53 40 48.519
Ridgeway and Drummondville	55 32 41.027	34_3	24	3.8	1	+0.188	−0.128	55 32 41.087
Drummondville and Ridgeway	304 27 18.598	34_{-3}	24	3.6	1	+0.187	+0.128	304 27 18.913
Drummondville and Tonawanda	33 52 09.146	34_4	24	3.1	1	+0.062	−0.782	33 52 08.426
Buffalo Plains and Drummondville	287 51 43.622	$34_{-4-5-6-7}$	24	6.4	1	+0.062	+1.160	287 51 44.844
Tonawanda and West Base	3 33 27.755	34_5	25	5.2	1	+0.062	+0.136	3 33 27.953
West Base and East Base	26 19 44.087	34_6	24	4.5	1	+0.062	+0.511	26 19 44.660
East Base and Buffalo Plains	8 22 55.080	34_7	25	8.4	1	+0.062	−1.025	8 22 54.117

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3(34_1)+2(34_2)\qquad -0.336=0$$
$$2(34_1)+3(34_2)\qquad -0.336=0$$
$$2(34_3)\qquad -0.375=0$$
$$2(34_4)+(34_5)+(34_6)+(34_7)-0.310=0$$
$$(34_4)+2(34_5)+(34_6)+(34_7)-0.310=0$$
$$(34_4)+(34_5)+2(34_6)+(34_7)-0.310=0$$
$$(34_4)+(34_5)+(34_6)+2(34_7)-0.310=0$$

NOTE.—34_1, 34_{1+2}, 34_{-1-2}, and 34_2 were read by Mr. Darling with the Repsold instrument, in November, 1875. The remainder were read by Mr. Metcalf with the Pistor & Martins instrument, in September and October, 1875.

SECTION XIII.—*Triangulation from the line Falkirk-Pekin to the line Grand River-Westfield*—Continued.

BUFFALO PLAINS—35.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms 14-inch theodolite No. 1. Date, October, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Buffalo and West Base	63 33 31. 558	35_1	16	3. 7	1	+0. 030	−0. 871	63 33 30. 717
West Base and Tonawanda	36 33 28. 541	35_2	16	3. 4	1	+0. 031	+0. 180	36 33 28. 752
Tonawanda and Middle Base	3 39 43. 466	35_3	16	4. 7	1	+0. 030	+0. 451	3 39 43. 947
Middle Base and East Base	51 00 04. 288	35_4	16	3. 0	1	+0. 031	−0. 489	51 00 03. 830
East Base and Buffalo	205 13 11. 995	35_5	16	3. 8	1	+0. 030	+0. 729	205 13 12. 754

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(35_1)+(35_2)+(35_3)+(35_4)-0.152=0$$
$$(35_1)+2(35_2)+(35_3)+(35_4)-0.152=0$$
$$(35_1)+(35_2)+2(35_3)+(35_4)-0.152=0$$
$$(35_1)+(35_2)+(35_3)+2(35_4)-0.152=0$$

EAST BASE—36.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms 14-inch theodolite No. 1. Date, September, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Buffalo Plains and Buffalo	16 50 19. 156	36_1	16	4. 0	1	−0. 080	−0. 398	16 50 18. 678
Buffalo and Middle Base	31 18 00. 494	36_2	16	5. 0	1	−0. 080	+0. 222	31 18 00. 636
Middle Base and West Base	0 00 00. 172	36_3	16	2. 8	1	−0. 080	+0. 192	0 00 00. 284
West Base and Tonawanda	44 35 02. 372	36_4	16	3. 3	1	−0. 080	−0. 105	44 35 02. 187
Tonawanda and Buffalo Plains	267 16 38. 206	36_5	16	4. 0	1	−0. 080	+0. 089	267 16 38. 215

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(36_1)+(36_2)+(36_3)+(36_4)+0.400=0$$
$$(36_1)+2(36_2)+(36_3)+(36_4)+0.400=0$$
$$(36_1)+(36_2)+2(36_3)+(36_4)+0.400=0$$
$$(36_1)+(36_2)+(36_3)+2(36_4)+0.400=0$$

WEST BASE—37.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms 14-inch theodolite No. 1. Date, September, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
East Base and Buffalo Plains	40 38 23. 731	37_{1+2}	16	3. 8	1	+0. 065	+0. 134	40 38 23. 930
Middle Base and Buffalo Plains	40 38 23. 336	37_2	16	1. 7	1	0. 000	+0. 295	40 38 23. 631
Buffalo Plains and Buffalo	81 43 50. 576	37_3	16	4. 3	1	+0. 065	−0. 034	81 43 50. 607
Buffalo and Ridgeway	59 34 41. 699	37_4	16	4. 0	1	+0. 065	−0. 686	59 34 41. 078
Ridgeway and Tonawanda	111 19 40. 533	37_5	16	1. 7	1	+0. 065	+0. 493	111 19 41. 091
Tonawanda and East Base	66 43 23. 136	37_6	16	1. 9	1	+0. 065	+0. 093	66 43 23. 294

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(37_{1+2})+(37_3)+(37_4)+(37_5)-0.325=0$$
$$(37_{1+2})+2(37_3)+(37_4)+(37_5)-0.325=0$$
$$(37_{1+2})+(37_3)+2(37_4)+(37_5)-0.325=0$$
$$(37_{1+2})+(37_3)+(37_4)+2(37_5)-0.325=0$$

SECTION XIII.—*Triangulation from the line Falkirk-Pekin to the line Grand River-Westfield*—Continued.

MIDDLE BASE—38.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms 14-inch theodolite No. 1. Date, October, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
West Base and East Base	179 59 59. 643	38_1	16	1. 6	1	−0. 124	−0. 102	179 59 59. 417
East Base and Buffalo Plains	80 51 37. 429	38_2	16	3. 3	1	−0. 125	−0. 419	80 51 36. 885
Buffalo Plains and West Base	99 08 23. 302	38_3	16	3. 9	1	−0. 125	+0. 521	99 08 23. 698

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(38_2)+(38_3)+0.374=0$$
$$(38_2)+2(38_3)+0.374=0$$

HAMBURG—39.

[Observer, T. Russell. Instrument, Gambey theodolite No. 1. Date, October, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Sturgeon Point and Ridgeway	65 45 29. 989	39_1	17	1. 4	1	+0. 419	−0. 340	65 45 30. 068
Ridgeway and Buffalo	49 15 26. 218	39_2	17	3. 6	1	+0. 419	+0. 235	49 15 26. 872
Buffalo and Sturgeon Point	244 59 02. 536	39_3	17	2. 7	1	+0. 419	+0. 105	244 59 03. 060

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(39_1)+(39_2)-1.257=0$$
$$(39_1)+2(39_2)-1.257=0$$

FONT HILL—40.

[Observer, R. S. Woodward. Instrument, Troughton & Simms 12-inch theodolite No. 2. Dates, August and September, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Drummondville and Ridgeway	57 52 21. 275	40_1	24	10. 4	1	+0. 099	+0. 569	57 52 21. 943
Ridgeway and Sugar Loaf	39 54 28. 595	40_2	24	9. 1	1	+0. 099	−0. 103	39 54 28. 591
Sugar Loaf and Grand River	55 25 23. 697	40_3	24	11. 7	1	+0. 099	−0. 372	55 25 23. 424
Grand River and Drummondville	206 47 46. 037	40_4	24	9. 8	1	+0. 099	−0. 094	206 47 46. 042

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(40_1)+(40_2)+(40_3)-0.396=0$$
$$(40_1)+2(40_2)+(40_3)-0.396=0$$
$$(40_1)+(40_2)+2(40_3)-0.396=0$$

SECTION XIII.—*Triangulation from the line Falkirk-Pekin to the line Grand River-Westfield*—Continued.

SUGAR LOAF—41.

[Observer, G. A. Marr. Instrument, Pistor and Martins 14-inch theodolite No. 2. Dates, May and June, 1876.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Font Hill and Drummondville	38 30 00.340	41_1	23	8.8	1	+0.290	−0.119	38 30 00.511
Drummondville and Ridgeway	56 55 47.188	41_2	21	5.0	1	+0.290	+0.679	56 55 48.157
Ridgeway and Sturgeon Point	47 55 45.200	41_3	21	4.4	1	+0.290	−0.132	47 55 45.358
Sturgeon Point and Silver Creek	36 26 39.381	41_4	20	3.3	1	+0.291	+0.369	36 26 40.041
Silver Creek and Westfield	29 52 10.264	41_5	21	5.7	1	+0.290	−1.351	29 52 09.203
Westfield and Grand River	61 06 29.172	41_6	18	6.1	1	+0.290	+0.189	61 06 29.651
Grand River and Font Hill	89 13 06.423	41_7	22	6.7	1	+0.291	+0.365	89 13 07.079

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(41_1)+(41_2)+(41_3)+(41_4)+(41_5)+(41_6)-2.032=0$$
$$(41_1)+2(41_2)+(41_3)+(41_4)+(41_5)+(41_6)-2.033=0$$
$$(41_1)+(41_2)+2(41_3)+(41_4)+(41_5)+(41_6)-2.032=0$$
$$(41_1)+(41_2)+(41_3)+2(41_4)+(41_5)+(41_6)-2.032=0$$
$$(41_1)+(41_2)+(41_3)+(41_4)+2(41_5)+(41_6)-2.032=0$$
$$(41_1)+(41_2)+(41_3)+(41_4)+(41_5)+2(41_6)-2.032=0$$

STURGEON POINT—42.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Date, June, 1876.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Silver Creek and Sugar Loaf	99 26 38.009	42_1	22	13.4	1	+0.406	+0.300	99 26 38.715
Sugar Loaf and Ridgeway	40 25 20.551	42_2	22	5.9	1	+0.405	+0.277	40 25 21.233
Ridgeway and Buffalo	35 23 42.820	42_3	22	7.4	1	+0.406	−1.115	35 23 42.111
Ridgeway and Hamburg	59 36 08.457	42_{3+4}	38	13.2	2	0.000	+0.138	59 36 08.595
Buffalo and Silver Creek	184 44 16.998	42_{4+5}	22	14.2	1	+0.405	+0.538	184 44 17.941

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(42_1)+(42_2)+(42_3)-1.622=0$$
$$(42_1)+2(42_2)+(42_3)-1.622=0$$
$$(42_1)+(42_2)+2(42_3)-1.622=0$$

SILVER CREEK—43.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1. Dates, May and June, 1876.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Westfield and Grand River	87 42 19.668	43_1	19	10.1	1	+0.130	−0.356	87 42 19.442
Grand River and Sugar Loaf	35 10 45.699	43_2	19	6.9	1	+0.129	−0.172	35 10 45.656
Sugar Loaf and Ridgeway	25 29 51.060	43_3	19	5.3	1	+0.130	+1.205	25 29 52.395
Ridgeway and Sturgeon Point	18 36 51.514	43_4	19	7.7	1	+0.129	−1.141	18 36 50.502
Sturgeon Point and Westfield	193 00 11.411	43_5	17	6.3	1	+0.130	+0.464	193 00 12.005

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(43_1)+(43_2)+(43_3)+(43_4)-0.648=0$$
$$(43_1)+2(43_2)+(43_3)+(43_4)-0.648=0$$
$$(43_1)+(43_2)+2(43_3)+(43_4)-0.648=0$$
$$(43_1)+(43_2)+(43_3)+2(43_4)-0.648=0$$

SECTION XIII.—*Triangulation from the line Falkirk-Pekin to the line Grand River-Westfield*—Continued.

GRAND RIVER—44.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Date, August, 1876.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Font Hill and Sugar Loaf	35 21 30.513	44_1	17	5.0	1	−0.201	+0.506	35 21 30.908
Sugar Loaf and Silver Creek	53 50 39.025	44_2	16	6.7	1	−0.201	−0.518	53 50 38.306
Silver Creek and Westfield	42 17 37.987	44_3	16	4.5	1	−0.201	+0.491	42 17 38.277
Westfield and Long Point	52 26 09.982	44_4	14	7.2	1	−0.201	....	....
Long Point and Font Hill	176 04 03.699	44_5	8	7.6	0.5	−0.402	....	....

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$1.5(44_1)+0.5(44_2)+0.5(44_3)+0.5(44_4)+0.603=0$$
$$0.5(44_1)+1.5(44_2)+0.5(44_3)+0.5(44_4)+0.603=0$$
$$0.5(44_1)+0.5(44_2)+1.5(44_3)+0.5(44_4)+0.603=0$$
$$0.5(44_1)+0.5(44_2)+0.5(44_3)+1.5(44_4)+0.603=0$$

WESTFIELD—45.

[Observers, G. Y. Wisner and R. S. Woodward. Instruments, Troughton & Simms theodolites Nos. 1 and 3. Dates, June and October, 1876.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Erie and Long Point	64 51 01.072	45_1	14	5.7	1	−0.170	....	....
Long Point and Grand River	52 02 06.509	45_2	16	4.8	1	−0.171	....	....
Grand River and Sugar Loaf	22 45 18.919	45_3	9	3.8	0.5	−0.808	+0.178	22 45 18.289
Grand River and Silver Creek	50 00 07.618	45_{3+4}	18	5.2	1	+0.234	−0.257	50 00 07.595
Sugar Loaf and Silver Creek	27 14 50.145	45_4	16	2.1	1	−0.404	−0.435	27 14 49.306
Silver Creek and Erie	193 06 45.079	45_5	17	0.8	1	−0.171	....	....

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(45_1)+(45_2)+(45_3)+(45_4)+1.724=0$$
$$(45_1)+2(45_2)+(45_3)+(45_4)+1.724=0$$
$$(45_1)+(45_2)+2.5(45_3)+2(45_4)+3.170=0$$
$$(45_1)+(45_2)+(45_3)+3(45_4)+3.170=0$$

NOTE.—45_{3+4} and parts of 45_1, 45_2, and 45_5, were read by Mr. Woodward with the Troughton & Simms No. 3, in October, 1876. The remainder were read by Mr. Wisner with the Troughton and Simms No. 1, in June, 1876.

Numerical equations of condition in the triangulation from the line Falkirk-Pekin to the line Grand River-Westfield.

SIDE-EQUATIONS.

$$\begin{array}{llllll}
\text{IV.}\ (10) & +\ 0.0001\,[36_{1+2}] & -\ 18.8660\,[36_3] & +24.5309\,[37_1] & +\ 0.0001\,[37_2] & \\
 & -\ 3.3874\,[38_2] & -\ 3.3875\,[38_3] & & & +\ 7.918=0 \\
\text{VII.}\ (40) & -\ 55.1748\,[35_1] & -\ 44.7040\,[35_{2+3+4}] & -69.5686\,[36_1] & +34.6295\,[36_{2+3}] & \\
 & +\ 13.3469\,[37_{1+2}] & +\ 16.4070\,[37_3] & & & -\ 84.971=0 \\
\text{IX.}\ (25) & -\ 17.7558\,[31_{1+2+3+4+5}] & -\ 23.7022\,[31_6] & -40.9471\,[35_{1+2}] & -44.7040\,[35_{3+4}] & \\
 & -\ 69.5686\,[36_1] & +\ 5.2949\,[36_{2+3+4}] & & & -\ 32.466=0 \\
\text{XI.}\ (150) & +193.5262\,[31_{1+2+3+4+5+6}] & +217.2284\,[31_7] & +14.2277\,[35_1] & +\ 3.7569\,[35_2] & \\
 & -\ 3.0601\,[37_3] & -131.5433\,[37_{4+5}] & & & -\ 60.883=0 \\
\text{XIV.}\ (10) & +\ 18.2261\,[31_1] & +\ 14.9324\,[31_2] & +14.9324\,[31_3] & +14.9324\,[31_4] & \\
 & +\ 14.9324\,[31_5] & +\ 14.9324\,[31_6] & +14.9324\,[31_7] & -\ 1.3083\,[34_3] & \\
 & -\ 1.3083\,[34_4] & -\ 1.0930\,[34_5] & -12.3639\,[37_4] & -\ 8.2209\,[37_5] & -\ 14.154=0 \\
\text{XX.}\ (10) & -\ 18.2261\,[31_1] & +\ 5.5604\,[31_2] & -16.0762\,[32_2] & +19.1986\,[32_3] & \\
 & -\ 14.2313\,[34_3] & +\ 0.2153\,[34_4] & & & +\ 28.137=0
\end{array}$$

Numerical equations of condition, &c.—Continued.

SIDE-EQUATIONS—Continued.

XXIV.	(20)	$+1.6412\,[34_1]$	$+17.1192\,[34_2]$	$-9.4809\,[39_1]$	$+18.1376\,[39_2]$	
		$-29.6326\,[42_3]$	$+12.3519\,[42_{3+4}]$			$-59.630=0$
XXVI.	(20)	$-35.2748\,[32_3]$	$+25.7548\,[32_4]$	$-15.4780\,[34_2]$	$+14.4466\,[34_3]$	
		$-13.7101\,[41_2]$	$+19.0053\,[41_3]$	$-24.7201\,[42_2]$	$+29.6326\,[42_3]$	$+86.242=0$
XXIX.	(40)	$-16.9310\,[41_3]$	$+2.0743\,[41_4]$	$+24.9742\,[42_1]$	$+49.6943\,[42_2]$	
		$-44.1480\,[43_3]$	$+62.5125\,[43_4]$			$+100.239=0$
XXXIII.	(15)	$-23.1644\,[32_4]$	$+2.5904\,[32_5]$	$-13.2219\,[40_1]$	$+25.1746\,[40_2]$	
		$+2.0014\,[41_1]$	$+15.7115\,[41_2]$			$-13.403=0$
XXXV.	(25)	$-7.6098\,[33_7]$	$+21.3048\,[33_8]$	$-25.1746\,[40_2]$	$+14.5124\,[40_3]$	
		$-29.8704\,[43_2]$	$+44.1480\,[43_3]$	$-29.6731\,[44_1]$	$+15.3852\,[44_2]$	$-13.151=0$
XXXIX.	(30)	$+13.6133\,[43_1]$	$+43.4837\,[43_2]$	$-17.6495\,[44_2]$	$-2.2643\,[44_3]$	
		$-50.1986\,[45_3]$	$+40.8862\,[45_4]$			$+31.036=0$

NOTE.—In the solution for determining the general corrections, each of the side-equations was divided by the number inclosed in parenthesis and placed opposite it.

ANGLE-EQUATIONS.

I.	$[35_4]+[36_1]+[36_2]+[38_2]$	$+1.084=0$
II.	$[35_2]+[35_3]+[35_4]+[36_1]+[36_2]+[36_3]+[37_{1+2}]$	$-0.292=0$
III.	$[35_2]+[35_3]+[37_2]+[38_3]$	$-1.447=0$
V.	$[31_6]+[35_3]+[35_4]+[36_1]+[36_2]+[36_3]+[36_4]$	$-0.220=0$
VI.	$[34_6]+[34_7]+[35_1]+[37_3]$	$+1.419=0$
VIII.	$-[31_1]-[31_2]-[31_3]-[31_4]-[31_5]+[34_5]+[34_6]+[36_2]+[36_3]+[36_4]$	$+0.092=0$
X.	$-[31_1]-[31_2]-[31_3]-[31_4]-[31_5]-[31_6]+[34_5]+[34_6]+[31_7]+[35_1]+[35_2]$	$+2.461=0$
XII.	$-[31_2]-[31_3]-[31_4]-[31_5]-[31_6]-[31_7]+[33_2]+[37_5]$	$-1.347=0$
XIII.	$[33_3]+[34_3]+[34_4]+[34_5]+[37_4]$	$+2.556=0$
XV.	$[29_1]+[30_2]+[31_4]$	$-1.586=0$
XVI.	$[30_3]+[31_3]+[32_1]$	$-0.211=0$
XVII.	$[31_2]+[32_2]+[32_3]+[33_1]$	$-0.287=0$
XVIII.	$[32_3]+[33_1]+[33_2]+[33_3]+[34_3]$	$+0.384=0$
XIX.	$[31_1]+[33_2]+[33_3]+[34_3]+[34_4]$	$+0.783=0$
XXI.	$[33_4]+[34_1]+[34_2]+[39_2]$	$-0.522=0$
XXII.	$[33_4]+[33_5]+[34_2]+[42_3]$	$-0.465=0$
XXIII.	$[33_6]+[39_1]+[42_{3+4}]$	$-0.220=0$
XXV.	$[33_6]+[33_7]+[41_3]+[42_2]$	$-0.181=0$
XXVII.	$[33_7]+[41_3]+[41_4]+[43_3]$	$-1.273=0$
XXVIII.	$[41_4]+[42_1]+[43_3]+[43_4]$	$-0.734=0$
XXX.	$[32_4]+[32_5]-[33_1]-[33_2]-[33_3]-[33_4]-[33_5]-[33_6]-[33_7]-[33_8]+[40_1]$	$-0.980=0$
XXXI.	$[32_4]-[33_1]-[33_2]-[33_3]-[33_4]-[33_5]-[33_6]-[33_7]+[41_2]$	$+0.057=0$
XXXII.	$[32_5]+[40_1]+[40_2]+[41_1]$	$-0.649=0$
XXXIV.	$[40_3]-[41_1]-[41_2]-[41_3]-[41_4]-[41_5]-[41_6]+[44_1]$	$-0.590=0$
XXXVI.	$[41_5]+[43_1]+[43_2]+[45_4]$	$+2.315=0$
XXXVII.	$[41_6]+[44_2]+[44_3]+[45_3]$	$-0.340=0$
XXXVIII.	$[43_1]+[44_3]+[45_3]+[45_4]$	$+0.123=0$

General corrections in terms of the correlates.

$[29_1]$	$=+0.72727$ XV				
$[30_2]$	$=+0.75000$ XV				
$[30_3]$	$=+0.75000$ XVI				
$[31_1]$	$=-0.10145$ V	-0.50723 VIII	-0.26406 IX	-0.40578 X	$+0.22968$ XI
	$+0.59422$ XII	$+0.56548$ XIV	-0.02899 XV	-0.02899 XVI	-0.02899 XVII
	$+0.79710$ XIX	-1.46892 XX			

General corrections in terms of the correlates—Continued.

$[31_{1+2+3+4+5}]$	= -0.24637 V	-1.23178 VIII	-0.64122 IX	-0.98541 X	+0.55773 XI
	+0.01459 XII	+0.90272 XIV	+0.07244 XV	+0.07244 XVI	+0.07244 XVII
	+0.50723 XIX	-0.88418 XX			
$[31_{1+2+3+4+5+6}]$	= +0.20291 V	-0.98541 VIII	-0.89221 IX	-1.18832 X	+0.67261 XI
	-0.18832 XII	+1.02080 XIV	+0.05795 XV	+0.05795 XVI	+0.05795 XVII
	+0.40578 XIX	-0.70734 XX			
$[31_2]$	= -0.01449 V	-0.07244 VIII	-0.03770 IX	-0.05795 X	+0.03279 XI
	-0.05795 XII	+0.03371 XIV	-0.28986 XV	-0.28986 XVI	+0.71014 XVII
	-0.02899 XIX	+0.44771 XX			
$[31_{2+3+4+5+6+7}]$	= +0.20291 V	+0.01459 VIII	-0.18198 IX	-0.18832 X	+0.83063 XI
	-1.18832 XII	+0.69143 XIV	+0.05795 XV	+0.05795 XVI	+0.05795 XVII
	-0.59422 XIX	+1.11527 XX			
$[31_3]$	= -0.01449 V	-0.07244 VIII	-0.03770 IX	-0.05795 X	+0.03279 XI
	-0.05795 XII	+0.03371 XIV	-0.28986 XV	+0.71014 XVI	-0.28986 XVII
	-0.02899 XIX	-0.10833 XX			
$[31_4]$	= -0.01449 V	-0.07244 VIII	-0.03770 IX	-0.05795 X	+0.03279 XI
	-0.05795 XII	+0.03371 XIV	+0.71014 XV	-0.28986 XVI	-0.28986 XVII
	-0.02899 XIX	-0.10833 XX			
$[31_5]$	= -0.10145 V	-0.50723 VIII	-0.26406 IX	-0.40578 X	+0.22968 XI
	-0.40578 XII	+0.23611 XIV	-0.02899 XV	-0.02899 XVI	-0.02899 XVII
	-0.20290 XIX	+0.35369 XX			
$[31_6]$	= +0.44928 V	+0.24637 VIII	-0.25099 IX	-0.20291 X	+0.11488 XI
	-0.20291 XII	+0.11808 XIV	-0.01449 XV	-0.01449 XVI	-0.01449 XVII
	-0.10145 XIX	+0.17684 XX			
$[31_7]$	= -0.10145 V	+0.49277 VIII	+0.44617 IX	+0.[illegible]9422 X	+0.38770 XI
	-0.40578 XII	+0.23611 XIV	-0.02899 XV	-0.02899 XVI	-0.02899 XVII
	-0.20290 XIX	+0.35369 XX			
$[32_1]$	= +0.80000 XVI	-0.20000 XVII	-0.10000 XVIII	-0.03123 XX	-0.08118 XXVI
	-0.40000 XXX	-0.20000 XXXI	-0.20000 XXXII	+0.27432 XXXIII	
$[32_2]$	= -0.10000 XVI	+0.40000 XVII	-0.30000 XVIII	-1.70129 XX	+0.40035 XXVI
	-0.20000 XXX	-0.10000 XXXI	-0.10000 XXXII	+0.13716 XXXIII	
$[32_3]$	= -0.10000 XVI	+0.40000 XVII	+0.70000 XVIII	+1.82619 XX	-1.36339 XXVI
	-0.20000 XXX	-0.10000 XXXI	-0.10000 XXXII	+0.13716 XXXIII	
$[32_4]$	= -0.20000 XVI	-0.20000 XVII	-0.10000 XVIII	-0.03123 XX	+1.20656 XXVI
	+0.60000 XXX	+0.80000 XXXI	-0.20000 XXXII	-1.26997 XXXIII	
$[32_6]$	= -0.20000 XVI	-0.20000 XVII	-0.10000 XVIII	-0.03123 XX	-0.08118 XXVI
	+0.60000 XXX	-0.20000 XXXI	+0.80000 XXXII	+0.44701 XXXIII	
$[33_1]$	= -0.11538 XII	-0.11538 XIII	+0.88462 XVII	+0.65385 XVIII	-0.23077 XIX
	-0.07692 XXI	-0.19231 XXII	-0.11538 XXIII	-0.23077 XXV	-0.11538 XXVII
	-0.11538 XXX	-0.23077 XXXI	-0.06321 XXXV		
$[33_2]$	= +0.88462 XII	-0.11538 XIII	-0.11538 XVII	+0.65385 XVIII	+0.76923 XIX
	-0.07692 XXI	-0.19231 XXII	-0.11538 XXIII	-0.23077 XXV	-0.11538 XXVII
	-0.11538 XXX	-0.23077 XXXI	-0.06321 XXXV		
$[33_3]$	= -0.11538 XII	+0.88462 XIII	-0.11538 XVII	+0.65385 XVIII	+0.76923 XIX
	-0.07692 XXI	-0.19231 XXII	-0.11538 XXIII	-0.23077 XXV	-0.11538 XXVII
	-0.11538 XXX	-0.23077 XXXI	-0.06321 XXXV		
$[33_4]$	= -0.07692 XII	-0.07692 XIII	-0.07692 XVII	-0.23077 XVIII	-0.15385 XIX
	+0.61538 XXI	+0.53846 XXII	-0.07692 XXIII	-0.15385 XXV	-0.07692 XXVII
	-0.07692 XXX	-0.15385 XXXI	-0.04214 XXXV		
$[33_6]$	= -0.11538 XII	-0.11538 XIII	-0.11538 XVII	-0.34615 XVIII	-0.23077 XIX
	-0.07692 XXI	+0.80769 XXII	+0.88462 XXIII	-0.23077 XXV	-0.11538 XXVII
	-0.11538 XXX	-0.23077 XXXI	-0.06321 XXXV		

General corrections in terms of the correlates—Continued.

$[33_6]$	=−0.11538 XII	−0.11538 XIII	−0.11538 XVII	−0.34615 XVIII	−0.23077 XIX
	−0.07692 XXI	−0.19231 XXII	−0.11538 XXIII	+0.76923 XXV	−0.11538 XXVII
	−0.11538 XXX	−0.23077 XXXI	−0.06321 XXXV		
$[33_7]$	=−0.11538 XII	−0.11538 XIII	−0.11538 XVII	−0.34615 XVIII	−0.23077 XIX
	−0.07692 XXI	−0.19231 XXII	−0.11538 XXIII	+0.76923 XXV	+0.88462 XXVII
	−0.11538 XXX	−0.23077 XXXI	−0.36759 XXXV		
$[33_8]$	=−0.11538 XII	−0.11538 XIII	−0.11538 XVII	−0.34615 XVIII	−0.23077 XIX
	−0.07692 XXI	−0.19231 XXII	−0.11538 XXIII	−0.23077 XXV	−0.11538 XXVII
	−0.11538 XXX	+0.76923 XXXI	+0.78898 XXXV		
$[34_1]$	=+0.20000 XXI	−0.40000 XXII	−0.29315 XXIV	+0.30956 XXVI	
$[34_2]$	=+0.20000 XXI	+0.60000 XXII	+0.48075 XXIV	−0.46434 XXVI	
$[34_3]$	=+0.50000 XIII	−0.06542 XIV	+0.50000 XVIII	+0.50000 XIX	−0.71157 XX
	+0.36117 XXVI				
$[34_4]$	=−0.40000 VI	−0.40000 VIII	−0.60000 X	+0.60000 XIII	−0.08280 XIV
	+0.80000 XIX	+0.01722 XX			
$[34_5]$	=−0.40000 VI	+0.60000 VIII	+0.40000 X	+0.60000 XIII	−0.06127 XIV
	−0.20000 XIX	−0.00431 XX			
$[34_6]$	=+0.60000 VI	+0.60000 VIII	+0.40000 X	−0.40000 XIII	+0.04803 XIV
	−0.20000 XIX	−0.00431 XX			
$[34_7]$	=+0.60000 VI	−0.40000 VIII	+0.40000 X	−0.40000 XIII	+0.04803 XIV
	−0.20000 XIX	−0.00431 XX			
$[35_1]$	=−0.20000 I	−0.60000 II	−0.40000 III	−0.40000 V	+0.80000 VI
	−0.43294 VII	−0.26746 IX	+0.60000 X	+0.07087 XI	
$[35_{1+2}]$	=−0.40000 I	−0.20000 II	+0.20000 III	−0.80000 V	+0.60000 VI
	−0.60411 VII	−0.53492 IX	+1.20000 X	+0.07194 XI	
$[35_2]$	=−0.20000 I	+0.40000 II	+0.60000 III	−0.40000 V	−0.20000 VI
	−0.17117 VII	−0.26746 IX	+0.60000 X	+0.00107 XI	
$[35_{2+3+4}]$	=+0.40000 I	+1.20000 II	+0.80000 III	+0.80000 V	−0.60000 VI
	−0.51351 VII	−1.10294 IX	−0.20000 X	−0.04689 XI	
$[35_3]$	=−0.20000 I	+0.40000 II	+0.60000 III	+0.60000 V	−0.20000 VI
	−0.17117 VII	−0.41774 IX	−0.40000 X	−0.02398 XI	
$[35_{3+4}]$	=+0.60000 I	+0.80000 II	+0.20000 III	+1.20000 V	−0.40000 VI
	−0.34234 VII	−0.83548 IX	−0.80000 X	−0.04796 XI	
$[35_4]$	=+0.80000 I	+0.40000 II	−0.40000 III	+0.60000 V	−0.20000 VI
	−0.17117 VII	−0.41774 IX	−0.40000 X	−0.02398 XI	
$[36_1]$	=+0.60000 I	+0.40000 II	+0.37733 IV	+0.20000 V	−1.73767 VII
	−0.60000 VIII	−2.35327 IX			
$[36_2]$	=+0.60000 I	+0.40000 II	+0.37733 IV	+0.20000 V	+0.86729 VII
	+0.40000 VIII	+0.64127 IX			
$[36_{2+3}]$	=+0.20000 I	+0.80000 II	−1.13195 IV	+0.40000 V	+1.73458 VII
	+0.80000 VIII	+1.28254 IX			
$[36_{2+3+4}]$	=−0.20000 I	+0.20000 II	−0.75463 IV	+0.60000 V	+1.73613 VII
	+1.20000 VIII	+1.92381 IX			
$[36_3]$	=−0.40000 I	+0.40000 II	−1.50928 IV	+0.20000 V	+0.86729 VII
	+0.40000 VIII	+0.64127 IX			
$[36_4]$	=−0.40000 I	−0.60000 II	+0.37732 IV	+0.20000 V	+0.00155 VII
	+0.40000 VIII	+0.64127 IX			
$[37_{1+2}]$	=+0.80000 II	+1.96247 IV	−0.20000 VI	+0.18490 VII	+0.35486 XI
	−0.20000 XII	−0.20000 XIII	+0.41170 XIV		
$[37_2]$	=+1.00000 III	−2.45308 IV			
$[37_3]$	=−0.20000 II	−0.49062 IV	+0.80000 VI	+0.26141 VII	+0.33446 XI
	−0.20000 XII	−0.20000 XIII	+0.41170 XIV		

General corrections in terms of the correlates—Continued.

[37_4]	=−0.20000 II	−0.49062 IV	−0.20000 VI	−0.14877 VII	−0.52210 XI
	−0.20000 XII	+0.80000 XIII	−0.82469 XIV		
[37_{4+5}]	=−0.40000 II	−0.98124 IV	−0.40000 VI	−0.29754 VII	−1.04420 XI
	+0.60000 XII	+0.60000 XIII	−1.23508 XIV		
[37_5]	=−0.20000 II	−0.49062 IV	−0.20000 VI	−0.14877 VII	−0.52210 XI
	+0.80000 XII	−0.20000 XIII	−0.41039 XIV		
[38_2]	=+0.66667 I	−0.33333 III	−0.11291 IV		
[38_3]	=−0.33333 I	+0.66667 III	−0.11292 IV		
[39_1]	=−0.33333 XXI	+0.66667 XXIII	−0.61833 XXIV		
[39_2]	=+0.66667 XXI	−0.33333 XXIII	+0.76260 XXIV		
[40_1]	=+0.75000 XXX	+0.50000 XXXII	−1.08067 XXXIII	−0.25000 XXXIV	+0.10662 XXXV
[40_2]	=−0.25000 XXX	+0.50000 XXXII	+1.47910 XXXIII	−0.25000 XXXIV	−0.90036 XXXV
[40_3]	=−0.25000 XXX	−0.50000 XXXII	−0.19921 XXXIII	+0.75000 XXXIV	+0.68712 XXXV
[41_1]	=−0.14286 XXV	−0.03782 XXVI	−0.28571 XXVII	−0.14286 XXVIII	+0.05306 XXIX
	−0.14286 XXXI	+0.85714 XXXII	−0.03526 XXXIII	−0.14286 XXXIV	−0.14286 XXXVI
	−0.14286 XXXVII				
[41_2]	=−0.14286 XXV	−0.72333 XXVI	−0.28571 XXVII	−0.14286 XXVIII	+0.05306 XXIX
	+0.85714 XXXI	−0.14286 XXXII	+0.87874 XXXIII	−0.14286 XXXIV	−0.14286 XXXVI
	−0.14286 XXXVII				
[41_3]	=+0.85714 XXV	+0.91245 XXVI	+0.71429 XXVII	−0.14286 XXVIII	−0.37022 XXIX
	−0.14286 XXXI	−0.14286 XXXII	−0.16869 XXXIII	−0.14286 XXXIV	−0.14286 XXXVI
	−0.14286 XXXVII				
[41_4]	=−0.14286 XXV	−0.03782 XXVI	+0.71429 XXVII	+0.85714 XXVIII	+0.10492 XXIX
	−0.14286 XXXI	−0.14286 XXXII	−0.16869 XXXIII	−0.14286 XXXIV	−0.14286 XXXVI
	−0.14286 XXXVII				
[41_5]	=−0.14286 XXV	−0.03782 XXVI	−0.28571 XXVII	−0.14286 XXVIII	+0.05306 XXIX
	−0.14286 XXXI	−0.14286 XXXII	−0.16869 XXXIII	−0.14286 XXXIV	+0.85714 XXXVI
	−0.14286 XXXVII				
[41_6]	=−0.14286 XXV	−0.03782 XXVI	−0.28571 XXVII	−0.14286 XXVIII	+0.05306 XXIX
	−0.14286 XXXI	−0.14286 XXXII	−0.16869 XXXIII	−0.14286 XXXIV	−0.14286 XXXVI
	+0.85714 XXXVII				
[42_1]	=−0.25000 XXII	+0.37041 XXIV	−0.25000 XXV	−0.06141 XXVI	+0.75000 XXVIII
	+0.15768 XXIX				
[42_2]	=−0.25000 XXII	+0.37041 XXIV	+0.75000 XXV	−1.29742 XXVI	−0.25000 XXVIII
	+0.77568 XXIX				
[42_3]	=+0.75000 XXII	−1.11122 XXIV	−0.25000 XXV	+1.42023 XXVI	−0.25000 XXVIII
	−0.46668 XXIX				
[42_{3+4}]	=+0.50000 XXIII	+0.30880 XXIV			
[43_1]	=−0.20000 XXVII	−0.40000 XXVIII	−0.09182 XXIX	−0.11422 XXXV	+0.60000 XXXVI
	+0.80000 XXXVIII	+0.07313 XXXIX			
[43_2]	=−0.20000 XXVII	−0.40000 XXVIII	−0.09182 XXIX	−1.30904 XXXV	+0.60000 XXXVI
	−0.20000 XXXVIII	+1.05881 XXXIX			
[43_3]	=+0.80000 XXVII	+0.60000 XXVIII	−1.19552 XXIX	+1.65170 XXXV	−0.40000 XXXVI
	−0.20000 XXXVIII	−0.38065 XXXIX			
[43_4]	=−0.20000 XXVII	+0.60000 XXVIII	+1.47099 XXIX	−0.11422 XXXV	−0.40000 XXXVI
	−0.20000 XXXVIII	−0.38065 XXXIX			
[44_1]	=+0.83333 XXXIV	−0.98910 XXXV			
[44_2]	=+0.51284 XXXV	+0.83333 XXXVII	−0.49027 XXXIX		
[44_3]	=+0.83333 XXXVII	+0.83333 XXXVIII	−0.06290 XXXIX		
[45_3]	=+0.93333 XXXVII	+0.93333 XXXVIII	−1.56174 XXXIX		
[45_4]	=+0.73333 XXXVI	+0.73333 XXXVIII	+0.99944 XXXIX		

Normal equations for determining the correlates.

No. of equation.					
1.	0=+1.08400	+ 2.66667 I	+ 1.20000 II	− 0.73333 III	+ 0.64175 IV
		+ 1.00000 V	0.20000 VI	− 1.04155 VII	− 0.20000 VIII
		− 2.12974 IX	− 0.40000 X	− 0.02398 XI	
2.	0=−0.29200	+ 1.20000 I	+ 3.20000 II	+ 0.80000 III	+ 1.20785 IV
		+ 1.40000 V	− 0.80000 VI	− 0.33170 VII	+ 0.20000 VIII
		− 2.17367 IX	− 0.20000 X	+ 0.30797 XI	− 0.20000 XII
		− 0.20000 XIII	+ 0.41170 XIV		
3.	0=−1.44700	− 0.73333 I	+ 0.80000 II	+ 2.86667 III	− 2.56600 IV
		+ 0.20000 V	− 0.40000 VI	− 0.34234 VII	− 0.68520 IX
		+ 0.20000 X	− 0.02291 XI		
4.	0=+0.79180	+ 0.64175 I	+ 1.20785 II	− 2.56600 III	+13.75563 IV
		− 0.37732 V	− 0.49062 VI	− 1.18266 VII	− 0.75464 VIII
		− 1.20984 IX	+ 0.87050 XI	− 0.49062 XII	− 0.49062 XIII
		+ 1.00994 XIV			
5.	0=−0.22000	+ 1.00000 I	+ 1.40000 II	+ 0.20000 III	− 0.37732 IV
		+ 2.44928 V	− 0.40000 VI	− 0.34388 VII	+ 0.84637 VIII
		− 1.51593 IX	− 1.00291 X	+ 0.06692 XI	− 0.20291 XII
		+ 0.11808 XIV	− 0.01449 XV	− 0.01449 XVI	− 0.01449 XVII
		− 0.10145 XIX	+ 0.17684 XX		
6.	0=+1.41900	− 0.20000 I	− 0.80000 II	− 0.40000 III	− 0.49062 IV
		− 0.40000 V	+ 2.80000 VI	− 0.17153 VII	+ 0.20000 VIII
		− 0.26746 IX	+ 1.40000 X	+ 0.40533 XI	− 0.20000 XII
		− 1.00000 XIII	+ 0.50776 XIV	− 0.40000 XIX	− 0.00862 XX
7.	0=−2.12428	− 1.04155 I	− 0.33170 II	− 0.34234 III	− 1.18266 IV
		− 0.34388 V	− 0.17153 VI	+ 5.86389 VII	+ 1.73612 VIII
		+ 6.80477 IX	− 0.60410 X	+ 0.21024 XI	− 0.14877 XII
		− 0.14877 XIII	+ 0.30624 XIV		
8.	0=+0.09200	− 0.20000 I	+ 0.20000 II	− 0.75464 IV	+ 0.84637 V
		+ 0.20000 VI	+ 1.73612 VII	+ 3.63178 VIII	+ 2.56503 IX
		+ 1.78541 X	− 0.55773 XI	− 0.01459 XII	+ 0.20000 XIII
		− 0.91596 XIV	− 0.07244 XV	− 0.07244 XVI	− 0.07244 XVII
		− 0.90723 XIX	+ 0.87556 XX		
9	0=−1.29864	− 2.12974 I	− 2.17367 II	− 0.68520 III	− 1.20984 IV
		− 1.51593 V	− 0.26746 VI	+ 6.80477 VII	+ 2.56503 VIII
		+10.01948 IX	+ 0.35730 X	− 0.53710 XI	+ 0.18202 XII
		− 0.75309 XIV	− 0.03771 XV	− 0.03771 XVI	− 0.03771 XVII
		− 0.26407 XIX	+ 0.46031 XX		
10.	0=+2.46400	− 0.40000 I	− 0.20000 II	+ 0.20000 III	− 1.00291 V
		+ 1.40000 VI	− 0.60410 VII	+ 1.78541 VIII	+ 0.35730 IX
		+ 3.58832 X	− 0.60067 XI	+ 0.18832 XII	− 0.20000 XIII
		− 0.98601 XIV	− 0.05795 XV	− 0.05795 XVI	− 0.05795 XVII
		− 1.00578 XIX	+ 0.69441 XX		
11.	0=−0.40589	− 0.02398 I	+ 0.30797 II	− 0.02291 III	+ 0.87050 IV
		+ 0.06692 V	+ 0.40533 VI	+ 0.21024 VII	− 0.55773 VIII
		− 0.53710 IX	− 0.60067 X	+ 2.34489 XI	− 1.35271 XII
		− 0.52210 XIII	+ 2.73365 XIV	+ 0.03278 XV	+ 0.03278 XVI
		+ 0.03278 XVII	+ 0.22969 XIX	− 0.40038 XX	
12.	0=−1.34700	− 0.20000 II	− 0.49062 IV	− 0.20291 V	− 0.20000 VI
		− 0.14877 VII	− 0.01459 VIII	+ 0.18202 IX	+ 0.18832 X
		− 1.35271 XI	+ 2.87294 XII	− 0.31538 XIII	− 1.10182 XIV
		− 0.05795 XV	− 0.05795 XVI	− 0.17333 XVII	+ 0.65385 XVIII
		+ 1.36345 XIX	− 1.11527 XX	− 0.07692 XXI	− 0.19231 XXII
		− 0.11538 XXIII	− 0.23077 XXV	− 0.11538 XXVII	− 0.11538 XXX
		− 0.23077 XXXI	− 0.06321 XXXV		

Normal equations for determining the correlates—Continued.

No. of equation.					
13.	0=+2.55600	− 0.20000 II	− 0.49062 IV	− 1.00000 VI	− 0.14877 VII
		+ 0.20000 VIII	− 0.20000 X	− 0.52210 XI	− 0.31538 XII
		+ 3.38462 XIII	− 1.03418 XIV	− 0.11538 XVII	+ 1.15385 XVIII
		+ 1.86923 XIX	− 0.69866 XX	− 0.07692 XXI	− 0.19231 XXII
		− 0.11538 XXIII	− 0.23077 XXV	+ 0.36117 XXVI	− 0.11538 XXVII
		− 0.11538 XXX	− 0.23077 XXXI	− 0.06321 XXXV	
14.	0=−1.41540	+ 0.41170 II	+ 1.00994 IV	+ 0.11808 V	+ 0.50776 VI
		+ 0.30624 VII	− 0.91596 VIII	− 0.75309 IX	− 0.98601 X
		+ 2.73365 XI	− 1.10182 XII	− 1.03418 XIII	+ 3.44623 XIV
		+ 0.03370 XV	+ 0.03370 XVI	+ 0.03370 XVII	− 0.06542 XVIII
		+ 0.41727 XIX	− 0.92059 XX	− 0.04725 XXVI	
15.	0=−1.58600	− 0.01449 V	− 0.07244 VIII	− 0.03771 IX	− 0.05795 X
		+ 0.03278 XI	− 0.05795 XII	+ 0.03370 XIV	+ 2.18741 XV
		− 0.28986 XVI	− 0.28986 XVII	− 0.02899 XIX	− 0.10833 XX
16.	0=−0.21100	− 0.01449 V	− 0.07244 VIII	− 0.03771 IX	− 0.05795 X
		+ 0.03278 XI	− 0.05795 XII	+ 0.03370 XIV	− 0.28986 XV
		+ 2.26014 XVI	− 0.48986 XVII	− 0.10000 XVIII	− 0.02899 XIX
		− 0.13956 XX	− 0.08118 XXVI	− 0.40000 XXX	− 0.20000 XXXI
		− 0.20000 XXXII	+ 0.27432 XXXIII		
17.	0=−0.28700	− 0.01449 V	− 0.07244 VIII	− 0.03771 IX	− 0.05795 X
		+ 0.03278 XI	− 0.17333 XII	− 0.11538 XIII	+ 0.03370 XIV
		− 0.28986 XV	− 0.48986 XVI	+ 2.39476 XVII	+ 1.05385 XVIII
		− 0.25976 XIX	+ 0.57261 XX	− 0.07692 XXI	− 0.19231 XXII
		− 0.11538 XXIII	− 0.23077 XXV	− 0.96304 XXVI	− 0.11538 XXVII
		− 0.51538 XXX	− 0.43077 XXXI	− 0.20000 XXXII	+ 0.27432 XXXIII
		− 0.06321 XXXV			
18.	0=+0.38400	+ 0.65385 XII	+ 1.15385 XIII	− 0.06542 XIV	− 0.10000 XVI
		+ 1.05385 XVII	+ 3.16155 XVIII	+ 1.80769 XIX	+ 1.11462 XX
		− 0.23076 XXI	− 0.57693 XXII	− 0.34614 XXIII	− 0.69231 XXV
		− 1.00222 XXVI	− 0.34614 XXVII	− 0.54614 XXX	− 0.79231 XXXI
		− 0.10000 XXXII	+ 0.13716 XXXIII	− 0.18963 XXXV	
19.	0=+0.78300	− 0.10145 V	− 0.40000 VI	− 0.90723 VIII	− 0.26407 IX
		− 1.00578 X	+ 0.22969 XI	+ 1.36345 XII	+ 1.86923 XIII
		+ 0.41727 XIV	− 0.02899 XV	− 0.02899 XVI	− 0.25976 XVII
		+ 1.80769 XVIII	+ 3.63556 XIX	− 2.16327 XX	− 0.15384 XXI
		− 0.38462 XXII	− 0.23076 XXIII	− 0.46154 XXV	+ 0.36117 XXVI
		− 0.23076 XXVII	− 0.23076 XXX	− 0.46154 XXXI	− 0.12642 XXXV
20.	0=+2.81370	+ 0.17684 V	− 0.00862 VI	+ 0.87556 VIII	+ 0.46031 IX
		+ 0.69441 X	− 0.40038 XI	− 1.11527 XII	− 0.69866 XIII
		− 0.92059 XIV	− 0.10833 XV	− 0.13956 XVI	+ 0.57261 XVII
		+ 1.11462 XVIII	− 2.16327 XIX	+10.18030 XX	− 3.77512 XXVI
		− 0.06245 XXX	− 0.03122 XXXI	− 0.03122 XXXII	+ 0.04283 XXXIII
21.	0=−0.52200	− 0.07692 XII	− 0.07692 XIII	− 0.07692 XVII	− 0.23076 XVIII
		− 0.15384 XIX	+ 1.68205 XXI	+ 0.73846 XXII	− 0.41025 XXIII
		+ 0.95020 XXIV	− 0.15385 XXV	− 0.15478 XXVI	− 0.07692 XXVII
		− 0.07692 XXX	− 0.15385 XXXI	− 0.04214 XXXV	
22.	0=−0.46500	− 0.19231 XII	− 0.19231 XIII	− 0.19231 XVII	− 0.57693 XVIII
		− 0.38462 XIX	+ 0.73846 XXI	+ 2.69615 XXII	+ 0.80770 XXIII
		− 0.63047 XXIV	− 0.63462 XXV	+ 0.95589 XXVI	− 0.19230 XXVII
		− 0.25000 XXVIII	− 0.46668 XXIX	− 0.19230 XXX	− 0.38462 XXXI
		− 0.10535 XXXV			

Normal equations for determining the correlates—Continued.

No. of equation.					
23.	0=−0.22000	− 0.11538 XII	− 0.11538 XIII	− 0.11538 XVII	− 0.34614 XVIII
		− 0.23076 XIX	− 0.41025 XXI	+ 0.80770 XXII	+ 2.05129 XXIII
		− 0.30953 XXIV	− 0.23077 XXV	− 0.11538 XXVII	− 0.11538 XXX
		− 0.23077 XXXI	− 0.06321 XXXV		
24.	0=−2.98150	+ 0.95020 XXI	− 0.63047 XXII	− 0.30953 XXIII	+ 3.20929 XXIV
		+ 0.37041 XXV	− 2.47631 XXVI	+ 0.37041 XXVIII	+ 0.69145 XXIX
25.	0=−0.18100	− 0.23077 XII	− 0.23077 XIII	− 0.23077 XVII	− 0.69231 XVIII
		− 0.46154 XIX	− 0.15385 XXI	− 0.63462 XXII	− 0.23077 XXIII
		+ 0.37041 XXIV	+ 3.14560 XXV	− 0.38497 XXVI	+ 1.48353 XXVII
		− 0.39286 XXVIII	+ 0.40546 XXIX	− 0.23076 XXX	− 0.60440 XXXI
		− 0.14286 XXXII	− 0.16869 XXXIII	− 0.14286 XXXIV	− 0.43080 XXXV
		− 0.14286 XXXVI	− 0.14286 XXXVII		
26.	0=+4.31210	+ 0.36117 XIII	− 0.04725 XIV	− 0.08118 XVI	− 0.96304 XVII
		− 1.00222 XVIII	+ 0.36117 XIX	− 3.77512 XX	− 0.15478 XXI
		+ 0.95589 XXII	− 2.47631 XXIV	− 0.38497 XXV	+ 9.64944 XXVI
		+ 0.87463 XXVII	− 0.09923 XXVIII	− 2.03838 XXIX	+ 1.12539 XXX
		+ 0.48323 XXXI	− 0.11900 XXXII	− 2.63999 XXXIII	− 0.03782 XXXIV
		− 0.03782 XXXVI	− 0.03782 XXXVII		
27.	0=−1.27300	− 0.11538 XII	− 0.11538 XIII	− 0.11538 XVII	− 0.34614 XVIII
		− 0.23076 XIX	− 0.07692 XXI	− 0.19230 XXII	− 0.11538 XXIII
		+ 1.48353 XXV	+ 0.87463 XXVI	+ 3.11320 XXVII	+ 1.31428 XXVIII
		− 1.46082 XXIX	− 0.11538 XXX	− 0.51649 XXXI	− 0.28572 XXXII
		− 0.33738 XXXIII	− 0.28572 XXXIV	+ 1.28411 XXXV	− 0.68572 XXXVI
		− 0.28572 XXXVII	− 0.20000 XXXVIII	− 0.38065 XXXIX	
28.	0=−0.73400	− 0.25000 XXII	+ 0.37041 XXIV	− 0.39286 XXV	− 0.09923 XXVI
		+ 1.31428 XXVII	+ 2.80714 XXVIII	+ 0.53807 XXIX	− 0.14286 XXXI
		− 0.14286 XXXII	− 0.16869 XXXIII	− 0.14286 XXXIV	+ 1.53748 XXXV
		− 0.94286 XXXVI	− 0.14286 XXXVII	− 0.40000 XXXVIII	− 0.76130 XXXIX
29.	0=+2.50598	− 0.46668 XXII	+ 0.69145 XXIV	+ 0.40546 XXV	− 2.03838 XXVI
		− 1.46082 XXVII	+ 0.53807 XXVIII	+ 4.84264 XXIX	+ 0.05306 XXXI
		+ 0.05306 XXXII	+ 0.06266 XXXIII	+ 0.05306 XXXIV	− 2.00149 XXXV
		− 0.13058 XXXVI	+ 0.05306 XXXVII	− 0.09182 XXXVIII	− 0.17476 XXXIX
30.	0=−0.98000	− 0.11538 XII	− 0.11538 XIII	− 0.40000 XVI	− 0.51538 XVII
		− 0.54614 XVIII	− 0.23076 XIX	− 0.06245 XX	− 0.07692 XXI
		− 0.19230 XXII	− 0.11538 XXIII	− 0.23076 XXV	+ 1.12539 XXVI
		− 0.11538 XXVII	+ 2.83462 XXX	+ 1.36923 XXXI	+ 1.10000 XXXII
		− 1.90363 XXXIII	− 0.25000 XXXIV	+ 0.04342 XXXV	
31.	0=+0.05700	− 0.23077 XII	− 0.23077 XIII	− 0.20000 XVI	− 0.43077 XVII
		− 0.79231 XVIII	− 0.46154 XIX	− 0.03122 XX	− 0.15385 XXI
		− 0.38462 XXII	− 0.23077 XXIII	− 0.60440 XXV	+ 0.48323 XXVI
		− 0.51649 XXVII	− 0.14286 XXVIII	+ 0.05306 XXIX	+ 1.36923 XXX
		+ 3.19560 XXXI	− 0.34286 XXXII	− 0.39123 XXXIII	− 0.14286 XXXIV
		+ 0.72578 XXXV	− 0.14286 XXXVI	− 0.14286 XXXVII	
32.	0=−0.64900	− 0.20000 XVI	− 0.20000 XVII	− 0.10000 XVIII	− 0.03122 XX
		− 0.14286 XXV	− 0.11900 XXVI	− 0.28572 XXVII	− 0.14286 XXVIII
		+ 0.05306 XXIX	+ 1.10000 XXX	− 0.34286 XXXI	+ 2.65714 XXXII
		+ 0.81018 XXXIII	− 0.64286 XXXIV	− 0.79374 XXXV	− 0.14286 XXXVI
		− 0.14286 XXXVII			
33.	0=−0.89353	+ 0.27432 XVI	+ 0.27432 XVII	+ 0.13716 XVIII	+ 0.04283 XX
		− 0.16869 XXV	− 2.63999 XXVI	− 0.33738 XXVII	− 0.16869 XXVIII
		+ 0.06266 XXIX	− 1.90363 XXX	− 0.39123 XXXI	+ 0.81018 XXXII
		+ 6.38907 XXXIII	− 0.36791 XXXIV	− 1.60506 XXXV	− 0.16869 XXXVI
		− 0.16869 XXXVII			

Normal equations for determining the correlates—Continued.

No. of equation.					
34.	0=−0.59000	− 0.14286 XXV	− 0.03782 XXVI	− 0.28572 XXVII	− 0.14286 XXVIII
		+ 0.05306 XXIX	− 0.25000 XXX	− 0.14286 XXXI	− 0.64286 XXXII
		− 0.36791 XXXIII	+ 2.44048 XXXIV	− 0.30198 XXXV	− 0.14286 XXXVI
		− 0.14286 XXXVII			
35.	0=−0.52604	− 0.06321 XII	− 0.06321 XIII	− 0.06321 XVII	− 0.18963 XVIII
		− 0.12642 XIX	− 0.04214 XXI	− 0.10535 XXII	− 0.06321 XXIII
		− 0.43080 XXV	+ 1.28411 XXVII	+ 1.53748 XXVIII	− 2.00149 XXIX
		+ 0.04342 XXX	+ 0.72578 XXXI	− 0.79374 XXXII	− 1.60506 XXXIII
		− 0.30198 XXXIV	+ 8.06020 XXXV	− 1.42326 XXXVI	+ 0.51284 XXXVII
		− 0.11422 XXXVIII	− 2.25095 XXXIX		
36.	0=+2.31500	− 0.14286 XXV	− 0.03782 XXVI	− 0.68572 XXVII	− 0.94286 XXVIII
		− 0.13058 XXIX	− 0.14286 XXXI	− 0.14286 XXXII	− 0.16869 XXXIII
		− 0.14286 XXXIV	− 1.42326 XXXV	+ 2.79048 XXXVI	− 0.14286 XXXVII
		+ 1.33333 XXXVIII	+ 2.14138 XXXIX		
37.	0=−0.34000	− 0.14286 XXV	− 0.03782 XXVI	− 0.28572 XXVII	− 0.14286 XXVIII
		+ 0.05306 XXIX	− 0.14286 XXXI	− 0.14286 XXXII	− 0.16869 XXXIII
		− 0.14286 XXXIV	+ 0.51284 XXXV	− 0.14286 XXXVI	+ 3.45714 XXXVII
		+ 1.76667 XXXVIII	− 2.11491 XXXIX		
38.	0=+0.12300	− 0.20000 XXVII	− 0.40000 XXVIII	− 0.09182 XXIX	− 0.11422 XXXV
		+ 1.33333 XXXVI	+ 1.76667 XXXVII	+ 3.30000 XXXVIII	− 0.55207 XXXIX
39.	0=+1.03453	− 0.38065 XXVII	− 0.76130 XXVIII	− 0.17476 XXIX	− 2.25095 XXXV
		+ 2.14138 XXXVI	− 2.11491 XXXVII	− 0.55207 XXXVIII	+ 5.85092 XXXIX

Values of the correlates and their logarithms.

I	=−0.2635	log 9.4208136$_{-}$	XXI	=−0.4676	log 9.6698373$_{-}$
II	=−0.9252	log 9.9662356$_{-}$	XXII	=+0.7274	log 9.8617793$_{+}$
III	=+0.6763	log 9.8301458$_{+}$	XXIII	=−0.1320	log 9.1206068$_{-}$
IV	=+0.1555	log 9.1916745$_{+}$	XXIV	=+0.6596	log 9.8193904$_{+}$
V	=−1.3431	log 0.1281115$_{-}$	XXV	=+0.3783	log 9.5778363$_{+}$
VI	=−0.8451	log 9.9269081$_{-}$	XXVI	=−0.9468	log 9.9762766$_{-}$
VII	=+1.7511	log 0.2433233$_{+}$	XXVII	=−0.5451	log 9.7364443$_{-}$
VIII	=+1.5360	log 0.1864054$_{+}$	XXVIII	=+0.6721	log 9.8274404$_{+}$
IX	=−1.8292	log 0.2622564$_{-}$	XXIX	=−1.4566	log 0.1633492$_{-}$
X	=−1.3397	log 0.1269978$_{-}$	XXX	=+1.0065	log 0.0028181$_{+}$
XI	=+0.0507	log 8.7048366$_{+}$	XXXI	=−0.1930	log 9.2855798$_{-}$
XII	=+0.0902	log 8.9550139$_{+}$	XXXII	=−0.4240	log 9.6273863$_{-}$
XIII	=−1.2666	log 0.1026360$_{-}$	XXXIII	=−0.0904	log 8.9560723$_{-}$
XIV	=−0.4297	log 9.6331451$_{-}$	XXXIV	=+0.0481	log 8.6823256$_{+}$
XV	=+0.6277	log 9.7977452$_{+}$	XXXV	=−0.5624	log 9.7500839$_{-}$
XVI	=+0.1617	log 9.2087906$_{+}$	XXXVI	=−1.6689	log 0.2224225$_{-}$
XVII	=−0.3586	log 9.5546224$_{-}$	XXXVII	=−0.1284	log 9.1085312$_{-}$
XVIII	=+1.0550	log 0.0232525$_{+}$	XXXVIII	=+0.7359	log 9.8667952$_{+}$
XIX	=−0.7116	log 9.8522604$_{-}$	XXXIX	=+0.2491	log 9.3963749$_{+}$
XX	=−0.9098	log 9.9589555$_{-}$			

Values of the general corrections.

″	″	″	″	″
$[29_1]=+0.457$	$[32_5]=+0.302$	$[34_6]=+0.511$	$[37_5]=+0.493$	$[42_1]=+0.300$
$[30_2]=+0.471$	$[33_1]=+0.523$	$[34_7]=-1.025$	$[38_2]=-0.419$	$[42_2]=+0.277$
$[30_3]=+0.121$	$[33_2]=+0.261$	$[35_1]=-0.871$	$[38_3]=+0.521$	$[42_3]=-1.115$
$[31_1]=+0.963$	$[33_3]=-1.096$	$[35_2]=+0.180$	$[39_1]=-0.340$	$[42_{3+4}]=+0.138$
$[31_2]=-0.833$	$[33_4]=+0.058$	$[35_3]=+0.451$	$[39_2]=+0.235$	$[43_1]=-0.356$
$[31_3]=+0.193$	$[33_5]=+0.422$	$[35_4]=-0.489$	$[40_1]=+0.569$	$[43_2]=-0.172$
$[31_4]=+0.659$	$[33_6]=+0.205$	$[36_1]=-0.398$	$[40_2]=-0.103$	$[43_3]=+1.205$
$[31_5]=+0.067$	$[33_7]=-0.169$	$[36_2]=+0.222$	$[40_3]=-0.372$	$[43_4]=-1.141$
$[31_6]=+0.348$	$[33_8]=-0.845$	$[36_3]=+0.192$	$[41_1]=-0.119$	$[44_1]=+0.596$
$[31_7]=-1.027$	$[34_1]=-0.871$	$[36_4]=-0.105$	$[41_2]=+0.679$	$[44_2]=-0.518$
$[32_1]=-0.103$	$[34_2]=+1.100$	$[37_{1+2}]=+0.134$	$[41_3]=-0.132$	$[44_3]=+0.491$
$[32_2]=+0.541$	$[34_3]=-0.128$	$[37_2]=+0.295$	$[41_4]=+0.369$	$[45_3]=+0.178$
$[32_3]=+0.056$	$[34_4]=-0.782$	$[37_3]=-0.034$	$[41_5]=-1.351$	$[45_4]=-0.435$
$[32_4]=-0.531$	$[34_5]=+0.136$	$[37_4]=-0.686$	$[41_6]=+0.189$	

Residuals resulting from substitution of general corrections in numerical equations of condition.

No. of equation.	Residual.	No. of equation.	Residual.
1	0.0000	21	0.0000
2	0.0000	22	0.0000
3	+0.0001	23	+0.0001
4	+0.0006	24	−0.0004
5	0.0000	25	0.0000
6	0.0000	26	+0.0002
7	+0.0024	27	+0.0001
8	+0.0001	28	0.0000
9	+0.0032	29	−0.0060
10	0.0000	30	+0.0001
11	−0.0120	31	+0.0001
12	0.0000	32	0.0000
13	0.0000	33	+0.0006
14	−0.0012	34	+0.0001
15	0.0000	35	+0.0017
16	0.0000	36	0.0000
17	−0.0001	37	0.0000
18	−0.0001	38	0.0000
19	0.0000	39	+0.0003
20	−0.0003		

PROBABLE ERRORS OF OBSERVED AND ADJUSTED ANGLES.

§ **5.** Let

m=whole number of observed angles in a section (one adjustment).

r=whole number of rigid conditions in a section.

n=number of triangles in principal chain.

$[pvv]$=sum of weighted squares of corrections to observed angles.

ρ_1=probable error of an observed angle of weight unity.

ρ_s=probable error of an observed angle of average weight in whole section.

ρ_s'=probable error of an adjusted angle of average weight in whole section.

p_s=average weight of an observed angle in whole section.

p_c=average weight of an observed angle in principal chain.

ρ_c=probable error of an observed angle of average weight in principal chain.

ρ_c'=probable error of an adjusted angle of average weight in principal chain.

$[vv]$=sum of squares of closing errors of triangles in principal chain.

ρ_t=probable error of an observed angle in principal chain as derived from the closing errors of triangles.

Proceeding as in Chapter XIV, C, § 8, there are found the following values:

FOR THE ENTIRE SECTIONS OF THIS CHAPTER.

Section.	Extent of section.	m	r	$[pvv]$	ρ_1	p_s	ρ_s	$\sqrt{\frac{m-r}{m}}$	ρ_s'
					$''$		$''$		$''$
XI	Chester-Willoughby to Grand River-Westfield	57	33	8.72	0.35	1.01	0.34	0.65	0.22
XII	Grand River-Westfield to Falkirk-Pekin	89	58	29.66	0.48	1.02	0.48	0.59	0.28

FOR THE PRINCIPAL CHAIN CONNECTING THE SANDUSKY AND BUFFALO BASES, GIVEN IN D, § 6, FOLLOWING.

Section.	Extent of principal chain in each section.	p_c	ρ_c	ρ_c'	From closing errors of triangles.				
					$[vv]$	n	ρ_t	Average error.	Greatest error.
			$''$	$''$			$''$	$''$	$''$
X	Sandusky Base to Chester-Willoughby	0.80	0.41	0.24	12.43	14	0.37	0.82	1.91
XI	Chester-Willoughby to Grand River-Westfield	0.99	0.35	0.23	12.49	12	0.40	0.81	2.33
XII	Grand River-Westfield to Buffalo Base	0.84	0.53	0.31	10.50	10	0.40	0.78	2.13
	Entire principal chain	......	.....	.....	35.42	36	0.39	0.80	2.33

D.—PRINCIPAL CHAIN OF TRIANGLES BETWEEN SANDUSKY AND BUFFALO BASES.

§ **6.** The principal chain of triangles joining Sandusky and Buffalo Bases has two triangles in common with the chain joining Sandusky and Chicago Bases, and five triangles in common with the chain joining Buffalo and Sandy Creek Bases. As the latter two chains were adjusted independently of the first, the complication to which the adjustment of the three chains would lead has been avoided as unimportant. The adjustment of the present chain, therefore, is made between the lines Danbury-Sandusky and Drummondville-Ridgeway, these being the lines separating this chain from the other two, the adjusted length of the first being taken from the chain joining Sandusky and Chicago Bases, and the adjusted length of the second from the chain joining Buffalo and Sandy Creek Bases. From Chapter XVII, D, the weighted mean value of the logarithm of the line Danbury-Sandusky is 4.6715463$\pm$12.21, the unit of length being the English foot, and the probable error being in units of the seventh decimal place. From Chapter XIX, D, the weighted mean value of the logarithm of the line Drummondville-Ridgeway is 4.8944527$\pm$22.45. These sides will not have their values changed in obtaining weighted mean sides for the intervening triangulation. The logarithm of the latter line as computed from the line Danbury-Sandusky and the connecting triangles is 4.8944500, giving a discrepancy of 27 units in the seventh decimal place. The triangles of the chain fall in three different sections of the general adjustment, the dividing lines of which between the two bases are Chester-Willoughby and Grand River-Westfield. The probable errors of observed angles of average weight in these parts of the principal chain are in order from Sandusky Base east, $\pm0''.41$, $\pm0''.35$, and $\pm0''.53$, respectively. (See Chapter XVIII, C, § 5.) With these data and those furnished in the following tables we have, using the notation of Chapter XIV, D,

$$d=+27$$

$$\frac{1}{p}+\frac{1}{p'}=\Sigma\,(\alpha^2+\beta^2)\,\rho^2+149+504=3579$$

From these quantities and the values of $\frac{1}{p}$ given in the tables, the corrections to the logarithms of the sides computed from the line Danbury-Sandusky are readily derived. The arrangement of the tables is the same as that of the tables in Chapter XIV, D, to which reference may be made for a detailed explanation. The line in the system having the minimum weight is Claridon-Little

Mountain, for which $\frac{1}{p}=1784$ and $\frac{1}{p'}=1795$. These values give for the probable error of the logarithm of this line in units of the seventh decimal place, ± 29.9, which corresponds to $\frac{1}{145190}$ part of the length of the line.

Principal chain of triangles between Sandusky and Buffalo Bases.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	α^2 and β^2	$\Sigma(\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Kelley's	39 46 52.697		4.6715463	640.09			4.6715463
Sandusky	42 52 18.183	+0.492	4.6982004				4.6982006
Danbury	97 20 49.668		4.8618821	7.29	647.38	258	4.8618823
Brownhelm	32 56 37.045		4.8618821	1056.25			4.8618823
Sandusky	81 34 32.572	+1.054	5.1217227				5.1217230
Kelley's	65 28 52.447		5.0853917	92.16	1795.79	452	5.0853920
Townsend	99 57 27.426		5.0853917	13.69			5.0853920
Brownhelm	47 23 46.434	+0.772	4.9588915				4.9588920
Sandusky	32 38 47.548		4.8239372	1075.84	2885.32	636	4.8239377
Camden	90 20 01.432		4.8239372	0.01			4.8239377
Brownhelm	50 46 05.363	+0.831	4.7130179				4.7130185
Townsend	38 53 53.716		4.6218618	681.21	3566.54	751	4.6218624
Elyria	38 49 27.958		4.6218618	686.44			4.6218624
Camden	43 06 01.664	−0.126	4.6592371				4.6592378
Brownhelm	98 04 30.823		4.8203112	9.00	4261.98	869	4.8203119
Grafton	45 27 23.821		4.8203112	428.49			4.8203119
Elyria	68 15 03.725	+0.863	4.9353229				4.9353236
Camden	66 17 33.684		4.9291042	86.49	4776.96	956	4.9291049
Olmsted	94 03 37.364		4.9291042	1.96			4.9291049
Grafton	38 49 36.401	−1.323	4.7274403				4.7274411
Elyria	47 06 47.018		4.7951201	384.16	5163.08	1021	4.7951209
Royalton	48 48 52.293		4.7951201	342.25			4.7951209
Olmsted	80 52 00.640	−0.260	4.9130257				4.9130266
Grafton	50 19 07.994		4.8048371	306.25	5811.58	1131	4.8048380
Rockport	73 38 29.580		4.8048371	38.44			4.8048380
Royalton	54 12 25.376	+1.058	4.7318772				4.7318781
Olmsted	52 09 05.685		4.7202109	265.69	6115.71	1182	4.7202118
Warrensville	37 10 08.443		4.7202109	772.84			4.7202118
Rockport	76 57 23.069	+0.290	4.9277013				4.9277023
Royalton	65 52 29.445		4.8993602	90.25	6978.80	1328	4.8993612
Willoughby	37 51 09.613		4.8993602	734.41			4.8993612
Warrensville	107 17 59.115	+0.854	5.0913480				5.0913492
Rockport	34 50 52.591		4.8683918	912.04	8625.25	1606	4.8683930
Chester	92 12 11.809		4.8683918	0.64			4.8683930
Willoughby	38 10 03.024	+0.331	4.6596747				4.6596760
Warrensville	49 37 45.774		4.7505938	320.41	8946.30	1660	4.7505951
Little Mountain	75 50 29.350		4.7505938	28.09			4.7505951
Chester	54 00 59.812	−1.503	4.6720400				4.6720413
Willoughby	50 08 31.317		4.6491459	309.76	337.85	1701	4.6491472
Claridon	40 25 02.596		4.6491459	610.09			4.6491472
Little Mountain	71 41 06.884	+1.247	4.8147600				4.8147613
Chester	67 53 51.156		4.8041874	72.25	1020.19	1784	4.8041887

Principal chain of triangles between Sandusky and Buffalo Bases—Continued.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	α^2 and β^2	$\Sigma\ (\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Thompson	57 19 50. 116		4. 8041874	182. 25			4. 8041887
Claridon	59 41 44. 690	−1. 178	4. 8151699				4. 8151713
Little Mountain	62 58 26. 067		4. 8287590	116. 61	1319. 08	1821	4 8287604
Mesopotamia	47 45 14. 930		4. 8287590	368. 64			4. 8287604
Thompson	25 50 49. 864	−0. 072	4. 5988292				4. 5988306
Claridon	106 23 55. 810		4. 9413347	38. 44	1726. 16	1870	4. 9413361
Andover	41 49 59. 375		4. 9413347	552. 25			4. 9413361
Thompson	70 00 41. 322	+0. 548	5. 0902511				5. 0902526
Mesopotamia	68. 09 21. 657		5. 0848754	70. 56	2318. 97	1946	5. 0848769
Conneaut	57 47 51. 151		5. 0848754	174. 24			5. 0848769
Andover	77 01 08. 719	+2. 330	5. 1461757				5. 1461772
Thompson	45 11 02. 977		5. 0082933	436. 81	2960. 02	2021	5. 0082948
Edinboro	42 42 44. 009		5. 0082933	519. 84			5. 0082948
Conneaut	85 07 58. 439	−0. 343	5. 1752941				5. 1752957
Andover	52 09 20. 392		5. 0743130	265 69	3745. 55	2116	5. 0743146
Erie	64 35 04. 213		5. 0743130	100. 00			5. 0743146
Conneaut	31 14 58. 161	−0. 347	4. 8334894				4. 8334910
Edinboro	84 09 59. 522		5. 1162654	4. 84	3850. 39	2129	5. 1162670
Houghton	31 08 13. 993		5. 1162654	1211. 04			5. 1162670
Erie	84 19 49. 168	+0. 781	5. 4005776				5. 4005793
Conneaut	64 32 03. 835		5. 3583181	100. 00	5161. 43	2287	5. 3583198
Long Point	88 43 45. 022		5. 3583181	0. 25			5. 3583198
Erie	37 51 30. 473	+0. 480	5. 1463849				5. 1463866
Houghton	53 24 50. 555		5. 2631177	243. 36	5405. 04	2318	5. 2631194
Westfield	64 51 00. 913		5. 2631177	98. 01			5. 2631194
Long Point	57 28 15. 490	−0. 591	5. 2322613				5. 2322631
Erie	57 40 49. 829		5. 2332702	176. 89	5679. 94	2352	5. 2332720
Grand River	52 26 09. 770		5. 2332702	262. 44			5. 2332720
Westfield	52 02 06. 329	−0. 341	5. 2309157				5. 2309175
Long Point	75 31 50. 547		5. 3201800	30. 25	5972. 63	2387	5. 3201818
Silver Creek	87 42 19. 442		5. 3201800	0. 81			5. 3201818
Grand River	42 17 38. 277	+0. 041	5. 1484973				5. 1484992
Westfield	50 00 07. 595		5. 2047928	309. 76	310. 57	2473	5. 2047947
Sugar Loaf	90 58 38. 854		5. 2047928	0. 16			5. 2047947
Silver Creek	35 10 45. 656	−1. 344	4. 9653795				4. 9653814
Grand River	53 50 38. 306		5. 1119505	234. 09	544. 82	2538	5. 1119524
Sturgeon Point	99 26 38. 715		5. 1119505	12. 25			5. 1119524
Sugar Loaf	36 26 40. 041	+1. 689	4. 8916934				4. 8916954
Silver Creek	44 06 42. 897		4. 9605237	470. 89	1027. 96	2671	4. 9605257
Ridgeway	91 38 54. 355		4. 9605237	0. 36			4. 9605257
Sugar Loaf	47 55 45. 358	+0. 699	4. 8312929				4. 8312950
Sturgeon Point	40 25 21. 233		4. 7725589	610. 09	1638. 41	2840	4. 7725610
Drummondville	39 15 59. 935		4. 7725589	660. 49			4. 7725610
Ridgeway	83 48 12. 997	+0. 099	4. 9686592				4. 9686615
Sugar Loaf	56 55 48. 157		4. 8944500	187. 69	2486. 59	3075	4. 8944527

CHAPTER XIX.

TRIANGULATION FROM BUFFALO BASE TO SANDY CREEK BASE.

A.—DESCRIPTIONS OF STATIONS.

NOTE RELATIVE TO ELEVATIONS.

§ **1.** The heights of ground at stations described in this chapter and in Chapter XVIII, from station Font Hill to station Oswego, both inclusive, depend upon connections with the surface of Lake Ontario made at Oswego by spirit-level and at Sodus by a single zenith distance over a line about 3 miles long, and upon connections with the surface of Lake Erie at West Base (Buffalo Base) by spirit-level and at Ridgeway by single zenith distances over two short lines. East of Oswego the heights depend on connections with the water surface at North and South Base (Sandy Creek Base) and Stony Point. Heights of stations not directly referred to the lake surface were computed mostly from non-simultaneous, reciprocal zenith distances. No exact adjustment of these heights has been made, but judging from discrepancies in heights computed by independent routes, the probable error of any height does not exceed ± 4 feet. Heights of stations on the islands of Lake Ontario and on the north shore were obtained chiefly from the detail topographical charts, and may be erroneous to an extent indicated by the above probable error.

DESCRIPTIONS OF STATIONS.

§ **2.** GASPORT, 1875, '78.*—This station is situated in Royalton Township, Niagara County, New York, about $1\frac{1}{2}$ miles east and one-fourth mile south of the village of Gasport, and about 15 metres south of the limestone ledge. The height of station used was 102 feet. The geodetic point is marked by a nail leaded into the solid rock, about 1 foot below the surface of the ground. A stone post is set directly over the geodetic point as a surface-mark, rising 8 inches above the ground. Three stone reference-posts are set as follows: One bearing north 8° 04′ west, distant 9 metres; one bearing north 49° 31′ east, distant 21.75 metres; and one bearing north 80° 11′ east, distant 22.11 metres from the geodetic point. The height of ground at the station above mean level of Lake Ontario is 397.8 feet.

BATAVIA, 1875.—This station is situated in the township of Batavia, Genesee County, New York, about 4 miles northwest of the city of Batavia, on a slight rise of ground about one-half mile north of an east-and-west highway. The height of station used was 89 feet. The geodetic point is marked by a hole drilled in the top of a stone post set 2 feet below the surface of the ground. A stone post is set directly over the geodetic point for a surface-mark. Three stone reference-posts are set as follows: One bearing south 80° 05′ east, distant 31.0 metres; one bearing south 36° 12′ east, distant 30.6 metres; and one bearing north 34° 55′ west, distant 21.9 metres from the geodetic point. The Blind Asylum at Batavia bears south 74° 38′ east, and is distant 3 or 4 miles; a beech tree 18 inches in diameter bears south 10° 20′ east, distant 8.3 metres; a hemlock tree 30 inches in diameter bears north 79° 05′ west, distant 12.7 metres; and a rail fence is distant 93 metres on the east. The height of ground at the station above mean level of Lake Ontario is 711.5 feet.

ALBION, 1875.—This station is the soldiers' monument in the cemetery 2 miles east of Albion, Orleans County, New York. The monument stands on the highest point in the cemetery, is built of sandstone, and its tower is 60.5 feet high. The geodetic point is on the west side of the center

* See note concerning topographical sketches of stations under Burnt Bluff, Chapter XV, A, § 2.

or "well" of the monument, and is marked by a one-fourth inch hole drilled in the coping-stone. A stone reference-post 2 feet long and 6 inches square is set with its top 4 feet below the surface of the ground vertically under a point 3.524 metres from the geodetic point on the line to Batavia station, bearing south 19° 54′ 46″.07 west. This reference-stone is in turn referred to three holes drilled in the southwest side of the monument, at a height of 3 feet above the ground, distant respectively 1.65 metres, 1.03 metres, and 1.47 metres, the first mentioned being farthest south. Distances from the reference-stone to marble monuments with square granite bases were taken as follows: Dyer, 3.9 metres; Hill, 34.78 metres; Smith, 25.0 metres; French, 29.3 metres. The height of ground at the station above mean level of Lake Ontario is 461.1 feet.

MORGANVILLE, 1875.—This station is situated in Stafford Township, Genesee County, New York, about 7 miles east of the city of Batavia and 1½ miles east of the village of Morganville. The height of station used was 75 feet. The geodetic point is marked by a hole drilled in the top of a stone post and filled with lead, set 2 feet below the surface. Three stone reference-posts are set as follows: One bearing north 62° 33′ east, distant 33.35 metres; one bearing north 2° 23′ east, distant 15.75 metres; and one bearing north 60° 59′ west, distant 30.1 metres from the geodetic point. The reference-stones are set along the road-fence on the north of the station. The height of the ground at the station above mean level of Lake Ontario is 635.1 feet.

BROCKPORT, 1875.—This station is situated in lot 13, section 7, district No. 3, Sweden Township, Monroe County, New York, about 1½ miles south and half a mile east of the village of Brockport. The height of station used was 38 feet. The geodetic point is marked by a cut marble post of the usual form, set 3 feet below the surface, with a stone of the same form set directly over it and projecting 6 inches above the surface. Two stone reference posts are set in the road to the south of the station as follows: One bearing south 22° 24′ west, distant 471.64 metres; and one bearing south 0° 31′ east, distant 451.51 metres from the geodetic point. The intersection of an east-and-west road about half a mile north of the station, and a north-and-south road, which is a prolongation of Main street, of Brockport, is at the corner of lots 5, 11, 8, and 14, and bears north 45° 55′ west, and is distant 1118.8 metres from the geodetic point. The height of ground at the station above mean level of Lake Ontario is 481.1 feet.

SCOTTSVILLE, 1875, '77.—This station is situated in Rush Township, Monroe County, New York, on a steep, wooded hill rising 150 feet above the general level, about 2½ miles southeast of the village of Scottsville and about half a mile east of Scottsville station, on the Rochester branch of the Erie Railway. The height of station used was 35 feet. The geodetic point is marked by a hole drilled in the top of a marble post set 1½ feet below the surface. A marble post is set directly over the geodetic point as a surface-mark. Three stone reference-posts are set as follows: One bearing south 24° 48′ west, distant 16.6 metres; one bearing south 72° 58′ west, distant 21.3 metres; and one bearing north 31° 08′ east, distant 16.2 metres from the geodetic point. The church at North Rush bears north 33° 24′ east, distant about one-third of a mile; an oak tree 18 inches in diameter bears south 76° 28′ east, distant 5.6 metres; and an oak tree 12 inches in diameter bears north 1° 32′ east, distant 6.1 metres from the geodetic point. The height of ground at the station above mean level of Lake Ontario is 545.7 feet.

PINNACLE HILL, 1875, '77.—This station is situated in the township of Brighton, Monroe County, New York, in a Catholic cemetery, on the summit of a hill known as Pinnacle Hill, about 2 miles southeast of the center of the city of Rochester. The height of station used was 33 feet. The geodetic point is marked by a cross and the letters U. S. cut in a bowlder 3 feet below the surface, with an ordinary marking-stone set directly over it, rising to the surface of the ground. Two stone reference-posts are set as follows: One bearing north 15° 23′ west, distant 22.56 metres; and one bearing south 13° 12′ east, distant 31.73 metres from the geodetic point. A large marble monument, marked "Mahon" on the base, bears south 80° 55′ west, distant 87.97 metres; a large granite monument, marked "Cummings" on the base, bears south 70° 26′ west, distant 117.75 metres from the geodetic point. A black-oak tree bears north 82° 02′ west, distant 25.54 metres. The Rochester court-house bears north 45° 38′ west, distant about 2 miles. The height of ground at the station above mean level of Lake Ontario is 502.1 feet.

TURK'S HILL, 1875, '77.—This station is situated near the southern boundary of Perrinton Township, Monroe County, New York, about 2 miles south of the village of Egypt, and about 3 miles northeast of Fisher's, a station on the Auburn branch of the New York Central Railroad. It is on a hill about 1 mile southeast of the hill generally known as Turk's Hill. The height of station used was 28 feet. The geodetic point is marked by a stone post of the usual form, set 2½ feet below the surface of the ground. Three stone reference-posts are set by the road-fence on the east of the station as follows: One bearing north 49° 23′ east, distant 21.2 metres; one bearing north 85° 10′ east, distant 17.9 metres; and one bearing south 56° east, approximately, distant 20.9 metres. The northeast corner of a house with permanent stone foundation is 16.4 metres distant on the south. The height of ground at the station above mean level of Lake Ontario is 680.9 feet.

WALWORTH, 1875, '77.—This station is situated in lot 121, Walworth Township, Wayne County, New York, about three-fourths of a mile northeast of the village of West Walworth. A tripod about 4 feet high was used to support the instrument while observing. The geodetic point is marked by a single stone post of the usual form, set 2½ feet below the surface of the ground. Three stone reference-posts are set as follows: One by the fence on the east of the station, bearing south 26° 51′ east, distant 82.39 metres; and two in the north-and-south road about one-quarter of a mile west of the station, one bearing south 62° 14′ west, distant 530.76 metres; and one bearing north 86° 19′ west, distant 480.9 metres. The height of ground at the station above the mean level of Lake Ontario is 406.2 feet.

PALMYRA, 1875, '77.—This station is situated in lot 29, Palmyra Township, Wayne County, New York, about 3 miles east of the village of Palmyra and 1½ miles southwest of East Palmyra, on the highest point of a long, narrow hill extending north and south. The height of station used was 49 feet. The geodetic point is marked by a stone post of the usual form, set 2½ feet below the surface of the ground. Three stone reference-posts are set along the line-fence west of the station as follows: One bearing north 60° 29′ west, 49.8 metres distant; one bearing south 77° 41′ west, 40 metres distant; and one bearing south 40° 17½′ west, 51.7 metres distant from the geodetic point. The distance from the northwest stone to the west stone is 32.8 metres, and from the west stone to the southwest stone is 31.6 metres. A hard-maple tree 9 inches in diameter stands 29.1 metres approximately north of the geodetic point. The height of ground at the station above mean level of Lake Ontario is 418.7 feet.

CLYDE, 1875.—This station is situated in lot 73, Galen Township, Wayne County, New York, about 2 miles south of the village of Clyde, on a high, narrow ridge running approximately north and south. The height of station used was 4 feet. The geodetic point is marked by a stone post of the usual form, set 2½ feet below the surface of the ground. Two stone reference-posts are set by the line-fence running north and south about 2 metres west of the station, as follows: One bearing north 23° 52′ west, distant 8.9 metres, and one bearing south 1° 30½′ west, distant 5.6 metres from the geodetic point, the distance between the two reference-stones being 14.88 metres. A third reference-stone is set by the road-fence west of the station, bearing north 89° 30′ west, distant 158 metres. The ground at the station is 168 feet above the road. A basswood tree, 14 inches in diameter, the only tree on the hill, stands 5.43 metres north of the geodetic point. The height of ground at the station above mean level of Lake Ontario is 391.9 feet.

VICTORY, 1875.—This station is situated near the eastern boundary of Victory Township, Cayuga County, New York, about half a mile southwest of Ira railway station, on the Southern Central Railroad. The height of station used was 85 feet. The geodetic point is marked by a cross cut in lead in the top of a stone post of the usual form, set 2 feet below the surface of the ground. Three stone reference-posts are set as follows, the bearings being approximate: One bearing south 22° 10′ east, distant 16.7 metres; one bearing north 31° east, distant 16.9 metres; and one bearing north 44° 57′ west, distant 15.3 metres from the geodetic point. The station stands on a hill about 75 feet above the general level of the ground. The height of ground at the station above mean level of Lake Ontario is 326 feet.

SODUS, 1875, '77.—This station is situated in lot 6, Sodus Township, Wayne County, New York, on Greene's Hill, about half a mile west of the village of Sodus and one-fourth mile south of the Lake Ontario Shore Railroad. The height of station used was 75 feet. The geodetic point is marked by a stone post of the usual form, set 3 feet below the surface of the ground, with a stone post set directly over it for a surface-mark. Two stone reference-posts are set as follows: One bearing north 47° 15′ west, distant 11.25 metres; and one bearing south 34° 50′ west, distant 12 metres from the geodetic point. Sodus Academy bears south 78° 28′ east, and is distant 815.3 metres. The height of ground at the station above mean level of Lake Ontario is 347.8 feet.

VANDERLIP, 1875, '77.—This station is situated about 450 metres from the shore of South Bay, Lake Ontario, in Prince-Edwards County, Province of Ontario, about 8 miles southeast of the town of Picton, half a mile south of the post-office of Cardwell, and half a mile southwest of the mouth of Black Creek. The height of station used was 75 feet. The geodetic point is marked by a dent in lead run into the solid rock about 1 foot below the surface of the ground. Three stone reference-posts are set as follows: One bearing north 18° 32′ east, distant 141.73 metres; one bearing north 33° 30′ west, distant 92.95 metres; and one bearing south 51° 26′ west, distant 29.5 metres. The height of ground at the station above Lake Ontario is about 150 feet.

SOUTH BASE, 1874, '75, '78.—This station, marking the south end of the Sandy Creek base-line, is situated close to the lake shore, near the southern end of a sand point lying between North Pond and Lake Ontario, in Sandy Creek Township, Oswego County, New York. The height of station used was 72 feet. The geodetic point is marked by a brass frustum leaded into a stone 8 inches by 8 inches by 3 feet, between the letters U. S. cut in the stone, the top of the stone being 3 feet below the surface of the ground. Two stone posts are set on a line to the east approximately at right angles to the base-line, one 3 feet long and set 3½ feet below the surface, distant 3.39 metres from the geodetic point, and one 5 feet long set 4 feet below the surface, and distant 7.98 metres from the geodetic point. Three stone reference-posts are set as follows: One bearing north 71° 38′ east, distant 34 metres; one near the hotel on the south side of Little Sandy Creek, bearing south 32° 01′ east, distant 543.6 metres; and one on the southeast side of North Pond, bearing south 70° 58′ east, distant 842.9 metres from the geodetic point. The height of ground at the station above the lake is 5.7 feet.

MIDDLE BASE, 1874.—This station is at the end of the 523d tube of the Sandy Creek base-line from South Base. A small tripod about 4 feet high was used to support the instrument during observations. The geodetic point is marked by a single stone of the usual form, 5 feet long, and set 5 feet below the surface. The height of ground at the station is about 11.5 feet above Lake Ontario.

NORTH BASE, 1874.—This station, marking the north end of the Sandy Creek base line, is situated on the lake shore, near the boundary line between Oswego and Jefferson Counties. The height of station used was 60 feet. The geodetic point is marked by a brass frustum leaded into a stone post 8 inches by 8 inches by 5 feet, set 5 feet below the surface of the ground. Two stone posts are set on a line through the geodetic point perpendicular to the direction of the base-line; one 5 feet long and set 5 feet below the surface, bearing north 87° 14′ east, distant 3.79 metres; and one 5 feet long and set 5 feet below the surface, bearing south 87° 14′ west, distant 3.70 metres from the geodetic point. Two stone reference-posts are set as follows: One bearing north 3° 09′ east, distant 60.17 metres; and one bearing south 82° 14′ west, distant 20.87 metres from the geodetic point. A tree blazed on the side toward the station bears north 2° 55′ east, distant 60.5 metres. The height of ground at the station above mean level of Lake Ontario is 11.9 feet.

SANDY CREEK, 1874.—This station is situated in Sandy Creek Township, Oswego County, New York, about 1 mile northwesterly from the village of Sandy Creek. The height of station used was 40 feet. The geodetic point is marked by a stone post of the usual form, set 3 feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Two stone reference posts are set as follows: One bearing south 29° 16′ west, distant 69.5 metres; and one bearing south 28° 59′ east, distant 65.5 metres. The reference-stones are set in a line of maple trees on the south side of the road running by the station on the south. The height of ground at the station above mean level of Lake Ontario is 235.7 feet.

MANNSVILLE, 1874, '75.—This station is situated about half a mile northeast of the village of Mannsville, in lot 177, Ellisburg Township, Jefferson County, New York, on the highest land in the immediate vicinity. A tripod about 4 feet high supported the instrument during observations. The geodetic point is marked by a single post of cut marble, set 18 inches below the surface of the ground. Three marble reference-posts are set as follows: One bearing north 67° 48′ east, distant 91.79 metres; one bearing south 54° 30′ east, distant 107.84 metres; and one bearing south 34° 06′ west, distant 393.1 metres from the geodetic point. The first two reference-stones mentioned are set close to a stone wall east of the station, and the third in the road (Lorain street) on the south of the station. The northeast corner of lot 177 bears north 23° 44′ east, and is distant 190.7 metres. A stone astronomical post, occupied in 1875, standing on the north side of Railroad street, near its junction with Lorain street, bears south 68° 28′ west, and is distant 630.98 metres from the geodetic point. The height of ground at the station above mean level of Lake Ontario is 474.8 feet.

OSWEGO, 1875, '78.—This station is situated in the southwest part of the city of Oswego, New York, one the grounds of the Orphan Asylum, corner of Ellen and West Seventh streets. The height of station used was 100 feet. The geodetic point is marked by a stone post of the usual form, set 2 feet below the surface of the ground, a stone post being set directly over it as a surface-mark. Three stone reference-posts are set as follows: one in the northeast corner of the lot on which the asylum stands, bearing north 50° east, distant 52.50 metres; one in the northwest corner of the lot, bearing north 54° west, distant 49.65 metres; and one bearing south 6° west, distant 49.38 metres from the geodetic point. The bearings given above are magnetic. The Oswego court-house bears north 22° 24′ east (true), and is distant 1627 metres. An astronomical post, occupied in 1868, bears north 16° 58′ east, distant 2242 metres from the geodetic point. The height of ground at the station above mean level of Lake Ontario is 168 feet.

STONY POINT, 1874, '75, '78.—This station is situated on Stony Point, Lake Ontario, in Henderson Township, Jefferson County, New York. The height of station used was 74 feet. The geodetic point is marked by a hole in the solid rock, filled with lead, 20 inches below the surface of the ground. A stone post is set over the geodetic point as a surface-mark. Three stone reference-posts are set as follows: One bearing south 69° 30′ east, distant 28.9 metres; one bearing north 63° 30′ east, distant 26.0 metres; and one bearing north 73° 48′ west, distant 30.6 metres from the geodetic point. An oak tree bears south 5° 30′ west, and is distant 29 metres; a bass-wood tree bears north 45° 30′ west, and is distant 19 metres. The lake shore on the west is distant 391 metres. Stony Point light-house bears south 38° 38′ west, and is distant 1025.8 metres. The height of ground at the station above the mean level of Lake Ontario from 1860 to 1875 is 26.3 feet.

GRENADIER, 1875.—This station is situated on the southwest side, near the west end of Grenadier Island, in the lower end of Lake Ontario. The station is about 170 metres back from the head of a small bay. The height of station used was 33.5 feet. The geodetic point is marked by a stone post of the usual form set 2 feet below the surface. Three stone reference-posts are set as follows: One approximately north, distant 40.05 metres; one northwest, distant 50.25 metres; and one northeast, distant 54.25 metres from the geodetic point. The northwest stone is distant 31.1 metres from the north stone, and the northeast stone is distant 46 metres from the north stone. The height of ground at the station above the lake is 18.4 feet.

DUCK ISLAND, 1874, '75.—This station is situated on the north side of Main Ducks Island. The height of station used was 45 feet. The geodetic point is marked by a hole drilled in the solid rock and filled with lead, about 1 foot below the surface of the ground. A stone post is set over the geodetic point as a surface-mark. Three stone reference-posts are set as follows: One bearing south 66° 41′ west, distant 28.3 metres; one bearing north 24° 34′ west, distant 35.4 metres; and and one bearing north 66° 07′ east, distant 33.2 metres from the geodetic point. False Ducks light-house bears north 83° 56′ west, distant about 9 miles. The height of ground at the station above the lake is 21.8 feet.

AMHERST, 1874.—This station is situated on the south side of Amherst Island, about midway between the east and west ends, and 116 metres from the lake shore. A roadway runs along the shore in front of the station at a height of 10 feet above the level of the water. The height of station used was 23.5 feet. The geodetic point is marked by a nail leaded into the solid rock about 18 inches below the surface of the ground. Three stone reference-posts are set along the fence southwest of the station, as follows: One bearing north 72° 13′ west, distant 14.2 metres; one bearing south 39° 48′ west, distant 9.1 metres; and one bearing south 16° 44′ east, distant 28.45 metres from the geodetic point. The distance from the intersection of the above fence with the road to a house owned by Mr. McCormick is 138.5 metres in a southwesterly direction along the road. The height of ground at the station above Lake Ontario is about 10 feet.

WOLFE, 1874.—This station is situated on the southeast shore of Wolfe or Long Island, about 135 metres back from the water's edge, in a sandy field on a bluff about 60 feet high. The height of station used was 33.5 feet. The geodetic point is marked by a stone set 1.7 feet below the surface of the ground. Three stone reference-posts are set by the road-fence northeast of the station, as follows: One bearing south 88° 28′ east, distant 100 metres; one bearing north 34° 17′ east, distant 85.17 metres; and one bearing north 1° 10′ west, distant 139.38 metres from the geodetic point. The height of ground at the station above the lake is 57.9 feet.

KINGSTON, 1874.—This station is the square tower of the Wesleyan Methodist Church, facing east, on the corner of Williams street and Sydenham Place, in the city of Kingston, Province of Ontario. The geodetic point is on the second stone pier from the south front pinnacle, and is marked by a hole drilled into the stone. On the stone is inscribed U. S. L. S., 1874. The height of the instrument above the ground was 108.5 feet. The northwest corner of the residence, on the southeast corner of Johnson street and Sydenham Place, bears north 54° 15′ east, and is distant 76.16 metres; the southeast corner of Regiopolis College, on the north side of Johnson street at the north end of Sydenham Place, bears north 37° 54′ east, and is distant 114.6 metres; the southwest corner of Regiopolis College bears north 32° 07′ east, and is distant 109.8 metres from the geodetic point. The transit-pier of the Kingston Observatory is situated south 527.2 metres and west 101.9 metres from the geodetic point.

CARLTON, 1874.—This station is situated on Carlton Island, in the upper end of the Saint Lawrence River, about one-half mile from the north shore of the island, in a piece of open woods on a ridge about 80 feet above the water. The height of station used was 33 feet. A triangular hole about 1 inch deep on the south side of a large irregularly-shaped hole cut in the solid rock marks the geodetic point. A stone post 5½ inches square, with a 1½-inch triangle drilled in the center, is placed over the geodetic point for a surface-mark. The station is referred to a blaze on a pine tree bearing south 70° 28′ west, distant 7.65 metres, and to a blaze on a stump bearing south 42° 32′ east, distant 4.65 metres from the geodetic point. An east-and-west rail-fence is south of the station 93 metres. The height of ground at the station above the Saint Lawrence River is about 80 feet.

SIR JOHN, 1874.—This station is situated on the southwest side of Howe's Island, Saint Lawrence River, in a pasture field about 60 metres northeast of a road running northwest and southeast, and about 535 metres from the intersection of the road with the water's edge. The height of station used was 23 feet. The geodetic point is marked by a square stone post rising to the surface of the ground. Three stone reference-posts are set as follows: One bearing south 86° 33′ west, distant 69.15 metres; one bearing south 60° 35′ west, distant 63.78 metres; and one bearing south 35° 28′ west, distant 71.91 metres from the geodetic point. The stones are set along the road-fence. The bearings given are approximate.

B.—STATIONS, SIGNALS, INSTRUMENTS, AND METHODS OF OBSERVATION.

§ **3.** See Chapter XVI, B.

C.—MEASURED AND ADJUSTED ANGLES BETWEEN THE LINES FALKIRK-PEKIN AND CARLTON-SIR JOHN.

§ **4.** The angles within the above limits were adjusted in one group. It is designated Section XIII. The sketch to accompany it is in Plate V. An abstract of the adjustment is given in the following tables. Weight unity was assigned to the following number of combined results for the respective instruments:

Troughton & Simms No. 1, 16.
Troughton & Simms No. 2, 24.
Troughton & Simms No. 3, 24 in 1877.
Troughton & Simms No. 3, 16 in 1878.
Pistor & Martins No. 2, 16.
Gambey No. 1, 16.
Repsold No. 1, 24.

For a detailed explanation of the tables see Chapter XIV, C, § 7, and see the remark in Chapter XV, C, § 6, relating to the column headed "No meas." The locally adjusted angles, with their resulting weights at stations Falkirk and Pekin, were used in computing the general adjustment which disregarded a sum-angle condition at each of those stations.

SECTION XIII.—*Triangulation from the line Sir John - Carlton to the line Falkirk - Pekin.*

SIR JOHN—1.

[Observer, T. Russell. Instrument, Gambey theodolite No. 1. Dates, June and July, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Carlton and Kingston	90 17 44.914	1_1+_2	20	5.4	1	0.000	+0.951	90 17 45.865
Wolfe and Kingston	56 24 09.774	1_2	20	5.0	1	0.000	+0.850	56 24 10.624

CARLTON—2.

[Observer, T. Russell. Instrument, Gambey theodolite No. 1. Date, June, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Wolfe and Sir John	120 48 06.544	2_1+_2	20	4.0	1	0.000	+1.342	120 48 07.886
Kingston and Sir John	62 03 27.564	2_2	20	4.1	1	0.000	+0.058	62 03 27.622

KINGSTON—3.

[Observer, T. Russell. Instrument, Gambey theodolite No. 1. Dates, July and August, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Sir John and Wolfe	64 40 50.914	3_1+_2	20	4.6	1	0.000	+1.228	64 40 52.142
Carlton and Wolfe	37 02 04.434	3_2	20	2.5	1	0.000	+0.915	37 02 05.349
Wolfe and Amherst	88 19 14.697	3_3	23	5.2	1	0.000	+1.316	88 19 16.013

WOLFE—4.

[Observer, T. Russell. Instrument, Gambey theodolite No. 1. Dates, June, August, and September, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Galloo and Carlton	225 32 01.901	$4_1+_2+_3+_4+_5$	16	3.9	1	0.000	0.000	225 32 01.901
Duck Island and Carlton	188 07 18.535	$4_2+_3+_4+_5$	21	5.4	1	0.000	−0.011	188 07 18.524
Amherst and Carlton	140 12 34.439	$4_3+_4+_5$	20	2.6	1	0.000	+1.327	140 12 35.766
Kingston and Carlton	84 13 14.344	4_4+_5	26	4.9	1	0.000	+0.397	84 13 14.741
Sir John and Carlton	25 18 16.804	4_5	21	3.6	1	0.000	+0.234	25 18 17.038

SECTION XIII.—*Triangulation from the line Sir John–Carlton to the line Falkirk–Pekin*—Continued.

AMHERST—5.

[Observer, T. Russell. Instrument, Gambey theodolite No. 1. Dates, October and November, 1874, and May, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Kingston and Wolfe	35 41 23. 025	5_1	16	7. 1	1	−0. 232	+0. 984	35 41 23. 677
Kingston and Duck Island	111 45 28. 463	5_{1+2+3}	20	3. 5	1	+0. 219	+0. 641	111 45 29. 323
Wolfe and Duck Island	76 04 06. 321	5_{2+3}	20	3. 4	1	−0. 332	−0. 343	76 04 05. 646
Grenadier and Duck Island	54 38 00. 343	5_3	18	2. 6	1	0. 000	−0. 636	54 37 59. 707
Duck Island and Vanderlip	71 15 25. 434	5_4	21	3. 4	1	−0. 112	−0. 951	71 15 24. 371
Vanderlip and Kingston	176 59 06. 108	5_5	20	3. 6	1	−0. 112	+0. 310	176 59 06. 306

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3(5_1)+2(5_{2+3})+(5_4)+1.771=0$$
$$2(5_1)+3(5_{2+3})+(5_4)+1.771=0$$
$$(5_1)+(5_{2+3})+2(5_4)+0.888=0$$

NOTE.—5_{1+3+2}, 5_{2+3}, and parts of 5_3 and 5_4 were read in 1874; the remainder in 1875.

DUCK ISLAND—6.

[Observers, G. Y. Wisner and J. H. Darling. Instruments, Troughton & Simms 14-inch theodolite No. 1 and Repsold theodolite No. 1. Dates, August, 1874, and June and July, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Oswego and Vanderlip	104 08 58. 927	6_1	40	10. 4	2	+0. 102	+0. 625	104 08 59. 654
Vanderlip and Amherst	70 26 31. 989	6_2	40	8. 4	2	+0. 101	−0. 854	70 26 31. 236
Amherst and Wolfe	56 01 12. 475	6_3	16	3. 8	1	+0. 203	+0. 203	56 01 12. 881
Wolfe and Grenadier	18 45 43. 359	6_4	16	4. 1	1	+0. 203	+0. 213	18 45 43. 775
Grenadier and Stony Point	49 53 12. 770	6_5	16	4. 5	1	+0. 203	+0. 856	49 53 13. 829
Stony Point and Oswego	60 44 19. 465	6_6	16	4. 5	1	+0. 203	−1. 044	60 44 18. 624

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3(6_1)+(6_2)+(6_3)+(6_4)+(6_5)-1.015=0$$
$$(6_1)+3(6_2)+(6_3)+(6_4)+(6_5)-1.015=0$$
$$(6_1)+(6_2)+2(6_3)+(6_4)+(6_5)-1.015=0$$
$$(6_1)+(6_2)+(6_3)+2(6_4)+(6_5)-1.015=0$$
$$(6_1)+(6_2)+(6_3)+(6_4)+2(6_5)-1.015=0$$

NOTE.—6_1 and 6_2 were read by Mr. Darling with the Repsold instrument in 1875, the remainder by Mr. Wisner, with the Troughton & Simms instrument in 1874.

SECTION XIII.—*Triangulation from the line Sir John–Carlton to the line Falkirk–Pekin*—Continued.

GRENADIER—7.

[Observers, T. Russell and W. A. Metcalf. Instruments, Gambey theodolite No. 1 and Pistor & Martins 14-inch theodolite No. 2. Dates, May and September, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Stony Point and Galloo	29 42 14. 537	7_1	11	4. 7	0. 5	−0. 042	0. 000	29 42 14. 495
Stony Point and Duck Island	78 13 33. 638	7_{1+2}	27	7. 6	1. 5	+0. 170	0. 000	78 13 33. 808
Galloo and Duck Island	48 31 19. 355	7_2	11	5. 8	0. 5	−0. 042	0. 000	48 31 19. 313
Duck Island and Amherst	50 35 04. 280	7_3	18	7. 1	1	−0. 065	+0. 629	50 35 04. 844
Duck Island and Stony Point	281 46 25. 893	7_{3+4}	16	2. 7	1	+0. 299	0. 000	281 46 26. 192
Amherst and Stony Point	231 11 22. 043	7_4	15	5. 8	1	−0. 066	−0. 629	231 11 21. 348

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$4.0(7_1)+3.5(7_2)+\ (7_3)+0.381=0$$
$$3.5(7_1)+4.0(7_2)+\ (7_3)+0.381=0$$
$$(7_1)+\ (7_2)+2(7_3)+0.215=0$$

NOTE.—7_{3+4} and part of 7_{1+2} were read by Mr. Russell with the Gambey instrument in September, 1875; the remainder were read by Mr. Metcalf with the Pistor & Martins instrument in May, 1875.

STONY POINT—8.

[Observers, G. Y. Wisner, T. Russell, J. H. Darling, and R. S. Woodward. Instruments, Troughton & Simms theodolites Nos. 1 and 3, Gambey theodolite No. 1, and Repsold theodolite No. 1. Dates, August and September, 1874, May and September, 1875, and September, 1878.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Mannsville and Sandy Creek	17 28 38. 635	8_1	24	4. 7	1. 5	−0. 526	+0. 272	17 28 38. 381
Mannsville and North Base	31 15 26. 612	8_{1+2}	16	4. 2	1	−0. 169	+0. 535	31 15 26. 978
Mannsville and South Base	34 52 40. 320	8_{1+2+3}	16	5. 0	1	+0. 025	+0. 433	34 52 40. 778
Mannsville and Oswego	74 37 34. 739	$8_{1+2+3+4}$	2	0. 9	0. 1	+1. 518	+0. 549	74 37 36. 806
Sandy Creek and North Base	13 46 48. 588	8_2	48	6. 7	2. 5	−0. 254	+0. 263	13 46 48. 597
South Base and Sandy Creek	342 35 57. 660	8_{-2-3}	30	7. 2	1. 5	+0. 104	−0. 161	342 35 57. 603
North Base and South Base	3 37 13. 905	8_3	66	9. 1	3. 5	−0. 003	−0. 102	3 37 13. 800
North Base and Oswego	43 22 10. 606	8_{3+4}	16	3. 6	1	−0. 792	+0. 014	43 22 09. 828
South Base and Oswego	39 44 55. 496	8_4	21	5. 9	1. 25	+0. 416	+0. 116	39 44 56. 028
South Base and Duck Island	128 06 57. 374	8_{4+5}	14	5. 4	1	−0. 658	−0. 303	128 06 56. 413
Oswego and Duck Island	88 22 00. 858	8_5	37	5. 7	2. 25	−0. 054	−0. 419	88 22 00. 385
Duck Island and Grenadier (W.)	51 53 11. 681	8_6	16	3. 0	1	−0. 059	−0. 065	51 53 11. 557
Duck Island and Grenadier	51 53 12. 597	8_{6+7}	17	2. 9	1	+0. 096	+0. 927	51 53 13. 620
Grenadier and Duck Island	308 06 47. 211	8_{-6-7}	17	3. 6	1	+0. 096	−0. 927	308 06 46. 380
Duck Island and Mannsville	197 00 23. 659	8_{6+7+8}	16	7. 4	1	−0. 720	−0. 130	197 00 22. 809
Grenadier (W.) and Mannsville	145 07 11. 375	8_{7+8}	16	3. 6	1	−0. 058	−0. 065	145 07 11. 252

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$5.6(8_1)+4.1(8_2)+3.1(8_3)+2.10(8_4)+2.00(8_5)+\ (8_6)\quad +3.286=0$$
$$4.1(8_1)+8.1(8_2)+4.6(8_3)+2.10(8_4)+2.00(8_5)+\ (8_6)\quad +3.516=0$$
$$3.1(8_1)+4.6(8_2)+9.1(8_3)+3.10(8_4)+2.00(8_5)+\ (8_6)\quad +1.700=0$$
$$2.1(8_1)+2.1(8_2)+3.1(8_3)+5.35(8_4)+3.00(8_5)+\ (8_6)\quad -0.357=0$$
$$2.0(8_1)+2.0(8_2)+2.0(8_3)+3.00(8_4)+5.25(8_5)+\ (8_6)\quad +0.659=0$$
$$(8_1)+\ (8_2)+\ (8_3)+\ (8_4)+\ (8_5)+2(8_6)\quad +0.538=0$$
$$2(8_{6+7})-0.192=0$$

NOTE.—8_{1+2}, 8_{3+4}, 8_6, 8_{7+8}, and part of 8_5 were read by Mr. Wisner with the Troughton & Simms instrument No. 1 in 1874. 8_{-2-3}, and parts of 8_2 and 8_3 were read by Mr. Darling, with the Repsold instrument in May, 1875. 8_{6+7} and 8_{-6-7} were read by Mr. Russell with the Gambey instrument in September, 1875. The remainder were read by Mr. Woodward in 1878.

SECTION XIII.—*Triangulation from the line Sir John–Carlton to the line Falkirk–Pekin*—Continued.

OSWEGO—9.

[Observers, G. Y. Wisner and R. S. Woodward. Instruments, Troughton & Simms theodolites Nos. 1 and 3. Dates, June and July, 1875, and July, August and September, 1878.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	$[v]$	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Victory and Sodus	46 52 09.575	9_1	40	6.0	2.5	+0.038	−0.058	46 52 09.555
Victory and Vanderlip	127 21 57.017	9_{1+2}	6	2.9	0.5	−0.169	−0.893	127 21 55.955
Victory and Duck Island	154 11 14.118	9_{1+2+3}	8	6.2	0.5	+0.494	−1.000	154 11 13.612
Sodus and Vanderlip	80 29 46.104	9_2	20	4.2	1	+1.131	−0.835	80 29 46.400
Sodus and Duck Island	107 19 03.283	9_{2+3}	3	3.7	0.2	+1.716	−0.942	107 19 04.057
Sodus and Stony Point	138 12 49.435	9_{2+3+4}	16	6.6	1	−0.683	−0.232	138 12 48.520
Vanderlip and Duck Island	26 49 16.607	9_3	21	5.9	1	+1.157	−0.107	26 49 17.657
Vanderlip and Stony Point	57 43 01.960	9_{3+4}	4	4.9	0.25	−0.443	+0.603	57 43 02.120
Duck Island and Stony Point	30 53 42.880	9_4	33	7.8	2	+0.873	+0.710	30 53 44.463
Stony Point and North Base	22 11 56.638	9_5	16	2.8	1	−0.359	+0.166	22 11 56.445
Stony Point and South Base	28 22 51.215	9_{5+6a}	6	3.6	0.5	0.000	+0.113	28 22 51.328
Stony Point and Mannsville	28 57 58.515	$9_{5+6a+6b}$	24	5.5	1.5	−0.264	+0.047	28 57 58.298
Stony Point and Victory	174 55 00.285	$9_{5+6a+6b+7+8}$	12	10.1	0.75	+1.350	+0.290	174 55 01.925
Stony Point and Sodus	221 47 10.554	$9_{5+6a+6b+7+8+1}$	18	9.1	1	+0.694	+0.232	221 47 11.480
North Base and Mannsville	6 46 02.177	9_{6a+6b}	28	4.7	1.75	−0.205	−0.119	6 46 01.853
Mannsville and Sandy Creek	4 16 06.292	9_7	30	5.0	2	−0.612	+0.526	4 16 06.206
Mannsville and Victory	145 57 02.449	9_{7+8}	6	1.8	0.5	+0.935	+0.243	145 57 03.627
Sandy Creek and Victory	141 40 58.931	9_8	16	5.0	1	−1.227	−0.283	141 40 57.421

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$5.75(9_1)+3.25(9_2)+2.75(9_3)+2.25(9_4)+1.50(9_{5+6a+6b})+(9_7)-8.028=0$$
$$3.25(9_1)+6.45(9_2)+4.95(9_3)+4.25(9_4)+1.50(9_{5+6a+6b})+(9_7)-15.841=0$$
$$2.75(9_1)+4.95(9_2)+6.20(9_3)+4.50(9_4)+1.50(9_{5+6a+6b})+(9_7)-15.790=0$$
$$2.25(9_1)+4.25(9_2)+4.50(9_3)+6.50(9_4)+1.50(9_{5+6a+6b})+(9_7)-14.760=0$$
$$2.75(9_5)-1.75(9_{5+6a+6b})+0.525=0$$
$$1.50(9_1)+1.50(9_2)+1.50(9_3)+1.50(9_4)-1.75(9_5)+4.75(9_{5+6a+6b})+(9_7)-3.556=0$$
$$(9_1)+(9_2)+(9_3)+(9_4)+(9_{5+6a+6b})+3(9_7)-1.096=0$$

NOTE.—9_2, 9_3, 9_5, 9_8, and parts of 9_1, 9_4, 9_{6a+6b}, and 9_7 were read by Mr. Wisner with the Troughton & Simms instrument No. 1 in 1875. The remainder were read by Mr. Woodward with the Troughton & Simms instrument No. 3 in 1878.

MANNSVILLE—10.

[Observers, G. Y. Wisner and R. S. Woodward. Instruments, Troughton & Simms theodolites, 14-inch No. 1, and 12-inch No. 2. Dates, October, 1874, and May, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	$[v]$	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Sandy Creek and Oswego	22 04 07.750	10_1	16	5.3	1	+0.186	−0.229	22 04 07.707
Sandy Creek and South Base	23 23 57.446	10_{1+2}	25	7.9	1	−0.435	+0.467	23 23 57.478
Oswego and North Base	20 05 15.706	10_{2+3}	16	6.3	1	+0.186	+0.757	20 05 16.649
South Base and North Base	18 45 27.252	10_3	25	6.6	1	−0.435	+0.061	18 45 26.878
North Base and Stony Point	56 19 10.600	10_4	16	2.7	1	−0.249	+0.731	56 19 11.082
Stony Point and Sandy Creek	261 31 26.060	10_5	16	3.5	1	−0.248	−1.259	261 31 24.562

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$+2(10_1)-(10_{1+2})-(10_3)-1.242=0$$
$$-(10_1)+3(10_{1+2})+2(10_3)+(10_4)+2.609=0$$
$$-(10_1)+2(10_{1+2})+3(10_3)+(10_4)+2.609=0$$
$$+(10_{1+2})+(10_3)+2(10_4)+1.367=0$$

NOTE.—10_{1+2} and 10_3 were read by Mr. Woodward with the Troughton & Simms instrument No. 2 in 1875. The remainder were read by Mr. Wisner with the Troughton & Simms instrument No. 1 in 1874.

SECTION XIII.—*Triangulation from the line Sir John–Carlton to the line Falkirk–Pekin*—Continued.

SANDY CREEK—11.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms 14-inch theodolite No. 1. Date, October, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Oswego and South Base	20 47 17.225	11_1	16	5.3	1	−0.051	+0.312	20 47 17.486
South Base and North Base	34 33 10.406	11_2	16	2.3	1	−0.051	+0.404	34 33 10.759
North Base and Stony Point	34 16 31.912	11_3	16	4.2	1	−0.051	−0.318	34 16 31.543
Stony Point and Mannsville	64 02 45.975	11_4	16	4.1	1	−0.051	+0.741	64 02 46.665
Mannsville and Oswego	206 20 14.737	11_5	16	4.3	1	−0.051	−1.139	206 20 13.547

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(11_1)+(11_2)+(11_3)+(11_4)+0.255=0$$
$$(11_1)+2(11_2)+(11_3)+(11_4)+0.255=0$$
$$(11_1)+(11_2)+2(11_3)+(11_4)+0.255=0$$
$$(11_1)+(11_2)+(11_3)+2(11_4)+0.255=0$$

NORTH BASE—12.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms 14-inch theodolite No. 1. Date, September, 1874.]

Angle as measured between—		Notation.	No meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
South Base and Oswego	46 58 56.275	12_1	16	3.9	1	+0.053	+0.247	46 58 56.575
Oswego and Stony Point	114 25 55.644	12_2	16	4.1	1	+0.052	−0.287	114 25 55.409
Stony Point and Mannsville	92 25 22.631	12_3	16	5.5	1	+0.053	−0.111	92 25 22.573
Mannsville and Sandy Creek	39 31 17.700	12_4	16	3.6	1	+0.052	−0.146	39 31 17.606
Sandy Creek and South Base	66 38 27.487	12_{5+6}	16	3.0	1	+0.053	+0.297	66 38 27.837
Middle Base and South Base	0 51 04.800	12_6	16	1.6	1	0.000	+0.172	0 51 04.972

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(12_1)+(12_2)+(12_3)+(12_4)-0.263=0$$
$$(12_1)+2(12_2)+(12_3)+(12_4)-0.263=0$$
$$(12_1)+(12_2)+2(12_3)+(12_4)-0.263=0$$
$$(12_1)+(12_2)+(12_3)+2(12_4)-0.263=0$$

MIDDLE BASE—13.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms 14-inch theodolite No. 1. Date, October, 1874.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angle.
	° ′ ″			″		″	″	° ′ ″
South Base and North Base	178 15 30.533	13_1	16	1.5	1	0.000	+0.172	178 15 30.705

SECTION XIII.—*Triangulation from the line Sir John-Carlton to the line Falkirk-Pekin*—Continued.

SOUTH BASE—14.

[Observers, G. Y. Wisner and R. S. Woodward. Instruments, Troughton and Simms theodolites Nos. 1, 2, and 3. Dates, October, 1874, May 1875, and October, 1878.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
North Base and Middle Base	0 53 24.150	14_1	16	1.8	1	0.000	+0.172	0 53 24.322
North Base and Mannsville	55 04 47.942	14_{1+2}	20	0.3	1.25	−0.018	−0.096	55 04 47.828
North Base and Sandy Creek	78 48 21.455	14_{1+2+3}	26	7.6	1.5	−0.044	+0.090	78 48 21.501
Mannsville and Sandy Creek	23 43 33.285	14_3	45	7.6	2.25	+0.202	+0.186	23 43 33.673
Sandy Creek and Oswego	154 21 28.845	14_{4a}	16	4.2	1	+0.367	+0.417	154 21 29.629
Sandy Creek and Stony Point	266 13 44.369	14_{4a+4b}	16	5.9	1	+0.022	−0.186	266 13 44.205
Oswego and Stony Point	111 52 14.812	14_{4b}	16	5.2	1	+0.367	−0.603	111 52 14.576
Stony Point and North Base	14 57 54.258	14_5	22	3.9	1.5	−0.060	+0.096	14 57 54.294
Stony Point and Mannsville	70 02 41.646	14_{5+1+2}	16	9.0	1	+0.476	0.000	70 02 42.122

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$4.25(14_{1+2})+3.00(14_3)+1.5(14_{4a})+1.5(14_{4b})-1.629=0$$
$$3.00(14_{1+2})+6.25(14_3)+2.5(14_{4a})+2.5(14_{4b})-3.041=0$$
$$1.50(14_{1+2})+2.50(14_3)+4.5(14_{4a})+3.5(14_{4b})-3.411=0$$
$$1.50(14_{1+2})+2.50(14_3)+3.5(14_{4a})+4.5(14_{4b})-3.411=0$$

NOTE.—14_1, 14_{4a+4b}, and parts of 14_{1+2+3} and 14_5 were read by Mr. Wisner with the Troughton and Simms instrument No. 1 in 1874. Part of 14_3 was read by Mr. Woodward with the Troughton & Simms No. 2 in 1875. The remainder were read by Mr. Woodward in 1878 with the Troughton & Simms No. 3.

VANDERLIP—15.

[Observers, T. Russell and R. S. Woodward. Instruments, Gambey theodolite No. 1 and Troughton and Simms theodolite No. 3. Dates, June and July, 1875, and May and June, 1877.]

Angle as measured between—		Notation.	No meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ″ ″
Amherst and Duck Island	38 18 07.120	15_1	47	8.7	2	+0.094	−1.116	38 18 06.098
Amherst and Oswego	87 19 53.467	15_{1+2}	6	5.2	0.25	−0.546	+0.355	87 19 53.276
Duck Island and Oswego	49 01 45.538	15_2	44	11.9	2	+0.169	+1.471	49 01 47.178
Duck Island and Sodus	87 59 12.554	15_{2+3}	16	4.9	0.75	−0.200	+0.565	87 59 12.919
Oswego and Sodus	38 57 26.546	15_3	40	13.3	2	+0.101	−0.906	38 57 25.741
Sodus and Amherst	233 42 40.406	15_4	39	14.8	2	+0.026	+0.551	233 42 40.983

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$4.25(15_1)+2.25(15_2)+2.00(15_3)-0.982=0$$
$$2.25(15_1)+5.00(15_2)+2.75(15_3)-1.335=0$$
$$2.00(15_1)+2.75(15_2)+4.75(15_3)-1.133=0$$

NOTE.—15_1, 15_2, 15_{2+3}, 15_3, and 15_4 were partly read by Mr. Russell with the Gambey instrument in 1875. The remainder of the angles were read by Mr. Woodward with the Troughton and Simms instrument in 1877.

SECTION XIII.—*Triangulation from the line Sir John–Carlton to the line Falkirk–Pekin*—Continued.

SODUS—16.

[Observer, R. S. Woodward. Instruments, Troughton & Simms 12-inch theodolite No. 2, and 14-inch theodolite No. 3. Dates, June an July, 1875, and July, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	$[v]$	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Vanderlip and Oswego	60 32 57.548	16_1	28	10.7	1.5	+0.095	−0.684	60 32 56.959
Oswego and Victory	29 42 35.931	16_2	24	8.6	1	−0.103	+0.466	29 42 36.294
Victory and Clyde	46 43 03.786	16_3	25	11.3	1	−0.103	−0.384	46 43 03.299
Clyde and Palmyra	59 50 58.700	16_4	26	11.1	1.5	−0.068	−0.394	59 50 58.238
Palmyra and Turk's Hill	32 05 38.709	16_{5a}	24	6.0	1	−0.171	−0.069	32 05 38.469
Palmyra and Walworth	47 07 14.585	16_{5a+5b}	26	7.4	1.5	+0.291	+0.020	47 07 14.896
Walworth and Palmyra	312 52 44.821	$16-_{5a}-_{5b}$	24	5.0	1	+0.303	−0.020	312 52 45.104
Palmyra and Pinnacle Hill	55 19 10.358	$16_{5a+5b+6a}$	8	4.0	0.25	−0.073	+0.658	55 19 10.943
Pinnacle Hill and Palmyra	304 40 49.523	$16-_{5a}-_{5b}-_{6a}$	8	4.0	0.25	+0.192	−0.658	304 40 49.057
Turk's Hill and Walworth	15 01 36.509	16_{5b}	24	3.4	1	−0.171	+0.089	15 01 36.427
Walworth and Pinnacle Hill	8 11 55.277	16_{6a}	12	3.6	0.5	+0.132	+0.638	8 11 56.047
Walworth and Vanderlip	116 03 09.242	16_{6a+6b}	27	8.5	1.5	+0.096	+0.976	116 03 10.314
Walworth and Oswego	176 36 07.471	$16_{6a+6b+1}$	10	3.6	0.5	−0.490	+0.292	170 36 07.273

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3.0(16_1)+1.5(16_2)+1.5(16_3)+1.5(16_4)+1.5(16_{5a})+1.5(16_{5b}) \qquad +0.637=0$$
$$1.5(16_1)+3.0(16_2)+2.0(16_3)+2.0(16_4)+2.0(16_{5a})+2.0(16_{5b}) \qquad +1.190=0$$
$$1.5(16_1)+2.0(16_2)+3.0(16_3)+2.0(16_4)+2.0(16_{5a})+2.0(16_{5b}) \qquad +1.190=0$$
$$1.5(16_1)+2.0(16_2)+2.0(16_3)+3.5(16_4)+2.0(16_{5a})+2.0(16_{5b}) \qquad +1.190=0$$
$$1.5(16_1)+2.0(16_2)+2\ 0(16_3)+2.0(16_4)+6.0(16_{5a})+5.0(16_{b5})+0.5(16_{6a})+2.218=0$$
$$1.5(16_1)+2.0(16_2)+2.0(16_3)+2.0(16_4)+5.0(16_{5a})+6.0(16_{5b})+0.5(16_{6a})+2.218=0$$
$$0.5(16_{5a})+0.5(16_{5b})+\quad(16_{6a})+0.039=0$$

NOTE.—16_1, 16_2, 16_3, 16_4, 16_{5a+5b}, 16_{6a+6b}, and $16_{6a+6b+1}$ were read in 1875 with the Troughton & Simms No. 2; the remainder in 1877 with the Troughton & Simms No. 3.

VICTORY—17.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms 14-inch theodolite No. 1. Date, July, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	$[v]$	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Clyde and Sodus	44 41 46.463	17_1	16	6.8	1	+0.121	−0.949	44 41 45.635
Sodus and Oswego	103 25 16.606	17_2	16	6.3	1	+0.121	−0.099	103 25 16.628
Oswego and Clyde	211 52 56.569	17_3	16	3.8	1	+0.120	+1.048	211 52 57.737

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(17_1)+\ (17_2)-0.362=0$$
$$(17_1)+2(17_2)-0.362=0$$

CLYDE—18.

[Observer, W. A. Metcalf. Instrument, Pistor & Martins 14-inch theodolite No. 2. Dates, June and July, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	$[v]$	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Palmyra and Sodus	45 23 43.875	18_1	16	2.0	1	+0.489	−0.804	45 23 43.560
Sodus and Victory	88 35 13.095	18_2	16	7.3	1	+0.489	−0.597	88 35 12.987
Victory and Palmyra	226 01 01.563	18_3	16	5.1	1	+0.489	+1.401	226 01 03.453

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(18_1)+\ (18_2)-1.467=0$$
$$(18_1)+2(18_2)-1.467=0$$

SECTION XIII.—*Triangulation from the line Sir John–Carlton to the line Falkirk–Pekin*—Continued.

PALMYRA—19.

[Observers, W. A. Metcalf and R. S. Woodward. Instruments, Pistor & Martins 14-inch theodolite No. 2, and Troughton & Simms 14-inch theodolite No. 3. Dates, June, 1875, and July, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Turk's Hill and Walworth	44 12 02.712	19_1	40	5.9	2	+0.036	+0.672	44 12 03.420
Walworth and Sodus	74 25 05.020	19_2	40	7.1	2	+0.036	−0.397	74 25 04.659
Sodus and Clyde	74 45 20.509	19_3	16	5.9	1	+0.118	−1.240	74 45 19.387
Sodus and Turk's Hill	241 22 52.242	19_{3+4}	24	3.0	1	−0.046	−0.275	241 22 51.921
Clyde and Turk's Hill	166 37 31.452	19_4	16	5.7	1	+0.117	+0.965	166 37 32.534

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$4(19_1)+2(19_2)+(19_3)-0.333=0$$
$$2(19_1)+4(19_2)+(19_3)-0.333=0$$
$$(19_1)+(19_2)+2(19_3)-0.307=0$$

NOTE.—19_3, 19_4, and parts of 19_1 and 19_2 were read by Mr. Metcalf with the Pistor & Martins instrument in 1875. The remainder wer read by Mr. Woodward with the Troughton & Simms instrument in 1877.

WALWORTH—20.

[Observer, R. S. Woodward. Instruments, Troughton & Simms 12-inch theodolite No. 2, and 14-inch theodolite No. 3. Dates, July, 1875, and July, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Sodus and Palmyra	58 27 41.923	20_1	49	9.7	2	−0.011	−0.629	58 27 41.283
Palmyra and Turk's Hill	83 52 54.744	20_2	49	10.0	2	−0.011	+0.343	83 52 55.076
Turk's Hill and Pinnacle Hill	54 48 16.562	20_3	48	8.1	2	−0.011	+0.016	54 48 16.567
Pinnacle Hill and Sodus	162 51 06.814	20_4	48	8.2	2	−0.010	+0.270	162 51 07.074

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$4(20_1)+2(20_2)+2(20_3)+0.086=0$$
$$2(20_1)+4(20_2)+2(20_3)+0.086=0$$
$$2(20_1)+2(20_2)+4(20_3)+0.086=0$$

NOTE.—Parts of all the angles at this station were read in 1875 with the Troughton & Simms instrument No. 2, and the remainder in 1877 with the Troughton & Simms No. 3.

TURK'S HILL—21.

[Observers, W. A. Metcalf and R. S. Woodward. Instruments, Pistor & Martins theodolite No. 2, and Troughton & Simms theodolites Nos. 2 and 3. Dates, July and September, 1875, and July, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Scottsville and Pinnacle Hill	49 25 11.353	21_1	64	7.5	3	−0.030	+0.418	49 25 11.741
Pinnacle Hill and Walworth	79 00 51.562	21_2	64	8.0	3	−0.030	−0.135	79 00 51.397
Walworth and Sodus	22 37 46.702	21_{3a}	24	4.6	1	+0.637	+0.407	22 37 47.746
Walworth and Palmyra	51 55 02.534	21_{3a+3b}	40	9.9	2	−0.363	−0.009	51 55 02.162
Sodus and Palmyra	29 17 14.195	22_{3b}	24	4.4	1	+0.637	−0.416	29 17 14.416
Palmyra and Scottsville	179 38 55.004	21_4	64	8.8	3	−0.030	−0.274	179 38 54.700

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$6(21_1)+3(21_2)+3(21_{3a})+3(21_{3b})-3.552=0$$
$$3(21_1)+6(21_2)+3(21_{3a})+3(21_{3b})-3.552=0$$
$$3(21_1)+3(21_2)+6(21_{3a})+5(21_{3b})-6.826=0$$
$$3(21_1)+3(21_2)+5(21_{3a})+6(21_{3b})-6.826=0$$

NOTE.—21_1, 21_2, 21_{3a+3b}, and 21_4 were partly measured by Mr. Metcalf with the Pistor & Martins instrument in July, 1875. 21_1, 21_2, and 21_4 were partly measured and the remainder of 21_{3a+3b} was measured by Mr. Woodward with the Troughton & Simms instrument No. 2 in September, 1875. The remainder of the angles were read by Mr. Woodward with the Troughton & Simms instrument No. 3 in July, 1877.

SECTION XIII.—*Triangulation from the line Sir John-Carlton to the line Falkirk-Pekin*—Continued.

PINNACLE HILL—22.

[Observers, G. Y. Wisner and R. S. Woodward. Instruments, Troughton & Simms theodolites Nos. 1, 2, and 3. Dates, July and October, 1875, and August, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Walworth and Turk's Hill	46 10 53.196	22_1	40	6.5	2	−0.316	−0.192	46 10 52.688
Turk's Hill and Scottsville	84 22 48.171	22_2	64	12.7	3	+0.037	+0.510	84 22 48.718
Scottsville and Brockport	68 35 26.907	22_3	16	4.9	1	+0.264	−0.658	68 35 26.513
Scottsville and Sodus	220 29 21.760	22_{3+4a}	10	3.2	0.5	+0.536	−0.930	220 29 21.366
Scottsville and Walworth	229 26 19.581	$22_{3+4a+4b}$	18	4.1	0.75	−0.669	−0.318	229 26 18.594
Brockport and Walworth	160 50 51.476	22_{4a+4b}	16	4.7	1	+0.265	+0.340	160 50 52.081
Sodus and Walworth	8 56 57.405	22_{4b}	14	3.5	0.5	−0.789	+0.612	8 56 57.228
Sodus and Turk's Hill	55 07 48.053	22_{4b+1}	6	5.2	0.25	+1.443	+0.420	55 07 49.916
Turk's Hill and Sodus	304 52 12.038	22_{-1b-1}	6	4.2	0.25	−1.534	−0.420	304 52 10.084
Sodus and Scottsville	139 30 38.112	22_{4b+1+2}	4	3.0	0.2	−0.408	+0.930	139 30 38.634

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$4.95(22_1)+2.45(22_2)+(22_3)+1.2(22_{4b})+2.156=0$$
$$2.45(22_1)+5.45(22_2)+(22_3)+0.7(22_{4b})+0.859=0$$
$$(22_1)+(22_2)+2(22_3)\qquad -0.250=0$$
$$1.20(22_1)+0.70(22_2)\qquad +1.7(22_{4b})+1.695=0$$

NOTE.—22_3, 22_{4a+4b}, and parts of 22_1 and 22_2 were read by Mr. Wisner with the Troughton & Simms instrument No. 1 in July, 1875. Part of 22_2 was read by Mr. Woodward with the Troughton & Simms instrument No. 2 in October, 1875. The remainder of the angles were read by Mr. Woodward with the Troughton & Simms instrument No. 3 in 1877.

SCOTTSVILLE—23.

[Observers, J. H. Darling and R. S. Woodward. Instruments, Repsold theodolite No. 1, and Troughton & Simms theodolites Nos. 2 and 3. Dates, July and October, 1875, and August, 1877.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″					″	″	° ′ ″
Morganville and Brockport	46 45 36.095	23_1	34	7.1	1.5	−0.531	+0.627	46 45 36.191
Brockport and Pinnacle Hill	74 23 46.295	23_2	34	9.2	1.5	−0.531	−0.894	74 23 44.870
Pinnacle Hill and Turk's Hill	46 11 59.830	23_3	66	8.9	2.75	+0.004	+0.495	46 12 00.329
Turk's Hill and Pinnacle Hill	313 47 59.091	23_{-3}	16	4.4	0.75	+1.075	−0.495	313 47 59.671
Turk's Hill and Morganville	192 38 39.368	23_4	36	10.9	1.5	−0.530	−0.228	192 38 38.610

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3.0(23_1)+1.5(23_2)+1.5(23_3)+2.382=0$$
$$1.5(23_1)+3.0(23_2)+1.5(23_3)+2.382=0$$
$$1.5(23_1)+1.5(23_2)+5.0(23_3)+1.572=0$$

NOTE.—23_1, 23_2, 23_4, and part of 23_3 were read by Mr. Darling with the Repsold theodolite, in July, 1875. Part of 23_3 was read by Mr. Woodward with the Troughton & Simms instrument No. 2, in October, 1875. 23_{-3}, and the remainder of 23_3 were read by Mr. Woodward with the Troughton & Simms instrument No. 3, in 1877.

BROCKPORT—24.

[Observer, R. S. Woodward. Instrument, Troughton & Simms 12-inch theodolite No. 2. Dates, July and August, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Pinnacle Hill and Scottsville	37 00 50.270	24_1	27	9.3	1.5	+0.118	−0.532	37 00 49.856
Scottsville and Morganville	65 53 17.304	24_2	27	13.6	1.5	+0.118	+0.988	65 53 18.410
Morganville and Albion	81 28 42.767	24_3	28	11.7	1.5	+0.118	−0.590	81 28 42.295
Albion and Pinnacle Hill	175 37 09.188	24_4	28	8.5	1.5	+0.117	+0.134	175 37 09.439

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3.0(24_1)+1.5(24_2)+1.5(24_3)-0.706=0$$
$$1.5(24_1)+3.0(24_2)+1.5(24_3)-0.706=0$$
$$1.5(24_1)+1.5(24_2)+3.0(24_3)-0.706=0.$$

SECTION XIII.—*Triangulation from the line Sir John-Carlton to the line Falkirk-Pekin*—Continued.

MORGANVILLE—25.

[Observer, T. Russell. Instrument, Gambey theodolite No. 1. Date, July, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Batavia and Albion	67 24 57.250	25_1	18	6.1	1	−0.053	+0.171	67 24 57.368
Batavia and Brockport	111 12 48.354	25_{1+2}	6	2.6	0.5	−0.459	−0.778	111 12 47.117
Albion and Brockport	43 47 50.751	25_2	18	3.0	1	−0.053	−0.949	43 47 49.749
Brockport and Scottsville	67 21 06.087	25_3	16	4.4	1	−0.283	+1.030	67 21 06.834
Scottsville and Batavia	181 26 06.585	25_4	16	6.0	1	−0.284	−0.252	181 26 06.049

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.5(25_1)+1.5(25_2)+(25_3)+0.496=0$$
$$1.5(25_1)+2.5(25_2)+(25_3)+0.496=0$$
$$(25_1)+(25_2)+2(25_3)+0.673=0$$

ALBION—26.

[Observer, W. A. Metcalf. Instrument, Pistor & Martins 14-inch theodolite No. 2. Dates, July and August, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles
	° ′ ″			″		″	″	° ′ ″
Brockport and Morganville	54 43 28.981	26_1	16	5.5	1	+0.459	−0.444	54 43 28.996
Morganville and Batavia	38 38 19.080	26_2	16	4.3	1	+0.459	+0.676	38 38 20.215
Batavia and Gasport	62 03 57.772	26_3	16	6.0	1	+0.459	−0.872	62 03 57.359
Gasport and Brockport	204 34 12.332	26_4	16	5.0	1	+0.458	+0.640	204 34 13.430

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(26_1)+(26_2)+(26_3)-1.835=0$$
$$(26_1)+2(26_2)+(26_3)-1.835=0$$
$$(26_1)+(26_2)+2(26_3)-1.835=0$$

BATAVIA—27.

[Observer, J. H. Darling. Instrument, Repsold theodolite No. 1. Dates, July and August, 1875.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Falkirk and Gasport	40 29 30.684	27_1	30	6.6	1.5	+0.108	+0.072	40 29 30.720
Gasport and Albion	68 48 39.518	27_2	28	8.0	1.5	+0.108	+0.747	68 48 28.879
Albion and Morganville	73 56 42.919	27_3	26	9.2	1	+0.163	+0.427	73 56 43.509
Morganville and Falkirk	176 45 06.337	27_4	24	4.4	1	+0.163	+0.392	176 45 06.892

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.5(27_1)+(27_2)+(27_3)-0.542=0$$
$$(27_1)+2.5(27_2)+(27_3)-0.542=0$$
$$(27_1)+(27_2)+2(27_3)-0.542=0$$

SECTION XIII.—*Triangulation from the line Sir John-Carlton to the line Falkirk-Pekin*—Continued.

GASPORT—28.

[Observer, R. S. Woodward. Instruments, Troughton & Simms theodolites Nos. 2 and 3. Dates, August, 1875, and October and November, 1878.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			″		″	″	° ′ ″
Albion and Batavia.................. 49 07 26.051	28_1	48	9.1	2.5	−0.071	−0.387	49 07 25.593
Batavia and Falkirk.................. 35 16 09.094	28_2	48	7.8	2.5	−0.057	+0.007	35 16 09.044
Batavia and Pekin.....................132 38 32.257	28_{2+3}	4	2.8	0.25	−0.132	+0.113	132 38 32.238
Falkirk and Pekin 97 22 23.145	28_3	48	12.6	2.5	−0.057	+0.106	97 22 23.194
Pekin and Albion.................... 178 14 01.984	28_4	43	6.3	2	−0.089	+0.274	178 14 02.169

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$4.5(28_1)+2.00(28_2)+2.00(28_3)+0.548=0$$
$$2.0(28_1)+4.75(28_2)+2.25(28_3)+0.543=0$$
$$2.0(28_1)+2.25(28_2)+4.75(28_3)+0.543=0$$

NOTE.—Parts of 28_1, 28_2, 28_3, and 28_4 were read with the Troughton & Simms No. 2, in 1875; the remainder of the angles in 1878, with Troughton & Simms No. 3.

FALKIRK—29.

[Observer, T. Russell. Instrument, Gambey theodolite No. 1. Dates, July and August, 1875.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			″		″	″	° ′ ″
Tonawanda and Pekin 31 23 10.691	29_1	17	3.1	1	+0.082	....	....
Pekin and Gasport.................... 48 49 46.890	29_2	19	3.9	1	+0.082	−0.185	48 49 46.787
Gasport and Batavia 104 14 21.342	29_3	24	6.5	1.5	+0.054	−0.273	104 14 21.123
Batavia and Tonawanda 175 32 40.778	29_4	18	3.9	1	+0.081	....	....

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(29_1)+(29_2)+(29_3)-0.299=0$$
$$(29_1)+2(29_2)+(29_3)-0.299=0$$
$$(29_1)+(29_2)+2.5(29_3)-0.299=0$$

PEKIN—30.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms 14-inch theodolite No. 1. Dates, July and August, 1875.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angle.
° ′ ″			″		″	″	° ′ ″
Gasport and Falkirk.................. 33 47 51.526	30_1	18	4.0	1	+0.065	−0.190	33 47 51.401
Falkirk and Tonawanda 66 25 49.294	30_2	18	5.9	1	+0.065	....	....
Tonawanda and Drummondville 59 58 19.872	30_3	18	5.6	1	+0.065	....	....
Drummondville and Gasport........ 199 47 59.017	30_4	18	4.6	1	+0.066	....	....

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(30_1)+(30_2)+(30_3)-0.261=0$$
$$(30_1)+2(30_2)+(30_3)-0.261=0$$
$$(30_1)+(30_2)+2(30_3)-0.261=0$$

Numerical equations of condition in the triangulation from the line Sir John – Carlton to the line Falkirk – Pekin.

SIDE-EQUATIONS.

I. (10)	$+13.9876\,[1_2]$	$+0.1087\,[1_{1+2}]$	$-12.7795\,[2_{1+2}]$	$+23.9476\,[2_2]$	
	$-10.5623\,[4_{4+5}]$	$+12.6933\,[4_5]$			$+4.987=0$
VIII. (20)	$-14.9447\,[5_3]$	$+7.1444\,[5_4]$	$-4.3887\,[7_1]$	$-4.3887\,[7_2]$	
	$+17.3045\,[7_3]$	$-0.6003\,[8_5]$	$+16.5172\,[8_{6+7}]$	$-41.6432\,[9_3]$	
	$+35.1870\,[9_4]$	$-26.6586\,[15_1]$	$+18.2841\,[15_2]$		$-115.270=0$
XV. (20)	$-5.7890\,[8_1]$	$+7.8064\,[8_2]$	$+7.8064\,[8_3]$	$+7.8064\,[8_4]$	
	$-57.0246\,[10_1]$	$+5.0910\,[10_{1+2}]$	$+5.0910\,[10_3]$	$+5.0910\,[10_4]$	
	$-42.6729\,[11_1]$	$-42.6729\,[11_2]$	$-42.6729\,[11_3]$	$-42.5320\,[11_4]$	$+28.443=0$
XVIII. (20)	$+13.5954\,[8_2]$	$+13.5954\,[8_3]$	$-11.7217\,[8_4]$	$+55.3220\,[11_1]$	
	$-0.1409\,[11_2]$	$-0.1409\,[11_3]$	$+43.8633\,[14_{4a}]$	$-8.4517\,[14_{4b}]$	$-41.468=0$
XXIII. (50)	$+109.5078\,[10_1]$	$-57.5742\,[10_{1+2}]$	$-57.5742\,[10_3]$	$+57.0890\,[11_1]$	
	$+57.0890\,[11_2]$	$+42.5320\,[11_3]$	$+42.5320\,[11_4]$	$+32.3738\,[12_2]$	
	$+32.3738\,[12_3]$	$-9.2090\,[12_4]$			$+8.165=0$
XXVI. (20)	$+3.1378\,[10_{1+2}]$	$+3.1378\,[10_3]$	$+17.1695\,[10_4]$	$-30.8941\,[11_3]$	
	$+10.2483\,[11_4]$	$-18.0304\,[12_3]$	$-18.9213\,[12_4]$		$-36.379=0$
XXVII. (10)	$+3.1378\,[10_{1+2}]$	$+8.7492\,[10_3]$	$+8.7492\,[10_4]$	$-8.1548\,[11_2]$	
	$-8.1548\,[11_3]$	$+10.2483\,[11_4]$	$+7.6447\,[14_3]$	$+9.0325\,[14_{4a}]$	
	$+9.0325\,[14_{4b}]$				$-15.027=0$
XXIX. (20)	$+19.5482\,[11_2]$	$+16.4685\,[11_3]$	$+16.4685\,[11_4]$	$+6.1022\,[12_1]$	
	$+6.1022\,[12_2]$	$+6.1022\,[12_3]$	$-25.5224\,[12_4]$	$-14.6993\,[14_{1+2}]$	
	$+47.9060\,[14_3]$				$-27.977=0$
XXXVII. (25)	$+29.3673\,[16_{5b}]$	$-49.0652\,[16_{6a}]$	$+27.2850\,[20_1]$	$+27.2850\,[20_2]$	
	$+14.8503\,[20_3]$	$-5.5327\,[22_1]$	$+14.6717\,[22_{4b}]$		$+26.223=0$
XXXIX. (30)	$+11.4886\,[19_1]$	$+17.3602\,[19_2]$	$-40.2070\,[20_1]$	$-27.2850\,[20_2]$	
	$-50.5077\,[21_{3a}]$	$+37.5392\,[21_{3b}]$			$+19.378=0$

NOTE.—In the solution for determining the general corrections each of the side-equations was divided by the number inclosed in parenthesis and placed opposite it.

ANGLE-EQUATIONS.

II.	$[1_2]$	$+[3_{1+2}]$	$+[4_{4+5}]$	$-[4_5]$						$-2.240=0$
III.	$[1_{1+2}]$	$+[2_2]$	$+[3_{1+2}]$	$-[3_2]$						$-1.322=0$
IV.	$[2_{1+2}]$	$-[2_2]$	$+[3_2]$	$+[4_{4+5}]$						$-2.595=0$
V.	$[3_3]$	$+[4_{3+4+5}]$	$-[4_{4+5}]$	$+[5_1]$						$-3.231=0$
VI.	$[4_{2+3+4+5}]$	$-[4_{3+4+5}]$	$+[5_{2+3}]$	$+[6_3]$						$+1.478=0$
VII.	$[5_3]$	$+[6_3]$	$+[6_4]$	$+[7_3]$						$-0.409=0$
IX.	$[5_4]$	$+[6_2]$	$+[15_1]$							$+2.920=0$
X.	$[6_1]$	$+[9_3]$	$+[15_2]$							$-1.989=0$
XI.	$-[6_1]$	$-[6_2]$	$-[6_3]$	$-[6_4]$	$-[6_5]$	$+[8_5]$	$+[9_4]$			$+0.753=0$
XII.	$[6_5]$	$+[7_1]$	$+[7_2]$	$+[8_{6+7}]$						$-1.782=0$
XIII.	$[8_1]$	$+[8_2]$	$+[8_3]$	$+[10_3]$	$+[10_4]$	$-[14_3]$	$-[14_{4a}]$	$-[14_{4b}]$		$-1.226=0$
XIV.	$[8_1]$	$+[8_2]$	$+[8_3]$	$+[8_1]$	$+[9_{5+6a+6b}]$	$-[10_1]$	$+[10_{1+2}]$	$+[10_3]$	$+[10_4]$	$-2.084=0$
XVI.	$[8_1]$	$+[8_2]$	$+[10_4]$	$+[12_3]$						$-1.155=0$
XVII.	$[8_2]$	$+[8_3]$	$+[8_4]$	$+[9_{5+6a+6b}]$	$+[9_7]$	$+[11_1]$	$+[11_2]$	$+[11_3]$		$-1.248=0$
XIX.	$[8_3]$	$+[8_4]$	$+[9_5]$	$+[12_2]$						$+0.107=0$
XX.	$[8_4]$	$+[9_{5+6a}]$	$+[14_{4b}]$							$+0.374=0$
XXI.	$[9_1]$	$+[16_2]$	$+[17_2]$							$-0.309=0$
XXII.	$[9_7]$	$+[10_1]$	$+[11_1]$	$+[11_2]$	$+[11_3]$	$+[11_4]$				$-1.436=0$
XXIV.	$[9_2]$	$+[15_3]$	$+[16_1]$							$+2.425=0$
XXV.	$[10_{1+2}]$	$+[10_3]$	$+[11_3]$	$+[11_4]$	$+[12_4]$					$-0.806=0$
XXVIII.	$[10_{1+2}]$	$+[11_2]$	$+[11_3]$	$+[11_4]$	$+[14_3]$					$-1.480=0$

Numerical equations of condition, &c.—Continued.

ANGLE-EQUATIONS—Continued.

XXX.	$[11_2] -[12_1] -[12_2] -[12_3] -[12_4] +[14_{1+2}] +[14_3]$	$-0.790=0$
XXXI.	$[12_6] +[13_1] +[14_1]$	$-0.517=0$
XXXII.	$[16_3] +[17_1] +[18_2]$	$+1.930=0$
XXXIII.	$[16_4] +[18_1] +[19_3]$	$+2.438=0$
XXXIV.	$[16_{5a}] +[16_{5b}] +[19_2] +[20_1]$	$+1.007=0$
XXXV.	$[16_{5a}] +[19_1] +[19_2] +[21_{3b}]$	$+0.210=0$
XXXVI.	$[16_{5b}] +[16_{6a}] +[21_2] +[21_{3a}] +[22_1] +[22_{4b}]$	$-1.418=0$
XXXVIII.	$[19_1] +[20_2] +[21_{3a}] +[21_{3b}]$	$-1.006=0$
XL.	$[20_3] +[21_2] +[22_1]$	$+0.311=0$
XLI.	$[21_1] +[22_2] +[23_3]$	$-1.423=0$
XLII.	$[22_3] +[23_2] +[24_1]$	$+2.083=0$
XLIII.	$[23_1] +[24_2] +[25_3]$	$-2.645=0$
XLIV.	$[24_3] +[25_2] +[26_1]$	$+1.983=0$
XLV.	$[25_1] +[26_2] +[27_3]$	$-1.274=0$
XLVI.	$[26_3] +[27_2] +[28_1]$	$+2.006=0$
XLVII.	$[27_1] +[28_2] +[29_3]$	$+0.337=0$
XLVIII.	$[28_3] +[29_2] +[30_1]$	$+0.269=0$

General corrections in terms of the correlates.

$[1_{1+2}]$	=+0.01087 I	+1.00000 III			
$[1_2]$	=+1.39876 I	+1.00000 II			
$[2_{1+2}]$	=−1.27795 I	+1.00000 IV			
$[2_2]$	=+2.39476 I	+1.00000 III	−1.00000 IV		
$[3_{1+2}]$	=+1.00000 II	+1.00000 III			
$[3_2]$	=−1.00000 III	+1.00000 IV			
$[3_3]$	=+1.00000 V				
$[4_{2+3+4+5}]$	=+1.00000 VI				
$[4_{3+4+5}]$	=+1.00000 V	−1.00000 VI			
$[4_{4+5}]$	=−1.05623 I	+1.00000 II	+1.00000 IV	−1.00000 V	
$[4_5]$	=+1.26933 I	−1.00000 II			
$[5_1]$	=+0.62500 V	−0.37500 VI	−0.04465 VIII	−0.12500 IX	
$[5_{2+3}]$	=−0.37500 V	+0.62500 VI	−0.04465 VIII	−0.12500 IX	
$[5_3]$	=+1.00000 VII	−0.74724 VIII			
$[5_4]$	=−0.12500 V	−0.12500 VI	+0.22326 VIII	+0.62500 IX	
$[6_1]$	=−0.10000 VI	−0.20000 VII	−0.05000 IX	+0.45000 X	−0.10000 XI
	−0.10000 XII				
$[6_2]$	=−0.10000 VI	−0.20000 VII	+0.45000 IX	−0.05000 X	−0.10000 XI
	−0.10000 XII				
$[6_3]$	=+0.80000 VI	+0.60000 VII	−0.10000 IX	−0.10000 X	−0.20000 XI
	−0.20000 XII				
$[6_4]$	=−0.20000 VI	+0.60000 VII	−0.10000 IX	−0.10000 X	−0.20000 XI
	−0.20000 XII				
$[6_5]$	=−0.20000 VI	−0.40000 VII	−0.10000 IX	−0.10000 X	−0.20000 XI
	+0.80000 XII				
$[7_1]$	=−0.07692 VII	−0.10032 VIII	+0.15385 XII		
$[7_{1+2}]$	=−0.15384 VII	−0.20064 VIII	+0.30770 XII		
$[7_2]$	=−0.07692 VII	−0.10032 VIII	+0.15385 XII		
$[7_3]$	=+0.57692 VII	+0.53293 VIII	−0.15385 XII		
$[8_1]$	=+0.00109 VIII	−0.03628 XI	+0.17124 XIII	+0.13744 XIV	−0.16058 XV
	+0.19078 XVI	−0.17771 XVII	−0.07802 XVIII	−0.05314 XIX	−0.03380 XX

General corrections in terms of the correlates—Continued.

$[8_{2}]$	=+0.00052 VIII	−0.01734 XI	+0.03287 XIII	+0.04609 XIV	+0.10267 XV
	+0.10621 XVI	+0.17066 XVII	+0.09928 XVIII	−0.06012 XIX	+0.01322 XX
$[8_{2+3}]$	=+0.00022 VIII	−0.00739 XI	+0.11642 XIII	+0.06062 XIV	+0.12149 XV
	+0.01353 XVI	+0.20453 XVII	+0.20969 XVIII	+0.04709 XIX	−0.05580 XX
$[8_{2+3+4}]$	=+0.00447 VIII	−0.14911 XI	+0.02682 XIII	+0.29405 XIV	+0.23558 XV
	−0.00705 XVI	+0.47176 XVII	−0.01756 XVIII	+0.30110 XIX	−0.26723 XX
$[8_{3}]$	=−0.00030 VIII	+0.00995 XI	+0.08355 XIII	+0.01453 XIV	+0.01882 XV
	−0.09268 XVI	+0.03387 XVII	+0.11041 XVIII	+0.10721 XIX	−0.06902 XX
$[8_{4}]$	=+0.00425 VIII	−0.14172 XI	−0.08960 XIII	+0.23343 XIV	+0.11409 XV
	−0.02058 XVI	+0.26723 XVII	−0.22725 XVIII	+0.25401 XIX	+0.32303 XX
$[8_{5}]$	=−0.00897 VIII	+0.29891 XI	−0.04367 XIII	−0.18539 XIV	−0.04771 XV
	−0.05362 XVI	−0.14911 XVII	+0.07803 XVIII	−0.13177 XIX	−0.14172 XX
$[8_{6}]$	=+0.00170 VIII	−0.05676 XI	−0.07719 XIII	−0.12304 XIV	−0.01364 XV
	−0.06496 XVI	−0.07246 XVII	+0.00879 XVIII	−0.05808 XIX	−0.04585 XX
$[8_{6+7}]$	=+0.41293 VIII	+0.50000 XII			
$[9_{1}]$	=+0.06289 VIII	−0.02208 X	+0.00962 XI	−0.05019 XIV	−0.08083 XVII
	−0.03194 XIX	+0.25706 XXI	−0.03064 XXII	−0.10250 XXIV	
$[9_{2}]$	=+0.39702 VIII	−0.26327 X	−0.08591 XI	−0.00545 XIV	−0.00878 XVII
	−0.00347 XIX	−0.10250 XXI	−0.00333 XXII	+0.46712 XXIV	
$[9_{3}]$	=−1.34975 VIII	+0.50835 X	−0.16556 XI	−0.02029 XIV	−0.03267 XVII
	−0.01291 XIX	−0.02208 XXI	−0.01238 XXII	−0.26327 XXIV	
$[9_{4}]$	=+0.92812 VIII	−0.16556 X	+0.33160 XI	−0.03169 XIV	−0.05104 XVII
	−0.02017 XIX	+0.00962 XXI	−0.01935 XXII	−0.08591 XXIV	
$[9_{5}]$	=−0.00860 VIII	−0.01291 X	−0.02017 XI	+0.21685 XIV	+0.16740 XVII
	+0.50164 XIX	−0.03194 XXI	−0.04945 XXII	−0.00347 XXIV	
$[9_{5+6a}]$	=+2.00000 XX				
$[9_{5+6a+6b}]$	=−0.01349 VIII	−0.02029 X	−0.03169 XI	+0.34077 XIV	+0.26306 XVII
	+0.21685 XIX	−0.05019 XXI	−0.07771 XXII	−0.00545 XXIV	
$[9_{7}]$	=−0.00826 VIII	−0.01238 X	−0.01935 XI	−0.07771 XIV	+0.30343 XVII
	−0.04945 XIX	−0.03064 XXI	+0.38114 XXII	−0.00333 XXIV	
$[10_{1}]$	=−0.50000 XIV	−1.85841 XV	−0.16667 XVI	+0.66667 XXII	+1.07629 XXIII
	+0.33333 XXV	−0.09078 XXVI	+0.05230 XXVII	+0.16667 XXVIII	
$[10_{1+2}]$	=−0.50000 XIII	−0.43278 XV	−0.16667 XVI	+0.16667 XXII	−0.01879 XXIII
	+0.33333 XXV	−0.09079 XXVI	−0.22827 XXVII	+0.66667 XXVIII	
$[10_{1+2+3}]$	=−0.85556 XV	−0.33333 XVI	+0.33333 XXII	−0.03758 XXIII	+0.66667 XXV
	−0.18158 XXVI	+0.10460 XXVII	+0.33333 XXVIII		
$[10_{1+2+3+4}]$	=+0.50000 XIII	+0.50000 XIV	−0.30550 XV	+0.33333 XVI	+0.16667 XXII
	−0.01879 XXIII	+0.33333 XXV	+0.33844 XXVI	+0.48976 XXVII	+0.16667 XXVIII
$[10_{3}]$	=+0.50000 XIII	−0.43278 XV	−0.16667 XVI	+0.16667 XXII	−0.01879 XXIII
	+0.33333 XXV	−0.09079 XXVI	+0.33287 XXVII	−0.33333 XXVIII	
$[10_{3+4}]$	=+1.00000 XIII	+0.50000 XIV	+0.12728 XV	+0.50000 XVI	+0.42923 XXVI
	+0.71803 XXVII	−0.50000 XXVIII			
$[10_{4}]$	=+0.50000 XIII	+0.50000 XIV	+0.56006 XV	+0.66667 XVI	−0.16667 XXII
	+0.01880 XXIII	−0.33333 XXV	+0.52002 XXVI	+0.38516 XXVII	−0.16667 XXVIII
$[11_{1}]$	=−0.42814 XV	+0.40000 XVII	+2.21570 XVIII	+0.20000 XXII	+0.34481 XXIII
	−0.40000 XXV	+0.20646 XXVI	+0.12123 XXVII	−0.60000 XXVIII	−0.52485 XXIX
	−0.20000 XXX				
$[11_{1+2+3}]$	=−1.28442 XV	+1.20000 XVII	+1.10082 XVIII	+0.60000 XXII	+0.74329 XXIII
	−0.20000 XXV	−0.92533 XXVI	−1.26727 XXVII	+0.20000 XXVIII	+0.22629 XXIX
	+0.40000 XXX				
$[11_{2}]$	=−0.42814 XV	+0.40000 XVII	−0.55741 XVIII	+0.20000 XXII	+0.34481 XXIII
	−0.40000 XXV	+0.20646 XXVI	−0.69425 XXVII	+0.40000 XXVIII	+0.45256 XXIX
	+0.80000 XXX				

General corrections in terms of the correlates—Continued.

$[11_3]$	=−0.42814 XV	+0.40000 XVII	−0.55744 XVIII	+0.20000 XXII	+0.05367 XXIII
	+0.60000 XXV	−1.33825 XXVI	−0.69425 XXVII	+0.40000 XXVIII	+0.29858 XXIX
	−0.20000 XXX				
$[11_4]$	=−0.42109 XV	−0.60000 XVII	−0.55039 XVIII	+0.20000 XXII	+0.05367 XXIII
	+0.60000 XXV	+0.71888 XXVI	+1.14606 XXVII	+0.40000 XXVIII	+0.29858 XXIX
	−0.20000 XXX				
$[12_1]$	=−0.20000 XVI	−0.20000 XIX	−0.22216 XXIII	−0.20000 XXV	+0.36952 XXVI
	+0.37727 XXIX	−0.20000 XXX			
$[12_{1+2+3}]$	=+0.40000 XVI	+0.40000 XIX	+0.62848 XXIII	−0.60000 XXV	+0.20704 XXVI
	+1.13181 XXIX	−0.60000 XXX			
$[12_2]$	=−0.20000 XVI	+0.80000 XIX	+0.42532 XXIII	−0.20000 XXV	+0.36952 XXVI
	+0.37727 XXIX	−0.20000 XXX			
$[12_{2+3}]$	=+0.60000 XVI	+0.60000 XIX	+0.85064 XXIII	−0.40000 XXV	−0.16248 XXVI
	+0.75454 XXIX	−0.40000 XXX			
$[12_3]$	=+0.80000 XVI	−0.20000 XIX	+0.42532 XXIII	−0.20000 XXV	−0.53200 XXVI
	+0.37727 XXIX	−0.20000 XXX			
$[12_4]$	=−0.20000 XVI	−0.20000 XIX	−0.40634 XXIII	+0.80000 XXV	−0.57655 XXVI
	−1.20396 XXIX	−0.20000 XXX			
$[12_6]$	=+1.00000 XXXI				
$[13_1]$	=+1.00000 XXXI				
$[14_1]$	=+1.00000 XXXI				
$[14_{1+2}]$	=+0.19424 XIII	−0.03183 XVIII	−0.01798 XX	−0.15348 XXVII	−0.15828 XXVIII
	−0.64350 XXIX	+0.20144 XXX			
$[14_3]$	=−0.16545 XIII	−0.10403 XVIII	−0.05876 XX	+0.11018 XXVII	+0.28297 XXVIII
	+0.79412 XXIX	+0.12469 XXX			
$[14_{4a}]$	=−0.10970 XIII	+1.45700 XVIII	−0.41577 XX	+0.10724 XXVII	−0.05876 XXVIII
	−0.12752 XXIX	−0.07674 XXX			
$[14_{4a+4b}]$	=−0.21940 XIII	+0.29828 XVIII	+0.16846 XX	+0.21448 XXVII	−0.11752 XXVIII
	−0.25504 XXIX	−0.15348 XXX			
$[14_{4b}]$	=−0.10970 XIII	−1.15872 XVIII	+0.58423 XX	+0.10724 XXVII	−0.05876 XXVIII
	−0.12752 XXIX	−0.07674 XXX			
$[15_1]$	=−0.53170 VIII	+0.32702 IX	−0.10480 X	−0.07702 XXIV	
$[15_2]$	=+0.43866 VIII	−0.10480 IX	+0.32702 X	−0.14520 XXIV	
$[15_3]$	=−0.03008 VIII	−0.07702 IX	−0.14520 X	+0.32702 XXIV	
$[16_1]$	=−0.10598 XXI	+0.49098 XXIV	−0.10598 XXXII	−0.07064 XXXIII	−0.03262 XXXIV
	−0.01631 XXXV	−0.05115 XXXVII			
$[16_2]$	=+0.73506 XXI	−0.10598 XXIV	−0.26494 XXXII	−0.17663 XXXIII	−0.08152 XXXIV
	−0.04076 XXXV	−0.12787 XXXVII			
$[16_3]$	=−0.26494 XXI	−0.10598 XXIV	+0.73506 XXXII	−0.17663 XXXIII	−0.08152 XXXIV
	−0.04076 XXXV	−0.12787 XXXVII			
$[16_4]$	=−0.17663 XXI	−0.07064 XXIV	−0.17663 XXXII	+0.54890 XXXIII	−0.05434 XXXIV
	−0.02717 XXXV	−0.08525 XXXVII			
$[16_{5a}]$	=−0.04076 XXI	−0.01631 XXIV	−0.04076 XXXII	−0.02717 XXXIII	+0.14130 XXXIV
	+0.57065 XXXV	−0.50000 XXXVI	−0.36570 XXXVII		
$[16_{5b}]$	=−0.04076 XXI	−0.01631 XXIV	−0.04076 XXXII	−0.02717 XXXIII	+0.14130 XXXIV
	−0.42935 XXXV	+0.50000 XXXVI	+0.80900 XXXVII		
$[16_{6a}]$	=+0.04076 XXI	+0.01631 XXIV	+0.04076 XXXII	+0.02717 XXXIII	−0.14130 XXXIV
	−0.07065 XXXV	+1.00000 XXXVI	−2.18425 XXXVII		
$[17_1]$	=−0.33333 XXI	+0.66667 XXXII			
$[17_2]$	=+0.66667 XXI	−0.33333 XXXII			
$[18_1]$	=−0.33333 XXXII	+0.66667 XXXIII			
$[18_2]$	=+0.66667 XXXII	−0.33333 XXXIII			

General corrections in terms of the correlates—Continued.

$[19_1]$	=−0.10000 XXXIII	−0.15000 XXXIV	+0.20000 XXXV	+0.35000 XXXVIII	+0.04723 XXXIX
$[19_2]$	=−0.10000 XXXIII	+0.35000 XXXIV	+0.20000 XXXV	−0.15000 XXXVIII	+0.14509 XXXIX
$[19_3]$	=+0.60000 XXXIII	−0.10000 XXXIV	−0.20000 XXXV	−0.10000 XXXVIII	−0.09617 XXXIX
$[20_1]$	=+0.37500 XXXIV	+0.19861 XXXVII	−0.12500 XXXVIII	−0.38890 XXXIX	−0.12500 XL
$[20_{1+2}]$	=+0.25000 XXXIV	+0.39722 XXXVII	+0.25000 XXXVIII	−0.56243 XXXIX	−0.25000 XL
$[20_2]$	=−0.12500 XXXIV	+0.19861 XXXVII	+0.37500 XXXVIII	−0.17353 XXXIX	−0.12500 XL
$[20_3]$	=−0.12500 XXXIV	−0.05009 XXXVII	−0.12500 XXXVIII	+0.28122 XXXIX	+0.37500 XL
$[21_1]$	=−0.04762 XXXV	−0.12699 XXXVI	−0.09524 XXXVIII	+0.02058 XXXIX	−0.07937 XL
	+0.25397 XLI				
$[21_2]$	=−0.04762 XXXV	+0.20635 XXXVI	−0.09524 XXXVIII	+0.02058 XXXIX	+0.25397 XL
	−0.07937 XLI				
$[21_{3a}]$	=−0.42857 XXXV	+0.52381 XXXVI	+0.14286 XXXVIII	−1.49833 XXXIX	−0.04762 XL
	−0.04762 XLI				
$[21_{3b}]$	=+0.57143 XXXV	−0.47619 XXXVI	+0.14286 XXXVIII	+1.43657 XXXIX	−0.04762 XL
	−0.04762 XLI				
$[22_1]$	=+0.13427 XXXVI	−0.17790 XXXVII	+0.31763 XL	−0.09920 XLI	−0.10922 XLII
$[22_2]$	=−0.13026 XXXVI	+0.00372 XXXVII	−0.09920 XL	+0.24550 XLI	−0.07315 XLII
$[22_3]$	=−0.00201 XXXVI	+0.08709 XXXVII	−0.10922 XL	−0.07315 XLI	+0.59118 XLII
$[22_{4b}]$	=+0.54710 XXXVI	+0.46926 XXXVII	−0.18336 XL	−0.3106 XLI	+0.10721 XLII
$[23_1]$	=−0.08333 XLI	−0.19444 XLII	+0.47222 XLIII		
$[23_2]$	=−0.08333 XLI	+0.47222 XLII	−0.19444 XLIII		
$[23_3]$	=+0.25000 XLI	−0.08333 XLII	−0.08333 XLIII		
$[24_1]$	=+0.50000 XLII	−0.16667 XLIII	−0.16667 XLIV		
$[24_2]$	=−0.16667 XLII	+0.50000 XLIII	−0.16667 XLIV		
$[24_3]$	=−0.16667 XLII	−0.16667 XLIII	+0.50000 XLIV		
$[25_1]$	=−0.16667 XLIII	−0.33333 XLIV	+0.66667 XLV		
$[25_2]$	=−0.16667 XLIII	+0.66667 XLIV	−0.33333 XLV		
$[25_3]$	=+0.66667 XLIII	−0.16667 XLIV	−0.16667 XLV		
$[26_1]$	=+0.75000 XLIV	−0.25000 XLV	−0.25000 XLVI		
$[26_2]$	=−0.25000 XLIV	+0.75000 XLV	−0.25000 XLVI		
$[26_3]$	=−0.25000 XLIV	−0.25000 XLV	+0.75000 XLVI		
$[27_1]$	=−0.20000 XLV	−0.13333 XLVI	+0.53333 XLVII		
$[27_2]$	=−0.20000 XLV	+0.53333 XLVI	−0.13333 XLVII		
$[27_3]$	=+0.70000 XLV	−0.20000 XLVI	−0.20000 XLVII		
$[28_1]$	=+0.29788 XLVI	−0.08511 XLVII	−0.08511 XLVIII		
$[28_2]$	=−0.08511 XLVI	+0.29574 XLVII	−0.10426 XLVIII		
$[28_3]$	=−0.08511 XLVI	−0.10426 XLVII	+0.29574 XLVIII		
$[29_2]$	=+0.27727 XLVIII				
$[29_3]$	=+0.54545 XLVII				
$[30_1]$	=+0.75000 XLVIII				

Normal equations for determining the correlates.

No. of equation.						
1.	0=+0.49870	+12.05150 I	−0.92680 II	+2.40563 III	−4.72894 IV	
		+ 1.05623 V				
2.	0=−2.24000	− 0.92680 I	+4.00000 II	+1.00000 III	+1.00000 IV	
		− 1.00000 V				
3.	0=−1.32200	+ 2.40563 I	+1.00000 II	+4.00000 III	−2.00000 IV	
4.	0=−2.59500	− 4.72894 I	+1.00000 II	−2.00000 III	+4.00000 IV	
		− 1.00000 V				
5.	0=−3.23100	+ 1.05623 I	−1.00000 II	−1.00000 IV	+3.62500 V	
		− 1.37500 VI	−0.04465 VIII	−0.12500 IX		

Normal equations for determining the correlates—Continued.

No. of equation.					
6.	0=+1.47800	−1.37500 V	+3.42500 VI	+0.60000 VII	− 0.04465 VIII
		−0.22500 IX	−0.10000 X	−0.20000 XI	− 0.20000 XII
7.	0=−0.40900	+0.60000 VI	+2.77692 VII	−0.21431 VIII	− 0.20000 IX
		−0.20000 X	−0.40000 XI	−0.55385 XII	
8.	0=−5.76345	−0.04465 V	−0.04465 VI	−0.21431 VII	+ 7.03756 VIII
		−0.30844 IX	−0.91109 X	+0.91915 XI	+ 0.21229 XII
		+0.00131 XIII	−0.00793 XIV	+0.00143 XV	+ 0.00161 XVI
		−0.01728 XVII	−0.00234 XVIII	−0.00465 XIX	+ 0.00425 XX
		+0.06289 XXI	−0.00826 XXII	+0.36694 XXIV	
9.	0=+2.92000	−0.12500 V	−0.22500 VI	−0.20000 VII	− 0.30844 VIII
		+1.40202 IX	−0.15480 X	−0.10000 XI	− 0.10000 XII
		−0.07702 XXIV			
10.	0=−1.98900	−0.10000 VI	−0.20000 VII	−0.91109 VIII	− 0.15480 IX
		+1.28537 X	−0.26556 XI	−0.10000 XII	− 0.02029 XIV
		−0.03267 XVII	−0.01291 XIX	−0.02208 XXI	− 0.01238 XXII
		−0.40847 XXIV			
11.	0=+0.75300	−0.20000 VI	−0.40000 VII	+0.91915 VIII	− 0.10000 IX
		−0.26556 X	+1.43051 XI	−0.20000 XII	− 0.04367 XIII
		−0.21708 XIV	−0.04771 XV	−0.05362 XVI	− 0.20015 XVII
		+0.07803 XVIII	−0.15194 XIX	−0.14172 XX	+ 0.00962 XXI
		−0.01935 XXII	−0.08591 XXIV		
12.	0=−1.78200	−0.20000 VI	−0.55385 VII	+0.21229 VIII	− 0.10000 IX
		−0.10000 X	−0.20000 XI	+1.60770 XII	
13.	0=−1.22600	+0.00131 VIII	−0.04367 XI	+1.67251 XIII	+ 0.69806 XIV
		+0.08819 XV	+0.70411 XVI	+0.02682 XVII	− 0.06278 XVIII
		−0.00605 XIX	−0.19930 XX	+0.42923 XXVI	+ 0.39337 XXVII
		−0.66545 XXVIII	−0.53908 XXIX	+0.02879 XXX	
14.	0=−2.08400	−0.00793 VIII	−0.02029 X	−0.21708 XI	+ 0.69806 XIII
		+1.77226 XIV	+1.62791 XV	+0.68353 XVI	+ 0.55711 XVII
		−0.09558 XVIII	+0.46481 XIX	+0.23343 XX	− 0.05019 XXI
		−0.57771 XXII	−1.09508 XXIII	−0.00545 XXIV	+ 0.42922 XXVI
		+0.43746 XXVII			
15.	0=+1.42215	+0.00143 VIII	−0.04771 XI	+0.08819 XIII	+ 1.62791 XIV
		+8.99543 XV	+0.50215 XVI	−1.04884 XVII	− 1.16257 XVIII
		+0.13291 XIX	+0.11409 XX	−3.56392 XXII	− 4.77359 XXIII
		−1.71479 XXV	+0.79055 XXVI	+0.24225 XXVII	− 1.71015 XXVIII
		−1.11778 XXIX	−0.42814 XXX		
16.	0=−1.15500	+0.00161 VIII	−0.05362 XI	+0.70411 XIII	+ 0.68353 XIV
		+0.50215 XV	+1.76346 XVI	−0.00705 XVII	+ 0.02126 XVIII
		−0.31326 XIX	−0.02058 XX	−0.16667 XXII	+ 0.41412 XXIII
		−0.53333 XXV	−0.01198 XXVI	+0.38516 XXVII	− 0.16667 XXVIII
		+0.37727 XXIX	−0.20000 XXX		
17.	0=−1.24800	−0.01728 VIII	−0.03267 X	−0.20015 XI	+ 0.02682 XIII
		+0.55711 XIV	−1.04884 XV	−0.00705 XVI	+ 2.23825 XVII
		+1.08326 XVIII	+0.46850 XIX	+0.26723 XX	− 0.08083 XXI
		+0.90343 XXII	+0.74329 XXIII	−0.00878 XXIV	− 0.20000 XXV
		−0.92533 XXVI	−1.26727 XXVII	+0.20000 XXVIII	+ 0.22629 XXIX
		+0.40000 XXX			
18.	0=−2.07340	−0.00234 VIII	+0.07803 XI	+0.06258 XIII	+ 0.09558 XIV
		−1.16257 XV	+0.02126 XVI	+1.08326 XVII	+10.09750 XVIII
		−0.11684 XIX	−1.38597 XX	+0.55043 XXII	+ 0.95097 XXIII
		−1.10783 XXV	+0.57907 XXVI	+0.53499 XXVII	− 1.76930 XXVIII
		−1.68286 XXIX	−0.69330 XXX		

Normal equations for determining the correlates—Continued.

No. of equation.					
19.	0=+0.10700	−0.00465 VIII	−0.01291 X	−0.15194 XI	−0.00605 XIII
		+0.46481 XIV	+0.13291 XV	−0.31326 XVI	+0.46850 XVII
		−0.11684 XVIII	+1.66286 XIX	+0.25401 XX	−0.03194 XXI
		−0.04945 XXII	+0.42532 XXIII	−0.00347 XXIV	−0.20000 XXV
		+0.36952 XXVI	+0.37727 XXIX	−0.20000 XXX	
20.	0=+0.37400	+0.00425 VIII	−0.14172 XI	−0.19930 XIII	+0.23343 XIV
		+0.11409 XV	−0.02058 XVI	+0.26723 XVII	−1.38597 XVIII
		+0.25401 XIX	+2.90726 XX	+0.10724 XXVII	−0.05876 XXVIII
		−0.12752 XXIX	−0.07674 XXX		
21.	0=−0.30900	+0.06289 VIII	−0.02208 X	+0.00962 XI	0.05019 XIV
		−0.08083 XVII	−0.03194 XIX	+1.65879 XXI	−0.03064 XXII
		−0.20848 XXIV	−0.59827 XXXII	−0.17663 XXXIII	−0.08152 XXXIV
		−0.04076 XXXV	−0.12787 XXXVII		
22.	0=−1.43600	−0.00826 VIII	−0.01238 X	−0.01935 XI	−0.57771 XIV
		−3.56392 XV	−0.16667 XVI	+0.90343 XVII	+0.55043 XVIII
		−0.04945 XIX	−0.03064 XXI	+1.84781 XXII	+1.87325 XXIII
		−0.00333 XXIV	+0.73333 XXV	−0.29723 XXVI	−0.06891 XXVII
		+0.76667 XXVIII	+0.52487 XXIX	+0.20000 XXX	
23.	0=+0.16330	−1.09508 XIV	−4.77359 XV	+0.44412 XVI	+0.74329 XVII
		+0.95097 XVIII	+0.42532 XIX	+1.87325 XXII	+3.90483 XXIII
		−0.33658 XXV	−0.04414 XXVI	−0.27582 XXVII	+0.43336 XXVIII
		+1.13573 XXIX	+0.12267 XXX		
24.	0=+2.42500	+0.36694 VIII	−0.07702 IX	−0.40847 X	−0.08591 XI
		−0.00545 XIV	−0.00878 XVII	−0.00347 XIX	−0.20848 XXI
		−0.00333 XXII	+1.28512 XXIV	−0.10598 XXXII	−0.07064 XXXIII
		−0.03262 XXXIV	−0.01631 XXXV	−0.05115 XXXVII	
25.	0=−0.80600	−1.71479 XV	−0.53333 XVI	−0.20000 XVII	−1.10783 XVIII
		−0.20000 XIX	+0.73333 XXII	−0.33658 XXIII	+2.66667 XXV
		−1.37750 XXVI	+0.55641 XXVII	+1.13333 XXVIII	−0.60680 XXIX
		−0.60000 XXX			
26.	0=−1.81890	+0.42923 XIII	+0.42922 XIV	+0.79055 XV	−0.01198 XVI
		−0.92533 XVII	+0.57907 XVIII	+0.36952 XIX	−0.29723 XXII
		−0.04414 XXIII	−1.37750 XXV	+3.87859 XXVI	+2.00673 XXVII
		−0.50370 XXVIII	+0.49069 XXIX	+0.57597 XXX	
27.	0=−1.50270	+0.39337 XIII	+0.43746 XIV	+0.24225 XV	+0.38516 XVI
		−1.26727 XVII	+0.53499 XVIII	+0.10724 XX	−0.06891 XXII
		−0.27582 XXIII	+0.55641 XXV	+2.00673 XXVI	+3.14136 XXVII
		−0.36053 XXVIII	+0.07017 XXIX	−0.73755 XXX	
28.	0=−1.48000	−0.66545 XIII	−1.71015 XV	−0.16667 XVI	+0.20000 XVII
		−1.76930 XVIII	−0.05876 XX	+0.76667 XXII	+0.43336 XXIII
		+1.13333 XXV	−0.50370 XXVI	−0.36053 XXVII	+2.14964 XXVIII
		+1.84384 XXIX	+0.52469 XXX		
29.	0=−1.39885	−0.53908 XIII	−1.11778 XV	+0.37727 XVI	+0.22629 XVII
		−1.68286 XVIII	+0.37727 XIX	−0.12752 XX	+0.52487 XXII
		+1.13573 XXIII	−0.60680 XXV	+0.49069 XXVI	+0.07017 XXVII
		+1.84384 XXVIII	+5.19089 XXIX	+0.67533 XXX	
30.	0=−0.79000	+0.02879 XIII	−0.42814 XV	−0.20000 XVI	+0.40000 XVII
		−0.69330 XVIII	−0.20000 XIX	−0.07674 XX	+0.20000 XXII
		+0.12267 XXIII	−0.60000 XXV	+0.57597 XXVI	−0.73755 XXVII
		+0.52469 XXVIII	+0.67533 XXIX	+1.92613 XXX	
31.	0=−0.51700	+3.00000 XXXI			
32.	0=+1.93000	−0.59827 XXI	−0.10598 XXIV	+2.06840 XXXII	−0.50996 XXXIII
		−0.08152 XXXIV	−0.04076 XXXV	−0.12787 XXXVII	

Normal equations for determining the correlates—Continued.

No. of equation.					
33.	0=+2.43800	−0.17663 XXI	−0.07064 XXIV	−0.50996 XXXII	+1.81557 XXXIII
		−0.15434 XXXIV	−0.22717 XXXV	−0.08525 XXXVII	−0.10000 XXXVIII
		−0.09617 XXXIX			
34.	0=+1.00700	−0.08152 XXI	−0.03262 XXIV	−0.08152 XXXII	−0.15434 XXXIII
		+1.00760 XXXIV	+0.34130 XXXV	+0.64191 XXXVII	−0.27500 XXXVIII
		−0.24381 XXXIX	−0.12500 XL		
35.	0=+0.21000	−0.04076 XXI	−0.01631 XXIV	−0.04076 XXXII	−0.22717 XXXIII
		+0.34130 XXXIV	+1.54208 XXXV	−0.97619 XXXVI	−0.36570 XXXVII
		+0.34286 XXXVIII	+1.62889 XXXIX	−0.04762 XL	−0.04762 XLI
36.	0=−1.41800	−0.97619 XXXV	+2.91153 XXXVI	−1.08389 XXXVII	+0.04762 XXXVIII
		−1.47775 XXXIX	+0.34062 XL	−0.25725 XLI	−0.00201 XLII
37.	0=+1.04892	−0.12787 XXI	−0.05115 XXIV	−0.12787 XXXII	−0.08525 XXXIII
		+0.64191 XXXIV	−0.36570 XXXV	−1.08389 XXXVI	+5.95568 XXXVII
		+0.19861 XXXVIII	−0.44679 XXXIX	−0.22799 XL	+0.00372 XLI
		+0.08709 XLII			
38.	0=−1.00600	−0.10000 XXXIII	−0.27500 XXXIV	+0.34286 XXXV	+0.04762 XXXVI
		+0.19861 XXXVII	+1.01072 XXXVIII	−0.18806 XXXIX	−0.22024 XL
		−0.09524 XLI			
39.	0=+0.64600	−0.09617 XXXIII	−0.24381 XXXIV	+1.62889 XXXV	−1.47775 XXXVI
		−0.44679 XXXVII	−0.18806 XXXVIII	+5.10126 XXXIX	+0.30180 XL
		+0.02058 XLI			
40.	0=+0.31100	−0.12500 XXXIV	−0.04762 XXXV	+0.34062 XXXVI	−0.22799 XXXVII
		−0.22024 XXXVIII	+0.30180 XXXIX	+0.94660 XL	−0.17857 XLI
		−0.10922 XLII			
41.	0=−1.42300	−0.04762 XXXV	−0.25725 XXXVI	+0.00372 XXXVII	−0.09524 XXXVIII
		+0.02058 XXXIX	−0.17857 XL	+0.74947 XLI	−0.15648 XLII
		−0.08333 XLIII			
42.	0=+2.08300	−0.00201 XXXVI	+0.08709 XXXVII	−0.10922 XL	−0.15648 XLI
		+1.56340 XLII	−0.36111 XLIII	−0.16667 XLIV	
43.	0=−2.64500	−0.08333 XLI	−0.36111 XLII	+1.63889 XLIII	−0.33333 XLIV
		−0.16667 XLV			
44.	0=+1.98300	−0.16667 XLII	−0.33333 XLIII	+1.91667 XLIV	−0.58333 XLV
		−0.25000 XLVI			
45.	0=−1.27400	−0.16667 XLIII	−0.58333 XLIV	+2.11667 XLV	−0.45000 XLVI
		−0.20000 XLVII			
46.	0=+2.00600	−0.25000 XLIV	−0.45000 XLV	+1.58121 XLVI	−0.21844 XLVII
		−0.08511 XLVIII			
47.	0=+0.33700	−0.20000 XLV	−0.21844 XLVI	+1.37452 XLVII	−0.10426 XLVIII
48.	0=+0.26900	−0.08511 XLVI	−0.10426 XLVII	+1.77301 XLVIII	

Values of the correlates and their logarithms.

I	=+0.4062	log 9.6087185+	XXV	=+1.4399	log 0.1583444+
II	=+0.2814	log 9.4493858+	XXVI	=+1.5074	log 0.1782314+
III	=+0.9463	log 9.9760197+	XXVII	=−0.8934	log 9.9510411−
IV	=+1.8608	log 0.2697090+	XXVIII	=−0.0834	log 8.9212181−
V	=+1.3164	log 0.1193945+	XXIX	=+0.4205	log 9.6237660+
VI	=−0.0107	log 8.0301948−	XXX	=+0.1777	log 9.2496141+
VII	=+0.2413	log 9.3825213+	XXXI	=+0.1723	log 9.2363609+
VIII	=+1.1743	log 0.0697939+	XXXII	=−1.9971	log 0.3003976−
IX	=−1.6794	log 0.2251515−	XXXIII	=−2.2044	log 0.3432806−
X	=+1.2787	log 0.1067755+	XXXIV	=−2.4332	log 0.3861850−
XI	=−1.0161	log 0.0069536−	XXXV	=+1.1281	log 0.0523322+
XII	=+0.8838	log 9.9463540+	XXXVI	=+1.0468	log 0.0198844+
XIII	=+0.0123	log 8.0899051+	XXXVII	=+0.2034	log 9.3084150+
XIV	=−0.3760	log 9.5752340−	XXXVIII	=−0.3553	log 9.5506441−
XV	=−0.2722	log 9.4348403−	XXXIX	=−0.3079	log 9.4883815−
XVI	=+1.9104	log 0.2811175+	XL	=−0.6287	log 9.7984573−
XVII	=+0.3982	log 9.6001340+	XLI	=+2.0764	log 0.3173068+
XVIII	=+0.4117	log 9.6146020+	XLII	=−0.9979	log 9.9990827−
XIX	=+0.3747	log 9.5737185+	XLIII	=+1.2825	log 0.1080506+
XX	=+0.0567	log 8.7535065+	XLIV	=−1.0850	log 0.0354457−
XXI	=−1.1471	log 0.0595899−	XLV	=+0.0350	log 8.5438198+
XXII	=+0.9359	log 9.9712434+	XLVI	=−1.5129	log 0.1798217−
XXIII	=−1.4037	log 0.1472898−	XLVII	=−0.4998	log 9.6987702−
XXIV	=−2.4911	log 0.3963929−	XLVIII	=−0.2537	log 9.4043798−

Values of the general corrections.

	″		″		″		″		″
$[1_{1+2}]$	=+0.951	$[7_3]$	=+0.629	$[11_3]$	=−0.318	$[16_{6a}]$	=+0.638	$[24_1]$	=−0.532
$[1_2]$	=+0.850	$[8_1]$	=+0.272	$[11_4]$	=+0.741	$[17_1]$	=−0.949	$[24_2]$	=+0.988
$[2_{1+2}]$	=+1.342	$[8_2]$	=+0.263	$[12_1]$	=+0.247	$[17_2]$	=−0.099	$[24_3]$	=−0.590
$[2_2]$	=+0.058	$[8_3]$	=−0.102	$[12_2]$	=−0.287	$[18_1]$	=−0.804	$[25_1]$	=+0.171
$[3_{1+2}]$	=+1.228	$[8_4]$	=+0.116	$[12_3]$	=−0.111	$[18_2]$	=−0.597	$[25_2]$	=−0.949
$[3_2]$	=+0.915	$[8_5]$	=−0.419	$[12_4]$	=−0.146	$[19_1]$	=+0.672	$[25_3]$	=+1.030
$[3_3]$	=+1.316	$[8_6]$	=−0.065	$[12_6]$	=+0.172	$[19_2]$	=−0.397	$[26_1]$	=−0.444
$[4_{2+3+4+5}]$	=−0.011	$[8_{6+7}]$	=+0.927	$[13_1]$	=+0.172	$[19_3]$	=−1.240	$[26_2]$	=+0.676
$[4_{3+4+5}]$	=+1.327	$[9_1]$	=−0.058	$[14_1]$	=+0.172	$[20_1]$	=−0.629	$[26_3]$	=−0.872
$[4_{4+5}]$	=+0.397	$[9_2]$	=−0.835	$[14_{1+2}]$	=−0.096	$[20_2]$	=+0.343	$[27_1]$	=−0.072
$[4_5]$	=+0.234	$[9_3]$	=−0.107	$[14_3]$	=+0.186	$[20_3]$	=+0.016	$[27_2]$	=−0.747
$[5_1]$	=+0.984	$[9_4]$	=+0.710	$[14_{4a}]$	=+0.417	$[21_1]$	=+0.418	$[27_3]$	=+0.427
$[5_{2+3}]$	=−0.343	$[9_5]$	=+0.166	$[14_{4b}]$	=−0.603	$[21_2]$	=−0.135	$[28_1]$	=−0.387
$[5_3]$	=−0.636	$[9_{5+6a}]$	=+0.113	$[15_1]$	=−1.116	$[21_{3a}]$	=+0.407	$[28_2]$	=+0.007
$[5_4]$	=−0.951	$[9_{5+6a+6b}]$	=+0.047	$[15_2]$	=+1.471	$[21_{3b}]$	=−0.416	$[28_3]$	=+0.106
$[6_1]$	=+0.625	$[9_7]$	=+0.526	$[15_3]$	=−0.906	$[22_1]$	=−0.192	$[29_2]$	=−0.185
$[6_2]$	=−0.854	$[10_1]$	=−0.229	$[16_1]$	=−0.684	$[22_2]$	=+0.510	$[29_3]$	=−0.273
$[6_3]$	=+0.203	$[10_{1+2}]$	=+0.467	$[16_2]$	=+0.466	$[22_3]$	=−0.658	$[30_1]$	=−0.190
$[6_4]$	=+0.213	$[10_3]$	=+0.061	$[16_3]$	=−0.384	$[22_{4b}]$	=+0.612		
$[6_5]$	=+0.856	$[10_4]$	=+0.731	$[16_4]$	=−0.394	$[23_1]$	=+0.627		
$[7_1]$	= 0.000	$[11_1]$	=+0.312	$[16_{5a}]$	=−0.069	$[23_2]$	=−0.894		
$[7_2]$	= 0.000	$[11_2]$	=+0.404	$[16_{5b}]$	=+0.089	$[23_3]$	=+0.495		

Residuals resulting from substitution of general corrections in numerical equations of condition.

No. of equation.	Residual.	No. of equation.	Residual.	No. of equation.	Residual.
1	-0.0003	17	—0.0001	33	0.0000
2	0.0000	18	—0.0048	34	0.0000
3	0.0000	19	0.0000	35	+0.0001
4	0.0000	20	0.0000	36	0.0000
5	0.0000	21	0.0000	37	—0.0010
6	0.0000	22	0.0000	38	+0.0001
7	0.0000	23	—0.0020	39	—0.0003
8	0.0000	24	+0.0001	40	0.0000
9	0.0000	25	0.0000	41	0.0000
10	0.0000	26	+0.0010	42	+0.0001
11	+0.0001	27	+0.0009	43	0.0000
12	0.0000	28	0.0000	44	0.0000
13	0.0000	29	+0.0008	45	0.0000
14	0.0000	30	—0.0001	46	0.0000
15	+0.0004	31	0.0000	47	—0.0001
16	0.0000	32	0.0000	48	0.0000

PROBABLE ERRORS OF OBSERVED AND ADJUSTED ANGLES.

§ **5.** Let

m=whole number of observed angles in a section (one adjustment).
r=whole number of rigid conditions in a section.
n=number of triangles in principal chain.
$[pvv]$=sum of weighted squares of corrections to observed angles.
ρ_1=probable error of an observed angle of weight unity.
ρ_s=probable error of an observed angle of average weight in whole section.
ρ_s'=probable error of an adjusted angle of average weight in whole section.
p_s=average weight of an observed angle in whole section.
p_c=average weight of an observed angle in principal chain.
ρ_c=probable error of an observed angle of average weight in principal chain.
ρ_c'=probable error of an adjusted angle of average weight in principal chain.
$[vv]$=sum of squares of closing errors of triangles in principal chain.
ρ_t=probable error of an observed angle in principal chain as derived from closing errors of triangles.

Proceeding as in Chapter XIV, C, § 8, there are found the following values:

FOR THE ENTIRE SECTION OF THIS CHAPTER.

Section.	Extent of section.	m	r	$[pvv]$	ρ_1	p_s	ρ_s	$\sqrt{\frac{m-r}{m}}$	ρ_s'
XIII	Falkirk-Pekin to Carlton-Sir John	170	112	90.61	″ 0.61	1.22	″ 0.55	0.58	″ 0.32

FOR THE PRINCIPAL CHAIN CONNECTING THE BUFFALO AND SANDY CREEK BASES, GIVEN IN D, § 6, FOLLOWING.

Section.	Extent of principal chain in each section.	p_c	ρ_c	ρ_c'	From closing errors of triangles.				
					$[vv]$	n	ρ_t	Average error.	Greatest error.
XII	Buffalo Base to Falkirk-Pekin	0.87	″ 0.52	″ 0.30	8.00	7	″ 0.42	″ 0.83	″ 2.13
XIII	Falkirk-Pekin to Sandy Creek Base	1.46	0.50	0.29	45.36	22	0.56	1.21	3.42
	Entire principal chain	......	......	......	53.36	29	0.53	1.11	3.42

D.—PRINCIPAL CHAIN OF TRIANGLES BETWEEN BUFFALO AND SANDY CREEK BASES.

§ **6.** In adjusting the sides of the principal triangles joining Buffalo and Sandy Creek Bases, the bases have been considered exact. The probable error of an observed angle of average weight in that part of this chain between Buffalo Base and the line Pekin–Falkirk is $\pm 0''.52$ (Chapter XIX, C, § 5), and for the remainder of the chain the corresponding probable error is $\pm 0''.50$ (Chapter XIX, C, § 5). The logarithm of the measured length of Sandy Creek Base with the English foot as unit is 4.2057301. As computed from Buffalo Base through the intervening triangulation, the logarithm of Sandy Creek base is 4.2057324.

The difference of these two logarithms gives, in the notation of Chapter XIV, D, § 10, $d = -23$. With the above probable errors, and the values of $(\alpha^2+\beta^2)$ for the several triangles given in the following tables, the constant

$$\frac{1}{p} + \frac{1}{p'} = \Sigma(\alpha^2+\beta^2)\rho^2 = 4249$$

These quantities and the values of $\frac{1}{p}$ supplied by the tables readily give the corrections to the logarithms of the sides computed from Buffalo Base. The line of least weight in the chain is Pinnacle Hill–Turk's Hill, for which $\frac{1}{p} = 2123$ and $\frac{1}{p'} = 2126$, giving for the probable error of the logarithm of its length ± 32.6 in units of the seventh decimal place. This probable error, which is independent of the probable errors of the bases, corresponds to $\frac{1}{133250}$ of the line's length, a fraction about seven times as great as the ratio of the probable error of either base to its length.

Principal chain of triangles between Buffalo and Sandy Creek Bases.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	α^2 and β^2	$\Sigma(\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Tonawanda	68 41 34.598		4.3472757	67.24	....	....	4.3472757
East Base	44 35 02.187	−0.403	4.2243328	....	....	....	4.2243328
West Base	66 43 23.294		4.3411538	82.81	150.05	40	4.3411538
Buffalo Plains	54 39 47.777		4.3411538	222.01	....	....	4.3411538
Tonawanda	32 36 50.514	+0.463	4.1611580	....	....	....	4.1611579
East Base	92 43 21.785		4.4290972	1.00	373.06	100	4.4290971
Buffalo	38 16 06.730		4.4290972	712.89	....	....	4.4290971
Tonawanda	41 36 53.978	−2.126	4.4594103	....	....	....	4.4594101
Buffalo Plains	100 06 59.469		4.6303575	14.44	1100.39	294	4.6303573
Ridgeway	41 27 59.899		4.6303575	566.44	....	....	4.6303573
Tonawanda	49 07 11.079	−0.507	4.6879462	....	....	....	4.6879460
Buffalo	89 24 49.513		4.8093565	0.04	1666.87	445	4.8093563
Drummondville	52 38 14.801		4.8093565	256.00	....	....	4.8093563
Tonawanda	75 12 23.540	+0.417	4.8944530	....	....	....	4.8944527
Ridgeway	52 09 22.600		4.8065475	265.69	2188.56	584	4.8065472
Pekin	59 58 20.058		4.8065475	148.84	....	....	4.8065472
Tonawanda	63 09 08.315	+0.270	4.8196056	....	....	....	4.8196052
Drummondville	56 52 32.461		4.7921164	190.44	2527.84	675	4.7921160
Falkirk	31 23 11.230		4.7921164	1190.25	....	....	4.7921160
Pekin	66 25 49.830	+1.606	5.0376086	....	....	....	5.0376081
Tonawanda	82 11 00.518		5.0713865	8.41	3726.50	995	5.0713860

Principal chain of triangles between Buffalo and Sandy Creek Bases—Continued.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	α^2 and β^2	$\Sigma\ (\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Gasport	97 22 23. 194		5. 0713865	7. 29			5 6713866
Falkirk	48 49 46. 787	—0. 179	4. 9516456				4 9516449
Pekin	33 47 51. 401		4. 8202694	985. 96	993. 25	1245	4 8262687
Batavia	46 29 36. 726		4. 8202694	616. 09			4. 8202687
Gasport	35 16 09. 044	—6. 233	4. 7692878				4. 7692870
Falkirk	104 14 21. 123		4. 9942461	28. 69	1631. 43	1466	4. 9942453
Albion	62 03 57. 359		4. 9942461	123. 21			4. 9942453
Batavia	68 48 38. 879	—1. 510	5. 0176445				5. 0176437
Gasport	49 07 25. 593		4. 9266391	331. 24	2085. 88	1521	4. 9266383
Morganville	67 24 57. 368		4. 9266391	77. 44			4. 9266383
Albion	38 38 26. 215	+1. 843	4. 7567579				4. 7567571
Batavia	73 56 43. 509		4. 9440111	36. 60	2199. 32	1549	4. 9440103
Brockport	81 28 42. 295		4. 9440111	9. 61			4. 9440163
Morganville	43 47 49. 749	—1. 459	4. 7896051				4. 7896642
Albion	54 43 28. 996		4. 8607279	222. 01	2430. 94	1668	4. 8607270
Scottsville	46 45 36. 191		4. 8607279	392. 04			4. 8607270
Brockport	65 53 18. 416	+1. 949	4. 9586568				4. 9586559
Morganville	67 21 06. 834		4. 9634528	77. 44	2900. 42	1726	4. 9634519
Pinnacle Hill	68 35 26. 513		4. 9634528	68. 89			4. 9634519
Scottsville	74 23 44. 870	—2. 233	4. 9781955				4. 9781945
Brockport	37 00 49. 856		4. 7741662	778. 41	3747. 72	1939	4. 7741652
Turk's Hill	49 25 11. 741		4. 7741062	324. 66			4. 7741052
Pinnacle Hill	84 22 48. 718	+1. 434	4. 8914878				4. 8914867
Scottsville	46 12 60. 329		4. 7519733	464. 01	4475. 73	2123	4. 7519722
Walworth	54 48 16. 567		4. 7519733	222. 61			4. 7519722
Turk's Hill	79 60 51. 397	—0. 668	4. 8316175				4. 8316163
Pinnacle Hill	46 10 52. 688		4. 6979065	408. 64	5165. 78	2282	4. 6979653
Palmyra	44 12 63. 420		4. 6979065	470. 89			4. 6979053
Walworth	83 52 55. 076	+0. 668	4. 8520833				4. 8520826
Turk's Hill	51 55 02. 162		4. 7506051	272. 25	5848. 92	2469	4. 7506038
Sodus	47 07 14. 896		4. 7506051	380. 25			4. 7506038
Palmyra	74 25 04. 659	—0. 690	4. 8693635				4. 8693621
Walworth	58 27 41. 283		4. 8162124	166. 41	6395. 58	2607	4. 8162110
Clyde	45 23 43. 566		4. 8162124	432. 64			4. 8162110
Sodus	59 56 58. 238	—1. 899	4. 9006210				4. 9006195
Palmyra	74 45 19. 387		4. 9481939	32. 49	6860. 71	2724	4. 9481924
Victory	44 41 45. 635		4. 9481939	453. 69			4. 9481924
Sodus	46 43 63. 299	—1. 423	4. 9631469				4. 9631454
Clyde	88 35 12. 987		5. 1008946	0. 25	7314. 65	2838	5. 1008931
Oswego	46 52 09. 555		5. 1608946	388. 69			5. 1008931
Sodus	29 42 36. 294	+0. 365	4. 9328329				4. 9328313
Victory	103 25 16. 628		5. 2256694	26. 01	7728. 75	2943	5. 2256678
Vanderlip	38 57 25. 741		5. 2256694	676. 00			5. 2256678
Oswego	80 29 46. 400	—1. 098	5. 4212037				5. 4212026
Sodus	60 32 56. 959		5. 3671106	141. 61	8546. 36	3149	5. 3671089

Principal chain of triangles between Buffalo and Sandy Creek Bases—Continued.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	α^2 and β^2	$\Sigma (\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Duck Island	104 08 59.654		5.3671106	28.09			5.3671089
Oswego	26 49 17.657	+3.417	5.0348665				5.0348647
Vanderlip	49 01 47.178		5.2584638	334.89	8909.34	3240	5.2584620
Stony Point	88 22 00.385		5.2584638	0.36			5.2584620
Oswego	30 53 44.463	+0.269	4.9691569				4.9691551
Duck Island	60 44 18.624		5.1993536	139.24	9048.94	3275	5.1993518
Sandy Creek	89 36 59.786		5.1993536	0.04			5.1993518
Stony Point	57 08 58.425	−0.572	5.1236876				5.1236857
Oswego	33 14 04.504		4.9381950	1036.84	10085.82	3537	4.9381931
Mannsville	98 28 35.438		4.9381950	9.61			4.9381931
Stony Point	17 28 38.381	+1.943	4.4205604				4.4205585
Sandy Creek	64 02 46.665		4.8967961	106.09	10201.52	3566	4.8967942
North Base	92 25 22.573		4.8967961	0.81			4.8967942
Mannsville	56 19 11.082	+0.790	4.8173834				4.8173813
Stony Point	31 15 26.978		4.6122550	1204.09	11406.42	3869	4.6122529
Sandy Creek	98 19 18.208		4.6122550	9.61			4.6122529
North Base	39 31 17.606	+1.127	4.4205604				4.4205582
Mannsville	42 09 24.356		4.4436788	542.89	11958.92	4009	4.4436766
South Base	78 48 21.501		4.4436788	17.64			4.4436766
North Base	66 38 27.837	+0.749	4.4148818				4.4148795
Sandy Creek	34 33 10.759		4.2057324	936.36	12912.92	4249	4.2057301

CHAPTER XX.

TRIANGULATION FROM CHICAGO BASE TO OLNEY BASE.

A.—DESCRIPTIONS OF STATIONS.

NOTE RELATIVE TO ELEVATIONS.

§ 1. The heights of ground at stations described in this chapter are all referred to the mean level of Lake Michigan, given in Chapter XXII, § 13. These heights were determined in the following manner: In the triangulation, reciprocal zenith distances were observed over nearly all lines, each mean zenith distance depending, as a rule, on eight to ten separate measures, made on two to six different days. The times of observation were not usually simultaneous, but were confined in nearly all cases to the period between 2 p. m. and 4 p. m. For each station in the triangulation between the lines Kankakee–Saint Anne and Casey–Belle Air, the last three stations included, a coefficient of refraction was computed on the assumption that such coefficient had the same value for all azimuths at a station. The mean of these coefficients, twenty-nine in number, assigning equal weights to individual values, is +0 059, the extreme values being +0.088 and +0.034. The observations in this part of the triangulation were made during August, September, and October. With these coefficients the relative heights of stations between the lines named were computed. For the triangulation north of the line Kankakee–Saint Anne and south of the line Casey–Belle Air, relative heights were computed on the assumption of equal coefficients of refraction at the two ends of a line. With the relative heights thus computed, and the heights of the stations of Chicago Base, the heights of all stations south of Chicago Base were computed successively, each adopted height being a weighted mean of the separate values derived by different routes from adopted heights of stations just preceding. Assuming errors in coefficients of refraction to preponderate, separate values for heights were weighted inversely as the fourth powers of the distances between any station and those stations from which its height was derived. The height of the stone marking West Base of Olney Base was found by this process to be 95.7 feet below the mean level of Lake Michigan, or 485.6 feet above mean tide sea-level. As explained in Chapter XII, § 3, the height of the stone at West Base was also determined by four lines of spirit leveling, and each result by the latter process was given the same weight as the result by zenith distances in deriving a mean height for West Base. This mean height is 491.4 feet, and differs 5.8 feet from that obtained by zenith distances alone. This discrepancy was distributed amongst the previously computed heights of stations between Chicago and Olney Bases proportionally to the distance from Chicago Base in deriving the finally adopted heights for these stations, heights of Chicago Base stations and the mean height for West Base (Olney) being taken as exact. Heights of stations south of Olney Base depend on the mean height adopted for West Base (Olney), the method followed in combining results obtained by different routes for any height being the same as that described above.

Although the heights thus computed can be regarded only as approximations to those which would have resulted from an exact adjustment, some idea of their precision may be formed from the errors of closure shown by summing relative heights about triangles. In the whole number of triangles (forty) giving such errors of closure, the maximum error was 9.6 feet and the average 1.6 feet. From independent errors of closure the probable error of a single relative height, without regard to lengths of triangle sides, was found to be somewhat less than one foot. Assuming ±1 foot as the probable error of any relative height, the probable error of any adopted height should not exceed ±1 foot multiplied by the square root of the number of relative heights added together to produce that height, supposing the route followed from a station of Chicago Base the shortest. This process would give a probable error of about ±4 feet for heights of stations in the vicinity of Olney Base, or of those most remote from Chicago Base.

DESCRIPTIONS OF STATIONS.

§ **2.** ORLAND,* 1879.—This station is situated in the road on the south side of section 28, Orland Township, Cook County, Illinois, about 3½ miles west of Bremen railway station on the Chicago, Rock Island and Pacific Railroad. The height of station used was 4 feet. The geodetic point is marked by a stone post of the usual form, set 4 feet below the surface, with a surface reference-stone set directly over it. Three stone reference-posts were set as follows: One on the north side of the road, bearing south 88° 56′ west, distant 201.15 metres; one on the south side of the road, bearing south 2° 58′ west, distant 14.6 metres; and one on the north side of the road, bearing north 88° 20′ east, distant 201.15 metres from the geodetic point. The corner of sections 27, 28, 33, and 34 bears south 89° 57′ east, and is distant 285.88 metres. The height of ground at the station above mean level of Lake Michigan is 186.4 feet.

CRETE, 1879.—This station is situated in the southeast quarter of the southwest quarter of section 20, township 34 north, range 11 west, Crete Township, Will County, Illinois, about 2¼ miles south of the railway station, Crete, on the Chicago and Eastern Illinois Railroad. The height of station used was 75 feet. The geodetic point is marked by a stone of the usual form, set 3 feet below the surface, with a surface reference-post set directly over it. Three stone reference-posts were set as follows: One near the fence on the west of the station, bearing north 72° 51′ west, distant 191.44 metres; and two on the south line of section 20, one bearing south 19° 49′ west, distant 193.27 metres, and one bearing south 14° 03′ east, distant 186.29 metres from the geodetic point. The southwest corner of section 20 bears south 72° 16′ west, and is distant 613 metres. The height of ground at the station above mean level of Lake Michigan is 209.7 feet.

GARDEN, 1879.—This station is situated in the northeast quarter of the northeast quarter of section 15, township 34 north, range 13 west, Green Garden Township, Will County, Illinois, about 1½ miles north and 4 miles west of Monee railway station on the Illinois Central Railroad. The height of station used was 50 feet. The geodetic point is marked by a stone post of the usual form, set about 3 feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Three stone reference-posts were set as follows: One on the west side of the road just east of the station, bearing south 57° 22′ east, distant 72.8 metres; one near the southeast corner of the crossing of two roads on section-lines, at the corner of sections 10, 11, 14, and 15, bearing north 49° 28′ east, distant 103.3 metres; and one on the north side of the road north of the station, bearing north 17° 07′ west, distant 86.6 metres from the geodetic point. The corner of sections 10, 11, 14, and 15 bears north 41° 13′ east, and is distant 103.0 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 218.3 feet.

GRANT, 1879.—This station is situated in the southeast quarter of the northeast quarter of section 19, township 32 north, range 11 west, Yellowhead Township, Kankakee County, Illinois, about 400 metres north of Grant railway station on the Chicago and Eastern Illinois Railroad. The height of station used was 50 feet. The geodetic point is marked by a stone post of the usual form, set 3 feet below the surface, with a stone post set directly over it as a surface-mark. Three stone reference-posts were set as follows: Two on the west side of the road just east of the station, one bearing north 60° 38′ east, distant 206.28 metres; and one bearing south 62° 15′ east, distant 205.0 metres; and one on the east side of the same road, bearing south 86° 19′ east, and distant 199.84 metres from the geodetic point. The northeast corner of section 20 bears north 73° 17′ east, and is distant 1853 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 134.0 feet.

MANTENO, 1879.—This station is situated in the northwest quarter of section 22, township 32 north, range 12 east, of the third principal meridian, in the village of Manteno, Kankakee County, Illinois, on Chestnut street, between South First and South Second streets, near the track of the

* See note relative to topographical sketches of stations, under Burnt Bluff, Chap. XV, A, § 2.

Illinois Central Railroad. The height of the station used was 60 feet. The geodetic point is marked by a hole in the top of a stone post, set 2½ feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Three stone reference-posts are set as follows: One bearing south 18° 47′ west, distant 19.0 metres; one on the northwest side of Chestnut street, bearing north 71° 38′ west, distant 27.1 metres; and one on the southeast side of Oak street, bearing south 57° 26′ east, distant 67.0 metres from the geodetic point. The railroad track is 18.6 metres distant on the southeast; the southwest corner of the passenger depot bears north 45° 52′ east, and is distant 60.9 metres. The corner of sections 15, 16, 21, and 22 bears north 52° 52′ west, and is distant 352.9 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 109.0 feet.

SAINT ANNE, 1879.—This station is situated near the center of the village of Saint Anne, a station on the Chicago and Eastern Illinois Railroad, in section 4, township 29 north, range 12 west, Saint Anne Township, Kankakee County, Illinois. The height of station used was 50 feet. The geodetic point is marked by a stone post of the usual form, set 3 feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Three stone reference-posts were set near the fence of the street nearest the station on the southeast, as follows: One bearing south 20° 48′ west, distant 20.4 metres; one bearing south 57° 27′ east, distant 9.75 metres; and one bearing north 46° 24′ east, distant 26.00 metres from the geodetic point. The southwest corner of section 4 bears south 82° 32′ west, and is distant 961.9 metres. The height of ground at the station above mean level of Lake Michigan is 94.5 feet.

KANKAKEE, 1879.—This station is the cupola of the high-school building at the southeast corner of Merchant street and Indiana avenue, in Kankakee, Kankakee County, Illinois. The height of instrument above the ground was 89 feet. The geodetic point is the intersection of two lines between two pairs of screws set in the flanges of the scuttle-hole of the cupola. The following points among others were located as reference-marks: The southwest corner of the water-table of the court-house on the square bounded by Merchant and Court streets and Harrison and Indiana avenues, bearing north 25° 11′ east, and distant 96.98 metres; the northeast corner of the high-school water-table, bearing north 64° 15′ east, and distant 24.36 metres; and the spire of the First Baptist Church on the northeast corner of Court street and Indiana avenue, bearing north 9° 19′ east, and distant 207.23 metres from the geodetic point. The quarter-section corner at the center of section 8, township 30 north, range 13 west, bears south 3° 47′ 13″ east, and is distant 2175.7 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 59.9 feet.

WATSEKA, 1879.—This station is situated in the southwest quarter of the northeast quarter of section 11, township 26 north, range 13 west, Crescent Township, Iroquois County, Illinois, about 5 miles west and 1½ miles south of Watseka railway station on the Toledo, Peoria and Warsaw Railroad. The height of station used was 86 feet. The geodetic point is marked by a stone post, set 3 feet below the surface, with a stone post set directly over it as a surface-mark. Three stone reference-posts are set on the west side of the road east of the station as follows: One bearing north 50° 53′ east, distant 233.71 metres; one bearing north 75° 50′ east, distant 191.24 metres; and one bearing south 68° 37′ east, and distant 202.88 metres from the geodetic point. The northeast corner of section 2 (above township) bears north 3° 39′ 26″ east, and is distant 2174.4 metres; the southeast corner of section 35, township 27 north, range 13 west, Iroquois Township, bears north 3° 01′ 22″ east, and is distant 2172.2 metres from the geodetic point. The height of ground at the station above the mean level of Lake Michigan is 87.6 feet.

CLIFTON, 1879.—This station is situated in the northwest quarter of section 3, township 28 north, range 14 west of the second principal meridian, in the village of Clifton, Chebanse Township, Iroquois County, Illinois, in the block bounded by Forest street and Lincoln, Maple, and West avenues. The height of station used was 75 feet. The geodetic point is marked by a hole in the top of a stone post set 2½ feet below the ground with a surface reference-stone set directly over it. Three stone reference-posts are set as follows: Two in the corners of the lot in which the station stands, on the west side of Forest street, one bearing south 88° 34′ east, distant 36.0 metres, and one bearing south 12° 16′ east, distant 72.6 metres; and one on the west side of Maple avenue, close to the fence, bearing north 89° 18′ west, distant 69.6 metres. The northeast corner of the Congregational Church bears south 45° 54′ east, and is distant 251 metres from the geodetic point. The section-corner between sections 33 and 34, township 29 north, range 14 west, and sections 3 and 4, township 28 north, range 14 west, bears north 34° 51′ west, and is distant 809.5 metres. The height of ground at the station above mean level of Lake Michigan is 79.8 feet.

Ash Grove, 1879.—This station is situated in the northeast quarter of the northwest quarter of section 35, township 25 north, range 13 west, in Ash Grove Township, Iroquois County, Illinois. The height of station used was 85 feet. The geodetic point is marked by a one-fourth inch hole in the top of a stone post 2½ feet long, set 34 inches below the surface of the ground. A stone post of the same form is set directly over the geodetic point as a surface-mark. Three stone reference-posts marked U. S. on top are set as follows: Two on the east side of the road on the west of the station, one bearing south 41° 47′ west, distant 256.2 metres, and one bearing north 87° 11′ west, distant 175.7 metres; and one on the south side of a fence on the section-line, bearing north 0° 53′ west, distant 12.9 metres. The northeast corner of section 35 bears north 87° 43′ east, and is distant 1003.8 metres. The height of ground at the station above mean level of Lake Michigan is 84.5 feet.

Spring Creek, 1879.—This station is situated in section 6, township 25 north, range 14 west, Onarga Township, Iroquois County, Illinois, about 1½ miles east of the railway station, Spring Creek, on the Chicago branch of the Illinois Central Railroad. The height of station used was 85 feet. The geodetic point is marked by a stone post of the usual form, set 3 feet below the surface, with a surface-stone set directly over it. Three stone reference-posts were set in the road running by the station as follows: One on the east side, bearing north 21° 05′ east, distant 104.85 metres; one in the angle of a turn of the road to the west, bearing south 14° 26′ east, distant 102.16 metres; and one in the angle of a turn of the road to the south, bearing south 35° 48′ west, and distant 118.67 metres from the geodetic point. The half-section stake at the middle point of the south side of section 31, township 26 north, range 14 west, bears north 26° 00′ 28″ east, and is distant 1588.9 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 84.2 feet.

Butler, 1879.—This station is situated near the dividing line between the southeast quarter and the northeast quarter of the southeast quarter of section 35, township 23 north, range 13 west, in Butler Township, Vermillion County, Illinois, about 4 miles south of East Lynn, a station on the LaFayette, Bloomington and Mississippi Railroad. The height of station used was 61 feet. The geodetic point is marked by a stone post, set 3 feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Three stone reference-posts are set near the fence on the west side of the road on the east of the station as follows: One bearing north 55° 41′ east, distant 197.52 metres; one bearing south 86° 42′ east, distant 169.12 metres; and one bearing south 58° 06′ east, distant 201.70 metres. The southeast corner of section 36, above township, bears south 78° 15′ east, and is distant 1820.15 metres. The southeast corner of section 35 bears south 25° 04′ 56″ east, and is distant 433.15 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 205.3 feet.

Paxton, 1879.—This station is situated in the southwest quarter of section 14, township 23 north, range 10 east, Paxton Township, Ford County, Illinois. The height of station used was 50 feet. The geodetic point is marked in the usual manner by two stone posts set one above the other. Two stone reference-posts are set on the south side of the road south of the station, one bearing south 28° 45′ west, distant 29.35 metres; and one bearing south 50° 12′ east, distant 39.6 metres. The corner of sections 14, 15, 22, and 23 bears south 87° 28′ west, and is distant 460.4 metres. A land-survey mark one-quarter mile east of the above section-corner bears south 73° 11′ west, and is distant 59.6 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 221.9 feet.

Pilot Grove, 1879.—This station is situated in the north half of section 16, township 20 north, range 13 west, of the second principal meridian, in Pilot Township, Vermillion County, Illinois. The height of station used was 65 feet. The geodetic point is marked by a one-quarter inch hole in the top of a stone post set 2½ feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Three stone reference-posts with the letters U. S. cut on their tops are set as follows: Two along the south side of the section-line road on the north of the station, one bearing north 21° 20′ west, distant 131 metres, and one bearing north 14° 08′ east, distant 128.9 metres; and one by the fence just east of the station, bearing north 86° 32′ east, and distant 33.1 metres. The corner of sections 8, 9, 16, and 17 bears north 82° 03½′ west, and is distant 774.3 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 197.5 feet.

RANTOUL, 1879.—This station is situated in the southeast quarter of the northwest quarter of section 35, township 22 north, range 10 east, Harwood Township, Champaign County, Illinois, about 1 mile northwest of Gifford, a station on the Havana, Rantoul and Eastern Railroad. The height of station used was 50 feet. The geodetic point is marked by a stone post of the usual form, set 3 feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Three stone reference-posts were set as follows: One by the quarter-section line south of the station, bearing south 24° 20′ west, and distant 55.3 metres; one on the south side of the section-line road south of the station, bearing south 26° 26′ west, and distant 968.4 metres; and one on the north side of the same road near the quarter-section stone, bearing south 16° 42′ east, and distant 886.1 metres from the geodetic point. The center of section 35 bears south 79° 22′ east, and is distant 274.67 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 238.7 feet.

FAIRMOUNT, 1879.—This station is situated near the south line of the southeast quarter of the southeast quarter of section 8, township 18 north, range 13 west, Vance Township, Vermillion County, Illinois, about 2½ miles southwest of Fairmount, a station on the Wabash Railway. The height of station used was 60 feet. The geodetic point is marked by a stone post set 3 feet below the surface, with a surface reference-stone set directly over it. Three stone reference-posts are set along the north side of the road just south of the station as follows: One bearing south 67° 43′ east, distant 24.6 metres; one bearing south 2° 40′ west, distant 9.1 metres; and one bearing south 65° 22′ west, distant 22.9 metres from the geodetic point. The southeast corner of section 8 bears south 86° 50′ east, and is distant 278.5 metres. An oak latitude-post 19 inches in diameter, occupied in 1880, bears north 82° 26′ east, and is distant 8.034 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 122.5 feet.

MAYVIEW, 1879.—This station is situated in the southwest quarter of section 7, township 19 north, range 10 east, of the third principal meridian, in Saint Joseph Township, Champaign County, Illinois, about 5 miles east of Urbana and 3 miles west of Saint Joseph, stations on the Indiana, Bloomington, and Western Railroad. The height of station used was 85 feet. The geodetic point is marked by a stone post of the usual form, set 3½ feet below the surface, with another stone set directly over it, 14 inches below the surface. Three stone reference-posts are set as follows: Two on the north side of and near the track of the Indiana, Bloomington and Western Railroad, one bearing south 26° 20′ east, distant 176.45 metres, and one bearing south 2° 44′ east, distant 157.65 metres; and one on the east side of the road west of the station, bearing north 87° 52′ west, distant 173.54 metres from the geodetic point. The section-corner between sections 7 and 18, township 19 north, range 10 east, and sections 12 and 13, township 19 north, range 9 east, bears south 48° 12′ west, and is distant 236.21 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 123.9 feet.

PALERMO, 1879.—This station is situated in the northeast quarter of section 6, township 16 north, range 13 west, of the second principal meridian, in Young America Township, Edgar County, Illinois. The height of station used was 50 feet. The geodetic point is marked by a one-fourth inch hole in the top of a stone post set 2½ feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Three stone reference-posts, marked U. S. on their tops, were set as follows: One bearing south 6° 47′ west, distant 76.8 metres; one bearing north 45° 05′ west, distant 11.4 metres; and one bearing north 82° 55′ east, distant 78.0 metres from the geodetic point. The corner of sections 5, 6, 7, and 8 bears south 23° 06′ east, and is distant 1283.6 metres. The center of section 5 bears south 39° 26′ west, and is distant 487.7* metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 160.4 feet.

LYNN GROVE, 1879.—This station is situated in the southwest quarter of the southeast quarter of section 31, township 18 north, range 10 east, Sidney Township, Champaign County, Illinois, about 3 miles southeast of Philo, a station on the Wabash Railway. The height of station used was 60 feet. The geodetic point is marked by a stone post of the usual form set 3 feet below the surface, with a stone post set directly over it as a surface-mark. Three stone reference-posts are set as follows: One near the fence on the east of the station, bearing south 86° 25′ east, distant 19.5 metres; and two on the north side of the town-line road on the south of the station, one bearing south 6° 41′ east, distant 204.54 metres, and the other near the southeast corner of a cemetery, bearing south 45° 21′ west, distant 287.04 metres from the geodetic point. The township corner, lying between townships 18 and 17 north, range 10 east, and townships 17 and 18 north, range 9 east, bears south 79° 45′ 32″ west, and is distant 1144.4 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 190.3 feet.

KANSAS, 1879.—This station is situated in the southeast quarter of the southwest quarter of section 4, township 12 north, range 13 west, Grandview Township, Edgar County, Illinois, about 4 miles south and 1½ miles west of Dudley, a station on the Indianapolis and Saint Louis Railroad. The height of station used was 89 feet. The geodetic point is marked by a stone post of the usual form, set 3 feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Three stone reference-posts are set by the fence west of the station as follows: One bearing north 43° 44′ west, distant 64.5 metres; one bearing north 83° 34′ west, distant 43.9 metres; and one bearing south 44° 46′ west, distant 60.7 metres from the geodetic point. The southwest corner of section 4 bears south 58° 55′ west, and is distant 510.2 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 256.8 feet.

OAKLAND, 1879.—This station is situated in the northwest quarter of section 36, township 15 north, range 10 east, of the third principal meridian, in Sargent Township, Douglas County, Illinois. The height of station used was 85 feet. The geodetic point is marked by a stone post of the usual form, set 3 feet below the surface of the ground, with a stone post set directly over it as a surface mark. Three stone reference-posts are set as follows: Two on the northern side of the road running a little north of west on the south of the station, one bearing south 22° 12′ east, distant 96.2 metres, and one bearing south 42° 43′ west, distant 91.0 metres; and one on the southern side of the same road, bearing south 8° 26′ west, distant 94.37 metres from the geodetic point. The land-survey mark on the line between sections 26 and 35, one-quarter mile west of the corner of sections 25, 26, 35, and 36, bears north 70° 59′ 44″ west, and is distant 598.6 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 111.0 feet.

MARTINSVILLE, 1879.—This station is situated in the southwest quarter of section 16, township 10 north, range 13 west, Martinsville Township, Clark County, Illinois, about 2 miles southeast of Martinsville, a station on the Vandalia Railway line. The height of station used was 85 feet. The geodetic point is marked by two stone posts, set one above the other in the usual manner. Three stone reference-posts are set as follows: One bearing south 1° 28½′ west, distant 209 metres; one set by a land-survey stone one-quarter mile north of the land-survey stone at the middle of the south side of section 16, bearing south 31° 24½′ east, distant 237 metres; and one set by a land-survey stone at the center of section 16, bearing north 29° 32′ east, distant 237 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 96.2 feet.

WESTFIELD, 1879.—This station is situated near the southwest corner of the northwest quarter of the southwest quarter of section 25, township 12 north, range 10 east, Hutton Township, Coles County, Illinois, about 2½ miles west of Westfield, a station on a branch of the Indianapolis and Saint Louis Railroad. The height of station used was 90 feet. The geodetic point is marked by a stone post of the usual form, set 3 feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Three stone reference-posts were set in the section-line road west of the station as follows: One on the west side of the road, bearing north 21° 16′ west, distant 92.9 metres; and two on the east side of the road, one bearing south 81° 42′ west, distant 23.13 metres, and one bearing south 13° 25′ west, distant 103.09 metres from the geodetic point. The corner of sections 25, 26, 35, and 36, bears south 3° 25′ west, and is distant 531.7 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 191.2 feet.

BELLE AIR, 1879.—This station is situated in section 32, township 9 north, range 13 west, Orange Township, Clark County, Illinois, a few metres from the southern boundary of the county. The height of station used was 100 feet. The geodetic point is marked by a one-fourth inch hole in the top of a stone post set $2\frac{1}{2}$ feet below the surface of the ground, a stone post being set directly over it as a surface-mark. Three stone reference-posts, marked U. S. on top, are set as follows: Two by the fence on the south side of the road south of the station, one bearing south 38° 07′ west, distant 41.4 metres, and one bearing south 58° 08′ east, distant 56.6 metres; and one on the north side of the same road, bearing south 49° 50′ east, distant 17.8 metres from the geodetic point. The section-corner of sections 4 and 5, township 8 north, range 13 west, on the south line of section 32, township 9 north, range 13 west, bears south 86° 28′ west, and is distant 559 metres from the geodetic point. The height of ground at the station above mean level of Lake Michigan is 6.0 feet.

CASEY, 1879.—This station is situated in section 18, township 9 north, range 11 east, of the third principal meridian, in Crooked Creek Township, Cumberland County, Illinois, about 4 miles southwesterly from the village of Casey, a station on the Vandalia Railway Line. The height of station used was 86 feet. The geodetic point is marked by a stone post of the usual form, set 3 feet below the surface of the ground, with a stone post 6 inches below the surface set directly over it as a surface-mark. Three stone reference-posts are set on the east side of the road just east of the station as follows: One bearing north 35° 37′ east, distant 58.0 metres; one bearing north 88° 36′ east, distant 34.7 metres; and one bearing south 35° 53′ east, distant 60.5 metres from the geodetic point. The northeast corner of section 18, township 9 north, range 11 east, of the third principal meridian, bears north 47° 29′ east, and is distant 1092.1 metres from the geodetic point. The height of ground at the station above the mean level of Lake Michigan is 48.9 feet.

OBLONG, 1879.—This station is situated in the southeast quarter of the southeast quarter of section 32, township 7 north, range 13 west, Oblong Township, Crawford County, Illinois. The height of station used was 100 feet. The geodetic point is marked in the usual manner by two stone posts set one above the other. Three stone reference-posts are set along the east side of the road west of the station as follows: One bearing south 44° 15′ west, distant 125.7 metres; one bearing south 78° 32′ west, distant 90.0 metres; and one bearing north 65° 13′ west, distant 97.7 metres from the geodetic point. The first reference-stone mentioned is set near the land-survey stone on the south line of section 32, one-fourth mile west of the southeast corner of the section, the land-survey stone bearing south 46° 23′ west, and being distant 131 metres from the geodetic point. The southeast corner of section 32 bears south 73° 42′ east, and is distant 325.6 metres from the geodetic point. The ground at the station is 81.6 feet below the mean level of Lake Michigan.

HUNT CITY, 1879.—This station is situated in the northeast quarter of the northwest quarter of section 7, township 7 north, range 14 west, Grandville Township, Jasper County, Illinois, about 10 miles northeast of Newton, on the Grayville and Mattoon Railroad, and about three-fourths of a mile northeast of Hunt City. The height of station used was 75 feet. The geodetic point was marked by a stone post of the usual form, set 3 feet below the surface, with a stone post set directly over it as a surface-mark. Three stone reference-posts were set as follows: Two on the south side of the section-line road north of the station, one bearing north 33° 52′ east, distant 334.71 metres, and one bearing north 9° 54′ west, distant 282.62 metres; and one on the east side of the section-line road west of the station, bearing south 85° 32′ west, and distant 678.88 metres from the geodetic point. The section-corner at the northwest corner of section 7 and southwest corner of section 6 (above township) bears north 66° 46′ west, and is distant 749.0 metres. The section-corner at the southeast corner of section 6 and the northeast corner of section 7, township 7 north, fractional range 11 east, bears north 67° 05′ west, and is distant 747.0 metres from the geodetic point. These two section corners are 4.56 metres apart. The ground at the station is 30.7 feet below the mean level of Lake Michigan.

CLAREMONT, 1879.—This station is situated in section 29, township 4 north, range 14 west, German Township, Richland County, Illinois, about 3 miles northwesterly from the town of Claremont, a station on the Ohio and Mississippi Railroad. The height of station used was 80 feet. The geodetic point is marked by two stone posts set one above the other, in the usual manner. Three stone reference-posts are set as follows: One bearing north 67° 33′ west, distant 23.1 metres; one bearing north 0° 39′ west, distant 7.8 metres; and one bearing north 71° 45′ east, distant 24.6 metres from the geodetic point. The northwest corner of section 29 bears north 60° 03′ west, and is distant 847 metres from the geodetic point. The height of ground at the station above the mean level of Lake Michigan is 24.3 feet.

Mound, 1879.—This station is situated in section 1, near the line between sections 1 and 2, township 5 north, range 10 east, of the third principal meridian, Fox Township, Jasper County, Illinois, on a hill known as Buffalo Mound, about 2½ miles southwest of the village of Saint Marie. The height of station used was 101 feet. The geodetic point is marked in the usual manner by two stone posts set one above the other. Three stone reference-posts are set on the west side of the section-line road just west of the station, as follows: One bearing south 40° 46′ west, distant 44.4 metres; one bearing north 87° 19′ west, distant 28.9 metres; and one bearing north 38° 54′ west, distant 45.3 metres. The corner of sections 1, 2, 11, and 12, bears south 1° 29′ west, and is distant 966 metres from the geodetic point. The ground at the station is 60.4 feet below the mean level of Lake Michigan.

Onion Hill, 1879.—This station is situated in the northeast quarter of section 2, township 4 north, range 9 east, Denver Township, Richland County, Illinois, about 5 miles southwest of West Liberty, a station on the Grayville and Mattoon Railway, on Onion Hill. The height of station used was 4½ feet. The geodetic point is marked by a stone post of the usual form, set 3 feet below the surface, with a stone post set directly over it as a surface-mark. Three stone reference-posts were set as follows: One on the south side of the road north of the station bearing north 33° 02′ east, distant 205.68 metres; one on the north side of the same road bearing north 25° 31′ west, distant 181.04 metres; and one on the west side of the road west of the station bearing north 84° 35′ west, distant 354.02 metres from the geodetic point. The northeast corner of section 2 bears north 69° 25′ east, and is distant 502.7 metres from the geodetic point. The ground at the station is 13.4 feet below the mean level of Lake Michigan.

West Base, 1879.—This station, marking the west end of the Olney base-line, is situated in the northwest quarter of the northeast quarter of section 21, township 5 north, range 10 east, Fox Township, Jasper County, Illinois. The height of station used was 50 feet. The geodetic point is marked by a stone post of the usual form, set in a bed of brick-work 3 feet square, with its top 4 feet below the surface of the ground. Two additional stones are set on a line through the geodetic point perpendicular to the direction of the base-line and at a depth below the surface of the ground of about 4 feet, one bearing north 1° 30′ west, distant 8.02 metres from the geodetic point; and one bearing south 1° 30′ east, distant 8.06 metres. Three stone reference-posts are set as follows: Two on the south side of the road north of the station, one bearing north 2° 45′ west, distant 246.7 metres, and one bearing north 45° 32′ east, distant 356.0 metres; and one bearing south 61° 00′ east, distant 302.0 metres. An oak latitude-post 17 inches in diameter, occupied in 1880, bears south 88° 36′ east, and is distant 16.[illegible]9 metres. The northeast corner of section 21 bears north 67° 19′ east, and is distant 727 metres. There is an uncertainty of a metre or two in this last determination. The ground at the station is 85.9 feet below the mean level of Lake Michigan.

East Base, 1879.—This station, marking the east end of the Olney base-line is situated in section 19, township 5 north, fractional range 11 east, Saint Marie Township, Jasper County, Illinois, about 3½ miles east, and one-half mile north of the railway station of West Liberty, on the Grayville and Mattoon Railroad. The height of station used was 36 feet. The geodetic point is marked by a brass cylinder leaded into the top of a stone post of the usual form, set 2½ feet below the surface of the ground, and surrounded by brickwork 3 feet square and 3 feet deep. Two side stones are set on a line at right angles to the direction of the base-line, and at a depth below the surface of the ground of about 2½ feet, one bearing north 1° 28′ west, distant 7.91 metres, and one bearing south 1° 28′ east, distant 8.04 metres from the geodetic point. Three stone reference-posts are set as follows: One bearing north 49° 49′ east, distant 361 metres; one bearing south 58° 02′ east, distant 322 metres; and one bearing south 35° 50′ west, distant 208 metres from the geodetic point. The northwest corner of section 19, township 5 north, fractional range 11 east, bears north 77° 12′ west, and is distant 1054 metres from the geodetic point. There is an uncertainty of a metre or two in this last determination. The ground at the station is 95.3 feet below the mean level of Lake Michigan.

Middle Base, 1879.—This station, near the middle of the Olney base-line, is situated in the northwest quarter of section 23, township 5 north, range 10 east, Fox Township, Jasper County, Illinois, about 1.1 miles east and one-half mile north of West Liberty, a station on the Grayville and Mattoon Railroad. The height of station used was 5 feet. The geodetic point is marked by a stone post of the usual form, set $2\frac{1}{2}$ feet below the surface. The northeast corner of section 23 bears north 66° 18′ east, and is distant 712 metres from the geodetic point. There is an uncertainty of a metre or two in this determination. The ground at the station is 103.3 feet below the mean level of Lake Michigan.

Check Base, 1879.—This station is situated in section 6, township 4 north, range 11 east, Preston Township, Richland County, Illinois. The height of station used was 41 feet. The geodetic point is marked by a hole in the top of a stone post set $2\frac{1}{2}$ feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Three stone reference-posts are set as follows: One on the south side of the road on the south of the station, bearing south 12° 12′ west, distant 22.6 metres; one at the northeast corner of the cemetery just west of the station, bearing north 3° 35′ west, distant 73 metres; and one on the north side of the above road, bearing south 80° 21′ east, distant 53.5 metres. The southeast corner of the German Reformed Church bears north 53° 10′ west, and is distant 20.1 metres. The quarter-section stone of the west line of section 6 bears north 31° 44′ west, and is distant 943.9 metres from the geodetic point. The ground at the station is 60.2 feet below the mean level of Lake Michigan.

Denver, 1879.—This station is situated in the southwest quarter of the northeast quarter of section 21, township 4 north, range 9 east, Denver Township, Richland County, Illinois, about $5\frac{1}{2}$ miles north of station Noble on the Ohio and Mississippi Railroad. The height of station used was 74.5 feet. The geodetic point is marked by a stone post of the usual form, set 3 feet below the surface of the ground, with a stone post set directly over it as a surface-mark. Three stone reference-posts were set as follows: One on the north side of the road north of the station, bearing north 15° 27′ east, distant 344.92 metres; one on the east side of the road east of the station, bearing north 69° 35′ east, distant 578.78 metres; and one on the west side of the latter road, bearing south 70° 01′ east, and distant 568.15 metres from the geodetic point. The corner of sections 15, 16, 21, and 22 bears north 58° 51′ 58″ east, and is distant 628.32 metres. The ground at the station is 37.6 feet below the mean level of Lake Michigan.

Parkersburg, 1879.—This station is situated in fractional section 30, township 2 north, fractional range 11 east, Richland County, Illinois. The height of station used was 75 feet. The geodetic point is marked in the usual manner by two stone posts set one above the other. Four stone reference-posts are set as follows: Two on the south side of the road north of the station, one bearing south 69° 54′ east, distant 48.7 metres, and one bearing north 68° 27′ west, distant 74.0 metres; one on the north side of the same road, bearing north 83° 22′ east, distant 44.7 metres; and one bearing north 28° 40′ east, distant 93.0 metres from the geodetic point. The corner of sections 19, 24, 25, and 30 bears north 55° 39′ 46″ west, and is distant 848.5 metres. A latitude-post, occupied in 1879, bears north 87° 43′ 30″ west, and is distant 34.68 metres. The ground at the station is 13.6 feet below the mean level of Lake Michigan.

B.—STATIONS, SIGNALS, INSTRUMENTS, AND METHODS OF OBSERVATION.

§ **3.** See Chapter XVI, B.

C.—MEASURED AND ADJUSTED ANGLES BETWEEN THE LINES WILLOW SPRINGS-MORGAN PARK AND DENVER-PARKERSBURG.

§ **4.** This portion of the triangulation was adjusted in two sections, viz:
Section XIV extending from Willow Springs-Morgan Park to Oakland-Kansas and
Section XV extending from Oakland-Kansas to Denver-Parkersburg.

A sketch of it is given in Plate XXIII. Weight unity was assigned to 16 combined results for each of the instruments used in the measurement of the angles, viz., Troughton & Simms theodolites Nos. 1, 3, and 4, Pistor & Martin's theodolite No. 2, and Repsold theodolite No. 1. An abstract of the adjustment is given in the following tables. For a detailed explanation of the tables see Chapter XIV, C, § 7, and see the remark in Chapter XV, C, § 6, relating to the column headed "No. meas." The locally adjusted angles at stations Oakland and Kansas, with weights equal to those applied to their observed values, were used in computing the general adjustment in which three sum-angle conditions at Oakland and two at Kansas were discarded.

SECTION XIV.—*Triangulation from the line Willow Springs-Morgan Park to the line Oakland-Kansas.*

WILLOW SPRINGS—1.

[Observer, J. H. Darling. Instrument, Troughton & Simms theodolite No. 4. Date, July, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Morgan Park and Orland	76 05 20.915	1_1	24	5.8	1	+0.051	+0.227	76 05 21.193
Orland and Morgan Park	283 54 38.983	1_2	24	10.4	1	+0.051	−0.228	283 54 38.806

MORGAN PARK—2.

[Observer, G. Y. Wisner. Instrument, Pistor & Martins theodolite No. 2. Date, July, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Crete and Orland	53 27 03.475	2_1	19	5.4	1	+0.261	+0.250	53 27 03.986
Orland and Willow Springs	56 04 39.052	2_2	16	6.8	1	+0.261	+0.103	56 04 39.416
Willow Springs and Crete	250 28 16.689	2_3	16	6.8	1	+0.261	−0.353	250 28 16.597

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(2_1)+(2_2)-0.784=0$$
$$(2_1)+2(2_2)-0.784=0$$

ORLAND—3.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Date, July, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Willow Springs and Morgan Park	47 50 00.068	3_1	18	4.5	1	−0.113	+0.085	47 50 00.040
Willow Springs and Crete	133 45 33.355	3_{1+2}	8	3.1	0.5	+0.364	+0.231	133 45 33.950
Morgan Park and Crete	85 55 33.800	3_2	18	6.1	1	−0.036	+0.146	85 55 33.910
Morgan Park and Garden	121 55 25.109	3_{2+3}	8	3.4	0.5	−0.154	+0.170	121 55 25.125
Crete and Garden	35 59 51.044	3_3	18	5.1	1	+0.146	+0.024	35 59 51.214
Garden and Willow Springs	190 14 35.021	3_4	18	7.4	1	+0.669	−0.255	190 14 34.835

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.5(3_1)+1.5(3_2)+(3_3)+0.189=0$$
$$1.5(3_1)+3.0(3_2)+1.5(3_3)+0.057=0$$
$$(3_1)+1.5(3_2)+2.5(3_3)-0.199=0$$

CRETE—4.

[Observer, G. Y. Wisner. Instrument, Pistor & Martins theodolite No. 2. Date, July, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Grant and Manteno	40 16 54.098	4_1	16	8.2	1	−0.463	+0.442	40 16 54.077
Grant and Garden	99 51 35.209	4_{1+2}	8	6.6	0.5	−0.311	+0.579	99 51 35.477
Manteno and Garden	59 34 41.726	4_2	16	8.3	1	−0.463	+0.137	59 34 41.400
Garden and Orland	34 22 31.463	4_3	16	6.5	1	−0.619	−0.151	34 22 30.693
Orland and Morgan Park	40 37 23.898	4_4	16	7.6	1	−0.619	+0.087	40 37 23.366
Morgan Park and Grant	185 08 31.597	4_5	16	5.9	1	−0.619	−0.515	185 08 30.463

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.5(4_1)+1.5(4_2)+(4_3)+(4_4)+3.089=0$$
$$1.5(4_1)+2.5(4_2)+(4_3)+(4_4)+3.089=0$$
$$(4_1)+(4_2)+2(4_3)+(4_4)+2.782=0$$
$$(4_1)+(4_2)+(4_3)+2(4_4)+2.782=0$$

SECTION XIV.—*Triangulation from the line Willow Springs-Morgan Park to the line Oakland-Kansas*—Continued.

GARDEN—5.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Date, July, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Orland and Crete	109 37 38.727	5_1	18	3.4	1	−0.013	−0.071	109 37 38.643
Crete and Manteno	81 35 37.282	5_2	18	4.8	1	−0.013	+0.507	81 35 37.776
Manteno and Orland	168 46 44.031	5_3	18	6.6	1	−0.013	−0.436	168 46 43.582

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(5_1)+(5_2)+0.040=0$$
$$(5_1)+2(5_2)+0.040=0$$

GRANT—6.

[Observer, G. Y. Wisner. Instrument, Pistor & Martins theodolite No. 2. Dates, July and August, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
St. Anne and Kankakee	37 33 21.679	6_1	20	10.5	1	+0.348	−0.187	37 33 21.840
St. Anne and Manteno	76 37 28.222	6_{1+2}	8	3.9	0.5	−0.462	+0.367	76 37 28.127
Kankakee and Manteno	39 04 05.384	6_2	20	6.6	1	+0.348	+0.554	39 04 06.286
Manteno and Crete	90 01 34.248	6_3	16	6.4	1	+0.117	+0.440	90 01 34.805
Crete and St. Anne	193 20 57.758	6_4	16	4.2	1	+0.117	−0.807	193 20 57.068

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.5(6_1)+1.5(6_2)+(6_3)-1.510=0$$
$$1.5(6_1)+2.5(6_2)+(6_3)-1.510=0$$
$$(6_1)+(6_2)+2(6_3)-0.931=0$$

MANTENO—7.

[Observer, J. H. Darling. Instrument, Troughton & Simms theodolite No. 4. Date, July, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Garden and Crete	38 49 41.581	7_1	16	6.0	1	+0.144	−0.074	38 49 41.651
Garden and Grant	88 31 13.423	7_{1+2}	16	7.5	1	−0.065	+0.157	88 31 13.515
Crete and Grant	49 41 31.488	7_2	16	5.7	1	+0.145	+0.231	49 41 31.864
Grant and St. Anne	68 35 47.213	7_3	19	8.2	1	−0.074	−0.161	68 35 46.978
Grant and Kankakee	98 03 26.888	7_{3+4}	16	5.5	1	+0.152	+0.703	98 03 27.743
St. Anne and Kankakee	29 27 39.975	7_4	20	5.2	1	−0.074	+0.864	29 27 40.765
Kankakee and Garden	173 25 19.523	7_5	18	5.8	1	+0.079	−0.860	173 25 18.742

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3(7_1)+2(7_2)+(7_3)+(7_4)+0.574=0$$
$$2(7_1)+3(7_2)+(7_3)+(7_4)+0.574=0$$
$$(7_1)+(7_2)+3(7_3)+2(7_4)+0.080=0$$
$$(7_1)+(7_2)+2(7_3)+3(7_4)+0.080=0$$

SECTION XIV.—*Triangulation from the line Willow Springs–Morgan Park to the line Oakland–Kansas*—Continued.

SAINT ANNE—8.

[Observer, G. Y. Wisner. Instrument, Pistor & Martins theodolite No. 2. Date, August, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Watseka and Clifton	52 04 50.631	8_1	16	6.4	1	−0.329	−0.051	52 04 50.251
Clifton and Kankakee	69 29 56.377	8_2	16	5.8	1	−0.109	−0.680	69 29 55.588
Clifton and Grant	131 09 07.965	8_{2+3+4}	16	5.0	1	−0.220	+0.409	131 09 08.154
Kankakee and Manteno	26 52 25.967	8_3	16	4.9	1	−0.109	+0.809	26 52 26.667
Manteno and Grant	34 46 45.729	8_4	16	5.3	1	−0.109	+0.279	34 46 45.899
Grant and Watseka	176 46 02.282	8_5	16	5.1	1	−0.329	−0.358	176 46 01.595

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(8_1)+(8_2)+(8_3)+(8_4)+0.986=0$$
$$(8_1)+3(8_2)+2(8_3)+2(8_4)+1.094=0$$
$$(8_1)+2(8_2)+3(8_3)+2(8_4)+1.094=0$$
$$(8_1)+2(8_2)+2(8_3)+3(8_4)+1.094=0$$

KANKAKEE—9.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Date, August, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Manteno and Grant	42 52 25.701	9_1	18	4.5	1	+0.192	+0.658	42 52 26.551
Manteno and Saint Anne	123 39 51.918	9_{1+2}	8	1.7	0.5	+0.179	+0.963	123 39 53.060
Grant and Saint Anne	80 47 26.341	9_2	18	5.0	1	−0.136	+0.305	80 47 26.510
Grant and Clifton	146 24 30.026	9_{2+3}	8	5.1	0.5	+0.655	−0.107	146 24 30.574
Saint Anne and Clifton	65 37 04.708	9_3	18	4.7	1	−0.231	−0.412	65 37 04.065
Saint Anne and Manteno	236 20 07.613	9_{3+4}	8	7.1	0.5	+0.290	−0.963	236 20 06.940
Clifton and Manteno	170 43 03.330	9_4	18	6.5	1	+0.096	−0.551	170 43 02.875

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3(9_1)+2.0(9_2)+(9_3)-0.070=0$$
$$2(9_1)+3.5(9_2)+1.5(9_3)+0.441=0$$
$$(9_1)+1.5(9_2)+2.5(9_3)+0.591=0$$

WATSEKA—10.

[Observer, G. Y. Wisner. Instrument, Pistor & Martins theodolite No. 2. Date, August, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Ash Grove and Spring Creek	58 42 01.740	10_1	16	7.6	1	+0.110	+0.995	58 42 02.845
Spring Creek and Clifton	87 19 51.706	10_2	16	9.2	1	+0.110	−0.791	87 19 51.025
Clifton and Saint Anne	42 13 18.502	10_3	16	6.0	1	+0.110	+0.052	42 13 18.664
Saint Anne and Ash Grove	171 44 47.612	10_4	16	2.3	1	+0.110	−0.256	171 44 47.466

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(10_1)+(10_2)+(10_3)+0.440=0$$
$$(10_1)+2(10_2)+(10_3)+0.440=0$$
$$(10_1)+(10_2)+2(10_3)+0.440=0$$

SECTION XIV.—*Triangulation from the line Willow Springs-Morgan Park to the line Oakland-Kansas*—Continued.

CLIFTON—11.

[Observer, J. H. Darling. Instrument, Troughton & Simms theodolite No. 4. Date, August, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Kankakee and Saint Anne	44 53 01.121	11_1	24	6.0	1	−0.080	+0.083	44 53 01.124
Saint Anne and Watseka	85 41 52.245	11_2	24	8.0	1	−0.289	+0.382	85 41 52.338
Saint Anne and Spring Creek	125 57 01.415	11_{2+3}	20	7.2	1	+0.209	−0.079	125 57 01.545
Watseka and Spring Creek	40 15 09.957	11_3	24	8.6	1	−0.289	−0.461	40 15 09.207
Spring Creek and Kankakee	189 09 57.416	11_4	24	8.8	1	−0.080	−0.004	189 09 57.332

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(11_1)+(11_2)+(11_3)+0.739=0$$
$$(11_1)+3(11_2)+2(11_3)+1.526=0$$
$$(11_1)+2(11_2)+3(11_3)+1.526=0$$

ASH GROVE—12.

[Observer, J. H. Darling. Instrument, Troughton & Simms theodolite No. 4. Date, August, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Butler and Paxton	55 24 53.835	12_1	20	10.0	1	+0.214	−0.345	55 24 53.704
Butler and Spring Creek	121 34 46.752	12_{1+2}	18	4.8	1	+0.123	−0.331	121 34 46.541
Paxton and Spring Creek	66 09 52.680	12_2	20	7.2	1	+0.146	+0.014	66 09 52.840
Paxton and Watseka	130 39 32.182	12_{2+3}	16	7.9	1	+0.068	+0.536	130 39 32.786
Spring Creek and Watseka	64 29 39.155	12_3	18	5.5	1	+0.269	+0.522	64 29 39.946
Watseka and Butler	173 55 33.363	12_4	18	8.6	1	+0.338	−0.191	173 55 33.510

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3(12_1)+2(12_2)+(12_3)-1.204=0$$
$$2(12_1)+4(12_2)+2(12_3)-1.551=0$$
$$(12_1)+2(12_2)+3(12_3)-1.314=0$$

SPRING CREEK—13.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Dates, August and September, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Clifton and Watseka	52 25 01.799	13_1	14	4.5	1	−0.073	−0.756	52 25 00.970
Watseka and Ash Grove	56 48 17.029	13_2	16	4.7	1	−0.073	+1.030	56 48 17.986
Ash Grove and Butler	30 41 45.312	13_3	16	5.7	1	−0.275	−0.326	30 41 44.711
Ash Grove and Paxton	70 48 21.387	13_{3+4}	16	3.0	1	+0.202	−0.054	70 48 21.535
Butler and Paxton	40 06 36.828	13_4	16	7.0	1	−0.275	+0.272	40 06 36.825
Paxton and Clifton	179 58 19.801	13_5	16	5.3	1	−0.073	−0.220	179 58 19.508

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(13_1)+(13_2)+(13_3)+(13_4)+0.769=0$$
$$(13_1)+2(13_2)+(13_3)+(13_4)+0.769=0$$
$$(13_1)+(13_2)+3(13_3)+2(13_4)+1.522=0$$
$$(13_1)+(13_2)+2(13_3)+3(13_4)+1.522=0$$

SECTION XIV.—*Triangulation from the line Willow Springs–Morgan Park to the line Oakland–Kansas*—Continued.

BUTLER—14.

[Observer, G. Y. Wisner. Instrument, Pistor & Martins theodolite No. 2. Dates, August and September, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Pilot Grove and Rantoul	54 47 29.819	14_1	22	8.8	1	−0.596	−0.150	54 47 29.073
Rantoul and Paxton	35 09 27.887	14_2	16	6.5	1	−0.596	+0.031	35 09 27.322
Paxton and Spring Creek	46 58 38.811	14_3	17	6.6	1	−0.006	−0.495	46 58 38.310
Paxton and Ash Grove	74 42 08.520	14_{3+4}	16	4.9	1	−0.589	−0.054	74 42 07.877
Spring Creek and Ash Grove	27 43 29.134	14_4	17	8.2	1	−0.007	+0.440	27 43 29.567
Ash Grove and Pilot Grove	195 20 56.150	14_5	16	4.8	1	−0.596	+0.173	195 20 55.727

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(14_1)+(14_2)+(14_3)+(14_4)+1.801=0$$
$$(14_1)+2(14_2)+(14_3)+(14_4)+1.801=0$$
$$(14_1)+(14_2)+3(14_3)+2(14_4)+1.226=0$$
$$(14_1)+(14_2)+2(14_3)+3(14_4)+1.226=0$$

PAXTON—15.

[Observer, A. R. Flint. Instrument, Troughton & Simms theodolite No. 1. Dates, August and September, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Spring Creek and Ash Grove	43 01 46.811	15_1	17	5.2	1	−0.027	−0.047	43 01 46.737
Ash Grove and Butler	49 52 59.669	15_2	20	6.7	1	−0.027	−0.124	49 52 59.518
Butler and Pilot Grove	46 28 16.388	15_3	17	6.3	1	−0.118	+0.179	46 28 16.449
Butler and Rantoul	78 04 57.014	15_{3+4}	17	3.4	1	+0.092	+0.050	78 04 57.156
Pilot Grove and Rantoul	31 36 40.955	15_4	16	5.9	1	−0.119	−0.130	31 36 40.707
Rantoul and Spring Creek	189 00 16.495	15_5	16	4.1	1	−0.027	+0.121	189 00 16.589

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(15_1)+(15_2)+(15_3)+(15_4)+0.318=0$$
$$(15_1)+2(15_2)+(15_3)+(15_4)+0.318=0$$
$$(15_1)+(15_2)+3(15_3)+2(15_4)+0.647=0$$
$$(15_1)+(15_2)+2(15_3)+3(15_4)+0.647=0$$

PILOT GROVE—16.

[Observer, J. H. Darling. Instrument, Troughton & Simms theodolite No. 4. Date, September, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Fairmount and Palermo	1 37 45.272	16_1	20	6.1	1	+0.294	+0.153	1 37 45.719
Fairmount and Lynn Grove	39 44 54.319	16_{1+2}	16	5.4	1	−0.396	+0.193	39 44 54.116
Fairmount and Mayview	66 04 31.691	16_{1+2+3}	16	6.7	1	+0.118	+0.445	66 04 32.254
Palermo and Lynn Grove	38 07 08.063	16_2	20	6.1	1	+0.294	+0.040	38 07 08.397
Lynn Grove and Mayview	26 19 37.988	16_3	16	7.6	1	−0.102	+0.252	26 19 38.138
Mayview and Rantoul	59 45 02.209	16_4	16	3.4	1	+0.016	+0.129	59 45 02.354
Rantoul and Paxton	19 42 34.103	16_5	16	4.3	1	+0.065	+0.005	19 42 34.173
Rantoul and Butler	63 17 23.027	16_{5+6}	16	5.0	1	−0.049	−0.359	63 17 22.619
Paxton and Butler	43 34 48.744	16_6	16	6.4	1	+0.065	−0.364	43 34 48.445
Butler and Fairmount	170 53 02.973	16_7	16	6.3	1	+0.016	−0.215	170 53 02.774

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$4(16_1)+3(16_2)+2(16_3)+(16_4)+(16_5)+(16_6)-2.000=0$$
$$3(16_1)+4(16_2)+2(16_3)+(16_4)+(16_5)+(16_6)-2.000=0$$
$$2(16_1)+2(16_2)+3(16_3)+(16_4)+(16_5)+(16_6)-1.016=0$$
$$(16_1)+(16_2)+(16_3)+2(16_4)+(16_5)+(16_6)-0.648=0$$
$$(16_1)+(16_2)+(16_3)+(16_4)+3(16_5)+2(16_6)-0.828=0$$
$$(16_1)+(16_2)+(16_3)+(16_4)+2(16_5)+3(16_6)-0.828=0$$

SECTION XIV.—*Triangulation from the line Willow Springs-Morgan Park to the line Oakland-Kansas*—Continued.

RANTOUL—17.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Dates, September and October, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Paxton and Butler	66 45 36.243	17_1	16	5.5	1	−0.175	+0.206	66 45 36.274
Paxton and Pilot Grove	128 40 44.634	17_{1+2}	4	2.1	0.25	+1.063	+0.006	128 40 45.703
Butler and Pilot Grove	61 55 09.856	17_2	16	7.0	1	−0.226	−0.200	61 55 09.430
Butler and Mayview	131 54 53.280	$17_{2+3+4+5}$	4	2.4	0.25	+0.205	−0.399	131 54 53.086
Pilot Grove and Fairmount	25 37 44.008	17_3	14	5.9	1	+0.016	−0.476	25 37 43.548
Pilot Grove and Mayview	69 59 43.832	17_{3+4+5}	16	4.6	1	+0.023	−0.198	69 59 43.657
Fairmount and Lynn Grove	35 34 07.260	17_4	14	5.0	1	+0.016	−0.277	35 34 06.099
Lynn Grove and Mayview	8 47 52.539	17_5	16	2.3	1	+0.016	+0.555	8 47 53.110
Mayview and Paxton	161 19 30.357	17_6	16	3.9	1	+0.091	+0.192	161 19 30.640

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.25(17_1)+1.25(17_2)+\quad(17_3)+\quad(17_4)+\quad(17_5)+0.629=0$$
$$1.25(17_1)+2.50(17_2)+1.25(17_3)+1.25(17_4)+1.25(17_5)+0.725=0$$
$$(17_1)+1.25(17_2)+3.25(17_3)+2.25(17_4)+2.25(17_5)+0.334=0$$
$$(17_1)+1.25(17_2)+2.25(17_3)+3.25(17_4)+2.25(17_5)+0.334=0$$
$$(17_1)+1.25(17_2)+2.25(17_3)+2.25(17_4)+3.25(17_5)+0.334=0$$

FAIRMOUNT—18.

[Observers, G. Y. Wisner. Instrument, Pistor & Martins theodolite No. 2. Date, September, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Palermo and Lynn Grove	68 16 31.729	18_1	16	7.8	1	+0.347	+0.250	68 16 32.326
Palermo and Mayview	106 37 36.272	18_{1+2}	6	4.4	0.5	−1.031	+0.040	106 37 35.281
Lynn Grove and Mayview	38 21 03.013	18_2	22	6.8	1	+0.152	−0.210	38 21 02.955
Lynn Grove and Rantoul	79 40 16.866	18_{2+3}	16	3.1	1	+0.195	−0.918	79 40 16.143
Mayview and Rantoul	41 19 14.620	18_3	15	3.9	1	−0.725	−0.708	41 19 13.187
Mayview and Pilot Grove	69 51 55.142	18_{3+4}	22	7.8	1	+0.361	+0.381	69 51 55.884
Rantoul and Pilot Grove	28 32 42.138	18_4	16	6.6	1	−0.530	+1.089	28 32 42.697
Pilot Grove and Palermo	183 30 29.424	18_5	16	4.0	1	−0.168	−0.421	183 30 28.835

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.5(18_1)+1.5(18_2)+\ (18_3)+\ (18_4)+0.159=0$$
$$1.5(18_1)+3.5(18_2)+2(18_3)+\ (18_4)+0.926=0$$
$$(18_1)+2.0(18_2)+4(18_3)+2(18_4)+3.307=0$$
$$(18_1)+\quad(18_2)+2(18_3)+3(18_4)+2.540=0$$

MAYVIEW—19.

[Observer, A. R. Flint. Instrument, Troughton & Simms theodolite No. 1. Date, September, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Rantoul and Pilot Grove	50 15 16.085	19_1	20	5.0	1	+0.179	−1.047	50 15 15.217
Pilot Grove and Fairmount	44 03 31.885	19_2	17	7.3	1	+0.179	+0.975	44 03 33.039
Fairmount and Lynn Grove	63 43 55.482	19_3	17	4.3	1	+0.179	+0.091	63 43 55.752
Lynn Grove and Rantoul	201 57 15.834	19_4	16	4.1	1	+0.178	−0.019	201 57 15.993

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(19_1)+\ (19_2)+\ (19_3)-0.714=0$$
$$(19_1)+2(19_2)+\ (19_3)-0.714=0$$
$$(19_1)+\ (19_2)+2(19_3)-0.714=0$$

SECTION XIV.—*Triangulation from the line Willow Springs-Morgan Park to the line Oakland-Kansas*—Continued.

PALERMO—20.

[Observer, J. H. Darling. Instrument, Troughton & Simms theodolite No. 4. Dates, September and October, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Kansas and Oakland	39 34 57.582	20_1	19	5.3	1	+0.252	−0.086	39 34 57.748
Oakland and Lynn Grove	77 34 05.829	20_2	16	6.0	1	−0.060	−0.213	77 34 05.556
Oakland and Fairmount	146 57 25.528	20_{2+3+4}	12	3.7	1	+0.312	+0.119	146 57 25.959
Lynn Grove and Pilot Grove	67 30 37.143	20_3	16	7.2	1	−0.225	+0.320	67 30 37.238
Lynn Grove and Fairmount	69 23 19.907	20_{3+4}	16	3.8	1	−0.164	+0.332	69 23 20.403
Pilot Grove and Fairmount	1 52 43.378	20_4	16	2.6	1	−0.225	+0.012	1 52 43.165
Fairmount and Kansas	173 27 36.075	20_5	18	4.3	1	+0.252	−0.033	173 27 36.294

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(20_1)+\ (20_2)+\ (20_3)+\ (20_4)+0.007=0$$
$$(20_1)+3(20_2)+2(20_3)+2(20_4)+0.829=0$$
$$(20_1)+2(20_2)+4(20_3)+3(20_4)+1.443=0$$
$$(20_1)+2(20_2)+3(20_3)+4(20_4)+1.443=0$$

LYNN GROVE—21.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Date, September, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Mayview and Rantoul	13 09 23.188	21_1	16	3.9	1	+0.025	+0.040	13 09 23.253
Rantoul and Pilot Grove	32 43 31.249	21_2	16	5.0	1	+0.025	−0.431	32 43 30.843
Pilot Grove and Fairmount	32 02 08.605	21_3	16	8.4	1	+0.026	−0.497	32 02 08.134
Fairmount and Palermo	42 20 07.326	21_4	16	4.2	1	+0.025	+0.846	42 20 08.197
Palermo and Oakland	52 16 56.483	21_5	18	5.1	1	+0.025	−0.009	52 16 56.499
Oakland and Mayview	187 27 52.997	21_6	16	4.3	1	+0.025	+0.051	187 27 53.073

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(21_1)+\ (21_2)+\ (21_3)+\ (21_4)+\ (21_5)-0.152=0$$
$$(21_1)+2(21_2)+\ (21_3)+\ (21_4)+\ (21_5)-0.152=0$$
$$(21_1)+\ (21_2)+2(21_3)+\ (21_4)+\ (21_5)-0.152=0$$
$$(21_1)+\ (21_2)+\ (21_3)+2(21_4)+\ (21_5)-0.152=0$$
$$(21_1)+\ (21_2)+\ (21_3)+\ (21_4)+2(21_5)-0.152=0$$

KANSAS—22.

[Observer, G. Y. Wisner. Instrument, Pistor & Martins theodolite No. 2. Dates, September and October, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angle.
	° ′ ″			″		″	″	° ′ ″
Oakland and Palermo	34 59 55.996	22_4	16	4.1	1	+0.276	−0.053	34 59 56.219

NOTE.—See section XV of the adjustment for normal equations for local adjustment, and local correction to 22_4.

SECTION XIV.—*Triangulation from the line Willow Springs–Morgan Park to the line Oakland–Kansas*—Continued.

OAKLAND—23.

[Observer, A. R. Flint. Instrument, Troughton & Simms theodolite No. 1. Date, September, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Lynn Grove and Palermo	50 08 59.831	23_1	16	3.5	1	−0.453	−0.060	50 08 59.318
Palermo and Kansas	105 25 07.523	23_2	16	4.5	1	+0.115	−0.053	105 25 07.585
Palermo and Westfield	141 21 09.518	23_{2+3}	8	5.1	0.5	−0.323	....	....
Kansas and Westfield	35 56 01.443	23_3	16	3.4	1	+0.115	+0.087	35 56 01.645
Westfield and Lynn Grove	168 29 51.880	23_4	16	5.3	1	−0.453	....	....
Westfield and Palermo	218 38 50.669	23_{4+1}	8	4.2	0.5	+0.130	....	....

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(23_1)+(23_2)+(23_3)+0.677=0$$
$$(23_1)+3(23_2)+2(23_3)-0.120=0$$
$$(23_1)+2(23_2)+3(23_3)-0.120=0$$

NOTE.—The general correction of 23_3 is taken from Section XV of the adjustment.

Numerical equations of condition in the triangulation from the line Willow Springs–Morgan Park to the line Oakland–Kansas.

SIDE-EQUATIONS.

IX. (12) $-11.23363\,[7_3]-2.98067\,[7_4]-11.35925\,[8_3]+18.95853\,[8_4]-22.67882\,[9_1]+3.41374\,[9_2]+18.546=0$

XIV. (12) $-14.51686\,[12_1]+9.30196\,[12_2]-7.32972\,[13_3]+17.66514\,[13_4]-13.89060\,[14_3]+5.75918\,[14_4]-21.739=0$

XXI. (10) $-15.55703\,[15_3]+4.44373\,[15_4]-10.59439\,[16_5]+11.53102\,[16_6]-9.04167\,[17_1]+11.23329\,[17_2]+11.730=0$

XXII. (11) $-9.34115\,[16_1]-9.34115\,[16_2]-9.34115\,[16_3]+12.27872\,[16_4]-7.66533\,[17_3]+13.86060\,[17_4]+13.86060\,[17_5]-16.22979\,[18_3]+7.71952\,[18_4]-24.833=0$

XXIV. (8) $-25.31763\,[16_1]+1.51673\,[16_2]-8.38928\,[18_1]-6.92923\,[18_2]-6.92923\,[18_3]-6.92923\,[18_4]-0.79815\,[20_3]+7.91878\,[20_4]+7.251=0$

XXIX. (7) $-7.91763\,[17_4]+21.52593\,[17_5]+1.58811\,[19_1]+1.58811\,[19_2]+10.39144\,[19_3]-4.50710\,[21_1]+5.41820\,[21_2]+5.41820\,[21_3]-9.781=0$

XXXIII. (18) $+6.92923\,[18_2]+14.64875\,[18_3]+14.64875\,[18_4]-28.51493\,[19_2]-6.75645\,[19_3]-20.41688\,[21_1]-20.41688\,[21_2]+33.64864\,[21_3]+33.018=0$

NOTE.—In the solution for determining the general corrections, each of the side-equations was divided by the number inclosed in parenthesis and placed opposite it.

ANGLE-EQUATIONS.

I. $[1_1]+[2_2]+[3_1]-0.415=0$

II. $[2_1]+[3_2]+[4_4]-0.483=0$

III. $[3_3]+[4_3]+[5_1]+0.198=0$

IV. $[4_1]+[6_3]+[7_2]-1.114=0$

V. $[4_2]+[5_2]+[7_1]-0.570=0$

VI. $[6_1]+[6_2]+[7_3]+[8_4]-0.485=0$

VII. $[6_1]+[8_3]+[8_4]+[9_2]-1.207=0$

VIII. $[6_2]+[7_3]+[7_4]+[9_1]-1.915=0$

X. $[8_1]+[10_3]+[11_2]-0.383=0$

XI. $[8_2]+[9_3]+[11_1]+1.009=0$

XII. $[10_1]+[12_3]+[13_2]-2.547=0$

Numerical equations of condition, &c.—Continued.

ANGLE-EQUATIONS—Continued.

XIII.	$[10_2]+[11_3]+[13_1]$	$+2.009=0$
XV.	$[12_1]+[14_4]+[14_1]+[15_2]$	$+0.523=0$
XVI.	$[12_2]+[13_3]+[13_4]+[15_1]$	$+0.086=0$
XVII.	$[13_4]+[14_3]+[15_1]+[15_2]$	$+0.393=0$
XVIII.	$[14_1]+[14_2]+[15_3]+[16_6]$	$+0.303=0$
XIX.	$[14_1]+[16_5]+[16_6]+[17_2]$	$+0.709=0$
XX.	$[14_2]+[15_3]+[15_4]+[17_1]$	$-0.287=0$
XXIII.	$[16_1]+[16_2]+[16_5]+[18_3]+[18_4]+[19_2]$	$-1.801=0$
XXV.	$[16_2]+[20_5]+[21_3]+[21_4]$	$-0.710=0$
XXVI.	$[16_3]+[16_4]+[17_3]+[17_4]+[21_2]$	$+0.803=0$
XXVII.	$[16_4]+[17_3]+[17_4]+[17_5]+[19_1]$	$+1.116=0$
XXVIII.	$[17_4]+[17_5]+[18_3]+[19_1]+[19_2]$	$+0.502=0$
XXX.	$[17_4]+[18_2]+[18_4]+[21_2]+[21_3]$	$+2.123=0$
XXXI.	$[18_1]+[20_3]+[20_4]+[21_4]$	$-1.428=0$
XXXII.	$[18_2]+[19_3]+[21_1]+[21_2]+[21_3]$	$+1.007=0$
XXXIV.	$[20_1]+[22_4]+[23_2]$	$+0.192=0$
XXXV.	$[20_2]+[21_5]+[23_1]$	$+0.282=0$

General corrections in terms of the correlates.

$[1_1]$	=+0.50000 I				
$[2_1]$	=−0.33333 I	+0.66667 II			
$[2_2]$	=+0.66667 I	−0.33333 II			
$[3_1]$	=+0.58333 I	−0.25000 II	−0.08333 III		
$[3_2]$	=−0.25000 I	+0.58333 II	−0.25000 III		
$[3_3]$	=−0.08333 I	−0.25000 II	+0.58333 III		
$[4_1]$	=−0.12500 II	−0.12500 III	+0.68750 IV	−0.31250 V	
$[4_2]$	=−0.12500 II	−0.12500 III	−0.31250 IV	+0.68750 V	
$[4_3]$	=−0.25000 II	−0.75000 III	−0.12500 IV	−0.12500 V	
$[4_4]$	=+0.75000 II	−0.25000 III	−0.12500 IV	−0.12500 V	
$[5_1]$	=+0.66667 III	−0.33333 V			
$[5_2]$	=−0.33333 III	+0.66667 V			
$[6_1]$	=−0.16667 IV	+0.33333 VI	+0.66667 VII	−0.33333 VIII	
$[6_2]$	=−0.16667 IV	+0.33333 VI	−0.33333 VII	+0.66667 VIII	
$[6_3]$	=+0.66667 IV	−0.33333 VI	−0.16667 VII	−0.16667 VIII	
$[7_1]$	=−0.38095 IV	+0.61905 V	−0.04762 VI	−0.09524 VIII	+0.05641 IX
$[7_2]$	=+0.61905 IV	−0.38095 V	−0.04762 VI	−0.09524 VIII	+0.05641 IX
$[7_3]$	=−0.04762 IV	−0.04762 V	+0.61905 VI	+0.23810 VIII	−0.48489 IX
$[7_4]$	=−0.04762 IV	−0.04762 V	−0.38095 VI	+0.23810 VIII	+0.20286 IX
$[8_1]$	=−0.09091 VI	−0.18182 VII	−0.05758 IX	+0.63636 X	−0.09091 XI
$[8_2]$	=−0.27273 VI	−0.54545 VII	−0.17272 IX	−0.09091 X	+0.72727 XI
$[8_3]$	=−0.27273 VI	+0.45455 VII	−1.11931 IX	−0.09091 X	−0.27273 XI
$[8_4]$	=+0.72727 VI	+0.45455 VII	+1.10716 IX	−0.09091 X	−0.27273 XI
$[9_1]$	=−0.29167 VII	+0.54167 VIII	−1.10667 IX	−0.04167 XI	
$[9_2]$	=+0.54167 VII	−0.29167 VIII	+0.70531 IX	−0.20833 XI	
$[9_3]$	=−0.20833 VII	−0.04167 VIII	+0.01948 IX	+0.54167 XI	
$[10_1]$	=−0.25000 X	+0.75000 XII	−0.25000 XIII		
$[10_2]$	=−0.25000 X	−0.25000 XII	+0.75000 XIII		
$[10_3]$	=+0.75000 X	−0.25000 XII	−0.25000 XIII		
$[11_1]$	=−0.12500 X	+0.62500 XI	−0.12500 XIII		
$[11_2]$	=+0.62500 X	−0.12500 XI	−0.37500 XIII		

General corrections in terms of the correlates—Continued.

[11_3]	=—0.37500 X	—0.12500 XI	+0.62500 XIII		
[12_1]	=—0.79866 XIV	+0.50000 XV	—0.25000 XVI		
[12_2]	=—0.25000 XII	+0.69002 XIV	—0.25000 XV	+0.50000 XVI	
[12_3]	=+0.50000 XII	—0.19379 XIV	—0.25000 XVI		
[13_1]	=—0.27273 XII	+0.72727 XIII	—0.07830 XIV	—0.18182 XVI	—0.09091 XVII
[13_2]	=+0.72727 XII	—0.27273 XIII	—0.07830 XIV	—0.18182 XVI	—0.09091 XVII
[13_3]	=—0.09091 XII	—0.09091 XIII	—0.92401 XIV	+0.27273 XVI	—0.36364 XVII
[13_4]	=—0.09091 XII	—0.09091 XIII	+1.15889 XIV	+0.27273 XVI	+0.63636 XVII
[14_1]	=+0.06160 XIV	—0.18182 XV	—0.09091 XVII	+0.45455 XVIII	+0.72727 XIX
	—0.27273 XX				
[14_2]	=+0.06160 XIV	—0.18182 XV	—0.09091 XVII	+0.45455 XVIII	—0.27273 XIX
	+0.72727 XX				
[14_3]	=—0.91114 XIV	+0.27273 XV	+0.63636 XVII	—0.18182 XVIII	—0.09091 XIX
	—0.09091 XX				
[14_4]	=+0.72634 XIV	+0.27273 XV	—0.36364 XVII	—0.18182 XVIII	—0.09091 XIX
	—0.09091 XX				
[15_1]	=—0.27273 XV	+0.72727 XVI	+0.45455 XVII	—0.09091 XVIII	—0.18182 XX
	+0.10103 XXI				
[15_2]	=+0.72727 XV	—0.27273 XVI	+0.45455 XVII	—0.09091 XVIII	—0.18182 XX
	+0.10103 XXI				
[15_3]	=—0.09091 XV	—0.09091 XVI	—0.18182 XVII	+0.63636 XVIII	+0.27273 XX
	—1.15158 XXI				
[15_4]	=—0.09091 XV	—0.09091 XVI	—0.18182 XVII	—0.36364 XVIII	+027273 XX
	+0.84849 XXI				
[16_1]	=—0.01266 XVIII	—0.02532 XIX	—0.00119 XXI	—0.12839 XXII	+0.10127 XXIII
	—2.03492 XXIV	—0.37975 XXV	—0.17721 XXVI	—0.03797 XXVII	
[16_2]	=—0.01266 XVIII	—0.02532 XIX	—0.00119 XXI	—0.12839 XXII	+0.10127 XXIII
	+1.31938 XXIV	+0.62025 XXV	—0.17721 XXVI	—0.03797 XXVII	
[16_3]	=—0.03797 XVIII	—0.07595 XIX	—0.00356 XXI	—0.38516 XXII	+0.30380 XXIII
	+0.41425 XXIV	—0.13924 XXV	+0.46835 XXVI	—0.11392 XXVII	
[16_4]	=—0.10127 XVIII	—0.20253 XIX	—0.00948 XXI	+0.93838 XXII	—0.18987 XXIII
	+0.11298 XXIV	—0.03797 XXV	+0.58228 XXVI	+0.69620 XXVII	
[16_5]	=—0.36709 XVIII	+0.26582 XIX	—1.09382 XXI	—0.05929 XXII	—0.06329 XXIII
	+0.03766 XXIV	—0.01266 XXV	—0.13924 XXVI	—0.10127 XXVII	
[16_6]	=+0.63291 XVIII	+0.26582 XIX	+1.11872 XXI	—0.05929 XXII	—0.06329 XXIII
	+0.03766 XXIV	—0.01266 XXV	—0.13924 XXVI	—0.10127 XXVII	
[17_1]	=—0.26027 XIX	+0.64381 XX	—0.87448 XXI	—0.07494 XXII	—0.08218 XXVI
	—0.12327 XXVII	—0.08218 XXVIII	—0.07990 XXIX	—0.04109 XXX	
[17_2]	=+0.63286 XIX	—0.26027 XX	+0.94624 XXI	—0.12487 XXII	—0.13698 XXVI
	—0.20547 XXVII	—0.13698 XXVIII	—0.13315 XXIX	—0.06849 XXX	
[17_3]	=—0.06849 XIX	—0.04109 XX	—0.03978 XXI	—1.19637 XXII	+0.45206 XXVI
	+0.17809 XXVII	—0.54794 XXVIII	—0.53261 XXIX	—0.27397 XXX	
[17_4]	=—0.06849 XIX	—0.04109 XX	—0.03978 XXI	+0.76053 XXII	+0.45206 XXVI
	+0.17809 XXVII	+0.45206 XXVIII	—1.66370 XXIX	+0.72603 XXX	
[17_5]	=—0.06849 XIX	—0.04109 XX	—0.03978 XXI	+0.76053 XXII	—0.54794 XXVI
	+0.17809 XXVII	+0.45206 XXVIII	+2.54252 XXIX	—0.27397 XXX	
[18_1]	=—0.16586 XXII	—0.10145 XXIII	—0.31921 XXIV	+0.04348 XXVIII	—0.18840 XXX
	+0.57971 XXXI	—0.23188 XXXII	—0.17183 XXXIII		
[18_2]	=+0.36143 XXII	—0.15942 XXIII	—0.04556 XXIV	—0.21739 XXVIII	+0.27537 XXX
	—0.23188 XXXI	+0.49276 XXXII	+0.05995 XXXIII		
[18_3]	=—0.88871 XXII	+0.21739 XXIII	—0.04560 XXIV	+0.47826 XXVIII	+0.26087 XXX
	+0.04348 XXXI	—0.21739 XXXII	+0.09324 XXXIII		

General corrections in terms of the correlates—Continued.

[18_4]	=+0.76121 XXII	+0.27536 XXIII	−0.13674 XXIV	−0.26087 XXVIII	−0.20290 XXX
	−0.14493 XXXI	+0.05797 XXXII	+0.24641 XXXIII		
[19_1]	=−0.25000 XXIII	+0.75000 XXVII	+0.50000 XXVIII	−0.25769 XXIX	0.25000 XXXII
	+0.48988 XXXIII				
[19_2]	=+0.75000 XXIII	−0.25000 XXVII	+0.50000 XXVIII	−0.25769 XXIX	−0.25000 XXXII
	−1.09428 XXXIII				
[19_3]	=−0.25000 XXIII	−0.25000 XXVII	−0.50000 XXVIII	+0.99993 XXIX	+0.75000 XXXII
	+0.11452 XXXIII				
[20_1]	=−0.04239 XXIV	−0.04762 XXV	−0.09524 XXXI	+0.61905 XXXIV	−0.14286 XXXV
[20_2]	=−0.12716 XXIV	−0.14286 XXV	−0.28571 XXXI	−0.14286 XXXIV	+0.57143 XXXV
[20_3]	=−0.43885 XXIV	+0.61905 XXV	+0.23810 XXXI	−0.04762 XXXIV	−0.14286 XXXV
[20_4]	=+0.65077 XXIV	−0.38095 XXV	+0.23810 XXXI	−0.04762 XXXIV	−0.14286 XXXV
[21_1]	=−0.33333 XXV	−0.16667 XXVI	−0.79457 XXIX	−0.33333 XXX	−0.16667 XXXI
	+0.50000 XXXII	−1.06774 XXXIII	−0.16667 XXXV		
[21_2]	=−0.33333 XXV	+0.83333 XXVI	+0.62333 XXIX	+0.66667 XXX	−0.16667 XXXI
	+0.50000 XXXII	−1.06774 XXXIII	−0.16667 XXXV		
[21_3]	=+0.66667 XXV	−0.16667 XXVI	+0.62333 XXIX	+0.66667 XXX	−0.16667 XXXI
	+0.50000 XXXII	+1.93591 XXXIII	−0.16667 XXXV		
[21_4]	=+0.66667 XXV	−0.16667 XXVI	−0.15070 XXIX	−0.33333 XXX	+0.83333 XXXI
	−0.50000 XXXII	+0.06654 XXXIII	−0.16667 XXXV		
[21_5]	=−0.33333 XXV	−0.16667 XXVI	−0.15070 XXIX	−0.33333 XXX	−0.16667 XXXI
	−0.50000 XXXII	+0.06654 XXXIII	+0.83333 XXXV		
[22_4]	=+1.00000 XXXIV				
[23_1]	=+1.00000 XXXV				
[23_2]	=+1.00000 XXXIV				

Normal equations for determining the correlates.

No. of equation					
1.	0=−0.41500	+1.75000 I	−0.58333 II	−0.08333 III	
2.	0=−0.48300	−0.58333 I	+2.00000 II	−0.50000 III	−0.12500 IV
		−0.12500 V			
3.	0=+0.19800	−0.08333 I	−0.50000 II	+2.00000 III	−0.12500 IV
		−0.45833 V			
4.	0=−1.11400	−0.12500 II	−0.12500 III	+1.97322 IV	−0.69345 V
		−0.38095 VI	−0.16667 VII	−0.26191 VIII	+0.05641 IX
5	0=−0.57000	−0.12500 II	−0.45833 III	−0.69345 IV	+1.97322 V
		−0.04762 VI	−0.09524 VIII	+0.05641 IX	
6.	0=−0.48500	−0.38095 IV	−0.04762 V	+2.01299 VI	+0.78788 VII
		+0.57143 VIII	+0.92227 IX	−0.09091 X	−0.27273 XI
7.	0=−1.20700	−0.16667 IV	+0.78788 VI	+2.11743 VII	−0.62500 VIII
		+0.99316 IX	−0.18182 X	−0.75378 XI	
8.	0=−1.91500	−0.26191 IV	−0.09524 V	+0.57143 VI	−0.62500 VII
		+1.68453 VIII	−1.38870 IX	−0.04167 XI	
9.	0=+1.54550	+0.05641 IV	+0.05641 V	+0.92227 VI	+0.99316 VII
		−1.38870 VIII	+5.97836 IX	−0.05758 X	−0.15324 XI
10.	0=−0.38300	−0.09091 VI	−0.18182 VII	−0.05758 IX	+2.01136 X
		−0.21591 XI	−0.25000 XII	−0.62500 XIII	
11.	0=+1.00900	−0.27273 VI	−0.75378 VII	−0.04167 VIII	−0.15324 IX
		−0.21591 X	+1.89394 XI	−0.12500 XIII	
12.	0=−2.54700	−0.25000 X	+1.97727 XII	−0.52273 XIII	−0.27209 XIV
		−0.43182 XVI	−0.09091 XVII		
13.	0=+2.00900	−0.62500 X	−0.12500 XI	−0.52273 XII	+2.10227 XIII
		−0.07830 XIV	−0.18182 XVI	−0.09091 XVII	

Normal equations for determining the correlates—Continued.

No. of equation.					
14..	0=−1.81160	−0.27209 XII	−0.07830 XIII	+5.17471 XIV	0.98346 XV
		−0.92490 XVI	+0.24775 XVII	+0.12320 XVIII	+0.06160 XIX
		+0.06160 XX			
15.	0=+0.52300	−0.98346 XIV	+1.77273 XV	−0.52273 XVI	+0.72727 XVII
		−0.45455 XVIII	−0.18182 XIX	−0.36364 XX	+0.10103 XXI
16.	0=+0.08600	−0.43182 XII	−0.18182 XIII	+0.92490 XIV	−0.52273 XV
		+1.77273 XVI	+0.72727 XVII	−0.09091 XVIII	−0.18182 XX
		+0.10103 XXI			
17.	0=+0.39300	−0.09091 XII	−0.09091 XIII	+0.24775 XIV	+0.72727 XV
		+0.72727 XVI	+2.18182 XVII	−0.36364 XVIII	−0.09091 XIX
		−0.45455 XX	+0.20206 XXI		
18	0=+0.30300	+0.12320 XIV	−0.45455 XV	−0.09091 XVI	−0.36364 XVII
		+2.17837 XVIII	+0.72036 XIX	+0.72727 XX	−0.03286 XXI
		−0.05929 XXII	−0.06329 XXIII	+0.03766 XXIV	−0.01266 XXV
		−0.13924 XXVI	−0.10127 XXVII		
19.	0=+0.70900	+0.06160 XIV	−0.18182 XV	−0.09091 XVII	+0.72036 XVIII
		+1.89177 XIX	−0.53300 XX	+0.97114 XXI	−0.24345 XXII
		−0.12658 XXIII	+0.07532 XXIV	−0.02532 XXV	−0.41546 XXVI
		−0.40801 XXVII	−0.13698 XXVIII	−0.13315 XXIX	−0.06849 XXX
20.	0=+0.28700	+0.06160 XIV	−0.36364 XV	−0.18182 XVI	−0.45455 XVII
		+0.72727 XVIII	−0.53300 XIX	+1.91654 XX	−1.17757 XXI
		−0.07494 XXII	−0.08218 XXVI	−0.12327 XXVII	−0.08218 XXVIII
		−0.07990 XXIX	−0.04109 XXX		
21	0=+1.17300	+0.10103 XV	+0.10103 XVI	+0.20206 XVII	−0.03286 XVIII
		+0.97114 XIX	−1.17757 XX	+6.47100 XXI	−0.07807 XXII
		−0.00594 XXIII	+0.00353 XXIV	−0.00119 XXV	−0.09260 XXVI
		−0.12882 XXVII	−0.07956 XXVIII	−0.07733 XXIX	−0.03978 XXX
22.	0=−2.25800	−0.05929 XVIII	−0.24345 XIX	−0.07494 XX	−0.07807 XXI
		+6.18835 XXII	−0.76944 XXIII	+0.35328 XXIV	−0.12839 XXV
		+0.11738 XXVI	+1.26307 XXVII	+0.63235 XXVIII	+1.47850 XXIX
		+0.23325 XXX	−0.16586 XXXI	+0.36143 XXXII	+0.03535 XXXIII
23.	0=−1.80100	−0.06329 XVIII	−0.12658 XIX	−0.00594 XXI	−0.76944 XXII
		+1.74909 XXIII	−0.48363 XXIV	+0.10126 XXV	+0.11393 XXVI
		−0.43986 XXVII	+0.71739 XXVIII	−0.25769 XXIX	+0.05797 XXX
		−0.10145 XXXI	−0.40942 XXXII	−0.75463 XXXIII	
24	0=+0.90610	+0.03766 XVIII	+0.07532 XIX	+0.00353 XXI	+0.35328 XXII
		−0.48363 XXIII	+7.91013 XXIV	+0.88053 XXV	+0.52723 XXVI
		+0.11298 XXVII	−0.04560 XXVIII	−0.09116 XXX	−0.10729 XXXI
		−0.04556 XXXII	−0.16593 XXXIII	−0.04239 XXXIV	−0.12716 XXXV
25.	0=−0.71000	−0.01266 XVIII	−0.02532 XIX	−0.00119 XXI	−0.12839 XXII
		+0.10126 XXIII	+0.88053 XXIV	+2.57264 XXV	−0.51054 XXVI
		−0.03797 XXVII	+0.47263 XXIX	+0.33333 XXX	+0.90476 XXXI
		+2.00245 XXXIII	−0.04762 XXXIV	−0.47619 XXXV	
26	0=+0.80300	−0.13924 XVIII	−0.41546 XIX	−0.08218 XX	−0.09260 XXI
		+0.11738 XXII	+0.11393 XXIII	+0.52723 XXIV	−0.51054 XXV
		+2.78808 XXVI	+0.93846 XXVII	−0.09588 XXVIII	−1.57298 XXIX
		+1.11873 XXX	−0.16667 XXXI	+0.50000 XXXII	−1.06774 XXXIII
		−0.16667 XXXV			
27	0=+1.11600	−0.10127 XVIII	−0.40801 XIX	−0.12327 XX	−0.12882 XXI
		+1.26307 XXII	−0.43986 XXIII	+0.11298 XXIV	−0.03797 XXV
		+0.93846 XXVI	+1.98047 XXVII	+0.85618 XXVIII	+0.08852 XXIX
		+0.17809 XXX	−0.25000 XXXII	+0.48988 XXXIII	

Normal equations for determining the correlates—Continued.

No. of equation.					
28.	0=+0.50200	−0.13698 XIX	−0.08218 XX	−0.07956 XXI	+0.63235 XXII
		+0.71739 XXIII	−0.04560 XXIV	−0.09588 XXVI	+0.85618 XXVII
		+2.38238 XXVIII	+0.36344 XXIX	+0.71293 XXX	+0.04348 XXXI
		−0.71739 XXXII	−0.51116 XXXIII		
29.	0=−1.39730	−0.13315 XIX	−0.07990 XX	−0.07733 XXI	+1.47850 XXII
		−0.25769 XXIII	+0.47263 XXV	−1.57298 XXVI	+0.08852 XXVII
		+0.36344 XXVIII	+12.54440 XXIX	−0.41704 XXX	−0.15070 XXXI
		+1.45202 XXXII	+1.39236 XXXIII	−0.15070 XXXV	
30.	0=+2.12300	−0.06849 XIX	−0.04109 XX	−0.03978 XXI	+0.23325 XXII
		+0.05797 XXIII	−0.09116 XXIV	+0.33333 XXV	+1.11873 XXVI
		+0.17809 XXVII	+0.71293 XXXIII	−0.41704 XXIX	+2.59561 XXX
		−0.52174 XXXI	+1.27537 XXXII	+1.02136 XXXIII	−0.33333 XXXV
31.	0=−1.42800	−0.16586 XXII	−0.10145 XXIII	−0.10729 XXIV	+0.90476 XXV
		−0.16667 XXVI	+0.04348 XXVIII	−0.15070 XXIX	−0.52174 XXX
		+1.88924 XXXI	−0.73188 XXXII	−0.10529 XXXIII	−0.09524 XXXIV
		−0.45239 XXXV			
32.	0=+1.00700	+0.36143 XXII	−0.40942 XXIII	−0.04556 XXIV	+0.50000 XXVI
		−0.25000 XXVII	−0.71739 XXVIII	+1.45202 XXIX	+1.27537 XXX
		−0.73188 XXXI	+2.74276 XXXII	−0.02510 XXXIII	−0.50000 XXXV
33.	0=+1.83430	+0.03535 XXII	−0.75463 XXIII	−0.16593 XXIV	+2.00245 XXV
		−1.06774 XXVI	+0.48988 XXVII	−0.51116 XXVIII	+1.39236 XXIX
		+1.02136 XXX	−0.10529 XXXI	−0.02510 XXXII	+8.03114 XXXIII
		+0.06654 XXXV			
34.	0=+0.19200	−0.04239 XXIV	−0.04762 XXV	−0.09524 XXXI	+2.61905 XXXIV
		−0.14286 XXXV			
35.	0=+0.28200	−0.12716 XXIV	−0.47619 XXV	−0.16667 XXVI	−0.15070 XXIX
		−0.33333 XXX	−0.45239 XXXI	−0.50000 XXXII	+0.06654 XXXIII
		−0.14286 XXXIV	+2.40476 XXXV		

Values of the correlates and their logarithms.

I =+0.4553	log 9.6582690+	XIX =−0.3099	log 9.4912216−
II =+0.6023	log 9.7798201+	XX =−0.1295	log 9.1123368−
III =+0.3645	log 9.5617333+	XXI =−0.1608	log 9.2064211−
IV =+1.2477	log 0.0961241+	XXII =+0.7226	log 9.8589220+
V =+0.9425	log 9.9742952+	XXIII =+1.3188	log 0.1201822+
VI =−0.9194	log 9.9635139−	XXIV =−0.0949	log 8.9773120−
VII =+1.7227	log 0.2362223+	XXV =+0.2054	log 9.3125581+
VIII =+2.4643	log 0.3916971+	XXVI =+0.2069	log 9.3157185+
IX =+0.1541	log 9.1877463+	XXVII =−0.6694	log 9.8256662−
X =+0.3071	log 9.4872232+	XXVIII =−0.3736	log 9.5723720−
XI =+0.0868	log 8.9385197+	XXIX =+0.0644	log 8.8090881+
XII =+1.2501	log 0.0969517+	XXX =−0.7686	log 9.8857286−
XIII =−0.5362	log 9.7293592−	XXXI =+0.5902	log 9.7709992+
XIV =+0.4120	log 9.6149394+	XXXII =+0.0061	log 7.7860412+
XV =+0.0151	log 8.1792645+	XXXIII =−0.0214	log 8.3312248−
XVI =+0.0924	log 8.9658599+	XXXIV =−0.0529	log 8.7237019−
XVII =−0.2601	log 9.4151737	XXXV =−0.0600	log 8.7784269−
XVIII =−0.0126	log 8.1003705−		

Values of the general corrections.

	″		″		″		″
$[1_1]$	$=+0.227$	$[8_2]$	$=-0.680$	$[14_1]$	$=-0.150$	$[18_1]$	$=+0.250$
$[2_1]$	$=+0.250$	$[8_3]$	$=+0.810$	$[14_2]$	$=-0.031$	$[18_2]$	$=-0.210$
$[2_2]$	$=+0.103$	$[8_4]$	$=+0.280$	$[14_3]$	$=-0.495$	$[18_3]$	$=-0.708$
$[3_1]$	$=+0.085$	$[9_1]$	$=+0.658$	$[14_4]$	$=+0.440$	$[18_4]$	$=+1.089$
$[3_2]$	$=+0.146$	$[9_2]$	$=+0.305$	$[15_1]$	$=-0.047$	$[19_1]$	$=-1.047$
$[3_3]$	$=+0.024$	$[9_3]$	$=-0.412$	$[15_2]$	$=-0.124$	$[19_2]$	$=+0.975$
$[4_1]$	$=+0.442$	$[10_1]$	$=+0.995$	$[15_3]$	$=+0.179$	$[19_3]$	$=+0.091$
$[4_2]$	$=+0.137$	$[10_2]$	$=-0.791$	$[15_4]$	$=-0.130$	$[20_1]$	$=-0.086$
$[4_3]$	$=-0.151$	$[10_3]$	$=+0.052$	$[16_1]$	$=+0.153$	$[20_2]$	$=-0.213$
$[4_4]$	$=+0.087$	$[11_1]$	$=+0.083$	$[16_2]$	$=+0.040$	$[20_3]$	$=+0.320$
$[5_1]$	$=-0.071$	$[11_2]$	$=+0.382$	$[16_3]$	$=+0.252$	$[20_4]$	$=+0.012$
$[5_2]$	$=+0.507$	$[11_3]$	$=-0.461$	$[16_4]$	$=+0.129$	$[21_1]$	$=+0.040$
$[6_1]$	$=-0.187$	$[12_1]$	$=-0.345$	$[16_5]$	$=+0.005$	$[21_2]$	$=-0.431$
$[6_2]$	$=+0.554$	$[12_2]$	$=+0.014$	$[16_6]$	$=-0.364$	$[21_3]$	$=-0.497$
$[6_3]$	$=+0.440$	$[12_3]$	$=+0.522$	$[17_1]$	$=+0.206$	$[21_4]$	$=+0.846$
$[7_1]$	$=-0.074$	$[13_1]$	$=-0.756$	$[17_2]$	$=-0.200$	$[21_5]$	$=-0.009$
$[7_2]$	$=+0.231$	$[13_2]$	$=+1.030$	$[17_3]$	$=-0.476$	$[22_4]$	$=-0.053$
$[7_3]$	$=-0.161$	$[13_3]$	$=-0.326$	$[17_4]$	$=-0.277$	$[23_1]$	$=-0.160$
$[7_4]$	$=+0.864$	$[13_4]$	$=+0.272$	$[17_5]$	$=+0.555$	$[23_2]$	$=-0.053$
$[8_1]$	$=-0.051$						

Residuals resulting from substitution of general corrections in numerical equations of condition.

No. of equation.	Residual.	No. of equation.	Residual.
1	0.0000	19	−0.0001
2	0.0000	20	0.0000
3	0.0000	21	0.0000
4	0.0000	22	−0.0016
5	0.0000	23	0.0000
6	0.0000	24	−0.0006
7	0.0000	25	0.0000
8	−0.0001	26	0.0000
9	+0.0007	27	−0.0001
10	0.0000	28	0.0000
11	0.0000	29	−0.0012
12	0.0000	30	0.0000
13	0.0000	31	0.0000
14	+0.0001	32	−0.0001
15	0.0000	33	−0.0007
16	0.0000	34	0.0000
17	0.0000	35	0.0000
18	0.0000		

SECTION XV.—*Triangulation from the line Oakland – Kansas to the line Denver – Parkersburg.*

KANSAS—22.

[Observer, G. Y. Wisner. Instrument, Pistor & Martins theodolite No. 2. Dates, September and October, 1879.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			″		″	″	° ′ ″
Martinsville and Casey 24 45 32.416	22_1	24	7.8	1	+0.220	−0.770	24 45 31.866
Martinsville and Westfield 68 59 53.026	22_{1+2}	16	4.3	1	−0.522	−0.020	68 59 52.484
Martinsville and Oakland........... 146 31 08.215	22_{1+2+3}	8	2.4	0.5	+0.367	+0.045	146 31 08.627
Casey and Westfield................. 44 14 19.648	22_2	24	9.7	1	+0.220	+0.750	44 14 20.618
Westfield and Oakland 77 31 15.986	22_3	16	5.1	1	+0.092	+0.065	77 31 16.143
Westfield and Palermo 112 31 13.142	22_{3+4}	8	6.0	0.5	−0.792	+0.012	112 31 12.362
Oakland and Palermo................ 34 59 55.996	22_4	16	4.1	1	+0.276	−0.053	34 59 56.219
Palermo and Martinsville 178 28 55.266	22_5	16	6.3	1	−0.120	....	....

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3.5(22_1)+2.5(22_2)+1.5(22_3)+\quad(22_4)-1.732=0$$
$$2.5(22_1)+3.5(22_2)+1.5(22_3)+\quad(22_4)-1.732=0$$
$$1.5(22_1)+1.5(22_2)+3.0(22_3)+1.5(22_4)-1.350=0$$
$$(22_1)+\quad(22_2)+1.5(22_3)+2.5(22_4)-1.268=0$$

NOTE.—The general correction [v] to 22_4 is taken from Section XIV of the adjustment.

OAKLAND—23.

[Observer, A. R. Flint. Instrument, Troughton & Simms theodolite No. 1. Date, September, 1879.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angle.
° ′ ″			″		″	″	° ′ ″
Kansas and Westfield 35 56 01.443	23_3	16	3.4	1	+0.115	+0.087	35 56 01.645

NOTE.—The local correction of 23_3 is taken from Section XIV of the adjustment.

MARTINSVILLE—24.

[Observer, G. Y. Wisner. Instrument, Pistor & Martins theodolite No. 2. Date, October, 1879.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			″		″	″	° ′ ″
Belle Air and Casey 50 09 38.937	24_1	16	5.0	1	−0.008	+0.096	50 09 39.025
Casey and Westfield................. 78 15 51.875	24_2	16	5.9	1	−0.009	−0.371	78 15 51.495
Westfield and Kansas 44 45 46.184	24_3	16	4.3	1	−0.008	−0.036	44 45 46.140
Kansas and Belle Air............... 186 48 43.038	24_4	16	6.7	1	−0.009	+0.311	186 48 43.340

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(24_1)+\quad(24_2)+\quad(24_3)+0.034=0$$
$$(24_1)+2(24_2)+\quad(24_3)+0.034=0$$
$$(24_1)+\quad(24_2)+2(24_3)+0.034=0$$

SECTION XV.—*Triangulation from the line Oakland – Kansas to the line Denver – Parkersburg*—Continued.

WESTFIELD—25.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Date, October, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Oakland and Kansas	66 32 43. 702	25_1	16	5. 2	1	−0. 240	−0. 133	66 32 43. 329
Kansas and Martinsville	66 14 22. 099	25_2	16	4. 1	1	+0. 004	+0. 161	66 14 22. 264
Kansas and Casey	108 02 29. 619	25_{2+3}	8	3. 4	0. 5	−0. 488	+0. 354	108 02 29. 485
Martinsville and Casey	41 48 07. 024	25_3	16	4. 6	1	+0. 004	+0. 193	41 48 07. 221
Casey and Oakland	185 24 47. 647	25_4	14	3. 9	1	−0. 240	−0. 221	185 24 47. 186

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(25_1)+(25_2)+(25_3)+0.472=0$$
$$(25_1)+2.5(25_2)+1.5(25_3)+0.224=0$$
$$(25_1)+1.5(25_2)+2.5(25_3)+0.224=0$$

BELLE AIR—26.

[Observer, J. H. Darling. Instrument, Troughton & Simms theodolite No. 4. Date, October, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Oblong and Hunt City	44 32 07. 589	26_1	16	5. 0	1	+0. 017	+0. 024	44 32 07. 630
Oblong and Casey	111 30 22. 168	26_{1+2}	12	4. 3	1	+0. 135	+0. 067	111 30 22. 370
Hunt City and Casey	66 58 14. 509	26_2	16	5. 2	1	+0. 188	+0. 043	66 58 14. 740
Hunt City and Martinsville	138 45 56. 669	26_{2+3}	16	3. 4	1	−0. 171	−0. 065	138 45 56. 433
Casey and Martinsville	71 47 41. 477	26_3	16	5. 4	1	+0. 324	−0. 108	71 47 41. 693
Martinsville and Oblong	176 41 55. 744	26_4	16	4. 9	1	+0. 152	+0. 041	176 41 55. 937

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3(26_1)+2(26_2)+(26_3)-0.751=0$$
$$2(26_1)+4(26_2)+2(26_3)-1.434=0$$
$$(26_1)+2(26_2)+3(26_3)-1.364=0$$

CASEY—27.

[Observer, A. R. Flint. Instrument, Troughton & Simms theodolite No. 1. Date, October, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Westfield and Kansas	27 43 10. 611	27_1	17	3. 9	1	+0. 052	+0. 278	27 43 10. 941
Westfield and Martinsville	59 56 02. 405	27_{1+2}	16	2. 4	1	−0. 072	−0. 079	59 56 02. 254
Kansas and Martinsville	32 12 51. 618	27_2	17	4. 4	1	+0. 052	−0. 357	32 12 51. 313
Martinsville and Belle Air	58 02 39. 812	27_3	16	3. 1	1	+0. 017	−0. 024	58 02 39. 805
Martinsville and Oblong	98 56 00. 024	27_{3+4}	17	5. 8	1	−0. 039	−0. 182	98 55 59. 803
Belle Air and Oblong	40 53 20. 134	27_4	8	3. 5	0. 5	+0. 022	−0. 158	40 53 19. 998
Belle Air and Hunt City	66 20 11. 094	27_{4+5}	18	4. 3	1	−0. 319	+0. 195	66 20 10. 970
Belle Air and Westfield	242 01 17. 514	27_{4+5+6}	16	4. 1	1	+0. 324	+0. 103	242 01 17. 941
Oblong and Hunt City	25 26 50. 646	27_5	17	4. 2	1	−0. 027	+0. 353	25 26 50. 972
Hunt City and Westfield	175 41 07. 409	27_6	18	3. 2	1	−0. 346	−0. 092	175 41 06. 971

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$4(27_1)+3(27_2)+2(27_3)+(27_4)+(27_5)-0.391=0$$
$$3(27_1)+4(27_2)+2(27_3)+(27_4)+(27_5)-0.391=0$$
$$2(27_1)+2(27_2)+4(27_3)+2.0(27_1)+(27_5)-0.293=0$$
$$(27_1)+(27_2)+2(27_3)+3.5(27_4)+2(27_5)-0.162=0$$
$$(27_1)+(27_2)+(27_3)+2.0(27_4)+3(27_5)-0.084=0$$

SECTION XV.—*Triangulation from the line Oakland - Kansas to the line Denver - Parkersburg*—Continued.

OBLONG—28.

[Observer, G. Y. Wisner. Instrument, Pistor & Martins theodolite No. 2. Dates, October and November, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Claremont and Mound	34 36 31.101	28_1	20	4.8	1	+0.099	−0.157	34 36 31.043
Mound and Hunt City	65 50 49.477	28_2	16	3.5	1	+0.100	+0.404	65 50 49.981
Hunt City and Casey	32 06 47.113	28_3	16	5.4	1	+0.143	+0.389	32 06 47.645
Hunt City and Belle Air	59 43 05.962	28_{3+4}	8	3.7	0.5	−0.088	+0.059	59 43 05.933
Casey and Belle Air	27 36 18.474	28_4	16	3.6	1	+0.144	−0.330	27 36 18.288
Belle Air and Claremont	199 49 33.249	28_6	16	5.9	1	+0.100	−0.306	199 49 33.043

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(28_1)+(28_2)+(28_3)+(28_4)-0.586=0$$
$$(28_1)+2(28_2)+(28_3)+(28_4)-0.586=0$$
$$(28_1)+(28_2)+2.5(28_3)+1.5(28_4)-0.773=0$$
$$(28_1)+(28_2)+1.5(28_3)+2.5(28_4)-0.773=0$$

HUNT CITY—29.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Date, October, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Casey and Belle Air	46 41 34.723	29_1	16	4.7	1	−0.054	+0.198	46 41 34.867
Belle Air and Oblong	75 44 47.237	29_2	16	4.5	1	−0.203	+0.026	75 44 47.060
Belle Air and Mound	145 05 08 755	29_{2+3}	12	1.7	1	+0.150	+0.179	145 05 09.084
Oblong and Mound	69 20 21.828	29_3	16	6.1	1	+0.043	+0.153	69 20 22.024
Oblong and Casey	237 33 38.790	29_{3+4}	8	2.2	0.5	−0.493	−0.224	237 33 38.073
Mound and Casey	168 13 16.233	29_4	16	7.3	1	+0.193	−0.377	168 13 16.049

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.5(29_1)+1.5(29_2)+(29_3)+0.396=0$$
$$1.5(29_1)+3.5(29_2)+2(29_3)+0.706=0$$
$$(29_1)+2(29_2)+3(29_3)+0.331=0$$

CLAREMONT—30.

[Observer, G. Y. Wisner. Instrument, Pistor & Martins theodolite No. 2. Date, November, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Parkersburg and Denver	85 42 19.356	30_1	16	7.7	1	−0.215	+0.198	85 42 19.339
Parkersburg and Mound	152 31 16.747	$30_{1+2+3+4+5}$	8	4.8	0.5	+0.548	−0.272	152 31 17.023
Denver and Onion Hill	17 49 15.536	30_2	16	8.2	1	−0.144	−0.437	17 49 14.955
Denver and Mound	66 48 58.298	$30_{2+3+4+5}$	8	4.8	0.5	−0.144	−0.470	66 48 57.684
Onion Hill and West Base	28 12 13.803	30_3	16	5.6	1	−0.144	−0.286	28 12 13.373
West Base and Check Base	7 24 42.167	30_4	24	6.2	1	−0.144	+0.220	7 24 42.243
Check Base and Mound	13 22 47.112	30_5	16	8.8	1	−0.032	+0.033	13 22 47.113
Check Base and East Base	18 30 33.645	30_{5+6}	8	4.8	0.5	−0.222	+0 028	18 30 33.451
Mound and East Base	5 07 46.173	30_6	16	7.6	1	+0.170	−0.005	5 07 46.338
East Base and Oblong	34 36 06.997	30_7	16	8.8	1	+0.059	+0.212	34 36 07.268
Oblong and Parkersburg	167 44 49.247	30_8	16	7.2	1	+0.059	+0.065	167 44 49.371

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.5(30_1)+1.5(30_2)+1.5(30_3)+1.5(30_4)+1.5(30_5)+(30_6)+(30_7)+1.004=0$$
$$1.5(30_1)+3.0(30_2)+2.0(30_3)+2.0(30_4)+2.0(30_5)+(30_6)+(30_7)+1.164=0$$
$$1.5(30_1)+2.0(30_2)+3.0(30_3)+2.0(30_4)+2.0(30_5)+(30_6)+(30_7)+1.164=0$$
$$1.5(30_1)+2.0(30_2)+2.0(30_3)+3.0(30_4)+2.0(30_5)+(36_6)+(30_7)+1.164=0$$
$$1.5(30_1)+2.0(30_2)+2.0(30_3)+2.0(30_4)+3.5(30_5)+1.5(30_6)+(30_7)+0.984=0$$
$$(30_1)+(30_2)+(30_3)+(30_4)+1.5(30_5)+2.5(30_6)+(30_7)+0.211=0$$
$$(30_1)+(30_2)+(30_3)+(30_4)+(30_5)+(30_6)+2(30_7)+0.391=0$$

SECTION XV.—*Triangulation from the line Oakland-Kansas to the line Denver-Parkersburg*—Continued.

MOUND—31.

[Observer, A. R. Flint. Instrument, Troughton & Simms theodolite No. 1. Dates, October and November, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Hunt City and Oblong	44 48 48.472	31_1	16	4.1	1	−0.148	+0.302	44 48 48.626
Oblong and East Base	93 44 38.110	31_2	16	4.4	1	−0.016	−0.199	93 44 37.895
Oblong and Claremont	105 39 36.322	31_{2+3}	8	4.3	0.5	−0.064	−0.121	105 39 36.137
East Base and Claremont	11 54 58.147	31_3	16	2.6	1	+0.017	+0.078	11 54 58.242
East Base and Check Base	19 31 25.758	31_{3+4}	18	4.0	1	+0.171	−0.154	19 31 25.775
East Base and West Base	71 13 08.118	$31_{3+4+5+6}$	16	5.0	1	−0.400	−0.185	71 13 07.533
Claremont and Check Base	7 36 27.861	31_4	16	3.7	1	−0.096	−0.232	7 36 27.533
Claremont and Denver	61 34 30.826	$31_{4+5+6+7}$	16	3.6	1	+0.300	+0.099	61 34 31.225
Claremont and Onion Hill	65 25 55.665	$31_{4+5+6+7+8}$	8	3.7	0.5	−0.439	+0.073	65 25 55.299
Check Base and Middle Base	19 31 55.608	31_5	16	3.8	1	+0.077	−0.455	19 31 55.230
Middle Base and West Base	32 09 46.027	31_6	18	3.1	1	+0.077	+0.424	82 09 46 528
West Base and Denver	2 16 21.701	31_7	16	3.4	1	−0.129	+0.362	2 16 21.934
West Base and East Base	288 46 52.475	$31_{7+8+9+1+2}$	16	2.0	1	−0.193	+0.185	288 46 52.467
Denver and Onion Hill	3 51 24.315	31_8	18	3.4	1	−0.215	−0.026	3 51 24.074
Denver and Hunt City	147 57 03.807	31_{8+9}	16	4.2	1	+0.485	−0.280	147 57 04.012
Onion Hill and Hunt City	144 05 40.961	31_9	8	3.7	0.5	−0.769	−0.254	144 05 39.938

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.5(31_1)+1.5(31_2)+1.5(31_3)+1.5(31_4)+1.5(31_5)+1.5(31_6)+1.5(31_7)+0.5(31_8)+0.334=0$$
$$1.5(31_1)+3.0(31_2)+2.0(31_3)+1.5(31_4)+1.5(31_5)+1.5(31_6)+1.5(31_7)+0.5(31_8)+0.391=0$$
$$1.5(31_1)+2.0(31_2)+6.0(31_3)+4.5(31_4)+3.5(31_5)+3.5(31_6)+1.5(31_7)+0.5(31_8)+0.194=0$$
$$1.5(31_1)+1.5(31_2)+4.5(31_3)+7.0(31_4)+5.0(31_5)+5.0(31_6)+3.0(31_7)+(31_8)+0.521=0$$
$$1.5(31_1)+1.5(31_2)+3.5(31_3)+5.0(31_4)+6.0(31_5)+5.0(31_6)+3.0(31_7)+(31_8)+0.271=0$$
$$1.5(31_1)+1.5(31_2)+3.5(31_3)+5.0(31_4)+5.0(31_5)+6.0(31_6)+3.0(31_7)+(31_8)+0.271=0$$
$$1.5(31_1)+1.5(31_2)+1.5(31_3)+3.0(31_4)+3.0(31_5)+3.0(31_6)+4.0(31_7)+(31_8)+0.628=0$$
$$0.5(31_1)+0.5(31_2)+0.5(31_3)+(31_4)+(31_5)+(31_6)+(31_7)+2.0(31_8)+0.524=0$$

ONION HILL—32.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Date, November, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Mound and West Base	4 40 28.048	32_1	16	3.5	1	−0.087	−0.365	4 40 27.596
West Base and East Base	16 39 28.348	32_2	16	5.2	1	+0.083	−0.320	16 39 28.111
West Base and Check Base	43 46 06.687	32_{2+3}	16	5.3	1	−0.170	+0.424	43 46 06.941
East Base and Check Base	27 06 38.004	32_3	16	8.0	1	+0.082	+0.744	27 06 38.830
Check Base and Claremont	17 07 48.677	32_4	16	3.8	1	−0.087	−0.609	17 07 47.981
Claremont and Denver	101 24 49.225	32_5	16	3.7	1	−0.087	−0.441	101 24 48.697
Denver and Mound	193 00 47.882	32_6	16	4.4	1	−0.088	+0.991	193 00 48 785

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(32_1)+(32_2)+(32_3)+(32_4)+(32_5)+0.184=0$$
$$(32_1)+3(32_2)+2(32_3)+(32_4)+(32_5)-0.151=0$$
$$(32_1)+2(32_2)+3(32_3)+(32_4)+(32_5)-0.151=0$$
$$(32_1)+(32_2)+(32_3)+2(32_4)+(32_5)+0.184=0$$
$$(32_1)+(32_2)+(32_3)+(32_4)+2(32_5)+0.184=0$$

SECTION XV.—*Triangulation from the line Oakland - Kansas to the line Denver - Parkersburg*—Continued.

WEST BASE—33.

[Observers, R. S. Woodward and J. H. Darling. Instruments, Troughton & Simms theodolites Nos. 3 and 4. Dates, October and November, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″		″	″	° ′ ″
Mound and East Base	47 46 00.480	33_1	16	4.6	1	+0.049	+0.065	47 46 00.594
Mound and Middle Base	47 46 02.866	33_{1+2}	16	4.9	1	+0.307	+0.436	47 46 03.609
East Base and Middle Base	0 00 02.638	33_2	8	1.4	0.5	+0.006	+0.371	0 00 03.015
East Base and Check Base	46 45 34.513	33_{2+3}	16	6.0	1	−0.328	+0.068	46 45 34.253
East Base and Claremont	52 08 20.994	33_{2+3+4}	16	5.3	1	+0.221	−0.230	52 08 20.985
Middle Base and Check Base	46 45 31.232	33_3	16	5.7	1	+0.309	−0.303	46 45 31.238
Check Base and Denver	88 45 13.338	33_{4+5}	16	5.2	1	−0.049	−0.074	88 45 13.215
Check Base and Onion Hill	96 16 39.302	33_{4+5+6}	16	4.1	1	+0.029	−0.595	96 16 38.736
Claremont and Denver	83 22 26.038	33_4	16	6.4	1	+0.221	+0.224	83 22 26.483
Denver and Onion Hill	7 31 26.225	33_6	16	4.7	1	−0.183	−0.521	7 31 25.521
Denver and Mound	176 43 11.642	33_{6+7}	16	6.2	1	+0.355	−0.059	176 43 11.938
Onion Hill and East Base	216 57 46.637	33_{7+1}	16	4.6	1	−0.153	+0.527	216 57 47.011

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$3(33_1)+2.0(33_2)+(33_3)+(33_{4+5})-0.418=0$$
$$2(33_1)+5.5(33_2)+4(33_3)+3(33_{4+5})-(33_5)+(33_6)-0.815=0$$
$$(33_1)+4.0(33_2)+5(33_3)+3(33_{4+5})-(33_5)+(33_5)-1.067=0$$
$$(33_1)+3.0(33_2)+3(33_3)+5(33_{4+5})-(33_5)+2(33_6)-0.163=0$$
$$-(33_2)-(33_3)-(33_{4+5})+2(33_5)-0.176=0$$
$$+(33_2)+(33_3)+2(33_{4+5})+3(33_5)+0.331=0$$

NOTE.—33_{2+3+4}, 33_{4+5+6}, 33_5, 33_6, and 33_{7+1} were measured by Mr. Darling with the Troughton & Simms No. 4. The remainder were read by Mr. Woodward with the Troughton & Simms No. 3.

EAST BASE—34.

[Observer, J. H. Darling. Instrument, Troughton & Simms theodolite No. 4. Date, November, 1879.]

Angle as measured between—		Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
	° ′ ″			″			″	° ′ ″
Claremont and Check Base	18 20 42.331	34_1	16	2.5	1	−0.220	+0.708	18 20 42.819
Claremont and West Base	101 56 22.781	$34_{1+2+3+4}$	16	5.0	1	+0.220	+0.516	101 56 23.517
Check Base and Onion Hill	63 17 21.868	34_2	28	7.3	1	−0.439	+0.287	63 17 21.716
Check Base and Middle Base	83 35 37.101	34_{2+3}	16	6.1	1	+0.830	−0.242	83 35 37.689
Check Base and West Base	83 35 40.512	34_{2+3+4}	34	7.8	2	+0.378	−0.192	83 35 40.698
Onion Hill and West Base	20 18 19.900	34_{3+4}	28	6.2	1	−0.439	−0.479	20 18 18.982
Middle Base and West Base	0 00 02.497	34_4	36	3.9	2	+0.462	+0.050	0 00 03.009
Middle Base and Mound	61 00 54.943	34_{4+5}	20	5.3	1	−0.094	+0.108	61 00 54.957
West Base and Mound	61 00 51.467	34_5	16	4.3	1	+0.423	+0.058	61 00 51.948
West Base and Check Base	276 24 18.069	34_{5+6+1}	28	9.2	1	+1.041	+0.192	276 24 19.302
Mound and Check Base	215 23 26.890	34_{6+1}	20	5.7	1	+0.330	+0.134	215 23 27.354

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(34_1)+(34_{2+3})+(34_4)-0.852=0$$
$$+2(34_2)-(34_{2+3})-(34_4)+2.170=0$$
$$(34_1)-(34_2)+7(34_{2+3})+8(34_4)+(34_5)-9.228=0$$
$$(34_1)-(34_2)+6(34_{2+3})+9(34_4)+2(34_5)-10.207=0$$
$$(34_{2+3})+2(34_4)+3(34_5)-3.024=0$$

SECTION XV.—*Triangulation from the line Oakland - Kansas to the line Denver - Parkersburg*—Continued.

MIDDLE BASE—35.

[Observer, E. S. Wheeler. Instrument, Repsold theodolite No. 1. Date, October, 1879.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			″		″	″	° ′ ″
West Base and Mound 100 04 09.081	35_1	16	7.8	1	+0.152	+0.669	100 04 09.902
Mound and East Base 79 55 44.134	35_2	16	4.2	1	+0.152	−0.210	79 55 44.076
East Base and West Base 180 00 06.328	35_3	16	9.2	1	+0.153	−0.459	180 00 06.022

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(35_1)+(35_2)-0.457=0$$
$$(35_1)+2(35_2)-0.457=0$$

CHECK BASE—36.

[Observer, J. H. Darling. Instrument, Troughton & Simms theodolite No. 4. Dates, November and December, 1879.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			″		″	″	° ′ ″
Claremont and Onion Hill 127 15 17.079	36_1	16	4.2	1	+0.090	−0.611	127 15 16.558
Onion Hill and West Base 39 57 14.179	36_2	16	4.4	1	+0.380	−0.062	39 57 14.497
Onion Hill and East Base 89 36 00.229	36_{2+3+4}	8	4.5	0.5	−0.581	+0.005	89 35 59.653
West Base and Mound 33 46 43.582	36_3	21	6.7	1	+0.132	−0.185	33 46 43.529
West Base and East Base 49 38 44.842	36_{3+4}	12	6.5	1	+0.247	+0.067	49 38 45.156
Mound and East Base 15 52 01.243	36_4	21	4.7	1	+0.132	+0.252	15 52 01.627
East Base and Claremont 143 08 43.092	36_5	16	6.1	1	+0.091	+0.606	143 08 43.789

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2(36_1)+(36_2)+(36_3)+(36_4)-0.825=0$$
$$(36_1)+2.5(36_2)+1.5(36_3)+1.5(36_4)-1.437=0$$
$$(36_1)+1.5(36_2)+3.5(36_3)+2.5(36_4)-1.454=0$$
$$(36_1)+1.5(36_2)+2.5(36_3)+3.5(36_4)-1.454=0$$

DENVER—37.

[Observer, R. S. Woodward. Instrument, Troughton & Simms theodolite No. 3. Date, November, 1879.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			″		″	″	° ′ ″
Onion Hill and Mound 9 09 24.748	37_1	16	4.8	1	+0.014	−0.003	9 09 24.759
Mound and West Base 1 00 26.277	37_2	16	2.1	1	+0.014	−0.153	1 00 26.138
West Base and Claremont 50 36 06.370	37_3	16	5.8	1	+0.014	−0.684	50 36 05.700
Claremont and Parkersburg 48 36 58.482	37_4	16	3.5	1	−0.028	+0.278	48 36 58.732
Claremont and Onion Hill 299 14 02.480	37_{4+5}	8	3.6	0.5	+0.083	+0.840	299 14 03.403
Parkersburg and Onion Hill 250 37 04.167	37_5	8	2.9	0.5	−0.058	+0.562	250 37 04.671

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$2.0(37_1)+(37_2)+(37_3)+0.5(37_4)-0.040=0$$
$$(37_1)+2.0(37_2)+(37_3)+0.5(37_4)-0.040=0$$
$$(37_1)+(37_2)+2.0(37_3)+0.5(37_4)-0.040=0$$
$$0.5(37_1)+0.5(37_2)+0.5(37_3)+1.5(37_4)+0.022=0$$

Section XV.—*Triangulation from the line Oakland - Kansas to the line Denver - Parkersburg*—Continued.

PARKERSBURG—38.

[Observer, A. R. Flint. Instrument, Troughton & Simms theodolite No. 1. Date, November, 1879.]

Angle as measured between—	Notation.	No. meas.	Range.	Wt.	(v)	[v]	Corrected angles.
° ′ ″			″		″	″	° ′ ″
West Azimuth Mark and Denver... 31 43 41.471	38_1	16	4.2	1	+0.268	+0.001	31 43 41.740
West Azimuth Mark and East Azimuth Mark 178 33 56.512	38_{1+2+3}	30	6.9	1.5	−0.019	0.000	178 33 56.493
Denver and Claremont............ 45 40 42.617	38_2	16	6.0	1	+0.268	−0.002	45 40 42.883
Claremont and East Azimuth Mark. 101 09 31.601	38_3	16	4.4	1	+0.268	+0.001	101 09 31.870
East Azimuth Mark and West Azimuth Mark 181 26 03.345	38_4	30	5.7	1.5	+0.162	0.000	181 26 03.507

NORMAL EQUATIONS FOR LOCAL ADJUSTMENT.

$$4(38_1)+3(38_2)+3(38_3)-2.683=0$$
$$3(38_1)+4(38_2)+3(38_3)-2.683=0$$
$$3(38_1)+3(38_2)+4(38_3)-2.683=0$$

Numerical equations of condition in the triangulation from the line Oakland–Kansas to the line Denver–Parkersburg.

SIDE-EQUATIONS.

XXXIX. (25) $-45.6532\,[22_1]\ +21.6222\,[22_2]\ -18.0616\,[24_2]\ -13.6876\,[24_3]\ +6.8580\,[25_2]$
$+30.4054\,[25_3] \qquad -65.546=0$

XLV. (25) $+8.2965\,[26_1]\ +17.2466\,[26_2]\ -33.5478\,[28_3]\ +40.2660\,[28_4]\ -33.2287\,[29_1]$
$-13.3824\,[29_2] \qquad +32.349=0$

LI. (5) $+2.8748\,[31_4]\ +2.8748\,[31_5]\ +2.8748\,[31_6]\ -9.6256\,[31_7]\ -9.6256\,[31_8]$
$+9.5630\,[32_1]\ -2.1568\,[32_2]\ -2.1568\,[32_3]\ -2.1568\,[32_4]\ +3.6770\,[33_1]$
$+3.6770\,[33_2]\ +3.6770\,[33_3]\ +3.6770\,[33_{4+5}]\ -4.0070\,[33_5]\ -0.3300\,[33_6] \qquad +7.594=0$

LIV. (6) $-3.0785\,[30_3]\ -3.0785\,[30_4]\ -3.0785\,[30_5]\ +15.2276\,[30_6]\ -4.7264\,[31_3]$
$+4.8992\,[31_4]\ +4.8992\,[31_5]\ +4.8992\,[31_6]\ +4.8992\,[31_7]\ +4.8992\,[31_8]$
$-3.0963\,[34_1]\ -6.3107\,[34_2]\ +3.2144\,[34_{2+3}]\ +3.2144\,[34_4]\ +3.2144\,[34_5] \qquad +4.414=0$

LVI. (10) $-11.7198\,[32_2]\ +9.9009\,[32_3]\ +9.9009\,[32_4]\ -16.3678\,[33_2]\ -16.3678\,[33_3]$
$-16.3678\,[33_{4+5}]\ +16.0378\,[33_5]\ -0.3300\,[33_6]\ -7.5486\,[34_1]\ -3.0963\,[34_2]$
$-4.4523\,[34_{2+3}]\ -4.4523\,[34_4] \qquad -3.586=0$

LIX. (10) $-0.0004\,[33_1]\ +19.1134\,[33_2]\ -11.6638\,[34_4]\ +0.0004\,[34_5]\ +3.7388\,[35_1]$
$+3.7395\,[35_2] \qquad -8.210=0$

LXII. (10) $+11.0867\,[30_3]\ +11.0867\,[30_4]\ -18.3061\,[30_5]\ +9.6256\,[31_4]\ -3.6210\,[31_5]$
$-3.6210\,[31_6]\ -3.6210\,[31_7]\ -3.6210\,[31_8]\ +16.0135\,[36_1]\ +6.1438\,[36_2]$
$+6.1438\,[36_3] \qquad +15.960=0$

LXIV. (10) $+3.3848\,[31_5]\ +3.3848\,[31_6]\ -13.2466\,[31_7]\ -13.2466\,[31_8]\ +18.6655\,[32_1]$
$-3.3149\,[32_2]\ -3.3149\,[32_3]\ +1.6668\,[33_1]\ +1.6668\,[33_2]\ +1.6668\,[33_3]$
$-2.3162\,[33_{4+5}]\ -2.3162\,[33_6] \qquad +11.171=0$

LXVI. (10) $+14.1652\,[30_3]\ +14.1652\,[30_4]\ -15.2276\,[30_5]\ -15.2276\,[30_6]\ +3.0963\,[34_1]$
$-7.4983\,[34_2]\ +16.0135\,[36_1]\ +0.1470\,[36_2]\ +0.1470\,[36_3]\ +0.1470\,[36_4] \qquad +11.086=0$

LXIX. (6) $+1.7706\,[31_4]\ +1.7706\,[31_5]\ +1.7706\,[31_6]\ +1.7706\,[31_7]\ -9.6256\,[31_8]$
$+9.5630\,[32_1]\ +9.5630\,[32_2]\ +9.5630\,[32_3]\ +9.5630\,[32_4]\ +4.2506\,[32_5]$
$+11.7838\,[37_1]\ -4.8989\,[37_2]\ -4.8989\,[37_3] \qquad +2.637=0$

LXXI. (6) $-11.7198\,[32_2]\ -11.7198\,[32_3]\ -11.7198\,[32_4]\ -4.2506\,[32_5]\ -2.7759\,[33_5]$
$-0.3300\,[33_6]\ -11.7838\,[37_1]\ -11.7838\,[37_2]\ +5.5100\,[37_3] \qquad -1.661=0$

Note.—In the solution for determining the general corrections, each of the side-equations was divided by the number in parenthesis placed opposite it.

Numerical equations of condition, &c.—Continued.

ANGLE-EQUATIONS.

XXXVI.	$[22_3]+[23_3]+[25_1]$	$-0.019=0$
XXXVII.	$[22_1]+[22_2]+[24_3]+[25_2]$	$-0.105=0$
XXXVIII.	$[22_2]+[25_2]+[25_3]+[27_1]$	$-1.382=0$
XL.	$[24_2]+[25_3]+[27_1]+[27_2]$	$+0.257=0$
XLI.	$[24_1]+[26_3]+[27_3]$	$+0.036=0$
XLII.	$[26_2]+[27_4]+[27_5]+[29_1]$	$-0.436=0$
XLIII.	$[26_1]+[26_2]+[27_4]+[28_4]$	$+0.422=0$
XLIV.	$[26_1]+[28_3]+[28_4]+[29_2]$	$-0.108=0$
XLVI.	$[28_2]+[29_3]+[31_1]$	$-0.859=0$
XLVII.	$[28_1]+[30_6]+[30_7]+[31_2]+[31_3]$	$+0.071=0$
XLVIII.	$[30_3]+[30_4]+[30_5]+[31_4]+[31_5]+[31_6]+[31_7]+[31_8]+[32_1]+[32_2]+[32_3]+[32_4]$	$+0.509=0$
XLIX.	$[30_4]+[30_5]+[31_4]+[31_5]+[31_6]+[33_1]+[33_2]+[33_3]+[33_{4+5}]-[33_5]$	$+0.176=0$
L.	$[30_3]+[32_2]+[32_3]+[32_4]+[33_5]+[33_6]$	$+0.768=0$
LII.	$[30_3]+[30_4]+[30_5]+[30_6]+[32_3]+[32_4]+[34_1]+[34_2]$	$-1.092=0$
LIII.	$[31_3]+[31_4]+[31_5]+[31_6]+[31_7]+[31_8]+[32_1]+[32_2]-[34_2]+[34_{2+3}]+[34_4]+[34_5]$	$+0.954=0$
LV.	$[31_3]+[31_4]+[31_5]+[31_6]+[33_1]+[34_5]$	$+0.062=0$
LVII.	$[31_6]+[33_1]+[33_2]+[35_1]$	$-1.528=0$
LVIII.	$[31_3]+[31_4]+[31_5]+[34_4]+[34_5]+[35_2]$	$+0.711=0$
LX.	$[30_3]+[30_4]+[32_4]+[36_1]$	$+1.285=0$
LXI.	$[31_5]+[31_6]+[31_7]+[31_8]+[32_1]+[32_2]+[32_3]+[36_2]+[36_3]$	$-0.117=0$
LXIII.	$[32_2]+[32_3]+[33_{4+5}]+[33_6]+[36_2]$	$+0.234=0$
LXV.	$[32_3]+[34_2]+[36_2]+[36_3]+[36_4]$	$-1.035=0$
LXVII.	$[30_2]+[30_3]+[30_4]+[30_5]+[31_4]+[31_5]+[31_6]+[31_7]+[37_2]+[37_3]$	$+1.207=0$
LXVIII.	$[30_2]+[32_5]+[37_1]+[37_2]+[37_3]$	$+1.718=0$
LXX.	$[30_2]+[30_3]+[33_5]+[37_3]$	$+1.183=0$
LXXII.	$[30_1]+[37_4]+[38_2]$	$-0.475=0$

General corrections in terms of the correlates.

$[22_1]$	$=-0.09091$ XXXVI	$+0.27273$ XXXVII	-0.36364 XXXVIII	-1.47659 XXXIX	
$[22_2]$	$=-0.09091$ XXXVI	$+0.27273$ XXXVII	$+0.63636$ XXXVIII	$+1.21443$ XXXIX	
$[22_3]$	$=+0.72727$ XXXVI	-0.18182 XXXVII	-0.09091 XXXVIII	$+0.08739$ XXXIX	
$[23_3]$	$=+1.00000$ XXXVI				
$[24_1]$	$=-0.25000$ XXXVII	$+0.31749$ XXXIX	-0.25000 XL	$+0.75000$ XLI	
$[24_2]$	$=-0.25000$ XXXVII	-0.40497 XXXIX	$+0.75000$ XL	-0.25000 XLI	
$[24_3]$	$=+0.75000$ XXXVII	-0.23001 XXXIX	-0.25000 XL	-0.25000 XLI	
$[25_1]$	$=+0.66667$ XXXVI	-0.16667 XXXVII	-0.33333 XXXVIII	-0.24842 XXXIX	-0.16667 XL
$[25_2]$	$=-0.16667$ XXXVI	$+0.66667$ XXXVII	$+0.33333$ XXXVIII	-0.22253 XXXIX	-0.33333 XL
$[25_3]$	$=-0.16667$ XXXVI	-0.33333 XXXVII	$+0.33333$ XXXVIII	$+0.71937$ XXXIX	$+0.66667$ XL
$[26_1]$	$=-0.25000$ XLII	$+0.25000$ XLIII	$+0.50000$ XLIV	-0.00654 XLV	
$[26_2]$	$=-0.25000$ XLI	$+0.50000$ XLII	$+0.25000$ XLIII	-0.25000 XLIV	$+0.26196$ XLV
$[26_3]$	$=+0.50000$ XLI	-0.25000 XLII	-0.25000 XLIII	-0.17246 XLV	
$[27_1]$	$=+0.60571$ XXXVIII	$+0.21143$ XL	-0.10857 XLI	-0.02286 XLII	$+0.03429$ XLIII
$[27_2]$	$=-0.39429$ XXXVIII	$+0.21143$ XL	-0.10857 XLI	-0.02286 XLII	$+0.03429$ XLIII
$[27_3]$	$=-0.10857$ XXXVIII	-0.21714 XL	$+0.46286$ XLI	-0.16571 XLII	-0.25143 XLIII
$[27_4]$	$=+0.03429$ XXXVIII	$+0.06857$ XL	-0.25143 XLI	$+0.26286$ XLII	$+0.60571$ XLIII
$[27_5]$	$=-0.05714$ XXXVIII	-0.11429 XL	$+0.08571$ XLI	$+0.22857$ XLII	-0.34286 XLIII

General corrections in terms of the correlates—Continued.

$[28_1]$	=−0. 12500 XLIII	−0. 25000 XLIV	−0. 03359 XLV	−0. 25000 XLVI	+0. 75000 XLVII
$[28_2]$	=−0. 12500 XLIII	−0. 25000 XLIV	−0. 03359 XLV	+0. 75000 XLVI	−0. 25000 XLVII
$[28_3]$	=−0. 31250 XLIII	+0. 37500 XLIV	−1. 42588 XLV	−0. 12500 XLVI	−0. 12500 XLVII
$[28_4]$	=+0. 68750 XLIII	+0. 37500 XLIV	+1. 52666 XLV	−0. 12500 XLVI	−0. 12500 XLVII
$[29_1]$	=+0. 54167 XLII	−0. 20833 XLIV	−0. 60844 XLV	−0. 04167 XLVI	
$[29_2]$	=−0. 20833 XLII	−0. 54167 XLIV	−0. 01305 XLV	−0. 29167 XLVI	
$[29_3]$	=−0. 04167 XLII	−0. 29167 XLIV	+0. 21151 XLV	+0. 54167 XLVI	
$[30_1]$	=−0. 20054 XLVII	−0. 26558 XLVIII	−0. 15718 XLIX	−0. 10840 L	−0. 33604 LII
	−0. 04255 LIV	−0. 21680 LX	−0. 15106 LXII	−0. 12553 LXVI	−0. 37398 LXVII
	−0. 10840 LXVIII	−0. 21680 LXX	+0. 70461 LXXII		
$[30_2]$	=−0. 04607 XLVII	−0. 57452 XLVIII	−0. 36043 XLIX	−0. 21409 L	−0. 56368 LII
	+0. 32228 LIV	−0. 42818 LX	−0. 20682 LXII	−0. 40019 LXVI	+0. 21139 LXVII
	+0. 78591 LXVIII	+0. 57182 LXX	−0. 10840 LXXII		
$[30_3]$	=−0. 04607 XLVII	+0. 42548 XLVIII	−0. 36043 XLIX	+0. 78591 L	+0. 43632 LII
	−0. 19080 LIV	+0. 57182 LX	+0. 90784 LXII	+1. 01633 LXVI	+0. 21139 LXVII
	−0. 21409 LXVIII	+0. 57182 LXX	−0. 10840 LXXII		
$[30_{3+4}]$	=−0. 09214 XLVII	+0. 85096 XLVIII	+0. 27914 XLIX	+0. 57182 L	+087264 LII
	−0. 38160 LIV	+1. 14364 LX	+1. 80368 LXII	+2. 03266 LXVI	+0. 42278 LXVII
	−0. 42818 LXVIII	+0. 14364 LXX	−0. 21680 LXXII		
$[30_{3+4+5}]$	=−0. 26287 XLVII	+1. 19243 XLVIII	+0. 76695 XLIX	+0. 42548 L	+1. 01899 LII
	−1. 05201 LIV	+0. 85096 LX	+0. 31832 LXII	+0. 94954 LXVI	+0. 61791 LXVII
	−0. 57452 LXVIII	−0. 14904 LXX	−0. 26558 LXXII		
$[30_4]$	=−0. 04607 XLVII	+0. 42548 XLVIII	+0. 63957 XLIX	−0. 21409 L	+0. 43632 LII
	−0. 19080 LIV	+0. 57182 LX	+0. 90184 LXII	+1. 01633 LXVI	+0. 21139 LXVII
	−0. 21409 LXVIII	−0. 42818 LXX	−0. 10840 LXXII		
$[30_5]$	=−0. 17073 XLVII	+0. 34147 XLVIII	+0. 48781 XLIX	−0. 14634 L	+0. 14635 LII
	−0. 67041 LIV	−0. 29268 LX	−1. 48536 LXII	−1. 08312 LXVI	+0. 19513 LXVII
	−0. 14634 LXVIII	−0. 29268 LXX	−0. 04878 LXXII		
$[30_{5+6}]$	=+0. 24933 XLVII	+0. 16803 XLVIII	+0. 30353 XLIX	−0. 13550 L	+0. 57996 LII
	+0. 95923 LIV	−0. 27100 LX	−1. 10413 LXII	−1. 67968 LXVI	+0. 03253 LXVII
	−0. 13550 LXVIII	−0. 27100 LXX	−0. 11924 LXXII		
$[30_6]$	=+0. 42006 XLVII	−0. 17344 XLVIII	−0. 18428 XLIX	+0. 01084 L	+0. 43361 LII
	+1. 62964 LIV	+0. 02168 LX	+0. 38123 LXII	−0. 59656 LXVI	−0. 16260 LXVII
	+0. 01084 LXVIII	+0. 02168 LXX	−0. 07046 LXXII		
$[30_7]$	=+0. 54472 XLVII	−0. 08943 XLVIII	−0. 03252 XLIX	−0. 05691 L	−0. 27642 LII
	−0. 42868 LIV	−0. 11382 LX	−0. 17084 LXII	+0. 08637 LXVI	−0. 14634 LXVII
	−0. 05691 LXVIII	−0. 11382 LXX	−0. 13008 LXXII		
$[31_1]$	=+0. 64994 XLVI	−0. 27204 XLVII	−0. 16230 XLVIII	−0. 00236 XLIX	+0. 30655 LI
	−0. 22032 LIII	−0. 08684 LIV	−0. 06038 LV	−0. 01256 LVII	−0. 04782 LVIII
	−0. 18506 LXI	+0. 08892 LXII	+0. 20336 LXIV	−0. 13564 LXVII	+0. 00274 LXIX
$[31_2]$	=−0. 21402 XLVI	+0. 40402 XLVII	−0. 06637 XLVIII	+0. 10533 XLIX	+0. 39110 LI
	−0. 24631 LIII	+0. 08754 LIV	−0. 07461 LV	+0. 00614 LVII	−0. 08075 LVIII
	−0. 15942 LXI	+0. 14730 LXII	+0. 23160 LXIV	−0. 03775 LXVII	+0. 03477 LXIX
$[31_3]$	=−0. 05802 XLVI	+0. 24382 XLVII	−0. 12544 XLVIII	−0. 32066 XLIX	−0. 56019 LI
	+0. 29832 LIII	−0. 43624 LIV	+0. 10310 LV	−0. 04351 LVII	+0. 14661 LVIII
	+0. 10820 LXI	−0. 26407 LXII	−0. 28806 LXIV	−0. 15798 LXVII	−0. 09882 LXIX
$[31_4]$	=+0. 02276 XLVI	−0. 14059 XLVII	+0. 08519 XLVIII	+0. 24278 XLIX	+0. 44297 LI
	−0. 14845 LIII	+0. 25360 LIV	+0. 00914 LV	−0. 14962 LVII	+0. 15876 LVIII
	−0. 45683 LXI	+0. 68715 LXII	+0. 10746 LXIV	+0. 11146 LXVII	+0. 07504 LXIX
$[31_{4+5+6}]$	=−0. 00236 XLVI	−0. 21533 XLVII	+0. 17509 XLVIII	+0. 57258 XLIX	+1. 09443 LI
	−0. 14557 LIII	+0. 39554 LIV	+0. 25192 LV	+0. 16490 LVII	+0. 08702 LVIII
	−0. 06769 LXI	+0. 25825 LXII	+0. 63816 LXIV	+0. 24134 LXVII	+0. 17752 LXIX

General corrections in terms of the correlates—Continued.

$[31_{4+5+6+7}]$	=−0. 13564 XLVI	−0. 19573 XLVII	+0. 31936 XLVIII	+0. 24134 XLIX	−0. 01143 LI
	+0. 16138 LIII	+0. 38519 LIV	+0. 08336 LV	+0. 06494 LVII	+0. 01842 LVIII
	+0. 20790 LXI	+0. 03205 LXII	−0. 05939 LXIV	+0. 47302 LXVII	+0. 38612 LXIX
$[31_{4+5+6+7+8}]$	=−0. 16230 XLVI	−0. 19181 XLVII	+0. 74822 XLVIII	+0. 17509 XLIX	−1. 00266 LI
	+0. 62278 LIII	+0. 70975 LIV	+0. 04965 LV	+0. 04495 LVII	+0. 00470 LVIII
	+0. 66303 LXI	−0. 15803 LXII	−0. 72877 LXIV	+0. 31936 LXVII	−0. 59374 LXIX
$[31_5]$	=−0. 01256 XLVI	−0. 03737 XLVII	+0. 04495 XLVIII	+0. 16490 XLIX	+0. 32573 LI
	+0. 00144 LIII	+0. 07097 LIV	+0. 12139 LV	−0. 34274 LVII	+0. 46413 LVIII
	+0. 19457 LXI	−0. 21446 LXII	+0. 26535 LXIV	+0. 06494 LXVII	+0. 05124 LXIX
$[31_{5+6}]$	=−0. 02512 XLVI	−0. 07474 XLVII	+0. 08990 XLVIII	+0. 32980 XLIX	+0. 65146 LI
	+0. 00288 LIII	+0. 14194 LIV	+0. 24278 LV	+0. 31452 LVII	−0. 07174 LVIII
	+0. 38914 LXI	−0. 42892 LXII	+0. 53070 LXIV	+0. 12988 LXVII	+0. 10248 LXIX
$[31_{5+6+7+8}]$	=−0. 18506 XLVI	−0. 05122 XLVII	+0. 66303 XLVIII	−0. 06769 XLIX	−1. 44563 LI
	+0. 77123 LIII	+0. 45615 LIV	+0. 04051 LV	+0. 19457 LVII	−0. 15406 LVIII
	+1. 11986 LXI	−0. 84518 LXII	−0. 83623 LXIV	+0. 20790 LXVII	−0. 66878 LXIX
$[31_6]$	=−0. 01256 XLVI	−0. 03737 XLVII	+0. 04495 XLVIII	+0. 16490 XLIX	+0. 32573 LI
	+0. 00144 LIII	+0. 07097 LIV	+0. 12139 LV	+0. 65726 LVII	−0. 53587 LVIII
	+0. 19457 LXI	−0. 21446 LXII	+0. 26535 LXIV	+0. 06494 LXVII	+0. 05124 LXIX
$[31_7]$	=−0. 13328 XLVI	+0. 01960 XLVII	+0. 14427 XLVIII	−0. 33124 XLIX	−1. 10586 LI
	+0. 30695 LIII	−0. 01035 LIV	−0. 16856 LV	−0. 09996 LVII	−0. 06860 LVIII
	+0. 27559 LXI	−0. 22618 LXII	−0. 69755 LXIV	+0. 23168 LXVII	+0. 20860 LXIX
$[31_{7+8}]$	=−0. 15994 XLVI	+0. 02352 XLVII	+0. 57313 XLVIII	−0. 39749 XLIX	−2. 09709 LI
	+0. 76835 LIII	+0. 31421 LIV	−0. 20227 LV	−0. 11995 LVII	−0. 08232 LVIII
	+0. 73072 LXI	−0. 41626 LXII	−1. 36693 LXIV	+0. 07802 LXVII	−0. 77126 LXIX
$[31_8]$	=−0. 02666 XLVI	+0. 00392 XLVII	+0. 42886 XLVIII	−0. 06625 XLIX	−0. 99123 LI
	+0. 46140 LIII	+0. 32456 LIV	−0. 03371 LV	−0. 01999 LVII	−0. 01372 LVIII
	+0. 45513 LXI	−0. 19008 LXII	−0. 66938 LXIV	−0. 15366 LXVII	−0. 97986 LXIX
$[32_1]$	=+0. 42857 XLVIII	−0. 35714 L	+1. 65681 LI	−0. 28571 LII	+0. 71429 LIII
	−0. 19917 LVI	−0. 21429 LX	+0. 64286 LXI	−0. 14286 LXIII	+1. 51393 LXIV
	−0. 07143 LXV	−0. 21429 LXVIII	+0. 53126 LXIX	+0. 84941 LXXI	
$[32_{1+2+3+4}]$	=+1. 14286 XLVIII	+0. 71430 L	+0. 51156 LI	+0. 57144 LII	+0. 57144 LIII
	+0. 39834 LVI	+0. 42856 LX	+0. 71430 LXI	+0. 28570 LXIII	+0. 70523 LXIV
	+0. 14286 LXV	−0. 57144 LXVIII	+1. 41670 LXIX	−0. 99040 LXXI	
$[32_2]$	=+0. 14286 XLVIII	+0. 21429 L	−0. 22905 LI	−0. 42857 LII	+0. 57143 LIII
	−1. 17774 LVI	−0. 07143 LX	+0. 21429 LXI	+0. 28571 LXIII	−0. 22804 LXIV
	−0. 35714 LXV	−0. 07143 LXVIII	+0. 17709 LXIX	−0. 36796 LXXI	
$[32_{2+3}]$	=+0. 28572 XLVIII	+0. 42858 L	−0. 45810 LI	+0. 14286 LII	+0. 14286 LIII
	−0. 19341 LVI	−0. 14285 LX	+0. 42858 LXI	+0. 57142 LXIII	−0. 45608 LXIV
	+0. 28572 LXV	−0. 14286 LXVIII	+0. 35418 LXIX	−0. 73592 LXXI	
$[32_{2+3+4}]$	=+0. 71429 XLVIII	+1. 07144 L	−1. 14525 LI	+0. 85715 LII	−0. 14285 LIII
	+0. 59751 LVI	+0. 64285 LX	+0. 07144 LXI	+0. 42856 LXIII	−0. 80870 LXIV
	+0. 21429 LXV	−0. 35715 LXVIII	+0. 88544 LXIX	−1. 83981 LXXI	
$[32_3]$	=+0. 14286 XLVIII	+0. 21429 L	−0. 22905 LI	+0. 57143 LII	−0. 42857 LIII
	+0. 98433 LVI	−0. 07143 LX	+0. 21429 LXI	+0. 28571 LXIII	−0. 22804 LXIV
	+0. 64286 LXV	−0. 07143 LXVIII	+0. 17709 LXIX	−0. 36796 LXXI	
$[32_{3+4}]$	=+0. 57143 XLVIII	+0. 85715 L	−0. 91620 LI	+1. 28572 LII	−0. 71428 LIII
	+1. 77525 LVI	+0. 71428 LX	−0. 14285 LXI	+0. 14285 LXIII	−0. 58066 LXIV
	+0. 57143 LXV	−0. 28572 LXVIII	+0. 70835 LXIX	−1. 47185 LXXI	
$[32_4]$	=+0. 42857 XLVIII	+0. 64286 L	−0. 68715 LI	+0. 71429 LII	−0. 28571 LIII
	+0. 79092 LVI	+0. 78571 LX	−0. 35714 LXI	−0. 14286 LXIII	−0. 35262 LXIV
	−0. 07143 LXV	−0. 21429 LXVIII	+0. 53126 LXIX	−1. 10389 LXXI	

General corrections in terms of the correlates—Continued.

$[32_5]$	=−0.57143 XLVIII	−0.35714 L	−0.25579 LI	−0.28571 LII	−0.28571 LIII
	−0.19917 LVI	−0.21429 LX	−0.35714 LXI	−0.14286 LXIII	−0.35262 LXIV
	−0.07143 LXV	+0.78571 LXVIII	−0.35414 LXIX	+0.14098 LXXI	
$[33_1]$	=+0.38241 XLIX	−0.01338 L	+0.28212 LI	+0.49139 LV	+0.17882 LVI
	+0.23519 LVII	−0.48970 LIX	+0.02676 LXIII	+0.05085 LXIV	−0.10898 LXX
	+0.04516 LXXI				
$[33_{1+2+3}]$	=+0.34419 XLIX	+0.08796 L	+0.24732 LI	+0.34227 LV	−0.00605 LVI
	+0.31169 LVII	−0.05845 LIX	−0.17591 LXIII	+0.14209 LXIV	+0.00192 LXX
	−0.00563 LXXI				
$[33_{1+2+3+4+5}]$	=+0.40346 XLIX	−0.01912 L	+0.29797 LI	+0.27343 LV	−0.21221 LVI
	+0.19314 LVII	−0.15346 LIX	+0.03823 LXIII	+0.04883 LXIV	+0.13003 LXX
	−0.05197 LXXI				
$[33_2]$	=−0.16824 XLIX	+0.04589 L	−0.12675 LI	−0.25620 LV	−0.14549 LVI
	+0.33654 LVII	+1.13294 LIX	−0.09177 LXIII	+0.01616 LXIV	+0.08795 LXX
	−0.03838 LXXI				
$[33_{2+3+4+5}]$	=+0.02105 XLIX	−0.00574 L	+0.01585 LI	−0.21796 LV	−0.39103 LVI
	−0.04205 LVII	+0.33624 LIX	+0.01147 LXIII	−0.00202 LXIV	+0.23901 LXX
	−0.09713 LXXI				
$[33_3]$	=+0.13002 XLIX	+0.05545 L	+0.09195 LI	+0.10708 LV	−0.03938 LVI
	−0.26004 LVII	−0.70169 LIX	−0.11090 LXIII	+0.07508 LXIV	+0.02295 LXX
	−0.01241 LXXI				
$[33_{4+5}]$	=+0.05927 XLIX	−0.10708 L	+0.05065 LI	−0.06884 LV	−0.20616 LVI
	−0.11855 LVII	−0.09501 LIX	+0.21414 LXIII	−0.00326 LXIV	+0.12811 LXX
	−0.04634 LXXI				
$[33_{4+5+6}]$	=+0.03249 XLIX	+0.26386 L	+0.00648 LI	+0.02676 LV	−0.01811 LVI
	−0.06501 LVII	−0.17541 LIX	+0.47226 LXIII	−0.13871 LXIV	+0.00574 LXX
	−0.01686 LXXI				
$[33_5]$	=−0.48948 XLIX	+0.49714 L	−0.39276 LI	−0.10898 LV	+0.60640 LVI
	−0.02103 LVII	+0.16810 LIX	+0.00574 LXIII	−0.00100 LXIV	+0.61951 LXX
	−0.27989 LXXI				
$[33_6]$	=−0.02678 XLIX	+0.37094 L	−0.04417 LI	+0.09560 LV	+0.18805 LVI
	+0.05354 LVII	−0.08040 LIX	+0.25812 LXIII	−0.04545 LXIV	−0.12237 LXX
	+0.02948 LXXI				
$[34_1]$	=+0.50000 LII	−0.00438 LIII	−0.26038 LIV	+0.04386 LV	−0.35595 LVI
	+0.00877 LVIII	+0.04093 LIX	−0.04825 LXV	+0.20593 LXVI	
$[34_2]$	=+0.50000 LII	−0.49562 LIII	−0.52353 LIV	−0.04386 LV	−0.17630 LVI
	−0.00877 LVIII	−0.04093 LIX	+0.54825 LXV	−0.42603 LXVI	
$[34_{2+3}]$	=+0.09350 LIII	+0.05171 LIV	+0.03509 LV	−0.02734 LVI	−0.19298 LVIII
	+0.26302 LIX	+0.06140 LXV	−0.06505 LXVI		
$[34_{2+3+4}]$	=+0.00877 LIII	+0.00473 LIV	−0.08772 LV	−0.04296 LVI	−0.01755 LVIII
	−0.08185 LIX	+0.09649 LXV	−0.10222 LXVI		
$[34_{2+3+4+5}]$	=+0.36842 LIII	+0.19741 LIV	+0.31579 LV	−0.02343 LVI	+0.26315 LVIII
	+0.06141 LIX	+0.05263 LXV	−0.05575 LXVI		
$[34_4]$	=−0.08773 LIII	−0.04698 LIV	−0.12281 LV	−0.01562 LVI	+0.17543 LVIII
	−0.34787 LIX	+0.03509 LXV	−0.03717 LXVI		
$[34_5]$	=+0.35965 LIII	+0.19268 LIV	+0.40351 LV	+0.01953 LVI	+0.28070 LVIII
	+0.14326 LIX	−0.04386 LXV	+0.04647 LXVI		
$[35_1]$	=+0.66667 LVII	−0.33333 LVIII	+0.12460 LIX		
$[35_2]$	=−0.33333 LVII	+0.66667 LVIII	+0.12467 LIX		
$[36_1]$	=+0.65625 LX	−0.25000 LXI	+0.89729 LXII	−0.18750 LXIII	−0.31250 LXV
	+1.04629 LXVI				
$[36_2]$	=−0.18750 LX	+0.50000 LXI	+0.00694 LXII	+0.62500 LXIII	+0.37500 LXV
	−0.29474 LXVI				

General corrections in terms of the correlates—Continued.

$[36_{2+3}]$	=—0. 25000 LX	+1. 00000 LXI	+0. 21405 LXII	+0. 50000 LXIII	+0. 50000 LXV
	—0. 39298 LXVI				
$[36_{2+3+4}]$	=—0. 31250 LX	+0. 50000 LXI	—0. 19322 LXII	+0. 37500 LXIII	+0. 62500 LXV
	—0. 49122 LXVI				
$[36_3]$	=—0. 06250 LX	+0. 50000 LXI	+0. 20711 LXII	—0. 12500 LXIII	+0. 12500 LXV
	—0. 09824 LXVI				
$[36_4]$	=—0. 06250 LX	—0. 50000 LXI	—0. 40727 LXII	—0. 12500 LXIII	+0. 12500 LXV
	—0. 09824 LXVI				
$[37_1]$	=—0. 47619 LXVII	+0. 28571 LXVIII	+1. 88516 LXIX	—0. 23810 LXX	—1. 24740 LXXI
	—0. 09524 LXXII				
$[37_{1+2}]$	=+0. 04762 LXVII	+0. 57142 LXVIII	+0. 98987 LXIX	—0. 47620 LXX	—2. 49480 LXXI
	—0. 19048 LXXII				
$[37_2]$	=+0. 52381 LXVII	+0. 28571 LXVIII	—0. 89529 LXIX	—0. 23810 LXX	—1. 24740 LXXI
	—0. 09524 LXXII				
$[37_{2+3}]$	=+1. 04762 LXVII	+0. 57142 LXVIII	—1. 79058 LXIX	+0. 52380 LXX	+0. 38750 LXXI
	—0 19048 LXXII				
$[37_3]$	=+0. 52381 LXVII	+0. 28571 LXVIII	—0. 89529 LXIX	+0. 76190 LXX	+1. 63490 LXXI
	—0. 09524 LXXII				
$[37_4]$	=—0. 19048 LXVII	—0. 28571 LXVIII	—0. 03152 LXIX	—0. 09524 LXX	+0. 28663 LXXI
	+0. 76190 LXXII				
$[38_1]$	=—0. 30000 LXXII				
$[38_2]$	=+0. 70000 LXXII				
$[38_3]$	=—0. 30000 LXXII				

Normal equations for determining the correlates.

No. of equation.					
36.	0=—0. 01900	+2. 39394 XXXVI	—0. 34849 XXXVII	—0. 42424 XXXVIII	—0. 16103 XXXIX
		—0. 16667 XL			
37.	0=—0. 10500	—0. 34849 XXXVI	+1. 96213 XXXVII	+0. 60605 XXXVIII	—0. 71470 XXXIX
		—0. 58333 XL	—0. 25000 XLI		
38.	0=—1. 38200	—0. 42424 XXXVI	+0. 60605 XXXVII	+1. 90873 XXXVIII	+1. 71127 XXXIX
		+0. 54477 XL	—0. 10857 XLI	—0. 02286 XLII	+0. 03429 XLIII
39.	0=—2. 62184	—0. 16103 XXXVI	—0. 71470 XXXVII	+1. 71127 XXXVIII	+4. 97917 XXXIX
		+0. 31440 XL	+0. 31749 XLI		
40	0=+0. 25700	—0. 16667 XXXVI	—0. 58333 XXXVII	+0. 54477 XXXVIII	+0. 31440 XXXIX
		+1. 83953 XL	—0. 46714 XLI	—0. 04572 XLII	+0. 06858 XLIII
41.	0=+0. 03600	—0. 25000 XXXVII	—0. 10857 XXXVIII	+0. 31749 XXXIX	—0. 46714 XL
		+1. 71286 XLI	—0. 41571 XLII	—0. 50143 XLIII	—0. 17246 XLV
42.	0=—0. 43600	—0. 02286 XXXVIII	—0. 04572 XL	—0. 41571 XLI	+1. 53310 XLII
		+0. 51285 XLIII	—0. 45833 XLIV	—0. 34648 XLV	—0. 04167 XLVI
43.	0=+0. 42200	+0. 03429 XXXVIII	+0. 06858 XL	—0. 50143 XLI	+0. 51285 XLII
		+1. 79321 XLIII	+0. 62500 XLIV	+1. 78208 XLV	—0. 12500 XLVI
		—0. 12500 XLVII			
44.	0=—0. 10800	—0. 45833 XLII	+0. 62500 XLIII	+1. 79167 XLIV	+0. 08119 XLV
		—0. 54167 XLVI	—0. 25000 XLVII		
45.	0=+1. 29396	—0. 17246 XLI	—0. 34648 XLII	+1. 78208 XLIII	+0. 08119 XLIV
		+5. 36655 XLV	+0. 17792 XLVI	—0. 03359 XLVII	
46.	0=—0. 85900	—0. 04167 XLII	—0. 12500 XLIII	—0. 54167 XLIV	+0. 17792 XLV
		+1. 94161 XLVI	—0. 52204 XLVII	—0. 16230 XLVIII	—0. 00236 XLIX
		+0. 30655 LI	—0. 22032 LIII	—0. 08684 LIV	—0. 06038 LV
		—0. 01256 LVII	—0. 04782 LVIII	—0. 18506 LXI	+0. 08892 LXII
		+0. 20336 LXIV	—0. 13564 LXVII	+0. 00274 LXIX	

Normal equations for determining the correlates—Continued.

No. of equation.					
47.	0=+0.07100	—0.12500 XLIII	—0.25000 XLIV	—0.03359 XLV	—0.52204 XLVI
		+2.36262 XLVII	—0.45468 XLVIII	—0.43213 XLIX	—0.04607 L
		—0.16909 LI	+0.15719 LII	+0.05202 LIII	+0.85226 LIV
		+0.02849 LV	—0.03737 LVII	+0.06586 LVIII	—0.09214 LX
		—0.05122 LXI	+0.09362 LXII	—0.05646 LXIV	—0.51019 LXVI
		—0.50467 LXVII	—0.04607 LXVIII	—0.06405 LXIX	—0.09214 LXX
		—0.20054 LXXII			
48.	0=+0.50900	—0.16230 XLVI	—0.45468 XLVII	+3.08351 XLVIII	+0.94204 XLIX
		+1.13978 L	—0.49110 LI	+1.59043 LII	+1.19422 LIII
		—0.34226 LIV	+0.04965 LV	+0.39834 LVI	+0.04495 LVII
		+0.00470 LVIII	+1.27952 LX	+1.37733 LXI	+0.16029 LXII
		+0.28570 LXIII	—0.02354 LXIV	+0.14286 LXV	+0.94954 LXVI
		+0.93727 LXVII	—1.14596 LXVIII	+0.82296 LXIX	—0.14904 LXX
		—0.99040 LXXI	—0.26558 LXXII		
49.	0=+0.17600	—0.00236 XLVI	—0.43213 XLVII	+0.94204 XLVIII	+2.59290 XLIX
		—0.87669 L	+1.78516 LI	+0.58267 LII	—0.14557 LIII
		—0.46567 LIV	+0.63433 LV	—0.81861 LVI	+0.37907 LVII
		+0.08702 LVIII	—0.32156 LIX	+0.27914 LX	—0.06769 LXI
		—0.32529 LXII	+0.03249 LXIII	+0.68799 LXIV	—0.06679 LXVI
		+0.64786 LXVII	—0.36043 LXVIII	+0.17752 LXIX	—1.21034 LXX
		+0.22792 LXXI	—0.15718 LXXII		
50.	0=+0.76800	—0.04607 XLVII	+1.13978 XLVIII	—0.87669 XLIX	+2.72543 L
		—1.58218 LI	+1.29347 LII	—0.14285 LIII	—0.19080 LIV
		—0.01338 LV	+1.39196 LVI	+0.03251 LVII	+0.08770 LIX
		+1.21467 LX	+0.07144 LXI	+0.90184 LXII	+0.69242 LXIII
		—0.85515 LXIV	+0.21429 LXV	+1.01633 LXVI	+0.21139 LXVII
		—0.57124 LXVIII	+0.88544 LXIX	+1.06896 LXX	—2.09022 LXXI
		—0.10840 LXXII			
51.	0=+1.51880	+0.30655 XLVI	—0.06909 XLVII	—0.49110 XLVIII	+1.78516 XLIX
		—1.58218 L	+8.86605 LI	—0.91620 LII	—0.13509 LIII
		—0.37747 LIV	+0.81636 LV	—1.29311 LVI	+0.48110 LVII
		+0.20851 LVIII	—0.24226 LIX	—0.68715 LX	—0.24692 LXI
		+0.94982 LXII	—0.45162 LXIII	+6.28251 LXIV	—0.22905 LXV
		—0.01143 LXVII	—0.25579 LXVIII	+2.22099 LXIX	—0.39276 LXX
		+2.60233 LXXI			
52.	0=—1.09200	+0.15719 XLVII	+1.59043 XLVIII	+0.58267 XLIX	+1.29347 L
		—0.91620 LI	+3.73832 LII	—1.21428 LIII	—0.20628 LIV
		+1.24300 LVI	+1.58692 LX	—0.14285 LXI	+0.69955 LXII
		+0.14285 LXIII	—0.58066 LXIV	+1.07143 LXV	+0.13288 LXVI
		+0.45531 LXVII	—0.84940 LXVIII	+0.70835 LXIX	—0.12736 LXX
		—1.47185 LXXI	—0.33604 LXXII		
53.	0=+0.95400	—0.22032 XLVI	+0.05201 XLVII	+1.19422 XLVIII	—0.14557 XLIX
		—0.14285 L	—0.13509 LI	—1.21428 LII	+3.07086 LIII
		+0.99445 LIV	+0.51240 LV	—1.22404 LVI	+0.00144 LVII
		+0.42323 LVIII	+0.10234 LIX	—0.28572 LX	+1.62838 LXI
		—0.42210 LXII	+0.14285 LXIII	+0.26906 LXIV	—0.92419 LXV
		+0.37028 LXVI	+0.16138 LXVII	—0.28572 LXVIII	+0.01579 LXIX
		+0.48145 LXXI			
54.	0=+0.73567	—0.08684 XLVI	+0.85226 XLVII	—0.34226 XLVIII	—0.46567 XLIX
		—0.19080 L	—0.37747 LI	—0.20628 LII	+0.99445 LIII
		+6.38962 LIV	+0.15198 LV	+0.35657 LVI	+0.07097 LVII
		+0.03403 LVIII	+0.05483 LIX	—0.38160 LX	+0.45615 LXI
		+0.88320 LXII	—0.36815 LXIV	—0.52353 LXV	—1.68927 LXVI
		—0.34454 LXVII	+0.32228 LXVIII	—0.40696 LXIX	+0.13148 LXX
		—0.04255 LXXII			

Normal equations for determining the correlates—Continued.

No. of equation.					
55.	0=+0.06200	−0.06038 XLVI	+0.02849 XLVII	+0.04965 XLVIII	+0.63433 XLIX
		−0.01338 L	+0.81636 LI	+0.51240 LIII	+0.15198 LIV
		+1.24992 LV	+0.19835 LVI	+0.35658 LVII	+0.51433 LVIII
		−0.34644 LIX	+0.04051 LXI	−0.00584 LXII	+0.02676 LXIII
		+0.40095 LXIV	−0.04386 LXV	+0.04647 LXVI	+0.08336 LXVII
		+0.07870 LXIX	−0.10898 LXX	+0.04516 LXXI	
56.	0=−0.35860	+0.39834 XLVIII	−0.81861 XLIX	+1.39196 L	−1.29311 LI
		+1.24300 LII	−1.22404 LIII	+0.35657 LIV	+0.19835 LV
		+5.08671 LVI	+0.03333 LVII	+0.00391 LVIII	−0.25989 LIX
		+0.79092 LX	−0.39258 LXI	−0.21152 LXIII	−0.30444 LXIV
		+0.80803 LXV	+0.02197 LXVI	−0.19917 LXVIII	+0.49378 LXIX
		+0.60640 LXX	−1.31689 LXXI		
57.	0=−1.52800	−0.01256 XLVI	−0.03737 XLVII	+0.04495 XLVIII	+0.37907 XLIX
		+0.03251 L	+0.48110 LI	+0.00144 LIII	+0.07097 LIV
		+0.35658 LV	+0.03333 LVI	+1.89566 LVII	−0.86920 LVIII
		+0.76784 LIX	+0.19457 LXI	−0.21446 LXII	−0.06501 LXIII
		+0.33236 LXIV	+0.06494 LXVII	+0.05124 LXIX	−0.02103 LXX
		+0.00678 LXXI			
58.	0=+0.71100	−0.01782 XLVI	+0.06586 XLVII	+0.00470 XLVIII	+0.08702 XLIX
		+0.20851 LI	+0.42323 LIII	+0.03403 LIV	+0.51433 LV
		+0.00391 LVI	−0.86920 LVII	+1.89230 LVIII	−0.07994 LIX
		−0.15406 LXI	+0.20862 LXII	+0.08475 LXIV	−0.00877 LXV
		+0.00930 LXVI	+0.01842 LXVII	+0.02746 LXIX	
59.	0=−0.82100	−0.32156 XLIX	+0.08770 L	−0.24226 LI	+0.10234 LIII
		+0.05483 LIV	−0.34644 LV	−0.25989 LVI	+0.76784 LVII
		−0.07994 LVIII	+2.66442 LIX	−0.17541 LXIII	+0.03089 LXIV
		−0.04093 LXV	+0.04335 LXVI	+0.16810 LXX	−0.07336 LXXI
60.	0=+1.28500	−0.09214 XLVII	+1.27952 XLVIII	+0.27914 XLIX	+1.21467 L
		−0.68715 LI	+1.58692 LII	−0.28572 LIII	−0.38160 LIV
		+0.79092 LVI	+2.58560 LX	−0.60714 LXI	+2.70097 LXII
		−0.33036 LXIII	−0.35262 LXIV	−0.38393 LXV	+3.07895 LXVI
		+0.42278 LXVII	−0.64247 LXVIII	+0.53126 LXIX	+0.14364 LXX
		−1.10389 LXXI	−0.21680 LXXII		
61.	0=−0.11700	−0.18506 XLVI	−0.05122 XLVII	+1.37733 XLVIII	−0.06769 XLIX
		+0.07144 L	−0.24692 LI	−0.14285 LII	+1.62838 LIII
		+0.45615 LIV	+0.04051 LV	−0.39258 LVI	+0.19457 LVII
		−0.15406 LVIII	−0.60714 LX	+3.19130 LXI	−0.63113 LXII
		+0.92856 LXIII	+0.22162 LXIV	+0.71429 LXV	−0.39298 LXVI
		+0.20790 LXVII	−0.35715 LXVIII	+0.21666 LXIX	+0.11349 LXXI
62.	0=+1.59600	+0.08892 XLVI	+0.09362 XLVII	+0.16029 XLVIII	−0.32529 XLIX
		+0.90184 L	+0.94982 LI	+0.69955 LII	−0.42210 LIII
		+0.88320 LIV	−0.00584 LV	−0.21446 LVII	+0.20862 LVIII
		+2.70097 LX	−0.63113 LXI	+7.25466 LXII	+0.00694 LXIII
		+0.40624 LXIV	−0.19322 LXV	+5.67036 LXVI	+0.14355 LXVII
		−0.20682 LXVIII	+0.31440 LXIX	+0.69502 LXX	−0.15106 LXXII
63.	0=+0.23400	+0.28570 XLVIII	+0.03249 XLIX	+0.69242 L	−0.45162 LI
		+0.14285 LII	+0.14285 LIII	+0.02676 LV	−0.21152 LVI
		−0.06501 LVII	−0.17541 LIX	−0.33036 LX	+0.92856 LXI
		+0.00694 LXII	+1.66868 LXIII	−0.59479 LXIV	+0.66072 LXV
		−0.29474 LXVI	−0.14286 LXVIII	+0.35418 LXIX	+0.00574 LXX
		−0.75278 LXXI			

Normal equations for determining the correlates—Continued.

No. of equation.					
64.	0=+1.11710	−0.20336 XLVI	−0.05646 XLVII	−0.02354 XLVIII	+ 0.68799 XLIX
		−0.85515 L	+6.28251 LI	−0.58066 LII	+ 0.26906 LIII
		−0.36815 LIV	+0.40095 LV	−0.30444 LVI	+ 0.33236 LVII
		+0.08475 LVIII	+0.03089 LIX	−0.35262 LX	+ 0.22162 LXI
		+0.40624 LXII	−0.59479 LXIII	+5.02318 LXIV	− 0.22804 LXV
		−0.05939 LXVII	−0.35262 LXVIII	+1.93056 LXIX	− 0.00100 LXX
		+1.83238 LXXI			
65.	0=−1.03500	+0.14286 XLVIII	+0.21429 L	−0.22905 LI	+ 1.07143 LII
		−0.92419 LIII	−0.52353 LIV	−0.04386 LV	+ 0.80803 LVI
		−0.00877 LVIII	−0.04093 LIX	−0.38393 LX	+ 0.71429 LXI
		−0.19322 LXII	+0.66072 LXIII	−0.22804 LXIV	+ 1.81611 LXV
		−0.91725 LXVI	−0.07143 LXVIII	+0.17709 LXIX	− 0.36796 LXXI
66.	0=+1.10860	−0.51019 XLVII	+0.94954 XLVIII	−0.06679 XLIX	+ 1.01633 L
		+0.13288 LII	+0.37028 LIII	−1.68927 LIV	+ 0.04647 LV
		+0.02197 LVI	+0.00930 LVIII	+0.04335 LIX	+ 3.07895 LX
		−0.39298 LXI	+5.67036 LXII	−0.29474 LXIII	− 0.91725 LXV
		+7.48852 LXVI	+0.54935 LXVII	−0.40019 LXVIII	+ 0.61614 LXX
		−0.12553 LXXII			
67.	0=+1.20700	−0.13564 XLVI	−0.50467 XLVII	+0.93727 XLVIII	+ 0.64786 XLIX
		+0.21139 L	−0.01143 LI	+0.45531 LII	+ 0.16138 LIII
		−0.34454 LIV	+0.08336 LV	+0.06494 LVII	+ 0.01842 LVIII
		+0.42278 LX	+0.20790 LXI	+0.14355 LXII	− 0.05939 LXIV
		+0.54935 LXVI	+2.34994 LXVII	+0.78281 LXVIII	− 1.40446 LXIX
		+0.94658 LXX	+0.38750 LXXI	−0.56446 LXXII	
68.	0=+1.71800	−0.04607 XLVII	−1.14596 XLVIII	−0.36043 XLIX	− 0.57124 L
		−0.25579 LI	−0.84940 LII	−0.28572 LIII	+ 0.32228 LIV
		−0.19917 LVI	−0.64247 LX	−0.35715 LXI	− 0.20682 LXII
		−0.14286 LXIII	−0.35262 LXIV	−0.07143 LXV	− 0.40019 LXVI
		+0.78281 LXVII	−2.42875 LXVIII	−0.25956 LXIX	+ 0.85752 LXX
		−0.71892 LXXI	−0.39412 LXXII		
69.	0=+0.43950	+0.00274 XLVI	−0.06405 XLVII	+0.82296 XLVIII	+ 0.17752 XLIX
		+0.88544 L	+2.22099 LI	+0.70835 LII	+ 0.01579 LIII
		−0.40696 LIV	+0.07870 LV	+0.49378 LVI	+ 0.05124 LVII
		+0.02746 LVIII	+0.53126 LX	+0.21666 LXI	+ 0.31440 LXII
		+0.35418 LXIII	+1.93056 LXIV	+0.17709 LXV	− 1.40446 LXVII
		−0.25956 LXVIII	+8.85737 LXIX	−0.89529 LXX	− 4.24491 LXXI
		−0.03152 LXXII			
70.	0=+1.18300	−0.09214 XLVII	−0.14904 XLVIII	−1.21034 XLIX	+ 1.06896 L
		−0.39276 LI	−0.12736 LII	+0.13148 LIV	− 0.10898 LV
		+0.60640 LVI	−0.02103 LVII	+0.16810 LIX	+ 0.14364 LX
		+0.69502 LXII	+0.00574 LXIII	−0.00100 LXIV	+ 0.61614 LXVI
		+0.94658 LXVII	+0.85752 LXVIII	−0.89529 LXIX	+ 2.52505 LXX
		+1.35501 LXXI	−0.31204 LXXII		
71.	0=−0.27683	−0.99040 XLVIII	+0.22792 XLIX	−2.09022 L	+ 2.60233 LI
		−1.47185 LII	+0.48145 LIII	+0.04516 LV	− 1.31689 LVI
		+0.00678 LVII	−0.07336 LIX	−1.10389 LX	+ 0.11349 LXI
		−0.75278 LXIII	+1.83238 LXIV	−0.36796 LXV	+ 0.38750 LXVII
		−0.71892 LXVIII	−4.24491 LXIX	+1.35501 LXX	+10.02279 LXXI
		+0.28663 LXXII			
72.	0=−0.47500	−0.20054 XLVII	−0.26558 XLVIII	−0.15718 XLIX	− 0.10840 L
		−0.33604 LII	−0.04255 LIV	−0.21680 LX	− 0.15106 LXII
		−0.12553 LXVI	−0.56446 LXVII	−0.39412 LXVIII	− 0.03152 LXIX
		−0.31204 LXX	+0.28663 LXXI	+2.16651 LXXII	

Values of the correlates and of their logarithms.

XXXVI	=+0.0873 log 8.9410640+		LV	=+0.7732 log 9.8883199+
XXXVII	=−0.1668 log 9.2223002−		LVI	=−0.2766 log 9.4418993−
XXXVIII	=+0.6348 log 9.8026301+		LVII	=+1.1760 log 0.0704258+
XXXIX	=+0.3291 log 9.5173147+		LVIII	=+0.2972 log 9.4729903+
XL	=−0.4448 log 9.6481550−		LIX	=−0.1318 log 9.1199813−
XLI	=−0.2153 log 9.3330440−		LX	=−2.8810 log 0.4595373−
XLII	=+0.6408 log 9.8066954+		LXI	=−0.4526 log 9.6557433−
XLIII	=−0.6142 log 9.7882886−		LXII	=+0.0247 log 8.3919931+
XLIV	=+0.6751 log 9.8293874+		LXIII	=+0.0559 log 8.7474118+
XLV	=−0.0355 log 8.5504730−		LXIV	=+1.2300 log 0.0899051+
XLVI	=+0.7093 log 9.8508361+		LXV	=−0.2997 log 9.4766867−
XLVII	=+0.1479 log 9.1699094+		LXVI	=+1.0147 log 0.0063205+
XLVIII	=+0.5721 log 9.7574644+		LXVII	=+0.3963 log 9.5980241+
XLIX	=−0.4693 log 9.6714320−		LXVIII	=−1.8415 log 0.2651811−
L	=−1.5482 log 0.1898383−		LXIX	=+0.1964 log 9.2930751+
LI	=−1.5331 log 0.1855733−		LXX	=+0.5724 log 9.7577376+
LII	=+0.8096 log 9.9082920+		LXXI	=−0.3828 log 9.5829833−
LIII	=−1.1513 log 0.0611988−		LXXII	=−0.0023 log 7.3677369−
LIV	=+0.2096 log 9.3214949+			

Values of the general corrections.

	″		″		″
[22_1]	=−0.770	[29_2]	=+0.026	[33_2]	=+0.371
[22_2]	=+0.750	[29_3]	=+0.153	[33_3]	=−0.303
[22_3]	=+0.065	[30_1]	=+0.198	[33_{4+5}]	=−0.074
[23_3]	=+0.087	[30_2]	=−0.437	[33_5]	=+0.224
[24_1]	=+0.096	[30_3]	=−0.286	[33_6]	=−0.521
[24_2]	=−0.371	[30_4]	=+0.220	[34_1]	=+0.708
[24_3]	=−0.036	[30_5]	=+0.033	[34_2]	=+0.287
[25_1]	=−0.133	[30_6]	=−0.005	[34_{2+3}]	=−0.242
[25_2]	=+0.161	[30_7]	=+0.212	[34_4]	=+0.050
[25_3]	=+0.193	[31_1]	=+0.302	[34_5]	=+0.058
[26_1]	=+0.024	[31_2]	=−0.199	[35_1]	=+0.669
[26_2]	=+0.043	[31_3]	=+0.078	[35_2]	=−0.210
[26_3]	=−0.108	[31_4]	=−0.232	[36_1]	=−0.611
[27_1]	=+0.278	[31_5]	=−0.455	[36_2]	=−0.062
[27_2]	=−0.357	[31_6]	=+0.424	[36_3]	=−0.185
[27_3]	=−0.024	[31_7]	=+0.362	[36_4]	=+0.252
[27_4]	=−0.158	[31_8]	=−0.026	[37_1]	=−0.003
[27_5]	=+0.353	[32_1]	=−0.365	[37_2]	=−0.153
[28_1]	=−0.157	[32_2]	=−0.320	[37_3]	=−0.684
[28_2]	=+0.404	[32_3]	=+0.744	[37_4]	=+0.278
[28_3]	=+0.389	[32_4]	=−0.609	[38_1]	=+0.001
[28_4]	=−0.330	[32_5]	=−0.441	[38_2]	=−0.002
[29_1]	=+0.198	[33_1]	=+0.065	[38_3]	=+0.001

Residuals resulting from substitution of general corrections in numerical equations of condition.

No. of equation.	Residual.	No. of equation.	Residual.
36	0.0000	55	0.0000
37	0.0000	56	−0.0013
38	0.0000	57	+0.0001
39	+0.0007	58	0.0000
40	0.0000	59	−0.0001
41	0.0000	60	−0.0001
42	0.0000	61	−0.0001
43	0.0000	62	−0.0007
44	0.0000	63	−0.0001
45	0.0000	64	+0.0013
46	0.0000	65	−0.0001
47	0.0000	66	+0.0003
48	0.0000	67	0.0000
49	+0.0001	68	−0.0001
50	0.0000	69	+0.0004
51	+0.0011	70	0.0000
52	−0.0001	71	−0.0005
53	0.0000	72	0.0000
54	−0.0007		

PROBABLE ERRORS OF OBSERVED AND ADJUSTED ANGLES.

§ **5.** Let—

m = whole number of observed angles in a section (one adjustment).
r = whole number of rigid conditions in a section.
n = number of triangles in principal chain.
$[pvv]$ = sum of weighted squares of corrections to observed angles.
ρ_1 = probable error of an observed angle of weight unity.
ρ_s = probable error of an observed angle of average weight in whole section.
ρ_s' = probable error of an adjusted angle of average weight in whole section.
p_s = average weight of an observed angle in whole section.
p_c = average weight of an observed angle in principal chain.
ρ_c = probable error of an observed angle of average weight in principal chain.
ρ_c' = probable error of an adjusted angle of average weight in principal chain.
$[vv]$ = sum of squares of closing errors of triangles in principal chain.
ρ_t = probable error of an observed angle in principal chain as derived from the closing errors of triangles.

Proceeding as in Chapter XIV, C, § 8, there are found the following values:

FOR THE ENTIRE SECTIONS OF THIS CHAPTER.

Section.	Extent of section.	m	r	$[pvv]$	ρ_1	p_s	ρ_s	$\sqrt{\frac{m-r}{m}}$	ρ_s'
					″		″		″
XIV	Willow Springs-Morgan Park to Oakland-Kansas...	125	83	30.18	0.41	0.96	0.42	0.58	0.24
XV	Oakland-Kansas to Denver-Parkersburg............	121	89	19.69	0.32	0.95	0.33	0.51	0.17

FOR THE PRINCIPAL CHAIN CONNECTING THE CHICAGO AND OLNEY BASES, GIVEN IN D, § 6, FOLLOWING.

Section.	Extent of principal chain in each section.	p_c	ρ_c	ρ_c'	From closing errors of triangles.				
					$[vv]$	n	ρ_t	Average error.	Greatest error.
			″	″			″	″	″
VII	Chicago Base to Willow Springs-Morgan Park	0.63	0.57	0.35	2.89	2	0.47	1.06	1.63
XIV	Willow Springs-Morgan Park to Oakland-Kansas	0.98	0.41	0.24	40.70	21	0.54	1.11	2.85
XV	Oakland-Kansas to Olney Base	0.89	0.34	0.17	2.96	12	0.19	0.34	1.29
	Entire principal chain				46.55	35	0.45	0.84	2.85

D.—PRINCIPAL CHAIN OF TRIANGLES BETWEEN CHICAGO AND OLNEY BASES.

§ **6.**—The two principal chains of triangles connecting Chicago Base with Sandusky and Olney Bases diverge from the line Morgan Park-Willow Springs. In adjusting the sides of the chain joining Chicago and Olney Bases, it has been deemed sufficiently accurate to take this common line in place of Chicago Base as the terminal line, and use its adjusted length, derived from the chain joining Chicago and Sandusky Bases, Chapter XVII, D, § 6. The last-named chain gives for the weighted mean logarithm of the line Morgan Park-Willow Springs, expressed in feet 4.7020318±15.07, the probable error being in units of the seventh decimal place. With this logarithm and the angles of the triangles given in the following table, the logarithm of the Olney Base is found to be 4.3349234. The logarithm of the measured length of Olney Base, expressed in feet, is from Chapter XII, § 5, 4.3349191. The discrepancy between this and the value derived by the triangulation is −43 units in the seventh place of decimals. The probable errors of observed angles of average weight in the chain are for the parts lying north and south of the line Kansas-Oakland, ±0″.41 and ±0″.34, respectively. (See Chapter XX, C, § 5.) With these values and the values of α^2 and β^2, given in the tables which follow, there results for the entire chain, using the notation of Chapter XIV, D,

$$\Sigma\,(\alpha^2+\beta^2)\,\rho^2=2745.$$

Considering Olney Base as exact, the constant for the system—

$$\frac{1}{p}+\frac{1}{p'}=2745+(15.07)^2=2972.$$

The logarithmic discrepancy derived above is $d=\,-43$.

From these data and the values of $\frac{1}{p}$ given in the tables, the corrections to the logarithms of the several sides as computed from the line Morgan Park-Willow Springs are readily derived. The logarithm of the line Morgan Park-Willow Springs is, however, left unchanged.

The arrangement of the tables is the same as that of the tables in Chapter XIV, D, to which reference may be made for a detailed explanation.

The line in the system having the maximum probable error is Ash Grove-Spring Creek, for which $\frac{1}{p}=1466$ and $\frac{1}{p'}=1506$, giving for the probable error of the logarithm of the line, ± 27.26 in units of the seventh decimal place. This probable error corresponds to the $\frac{1}{159340}$ part of the line's length.

Principal chain of triangles between Chicago and Olney Bases.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	α^2 and β^2	$\Sigma\,(\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Orland	47 50 00.040		4.7020318	364.81	....	......	4.7020318
Morgan Park	56 04 39.416	+0.614	4.7510696	....	....	......	4.7510692
Willow Springs	76 05 21.193		4.8191715	27.04	391.85	203	4.8191711
Crete	40 37 23.366		4.8191715	600.25	....	......	4.8191711
Orland	85 55 33.910	+0.089	5.0044387	....	....	......	5.0044381
Morgan Park	53 27 03.986		4.9104411	243.36	1235.46	436	4.9104405
Garden	109 37 38.643		4.9104411	56.25	....	......	4.9104405
Creto	34 22 30.693	−0.684	4.6881854	....	....	......	4.6881846
Orland	35 59 51.214		4.7056303	841.00	2132.71	587	4.7056295
Manteno	38 49 41.651		4.7056303	681.21	....	......	4.7056295
Crete	59 34 41.400	+0.238	4.8440404	....	....	......	4.8440394
Garden	81 35 37.776		4.9036808	9.61	2823.53	704	4.9036798
Grant	90 01 34.805		4.9036808	0.01	....	......	4.9036798
Manteno	49 41 31.863	+0.911	4.7859658	....	....	......	4.7859646
Crete	40 16 54.077		4.7142796	615.04	3438.58	808	4.7142784
Kankakee	42 52 26.551		4.7142796	515.29	....	......	4.7142784
Grant	39 04 06.286	+2.607	4.6810337	....	....	......	4.6810324
Manteno	98 03 27.743		4.8772140	9.00	3962.87	896	4.8772127
St. Anne	61 39 12.566		4.8772140	127.69	....	......	4.8772127
Kankakee	80 47 26.510	+1.200	4.9270517	....	....	......	4.9270502
Grant	37 33 21.840		4.7176857	750.76	4841.32	1045	4.7176842
Clifton	44 53 01.124		4.7176857	445.21	....	......	4.7176842
St. Anne	69 29 55.588	+1.429	4.8406689	....	....	......	4.8406673
Kankakeo	65 37 04.065		4.8285135	92.16	5378.69	1136	4.8285119
Watseka	42 13 18.664		4.8285135	542.89	....	......	4.8285119
Clifton	85 41 52.338	−0.125	4.9999177	....	....	......	4.9999159
St. Anne	52 04 50.251		4.8981514	268.96	6190.54	1272	4.8981496
Spring Creek	52 25 00.970		4.8981514	262.44	....	......	4.8981496
Watseka	87 19 51.025	−2.260	4.9986978	....	....	......	4.9986957
Clifton	40 15 09.207		4.8095070	615.04	7068.02	1421	4.8095049
Ash Grove	64 29 39.946		4.8095070	102.01	....	......	4.8095049
Spring Creek	56 48 17.986	+2.853	4.7766667	....	....	......	4.7766646
Watseka	58 42 02.845		4.7857336	163.84	7333.87	1466	4.7857315
Paxton	43 01 46.737		4.7857336	510.76	....	......	4.7857315
Ash Grove	66 09 52.840	+0.234	4.9129937	....	....	......	4.9129914
Spring Creek	70 48 21.535		4.9268709	53.29	7897.92	1561	4.9268686
Butler	74 42 07.877		4.9268709	32.49	....	......	4.9268686
Paxton	49 52 59.518	−0.925	4.8260474	....	....	......	4.8260451
Ash Grove	55 24 53.703		4.8580877	210.25	8140.66	1602	4.8580854
Rantoul	66 45 36.274		4.8580877	81.00	....	......	4.8580854
Butler	35 09 27.322	−0.393	4.6551298	....	....	......	4.6551275
Paxton	78 04 57.155		4.8853751	20.25	8241.91	1619	4.8853728
Pilot Grove	63 17 22.618		4.8853751	112.36	....	......	4.8853728
Rantoul	61 55 09.429	−1.582	4.8799917	....	....	......	4.8799893
Butler	54 47 29.073		5.8466356	219.04	8573.31	1675	4.8466332

Principal chain of triangles between Chicago and Olney Bases—Continued.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	α^2 and β^2	$\Sigma\ (\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Mayview	50 15 15.217		4.8466356	306.25			4.8466332
Pilot Grove	59 45 02.354	−0.898	4.8972061				4.8972036
Rantoul	69 59 43.657		4.9337457	59.29	8938.85	1737	4.9337432
Fairmount	69 51 55.884		4.9337457	59.29			4.9337432
Mayview	44 03 33.039	+2.459	4.8033669				4.8033644
Pilot Grove	66 04 32.254		4.9221171	88.36	9086.50	1762	4.9221146
Lynn Grove	77 55 02.230		4.9221171	20.25			4.9221146
Fairmount	38 21 02.955	−0.601	4.7245699				4.7245673
Mayview	63 43 55.751		4.8845103	108.16	9214.91	1784	4.8845077
Palermo	69 23 20.403		4.8845103	62.41			4.8845077
Lynn Grove	42 20 08.198	+1.965	4.7415573				4.7415547
Fairmount	68 16 32.326		4.8812423	70.56	9347.88	1806	4.8812397
Oakland	50 08 59.318		4.8812423	309.76			4.8812397
Palermo	77 34 05.556	−0.770	4.9857345				4.9857317
Lynn Grove	52 16 56.499		4.8942340	265.69	9923.33	1903	4.8942312
Kansas	34 59 56.219		4.8942340	906.01			4.8942312
Oakland	105 25 07.585	+0.451	5.1197367				5.1197336
Palermo	39 34 57.748		4.9399242	650.25	11479.59	2166	4.9399211
Westfield	66 32 43.329		4.9399242	82.81			4.9399211
Kansas	77 31 16.143	−0.014	4.9669942				4.9669909
Oakland	35 56 01.645		4.7459034	846.81	929.62	2271	4.7459001
Martinsville	44 45 46.140		4.7459034	449.44			4.7459001
Westfield	66 14 22.264	−0.421	4.8597580				4.8597546
Kansas	68 59 52.484		4.8683697	64.00	1443.06	2329	4.8683663
Casey	59 56 02.254		4.8683697	148.84			4.8683663
Martinsville	78 15 51.495	−0.334	4.9219541				4.9219506
Westfield	41 48 07.221		4.7549664	556.96	2148.86	2409	4.7549629
Belle Air	71 47 41.693		4.7549664	47.61			4.7549629
Casey	58 02 39.805	+0.297	4.7058987				4.7058952
Martinsville	50 09 39.025		4.6625421	309.76	2506.23	2449	4.6625386
Hunt City	46 41 34.867		4.6625421	392.04			4.6625386
Belle Air	66 58 14.740	+0.251	4.7645284				4.7645248
Casey	66 20 10.970		4.7624528	84.64	2982.91	2503	4.7624492
Oblong	59 43 05.933		4.7624528	151.29			4.7624492
Hunt City	75 44 47.060	−0.165	4.8125824				4.8125787
Belle Air	44 32 07.630		4.6720968	457.96	3592.16	2572	4.6720931
Mound	44 48 48.626		4.6720968	449.44			4.6720931
Oblong	65 50 49.981	+0.854	4.7842430				4.7842392
Hunt City	69 20 22.024		4.7951610	62.41	4104.01	2629	4.7951572
Claremont	39 43 53.606		4.7951610	645.16			4.7951572
Mound	105 39 36.137	+0.193	4.9731031				4.9730990
Oblong	34 36 31.043		4.7438535	930.25	5679.42	2807	4.7438494
Onion Hill	65 34 22.518		4.7438535	90.25			4.7438494
Mound	65 25 55.299	−1.293	4.7433668				4.7433627
Claremont	48 59 42.729		4.6623272	338.56	6108.23	2856	4.6623231

Principal chain of triangles between Chicago and Olney Bases—Continued.

Stations.	Angles.	Errors of closure.	Logarithms of sides in feet.	α^2 and β^2	$\Sigma(\alpha^2+\beta^2)$	$\frac{1}{p}$	Weighted mean logarithms of sides in feet.
	° ′ ″	″					
Check Base	73 43 58.026		4.6623272	37.21	....	......	4.6623231
Onion Hill	48 26 34.537	+0.029	4.5541443	....	....	......	4.5541401
Mound	57 49 27.766		4.6076572	176.89	6322.33	2880	4.6076539
West Base	96 16 38.736		4.6076572	5.29	....	......	4.6076530
Check Base	39 57 14.497	+0.006	4.4179207	....	....	......	4.4179165
Onion Hill	43 46 06.941		4.4502164	484.00	6811.62	2935	4.4502122
East Base	83 35 40.698		4.4502164	5.76	....	......	4.4502122
West Base	46 45 34.253	+0.240	4.3153564	....	....	......	4.3153521
Check Base	49 38 45.156		4.3349234	320.41	7137.79	2972	4.3349191

CHAPTER XXI.

TRIANGULATION NOT FORMING A PART OF THE MAIN SYSTEM.

§ **1**. Besides the main primary triangulation for which the adjusted angles and mean sides have already been given in Chapters XIV to XX, there are two lateral chains depending on these mean sides, and one independent triangulation depending on a base measured with the primary base-apparatus of Würdemann. The lateral chains are: One in the north end of Lake Michigan, extending from the line Door Bluff–Cedar River of the main triangulation in Green Bay to Spectacle Reef in Lake Huron; and the other in Lake Superior, extending from the line Vulcan–St. Ignace of the main triangulation east to station Mamainse. In both chains there are many angles not well measured. The independent triangulation is in Saginaw Bay.

In these triangulations each triangle has been adjusted by making the sum of its angles equal to 180° plus the spherical excess, and for some stations, where sum-angles were read, a local adjustment was made.

§ **2**. The triangulation extending from the line Door Bluff–Cedar River, in Green Bay, to Spectacle Reef, in Lake Huron, was measured in three sections. The section between Spectacle Reef and High Island had its angles read by Captain J. N. Macomb, between 1849 and 1855, with the 10-inch repeating theodolite Gambey No. 1, and the angles were well measured. The section between the line Hat Island–Pointe aux Chênes and Pointe aux Becs Sciés was measured in 1860 by Lieutenant J. L. K. Smith and Assistant James Carr, the theodolites used being Gambey No. 1 and Würdemann No. 65. The section between the lines South Fox–North Manitou and Cedar River–Door Bluff was measured in 1864, the observers being O. B. Wheeler, O. N. Chaffee, G. E. Swinscoe, and W. T. Casgrain. The theodolites were Gambey Nos. 1 and 2, Würdemann-Gambey No. 1, and Würdemann No. 65.

A base-line, about 4 miles long, in the first section, was measured by Captain T. J. Lee, topographical engineers, in 1854, on the south side of the Straits of Mackinac. (See Report of Chief of Topographical Engineers, U. S. A., for 1854.) Its length depended on that of the 15-feet brass bar of the Lake Survey, whose length at the time was not known with great precision. Würdemann had assigned a length and a coefficient of expansion to this bar, but nothing is known of the methods by which he determined them, and it is now known that the expansion assigned by him, namely, $0^{in}.0017$ per degree Fahrenheit, is largely in error. As the length of this bar has been changed since the measurement of the base by the insertion of agate plates in its ends, it is impossible now to determine its original length. Accordingly, the sides of the triangulation have been made to depend on the side Door Bluff–Cedar River of the main triangulation, both for length and azimuth, this being the nearest side for which identity between the old and new stations was certain. The azimuth Door Bluff–Cedar River is given in Chapter XXVII, § 3, as 126° 05′ 24″.45, and the logarithm of the length in feet (Chapter XV, D, § 8) as 4.9190836.

§ **3**. In the following table, the first column gives the names of stations in groups of threes for each triangle; the second gives the date; the third the observer; the fourth gives the seconds of the mean observed angle at the station for that triangle; the fifth gives the adjusted angle; the sixth gives the logarithm of the side in feet which is opposite to the station on the same horizontal line.

Triangulation from Green Bay to Lake Huron.

Name of station.	Date.	Observer.	Seconds of measured angle.	Adjusted spherical angle.	Logarithms of sides in feet.
			"	° ′ "	
Boyer's Bluff	1864	O. N. Chaffee	00. 60	55 57 01. 14	4. 9190835
Cedar River	1864	......do	08. 24	33 58 06. 56	4. 7479703
Door Bluff	1864	......do	55. 72	90 04 53. 40	5. 0007638
Bark River	1864	O. N. Chaffee	05. 28	76 48 06. 08	5. 0007638
Boyer's Bluff	1864	......do	58. 73	33 50 00. 63	4. 7580733
Cedar River	1864	......do	54. 99	69 21 54. 55	4. 9835935
Burnt Bluff	1864	O. N. Chaffee	23. 65	42 42 33. 60	4. 9835935
Bark River	1864	......do	49. 33	52 37 49. 15	5. 0524311
Boyer's Bluff	1864	......do	50. 89	84 39 49. 80	5. 1503232
Rock Island	1864	O. N. Chaffee	59. 47	76 48 58. 43	5. 1503232
Burnt Bluff	1864	......do	38. 05	58 14 37. 01	5. 0914914
Bark River	1864	......do	26. 50	44 56 27. 47	5. 0109587
South Fox	1864	O. B. Wheeler	58. 00	24 22 59. 18	5. 0109514
Rock Island	1864	O. N. Chaffee	00. 28	74 16 00. 44	5. 3786337
Burnt Bluff	1864	W. T. Casgrain	04. 90	81 20 06. 08	5. 3901967
Northwest Manitou	1864	G. E. Swinscoe	57. 75	91 02 58. 39	5. 3901967
South Fox	1864	O. B. Wheeler	31. 19	63 21 32. 42	5. 3415242
Rock Island	1864	O. N. Chaffee	34. 00	25 35 34. 68	5. 0257201
Cat Head*	....	....	....	58 41 40. 44	5. 0257201
South Fox	1864	O. B. Wheeler	32. 32	63 10 33. 40	5. 0446113
Northwest Manitou	1864	G. E. Swinscoe	48. 50	58 07 48. 50	5. 0230893
North Manitou	1860, '64	James Carr and O. B. Wheeler.	19. 27	50 48 19. 24	5. 0230893
Cat Head	1860	Lieut. J. L. K. Smith	38. 50	77 38 33. 59	5. 1236067
South Fox	1860	James Carr	09. 53	51 33 09. 75	5. 0276476
Pyramid Point	1860	James Carr	25. 54	72 46 26. 24	5. 0276476
North Manitou	1860	......do	37. 71	86 23 38. 19	5. 0467182
Cat Head	1860	Lieut. J. L. K. Smith	55. 79	20 49 56. 56	4. 5985820
South Manitou	1860	James Carr	27. 10	41 01 32. 01	4. 5985820
Pyramid Point	1860	......do	23. 71	47 40 26. 40	4. 6502522
North Manitou	1860	......do	59. 16	91 18 02. 03	4. 7813048
Sleeping Bear	1860	James Carr	02. 49	70 06 02. 13	4. 7813048
South Manitou	1860	......do	23. 55	52 27 28. 11	4. 7072630
Pyramid Point	1860	......do	26. 99	57 26 30. 36	4. 7337897
Becs Sciés	1860	James Carr	02. 55	23 26 01. 55	4. 7337897
Sleeping Bear	1860	......do	39. 00	120 49 38. 00	5. 0680981
South Manitou	1860	......do	23. 87	35 44 21. 32	4. 9007324
Beaver Island	1860	Lieut. J. L. K. Smith	42. 90	41 55 47. 04	5. 0230893
Cat Head	1860	......do	36. 57	47 28 35. 92	5. 0656396
South Fox	1860	James Carr	38. 20	90 35 39. 93	5. 1981495
Pine River	1860	Lieut. J. L. K. Smith	03. 50	87 04 03. 42	5. 1981495
Beaver Island	1860	......do	22. 50	44 05 23. 51	5. 0411919
Cat Head	1860	......do	37. 00	48 50 36. 16	5. 0754617

*Angle not measured.

Triangulation from Green Bay to Lake Huron—Continued.

Name of station.	Date.	Observer.	Seconds of measured angle.	Adjusted spherical angle.	Logarithms of sides in feet.
			″	° ′ ″	
Middle Village	1860	Lieut. J. L. K. Smith	07. 20	73 42 08. 08	5. 0754617
Beaver Island	1860	do	56. 24	51 32 56. 80	4. 9871130
Pine River	1860	do	56. 40	54 44 57. 34	5. 0053005
Hat Island	1860	Lieut. J. L. K. Smith	53. 25	63 23 51. 98	5. 0053005
Middle Village	1860	do	25. 45	54 09 25. 58	4. 9627166
Beaver Island	1860	do	42. 00	62 26 44. 38	5. 0016108
Pointe aux Chênes	1860	Lieut. J. L. K. Smith	28. 98	46 53 31. 07	5. 0016108
Hat Island	1860	do	32. 25	82 31 34. 83	5. 1345446
Middle Village	1860	do	52. 50	50 34 56. 59	5. 0261687
d	1853	Capt. J. N. Macomb	17. 90	82 45 18. 89	5. 0261687
Hat Island	1853	do	16. 15	35 10 16. 76	4. 7900886
Pointe aux Chênes	1853	do	25. 12	62 04 25. 71	4. 9758813
Manitou Payment	1853	Capt. J. N. Macomb	07. 50	56 03 07. 38	4. 9758813
Hat Island	1853	do	39. 90	68 02 40. 29	5. 0243436
d	1853	do	14. 28	55 54 14. 28	4. 9751235
Biddle Point	1854	Capt. J. N. Macomb	13. 20	65 20 13. 20	4. 9751235
Hat Island	1853	do	51. 25	46 40 51. 25	4. 8785246
Manitou Payment*				67 58 57. 11	4. 9837783
Point Patterson	1854	Capt. J. N. Macomb	26. 25	60 30 27. 29	4. 9837783
Hat Island	1853	do	02. 85	46 35 03. 49	4. 9052163
Biddle Point	1854	do	30. 30	72 54 30. 96	5. 0244333
Garden Island	1854	Capt. J. N. Macomb	09. 30	124 19 09. 31	5. 0244333
Point Patterson	1854	do	05. 40	26 09 04. 06	4. 7516827
Hat Island	1853	do	47. 40	29 31 47. 32	4. 8002380
High Island	1855	Capt. J. N. Macomb	35. 85	46 37 37. 43	4. 8002380
Garden Island	1855	do	49. 03	95 40 50. 87	4. 9366262
Point Patterson	1854	do	30. 90	37 41 32. 49	4. 7251043
Scott's Point*				39 36 42. 88	4. 7251043
High Island	1855	Capt. J. N. Macomb	17. 23	53 42 17. 18	4. 8268900
Garden Island	1855	do	00. 83	86 41 00. 78	4. 9198396
Seul Choix*				30 06 04. 38	4. 7251043
High Island	1855	Capt. J. N. Macomb	47. 03	91 36 46. 77	5. 0246374
Garden Island	1855	do	10. 00	58 17 09. 98	4. 9545771
Gull Island	1855	Capt. J. N. Macomb	35. 27	88 21 35. 25	4. 9545771
Seul Choix*				26 07 58. 56	4. 5986558
High Island	1855	Capt. J. N. Macomb	27. 02	65 30 26. 95	4. 9138036
Gros Cap	1853	Capt. J. N. Macomb	26. 59	48 32 28. 02	4. 97 8813
d	1853	do	10. 23	107 08 11. 91	5. 0814281
Hat Island	1853	do	20. 95	24 19 21. 18	4. 7159120
West Base	1852	Capt. J. N. Macomb	26. 11	78 56 26. 32	4. 7159120
Gros Cap	1852	do	59. 20	54 44 59. 45	4. 6360837
d	1853	do	34. 45	46 18 34. 62	4. 5832411

*Not measured.

Triangulation from Green Bay to Lake Huron—Continued.

Name of station.	Date.	Observer.	Seconds of measured angle.	Adjusted spherical angle.	Logarithms of sides in feet.
			"	° ' "	
Mackinac Island	1852	Capt. J. N. Macomb	06.06	42 04 06.18	4.5832411
West Base	1852	do	13.75	81 35 13.84	4.7524569
Gros Cap	1853	do	40.30	56 20 40.40	4.6774798
B	1852	Capt. J. N. Macomb	13.75	76 01 13.18	4.6774798
West Base	1852	do	29.80	64 29 30.82	4.6459962
Mackinac Island	1851	do	15.62	39 29 16.32	4.4939361
St. Ignace	1853	Capt. J. N. Macomb	28.95	80 06 28.27	4.6459962
B	1852	do	23.15	32 18 24.04	4.3804089
Mackinac Island	1851	do	08.06	67 35 07.93	4.6183846
East Base	1852	Capt. J. N. Macomb	26.90	33 58 27.14	4.3804089
St. Ignace	1853	do	30.30	90 57 29.56	4.6330767
Mackinac Island	1852	do	03.15	55 04 03.51	4.5468605
West Base	1853	Capt. J. N. Macomb	32.84	87 40 35.18	4.5468605
East Base	1852	do	42.80	54 33 44.75	4.4582410
St. Ignace	1853	do	38.18	37 45 40.22	4.3342326
Rabbit's Back	1851	Capt. J. N. Macomb	56.62	42 25 56.05	4.3804089
Mackinac Island	1851	do	06.32	45 29 05.65	4.4044164
St. Ignace	1851	do	58.98	92 04 58.44	4.5509998
H	1851	Capt. J. N. Macomb	42.71	31 14 42.73	4.5509998
Mackinac Island	1851	do	53.88	45 20 54.03	4.6881917
Rabbit's Back	1851	do	23.75	103 24 23.64	4.8240836
Point St. Martin	1851	Capt. J. N. Macomb	58.00	107 13 57.22	4.8240836
Mackinac Island	1851	do	27.04	31 36 26.83	4.5634411
H	1851	do	36.74	41 09 36.33	4.6623647
Pointe Fuyard	1851	Capt. J. N. Macomb	49.55	43 30 49.55	4.6623647
Mackinac Island	1851	do	27.67	37 45 27.75	4.6114235
Point St. Martin's	1851	do	44.48	98 43 43.14	4.8193836
Bois Blanc Island	1851	Capt. J. N. Macomb	55.00	58 35 55.49	4.8193836
Pointe Fuyard	1851	do	50.00	71 40 51.39	4.8655733
Mackinac Island	1851	do	13.69	49 43 13.99	4.7706277
Beaver Tail Point	1851	Capt. J. N. Macomb	41.80	43 34 41.95	4.7706277
Bois Blanc Island	1851	do	01.03	39 39 01.05	4.7370795
Pointe Fuyard	1851	do	17.49	96 46 17.75	4.9291512
Spectacle Reef*				81 30 20.81	4.9291512
Bois Blanc Island	1851	Capt. J. N. Macomb	12.08	57 44 12.12	4.8611081
Beaver Tail Point	1851	do	28.06	40 45 28.02	4.7487625
Mast	1851	Capt. J. N. Macomb	59.63	38 12 59.61	†4.7487619
Bois Blanc Island	1851	do	17.50	51 07 17.51	4.8485743
Spectacle Reef*				90 39 43.81	4.9572988
Astronomical Station on Round Island	1853	Capt. J. N. Macomb	34.63	69 11 38.90	4.3804089
Mackinac Island	1852	do	48.43	91 49 51.10	4.4094734
St. Ignace	1853	do	28.85	18 58 30.05	3.9217867

*Not measured. †Mean value.

§ **4.** The angles of the lateral chain in Lake Superior, extending from the line Vulcan–Saint Ignace eastward to Mamainse, were measured in 1869 and 1871 by Assistant Engineers O. B. Wheeler, G. Y. Wisner, A. R. Flint, and G. A. Marr. The instruments used were Coast-Survey theodolite No. 93, by Brunner, Coast-Survey theodolite No. 30, and Gambey No. 1, and the angles were observed by repetitions, usually in sets of 5. They have been adjusted by closing the triangles, and the lengths and directions of sides depend on those of the line Vulcan–Saint Ignace, given in Chapter XXVII, § 3, as

Log. length in feet=5.6900927
Azimuth =178° 21′ 39″.93 west of south.

The following table, whose form is the same as that in § 3, gives the angles and logarithms of the sides in feet.

Triangulation in Lake Superior from Vulcan to Mamainse.

Name of station.	Date.	Observer.	Seconds of measured angle.	Adjusted spherical angle.	Logarithms of sides in feet.
			″	° ′ ″	
Tip Top	1871	G. Y. Wisner	15.68	57 15 15.59	5.6900927
Vulcan	1871	A. R. Flint	24.66	56 30 24.57	5.6863956
St. Ignace	1871,'72	Gen. C. B. Comstock and G. A. Marr.	11.20	66 15 11.10	5.7268419
Michipicoten	1869	O. B. Wheeler	39.71	93 55 41.29	5.7268419
Tip Top	1871	G. Y. Wisner	31.40	65 07 32.97	5.6855844
Vulcan	1871	A. R. Flint	05.91	20 57 07.49	5.2812042
Paugon	1869	G. A. Marr	15.49	92 07 17.19	5.2812042
Michipicoten	1869	O. B. Wheeler	21.11	54 31 22.79	5.1923103
Tip Top	1869	A. R. Flint	22.19	33 21 23.87	5.0217410
Gargantua	1869	A. R. Flint	17.44	27 00 22.37	5.0217410
Michipicoten	1869	O. B. Wheeler	24.49	60 14 29.42	5.3031891
Paugon	1869	G. A. Marr	08.27	92 45 13.18	5.3641071
Mamainse	1869	G. A. Marr	32.35	24 48 33.88	5.3641071
Michipicoten	1869	O. B. Wheeler	29.26	23 42 30.80	5.3455870
Gargantua	1869	A. R. Flint	02.82	131 29 04.36	5.6158495

§ **5.** For the triangulation of Saginaw Bay, a base-line about four miles long was measured with the Würdemann base-apparatus in 1857 by Captain G. G. Meade. An account of the measurement may be found in the report of the Chief Topographical Engineer for that year. On this base depended a triangulation covering Saginaw Bay, whose angles were measured in 1856, 1857, and 1858, by Lieutenant O. M. Poe and Assistant James Carr. The angles were observed by repetitions in sets usually of 5 or 6, from two to eight sets being obtained for each angle. The theodolites used were Gambey No. 1 and Würdemann No. 65, both having 10-inch limbs.

The following table, whose arrangement is like that in § 3, gives the stations, angles, and sides of the Saginaw Bay triangulation. The length of the base is that adopted at the time, as the base depended on the 15-feet brass bar prior to the insertion of the agates in its ends. The values for the sides and angles adopted at the time are retained. When the length of a side could be obtained by different routes the mean result was used.

Triangulation of Saginaw Bay.

Name of station.	Date.	Observer.	Seconds of measured angle.	Adjusted plane angle.	Logarithms of sides in feet.
			″	° ′ ″	
Sebouin	1857	Lieut. O. M. Poe	13.08	23 54 11.50	4.3221344
East Base	1857	do	05.34	26 29 05.34	4.3637724
West Base	1857	do	44.75	129 36 43.16	4.6011805
Wild Fowl	1857	Lieut. O. M. Poe	11.75	54 04 13.45	4.3221344
East Base	1857	do	56.75	60 11 58.46	4.3521900
West Base	1857	do	48.09	65 43 48.09	4.3736028
C	1857	Lieut. O. M. Poe	58.37	74 53 57.76	4.3521900
Wild Fowl	1857	do	25.23	32 54 24.62	4.1024707
West Base	1857	do	38.23	72 11 37.62	4.3461321
Observatory	1857	Lieut. O. M. Poe	35.69	72 31 35.69	4.3461321
C	1857	do	19.88	62 43 20.53	4.3154513
Wild Fowl	1857	do	03.13	44 45 03.78	4.2142388
Little Charity	1857	Lieut. O. M. Poe	10.63	45 40 09.63	4.6011805
Sebouin	1857	do	56.12	64 03 55.12	4.7005855
East Base	1857	do	56.25	70 15 55.25	4.7203892
Oak Point	1857	Lieut. O. M. Poe	35.00	65 05 31.62	4.7005855
Little Charity	1857	do	48.25	28 43 47.96	4.4248571
East Base	1857	do	38.75	86 10 40.42	4.7420190
West Base	1857	Lieut. O. M. Poe	41.33	85 31 41.91	4.7420190
Little Charity	1857	do	20.69	51 15 18.59	4.6354105
Oak Point	1857	do	59.50	43 12 59.50	4.5788666
Point aux Gres	1857	Lieut. O. M. Poe	26.45	42 59 26.29	4.7203892
Sebouin	1857	do	05.14	46 01 04.05	4.7437459
Little Charity	1857	do	26.90	90 59 29.66	4.8866178
Gravelly Point	1857	Lieut. O. M. Poe	10.25	110 11 11.17	4.8866178
Point aux Gres	1857	do	46.04	35 08 49.83	4.5314505
Little Charity	1857	do	58.94	34 39 59.00	4.5262366
Big Charity	1858	James Carr	22.62	68 38 21.70	4.5314572
Gravelly Point	1857,'58	Lieut. O. M. Poe and James Carr.	03.75	23 50 02.85	4.1688279
Little Charity	1857	Lieut. O. M. Poe	36.37	87 31 35.45	4.5619598
Fish Point	1857	Lieut. O. M. Poe	11.13	42 35 11.28	4.8866178
Sebouin	1857	do	31.25	107 26 31.40	5.0357773
Point aux Gres	1857	do	17.16	29 58 17.32	4.7548150
Nyaquonk	1857	Lieut. O. M. Poe	14.66	63 01 12.04	5.0357773
Fish Point	1857	do	19.62	53 42 17.00	4.9921419
Point aux Gres	1857	do	39.71	63 16 30.96	5.0367569
Pine River	1857	Lieut. O. M. Poe	21.38	111 10 21.38	4.9921419
Nyaquonk	1857	do	33.87	26 16 33.87	4.6686010
Point aux Gres	1857	do	58.62	42 33 04.75	4.8526021
Quannakissee	1857	Lieut. O. M. Poe	48.00	96 04 47.24	5.0367569
Fish Point	1857	do	10.68	47 17 09.91	4.9053464
Nyaquonk	1857	do	03.62	36 38 02.85	4.8149650

Triangulation of Saginaw Bay—Continued.

Name of station.	Date.	Observer.	Seconds of measured angle.	Adjusted plane angle.	Logarithms of sides in feet.
			″	° ′ ″	
Saginaw Light-House*				92 50 46.44	5.0367569
Fish Point	1857	Lieut. O. M. Poe	36.86	31 15 36.86	4.7523981
Nyaquonk	1857	do	36.70	55 53 36.70	4.9553218
Gravelly Point	1858	James Carr	30.73	28 54 29.87	4.6354105
West Base	1858	do	18.64	100 15 18.61	4.9441016
Oak Point	1858	do	12.38	50 50 11.52	4.8405911
Tawas	1858	James Carr	57.38	49 41 56.54	4.9441016
Gravelly Point	1858	do	02.25	82 50 58.12	5.0583804
Oak Point	1858	do	02.88	47 27 05.34	4.9290594
Tawas	1858	James Carr	18.65	28 40 16.20	4.7420190
Little Charity	1858	do	25.94	96 15 26.62	5.0583804
Oak Point	1858	do	16.50	55 04 17.18	4.9747189
Whitestone Point	1858	James Carr	41.88	115 56 41.96	4.9747189
Little Charity	1858	do	16.46	38 06 16.48	4.8112103
Tawas	1858	do	03.33	25 57 01.56	4.6619249
Mason's Creek	1858	James Carr	40.36	110 34 39.27	4.9747189
Little Charity	1858	do	24.58	22 26 22.29	4.5850753
Tawas	1858	do	58.33	46 58 58.44	4.8673528
Pointe aux Barques	1858	James Carr	00.69	52 13 01.77	5.0583804
Tawas Point	1858	do	53.88	36 39 52.61	4.9366361
Oak Point	1858	do	04.87	91 07 05.62	5.1604856
Pointe au Sable	1858	James Carr	25.31	36 50 26.02	4.9366361
Oak Point	1858	do	27.08	74 57 26.76	5.1436388
Pointe aux Barques	1858	do	07.88	68 12 07.22	5.1265567
Tawas	1858	James Carr	23.00	74 08 22.41	5.1436388
Pointe au Sable	1858	do	31.67	89 52 32.14	5.1604856
Pointe aux Barques	1858	do	03.71	15 59 05.45	4.6004103

*Not measured.

CHAPTER XXII.

ELEVATIONS OF THE GREAT LAKES.

§ **1.** The elevations of the Great Lakes above mean tide sea-level, as determined by the Lake Survey, depend upon two distinct processes, viz., that of spirit-level measurements and that of water-level measurements.

By the first process, starting from a bench-mark of known height above sea-level at Greenbush, New York, the elevation of a bench-mark at Oswego, New York, near the east end of Lake Ontario, was found. In like manner the differences in elevation of bench-marks at the following pairs of points were determined: Port Dalhousie, Ontario, near the west end of Lake Ontario, and Port Colborne, Ontario, near the east end of Lake Erie; Rockwood, Michigan, near the west end of Lake Erie, and Lakeport, Michigan, near the south end of Lake Huron; Escanaba, Michigan, near the north end of Green Bay,* and Marquette, Michigan, on the south shore of Lake Superior.

By the second process, depending on the assumption that the mean surface of each lake is level, the relative heights of the pairs of bench-marks for the respective lakes were determined. For this purpose water-gauges were fixed near these bench-marks, and tri-daily observations of the height of the water-surface at each gauge were made during the months of May, June, July, and August, 1875, this series of observations being taken as of sufficient extent to give a reliable mean. Assuming, then, that the mean surface of each lake for this period of about four months was level, the differences of the gauge-readings gave the relative heights of the zero-points of the two gauges on each lake, and as these zero-points were carefully referred to the corresponding bench-marks, the relative heights of these bench-marks were also known.

As the surfaces of the lakes vary considerably in elevation from year to year, their mean elevations can only be found by observations extending over a series of years. Such observations, consisting of tri-daily gauge-readings, have been made on Lake Ontario at Charlotte and Sacket's Harbor, N. Y.; on Lake Erie at Cleveland, Ohio, and Erie, Pa.; on Lake Huron at Port Austin, Mich.; on Lake Michigan at Milwaukee, Wis.; and on Lake Superior at Marquette, Mich. By comparing the observations made at Oswego in 1875 with those made during the same time at Charlotte, the elevation of the bench-mark at the latter place, to which the surface of Lake Ontario has been referred, becomes known, and thus also the mean elevation of Lake Ontario for the period covered by the observations at Charlotte. Similarly the mean elevation of Lake Erie has been derived from the observations made at Cleveland, the mean elevation of Lakes Huron and Michigan from the observations made at Milwaukee, and the mean elevation of Lake Superior from the observations made at Marquette.

The methods used and the results derived thereby, of which the foregoing is a brief outline, will now be given somewhat in detail.

LEVELING BY MEANS OF THE SPIRIT-LEVEL.

§ **2.** For this work two parties were detailed, Assistant F. W. Lehnartz having charge of the first, and Assistant L. L. Wheeler of the second. During the year 1875 the lines from Greenbush to Oswego, and from Port Dalhousie to Port Colborne were leveled in duplicate, and a single line of levels was run from Gibraltar, near Rockwood, Mich., to Lakeport. The instruments used during this year were Stackpole level No. 1496, 11 inches focal length, object-glass $1\frac{1}{4}$ inches in

* For reasons given in the sequel it is assumed that Lakes Huron and Michigan and Green Bay have the same altitude.

diameter, and magnifying power of 24 diameters, and Würdemann level No. 2, 17 inches focal length, and object-glass $1\frac{1}{4}$ inches in diameter. The spirit-level in each instrument is attached to the lower side of the telescope. The reticule in each telescope is provided with one horizontal and one vertical thread. The values of one division of the level-tubes were determined by means of a level-trier at the beginning and at the end of the season's work. The adopted values of one division of the levels are—

Würdemann level No. 2, 1 division$=3''.17$.

Stackpole level No. 1496, 1 division$=6''.42$.

With these values of one division of the levels, tables of corrections were constructed having for arguments the inclinations of the levels in divisions and the distances of the rod from the instrument in feet, and each rod-reading has been corrected by the proper correction found in the table. The inclination of the level was not allowed to exceed five divisions.

The leveling rods were of the pattern known as New York rods, were graduated to hundredths, and read by verniers to thousandths of a foot. They were compared with a standard brass metre before leaving and after returning to the office. The brass metre with which the rods were compared has itself been compared with the standard metre (R 1876), and its length at 32° F. found to be $1000^{mm}.0$. The values adopted for mean length of one foot are—

Rod No. 1, 1 foot=1.00062 English feet.

Rod No. 2, 1 foot=1.00066 English feet.

All elevations determined with these rods have been reduced to English feet. The rods were supported while in use on steel pins eight inches long, driven into the ground. They were made vertical by means of a plumb-line.

§ **3.** The method of leveling during 1875 was as follows: The instrument was carefully adjusted before commencing work each day, and also at any time when for any reason it was supposed that it might be out of adjustment. Each time the instrument was set up it was carefully leveled so as to turn in azimuth with little displacement of the bubble, and the scale-readings at the time of pointing to the rod were noted and recorded. The instrument was sheltered from the sun by an umbrella. It was set up at a convenient distance from a bench-mark, and a reading taken on the bench-mark. The rodman then drove the steel pin at the same distance from the instrument as the bench-mark, and a reading was taken on the pin. The instrument was then set up beyond the pin, and the work continued in this manner. The length of sight, where possible, was 200 feet, determined by pacing, and backsights and foresights were always taken of equal length.

At the end of each day's work the first party established two permanent bench-marks; also one at midday if one could be obtained. Where it was not possible to establish permanent bench-marks at the end of a day's work, three stakes at least one foot long were driven into the ground 20 feet apart, until their tops were even with the surface of the ground, and used as stopping points. Permanent bench-marks consisted usually of marks placed on the masonry of canal-locks, railroad and road bridges, aqueducts, &c. A description of these bench-marks with their elevations was sent to the second party, who determined the elevations of the same bench-marks. When the difference between the two results for the difference of elevation between two successive bench-marks exceeded $0.1^{ft.} \sqrt{\text{distance in miles}}$, the line was releveled twice to ascertain which result was in error. This was necessary only twice throughout the season, but three other lines were releveled on account of discrepancies which nearly approached the limit.

§ **4.** The following is a brief history of operations during the season:

The party in charge of Assistant Engineer F. W. Lehnartz, left Detroit May 4, established the water-gauge stations at Port Colborne, Port Dalhousie, and Oswego, and commenced work at Greenbush May 13. Oswego was reached by this party August 15. Work was commenced at Port Dalhousie August 16, and Port Colborne reached September 2. Work was commenced at Rockwood September 7, and Lakeport was reached October 28. The average distance leveled by this party per day, including days on which no work could be done, was 1.97 miles.

The party in charge of Assistant Engineer L. L. Wheeler, left Detroit May 6, established water-gauge stations at Gibraltar, Rockwood, and Lakeport, examined the water-gauge station at Sacket's Harbor, and commenced work at Greenbush May 28. Oswego was reached October 15, and

Port Colborne October 29. The average distance leveled by this party per day, including days on which no work could be done, was 1.89 miles. This party was delayed by needed repairs to instrument, and by releveling of lines.

The route followed by both parties was along the Erie Canal to Higginsville, along wagon-roads to Fish Creek, and along the New York and Oswego Midland Railroad to Oswego. The Welland Railway was followed from Port Dalhousie to Port Colborne.

§ **5.** During the year 1876, the line from Escanaba to Marquette was leveled in duplicate, both parties commencing at Escanaba. The instruments used during this season were constructed by Kern, of Aarau, Switzerland, and were leveling-instruments Kern Nos. 1 and 2, and leveling-rods Nos. 2 and 3.

The leveling-instruments are alike in construction and the same description applies to both. The instrument is supported on the tripod by three foot-screws and is fastened to it by a hook passing up through the triangular opening in the head of the tripod. This hook is drawn downward by a coiled spring, which maintains a constant pressure on the leveling-screws. One of the leveling-screws terminates in a sphere which is firmly united to the frame of the tripod as a guard against accidents. The instrument revolves about a vertical axis in the usual manner, and is provided with a clamp- and a tangent-screw. To the upper end of the vertical axis a horizontal bar is attached. Above this bar another bar is attached in such a manner that one end may be moved in a vertical direction by means of a slow-motion screw, or held immovable by means of a clamp-screw. The other ends of the bars are joined in such a manner that two opposing screws act as pivots about which the vertical motion of the upper bar takes place. The screws also permit a small lateral motion of that end of the upper bar. The telescope is supported in wyes attached to the upper bar, which are closed by means of spring-catches. The diameter of the object-glass is $1\frac{7}{16}$ inches, the focal length $14\frac{1}{2}$ inches, and the magnifying power of the telescope is 50 diameters. The reticule is provided with one vertical and three horizontal threads. The angular distance of the extreme threads from the middle thread was determined by readings taken on the rod at distances varying from 10 metres to 100 metres. The angular distance of the upper thread from the middle is, for No. 1, 17′ 33″, and for No. 2, 17′ 31″. The angular distance of the lower thread from the middle is, for No. 1, 17′ 42″, and for No. 2, 17′ 43″. The distance between the extreme threads is, for No. 1, 35′ 15″, and for No. 2, 35′ 14″. From the results of these observations tables were formed showing the distance of the rod from the instrument for any observed difference of the readings of the extreme threads on the rod. The level is inclosed in a wooden case supported at the ends on wyes which rest on the pivots (rings) of the telescope, after the manner of the striding-level of an astronomical transit. One of the wyes may be adjusted in a horizontal and the other in a vertical direction, and from each project small steel pins which pass under the spring-catches, fastening the telescope in the wyes. By raising the spring-catches the level may be reversed on the telescope, thus determining the error of adjustment of the level, and the telescope may be reversed in its wyes. By a combination of these two operations the inequality of the pivots may be determined. Assistant Engineer Lehnartz determined the inequality of the pivots by a series of observations made in the Lake-Survey Observatory at Detroit, and found the pivot at eye-end of No. 1 to be $0''.234 \pm 0''.046$ larger, and of No. 2 to be $0''.553 \pm 0''.041$ smaller than pivot at object-end of telescope. The divisions of the level are etched on the glass tube. The values of the divisions were determined by Lieutenant T. N. Bailey, by means of a level-trier, and the following results obtained:

1 division No. $1 = 4''.87$.

1 division No. $2 = 2''.15$.

The upper portion of the level-case is of plate-glass, above which is a mirror moving on a hinge in such a manner that the observer may note the position of the level, his eye being near the eye-piece of the telescope.

The rods are constructed of well-seasoned fir-wood, and have a breadth of 8^{cm} and a thickness of $2^{cm}.5$. They are strengthened by a strip of wood extending the length of the rods having a breadth of 5^{cm} and a thickness of $2^{cm}.5$, attached to the rod in such a manner that the cross-section is T-shaped. The rods are divided into centimeters, the graduation extending three metres. The values of the divisions of the rods were determined by Assistant Engineer L. L. Wheeler, by comparisons with the standard brass metre-scale. The following results were obtained:

Mean length of 1^{m} of rod No. $2 = 999^{mm}.840$ at 82° F.

Mean length of 1^{m} of rod No. $3 = 999^{mm}.903$ at 84° F.

Each rod has a handle attached one metre above its foot to aid in holding it, and a watch-level attached 1.3 metres above its foot for keeping it in a vertical position. A plumb-line can also be attached for adjusting the watch-level. The rod is set on a cast-iron plate while in use, and is accompanied by a tripod for supporting it.

§ **6.** Before commencing work at any time a number of observations was made for determining the errors of adjustment of the instrument. For this purpose the rod was supported in its tripod in a vertical position determined by the plumb-line, and its level was adjusted. The collimation was then determined by noting the position of the three horizontal threads on the rod at a distance of about 50 metres, when the telescope was in its normal position, and when it was rotated 180° about its axis. The collimation of the mean of the threads was thus determined, and was never allowed to exceed $2^{mm}.5$ for a distance of 100 metres. The determination of the angle between the bubble and the feet supporting it, which will be called inclination, was made by reversing the level several times on the telescope, and was never allowed to exceed two divisions of the level. The observations for collimation and inclination were usually repeated at the close of a day's work.

When at work, the instrument was set up in such a position that the backsight and foresight should be nearly equal, the difference in length of sights being limited to 10 metres. The length of sight was not to exceed 100 metres. After having properly leveled the instrument, the observations were made in the following order: First, the level was read, the tenths of the divisions being estimated; then the positions of the three threads on the rod were noted, the millimeters being estimated; and finally, the level was again read. The recorder then took the difference between the readings of the middle and the extreme threads to guard against errors, and if these differences indicated any error the observations were repeated. Both parties connected with the same bench-marks when work was stopped for any cause, the bench-marks being usually spikes driven into the roots of stumps. Two bench-marks were usually established and the elevation of each determined. Whenever it was possible, permanent bench-marks were established. These bench-marks consist of a copper bolt 3 inches long and three-eighths inch in diameter, leaded into solid masonry or natural rock. A small hole, $\frac{1}{32}$ inch in diameter, drilled in the end of the bolt, is the point of reference. These permanent bench-marks have been designated by the letters U. S. B. M.

A reduction of the observations was made at the end of each day's work, and a comparison of the two lines of levels made. The limit of discrepancy between the two lines of levels was fixed at $10^{mm}\sqrt{\text{distance in kilometers}}$.

In reducing the observations, the mean of the readings of the three threads, the difference of the readings of the extreme threads, and the difference of the sums of the two level-readings were taken. In a table constructed for the purpose, and with the difference of the readings of the extreme threads as argument, the distance of the rod from the instrument was found. In another table, with this distance and the difference of the sums of the level-readings as arguments, the correction for the inclination of the line of sight was found and applied to the mean of the readings of the three threads. The sums of these corrected means for the backsights and foresights were then taken, and their difference gave the difference of elevation between the bench-marks, subject to four corrections. These corrections are for collimation, inclination, inequality of pivots, and absolute length of rod. The collimation and inclination are assumed to be constant between two bench-marks, and the corrections to be applied to the difference of elevation due to these quantities and to the inequalities of pivots are found by means of the difference of the sums of the differences between readings of extreme threads. The result was then corrected for absolute length of rod.

The parties worked together during the season, and were much delayed in the work by wind and unsteadiness of atmosphere. The route followed was along the Chicago and Northwestern Railroad, from Escanaba to U. S. B. M. 5, and along wagon-roads from that point to Marquette. Work was commenced at Escanaba August 11, and Marquette was reached October 28. The length of line leveled is 104.7 kilometers (65 miles), and the mean distance leveled per day for the days on which work could be done was 2.19 kilometers.

§ **7.** During the year 1877 the line from Gibraltar to Lakeport was leveled the second time, one party commencing at Gibraltar and the other at Lakeport, and working toward each other till they met. The instruments used were the same as used the preceding year, with the exception that the level-tube of Kern level No. 2 had been broken and was replaced by another. The value of one division of this level was determined by means of a level-trier, and the following result was obtained: 1 division Kern level No. $2=3''.05$. The methods of determining errors of adjustment, of observing, and of reducing the notes were the same as those employed the preceding year.

The route followed was along wagon-roads from Gibraltar to the nearest point on the Canada Southern Railroad, thence along that railroad to Detroit Junction, near Detroit, thence along the Grand Trunk Railroad to Fort Gratiot, thence along wagon-roads to Lakeport. Work was commenced April 17, and the two parties met May 22. The length of line leveled was 142 kilometers. The mean distance leveled per day for the days on which leveling was done by the party commencing at Gibraltar was 2.24 kilometers, and by the party commencing at Lakeport 3.81 kilometers.

§ **8.** The results of leveling with the spirit-level are given in the following tables.

Table I contains the results of levels between Greenbush and Oswego, N. Y. The first column contains a list of bench-marks, the differences of whose elevations have been determined. The second column contains the distance of the more advanced bench-mark from Greenbush. The third and fourth columns give the differences of elevation between successive bench-marks as determined by the first and the second parties respectively. The fifth column gives the mean differences of elevation between successive bench-marks, all determinations being given equal weight. The sixth column gives the partial excesses, obtained by subtracting the difference of elevation as determined by the first party from that as determined by the second. The seventh column gives the total excess for each bench-mark, the total excess at any bench-mark being the sum of all the partial excesses up to that point. The results given in this table make bench-mark A at Oswego 237.234 feet above Coast-Survey bench-mark on grist-mill at Greenbush, N. Y. On examining the table it will be seen that for the first 70 miles from Greenbush the discrepancies between the two lines of levels, over short distances of 2 or 3 miles, had no marked bias as to sign, and that at 70 miles the total discrepancy was but +0.13 foot. But for the rest of the distance to Oswego, 117 miles, the plus sign predominates, and the total discrepancy increases to +0.953 foot. This discrepancy, however, does not include any errors in the five lines that were each leveled four times, which have a total length of about 14 miles. The probable error of leveling these five lines, as deduced from the four measurements, is ± 0.037 foot. The probable error of leveling the remainder of the line, as deduced from the total discrepancy between the two lines of levels, is ± 0.32 foot. Combining these two probable errors we have ± 0.32 foot as the probable error of determining the difference of elevation of bench-mark A at Oswego and Coast-Survey bench-mark on grist-mill at Greenbush. Therefore, bench-mark A at Oswego is $237^{ft}.23 \pm 0^{ft}.32$ above bench-mark on grist-mill at Greenbush.

Table II contains the results of levels between Port Dalhousie and Port Colborne, Ontario, arranged in the same manner as Table I. The results given in this table make the bench-mark on the custom-house at Port Colborne $326^{ft}.59 \pm 0^{ft}.01$ above bench-mark B at Port Dalhousie, the probable error being derived from the discrepancy between the two results.

Table III contains the results of levels between Gibraltar and Lakeport, Mich. This line was first leveled in 1875, with an ordinary instrument and rod. In 1877 it was leveled with the improved instruments made by Kern. There not being sufficient data to establish the relative weights of results obtained with the different kinds of instruments, it was thought that the truth would be more nearly reached, in view of the superior instruments used in 1877, as well as the increased experience of the observers, by rejecting the results of 1875 and adopting those obtained in 1877. The partial and total excesses, however, of the results of 1877 over those of 1875 are given in the table. When two bench-marks have been established at the end of a day's work which were of such a nature as not to be likely to remain a number of years, the mean of their elevations has been used in the table. The results in this table make bench-mark 2 at Lakeport $5^{ft}.89 \pm 0^{ft}.09$ above bench-mark 2 at Gibraltar, the probable error being derived from the discrepancy ($0^{ft}.275$) between the determinations of 1875 and 1877, supposing them of equal weight.

Table IV contains the results of levels between Escanaba and Marquette, Mich. Where two bench-marks, not likely to be permanent, were established at the end of a day's work, their mean elevation has been used. Results marked with an asterisk (*) have been rejected on account of disagreement with mean of three other results. The results given in this table make bench-mark 1 at Marquette $16^{ft}.92 \pm 0^{ft}.03$ above bench-mark 1 at Escanaba, the probable error being derived from the difference between the two independent determinations.

TABLE I.—*Results of levels between Greenbush and Oswego, New York.*

Bench-marks.	Distance from Greenbush.	Difference of elevation.			Partial excess.	Total excess.
		First party.	Second party.	Mean.		
	Miles.	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*
B. M. 1—C. S. B. M	1.25	+12.237	+12.339	+12.288	+0.102	+0.102
B. M. 2—B. M. 1	2.25	− 9.418	− 9.434	− 9.426	−0.016	+0.086
3—B. M. 2	3.75	+ 9.385	+ 9.377	+ 9.381	−0.008	+0.078
4— 3	4.75	+ 2.757	+ 2.763	+ 2.760	+0.006	+0.084
5— 4	7.25	+ 0.854	+ 0.855	+ 0.854	+0.001	+0.085
B. M. — 5	8.45	+ 0.411	+ 0.408			
			+ 0.417	+ 0.405	....	+0.085
			+ 0.385			
6—B. M	9.55	+18.699	+18.703	+18.701	+0.004	+0.089
7— 6	10.10	+20.776	+20.778	+20.777	+0.002	+0.091
8— 7	11.60	+90.082	+90.149	+90.116	+0.067	+0.158
9— 8	13.60	+34.054	+34.080			
			+34.152	+34.100	....	+0.158
			+34.113			
10— 9	16.35	− 1.089	− 1.090	− 1.090	−0.001	+0.157
11—10	17.60	− 1.577	− 1.572	− 1.574	+0.005	+0.162
12—11	20.35	− 0.291	− 0.321	− 0.306	−0.030	+0.132
13—12	21.75	+10.494	+10.479	+10.486	−0.015	+0.117
14—13	23.75	+ 6.879	+ 6.863	+ 6.871	−0.016	+0.101
15—14	26.75	+ 8.911	+ 8.954	+ 8.933	+0.043	+0.144
16—15	29.75	+17.871	+17.932	+17.901	+0.061	+0.205
17—16	32.25	− 2.936	− 2.966	− 2.951	−0.030	+0.175
18—17	35.25	+18.483	+18.523	+18.503	+0.040	+0.215
19—18	38.25	+ 0.065	− 0.003	+ 0.031	−0.068	+0.147
20—19	40.75	+ 7.981	+ 7.919	+ 7.950	−0.062	+0.085
21—20	44.50	+ 4.818	+ 4.758	+ 4.788	−0.060	+0.025
22—21	45.00	+ 9.215	+ 9.205	+ 9.210	−0.010	+0.015
23—22	47.00	− 0.975	− 0.952	− 0.963	+0.023	+0.038
24—23	50.00	+ 7.935	+ 7.934	+ 7.934	−0.001	+0.037
25—24	53.00	+22.428	+22.463	+22.446	+0.035	+0.072
26—25	57.00	− 1.993	− 1.993	− 1.993	0.000	+0.072
28—26	63.75	+ 0.567	+ 0.610	+ 0.588	+0.043	+0.115
29—28	66.25	+ 4.440	+ 4.388	+ 4.414	−0.052	+0.063
30—29	67.75	− 1.523	− 1.547	− 1.535	−0.024	+0.039
31—30	69.50	+ 2.661	+ 2.703	+ 2.682	+0.042	+0.081
32—31	70.50	− 0.168	− 0.118	− 0.143	+0.050	+0.131
33—32	74.25	+ 4.690	+ 4.836	+ 4.763	+0.146	+0.277
34—33	77.75	+ 8.358	+ 8.383	+ 8.371	+0.025	+0.302
35—34	80.75	+ 8.201	+ 8 229	+ 8.215	+0.028	+0.330
36—35	83.50	+ 8.329	+ 8.403	+ 8.366	+0.074	+0.404
37—36	87.25	+ 7.378	+ 7.390	+ 7.384	+0.012	+0.416
38—37	90.50	+34.832	+34.887	+34.859	+0.055	+0.471
39—38	94.00	+15.142	+15.182	+15.162	+0.040	+0.511
40—39	96.50	+15.495	+15.518	+15.507	+0.023	+0.534
41—40	100.00	+ 9.745	+ 9.750	+ 9.747	+0 005	+0.539
42—41	104.00	+ 9.990	+10.053	+10.022	+0.063	+0.602
43—42	106.75	+ 0.005	− 0.030	− 0.013	−0.035	+0.567
44—43	109.75	+ 3.744	+ 3.734	+ 3.739	−0.010	+0.557

TABLE I.—*Results of levels between Greenbush and Oswego, New York*—Continued.

Bench-marks.	Distance from Greenbush.	Difference of elevation. First party.	Second party.	Mean.	Partial excess.	Total excess.
	Miles.	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*
45–44	114.25	+ 0.502	+ 0.629 + 0.680 + 0.673	+ 0.621	....	+0.557
46–45	118.75	− 0.129	− 0.179	− 0.154	−0.050	+0.507
47–46	120.00	− 0.418	− 0.420	− 0.419	−0.002	+0.505
48–47	124.25	− 1.160	− 1.211	− 1.185	−0.051	+0.454
49–48	126.25	+ 1.018	+ 1.032	+ 1.025	+0.014	+0.468
50–49	129.00	− 1.611	− 1.603	− 1.607	+0.008	+0.476
51–50	133.75	+ 0.133	+ 0.141	+ 0.137	+0.008	+0.484
52–51	136.75	−39.460	−39.372 −39.419 −39.388	−39.410	....	+0.484
53–52	140.00	−18.813	−18.724 −18.742 −18.712	−18.748	....	+0.484
54–53	144.00	+56.601	+56.704	+56.652	+0.103	+0.587
55–54	145.25	+10.801	+10.829	+10.815	+0.028	+0.615
56–55	147.25	−23.579	−23.543	−23.561	+0.036	+0.651
57–56	150.75	− 3.260	− 3.305	− 3.282	−0.045	+0.606
58–57	152.25	−12.282	−12.245	−12.264	+0.037	+0.643
59–58	156.25	−11.024	−10.966	−10.995	+0.058	+0.701
60–59	158.25	−11.284	−11.277	−11.280	+0.007	+0.708
61–60	160.50	+15.213	+15.280	+15.246	+0.067	+0.775
62–61	165.00	−17.847	−17.796	−17.821	+0.051	+0.826
63–62	167.50	+28.809	+28.765	+28.787	−0.044	+0.782
64–63	171.75	−25.942	−25.931	−25.937	+0.011	+0.793
65–64	174.25	− 4.834	− 4.761	− 4.797	+0.073	+0.866
66–65	176.50	−10.097	−10.101	−10.099	−0.004	+0.862
67–66	178.00	−30.988	−30.973	−30.981	+0.015	+0.877
69–67	185.25	−48.714	−48.632	−48.673	+0.082	+0.959
B. M. "A" Oswego—69	187.25	−33.093	−33.099	−33.096	−0.006	+0.953
B. M. "A" Oswego—C. S. B. M. Greenbush	187.25	+236.758	+237.711	+237.234	....	+0.953

TABLE II.—*Results of levels between Port Dalhousie and Port Colborne, Ontario.*

Bench-marks.	Distance from Port Dalhousie.	Difference of elevation. First party.	Second party.	Mean.	Partial excess.	Total excess.
	Miles.	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*
1—B. M. "B" Port Dalhousie	2.00	+ 60.394	+ 60.385	+ 60.390	−0.009	−0.009
2—1	3.60	+ 22.060	+ 22.081	+ 22.070	+0.021	+0.012
3—2	6.60	+127.290	+127.256	+127.273	−0.034	−0.022
4—3	9.60	+110.289	+110.337	+110.313	+0.048	+0.026
5—4	13.10	+ 13.140	+ 13.140	+ 13.140	0.000	+0.026
6—5	15.60	+ 9.210	+ 9.186	+ 9.198	−0.024	+0.002
7—6	18.20	− 8.032	− 8.038	− 8.035	−0.006	−0.004
8—7	21.80	− 12.596	− 12.576	− 12.586	+0.020	+0.016
B. M. Custom-House—8	24.55	+ 4.819	+ 4.831	+ 4.825	+0.012	+0.028
B. M. Custom-House—B. M. "B" Port Dalhousie.	24.55	+326.574	+326.602	+326.588	....	+0.028

TABLE III.—*Results of levels between Gibraltar and Lakeport, Michigan.*

Bench-marks.	Distance from Gibraltar.	Difference of elevation, 1877.	Partial excess over measurement in 1875.	Total excess over measurement in 1875.
	Miles.	*Feet.*	*Feet.*	*Feet.*
U. S. B. M. "Gibraltar"—2 (Gibraltar)	....	− 2.519	−0.002	−0.002
1—U. S. B. M. "Gibraltar"	0.31	+ 4.490	....	....
2a—1	....	+ 0.745	....	....
U. S B. M. "Trenton"—2a	....	+15.739	....	....
(2a and 2b)—U. S. B. M. "Trenton"	4.13	−15.780	−0.021	−0.023
(3 and 3a)—(2a and 2b)	7.41	− 9.507	+0.105	+0.082
(12 and 12a)—(3 and 3a)	....	+ 5.754	....	....
U. S. B. M. "Wyandotte"—(12 and 12a)	....	+ 2.374	....	....
(5 and 6a)—U. S. B. M. "Wyandotte"	15.07	− 3.047	+0.022	+0.104
(6 and 6a)—(5 and 5a)	16.41	+ 8.603	−0.004	+0.100
2—(6 and 6a)	....	+ 2.338	....	....
U. S. B. M. "Detroit Junction"—2	....	− 1.476	....	....
(7 and 7a)—U. S. B. M. "Detroit Junction"	19.88	+34.315	−0.006	+0.094
U. S. B. M. (a) "Milwaukee Junction"—(7 and 7a)	22.14	+ 2.674	+0.119	+0.213
U. S. B. M. (b) "Milwaukee Junction"—U. S. B. M. (a) "Milwaukee Junction"	22.14	+ 0.008	−0.004	+0.209
(8 and 8a)—U. S. B. M. (b) "Milwaukee Junction"	25.09	− 4.740	−0.020	+0.189
(9 and 9a)—(8 and 8a)	28.71	+ 2.594	0.000	+0.189
(10 and 10a)—(9 and 9a)	31.11	− 4.758	+0.014	+0.203
(11 and 11a)—(10 and 10a)	35.87	− 6.941	+0.059	+0.262
(12 and 12a)—(11 and 11a)	38.66	−13.468	+0.012	+0.274
(13 and 13a)—(12 and 12a)	45.19	+ 5.375	−0.006	+0.268
(14 and 14a)—(13 and 13a)	47.61	+17.573	−0.031	+0.237
(15 and 15a)—(14 and 14a)	48.56	− 4.794	+0.006	+0.243
16—(15 and 15a)	49.58	+ 7.556	−0.061	+0.182
U. S. B. M. "New Haven"—16	....	+ 2.639	....	....
17—U. S. B. M. "New Haven"	52.65	+25.595	+0.086	+0.268
(18 and 18a)—17	54.36	+30.091	−0.023	+0.245
(19 and 19a)—(18 and 18a)	56.78	+25.992	−0.033	+0.212
(20 and 20a)—(19 and 19a)	58.92	−33.856	−0.072	+0.140
(21 and 21a)—(20 and 20a)	59.92	+ 4.822	−0.035	+0.105
(22 and 22a)—(21 and 21a)	62.99	−12.117	−0.063	+0.042
(23 and 23a)—(22 and 22a)	66.06	−36.701	+0.043	+0.085
(24 and 24a)—(23 and 23a)	68.00	+ 2.394	+0.008	+0.093
26—(24 and 24a)	69.05	− 8.282	+0.005	+0.098
U. S. B. M. "Pine River"—25	....	+ 0.006	....	....
26—U. S. B. M. "Pine River"	71.73	+ 6.481	+0.022	+0.120
(28 and 28a)—26	75.80	−41.578	+0.076	+0.196
U. S. B. M. "Fort Gratiot"—(28 and 28a)	....	− 3.661	....	....
3—U. S. B. M. "Fort Gratiot"	87.46	+14.910	+0.086	+0.282
4—3	88.06	− 6.149	+0.007	+0.289
2 (Lakeport)—4	88.06	− 7.803	−0.014	+0.275
B. M. 2 (Lakeport)—B. M. 2 (Gibraltar)	88.06	+ 5.891	....	+0.275

TABLE IV.—*Results of levels between Escanaba and Marquette, Michigan.*

Bench-marks.	Distance from Escanaba.	Difference of elevation.			Partial excess.	Total excess.
		First party.	Second party.	Mean.		
	Kilometers.	*Metres.*	*Metres.*	*Metres.*	*Millimeters.*	*Millimeters.*
1—B. M. 1 (Escanaba)	1.2	+ 0.0048	+ 0.0023	+ 0.0036	− 2.5	− 2.5
2—1	2.6	+ 8.0393	+ 8.0395	+ 8.0394	+ 0.2	− 2.3
3—2	3.5	− 4.8253	− 4.8247	− 4.8250	+ 0.6	− 1.7
(4 and 4*a*)—3	4.7	− 2.1444	− 2.1403	− 2.1424	+ 4.1	+ 2.4
5—(4 and 4*a*)	5.9	+ 2.5967	+ 2.5944	+ 2.5955	− 2.3	+ 0.1
(6 and 6*a*)—5	7.7	+19.3436	+19.3452	+19.3444	+ 1.6	+ 1.7
7—(6 and 6*a*)	8.6	+ 8.9015	+ 8.9031	+ 8.9023	+ 1.6	+ 3.3
8—7	9.1	+ 1.7648	+ 1.7631	+ 1.7640	− 1.7	+ 1.6
9—8	9.4	− 0.1659	− 0.1661	− 0.1660	− 0.2	+ 1.4
10—9	11.3	+ 2.4787	+ 2.4709	+ 2.4748	− 7.8	− 6.4
(11 and 11*a*)—10	12.4	+ 1.8383	+ 1.8372	+ 1.8378	− 1.1	− 7.5
(12 and 12*a*)—(11 and 11*a*)	14.9	+ 3.1006	+ 3.1022	+ 3.1014	+ 1.6	− 5.9
13—(12 and 12*a*)	15.7	+ 2.6271	+ 2.6290	+ 2.6281	+ 2.0	− 3.9
(14 and 14*a*)—13	17.5	− 0.0949	− 0.0878	− 0.0914	+ 7.1	+ 3.2
(15 and 15*a*)—(14 and 14*a*)	18.4	+ 1.8849	+ 1.8843	+ 1.8846	− 0.5	+ 2.7
(16 and 16*a*)—(15 and 15*a*)	19.5	+ 0.2551	+ 0.2553	+ 0.2552	+ 0.2	+ 2.9
(17 and 17*a*)—(16 and 16*a*)	20.9	+ 0.8698 + 0.8724	+ 0.8783* + 0.8688	+ 0.8703	....	+ 2.9
(18 and 18*a*)—(17 and 17*a*)	22.1	− 0.7112	− 0.7086	− 0.7099	+ 2.6	+ 5.5
(19 and 19*a*)—(18 and 18*a*)	23.9	+ 7.7361	+ 7.7455	+ 7.7408	+ 9.4	+14.9
(20 and 20*a*)—(19 and 19*a*)	24.6	+ 2.2895	+ 2.2935	+ 2.2915	+ 3.9	+18.8
(21 and 21*a*)—(20 and 20*a*)	26.4	+ 4.9440	+ 4.9477	+ 4.9459	+ 3.7	+22.5
(22 and 22*a*)—(21 and 21*a*)	28.5	+ 8.0718	+ 8.0707	+ 8.0712	− 1.1	+21.4
(23 and 23*a*)—(22 and 22*a*)	29.3	+ 3.0147	+ 3.0109	+ 3.0128	− 3.8	+17.6
(24 and 24*a*)—(23 and 23*a*)	30.0	+ 0.6222	+ 0.6221	+ 0.6222	− 0.1	+17.5
(25 and 25*a*)—(24 and 24*a*)	30.6	+ 5.1350	+ 5.1365	+ 5.1357	+ 1.5	+19.0
(26 and 26*a*)—(25 and 25*a*)	33.7	+ 7.0095	+ 7.0008	+ 7.0052	− 8.7	+10.3
(27 and 27*a*)—(26 and 26*a*)	34.7	− 0.6993	− 0.7010	− 0.7002	− 1.7	+ 8.6
(28 and 28*a*)—(27 and 27*a*)	36.1	+11.8175	+11.8244	+11.8210	+ 6.9	+15.5
(29 and 29*a*)—(28 and 28*a*)	36.8	− 0.2208	+ 0.2195	− 0.2202	+ 1.3	+16.8
(30 and 30*a*)—(29 and 29*a*)	37.4	+ 5.4021	+ 5.4026	+ 5.4024	+ 0.5	+17.3
(31 and 31*a*)—(30 and 30*a*)	38.0	+ 5.4580	+ 5.4541	+ 5.4560	− 3.9	+13.4
(32 and 32*a*)—(31 and 31*a*)	39.0	+ 6.1564	+ 6.1597	+ 6.1581	+ 3.3	+16.7
(33 and 33*a*)—(32 and 32*a*)	39.5	− 2.9277	− 2.9283	− 2.9280	− 0.6	+16.1
(34 and 34*a*)—(33 and 33*a*)	42.5	+ 7.1745	+ 7.1758	+ 7.1751	+ 1.3	+17.4
(35 and 35*a*)—(34 and 34*a*)	44.3	+ 1.9044	+ 1.9043	+ 1.9044	− 0.1	+17.3
(36 and 36*a*)—(35 and 35*a*)	45.7	+ 3.6926	+ 3.6870	+ 3.6898	− 5.6	+11.7
(37 and 37*a*)—(36 and 36*a*)	47.3	+ 5.1474	+ 5.1465	+ 5.1469	− 0.9	+10.8
(38 and 38*a*)—(37 and 37*a*)	49.5	+ 9.5870	+ 9.5986	+ 9.5928	+11.6	+22.4
39—(38 and 38*a*)	50.9	+ 4.9243	+ 4.9184	+ 4.9214	− 5.8	+16.6
(40 and 40*a*)—39	52.5	+ 6.1196	+ 6.1283	+ 6.1240	+ 8.7	+25.3
(41 and 41*a*)—(40 and 40*a*)	54.2	+ 5.0896	+ 5.0914	+ 5.0905	+ 1.8	+27.1
(42 and 42*a*)—(41 and 41*a*)	56.0	+ 5.1542	+ 5.1564	+ 5.1553	+ 2.2	+29.3
(43 and 43*a*)—(42 and 42*a*)	57.9	+ 3.1316	+ 3.1362	+ 3.1339	+ 4.6	+33.9
(44 and 44*a*)—(43 and 43*a*)	59.1	+ 0.3826	+ 0.3843	+ 0.3834	+ 1.7	+35.6
(45 and 45*a*)—(44 and 44*a*)	62.5	− 4.2277	− 4.2251	− 4.2264	+ 2.6	+38.2
(46 and 46*a*)—(45 and 45*a*)	64.2	+ 3.3461	+ 3.3465	+ 3.3463	+ 0.4	+38.6
(47 and 47*a*)—(46 and 46*a*)	64.8	+ 0.6603	+ 0.6617	+ 0.6610	+ 1.5	+40.1
(48 and 48*a*)—(47 and 47*a*)	67.0	+ 0.6732	+ 0.6723	+ 0.6727	− 1.0	+39.1
(49 and 49*a*)—(48 and 48*a*)	67.7	+ 1.6066	+ 1.6032	+ 1.6049	− 3.4	+35.7
(50 and 50*a*)—(49 and 49*a*)	68.8	+ 1.3354	+ 1.3320	+ 1.3337	− 3.3	+32.4
(51 and 51*a*)—(50 and 50*a*)	70.3	+ 2.5550	+ 2.5616	+ 2.5583	+ 6.6	+39.0
(52 and 52*a*)—(51 and 51*a*)	73.8	+ 4.8420	+ 4.8450	+ 4.8435	+ 2.9	+41.9
53—(52 and 52*a*)	77.3	+ 7.1398* + 7.0370	+ 7.0333 + 7.0383	+ 7.0362	....	+41.9
(54 and 54*a*)—53	79.6	+ 1.9186* + 1.9341	+ 1.9343 + 1.9343	+ 1.9342	....	+41.9
U. S. B. M. 5—(54 and 54*a*)	80.5	+ 3.9978	+ 3.9949	+ 3.9963	− 3.0	+38.9
(56 and 56*a*)—U. S. B. M. 5	82.5	+ 7.7869	+ 7.7816	+ 7.7842	− 5.3	+33.6

*Rejected.

TABLE IV.—*Results of levels between Escanaba and Marquette, Michigan*—Continued.

Bench-marks.	Distance from Escanaba.	Difference of elevation.			Partial excess.	Total excess.
		First party.	Second party.	Mean.		
	Kilometers.	*Metres.*	*Metres.*	*Metres.*	*Millimeters.*	*Millimeters.*
(57 and 57*a*)—(56 and 56*a*)	85.2	−12.7977	−12.8012	−12.7994	− 3.5	+30.1
(58 and 58*a*)—(57 and 57*a*)	85.5	− 2.8394	− 2.8401	− 2.8397	− 0.6	+29.5
(59 and 59*a*)—(58 and 58*a*)	88.2	−21.9827	−21.9828	−21.9828	− 0.1	+29.4
(60 and 60*a*)—(59 and 59*a*)	90.6	−88.6850	−88.6833	−88.6841	+ 1.7	+31.1
(61 and 61*a*)—(60 and 60*a*)	93.1	−28.8549	−28.8572	−28.8560	− 2.3	+28.8
(62 and 62*a*)—(61 and 61*a*)	94.1	−11.3789	−11.3806	−11.3798	− 1.7	+27.1
(63 and 63*a*)—(62 and 62*a*)	95.1	− 7.3899	− 7.3888	− 7.3894	+ 1.1	+28.2
64—(63 and 63*a*)	97.1	− 9.5345	− 9.5344	− 9.5344	+ 0.2	+28.4
(65 and 65*a*)—64	101.6	− 0.7956	− 0.7926	− 0.7941	+ 3.1	+31.5
B. M. 1 (Marquette)—(65 and 65*a*)	104.7	− 3.9731* − 4.0278	− 4.0273 − 4.0268	− 4.0273		+31.5
B. M. 1 (Marquette)—B. M. 1 (Escanaba)	104.7			+ 5.1565 =16ft.918		+31.5

* Rejected.

LEVELING BY MEANS OF WATER-LEVEL OBSERVATIONS.

§ **9.** At each station where water-level observations were taken, three permanent bench-marks were established, and their elevations with reference to the zero of the gauge were determined. Readings of the height of the water with reference to the zero of gauge were taken at 7 a. m., 1 p. m., and 7 p. m. each day.

At Oswego the gauge consists of a strap of iron spiked to a post and graduated to tenths of a foot, the graduations commencing at a point below the surface of the water and extending upward. Readings at this station, therefore, show the height of the surface of the water above the zero of the gauge. At the other stations measurements were taken by measuring down from a fixed point to the surface of the water with a rod graduated to hundredths of a foot. These measurements, therefore, show the height of the fixed point (zero of gauge) above the surface of the water.

§ **10.** The following table shows the monthly mean readings of the gauges at the stations on Lake Ontario. Observations on this lake were made from May 11 to August 31, 1875, inclusive. The May record at Charlotte has been reduced to the new zero.* Since the readings at Oswego show how much the surface of the water is *above* the zero of the gauge, and at the other stations how much it is *below* the zero of the gauge, the readings at Oswego and Charlotte, or Oswego and Port Dalhousie, should show a constant sum, and the readings at Charlotte and Port Dalhousie should show a constant difference. In deriving the final means given in the tables the mean for each month has been assigned a weight proportional to the number of days during which observations were taken.

Table of mean gauge-readings, Lake Ontario.

Month.	No. of days.	Mean gauge-reading—		Sum.	Mean gauge-reading—		Sum.
		At Charlotte.	At Oswego.		At Oswego.	At Port Dalhousie.	
1875.		*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*
May	21	2.91	1.65	4.56	1.65	12.21	13.86
June	30	2.78	1.73	4.51	1.73	12.12	13.85
July	31	2.64	1.81	4.45	1.81	12.04	13.85
August	31	2.82	1.63	4.45	1.63	12.19	13.82
Means		2.72	1.71	4.49	1.71	12.13	13.84

* Established in May, 1875.

From the preceding table we have the following results for differences of elevation of the zeros of the gauges on Lake Ontario:

	Feet.
Port Dalhousie — Oswego	=+13.84
Charlotte — Oswego	=+ 4.49

The following table shows the monthly mean readings of the gauges at the stations on Lake Erie. Observations were made on this lake from May 19 to August 31, 1875, inclusive. In deriving the final means given in the table, the mean for each month has been assigned a weight proportional to the number of days during which observations were taken. Since the readings in every case are taken by measuring down from a fixed point to the surface of the water, the water-level readings at any two stations should show a constant difference.

Table of mean gauge-readings, Lake Erie.

Month.	No. of days.	Mean gauge-reading— At Cleveland.	Mean gauge-reading— At Port Colborne.	Difference.	Mean gauge-reading— At Port Colborne.	Mean gauge-reading— At Rockwood.	Difference.
1875.		Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
May	13	5.32	7.97	2.65	7.97	3.42	4.55
June	30	5.21	7.83	2.62	7.83	3.34	4.49
July	31	5.08	7.69	2.61	7.69	3.21	4.48
August	31	5.09	7.73	2.64	7.73	3.09	4.64
Means		5.15	7.78	2.63	7.78	3.24	4.54

From the above table we have the following results for differences of elevation of the zeros of the gauges on Lake Erie:

	Feet.
Cleveland — Port Colborne	=−2.63
Rockwood — Port Colborne	=−4.54

The following table shows the monthly mean readings of the gauges at the stations on Lakes Huron and Michigan. Observations were made from May 19 to August 31, 1875, inclusive. In deriving the final means given in the table, the mean for each month has been assigned a weight proportional to the number of days during which observations were made. Since the readings in every case are taken by measuring down from a fixed point to the surface of the water, the water-level observations at any two stations should show a constant difference.

Table of mean gauge-readings, Lakes Huron and Michigan.

Month.	No. of days.	Mean gauge-reading. At Lakeport.	Mean gauge-reading. At Escanaba.	Difference.	Mean gauge-reading. At Lakeport.	Mean gauge-reading. At Milwaukee.	Difference.
1875.		Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
May	13	6.35	3.74	2.61	6.35	3.92	2.43
June	30	6.22	3.59	2.63	6.22	3.80	2.42
July	31	6.06	3.56	2.50	6.06	3.77	2.29
August	31	6.06	3.50	2.56	6.06	3.66	2.40
Means		6.14	3.57	2.57	6.14	3.76	2.38

From the above table we have the following results for differences of elevation of zeros of gauges on Lake Huron and Michigan:

	Feet.
Milwaukee — Lakeport	=−2.38
Escanaba — Lakeport	=−2.57

§ **11.** In order to ascertain the probable effect of the winds on the surface of the water, a comparison has been made of the mean daily measurements at Oswego and Port Dalhousie, with the directions and velocities of the wind as shown by the Signal-Service record at Rochester, N. Y.; also of the mean daily measurements at Port Colborne and Rockwood, with the directions and velocities of the wind as shown by the Signal-Service record at Cleveland; also of the mean daily measurements at Lakeport and Escanaba, with the directions and velocities of the wind as shown by the Lake-Survey record kept at Port Austin.

§ **12.** The following table contains the mean results for Lake Ontario for winds from the eight principal points of the compass for the period May 11 to August 31, 1875. The quantities in the columns marked "Results" show how much the zero of the gauge at Port Dalhousie is above the zero of the gauge at Oswego. Days when the velocity of the wind exceeded ten miles per hour are considered as storm-days:

Direction of wind.	Rejecting storm-days.			Without rejecting storm-days.		
	No. of days.	Results.	Range.	No. of days.	Results.	Range.
		Feet.	*Feet.*		*Feet.*	*Feet.*
North	3	13.86	0.14	3	13.86	0.14
Northeast	2	13.82	0.18	4	13.86	0.22
East	4	13.81	0.19	4	13.81	0.19
Southeast	7	13.81	0.50	8	13.75	0.50
South	25	13.82	0.32	25	13.82	0.32
Southwest	31	13.81	0.38	34	13.83	0.43
West	20	13.89	0.39	28	13.89	0.39
Northwest	3	13.87	0.10	6	13.93	0.23
Calm	1	13.63	0.00	1	13.63	0.00

The length of Lake Ontario extending east and west, the east and west winds would have the greatest, and the north and south winds the least effect on the surface of the water. This is shown by the following means of the results for winds from the various directions:

Direction of wind.	Rejecting storm-days.		Storm-days not rejected.	
	No. of days.	Results.	No. of days.	Results.
		Feet.		*Feet.*
North, south, and calm	29	13.84	29	13.84
Northeast, east, and southeast	14	13.81	16	13.81
Southwest, west, and northwest	54	13.86	68	13.88

As it is not easy to define what shall constitute a storm-day, and as the results are but slightly changed by rejecting storm-days, they will be retained in deriving the mean differences in height.

The mean of all observations during the summer makes the zero of the gauge at Port Dalhousie 13.84 feet above the zero of the gauge at Oswego. Judging from the ranges in the results of the preceding tables, the probable error of this difference ($13^{ft}.84$) cannot exceed $\pm 0^{ft}.04$.

The following table contains the mean results for Lake Erie for winds from the eight principal points of the compass for the period May 19 to August 31, 1875. The quantities in the columns marked "Results" show how much the zero of the gauge at Port Colborne was above the zero of the gauge at Rockwood:

Direction of wind.	Rejecting storm-days.			Not rejecting storm-days.		
	No. of days.	Results.	Range.	No. of days.	Results.	Range.
		Feet.	*Feet.*		*Feet.*	*Feet.*
North	7	4.42	0.53	11	4.38	0.53
Northeast	5	4.95	0.98	5	4.95	0.98
East	10	4.80	0.56	10	4.80	0.56
Southeast	27	4.68	1.44	33	4.68	1.44
South	25	4.33	1.50	27	4.28	1.81
Southwest	11	4.48	0.93	13	4.45	1.01
West	4	4.25	0.70	5	4.20	0.70
Northwest	1	4.44	0.00	1	4.44	0.00

It is assumed in this discussion that, since the length of Lake Erie extends from east-northeast to west-southwest, southeast and northwest winds have the least effect on the level of the lake, and the following means have been taken with reference to this assumption:

Direction of wind.	Rejecting storm-days.		Not rejecting storm-days.	
	No. of days.	Results.	No. of days.	Results.
		Feet.		*Feet.*
Southeast and northwest	28	4.56	34	4.56
North, northeast, and east	22	4.72	26	4.71
South, southwest, and west	40	4.35	45	4.31

The mean of all observations during the summer makes the zero of the gauge at Port Colborne 4.54 feet above the zero of the gauge at Rockwood, with a probable error not exceeding $\pm 0^{ft}.10$.

The following table contains the mean results for Lakes Huron and Michigan for winds from the eight principal points of the compass for the period May 19 to August 31, 1875. The quantities in the columns marked "Results" show how much the zero of the gauge at Lakeport is above the zero of the gauge at Escanaba:

Direction of wind.	Rejecting storm-days.			Not rejecting storm-days.		
	No. of days.	Results.	Range.	No. of days.	Results.	Range.
		Feet.	*Feet.*		*Feet.*	*Feet.*
North	17	2.46	0.55	19	2.44	0.55
Northeast	12	2.51	0.36	13	2.49	0.36
East	11	2.63	0.39	11	2.63	0.39
Southeast	9	2.68	0.49	9	2.68	0.49
South	9	2.73	0.67	9	2.73	0.67
Southwest	16	2.66	0.54	16	2.66	0.54
West	11	2.61	0.29	11	2.61	0.29
Northwest	6	2.47	0.17	8	2.43	0.28
Calm	7	2.58	0.37	7	2.58	0.37

Since Lakeport is near the south end of Lake Huron, and Escanaba near the north end of Lake Michigan, and since the general direction of the length of these two lakes is north and south, east and west winds have the least effect, and north and south winds the greatest effect on the level of the lakes at Lakeport and Escanaba. The following means have been taken with reference to this assumption:

Direction of wind.	Rejecting storm-days.		Not rejecting storm-days.	
	No. of days.	Results.	No. of days.	Results.
		Feet.		*Feet.*
East and west	22	2.62	22	2.62
North, northwest, and northeast	35	2.48	40	2.45
South, southwest, and southeast	34	2.68	34	2.68
Calm	7	2.58	7	2.58

The mean of all observations makes zero of gauge at Lakeport 2.57 feet above zero of gauge at Escanaba, with a probable error not exceeding $\pm 0^{ft}.05$.

The above determination of the difference of elevation of the bench-marks at Lakeport and Escanaba is made on the assumption that Lakes Huron and Michigan were at the same mean level from May 19 to August 31, 1875. The slope of the St. Clair River, which carries off the surplus waters not only of Lake Huron, but also of Lakes Superior and Michigan, is, at St. Clair, about 0.50 inches per mile, the cross-section of the river there being 68150 square feet. Now at Mackinac the smallest water cross-section is about 1558800 square feet, or 22.87 times that of the St. Clair River at St. Clair, and this greater cross-section has to deliver the surplus water of but one of the three great lakes drained by the St. Clair River. If we suppose Lake Michigan has an outflow equal to one-third the discharge of the St. Clair River, then, from the approximate formula $V=B\sqrt{RI}$, where V is the mean velocity, R and I the mean radius and the slope, and B a coefficient determined by experiment, it is easy to see that I varies as $\frac{D^2}{C^2R}$, in which D is the discharge and C the cross-section. C, R, and D, for the St. Clair station, are respectively $\frac{1}{23}$, $\frac{20}{43}$, and 3 times as large as the corresponding values for Mackinac, so that the water slope at Mackinac would be about $\frac{1}{10000}$ that at St. Clair, or 0.00005 inch per mile. Computed with Hagen's empirical formula for large rivers, namely, $V=6\ R^{\frac{1}{2}}\ I^{\frac{1}{6}}$, where 1 foot is the unit, a still smaller slope will be found. As the narrow portion of the Straits of Mackinac is only 10 miles long, this slope gives an insignificant fall. This result is of course a very rough approximation, as there are no experimental data for the flow of water under such gentle slopes, but it shows that the permanent difference of level between Lakes Huron and Michigan is insignificant in comparison with the differences produced by winds or by different atmospheric pressures, and that its average value is probably less than 0.1 foot.

§ **13.** The elevation of the Coast-Survey bench-mark on grist-mill at Greenbush, N. Y., above mean tide at New York City, is given in a letter of the Superintendent of the United States Coast Survey, dated July 9, 1880, as 14.73 feet.

From the records kept at Charlotte from January 1, 1860, to December 31, 1875, the mean surface of Lake Ontario for that period is found to be below bench-mark on light-house at Charlotte, 36.62 feet. The zero of gauge at Charlotte in 1875 was below the same bench-mark, 34.53 feet.

From the Cleveland records the mean surface of Lake Erie for the period, January 1, 1860, to December 31, 1875, is found to be below the bench-mark on coping of Ohio Canal lock, 8.64 feet, and zero of gauge in 1875 below same bench-mark, 3.36 feet.

From the Milwaukee records the mean surface of Lakes Michigan and Huron for the period, January 1, 1860, to December 31, 1875, is found to be below bench-mark on Dr. I. A. Lapham's house, 11.39 feet. The zero of gauge at Milwaukee for 1875 was below the same bench-mark, 7.33 feet.

From the Marquette records the mean surface of Lake Superior for the period, January 1, 1871, to December 31, 1875, is found to be below bench-mark 1 at Marquette, 8.15 feet.

From an examination of the water-level curves, it is concluded that ± 0.10 foot is a sufficient allowance for the probable error of the determination of the mean level of a lake and its reference to a bench-mark.

The distances between the zeros of gauges and their respective bench-marks at Oswego, Charlotte, Port Colborne, Cleveland, Lakeport, Milwaukee, and Escanaba being short, the probable errors of the determinations of the differences of elevation between the zeros of gauges and their respective bench-marks may be neglected in comparison with the probable error of leveling the long lines. Bench mark 2 at Gibraltar is about $2\frac{1}{2}$ miles from the zero of gauge at Rockwood. The three determinations of the difference of elevation of the two points have a range of about 0.1 foot, and the probable error of their mean (+8.74 feet) may safely be taken as ± 0.04 foot.

From these data and the results given in §§ 8 and 12 the following table, giving the elevations of the principal bench-marks used in the two processes of leveling, and the elevations of the mean surfaces of the several lakes for the above-named periods, is constructed. No probable error is assigned by the Superintendent of the United States Coast Survey to the result for the elevation of the bench-mark on the grist-mill at Greenbush, N. Y. Hence probable errors are given in the table only to elevations of points above that bench-mark.

TABLE V.—*Elevations of the Great Lakes.*

	Difference of elevation between successive points.	Height above Coast-Survey bench-mark at Greenbush.	Height above mean tide at New York.
	Feet.	*Feet.*	*Feet.*
Coast-Survey bench-mark on grist-mill at Greenbush	....	....	14. 73
Bench-mark "A" at Oswego	+237. 23 ± 0. 32	237. 23 ± 0. 32	251. 96
Zero of gauge at Oswego	− 7. 75	229. 48 ± 0. 32	244. 21
Zero of gauge at Charlotte	+ 4. 49 ± 0. 04	233. 97 ± 0. 32	248. 70
Bench-mark on light-house at Charlotte	+ 34. 53	268. 50 ± 0. 32	283. 23
Mean surface of *Lake Ontario* from January 1, 1860, to December 31, 1875	− 36. 62 ± 0. 10	231. 88 ± 0. 34	246. 61
Zero of gauge at Oswego	....	229. 48 ± 0. 32	244. 21
Zero of gauge at Port Dalhousie	+ 13. 84 ± 0. 04	243. 32 ± 0. 32	258. 05
Bench-mark on custom-house at Port Colborne	+326. 59 ± 0. 01	569. 91 ± 0. 32	584. 64
Zero of gauge at Port Colborne	− 3. 87	566. 04 ± 0. 32	580. 77
Zero of gauge at Cleveland	− 2. 63 ± 0. 10	563. 41 ± 0. 34	578. 14
Bench-mark on coping of Ohio Canal	+ 3. 36	566. 77 ± 0. 34	581. 50
Mean surface of *Lake Erie* from January 1, 1860, to December 31, 1875	− 8. 64 ± 0. 10	558. 13 ± 0. 35	572. 86
Zero of gauge at Port Colborne	....	566. 04 ± 0. 32	580. 77
Zero of gauge at Rockwood	− 4. 54 ± 0. 10	561. 50 ± 0. 34	576. 23
Bench-mark 2 at Gibraltar	+ 8. 74 ± 0. 04	570. 24 ± 0. 34	584. 97
Benchmark 2 at Lakeport	+ 5. 89 ± 0. 09	576. 13 ± 0. 35	590. 86
Zero of gauge at Lakeport	− 3. 14	572. 99 ± 0. 35	587. 72
Zero of gauge at Milwaukee	− 2. 38 ± 0. 05	570. 61 ± 0. 36	585. 34
Bench-mark on Dr. I. A. Lapham's house	+ 7. 33	577. 94 ± 0. 36	592. 67
Mean surface of *Lakes Huron and Michigan* from January 1, 1860, to December 31, 1875	− 11. 39 ± 0. 10	566. 55 ± 0. 38	581. 28
Zero of gauge at Lakeport	....	572. 99 ± 0. 35	587. 72
Zero of gauge at Escanaba	− 2. 57 ± 0. 05	570. 42 ± 0. 36	585. 15
Bench-mark 1 at Escanaba	+ 7. 86	578. 28 ± 0. 36	593. 01
Bench-mark 1 at Marquette	+ 16. 92 ± 0. 03	595. 20 ± 0. 36	609. 93
Mean surface of *Lake Superior* from January 1, 1871, to December 31, 1875	− 8. 15 ± 0. 10	587. 05 ± 0. 38	601. 78

§ **14.** The following tables (VI to XVII, inclusive,) contain a list of permanent bench-marks with their descriptions and elevations. Bench-marks which were of such a nature that they could not be expected to be permanent have not been included in this list. The bench-marks between Greenbush and Oswego, given in this list, are along the Erie Canal. The year in which each bench-mark was established, when known, has been placed in a parenthesis after the designation of the bench-mark. The elevation of each bench-mark above the Coast-Survey bench-mark on grist-mill at Greenbush, N. Y., as determined by the Lake Survey, is first given, and its height above mean tide at New York is obtained by adding 14.730 feet (§ 13).

TABLE VI.—*Bench-marks established between Greenbush and Oswego, New York.*

Bench-marks.	Description.	Distance from Greenbush.	Elevation above Coast-Survey bench-mark.	Elevation above mean tide at New York.
		Miles.	*Feet.*	*Feet.*
C. S. B. M	A cross cut on the northwest side of the northeast corner of stone foundation of steam grist-mill at Greenbush, N. Y.	0.00	0.000	14.730
B. M. (1875)	Upper side of head of copper bolt leaded into springing-stone of north arch of culvert of Boston and Albany Railroad a few rods south of bridge over Second avenue, Greenbush. The bench-mark is on north side of culvert and west side of railroad.	0.50	0.134	14.864
B. M. 1 (1875)	Point on top of east shoulder of northeast end of southeast pier of upper railroad bridge across Hudson River at East Albany, marked B. M.	1.25	12.288	27.018
Miter-sill of Lock No. 1 at Albany.	Miter-sill of southwest or lower lock of Lock No. 1 of Erie Canal at Albany, N. Y. (According to letter of Horatio Seymour, jr., State engineer and surveyor, to James T. Gardner, director of State survey, dated May 7, 1880, the elevation of this mitre-sill has not been changed since 1839.)	....	20.99	6.26
B. M. 2 (1875)	Top of stone at center of cross cut into top of masonry at southwest corner of east wall of west lock (Lock No. 1) at Albany, marked B. M. U. S.	2.25	2.862	17.592
B. M. 3 (1875)	Top of coping at southeast end of northwest wall of northwest lock (Lock No. 2). It is 16 feet southeast from heel-post of gate and 1 foot from west face of wall, marked B. M. U. S.	3.75	12.243	26.973
B. M. 5 (1875)	Cross cut in second stone from south corner of fourth tier of stones of west abutment of horse-car bridge across canal at West Troy.	7.25	15.857	30.587
B. M. 5*a* (1875)	Cross cut into east face of third stone step on north wing of west abutment of horse-car bridge across canal at West Troy, marked B. M.	7.25	14.816	29.546
B. M. 6 (1875)	Cross cut in top of northwest end of east foot-iron on southeast end of southwest wall of southwest lock of Lock No. 4.	9.55	34.963	49.693
B. M. 6*a* (1875)	Southwest corner of heel of wall separating the two locks of Lock No. 4 (southeast corner of northeast wall of southwest lock), marked by three converging lines cut in stone.	9.55	34.887	49.617
B. M. 7 (1875)	Cross cut in top of middle foot-iron at south end of west wall of west lock of Lock No. 6.	10.10	55.740	70.470
B. M. 7*a* (1875)	Top of screw-bolt fastening down iron collar of south gate of west lock of Lock No. 6.	10.10	55.700	70.430
B. M. 8 (1875)	Cross cut on top of second foot-iron (clamp) on east wall (south end) of west Lock No. 15.	11.60	145.856	160.586
B. M. 8*a* (1875)	Top of point of coping southwest corner of east wall of west Lock No. 15.	11.60	145.762	160.492
B. M. 9 (1875)	Cross cut on southwest face of south wing of east abutment of bridge across canal. Cross is on third course of stones close to the corner, 3 feet above ground.	13.60	179.956	194.686
B. M. 9*a* (1875)	Highest point of corner of stone, southeast face, southeast corner of same wing wall. Stone marked B. M.	13.60	179.194	193.924
B. M. 10 (1875)	Top of projecting point of stone in third course of masonry, on northeast face, 1 foot from east corner of east wing wall of southeast abutment of canal bridge, marked B. M.	16.35	178.866	193.596

TABLE VI.—*Bench-marks established between Greenbush and Oswego, New York*—Continued.

Bench-marks.	Description.	Distance from Greenbush.	Elevation above Coast-Survey bench-mark.	Elevation above mean tide at New York.
		Miles.	*Feet.*	*Feet.*
B. M. 10*a* (1875)	Top of projecting point of stone in second course of masonry in southeast abutment of canal bridge, marked B. M.	16.35	177.500	192.230
B. M. 11 (1875)	Top of projection of stone in second course of masonry in southeast abutment of canal bridge, marked B. M.	17.60	177.292	192.022
B. M. 11*a* (1875)	Top of projection of stone in bottom course of masonry in east wing wall of southeast abutment of same bridge, about 3 feet from corner of abutment, marked B. M.	17.60	176.120	190.850
B. M. 12 (1875)	Top of projection of stone on second course of masonry at east corner of south abutment of second canal bridge below Lock No. 19.	20.35	176.986	191.716
B. M. 12*a* (1875)	Top of projection of stone on second course of stones at east corner of east wing wall of south abutment of same bridge.	20.35	176.867	191.597
B. M. 13 (1875)	On top of corner of stone in second course of stones on southeast wing wall of southwest abutment of first bridge across canal at Vischer's Ferry.	21.75	187.472	202.202
B. M. 13*a* (1875)	Cross on southeast corner of same abutment of same bridge.	21.75	188.901	203.631
B. M. 14 (1875)	South corner of east wall of west Lock No. 20	23.75	194.343	209.073
B. M. 14*a* (1875)	Top of screw-bolt fastening down collar of southwest gate of west Lock No. 20. Top of bolt with cross.	23.75	194.457	209.187
B. M. 15 (1875)	On coping 1.5 feet from south corner of west abutment of Rexford feeder bridge.	26.75	203.276	218.006
B. M. 15*a* (1875)	On east corner of coping of north wall of south Lock No. 21, top of stone steps.	26.75	205.801	220.531
B. M. 16 (1875)	On northwest abutment of New York Central Railroad bridge across canal at Schenectady, top of corner-stone in third course of stones on the northeast corner.	29.75	221.177	235.907
B. M. 16*a* (1875)	On same abutment, top of corner-stone in third course of stones on southwest corner.	29.75	221.357	236.087
B. M. 18 (1875)	Top of projection of stone (fourth stone from northwest corner) in second course of stones in face of northeast abutment of canal bridge, marked B. M.	35.25	236.729	251.459
B. M. 18*a* (1875)	Top of stone projection in second course of stones in face of southeast wing wall of northeast abutment of same bridge, marked B. M.	35.25	236.573	251.303
B. M. 19 (1875)	Top of projection of stone in third course of stones in southeast wing wall of east abutment of canal bridge next below Lock No. 25, marked B. M.	38.25	236.760	251.490
B. M. 19*a* (1875)	Top of projection of middle stone of bottom course of masonry in face of east abutment of same bridge, marked B. M.	38.25	234.026	248.756
B. M. 20 (1875)	Top of coping on corner of east wing wall of north abutment of canal bridge at Pattersonville, marked B. M.	40.75	244.710	259.440
B. M. 20*a* (1875)	Top of coping, corner of west wing wall of north abutment of same canal bridge, marked B. M.	40.75	244.048	258.778
B. M. 21 (1875)	On east corner of south wall of north Lock No. 26, marked B. M.	44.50	249.498	264.228
B. M. 21*a* (1875)	On east corner of north wall of south Lock No. 26, marked B. M.	44.50	249.534	264.264

TABLE VI.—*Bench-marks established between Greenbush and Oswego, New York*—Continued.

Bench-marks.	Description.	Distance from Greenbush.	Elevation above Coast-Survey bench-mark.	Elevation above mean tide at New York.
		Miles.	*Feet.*	*Feet.*
B. M. 23 (1875)	On top stone of east wall of waste-gate, marked B. M.	47.00	257.745	272.475
B. M. 23*a* (1875)	On top stone at northwest corner of west wall of waste-gate, marked B. M.	47.00	257.691	272.421
B. M. 24 (1875)	Point on east end of north wall of north Lock No. 28, marked B. M.	50.00	265.679	280.409
B. M. 24*a* (1875)	Top of iron bolt in top of coping on east wall of space between the two locks. Head of bolt marked with cross.	50.00	265.704	280.434
B. M. 25 (1875)	Cross on corner-stone, fifth course of stones, northeast corner of northwest abutment of first bridge above Schoharie Creek aqueduct, marked B. M.	53.00	288.125	302.855
B. M. 25*a* (1875)	Cross on southwest corner, fifth course of stones of same abutment, marked B. M.	53.00	288.118	302.848
B. M. 26 (1875)	Top of projection of stone in fourth course of stones in east wing of northeast abutment of bridge in Fultonville.	57.00	286.132	300.862
B. M. 26*a* (1875)	Top of projection of stone in bottom course, third stone from west corner of the same abutment. Face of abutment.	57.00	274.359	289.089
B. M. 28 (1875)	On top of corner of corner-stone, third course of stones, east wing of north abutment of bridge.	63.75	286.720	301.450
B. M. 28*a* (1875)	On top of projection of stone, second course of stones, face of north abutment of same bridge.	63.75	285.374	300.104
B. M. 29 (1875)	On top of iron bolt, west wing of north abutment of second bridge above Lock No. 31, at Spraker's Basin.	66.25	291.134	305.864
B. M. 29*a* (1875)	Top of projection of stone in face of north abutment of same bridge. Third stone from east corner in second course of stones.	66.25	291.167	305.897
B. M. 30 (1875)	On projection of stone in second course of stones northwest corner of northeast abutment of bridge.	67.75	289.599	304.329
B. M. 30*a* (1875)	On top of projection of stone on southeast corner of same abutment, second course of stones.	67.75	289.543	304.273
B. M. 31 (1875)	Cross cut on corner-stone at east corner, second course of stones from top, on south face of stone foundation of old barn near upper foot-bridge across canal at Canajoharie.	69.50	292.281	307.011
B. M. 31*a* (1875)	Cross cut on corner-stone, second course of stones, west corner of same face of same barn.	69.50	291.763	306.493
B. M. 32 (1875)	On corner of coping of southeast wing wall of northeast abutment of bridge.	70.50	292.138	306.868
B. M. 34 (1875)	Top of projection of stone in second course of stones, east wing, north abutment of first bridge above Lock No. 33.	77.75	305.272	320.002
B. M. 34*a* (1875)	On top of projection of stone in second course of stones, west wing of same abutment of same bridge.	77.75	305.231	319.961
B. M. 35 (1875)	On top of projecting point of stone in second course of stones on southeast wing of northeast abutment of bridge.	80.75	313.487	328.217
B. M. 35*a* (1875)	On top of projecting point of stone in second course of stones on northwest wing of abutment of same bridge.	80.75	314.026	328.756
B. M. 36 (1875)	On top of projection of stone, third course of stones, northwest wing of northeast abutment of bridge, about half a mile above Lock No. 35.	83.50	321.853	336.583

TABLE VI.—*Bench-marks established between Greenbush and Oswego, New York*—Continued.

Bench-marks.	Description.	Distance from Greenbush.	Elevation above Coast-Survey bench-mark.	Elevation above mean tide at New York.
		Miles.	*Feet.*	*Feet.*
B. M. 36*a* (1875)	On top of projection of stone, third course of stones, face of abutment of same bridge.	83. 50	321. 908	336. 638
B. M. 37 (1875)	Top of iron bolt in coping of southeast end of northeast wall of northeast Lock No. 36. Bolt marked with cross in top.	87. 25	329. 237	343. 967
B. M. 37*a* (1875)	Top of corner at southeast end of southwest wall of northeast Lock No. 36.	87. 25	329. 241	343. 971
B. M. 38 (1875)	Top of coping at corner of east wing of north abutment of bridge.	90. 50	364. 096	378. 826
B. M. 38*a* (1875)	On top of coping at corner of west wing wall of north abutment of same bridge.	90. 50	364. 040	378. 770
B. M. 39 (1875)	On top of corner of east wing of north abutment of first bridge above Lock No. 41.	94. 00	379. 258	393. 988
B. M. 39*a* (1875)	On top of corner of west wing of north abutment of same bridge.	94. 00	379. 301	394. 031
B. M. 40 (1875)	On top of corner of southeast wing of northeast abutment of bridge next above Lock No. 43.	96. 50	394. 765	409. 495
B. M. 40*a* (1875)	On top of corner of northwest wing of same abutment.	96. 50	394. 870	409. 600
B. M. 41 (1875)	On top of projection of stone in bottom course of stones on southeast corner of northeast abutment of bridge next below Lock No. 45, in Frankfort.	100. 00	404. 512	419. 242
B. M. 41*a* (1875)	Cross cut in corner-stone, sixth course of stones, northwest corner of same abutment.	100. 00	408. 539	423. 269
B. M. 42 (1875)	Top of projection of stone, second course of stones, southeast wing of northeast abutment of bridge.	104. 00	414. 534	429. 264
B. M. 42*a* (1875)	Top of projection of stone, second course of stones, northwest wing of same abutment.	104. 00	414. 350	429. 080
B. M. 43 (1875)	On top of projection of stone, lower course of stones, southwest wing of northeast abutment of bridge.	106. 75	414. 521	429. 251
B. M. 43*a* (1875)	On top of projection of stone, second course of stones, northwest wing of same abutment.	106. 75	415. 689	430. 419
B. M. 44 (1875)	Cross cut on northwest corner of northeast abutment of bridge about 2,000 feet below Lock No. 46, at Utica. Bench-mark in third course of stones.	109. 75	418. 260	432. 990
B. M. 44*a* (1875)	On corner of fourth step of northwest wing of northeast abutment of bridge.	109. 75	418. 047	432. 777
B. M. 45 (1875)	Cross cut on south corner of east abutment of bridge, third course of stones.	114. 25	418. 881	433. 611
B. M. 45*a* (1875)	Cross cut on north corner of same abutment, fourth course of stones.	114. 25	419. 075	433. 805
B. M. 46 (1875)	On top of projection of stone in second course of stones, east wing of north abutment of bridge.	118. 75	418. 727	433. 457
B. M. 46*a* (1875)	On top of projection of stone, bottom course, face of abutment of same bridge.	118. 75	417. 252	431. 982
B. M. 47 (1875)	Top of projection of stone in face of north abutment of bridge, second course of stones.	120. 00	418. 308	433. 038
B. M. 47*a* (1875)	Top of projection of stone, second course of stones, west wing of same abutment.	120. 00	418. 354	433. 084
B. M. 48 (1875)	Projection of stone, second course of stones, east wing of north abutment of bridge in Rome.	124. 25	417. 123	431. 853
B. M. 48*a* (1875)	Projection of stone, second course of stones, west wing of same abutment.	124. 25	418. 199	432. 929
B. M. 49 (1875)	Top of projection of stone, second course of stones, east wing of north abutment of bridge.	126. 25	418. 148	432. 878

TABLE VI.—*Bench-marks established between Greenbush and Oswego, New York*—Continued.

Bench-marks.	Description.	Distance from Greenbush.	Elevation above Coast-Survey bench-mark.	Elevation above mean tide at New York.
		Miles.	*Feet.*	*Feet.*
B. M. 49*a* (1875)	Top of projection of stone, third course of stones, face of abutment of same bridge.	126.25	419.359	434.089
B. M. 51 (1875)	Top of projection of stone, bottom course, east wing of north abutment of bridge about 1 mile below Higginsville.	133.75	416.678	431.408
B. M. 51*a* (1875)	Top of projection of stone, second course of stones, face of north abutment of same bridge.	133.75	418.450	433.180
B. M. "A" (Oswego)	Top of iron bolt in top of masonry of old government pier, 0.5 feet from east face of pier, 3.5 feet north of northwest corner of United States Engineer's storehouse, on United States reservation at foot of Third street. Marked, U. S. + B. M.	187.25	237.234	251.964
B. M. "B" (Oswego)	Top of stone post in prolongation south of west face of old stone pier. The stone marks southwest boundary of United States reservation, is 8 feet south of masonry of pier, 28 feet west of southwest corner of Engineer's storehouse, and surface is flush with ground. Marked, U. S.	....	237.747	252.477
B. M. "C" (Oswego)	Cross cut on shop of "Dry-dock of Marine Railway," on corner of Lake and Second streets. Cross in third course of stones from ground on west side of shop, 3 feet from southwest corner.	....	247.310	262.040

TABLE VII.—*Bench-marks at Sacket's Harbor, New York.*

Bench-marks.	Description.	Elevation above Coast-Survey bench-mark at Greenbush.	Elevation above mean tide at New York.
		Feet.	*Feet.*
B. M. 1 (1874)	A cross on the solid rock between the sidewalk and the water, N. 2° E. from the northwest corner of the Masonic Temple, and 96½ feet distant.	237.29	252.02
B. M. 2	The upper side at the outer edge of the water-table at the northeast corner of the stone Masonic Temple.	249.95	264.68
B. M. 3 (Check Point) (1875).	The intersection of two perpendicular lines in the head of a 7-inch screw-bolt leaded into the natural rock, situated about three-eighths of a mile down the bay from the United States naval quarters.	235.62	250.35

TABLE VIII.—*Bench-marks at Charlotte, New York.*

Bench-marks.	Description.	Elevation above Coast-Survey bench-mark at Greenbush.	Elevation above mean tide at New York.
		Feet.	*Feet.*
B. M. 1	The upper side of the water-table of the light-house, at the south-southeast angle east of the south window.	268.50	283.23
B. M. 2 (1874)	A bench-mark (B×M) on the top of the circular wall of the railroad turn-table, southwest part of wall.	238.71	253.44

TABLE IX.—*Bench-marks at Port Dalhousie, Ontario.*

Bench-marks.	Description.	Elevation above Coast-Survey bench-mark at Greenbush.	Elevation above mean tide at New York.
		Feet.	*Feet.*
B. M. "A"	Top of stone post buried under sidewalk, corner of Canal and Lock streets, 10½ feet from southeast corner of "Wood House" on perpendicular to east side of "Wood House" toward canal, about 110 feet west of heel-post of west gate of north end of canal lock.	249.30	264.03
B. M. "B"	Edge of cut in top course of masonry in north recess in east wall of canal, 20 feet north of northeast gate of lock.	243.32	258.05
B. M. "C"	Cross cut into stone of foundation of customs collector's office, third course of stones from top, north side, 1.4 feet from northwest corner.	243.50	258.23

TABLE X.—*Bench-marks at Port Colborne, Ontario.*

Bench-marks.	Description.	Elevation above Coast-Survey bench-mark at Greenbush.	Elevation above mean tide at New York.
		Feet.	*Feet.*
B. M. on custom-house (1875)	Top of point of iron bolt set in masonry of stone foundation of custom-house, west side, southwest corner.	569.91	584.64
B. M. on Baptist Church (1875).	Top of point of iron bolt in east end of window-sill in basement of steeple, south side of Baptist Church.	565.71	580.44
B. M. on Church of England (1875).	Top of point of iron bolt in stone foundation, lower tier of stones in south side of Church of England (street front, east side of entrance).	564.26	578.99

TABLE XI.—*Bench-marks at Cleveland, Ohio.*

Bench-marks.	Description.	Elevation above Coast-Survey bench-mark at Greenbush.	Elevation above mean tide at New York.
		Feet.	*Feet.*
B. M. 1	A mark [B×M] on the top of the northeast wall of the Ohio Canal lock, at the connection of the canal with the river.	566.77	581.50
B. M. 2	A cross (×) on the water-table, northeast corner of Johnson House block, southwest corner of Front and East River streets.	563.10	577.83
B. M. 3	A cross (×) on the stone water-table, southwest corner of brick block, northeast corner of River and Superior streets.	580.09	594.82

TABLE XII.—*Bench-marks at Erie, Pennsylvania.*

Bench-marks.	Description.	Elevation above Coast-Survey bench-mark at Greenbush.	Elevation above mean tide at New York.
		Feet.	*Feet.*
B. M. 1 (1873)	The highest point of a stone post 336 feet distant from the nearest point of the pier on which the water-gauge is situated, and on the north side. The northeast corner of the light-keeper's dwelling is 228 feet distant and bears S. 68° E.	560.87	575.60
B. M. 2 (1859)	A mark (×) on the beacon pier, being on the southwest corner of the cut-stone foundation of the beacon light.	568.36	583.09

TABLE XIII.—*Bench-marks between Gibraltar and Lakeport, Michigan.*

Bench-marks.	Description.	Elevation above Coast-Survey bench-mark at Greenbush.	Elevation above mean tide at New York.
		Feet.	*Feet.*
B. M. 1 (1875)	A small cross on the stone window-sill of the south window on the west side of the brick house of Mr. Craig, corner of Farnsworth and Adams streets, Gibraltar, Mich.	572. 22	586. 95
B. M. 2 (1875)	The southeast corner of the stone door-sill of the door in the southeast angle of the light-house at Gibraltar.	570. 24	584. 97
U. S. B. M. "Gibraltar" (1877).	Center of small hole in head of copper bolt leaded into stone in upper course of masonry of foundation-wall of light-house tower on east side, southeast corner, at Gibraltar.	567. 72	582. 45
U. S. B. M. "Trenton" (1877).	Center of small hole in head of copper bolt leaded into second stone from northwest corner in first course below water-table in foundation-wall on west side of "Dion's House," corner Washington and St. Joseph avenues, at Trenton, Mich.	588. 69	603. 42
U. S. B. M. "Wyandotte" (1877).	Center of small hole in head of copper bolt leaded into stone midway between basement windows, third course of stones below water-table in foundation-wall of north side of Union School building at Wyandotte, Mich.	571. 54	586. 27
U. S. B. M. "Detroit Junction" (1877).	Center of small hole in head of copper bolt leaded into capstone of foundation-wall of "planing mill and machine shop" of the Michigan Central Railroad at Detroit Junction, about 3 metres from southeast corner on east side of shop.	577. 95	592. 68
U. S. B. M. "Detroit" (1871).	A cross cut on the new light-house depot at Detroit, 8.2 feet below the outer edge of the water-table in the southern projection of the southeast door, on the cut-stone foundation.	570. 05	584. 78
U. S. B. M. "New Haven" (1877).	Center of small hole in head of copper bolt set horizontally in water-table on north side of station-house of Grand Trunk Railroad at New Haven, 0.88 metre from northwest corner.	615. 99	630. 72
U. S. B. M. "Pine River" (1877).	Top of copper bolt set vertically in center of upper surface of capstone at south end of west abutment of Grand Trunk Railroad bridge over Pine River.	613. 93	628. 66
U. S. B. M. "Fort Gratiot," (1877).	Center of small hole in head of copper bolt set in upper course of masonry in south foundation-wall of brick dwelling attached to light-house at Fort Gratiot, Mich. It is 0.65 metre from southeast corner and 0.18 metre below the water-table.	575. 17	589. 90
B. M. 1 (Lakeport) (1875)	A spike driven into a cedar post supporting the steam grist-mill of Mr. John Cole, at Lakeport. The post is on the north side of mill, and 12 feet from northwest corner.	579. 39	594. 12
B. M. 2 (Lakeport) (1875)	A cross cut in the stone on the east side of the engine-house of the grist-mill, and at the southeast corner.	576. 13	590. 86
B. M. 3 (Lakeport) (1875)	The highest point of the top stone of the foundation of the milk-house of Mr. Cole, Second street, at the southeast corner of the building.	583. 92	598. 65

TABLE XIV.—*Bench-mark at Port Austin, Michigan.*

Bench-mark.	Description.	Elevation above Coast-Survey bench-mark at Greenbush.	Elevation above mean tide at New York.
		Feet.	*Feet.*
B. M. (1873)	The head of a 6-inch bolt leaded into the rock 13 feet southwest of the gauge. The letters B. M. are cut in the rock near it.	575. 80	590. 53

TABLE XV.—*Bench-marks at Milwaukee, Wisconsin.*

Bench-marks.	Description.	Elevation above Coast-Survey bench-mark at Greenbush.	Elevation above mean tide at New York.
		Feet.	*Feet.*
B. M. 1	This bench-mark was formerly on house of Dr. I. A. Lapham, but has been destroyed by repairs to house.	577.94	592.67
B. M. 2	Stone monument in court-house square, near the southeast corner thereof, in the seventh ward.	620.97	635.70
B. M. 3	Stone monument on sidewalk at southeast corner of Eighth and Chestnut streets, second ward.	618.88	633.61
B. M. 4	The highest point of the stone water-table at the corner of the building, Ludington's block, northwest corner of East Water and Wisconsin streets.	579.22	593.95
B. M. 5 (1876)	A cross on the masonry of the Kilbourn grist-mill at foot of Poplar street. It is cut in the stone 10½ inches from the southeast corner on the east wall, and about 3 feet above the surface of the ground. (In Report of Chief of Engineers for 1877, page 1194, where this bench-mark is described, for 2.64 read 2.48, and for 5.69 read 5.85.)	575.46	590.19
B. M. 6 (Check Point) (1875).	Top of copper bolt, 1 inch in diameter, leaded into the north side of center pier of swing bridge over the river between Chestnut and Division streets.	571.47	586.20

TABLE XVI.—*Bench-marks between Escanaba and Marquette, Michigan.*

Bench-marks.	Description.	Elevation above Coast-Survey bench-mark at Greenbush.	Elevation above mean tide at New York.
		Feet.	*Feet.*
B. M. 1 (Escanaba) (1874)	The top of the water-table of the large brick building of S. Adler, on the northwest corner of Ludington street and Doneseman avenue, on the southeast corner of the building.	578.28	593.01
U. S. B. M. 3 (1876)	Center of small hole in copper bolt leaded into masonry foundation of Escanaba light-house, west side of light-house, near northwest corner.	571.79	586.52
U. S. B. M. 4 (1876)	Center of small hole in copper bolt leaded into natural rock on east side of Chicago and Northwestern Railroad, about 36 metres north of switch of siding leading to charcoal kilns at Maple Ridge.	943.91	958.64
U. S. B. M. 5 (1876)	Center of small hole in copper bolt leaded into natural rock, about 74 metres east and 53 metres south of switch at north end of siding at Sands.	1187.40	1202.13
U. S. B. M. 6 (1876)	Center of small hole in copper bolt leaded into third course of stones above water-table on north side of the Marquette, Houghton and Ontonagon Railroad general freight and ticket office at Marquette, about 1 foot from northeast corner.	613.12	627.85
B. M. 1 (Marquette) (1871)	Southeast corner of the top of the foundation stone of Grace furnace.	595.20	609.93
B. M. 2 (Marquette) (1874)	Cross on the window-sill of the Marquette City water-works. It is on the center window, west side of building, and north side of window.	594.70	609.43
B. M. 3 (Marquette) (1874)	Cross on the window-sill of the Marquette City water-works. It is on the north window, east side of building, and 6 inches from north end of sill.	594.62	609.35

TABLE XVII.—*Bench-marks at Sault Ste. Marie, Michigan.*

Bench-marks.	Description.	Elevation above Coast-Survey bench-mark at Greenbush.	Elevation above mean tide at New York.
		Feet.	*Feet.*
B. M. 1 (1867)	A cross cut on a stone near the Indian agency	574.47	589.20
B. M. 2	A bench-mark (B□M) on the coping of the south wall of the guard-gates of St. Mary's Canal. It is at the upper gate-recess, 2.5 feet from the foot of the gate-tower, and 0.5 foot from the face of the wall.	591.14	605.87

PART IV.

ASTRONOMICAL DETERMINATIONS.

CHAPTER XXIII.

LATITUDES.

INSTRUMENTS.

§ **1.** Latitudes have been determined exclusively by Talcott's method. This method, as is well known, consists in selecting pairs of stars culminating within a few minutes of each other in time, on opposite sides of the zenith, at distances from it not differing by more than twenty minutes of arc. A telescope that can be turned in azimuth about a vertical axis has a delicate level to measure any slight changes in the inclination of its line of sight. It has also a micrometer capable of measuring vertical angles of twenty or more minutes. On placing such a telescope in the meridian, and reading the micrometer on the star which first culminates, then turning the telescope 180° in azimuth by means of a horizontal circle, and reading the micrometer on the second star as it culminates, the difference of micrometer-readings, corrected for change of level and refraction, will give with precision the difference of zenith distances of the stars. Adding to half this difference the half sum of the declinations of the stars observed, the latitude results.

Two forms of latitude instrument have been used on the Lake Survey. The first is that devised by Captain Talcott, of the United States Engineers, of which a full description may be found in Chauvenet's Astronomy; the second is the portable transit of Ertel, having an azimuth circle reading to ten seconds, delicate finding-levels and a vertical micrometer being added to the transit as originally designed. The first form of instrument is called a zenith-telescope. The instruments used in the determination of latitudes given in this chapter are zenith-telescopes Nos. 1, 12, and 19, made by William Würdemann, and the portable transit No. 1, made by Pistor & Martins. The instruments have the following dimensions respectively:

Zenith-telescope No. 1: Focal length, 24 inches; diameter of object-glass, $2\frac{3}{8}$ inches.
Zenith-telescope No. 12: Focal length, 32 inches; diameter of object-glass, $2\frac{1}{2}$ inches.
Zenith-telescope No. 19: Focal length, 32 inches; diameter of object-glass, 3 inches.
Portable transit No. 1: Focal length, 24 inches; diameter of object-glass, $2\frac{1}{2}$ inches.

Zenith-telescope No. 1 and the portable transit have been used at but five stations.

The instruments have usually been mounted on wooden logs from two to three feet in diameter and set firmly in the ground to a depth of four or five feet. Exceptionally, they have been mounted on heavy stones resting on rock.

An idea of the precision of latitude determinations with these instruments can be formed from data which have been obtained in the following way: The error in a latitude from a single pair of stars observed with the zenith-telescope is of two parts, first, that arising from observing and instrumental errors in determining the half difference of the zenith distances; and second, that arising from errors in the half sum of the declinations adopted for the stars.

ERRORS OF OBSERVATION.

§ **2.** If the same pair of stars be observed on several nights, the differences between the results for each separate night, and the mean of such results, will not be affected by declination-errors, but will give by the method of least squares the probable error due to the instrument and observer, which may be called the error of observation. Denote for one pair of stars observed once the probable error in half the observed difference of zenith distances by E_0. If this pair be observed on n nights, the probable error of the mean half difference of zenith distances will be $\sqrt{\frac{E_0^2}{n}}$. Calling the probable error in the half sum of the declinations of this pair of stars E_δ, there follows for the probable error, E_φ, of the latitude resulting from n nights' observation on one pair of stars,

$$\text{(1)} \qquad E_\varphi=\sqrt{E_\delta^2+\frac{E_0^2}{n}}$$

or taking for its weight, p, the reciprocal of E_φ^2,

$$\text{(2)} \qquad p=\frac{n}{n\,E_\delta^2+E_0^2}$$

The value of E_0 may be found as follows: Observe each of many pairs of stars on several nights; the differences, $\varDelta$, between the mean of the latitudes resulting from any pair, and the separate latitudes from that pair, will give a series of equations of the form

$$\left.\begin{array}{l}\varDelta_1=-l_1+x \\ \varDelta_2=-l_2+x \\ \cdots\cdots\end{array}\right\}\text{1st pair,}$$

$$\left.\begin{array}{l}\varDelta_6=-l_6+y \\ \varDelta_7=-l_7+y \\ \cdots\cdots\end{array}\right\}\text{2d pair,}$$

&c., &c., &c.,

where l is the latitude resulting from one observation on a pair, and x, y, &c., are the means of the latitudes from each pair. If there are m observations on n pairs, we shall have for the probable observation-error for that instrument

$$\text{(3)} \qquad E_0=0.6745\sqrt{\frac{[\varDelta\varDelta]}{m-n}}$$

Chauvenet in his astronomy finds E_0, in an example taken from the Coast Survey, equal to 0''.30. The Coast-Survey Report for 1866 states that its usual value is below 0''.50. Professor Gould, however, in the Coast-Survey Report for 1865, gives a long series of observations with an average of seventeen nights' work on nineteen pairs, and finds E_0=0''.80. The following results have been obtained from the following Lake-Survey instruments: 32-inch zenith-telescope No. 12, about 15 years old, and 24-inch Pistor & Martins transit No. 1, twenty-three years old:

1.—ZENITH-TELESCOPE No. 12.

Station.	Date.	Observer.	No. of results.	No. of pairs.	Probable error of observation in the half difference of zenith distances.
					''
North Base, Minnesota Point	1871	G. Y. Wisner	55	23	0.89
South Base, Minnesota Point	1871	....do	60	30	0.78
Aminicon River	1871	....do	54	27	0.62
Isle St. Ignace, south latitude post	1871	G. A. Marr	73	33	0.62
Crebassa	1871	....do	128	47	0.54
South Base, Keweenaw Point	1871	A. R. Flint	29	14	0.95
Detroit	1872	C. B. Comstock	63	23	0.69
Sums			462	197	

Probable error of observation in the whole series, 0''.69.

2.—PISTOR & MARTINS TRANSIT No. 1.

Station.	Date.	Observer.	No. of results.	No. of pairs.	Probable error of observation in the half difference of zenith distances.
					$''$
Brulé River	1871	G. A. Marr	90	34	0.99
Isle St. Ignace, north latitude post	1871	....do	119	52	1.19
Detroit	1872	C. B. Comstock	79	30	0.89
Sums			288	116	

Probable error of observation in the whole series, 1″.05.

Subsequently to the above determinations of E_0 for zenith-telescope No. 12, it was found that its micrometer-screw was not of uniform value throughout its length. Determining and using its correct variable value, the following results were subsequently obtained for it and for zenith-telescope No. 19:

Instrument No.	Station.	Observer and date.	Pairs of stars.	No. of nights.	E_0	Mean E_0
					$''$	$''$
12	South Base, Keweenaw Point	G. Y. Wisner, 1873	51	4	0.48	0.45
12	North Base, Sandy Creek	G. Y. Wisner, 1874	32	4	0.43	
19	Minnesota Junction	A. R. Flint, 1873	36	5	0.41	0.43
19	Ford River	G. Y. Wisner, 1874	34	4	0.49	
19	Willow Springs	A. R. Flint, 1874	33	4	0.39	

ERRORS OF DECLINATION.

§ **3.** From (1) we have

$$E_\delta^2 \doteq E_\varphi^2 - \frac{E_0^2}{n} \tag{4}$$

If the errors of declination in a catalogue are accidental, we may find the probable error E_δ for that catalogue by observing stars contained in the catalogue and finding, first, E_φ, as follows: Observe many different pairs of stars an equal number of nights n at one or several stations, taking care that no star enters more than one pair, and call the mean of the results from a single pair an individual result; these individual results depending on equal numbers of observations will have equal weights. Take the differences Δ' between each individual result at a station, and the mean of them for that station. Suppose there are m' individual results or pairs, and n' stations, and for the same reason as in (3) we shall have

$$E_\varphi = 0.6745 \sqrt{\frac{[\Delta'\Delta']}{m'-n'}} \tag{5}$$

as the probable error of the latitude resulting from n observations on one pair of stars. Substituting the value of E_φ thus found and of n in (4), we have the value of E_δ for the catalogue used, and substituting it and the value of E_0 already found in (2), we have the combining weight to be given to a latitude result from one pair of stars of the given catalogue, when observed n times. Combining at a station the results from each pair with the weights thus determined, we have the final result for the latitude of the station.

It will be noticed that all the Δ' in (5) are independent of each other, not being connected either by observation or declination error. This would not be the case if any star had been used at two stations or in two or more pairs at the same station; and then the ordinary method of least squares could not have been applied. If pairs had been observed an unequal number of times, the individual results would have had different weights which could not then be assigned, as E_δ was as yet not known. On applying the above method to 132 stars whose declinations are given in "Mean Declinations of 981 stars, 1875," there is found $E_\delta = \pm 0''.53$. This value of E_δ gives $0''.75$ as the probable error of a declination in that catalogue. Another determination, in 1875, of E_δ from latitude-work at stations Ford River, Minnesota Junction, and Willow Springs gave $E_\delta = 0''.60$.

Taking for No. 12 the latest determination of E_0, namely, $\pm 0''.45$, and for E_δ, $\pm 0''.60$, there results for the probable error E_φ in the latitude derived from observing a single pair of stars once,

$$E_\varphi = \sqrt{(0''.45)^2 + (0''.60)^2} = \pm 0''.75$$

The determination of the values of one revolution at different parts of the micrometer-screw, and of its periodic inequality, is so easily made by observing transits near elongation of a close polar star across the horizontal, movable, micrometer-wire set in advance at every quarter-turn of the micrometer-head, that it should never be omitted when the highest precision is desired. The comparison of the mean value of the first quarter-turn derived from many turns of the screw, with similar values for the second, third, and fourth quarters, will indicate any large periodic inequality, while the comparison of the mean values of one turn derived from the first ten turns, with a similar value for the second, third, &c., ten turns, will show the irregularity of the screw if it is serious. As any errors in change of inclination of telescope are measured with the level, the values of one division of the level should be determined for different parts of the level-tube, with either a level-trier or the micrometer. If these differ much the separate values, and not their means, should be used.

To get the best declinations practicable, Prof. T. H. Safford was authorized in 1872 to prepare a list, already referred to, whose title is "Mean Declinations of 981 Stars for January 1, 1875," which has furnished the star places used in the computation of latitudes given in this chapter, unless otherwise specially stated.

In giving the results of the latitude work connected with the triangulation of Lake Superior, to avoid occupying too much space, the separate results will not be given, except for South Base, Keweenaw Point. In Chapter XXVIII all the latitudes observed near primary triangulation stations are collected to show the effect of local attractions.

LATITUDE OF ST. IGNACE.

§ **4.** Station St. Ignace, on the highest point of the island of that name, is 1263 feet above the level of Lake Superior. Its rocks are igneous, large masses of basalt being found in the immediate vicinity. The island is 15 miles long from east to west, seven miles wide, and the station is near the longitudinal axis of the island, at about 5 miles from its eastern end. To the south of the island Lake Superior deepens to 600 feet at a distance of fifty miles; on the north a shallow strait, from 6 to 10 miles wide, separates it from the main land, which rises as it recedes from the strait, in 3 or 4 miles, to hills 1500 feet in height. For further details Plate VI may be referred to, which gives outlines and contours that are, however, only roughly sketched, and shows the positions of the six points at which latitude was observed.

From this description it will be seen that in the surface of the earth in the vicinity of the station there are considerable irregularities, and that the geological formation would indicate unequal densities. For both reasons the station is one at which deviation of the plumb-line might be anticipated. In order to form an idea of the amount of local deviation of the plumb-line, latitude determinations have been made at six points in the vicinity of the trigonometrical station St. Ignace. The resulting latitudes of the station St. Ignace are given in the following table, which also gives the rectangular coördinates of the points at which latitude determinations were made, station St. Ignace being taken as the origin:

Table giving observed latitudes in vicinity of station St. Ignace.

Post.	Observer.	Date.	Zenith telescope No.	No. of pairs.	No. of results.	Latitude of post.	P. E.	Coördinates of post. Latitude.	Coördinates of post. Departure.	Resulting latitude of St. Ignace station.
							″	*Metres.*	*Metres.*	
St. Ignace...	....	......	....	......	......	....	......	000.00	000.00	
No. 1.....	O. B. Wheeler...	1866	12	17	41	48 47 28.55	±0.14	S. 16.89	W. 28.71	48 47 29.10
No. 2.....	G. Y. Wisner....	1867	12	21	56	48 47 25.03	±0.25	S. 93.76	W. 78.97	48 47 28.07
No. 3.....	....do	1867	12	16	33	48 47 27.67	±0.18	S. 16.98	W. 28.56	48 47 28.22
No. 4.....	O. B. Wheeler...	1866	12	16	29	48 46 15.60	±0.11	S. 2217.09	E. 3374.28	48 47 27.35
No. 4.....	G. A. Marr......	1871	12	5	5	48 46 16.65	±0.31	S. 2217.09	E. 3374.28	48 47 28.40
No. 5.....	....do	1871	12	25	40	48 44 52.00	±0.15	S. 4687.66	W. 1010.10	48 47 23.76
No. 6.....	....do	1871	{12 / P. & M. 1}	} 30	73	48 51 19.00	±0.17	N. 7104.03	E. 11.86	48 47 29.00

One revolution of micrometer-screw of zenith-telescope No. 12 was 63″.07 at 5th turn, and 63″.87 at 45th turn, the middle notch of comb-scale being taken as the 25th turn, and the value of a revolution was assumed to increase uniformly from the 5th turn to the 45th. One division of level equals 1″.00. One revolution of micrometer of Pistor & Martins No. 1 was taken as 85″.226 and constant, and one division of level equals 2″.29.

It will be seen that latitude posts Nos. 1, 2, and 3 were in the immediate vicinity of station St. Ignace, the most distant one being within 130 feet. Giving weights derived from their probable errors to the three results we have for their weighted mean

48° 47′ 28″.65

which is adopted as the observed latitude of station St. Ignace.

Post No. 5, nearly a mile south-southwest from St. Ignace, and about 1200 feet below it, being near the level of the lake, gives for the latitude of station St. Ignace a value 4″.89 less than that adopted above. Latitude post No. 6, 4½ miles north of station St. Ignace, gives for the latitude of station St. Ignace a value 0″.35 greater than that adopted above. Post No. 5 is one at which a deflection of the plumb-line to the north would be expected, since immediately to its north the island St. Ignace rises 1200 feet above it, and at a distance of 15 miles the high mainland rises; while to its south the depth of the lake becomes 500 feet within 10 miles, and for 60 miles averages 600 or 700 feet.

LATITUDE OF SOUTH BASE, KEWEENAW POINT.

§ **5.** This station, near the head of Keweenaw Bay, has within a radius of 3 or 4 miles only moderate elevations, not exceeding two or three hundred feet, and as small depressions below the water surface in the lake. But a radius of 25 miles would include much of the central ridge of Keweenaw Point, which rises to a height of about 1000 feet and contains eruptive rocks; it would include a part of the Huron Mountains to the east, rising to about the same height, the high ground to the south of Keweenaw Bay, and a portion of the lake, out to the depth of 400 feet.

Aside, then, from the eruptive character of a part of this region, which would give varying densities, the surface is itself quite irregular, so that a sensible deviation of the vertical from the normal to the mean ellipsoid is not improbable.

The following table gives the individual results for the latitude of South Base, Keweenaw Point. The star numbers are those of the British Association Catalogue unless otherwise indicated. In determining weights the values $E_\delta = \pm 0''.56$, and $E_0 = \pm 0''.48$, have been used. The instrument used was Würdemann zenith-telescope No. 12. The value of one revolution of micrometer was taken to vary uniformly from 63″.07 at the fifth turn to 63″.87 at the 45th turn, the middle notch of comb-scale being the 25th turn. One division of level equals 1″.00. The instrument was mounted on a post, which was 25.3 feet distant, bearing north 89° 28′ east from station South Base.

Latitude of South Base, Keweenaw Point.

[Instrument, Würdemann zenith-telescope No. 12. Observer, G. Y. Wisner.]

Pairs of stars observed.		Resulting latitude. July 23, 1873.	July 27, 1873.	July 29, 1873.	August 3, 1873.	Means.	Weights.
		° ′ 46 52 ″	″	″	″	″	
5568	5688	....	20.82	20.79	....	20.81	1.55
5568	5706	....	21.00	22.01	....	21.51	1.55
5873	7775	....	24.17	21.77	22.28	22.74	2.56
β Draco.	5944	....	23.66	22.59	21.79	22.68	2.56
6036	5990	....	....	21.89	22.66	22.27	2.33
6056	Gr. 2994	....	....	21.98	22.19	22.09	2.33
6129	6109	....	22.89	23.19	23.33	23.14	2.28
R. C. 3820	6109	....	23.47	....	....	23.47	1.23
6246	6203	24.75	22.03	22.34	22.07	22.79	2.69
6258	Gr. 2563	....	23.74	23.46	23.20	23.47	2.56
6368	α Lyrae	....	22.07	22.55	22.68	22.43	2.56
6419	6404	....	22.60	23.08	23.38	23.02	2.56
6452	Gr. 2693	....	21.14	22.93	22.40	22.16	2.56
6522	Gr. 2770	....	21.14	21.97	21.08	21.40	2.56
6626	Gr. 2844	....	20.93	22.63	22.23	21.93	2.56
XIX 193	6711	....	21.47	21.21	20.64	21.11	2.56
6741	6731	....	23.59	23.27	23.75	23.54	2.56
6852	6851	....	21.63	21.81	21.47	21.64	2.56
6867	6915	....	23.08	....	23.65	23.36	2.33
6959	6963	....	....	22.67	22.46	22.57	2.33
7035	7022	....	21.35	22.78	22.55	22.23	1.82
7035	6986	....	21.24	....	21.81	21.52	1.36
7035	7048	....	....	....	21.42	21.42	0.92
7182	7241	....	....	21.42	22.09	21.76	2.33
7243	7253	....	....	22.94	23.83	23.38	2.33
7278	XX 401	....	....	23.95	22.55	23.25	2.33
7294	7290	....	....	22.14	20.94	21.54	2.33
7294	7241	22.98	....	....	....	22.98	1.84
7377	7399	21.74	....	23.52	....	22.63	2.33
7411	7503	22.42	....	23.06	22.08	22.52	2.56
7431	7501	....	....	22.94	22.77	22.86	2.33
7560	7544	....	....	20.46	23.89	22.17	2.33
7589	7593	....	....	21.72	....	21.72	1.84
Gr. 3601	7614	....	....	....	20.36	20.36	1.84
3680	7695	....	....	....	21.22	21.22	1.23
7727	7695	....	....	....	21.64	21.64	1.23
7754	7777	22.30	....	22.60	22.11	22.34	2.56
7824	7803	....	....	23.33	....	23.33	1.84
7855	7913	22.51	....	....	....	22.51	1.84
7882	7894	21.78	....	....	....	21.78	1.84
7855	Gr. 3843	....	....	20.92	....	20.92	1.84
Gr. 3901	Gr. 3867	....	....	22.80	....	22.80	1.84
Gr. 3913	7948	....	....	22.72	....	22.72	1.84
8036	7983	22.59	....	21.07	....	21.83	2.33
7995	Gr. 3947	....	....	22.36	....	22.36	1.23
8082	8056	....	....	22.79	....	22.79	1.23
8082	8058	....	....	22.43	....	22.43	1.23
8107	8118	....	....	21.91	....	21.91	1.23
8107	8128	....	....	21.82	....	21.82	1.23
8231	8223	....	....	22.84	....	22.84	1.84
Gr. 4125	8245	....	....	23.76	....	23.76	1.84

	° ′ ″ ″
Weighted mean of results....................................	46 52 22.350 ± 0.072
Reduction to station	— 0.002
Latitude of station....................................	46 52 22.348 ± 0.072

Hence we have for the latitude of South Base, Keweenaw Point,

46° 52′ 22″.35±0″.07

NOTE.—In a few cases a single star on one side of the zenith has been used in combination with two or three on the other side. The weights of the results, derived by (2) § 2, have in such cases been multiplied by the factor $\frac{2}{n+1}$, n being the number of stars on the one side of the zenith combined with one on the other. There are also a few cases in which a latitude result has been obtained by observing a single star, first as a N. star and immediately afterwards as a S. star, or in the reverse order. The weight assigned to such a result is the weight of an ordinary result (derived by (2) § 2) multiplied by $\frac{E_\delta^2+E_0^2}{2E_\delta^2+E_0^2}$

LATITUDE OF NORTH BASE, MINNESOTA POINT.

§ **6.** This station is at the west end of Lake Superior. To the west and the north the ground soon rises to heights of 500 feet. To the south the country is more level, while to the east the lake deepens to 400 feet within 30 miles.

Latitude was observed here with zenith-telescope No. 12, by Assistant G. Y. Wisner, in 1871, on three nights. Twenty-three pairs of stars were observed and 46 results for latitude obtained. The value of one revolution of micrometer-screw was taken as increasing uniformly from 63″.19 at 5th turn to 63″.81 at 45th, the middle notch of comb-scale being the 25th turn. One division of level was equal to 1″.00. The instrument was mounted on a post 389.2 feet distant and bearing north 63° 53′ 24″ east from station North Base. Star places were taken from "Declinations of 981 Stars, 1875," and weights were determined as previously explained, the value of E_0 being ±0″.48, and of E_δ, ±0″.53. The resulting latitude of the post was 46° 45′ 30″.01±0″.17, and applying the correction −1″.69, to reduce to the trigonometrical station, there results for latitude of North Base, Minnesota Point,

46° 45′ 28″.32±0″.17

LATITUDE OF VULCAN.

§ **7.** This station is on the high promontory named Keweenaw Point, and is at an elevation of 726 feet above Lake Superior. To the north the lake deepens to 600 feet within 10 miles, and to the south it deepens to 300 feet within 18 miles.

Observations for latitude were made at this station by Lieutenant James Mercur, Corps of Engineers, in August, 1867, the instrument used being Würdemann zenith-telescope No. 1, a small instrument, whose object-glass is 2⅜ inches in diameter and focal length 24 inches. One revolution of micrometer-screw was taken as 69″.20, and one division of level as 1″.08. Two observation-posts, differing 1″.541 in latitude, were occupied. Post No. 1, 48.5 feet distant, and bearing north 83° 16′ west from station Vulcan, was occupied on August 9, 14, and 15, 1867. Twenty-seven pairs of stars were observed, giving 41 results for latitude. Star places were taken from Safford's Catalogue of 2018 stars, 1875.0. In weighting results, E_0 was taken as ±1″.35 and E_δ as ±0″.53.

The weighted mean latitude of Post No. 1 was		47° 26′ 44″.871±0″.27
Reduction to trigonometrical station Vulcan	−	0″.056
Latitude of Vulcan from Post No. 1		47° 26′ 44″.815±0″.27

At Post No. 2, 156.2 feet due south from Post No. 1, 27 pairs of stars were observed on July 24, 25, 26, August 2 and 4, 1867, giving 47 results for latitude.

The weighted mean latitude for Post No. 2 was		47° 26′ 42″.771±0″.32
Reduction to trigonometrical station Vulcan	+	1″.485
Latitude of Vulcan from Post No. 2		47° 26′ 44″.256±0″.32

Weighting the results according to their probable errors there results finally for latitude of station Vulcan,

47° 26′ 44″.582±0″.20

LATITUDE OF HURON MOUNTAINS.

§ **8.** This station is on the south shore of Lake Superior, about 35 miles northwest from Marquette. The shore rises rapidly to hills of a thousand feet in height, while to the north the lake reaches a depth of 300 feet within 15 miles.

Observations for latitude were made at a point 97.5 feet distant and bearing south 36° 30′ west from the station, which is 932 feet above the lake, by Mr. S. W. Robinson, in September, 1866. The instrument used was the Pistor & Martins broken transit previously described, one revolution of micrometer being 85″.22 and one division of level being 2″.29. Observations were made on 32 pairs of stars on September 20, 21, 24, and 25, 1866, but out of the 54 results for latitude all but 7 were on the last two nights. The star places were taken from Safford's Catalogue of 2018 Stars for 1875.0. The results from each pair were weighted by using the values $E_0 = \pm 1''.05$ and $E_\delta = \pm 0''.57$, although the latter is probably too large, giving too small weights.

The resulting latitude of the observing post is 46° 52′ 52″.30±0″.13. Applying correction to trigonometrical point, +0″.77, there results for latitude of trigonometrical station Huron Mountains,

46° 52′ 53″.07±0″.13

LATITUDE OF FORD RIVER.

§ **9.** Ford River station is situated on the west shore of Green Bay, about 4 miles south of Escanaba. In its immediate vicinity there is a thin surface-soil underlaid by limestone rock, whose strata are nearly horizontal. To the southwest, west, and north, within fifty miles, there are no marked surface irregularities. On the east and south, however, lies Green Bay, 15 to 20 miles wide and 40 to 120 feet deep, and within fifty miles the broken peninsula lying between Green Bay and Lake Michigan rises at intervals to a height of 200 feet and Lake Michigan attains a depth of 400 to 500 feet.

Latitude was observed by Assistant Engineer G. Y. Wisner, with Würdemann zenith-telescope No. 19, the instrument being mounted on a heavy wooden post 29.5 feet distant, bearing south 76° 00′ west from the station. One revolution of micrometer was taken as 62″.241, and one division of level as 0″.86. Observations were made on four different nights, in July, 1874, as shown by the table following. Thirty-four pairs of stars were observed, giving in all 115 individual results.

Latitude of Ford River.

[Instrument, Würdemann zenith-telescope No. 19. Observer, G. Y. Wisner.]

Pairs of stars observed.		Resulting latitude. July 3, 1874.	July 4, 1874.	July 7, 1874.	July 8, 1874.	Means.	Weights.
		° ′ 45 41					
		″	″	″	″	″	
XV 39	5076	06. 92	06. 64	06. 96	....	06. 84	3. 08
5113	5157	....	03. 21	02. 82	....	03. 02	2. 74
5168	5181	05. 58	05. 49	07. 65	05. 74	06. 11	3. 28
5210	5295	06. 46	06. 86	05. 48	05. 28	06. 02	3. 28
5338	5388	02. 49	04. 50	04. 29	05. 32	04. 15	3. 28
5463	5497	04. 99	05. 27	04. 55	04. 80	04. 90	3. 28
5523	5535	07. 19	07. 20	07. 38	07. 35	07. 28	3. 28
5546	5574	04. 13	....	01. 70	04. 71	03. 51	3. 08
5596	5644	04. 51	05. 20	04. 79	05. 58	05. 02	3. 28
5790	5795	05. 02	05. 49	06. 06	05. 93	05. 62	3. 28
5929	5937	05. 72	04. 69	04. 27	03. 98	04. 67	3. 28
5997	6036	04. 74	....	04. 56	05. 21	04. 84	3. 08
6068	6091	05. 88	05. 23	05. 45	05. 18	05. 44	3. 28
2494	6227	03. 31	02. 76	05. 61	04. 90	04. 15	3. 28
6109	6264	07. 06	05. 59	....	06. 86	06. 50	3. 08

Latitude of Ford River—Continued.

Pairs of stars observed.		Resulting latitude.				Means.	Weights.
		July 3, 1874.	July 4, 1874.	July 7, 1874.	July 8, 1874.		
		"	"	"	"	"	
6203	6252	03.53	03.98	03.63	05.03	04.04	3.28
6268	6335	05.09	06.22	05.06	05.25	05.41	3.28
6350	6355	07.16	06.72	05.66	05.14	06.14	3.28
6372	6391	....	....	....	05.50	05.50	2.07
6522	6556	04.65	05.47	04.68	04.97	04.94	3.28
6599	6623	05.21	04.65	05.13	05.24	05.06	3.28
6651	XIX 193	....	04.96	05.60	05.84	05.47	3.08
6721	6728	06.67	....	06.20	06.59	06.49	3.08
6741	6745	05.99	05.68	06.38	05.34	05.85	3.28
6763	6769	03.12	05.63	....	06.02	04.92	3.08
6806	6824	06.43	05.05	05.31	04.58	05.34	3.28
6851	XIX 370	05.80	05.92	04.66	05.98	05.59	3.28
6915	6924	....	03.40	03.56	04.36	03.77	3.08
6965	3110	....	06.40	05.88	06.12	06.13	3.08
6997	7035	....	04.42	03.72	04.76	04.30	3.08
7055	7084	....	04.50	05.51	05.15	05.05	3.08
7120	7158	....	05.16	05.94	....	05.55	2.74
7171	7198	....	06.08	06.14	05.84	06.02	3.08
7254	7268	....	05.65	04.71	....	05.18	2.74

		°	′	″	″
Weighted mean of results		45	41	05.265 ± 0.110	
Reduction to station	+			0.071	
Latitude of station		45	41	05.336 ± 0.110	

These results combined in the manner explained in §§ 2, 3, the values $E_0 = \pm 0''.487$ and $E_\delta = \pm 0''.496$ being derived from the observations, give 45° 41′ 05″.265 ± 0″.110 for the latitude of the observing-post. Adding to this value +0″.071, to reduce to the triangulation station, there results for the latitude of Ford River station,

45° 41′ 05″.34 ± 0″.11

LATITUDE AT FORT HOWARD.

§ **10.** Station Fort Howard is situated near the southern extremity of Green Bay, in Wisconsin. There is a limestone ledge about 4 miles southeast of the observing-post, which has a general direction of northeast by north, and an elevation of 50 to 200 feet above the bay. Between the astronomical post and this ledge, and for 25 miles or more to the west and north, the country is flat, rising nowhere higher than perhaps 25 feet above the bay. The ground at the post is about 20 feet above the bay.

Observations for latitude were made at a point nearly two miles northeast of the station, by James Carr, assistant engineer, in October, 1862, on seven nights, with zenith-telescope Würdemann No. 1, having a focal length of 24 inches and an aperture of 2⅜ inches. One revolution of the micrometer-screw was equal to 69″.14, and one division of the level to 1″.08. The instrument was mounted on a wooden post 21 inches in diameter, set 4 feet deep in sandy soil, which was made firm by wetting the sand as it was filled in round the post. The number of pairs of stars observed was 30, and 77 individual results for latitude were obtained.

In reducing the observations, declinations from Safford's Catalogue of 981 Stars (Washington, Government Printing Office, 1873) were used. Weights for the results from each pair of stars were assigned according to the method described in §§ 2 and 3, and note added to § 5, with $E_0 = \pm 1''.92$ and $E_\delta = \pm 0''.53$. The value of E_0 was derived from the observations at this station.

The following table gives the pairs of stars observed, the individual results, the mean result for each pair, and the weights assigned to each pair. The resulting latitude of the observing-post is 44° 31′ 18″.52 ± 0″.10.

The position of the observing-post, referred to station and other marks, is as follows: 8465.5 feet distant, bearing north 54° 44′ 43″ east from station Fort Howard; north, 2798.7 feet, west, 775.4 feet from court-house, Green Bay (center of cupola); 272.8 feet due north of a stone meridian-post; and 1612.6 feet due south of another stone meridian-post. Applying a correction of −48″.24 to the above result to reduce to the trigonometric station, there results for the latitude of station Fort Howard,

44° 30′ 30″.28±0″.10

Latitude at Fort Howard.

[Observer, James Carr. Instrument, Würdemann zenith-telescope No. 1.]

Pairs of stars with British Association Catalogue numbers.		Resulting latitude.							Means.	Weights.
		October 11, 1862.	October 14, 1862.	October 16, 1862.	October 17, 1862.	October 19, 1862.	October 21, 1862.	October 22, 1862.		
		° ′ 44 31 ″	″	″	″	″	″	″	″	
6865	6813		18.23						18.23	0.25
6928	6937		17.06						17.06	0.25
7076	7114	9.46	17.90				21.65		16.34	0.44
7085	7114	10.78	18.19				20.35		16.44	0.44
7166	7204	17.62					18.35		17.99	0.47
7233	7241	17.85	13.71		22.43		18.82		18.20	0.83
7254	7273	17.45							17.45	0.25
7278	7313				18.14	17.09	18.98		18.07	0.44
7294	7313	19.24			18.70	16.96	18.27		18.29	0.55
7365	7373	18.60			18.93	19.08			18.87	0.66
7477	7501				16.65				16.65	0.25
7560	7524	15.72			20.73	14.90	18.12		17.37	0.55
7560	7602	21.05			20.43	18.20			19.89	0.44
7746	7765	19.94		18.58	20.15	20.99			19.92	0.83
7787	7777	19.02			16.25	17.53			17.60	0.66
7824	7901	18.16				12.25			15.21	0.31
7888	7901	18.00		20.39	19.17	13.75		18.06	17.87	0.65
7847	XXII 113			17.51				14.00	15.75	0.31
7848	XXII 113			17.13	18.43			14.34	16.63	0.44
7999	7917	21.85							21.85	0.12
7999	7932	22.49							22.49	0.12
7999	7962	22.55			16.76			16.63	18.65	0.33
Ll 44750	7932				18.43			18.42	18.43	0.31
8118	8125				16.08	18.18		19.11	17.79	0.66
8224	8237	18.26			24.76	15.72		16.21	18.74	0.83
92	120					22.37		17.11	19.74	0.47
169	155				18.01	18.37		15.21	17.20	0.66
314	334				20.61			25.81	23.21	0.47
441	425				20.60			24.72	22.66	0.47
558	516				22.69			22.33	22.51	0.47

		° ′ ″ ″
Weighted mean		44 31 18.523 ± 0.103
Reduction to trigonometrical station	−	48.24
Latitude of Fort Howard		44 30 30.283 ± 0.103

LATITUDE OF MINNESOTA JUNCTION.

§ **11.** Minnesota Junction station is situated near the center of Dodge County, Wisconsin, on a mound rising about 40 feet above the general surface of the land, which, within a radius of 5 miles, is slightly rolling prairie. On the south, west, and north this prairie extends upwards of 20 miles. On the east the surface is more irregular. At a distance of 10 miles the limestone ledge extending approximately northeast by north one-fourth north, across the State of Wisconsin, rises to a height 120 feet greater than that at the station, while between the latter and the ledge the basin of Horicon Lake forms a depression of 250 feet. Between the ledge and Lake Michigan the surface is broken by numerous hills of drift origin. The mound at the station is 416 feet above the surface of Lake Michigan, which attains a depth of 350 feet within 60 miles of the station, the shore of the lake being about 45 miles from the station.

The latitude was observed by Assistant Engineer A. R. Flint, with Würdemann zenith-telescope No. 19, on five different nights, during August and September, 1873, as shown in the table following. The instrument was mounted on a heavy oak post. There were obtained in all 156 individual results, 36 different pairs of stars being observed. One revolution of the micrometer was taken as 62″.251, and one division of level as 0″.86. Star places were taken from "Mean Declinations of 981 Stars for January 1, 1875."

Latitude of Minnesota Junction.

[Instrument, Würdemann zenith-telescope No. 19. Observer, A. R. Flint.]

Pairs of stars observed.		Resulting latitude. August 26, 1873.	August 27, 1873.	August 28, 1873.	August 29, 1873.	September 1, 1873.	Means.	Weights.
		° ′ 43 28 ″	″	″	″	″	″	
6095	6162	....	33.43	33.03	33.38	33.77	33.40	2.74
Gr. 2597	6364	....	....	33.89	32.99	32.95	33.28	2.64
6392	6421	....	32.54	31.72	32.81	31.71	32.20	2.74
Gr. 2701	6475	33.92	32.86	32.19	31.92	....	32.72	2.74
6495	6516	31.08	30.60	30.20	31.37	30.37	30.72	2.81
6599	6626	29.60	29.97	28.57	29.75	29.37	29.45	2.81
6687	6698	30.48	31.59	32.42	....	31.24	31.43	2.74
6720	6728	31.28	31.39	30.81	30.70	30.84	31.00	2.81
6754	6769	32.19	32.49	29.15	32.17	31.96	31.59	2.81
6817	Gr. 2957	....	30.35	30.10	30.19	31.03	30.42	2.74
6851	6856	33.52	32.43	32.50	32.16	32.57	32.64	2.81
6881	6915	31.37	30.67	....	30.65	31.98	31.17	2.74
6962	6996	31.62	32.79	32.53	32.30	....	32.31	2.74
7061	7076	....	33.30	33.11	32.08	32.20	32.67	2.74
7112	7158	....	32.14	33.15	31.72	32.40	32.35	2.74
7260	7268	30.67	30.40	30.64	30.51	30.72	30.59	2.81
7274	7320	....	33.05	32.89	32.94	32.44	32.83	2.74
7333	7402	....	32.11	31.76	31.20	31.85	31.73	2.74
7548	7566	....	31.04	31.89	32.30	31.57	31.70	2.74
7598	7602	....	31.15	31.60	32.02	31.08	31.46	2.74
7679	7705	....	33.31	33.66	33.56	33.00	33.38	2.74
Gr. 3717	7770	29.74	30.44	31.14	31.33	30.45	30.62	2.81
7850	7894	31.28	31.40	31.40	31.53	31.80	31.48	2.81
7932	7950	....	33.13	32.63	32.94	33.08	32.94	2.74
7972	7983	....	32.89	32.27	32.42	30.87	32.11	2.74
8023	8056	....	30.80	30.30	29.98	30.32	30.35	2.74
8076	8110	....	32.10	31.28	31.48	32.75	31.90	2.74
Gr. 4052	Gr. 4074	32.70	33.28	32.07	....	33.59	32.91	2.74
8223	8237	33.29	32.33	30.68	33.29	31.05	32.11	2.81
8345	16	31.41	30.78	31.40	30.49	30.74	30.96	2.81
60	78	32.90	....	....	32.92	....	32.91	2.44
189	227	33.13	32.46	33.49	33.45	33.32	33.17	2.81
283	330	30.48	30.29	31.42	31.13	30.45	30.75	2.81
352	377	32.70	31.91	31.23	32.49	32.38	32.14	2.81
441	480	31.91	32.31	31.89	30.95	32.01	31.83	2.81
560	579	31.17	30.24	29.79	31.37	30.67	30.65	2.81

Weighted mean of results........................ 43° 28′ 31″.82 ± 0″.11

From the observations, the value ±0″.425 was found for E_0 (see §§ 2, 3) and $E_\delta = \pm 0''.565$ was derived from the observations at both Ford River and Minnesota Junction stations. The combining weights of the individual results being deduced from these values of E_0 and E_δ, there results for the latitude of Minnesota Junction station, the instrument being centered vertically over the triangulation station,

43° 28′ 31″.82±0″.11

LATITUDE OF WILLOW SPRINGS.

§ **12.** This station is situated in Cook County, Illinois, about 1 mile southeast of Mount Forest, a railway station on the Chicago and Alton Railway, and is about 16 miles southwest of Chicago. The ground at the station is about 150 feet above the surface of Lake Michigan, and is considerably undulating in the vicinity. To the northward from station Willow Springs the ground declines until it reaches a river and canal about 2 miles distant, where it is about 150 feet below the station. Continuing northward, the ground rises 30 feet or so above the river, and remains at about this elevation for many miles. To the southward from the station the land for a mile or so has about the same elevation as at the station, then there is a depression of from 50 to 100 feet, after which the ground gradually rises until at a distance of 10 miles it reaches the level of the station, and continues at about this level farther south.

Latitude was observed here by Assistant Engineer A. R. Flint in September and October, 1874, on five nights, with the Würdemann zenith-telescope No. 19, having a focal length of 32 inches and a 3-inch object-glass. One revolution of the micrometer-screw was equal to 62″.251, and one division of the level to 0″.861. The instrument rested on a solid oak stump, about 3 feet in diameter, around which a platform was built. It was situated 38.4 feet distant, bearing north 22° 50′ west from the station. The number of pairs of stars observed was 33, and 132 individual results for latitude were obtained.

For reducing the work, declinations were taken from Safford's Catalogue of 981 Stars. Weights were assigned to the results for each pair of stars according to the adopted method (§§ 2, 3) with $E_0 = \pm 0''.426$ and $E_\delta = \pm 0''.57$.

The following table gives in the successive columns the British Association Catalogue numbers of the stars observed, those forming a pair being placed on the same horizontal line; the individual results for each pair on the separate nights; the mean result for each pair; and the weights assigned to those means.

The resulting latitude of the observing-post is 41° 43′ 38″.979 ± 0″.117, and applying a correction of −0″.350 to reduce to the trigonometrical station, there results for the latitude of station Willow Springs,

41° 43′ 38″.629 ± 0″.117

Latitude of Willow Springs.

[Observer, A. R. Flint. Instrument, Würdemann zenith-telescope No. 19.]

Pairs of stars observed.		September 28, 1874.	September 29, 1874.	September 30, 1874.	October 1, 1874.	October 3, 1874.	Means.	Weights.
		° ′ 41 43						
		″	″	″	″	″	″	
6470	6426	....	39.78	40.48	40.15	....	40.10	1.30
6530	6571	....	....	39.63	39.97	....	39.80	1.22
6656	6624	....	39.27	....	40.15	....	39.71	1.22
6717	6698	....	40.32	40.33	40.20	....	40.28	1.30
6764	6784	....	37.83	39.08	38.17	....	38.36	1.30
6865	6800	....	37.69	38.90	38.06	....	38.22	1.30
6962	6937	39.17	....	40.30	40.58	40.14	40.05	1.37
6983	6997	....	38.55	36.95	36.94	36.31	37.19	1.37
7083	7061	....	....	38.54	39.75	39.76	39.35	1.30
7091	7103	....	....	37.81	37.41	36.70	37.31	1.30
7120	7164	38.69	39.35	39.08	39.46	40.25	39.37	1.39
7182	7204	....	38.24	38.16	37.59	39.38	38.34	1.37
7241	7260	38.09	37.89	....	37.24	37.95	37.79	1.37
7290	7297	37.16	36.91	36.32	37.28	37.34	37.00	1.39
7345	7373	39.64	39.10	39.62	40.20	40.10	39.73	1.39
7411	7399	38.83	39.53	39.58	39.81	39.62	39.47	1.39

Latitude of Willow Springs—Continued.

Pairs of stars observed.		September 28, 1874.	September 29, 1874.	September 30, 1874.	October 1, 1874.	October 3, 1874.	Means.	Weights.
		″	″	″	″	″	″	
7488	7465	37.73	38.37	38.65	38.56	....	38.33	1.37
7501	7505	37.85	....	39.75	38.87	37.75	38.56	1.37
7544	7550	38.92	38.30	39.30	38.49	38.70	38.74	1.39
7593	7565	38.65	38.28	39.69	38.28	....	38.71	1.37
7681	7614	40.31	40.64	40.04	40.12	40.62	40.35	1.39
7800	7770	39.13	39.63	40.11	39.92	40.69	39.90	1.39
7815	7843	37.80	39.76	38.12	38.60	39.00	38.66	1.39
7894	7858	38.33	39.87	38.05	38.47	38.88	38.72	1.39
7948	7915	39.92	39.78	39.67	39.28	39.13	39.56	1.39
7972	7962	37.33	38.51	37.85	37.98	....	37.92	1.37
8028	7994	40.79	39.10	39.15	39.17	38.90	39.42	1.39
8107	8141	37.12	37.87	37.58	37.83	37.20	37.52	1.39
8171	Gr. 4052	40.28	40.50	40.07	....	....	40.28	1.30
8223	8212	39.69	40.47	39.95	40.61	40.00	40.14	1.39
Gr. 4172	8345	38.97	38.58	37.31	38.32	....	38.30	1.37
16	52	38.15	39.47	39.45	39.69	....	39.19	1.37
165	158	....	40.88	39.70	....	....	40.29	1.22

		° ′ ″ ″
Weighted mean		41 43 38.979±0.117
Reduction to station	−	0.350
Latitude of Willow Springs		41 43 38.629±0.117

LATITUDE OF FAIRMOUNT.

§ **13.** This station is situated in Vermillion County, Illinois, about 2½ miles southwest of the village of Fairmount on the Wabash Railway. The ground in the vicinity of the station is some 20 feet higher than that of the surrounding country, sloping very gradually away from the station for about 2 miles when it reaches the general level. At a distance of 20 miles north of Fairmount the general level of the country is upwards of 50 feet above the land at Fairmount, mounds on the prairies rising about 25 feet above the general level. The land to the south of the station is probably 50 feet lower than at Fairmount, at a distance of 20 miles. These comparative elevations are based on zenith distances observed at Fairmount.

Latitude was observed here by Assistant Engineer G. Y. Wisner, in May, 1880, on four nights, with the Würdemann zenith-telescope No. 19, having a focal length of 32 inches and a 3-inch object-glass. One revolution of the micrometer-screw was equal to 62″.224, and one division of the level to 1″.292. The observations for latitude were made on a solid oak post, 19 inches in diameter, set 5 feet in the ground, with the earth well tamped around it. It was situated 26.4 feet distant, bearing north 82° 26′ east from the station. The number of pairs of stars observed was 33, and 104 individual results for latitude were obtained.

In reducing the work, Safford's Catalogue of 2018 Stars was used for declinations. Weights for the results from each pair of stars were assigned according to the adopted method (§§ 2, 3) with $E_0=\pm 0''.42$ and $E_\delta=\pm 0''.57$.

The following table gives in the first column the British Association Catalogue number of each star observed, the two which form a pair being placed on the same horizontal line; in the four succeeding columns the individual results on separate nights for each pair; in the next column the mean result for each pair; and in the last column the weights assigned to the mean results in the preceding column.

The resulting latitude of the observing-post is 40° 01′ 36″.737±0″.108, and applying a correction of −0″.033 to reduce to the trigonometrical station, there results for the latitude of station Fairmount,

40° 01′ 36″.704±0″.108

Latitude of Fairmount.

[Observer, G. Y. Wisner. Instrument, Würdemann zenith-telescope No. 19.]

Pairs of stars observed.		May 5, 1880.	May 6, 1880.	May 7, 1880.	May 10, 1880.	Means.	Weights.
		° ′ 40 01					
		″	″	″	″	″	
13	23	37.05	36.48	36.07	37.25	36.71	1.35
33	39	34.81	35.71	35.90	36.47	35.72	1.35
48	57	36.67	36.28	36.58	38.84	37.09	1.35
61	82	35.94	39.86	36.87	36.82	37.37	1.35
85	89	36.33	36.64	....	36.69	36.55	1.30
118	127	....	....	35.78	34.98	35.38	1.21
134	144	35.08	36.07	35.61	34.02	35.17	1.35
136	154	....	37.65	35.79	38.11	37.18	1.30
158	170	36.38	35.78	33.49	35.47	35.28	1.35
197	199	40.09	36.01	38.15	37.45	37.92	1.35
234	249	36.30	36.01	35.91	36.11	36.08	1.35
254	262	36.62	35.64	36.08	36.04	36.10	1.35
271	280	37.15	....	....	37.12	37.14	1.21
293	299	37.60	37.73	38.13	37.97	37.71	1.35
304	308	36.04	36.46	35.93	36.31	36.18	1.35
319	327	35.51	37.61	36.61	34.56	36.07	1.35
329	340	36.88	38.45	....	36.85	37.39	1.30
344	345	36.87	39.16	....	....	38.02	1.21
360	362	34.82	35.56	....	35.79	35.39	1.30
371	380	37.60	39.17	....	37.81	38.19	1.30
394	410	34.47	35.50	....	36.33	35.43	1.30
422	429	35.86	37.55	....	36.52	36.64	1.30
431	443	37.20	37.92	....	36.90	37.34	1.30
464	475	....	37.60	....	37.65	37.62	1.21
471	482	37.65	38.23	....	37.79	37.89	1.30
494	503	37.84	38.29	....	37.00	37.71	1.30
518	521	36.06	37.01	....	36.15	36.41	1.30
527	537	38.34	39.31	....	37.14	38.26	1.30
549	562	....	36.93	....	37.08	37.00	1.21
570	585	36.21	37.32	....	36.69	36.74	1.30
604	620	36.09	37.33	....	35.98	36.47	1.30
631	632	....	35.79	....	35.47	35.63	1.21
635	636	....	36.25	....	37.07	36.66	1.21

	° ′ ″ ″
Weighted mean	40 01 36.737 ± 0.108
Correction to station	− 0.033
Latitude of Fairmount	40 01 36.704 ± 0.108

LATITUDE OF WEST BASE, OLNEY BASE-LINE.

§ **14.** This station, forming the western extremity of the Olney base-line, is situated in Jasper County, Illinois, about 9 miles north of the town of Olney. The land in the vicinity of the station, and for several miles to both the north and the south, is very nearly level, and for 20 miles in every direction it probably does not vary 100 feet in elevation from that of this station.

Latitude was observed here in May, 1880, by Assistant Engineer G. Y. Wisner on four nights with the Würdemann zenith-telescope No. 19, having a focal length of 32 inches and a 3-inch objective. A revolution of the micrometer-screw was equal to 62″.224, and one division of the level to 1″.292.

The observations were made on a post of white oak, 17 inches in diameter, and set $5\frac{1}{2}$ feet in the ground, with the earth well tamped around it. This post was situated 53.5 feet distant, bearing north 88° 36′ east from the station. The number of pairs of stars observed was 30, and 115 individual results for latitude were obtained. Declinations were taken from Safford's Catalogue of 2018 Stars, and weights were assigned to the results from the different pairs according to the adopted method (§§ 2, 3) with $E_0 = \pm 0''.42$ and $E_\delta = \pm 0''.57$.

The following table gives in the successive columns the British Association Catalogue numbers of the stars observed, with those forming a pair placed on the same horizontal line, the individual results for each pair on the separate nights, the mean results for each pair, and the weights assigned to those means.

The resulting latitude for the observing post is 38° 51′ 41″.216±0″.064, and applying a correction of +0″.013 to reduce to the trigonometrical station, there results for the latitude of station West Base

38° 51′ 41″.229±0″.064

Latitude of West Base, Olney.

[Observer, G. Y. Wisner. Instrument, Würdemann zenith-telescope No. 19.]

Pairs of stars observed.		May 13, 1880.	May 14, 1880.	May 15, 1880.	May 16, 1880.	Means.	Weights.
		° ′ 38 51					
		″	″	″	″	″	
10	19	40.42	39.77	41.88	40.44	40.628	1.35
33	29	41.58	40.56	41.89	40.13	41.040	1.35
40	50	40.54	39.95	41.18	42.00	40.918	1.35
67	66	41.64	40.39	41.44	41.53	41.250	1.35
92	97	42.14	40.26	41.84	42.02	41.565	1.35
125	114	41.10	41.63	40.83	40.05	40.902	1.35
135	132	41.39	40.89	41.98	41.69	41.489	1.35
145	153	40.96	40.86	40.51	41.46	40.948	1.35
183	170	41.38	40.06	40.50	....	40.647	1.30
208	217	42.82	41.32	41.66	41.76	41.890	1.35
227	221	....	....	42.24	41.70	41.970	1.21
252	257	41.73	41.85	41.86	41.68	41.780	1.35
267	275	40.38	39.82	40.07	40.55	40.205	1.35
286	297	40.60	40.85	41.68	41.02	41.038	1.35
312	302	40.73	40.97	40.41	39.80	40.478	1.35
345	334	41.04	41.42	41.50	42.37	41.582	1.35
356	366	41.27	40.90	40.50	41.07	40.935	1.35
384	382	41.94	41.16	41.46	41.98	41.635	1.35
405	410	41.11	41.24	41.20	40.62	41.042	1.35
429	433	40.92	40.93	40.38	40.66	40.722	1.35
458	447	41.98	41.61	41.76	40.56	41.478	1.35
459	466	....	42.67	42.41	41.42	42.170	1.30
474	478	42.44	39.80	42.07	41.04	41.338	1.35
516	490	41.74	41.64	41.53	40.80	41.428	1.35
519	521	....	41.07	41.27	41.30	41.213	1.30
526	524	42.11	42.58	43.07	41.75	42.378	1.35
529	541	41.04	40.73	41.16	40.38	40.828	1.35
554	565	41.93	41.76	40.73	41.62	41.510	1.35
585	591	40.60	39.87	40.99	40.43	40.472	1.35
597	604	41.14	41.56	40.78	40.87	41.088	1.35

		° ′ ″ ″
Weighted mean		38 51 41.216 ± 0.064
Reduction to station	+	0.013
Latitude of West Base, Olney		38 51 41.229 ± 0.064

LATITUDE OF PARKERSBURG.

§ **15.** This is the terminal station at the south end of the triangulation extending south from Lake Michigan. It is situated about 11 miles south of Olney, Ill., on a rise of ground about 80 feet above the general level. The surface of the country in this region is level or rolling, there being no hills with an elevation exceeding one or two hundred feet within a radius of perhaps 50 miles.

Latitude was observed here by Lieutenant P. M. Price, in August, 1879, on five nights, with zenith-telescope No. 19, made by Würdemann, having a focal length of 32 inches and an aperture of 3 inches. The value of one revolution of its micrometer-screw was 62″.224, and of one division of its level, 1″.292. The instrument was mounted on a heavy oak post 22 inches in diameter, sunk 3 feet in the ground, situated 113.8 feet distant, bearing north 87° 43′ west from the station. The number of pairs of stars observed was 38, and 126 individual results for latitude were obtained.

In reducing the observations, declinations from Safford's Catalogue of 2018 Stars (Washington, Government Printing Office, 1879) were used, except for two stars not in that catalogue, which were taken from the United States Coast-Survey Catalogue, in Appendix 7 of the Report for 1876. The constants for reducing from mean to apparent declinations were taken from the American Ephemeris for 1879. Weights for the results from each pair of stars were assigned according to the adopted method, previously described (§§ 2, 3), with $E_0 = \pm 0''.42$ and $E_\delta = \pm 0''.57$.

Following herewith is the usual table giving the pairs of stars observed, with the British Association Catalogue number, the individual results, &c.

The resulting latitude of the observing-post is 38° 34′ 53″.249±0″.09, and applying a correction of −0″.045 to reduce to the trigonometrical station, there results for the latitude of Parkersburg,

38° 34′ 53″.204±0″.09

Latitude of Parkersburg.

[Observer, Lieutenant P. M. Price. Instrument, Würdemann zenith-telescope No. 19.]

Pairs of stars observed.		August 9, 1879.	August 10, 1879.	August 11, 1879.	August 12, 1879.	August 13, 1879.	Means.	Weights.
		° ′ 38 34 ″	″	″	″	″	″	
5552	5619	....	....	....	53.69	....	53.69	1.00
5652	5706	53.47	....	53.66	52.62	....	53.25	1.30
5775	5842	53.15	....	....	53.27	53.07	53.16	1.30
5874	5886	53.89	....	53.98	52.71	53.06	53.41	1.36
5962	5990	53.65	....	53.24	....	52.52	53.14	1.30
6068	6082	53.84	....	53.29	52.77	52.95	53.21	1.36
6129	6150	52.92	51.91	53.12	54.18	54.11	53.25	1.39
6218	6235	49.71	53.32	54.13	52.45	....	52.40	1.36
6355	6365	53.49	55.45	54.04	55.25	54.52	54.55	1.39
6429	6475	52.72	49.63	51.29	54.90	53.87	52.48	1.39
6581	6599	....	54.43	52.85	52.95	52.94	53.29	1.36
6656	6698	52.90	52.77	52.60	53.51	52.72	52.90	1.39
6714	6721	52.50	56.94	53.03	55.46	54.78	54.54	1.39
6745	6777	52.79	53.85	53.47	53.37	52.79	53.25	1.39
6817	6875	53.64	53.89	53.38	53.94	54.46	53.86	1.39
6996	7006	53.57	53.47	48.92	55.25	....	52.80	1.36
7073	7101	53.29	53.19	54.01	....	....	53.50	1.30
7112	7131	....	54.32	....	53.26	....	53.79	1.21
7161	7164	52.65	53.37	....	53.68	55.95	53.91	1.36
7204	7241	57.75	52.45	....	52.80	52.47	53.87	1.36
7313	7336	52.69	51.88	....	....	53.16	52.58	1.30
7383	C. S. 1924	....	51.43	....	....	....	51.43	1.00
7465	7501	51.67	51.45	....	52.80	....	51.97	1.30
7602	7614	53.84	52.20	....	....	....	53.02	1.21
7705	7721	....	53.10	....	53.95	55.17	54.07	1.30
7705	7731	50.28	....	....	....	....	50.28	1.00
7731	Gr. 3717	....	52.14	....	51.62	53.07	52.28	1.30
7879	7901	54.08	....	....	....	....	54.08	1.00
7880	7901	....	52.32	....	52.10	53.45	52.62	1.30
8058	8141	53.38	....	....	55.31	54.03	54.24	1.30
8211	8245	54.11	....	....	54.17	53.92	54.07	1.30
C. S. 2176	58	54.50	53.01	....	53.17	53.90	53.64	1.36
166	197	51.81	52.49	....	52.21	52.17	52.17	1.36
259	297	53.00	55.09	....	....	53.63	53.91	1.30
330	345	52.06	53.40	....	53.21	53.49	53.04	1.36
465	480	....	51.92	....	54.05	55.70	53.89	1.30
501	516	....	....	....	....	54.02	54.02	1.00
544	575	....	53.49	....	53.81	52.28	53.19	1.30

		° ′ ″ ″
Weighted mean		38 34 53.249±0.09
Reduction to station	—	0.045
Latitude of Parkersburg		38 34 53.204±0.09

LATITUDE OF TOLEDO.

§ **16.** Observations for latitude of Toledo, Ohio, were made in 1868 by Assistant Engineer O. B. Wheeler at a point 185.2 feet west and 188.4 feet north of the stone which marks the intersection of Monroe and Ontario streets in that city, and 54.5 feet east and 44.6 feet south of the longitude stone post occupied in 1881. The land is quite flat in this vicinity, and there is no marked elevation of ground within a radius of 50 miles. In the vicinity of Toledo the soil is deep and underlaid by limestone rock. The instrument used was Pistor & Martins transit No. 1, of 24 inches focal length and $2\frac{1}{2}$-inch object-glass. It was mounted on a large wooden post. One revolution of the micrometer was equal to $85''.220$, and one division of the level equals $2''.293$. Observations were made on five nights. Twenty-four pairs of stars were observed and 44 individual results were obtained. Declinations were taken from Safford's Catalogue of Mean Declinations of 2018 Stars (Washington, Government Printing Office, 1879). Weights were assigned to the result for each pair of stars by formula (2) § 2, with $E_0 = \pm 1''.05$ and $E_\delta = \pm 0''.53$.

Following is the usual table of results, giving the British Association Catalogue numbers of the stars, the individual and mean results for each pair, and the weight for each mean result.

The resulting latitude of the observing-post is $41° 39' 03''.21 \pm 0''.15$, and applying a correction of $+00''.44$ to reduce to the longitude-post of 1881, there results for latitude of Toledo (longitude-post of 1881)

$$41° 39' 03''.65 \pm 0''.15$$

Latitude of Toledo.

[Observer, O. B. Wheeler. Instrument, Pistor & Martins transit No. 1.]

Pairs of stars observed.		October 28, 1868.	October 29, 1868.	November 2, 1868.	November 4, 1868.	November 5, 1868.	Means.	Weights.
		° ′ 41 39						
		″	″	″	″	″	″	
6962	6997			02.44			02.44	0.72
6965	7006			01.26			01.26	0.72
7027	7100			04.50			04.50	0.72
7114	Gr. 3243			02.43			02.43	0.72
7194	7218			00.81			00.81	0.72
XX 401	7297			04.11			04.11	0.72
R. C. 5132	7373			03.65		03.84	03.74	1.20
7399	7411			04.40	03.91	04.47	04.26	1.54
7501	7566			04.69	04.81	03.62	04.37	1.54
7607	7637			02.98	02.85		02.92	1.20
7607	7642					03.30	03.30	0.72
7755	7706				02.54		02.54	0.72
7777	7800			00.34	02.36	01.89	01.53	1.54
7815	XXII 113				01.66	01.41	01.54	1.20
7850	7917			04.35	03.46	04.25	04.02	1.54
7962	8028		03.73		03.12	02.76	03.20	1.54
8083	8091	04.80	02.75				03.78	1.20
8118	8171	03.73			02.19	02.87	02.93	1.54
16	52	03.31	03.42				03.36	1.20
109	121	03.58	01.80				02.69	1.20
169	229	03.48	03.14			03.83	03.48	1.54
377	337		05.80				05.80	0.72
425	575		04.18				04.18	0.72
501	566		03.17				03.17	0.72

	° ′ ″ ″
Weighted mean	41 39 03.21 ± 0.15
Correction to reduce to longitude-post of 1881	+ 00.44
Latitude of Toledo (longitude-post of 1881)	41 39 03.65 ± 0.15

LATITUDE OF WEST BASE, SANDUSKY.

§ **17.** The observations from which the latitude of West Base was determined were made in the city of Sandusky, about $2\frac{1}{2}$ miles nearly south of station West Base. The post was situated 1022.7 feet south and 2599.5 feet west of the northeast corner of the custom-house.

In this vicinity the soil is underlaid by limestone rock, which reaches nearly to the surface of the ground. There is no marked elevation of ground for 40 or 50 miles in any direction. To the northward lie in succession, first, Sandusky Bay, then Marblehead and Cedar Point peninsulas, and then Lake Erie, with comparatively shallow water and numerous islands.

Latitude was observed by Assistant Engineer G. Y. Wisner on four nights in October, 1868. The instrument used was Würdemann zenith-telescope No. 12, having a focal length of 32 inches and a $2\frac{1}{2}$-inch object-glass. One revolution of the micrometer-head at turn No. 5 = 63''.07, and at turn No. 45 = 63''.87, and was assumed to change uniformly between these values. The numbering of the turns was such as to make the middle notch of the comb-scale No. 25. One division of the level = 1''.03. The instrument was mounted on a heavy wooden post. Thirty-one pairs of stars were observed, giving 69 individual results for latitude. In reducing the observations Safford's Catalogue of Mean Declinations of 981 Stars (Washington, Government Printing Office, 1873), was used for declinations. Weights were assigned to the results for each pair by the formula (2) § 2, with $E_0 = \pm 0''.80$ and $E_\delta = \pm 0''.53$. In one case a single star on one side of the zenith was combined with 4 stars on the other side, thus forming 4 pairs. In another instance 1 was combined with 3; and there were 6 cases where 1 was combined with 2. These were weighted according to the formula given in the note to § 5.

Following is the usual table, giving in its successive columns the pairs of stars with their British Association Catalogue numbers, the results for each pair on each night, the mean result for each pair, and the weights assigned to the mean results.

The resulting latitude of the observing-post is 41° 27′ 13''.49 ± 0''.15, and applying a correction of +1′ 51''.10 to reduce to West Base (determined by a secondary triangulation), there results for latitude of West Base, Sandusky,

41° 29′ 04''.59 ± 0''.15

Latitude of Sandusky.

[Observer, G. Y. Wisner. Instrument, Würdemann zenith-telescope No. 12.]

Pairs of stars.		October 10, 1868.	October 11, 1868.	October 12, 1868.	October 13, 1868.	Means.	Weights.
		° ′ 41 27					
		''	''	''	''	''	
7112	6967	17.23	....	....	....	17.23	0.44
7112	6997	15.76	....	....	....	15.76	0.44
7112	6969	16.14	....	....	....	16.14	0.44
7112	7084	....	14.32	....	14.23	14.28	0.67
7171	7167	13.28	13.34	14.38	13.42	13.61	2.27
7254	7336	11.96	13.04	12.46	12.04	12.38	2.27
7317	7337	14.24	14.18	11.69	12.65	13.19	2.27
7273	7320	....	13.60	13.12	12.55	13.09	2.03
7345	7373	12.43	11.41	13.31	....	12.38	1.35
R. 5132	7373	11.94	10.85	13.62	....	12.14	1.35
7411	7399	....	14.06	....	....	14.06	0.72
7431	7399	....	14.30	....	....	14.30	0.72
7480	7462	....	13.03	....	....	13.03	0.72
7469	7462	....	....	13.73	....	13.73	0.72
7501	7505	....	13.50	....	....	13.50	1.08

Latitude of Sandusky—Continued.

Pairs of stars.		October 10, 1868.	October 11, 1868.	October 12, 1868.	October 13, 1868.	Means.	Weights.
		″	″	″	″	″	
7503	7566	....	15.79	....	....	15.79	0.72
7503	R. 9430	....	13.01	....	....	13.01	0.72
7681	7602	16.08	14.99	16.15	....	15.74	2.03
7705	7614	11.64	13.54	14.47	....	13.22	2.03
7746	7721	16.27	22.43	09.11	....	15.94	2.03
7800	7777	13.71	13.38	13.98	....	13.69	2.03
7815	XXII 113	13.06	13.19	11.88	....	12.71	2.03
Gr. 3779	7843	13.58	14.36	....	....	13.97	1.66
7894	7879	12.35	11.75	11.35	....	11.82	1.35
7894	7880	12.84	....	12.43	....	12.64	1.11
7913	7901	13.58	13.34	12.40	....	13.11	2.03
8028	7932	13.03	12.68	12.50	....	12.74	1.01
8028	7962	12.68	12.03	12.27	....	12.33	1.01
8028	7994	13.63	12.87	12.31	....	12.94	1.01
8261	52	11.99	....	....	....	11.99	0.72
16	52	12.85	....	....	....	12.85	0.72

	° ′ ″	″
Weighted mean...	41 27 13.491	± 0.15
Correction to reduce to West Base...............................	+ 1 51.10	
Latitude of West Base..................................	41 29 04.59	± 0.15

LATITUDE OF TONAWANDA.

§ **18.** This station is situated in Erie County, New York, about 2 miles south of the village of Tonawanda. To the east, south, and west of the station, within a radius of 10 miles, there are no marked changes of elevation. Fifty miles to the south, however, the hills rise to a height 500 to 1000 feet greater than that at the station. Northward, the surface declines very gradually for twelve miles to the limestone ledge crossing Niagara River near Lewiston, N. Y., and visible in a line nearly parallel to the south shore of Lake Ontario, between Rochester, N. Y., and Hamilton, Ontario. From this ledge, which is about 250 feet high in the vicinity of Lewiston, the ground slopes off gradually to the basin of Lake Ontario.

Latitude was observed here by Assistant Engineer A. R. Flint in October, 1875, on six nights with the Würdemann zenith-telescope No. 19, having a focal length of 32 inches and a 3-inch object-glass. One revolution of the micrometer-screw was equal to 62″.224, and one division of the level to 1″.292. The first night's observations were made with the instrument supported by a wooden post used previously as an azimuth post (see Chapter XXIV, § 11). On the remaining nights a stone pier was used. This stone pier was situated 31.9 feet distant, bearing north 86° 30′ east from the station. The number of pairs of stars observed was 40, and 156 individual results for latitude were obtained. Declinations were taken from Safford's Catalogue of 981 Stars, and weights were assigned to results from each pair according to the adopted method (§§ 2, 3) with $E_0 = \pm 0''.42$ and $E_\delta = \pm 0''.57$.

Herewith is given the usual table, showing the British Association Catalogue numbers of stars, the individual results for each pair on separate nights, the mean result for each pair, and the weights assigned to the latter. In this table the individual results for October 2 are corrected to reduce them to the position of the instrument on the remaining nights.

The resulting latitude for the stone pier is 43° 00′ 07″.845±0″.085, and applying a correction of −0″.019 to reduce to the trigonometric station, there results for the latitude of Tonawanda

43° 00′ 07″.826±0″.085

Latitude of Tonawanda.

[Observer, A. R. Flint. Instrument, Würdemann zenith-telescope No. 19.]

Pairs of stars observed.		October 2, 1875.	October 14, 1875.	October 19, 1875.	October 20, 1875.	October 21, 1875.	October 22, 1875.	Means.	Weights.
		° ′ 43 00							
		″	″	″	″	″	″	″	
6556	6566	06.26	....	....	....	....	....	06.260	1.00
6603	6651	....	....	....	....	06.45	....	06.450	1.00
6711	6721	09.22	....	07.98	....	08.03	....	08.410	1.30
6728	6745	....	....	08.55	....	06.97	....	07.760	1.21
6769	6779	08.48	07.30	08.07	07.59	07.44	....	07.776	1.39
6799	6806	08.56	08.08	08.63	....	07.14	09.22	08.322	1.39
6799	6813	....	....	....	08.15	....	....	08.150	1.00
6830	6849	08.74	09.17	07.70	....	05.94	07.99	07.908	1.39
6876	Gr. 3013	09.32	08.69	08.44	07.81	07.61	08.54	08.401	1.41
6967	6985	08.55	07.92	07.79	08.75	07.53	....	08.108	1.39
7029	7035	07.76	08.90	08.19	06.91	07.45	07.91	07.853	1.41
7083	7114	07.95	09.70	07.53	08.97	07.33	06.96	08.070	1.41
7171	7174	....	08.30	07.09	07.70	07.81	06.44	07.468	1.39
7182	7213	05.96	07.89	07.26	08.25	07.83	07.90	07.515	1.41
7233	7260	06.76	....	07.95	05.49	....	07.55	06.940	1.36
7277	7306	08.70	08.33	08.73	07.86	08.98	07.96	08.427	1.41
7345	7398	....	06.84	07.87	08.19	07.44	07.30	07.528	1.39
7411	7462	....	07.21	....	07.78	09.00	07.94	07.983	1.36
7496	7505	....	08.82	....	08.22	08.23	08.68	08.488	1.36
7566	7598	....	08.05	07.19	06.39	07.61	07.80	07.408	1.39
7753	7787	....	08.46	....	08.19	....	08.17	08.274	1.30
7845	7858	....	07.48	....	07.52	....	07.18	07.394	1.30
7894	7932	....	07.01	....	06.85	08.69	07.92	07.618	1.36
7983	8023	....	07.21	....	07.13	09.33	07.57	07.810	1.36
8076	8076	....	09.00	....	08.26	09.03	06.91	08.300	0.72
8110	8128	....	10.34	....	08.52	06.27	08.84	08.493	1.36
8229	8237	....	08.62	....	07.76	08.07	07.31	07.940	1.36
16	28	....	08.63	....	07.34	07.51	07.45	07.733	1.36
60	60	....	08.06	....	08.25	07.13	....	07.813	0.70
120	123	....	07.00	....	....	....	....	07.000	1.00
120	146	....	....	....	06.86	07.03	07.65	07.180	1.30
166	169	....	08.03	....	06.63	06.57	05.88	06.783	1.36
245	259	....	07.67	....	06.90	08.70	06.00	07.318	1.36
310	314	....	07.34	....	06.42	08.35	06.82	07.232	1.36
337	352	....	06.94	....	07.52	06.87	06.97	07.075	1.36
294	294	....	08.32	....	....	10.01	09.87	09.400	0.70
299	299	....	08.94	....	....	09.11	08.75	08.933	0.70
425	Gr. 317	....	07.69	....	07.29	07.80	07.40	07.540	1.36
492	501	....	08.85	....	07.85	08.60	07.52	08.205	1.36
516	555	....	10.73	....	10.97	11.27	10.51	10.870	1.36

	° ′ ″ ″
Weighted mean	43 00 07.845 ± 0.850
Reduction to station	-- 0.019
Latitude of Tonawanda	43 00 07.826 ± 0.085

LATITUDE OF NORTH BASE, SANDY CREEK.

§ **19.** This station forms the northern extremity of the Sandy Creek base-line, and is situated on a sandy beach at the eastern end of Lake Ontario. The line of the beach at this point, and for 10 miles in either direction, runs nearly due north and south. To the west the lake bottom declines at the rate of about 40 feet to 1 mile. To the east there is a sand dune about 50 feet in height, 100 metres distant, then a pond and marsh extending about 1 mile, followed by rising ground. The ground at the station is scarcely above the surface of the lake.

Latitude was observed here by Assistant Engineer G. Y. Wisner, in September, 1874, on four nights, with Würdemann zenith-telescope No. 12, having a focal length of 32 inches and a $2\frac{1}{2}$-inch object-glass. One revolution of the micrometer at turn No. 0=63″.07, and at turn No. 40=63″.87, and was assumed to change uniformly between those values, the middle notch of the comb-scale being taken as the 20th turn. One division of the level was equal to 1″.00. The instrument rested on a large pine post set firmly in the ground, situated 81.1 feet distant, bearing south 13° 24′ 26″ west from the station. The number of pairs of stars observed was 32, and 120 individual results for latitude were obtained.

For reducing the work declinations from Safford's Catalogue of 981 Stars were used. Weights were assigned to the results for each pair according to the adopted method (§§ 2, 3) with $E_0 = \pm 0''.45$ and $E_\delta = \pm 0''.57$.

The accompanying table gives in the successive columns the British Association Catalogue numbers of the stars observed, those forming a pair being placed on the same horizontal line; the individual results for each pair on the separate nights; the mean result for each pair; and the weights assigned to those means.

The resulting latitude for the observing-post is 43° 40′ 40″.741 ± 0″.07, and applying a correction of +0″.779 to reduce to the trigonometrical station, there results for the latitude of North Base, Sandy Creek,

43° 40′ 41″.520 ± 0″.07

Latitude of North Base, Sandy Creek.

[Observer, G. Y. Wisner. Instrument, Würdemann zenith-telescope No. 12.]

Pairs stars observed.		September 23, 1874.	September 24, 1874.	September 25, 1874.	September 26, 1874.	Means.	Weights.
		° ′ 43 40					
		″	″	″	″	″	
6520	6493	41.02	41.07	40.31	40.05	40.61	1.33
6579	6599	39.98	40.15	38.49	39.79	39.60	1.33
6697	6651	40.70	40.12	40.94	39.57	40.33	1.33
6717	6711	40.80	40.24	40.48	39.93	40.36	1.33
6754	6745	39.15	41.50	41.09	40.69	40.61	1.33
6764	6771	....	40.62	41.20	....	40.91	1.18
Gr. 2957	6817	40.45	40.88	40.79	42.19	41.08	1.33
6865	6875	41.25	41.54	41.41	40.77	41.24	1.33
6959	6937	....	42.08	42.73	41.21	42.00	1.27
6985	6990	39.51	40.71	41.01	40.27	40.37	1.33
7062	7001	40.38	41.45	40.53	40.47	40.71	1.33
7083	Gr. 3215	40.99	41.31	41.13	40.64	41 02	1.33
7112	7114	41.77	....	....	40.17	40.97	1.18
7161	Gr. 3243	40.61	40.43	40.55	40.97	40.64	1.33
7215	7194	41.97	42.51	41.38	41.33	41.80	1.33
7253	7241	40.68	41.18	40.91	41.30	41.02	1.33
7268	ν Cygni	41.04	42.41	42.42	40.65	41.63	1.33
7317	7333	40.54	41.27	41.49	40.82	41.03	1.33
7345	7383	40.45	40.40	40.08	40.06	40.25	1.33
7448	7453	42.05	40.61	41.69	38.98	40.83	1.33
Gr. 3524	7505	41.93	40.86	40.80	41.72	41.33	1.33
7598	7602	40.90	41.81	41.15	39.49	40.84	1.33
7695	R.C. 5408	40.63	39.85	40.66	40.19	40.33	1.33
Gr. 3717	7737	40.53	39.72	40.05	40.45	40.19	1.33
7800	Gr. 3750	40.36	40.89	41.53	40.61	40.85	1.33
7813	7843	40.47	39.78	40.59	40.48	40.33	1.33
7906	Gr. 3843	38.78	40.69	40.61	39.26	39.83	1.33
Ll. 44750	7978	....	40.45	40.64	41.07	40.72	1.27
7099	7984	39.88	41.11	39.45	39.87	40.08	1.33
8058	8023	39.43	39.66	39.84	39.47	39.60	1.33
8110	8076	40.43	41.30	41.21	41.45	41.10	1.33
8223	8237	....	41.50	41.94	....	41.72	1.18

	° ′ ″ ″
Weighted mean	43 40 40.741 ± 0.07
Reduction to station	+ 0.779
Latitude of North Base	43 40 41.520 ± 0.07

CHAPTER XXIV.

AZIMUTHS.

§ **1.** In the triangulation between Keweenaw and Minnesota Point Bases, azimuth determinations were made at North Base, Minnesota Point (§ 2), at Aminicon (§ 3), and at South Base, Keweenaw Point (§ 4). In the triangulation between Keweenaw and Fond du Lac Bases, azimuth determinations were made at Ford River (§ 5), and at Bruce (§ 6); in that between Fond du Lac and Chicago Bases, at Minnesota Junction (§ 7); in that south of Chicago Base, at Willow Springs (§ 8), and at Parkersburg (§ 9). Azimuth determinations were also made at West Base, Sandusky (§ 10), and in the triangulation between Buffalo and Sandy Creek Bases, at Tonawanda (§ 11), and at North Base, Sandy Creek (§ 12).

AZIMUTH AT NORTH BASE, MINNESOTA POINT.

§ **2.** Assistant G. Y. Wisner observed at this station for azimuth on one night, the instrument used being Troughton & Simms 14-inch theodolite No. 1. The azimuth of the line North Base—South Base was to be found.

The instrument was on a stone post, firmly set in the ground, which had been previously used in longitude determinations. The azimuth mark observed was a light shining through a narrow slit in a board set up on the post, 3145 meters distant, which had previously served as support for a meridian mark. Time was given by Bond & Son's sidereal clock No. 256.

The observations consisted in measuring the angle between a close circumpolar star and the azimuth mark, and noting the time. The programme was to point alternately to the star and mark till from two to six pointings at the star had been obtained, then to reverse the telescope and get as many additional pointings. The level was read for each pointing at star. In some cases two pointings at the star were made for a single pointing at the azimuth mark. The instrument was very stable, the means of pointings at mark before and after reversal differing in no case by one second of arc. In computation, the pointing at each star was used separately to obtain a result. The mean of pointings at the mark before reversal was used with pointings at the star before reversal, and the mean after reversal with pointings at star after reversal. Azimuths were computed with the formula

$$\tan A = \frac{\sin t}{\cos \varphi \tan \delta - \sin \varphi \cos t}$$

in which φ, δ, and t are the latitude, star's declination, and star's hour-angle. Corrections for level were computed separately. Star places for Polaris and λ Ursæ Minoris were taken from the American Nautical Almanac, and for 6 Ursæ Minoris and 39 Cephei, which are not given in the American Ephemeris, from Safford's Catalogue of Time Stars. In a set, equal weights were given to means of results before and after reversal, although for 39 Cephei, on July 13, there were four pointings at star before reversal and but two after.

In combining the mean results derived from different sets of observations, a weight proportional to the total number of pointings at the star is attributed to each result. The probable error is derived from the discrepancies between the results from the different sets and their weighted mean.

The origin of the horizontal circle remained in essentially the same position throughout the work, but the microscopes were temporarily changed 180° in reversal. Periodic error is then only so far eliminated as is effected with three microscopes when turned 180°. The following table gives the results of the separate observations:

Azimuth at North Base, Minnesota Point.

AZIMUTH OF LINE AZIMUTH POST — MERIDIAN MARK.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms 14-inch theodolite No. 1.]

Star, date, &c.	Telescope—	Azimuth of mark.	Means.	Result for star.
		° ′ ″	° ′ ″	° ′ ″
Polaris, near Lower Culmination, July 13, 1871.	Direct	00 00 01.6		
$\alpha=1^h\ 11^m\ 44^s.4$		359 59 57.3	359 59 59.45	
$\delta=88°\ 37'\ 01''.2$	Reversed ...	00 00 01.0		
		359 59 59.1	00 00 00.05	359 59 59.75
39 Cephei, near East Elongation, July 13, 1871.	Direct	359 59 59.1		
$\alpha=23^h\ 28^m\ 01^s.4$		60.9		
$\delta=86°\ 35'\ 30''.84$		56.7		
		62.3	359 59 59.75	
	Reversed ...	58.0		
		57.2	359 59 57.60	359 59 58.68
6 Ursæ Minoris, near West Elongation, July 13, 1871.	Direct	359 59 59.0		
$\alpha=12^h\ 14^m\ 04^s.3$		58.7	359 59 58.85	
$\delta=88°\ 25'\ 09''.77$	Reversed ...	56.8		
		59.4	359 59 58.10	359 59 58.48
Polaris, near East Elongation, July 13, 1871.	Direct	359 59 60.0		
$\alpha=1^h\ 11^m\ 44^s.9$		61.6		
$\delta=88°\ 37'\ 01''.2$		56.0		
		59.8		
		59.1		
		60.9	359 59 59.57	
	Reversed ...	56.8		
		59.8		
		56.2		
		57.1		
		57.8		
		57.7	359 59 57.57	359 59 58.57
λ Ursæ Minoris, near Upper Culmination, July 13, 1871.	Direct	359 59 57.2		
$\alpha=19^h\ 54^m\ 17^s.2$		59.3	359 59 58.25	
$\delta=88°\ 55'\ 12''.7$	Reversed ...	59.2		
		58.9	359 59 59.05	359 59 58.65

The second set of observations on Polaris required the maximum time, sixty minutes, while the set on λ Ursæ Minoris required but sixteen minutes. The following table gives the mean result for each set of observations, the number of pointings to mark and to star in each set, and the weights attributed to the mean results. The results derived from Polaris and λ Ursæ Minoris are corrected to conform to Auwers' declinations (see § 4), and the result from 39 Cephei (which is not given in Auwers' Catalogue), is corrected to agree with the declinations as given in the General Bericht der Europäische Gradmessung for 1874. 6 Ursæ Minoris is given in neither of these catalogues, and receives no correction in the table. The results given in the table have also been corrected for diurnal aberration and for the effect of a correction, $+2''.34$, to the latitude, $46°\ 45'\ 27''.0$, with which the azimuths in the preceding table were computed.

Azimuth at North Base, Minnesota Point.

SUMMARY OF RESULTS.

AZIMUTH OF LINE AZIMUTH POST — MERIDIAN MARK.

Date.	Star, &c.	Position of telescope.	Number of pointings at— Mark.	Number of pointings at— Star.	Azimuth. Declinations from American Ephemeris.	Reduction to Auwers' declinations. $\frac{dA}{d\delta}$	$\Delta\delta$	ΔA	Corrected azimuth.	Weight.
					° ′ 359 59				° ′ 359 59	
1871.					″		″	″	″	
July 13	Polaris, near Lower Culmination.	Direct	2	2						
		Reversed ...	2	2	60.05	−0.04	+0.04	0.00	60.05	1
13	39 Cephei, near East Elongation.	Direct	4	4						
		Reversed ...	2	2	59.21	−1.44	+0.30*	−0.43*	58.78	1.5
13	6 Ursæ Minoris, near West Elongation.	Direct	1	2						
		Reversed ...	1	2	58.69	....	....	....	58.69	1
13	Polaris, near East Elongation.	Direct	3	6						
		Reversed ...	3	6	58.97	−1.44	+0.04	−0.06	58.91	3
13	λ Ursæ Minoris, near Upper Culmination.	Direct	2	2						
		Reversed ...	2	2	58.97	+0.01	+0.55	+0.01	58.98	1

Weighted mean.. 359° 59′ 59″.02±0″.14.

*Corrections to reduce to declination as given in General Bericht der Europäische Gradmessung for 1874.

In the previous longitude work at this station the instrument was set, each of six nights, at the beginning of the work, on the meridian mark. If the air had been steady, and the instrument had not changed its deviation during the night, a value of the azimuth of the meridian mark would have resulted from the time-reductions. The value which did result was 0″.22 west of south, differing 1″.09 from the value resulting from the azimuth work. It is a rough check on the latter.

The azimuth of the line Azimuth Post—Meridian Mark has now been given. From it is to be derived—

1. That of the line North Base—Meridian Mark, and
2. From this, that of North Base—South Base.

The distance of the azimuth post from North Base was measured five times with rods, depending on a standard yard, along a stretched wire, giving a mean distance of 186ft.651, the measures having a range of 0in.15. The angle Meridian Mark—Azimuth Post—North Base was measured with Troughton & Simms theodolite No. 1. This gave the means of computing with accuracy the correction to the azimuth of line Azimuth Post—Meridian Mark, to get that of line North Base—Meridian Mark. Its value is −51′ 04″.13. In addition the angle North Base—Meridian Mark—Azimuth Post was measured on a cloudy day, eight single or four combined measures being obtained with Troughton & Simms theodolite No. 1, giving in the mean 51′ 02″.23 ±0″.4. Adding 1″.58 for convergence of meridians at Meridian Mark and North Base, this becomes 51′ 03″.81 ±0″.4, differing but 0″.32 from the preceding value. Attributing double weight to the first value, there results for the mean correction −51′ 04″.02 ±0″.23. This gives for azimuth of line North Base to Meridian Mark,

359° 08′ 55″.00 ±0″.27

The angle at North Base between the meridian mark and South Base was measured with Troughton & Simms theodolite No. 1, sixteen single or eight combined measures being obtained. The mean value of the angle is,

35° 16′ 30″.68 ±0″.27

the probable error being derived from the discrepancies between the separate combined results and their mean. Subtracting this from the azimuth of North Base—Meridian Mark, there results for azimuth of line North Base—South Base,

323° 52′ 24″.32± 0″.38 west of south.

AZIMUTH AT STATION AMINICON.

§ **3.** At station Aminicon the same observer as at North Base, Minnesota Point, determined the azimuth of the triangle side Aminicon—Lester River, with the same instrument and essentially the same methods and programme. There were usually, however, for each pointing at the mark, two consecutive pointings at the star, and for these two pointings at the star the level was read but once.

The instrument was mounted on a wooden post sunk 5 feet in the ground. Time was given by a chronometer. The azimuth mark was a light on station Lester River, 13 miles distant. In reduction, the same method was followed as for North Base, Minnesota Point. The microscopes were turned 180° in azimuth by reversal, on July 22, but not on July 23. The shortest time occupied in a set of observations was 24 minutes, on 6 Ursæ Minoris, July 22, and the longest time was 57 minutes, on Polaris, on July 22.

The following table, arranged like that for North Base, gives the result for each observation on a star, and the mean result for each star.

Azimuth at Aminicon River.

AZIMUTH OF LINE, AZIMUTH POST — LESTER RIVER.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms 14-inch theodolite No. 1.]

Star, date, &c.	Telescope—	Azimuth of Lester River station.	Means.	Result for star.
		° ′ ″	° ′ ″	° ′ ″
39 Cephei, near East Elongation, July 22, 1871.	Direct	153 36 31.6		
$\alpha=23^h\ 28^m\ 03^s.6$		28.0		
$\delta=86°\ 35'\ 33''.17$		29.4		
		28.7		
		29.4		
		27.6	153 36 29.12	
	Reversed ...	27.3		
		28.8	28.05	153 36 28.58
6 Ursæ Minoris, near West Elongation, July 22, 1871.	Direct	27.0		
$\alpha=12^h\ 13^m\ 58^s.63$		30.2	28.60	
$\delta=88°\ 25'\ 07''.98$	Reversed ...	30.8		
		27.3		
		31.6		
		29.9	29.90	29.25
Polaris, near East Elongation, July 22, 1871.	Direct	28.8		
$\alpha=1^h\ 11^m\ 52^s.26$		26.1		
$\delta=88°\ 37'\ 02''.4$		30.1		
		27.1		
		28.3		
		30.0	28.40	
	Reversed ...	30.5		
		30.7		
		30.3		
		28.9		
		26.6		
		26.7		
		29.4	29.61	28.70
39 Cephei, near East Elongation, July 23, 1871.	Direct	28.1		
$\alpha=23^h\ 28^m\ 03^s.9$		27.4		
$\delta=86°\ 35'\ 33''.44$		30.8		
		33.4	29.92	
	Reversed ...	33.1		
		31.7		
		29.0		
		29.2	30.75	30.34

Azimuth at Aminicon River—Continued.

Star, date, &c.	Telescope—	Azimuth of Lester River station.	Means.	Result for star.
		° ′ ″	° ′ ″	° ′ ″
6 Ursæ Minoris, near West Elongation, July 23, 1871.	Direct	153 36 33.3		
$\alpha = 12^h\ 13^m\ 58^s.01$		37.2		
$\delta = 88°\ 25'\ 07''.77$		32.6		
		29.9	153 36 33.25	
	Reversed ...	28.7		
		28.5		
		26.6		
		28.4	28.05	153 36 30.65

Star, date, &c.	Telescope—	Azimuth of Buchanan station.	Means.	Result for star.
		° ′ ″	° ′ ″	° ′ ″
Polaris, near East Elongation, July 23, 1871.	Direct	191 23 12.3		
$\alpha = 1^h\ 11^m\ 53^s.0$		10.2		
$\delta = 88°\ 37'\ 02''.5$		9.0		
		10.8		
		10.4		
		12.7	191 23 10.90	
	Reversed ...	8.8		
		11.1		
		10.3		
		8.8		
		7.1		
		9.5	09.27	119 23 10.09

The results in the preceding table have each to be corrected for diurnal aberration and for the effect of a correction, $+4''.93$, to the latitude, $46°\ 41'\ 31''.2$, with which they were originally computed. Applying these corrections, and giving weights to the results from each set proportional to the number of pointings at a star in the set, the following table results. In it, instead of giving the weight 1.2 to Polaris on July 23, but half that weight has been assigned. The reason for this is that in this set the angle from the star was not read as in other sets to Lester River station since the light had become dim there, but to a light on Buchanan station. This azimuth was reduced to Lester River by using the finally adjusted angle given in Chapter XIV, C, between Lester River and Buchanan.

The results in the following table are also corrected for errors in declination, in the same manner as the results at Minnesota Point (see §§ 2 and 4).

Azimuth at Aminicon River.

SUMMARY OF RESULTS.

AZIMUTH OF LINE AZIMUTH POST—LESTER RIVER.

Date.	Star, &c.	Position of telescope.	Number of pointings at— Light.	Star.	Azimuth. Declinations from American Ephemeris.	Reduction to Auwers' declinations. $\frac{dA}{d\delta}$	$\Delta\delta$	ΔA	Corrected azimuth.	Weight.
					° ′				° ′	
1871.					153 36				153 36	
					″		″	″	″	
July 22	39 Cephei, near East Elongation.	Direct	4	6						
		Reversed ..	2	2	29.34	−1.43	+0.30*	−0.43*	28.91	.8
22	6 Ursæ Minoris, near West Elongation.	Direct	1	2						
		Reversed ...	2	4	29.35	+1.45	....	....	29.35	.6
22	Polaris, near East Elongation.	Direct	2	6						
		Reversed ...	3	7	29.03	−1.43	+0.04	−0.06	28.97	1.3
23	39 Cephei, near East Elongation.	Direct	3	4						
		Reversed ...	2	4	31.10	−1.43	+0.30*	−0.43*	30.67	.8
23	6 Ursæ Minoris, near West Elongation.	Direct	2	4						
		Reversed ...	2	4	30.75	+1.45	...	....	30.75	.8
23	Polaris, near East Elongation.	Direct	3	6						
		Reversed ...	3	6	27.94	−1.43	+0.04	−0.06	27.88	.6

Weighted mean .... $153°\ 36'\ 29.44'' \pm 0.29$

* Correction to reduce to declinations given in General Bericht der Europäische Gradmessung for 1874.

The azimuth just given is that of Lester River from the azimuth post on which the instrument stood. It is to be corrected so that the azimuth of Lester River from station Aminicon may result. To obtain this correction the distance from instrument on azimuth post to Aminicon station was measured as at North Base, three times, the range in measument being $0^{in}.10$, giving a mean distance of $30^{ft}.432$. The angle at azimuth post between Aminicon and Lester River was measured with the Troughton & Simms theodolite No. 1, giving $24^\circ\ 32'\ 42''.1$. The resulting correction is $+36''.71$. Applying it to the azimuth of line Azimuth Post to Lester River, there results for azimuth of the triangle side, Aminicon—Lester River,

$153^\circ\ 37'\ 06''.15 \pm 0''.29$ west of south.

AZIMUTH AT SOUTH BASE, KEWEENAW POINT.

§ **4.** The observer for azimuth and the instrument were the same at this station as at Aminicon, and the method was generally the same. But instead of pointing at the azimuth mark after each one or two pointings at the star, no pointings at the mark (with a single exception) were made during the pointings at the star in a given set. Instead, five or more pointings at the mark were made immediately before the first pointings at the star, and as many more after the last pointing at the star. The programme was then as follows:

1. Five or more pointings at mark.
2. Four or five pointings at the star, and levels read for each two pointings.
3. Reversal.
4. Four or five pointings at star, and levels read for each two pointings.
5. Five or more pointings at mark.

For one set, that on ζ Ursæ Minoris on July 25, there were in addition two pointings at mark immediately before and immediately after reversal. On the other nights the method of observation requires the assumption that any change of position of the instrument in the period, varying from 18 to 35 minutes, occupied by the star pointings was proportional to the time. Such an assumption is doubtless inadmissible for so long a period, and is probably one of the causes of the discrepancies in the results obtained in each set from observations made before and after reversal.

The instrument was mounted on a wooden post, 2 feet in diameter, sunk 5 feet in the ground, which had been used by Assistant Flint as a latitude post in 1871. The distance of the instrument on its azimuth post from station South Base was measured with a standard scale on the edge of a level board. The mean of three measures having a range of $0^{ft}.004$, was $26^{ft}.570$. The angle at South Base between North Base and Azimuth Post was measured three times with Troughton & Simms theodolite No. 1, and found to be $10^\circ\ 22'\ 46''$. These coördinates give a correction of $33''.92$ to reduce the observed azimuth of line Azimuth Post—North Base to that of South Base—North Base.

The difference between the mean pointings at the mark before and after star observations varied between $0''.72$ and $7''.6$, except on July 26 in the set of observations on Polaris, when it amounted to $13''.86$. If the change had been due to a twisting of the instrument in azimuth during the period of observation, it should have shown itself in comparing the five azimuths deduced from the five observations on the star before reversal with each other, or in a similar comparison of the azimuths resulting from observations after reversal. Giving seconds alone, the results from the five observations on Polaris before reversal and the mean of the five pointings at the mark before reversals were $29''.24$, $32''.19$, $32''.39$, $30''.49$, $28''.38$, while those from observations after reversal were $34''.34$, $34''.44$, $34''.64$, $34''.94$, $35''.54$. Neither set shows any marked change in azimuth with the time, as would be the case if the reading of the mark on the limb had been changing instead of fixed, as the computations assume. The difference of $13''.86$ in the mean pointings at the mark before and after reversal must then be attributed to instrumental errors and to disturbances arising from reversal or other cause. The reversal is effected by hand, and the telescope and vertical circle are very heavy, so that there is danger of disturbance. There was no certain disturbance in level. There was no systematic elimination of periodic error, the horizontal limb remaining in essentially the same position throughout the observations. The microscopes were turned 180° in azimuth in reversal on July 23, but not on July 25 and 26.

The following tables give the results for each observation on each star:

Azimuth at South Base, Keweenaw Point.

AZIMUTH OF LINE AZIMUTH POST — NORTH BASE.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms 14-inch theodolite No. 1.]

Star, date, &c.	Telescope—	Azimuth of North Base.	Means.	Result for star.
		° ′ ″	° ′ ″	° ′ ″
Polaris, near East Elongation, July 23, 1873.	Direct	199 10 33. 47		
$\alpha = 1^h 12^m 21^s.5$		31. 42		
$\delta = 88° 37' 44''.2$		35. 37		
		34. 07	199 10 33. 58	
	Reversed ...	31. 60		
		31. 17		
		31. 45		
		32. 46	31. 67	199 10 32. 63
Polaris, at East Elongation, July 25, 1873.	Reversed ...	34. 00		
$\alpha = 1^h 12^m 23^s.0$		35. 38		
$\delta = 88° 37' 44''.5$		29. 78		
		31. 28		
		33. 00	32. 69	
	Direct	35. 28		
		34. 91		
		34. 80		
		35. 86		
		35. 66	35. 30	34. 00
ζ Ursæ Minoris, at West Elongation, July 25, 1873.	Direct	34. 37		
$\alpha = 15^h 48^m 41^s.6$		35. 70		
$\delta = 78° 11' 12''.7$		31. 11		
		35. 51		
		31. 71	33. 68	
	Reversed ...	31. 37		
		35. 07		
		38. 47		
		36. 17		
		34. 87	35. 19	34. 44
ε Ursæ Minoris, at West Elongation, July 25, 1873.	Reversed ...	34. 12		
$\alpha = 16^h 59^m 09^s.1$		36. 42		
$\delta = 82° 14' 40''.9$		36. 08		
		35. 53		
		37. 60	35. 95	
	Direct	37. 04		
		32. 84		
		31. 54		
		31. 64		
		31. 20	32. 85	34. 40
δ Ursæ Minoris, at West Elongation, July 25, 1873.	Direct	36. 33		
$\alpha = 18^h 13^m 31^s.2$		42. 73		
$\delta = 86° 36' 29''.3$		40. 51		
		40. 87		
		35. 47	39. 18	
	Reversed ...	32. 30		
		30. 07		
		31. 84		
		30. 50		
		29. 57	30. 86	35. 02

Azimuth at South Base, Keweenaw Point—Continued.

Star, date, &c.	Telescope—	Azimuth of North Base.	Means.	Result for star.
		° ′ ″	° ′ ″	° ′ ″
Polaris at East Elongation, July 26, 1873.	Reversed ...	199 10 29.24		
$\alpha = 1^h 12^m 23^s.7$		32.19		
$\delta = 88° 37' 44''.65$		32.39		
		30.49		
		28.39	199 10 30.54	
	Direct	34.34		
		34.44		
		34.64		
		34.94		
		35.54	34.78	199 10 32.66
ζ Ursæ Minoris at West Elongation, July 26, 1873.	Direct	199 10 32.66		
$\alpha = 15^h 48^m 41^s.5$		35.56		
$\delta = 78° 11' 12''.8$		34.06		
		34.76	34.26	
	Reversed ...	28.26		
		26.41		
		29.81		
		33.06	29.39	31.82

An examination of this table shows considerable differences in the results on the same night, before and after reversal on a given star. Part of the discrepancies arise without doubt from the long interval between the pointings at the mark, during which it has to be assumed that the change in reading on the mark is proportional to the time. It is possible that a part may come from some other cause. The difference in the results from δ Ursæ Minoris, July 25, before and after reversal is so much greater than any possible error of observation that the results are not used. To the results from the other stars weights are assigned proportional to the number of pointings at the star.

The following table gives in the first six columns a summary of the results in the preceding table, the results in the sixth column being further corrected for diurnal aberration. The results in the sixth column depend upon the declinations given in the American Ephemeris, which has heretofore been the preferred authority for star-places in Lake-Survey reductions. There is, however, a later and thoroughly revised catalogue—the Fund. Cat. für die Zonen-Beobachtungen, Leipzig, 1879—compiled by Dr. Auwers under the direction of the Zonen-Commission der Astronomischen Gesellschaft, which may now be taken as the best authority available. Columns giving the quantities used in deriving corrections to reduce the results in the sixth column to conformity with Auwers' declinations, and a column of corrected results, are, therefore, added to the following table, the seventh column giving the differential coefficient of each azimuth result with respect to the declinations, denoted by $\frac{dA}{d\delta}$; the eighth column giving the corrections to the declinations given in the American Ephemeris to reduce them to Auwers' declinations, denoted by $\Delta\delta$; the ninth column giving the corresponding corrections to apply to the azimuth results in the sixth column, denoted by $\Delta A \left(=\frac{dA}{d\delta}\Delta\delta\right)$; and the tenth column giving the azimuth results corrected to conform to Auwers' declinations. The quantities $\frac{dA}{d\delta}$ can be used hereafter, if so desired, to further correct these azimuth results, when the star-places become known with greater precision. The probable error is derived from the differences between the separate results and their weighted mean.

Azimuth at South Base, Keweenaw Point.

SUMMARY OF RESULTS.

AZIMUTH OF LINE AZIMUTH POST — NORTH BASE.

Date.	Star, &c.	Position of telescope.	Number of pointings at— Light.	Star.	Azimuth. Declinations from American Ephemeris.	Reduction to Auwers' declinations. $\frac{dA}{d\delta}$	$\Delta\delta$	ΔA	Corrected azimuth.	Weight.
					° ′ 199 10				° ′ 199 10	
1873.					″		″	″	″	
July 23	Polaris, near East Elongation.	Direct	8	4						
		Reversed ...	5	4	32.63	−1.46	+0.04	−0.06	32.57	0.8
25	......do	Reversed ...	9	5						
		Direct	5	5	33.98	−1.46	+0.04	−0.06	33.92	1
26	......do	Reversed ...	5	5						
		Direct	5	5	32.66	−1.46	+0.04	−0.06	32.60	1
25	ζ Ursæ Minoris, near West Elongation.	Direct	7	5						
		Reversed ...	7	5	34.44	+1.50	+0.02	+0.03	34.47	1
26	......do	Direct	5	4						
		Reversed ...	5	4	31.82	+1.50	+0.02	+0.03	31.85	0.8
25	ε Ursæ Minoris, near West Elongation.	Reversed ...	5	5						
		Direct	5	5	34.40	+1.48	−0.05	−0.07	34.33	1

Weighted mean .... 199° 10′ 33″.37 ± 0″.30.

To reduce this azimuth to the azimuth of triangle-side South Base — North Base, a correction, +33″.92, is to be applied, derived from the coördinates of the azimuth post referred to South Base, already given. There results for azimuth of North Base, Keweenaw Point, from South Base, Keweenaw Point,

199° 11′ 07″.29±0″.30 west of south.

AZIMUTH AT FORD RIVER.

§ **5.** Observations for azimuth were made at Ford River station, July 10, 11, 12, and 13, 1874, by Assistant Engineer G. Y. Wisner. The instrument used was Troughton & Simms 14-inch theodolite No. 1. It was mounted on a heavy wooden post, set firmly in the ground. The stars observed were:

32 Camelopardalis, near Western Elongation.
Polaris, near Eastern Elongation.
ε Ursæ Minoris, near Western Elongation.
λ Ursæ Minoris, near Upper Culmination.

Their places were taken from the American Ephemeris. Time was given by a chronometer. The observations were made according to the following programme:

1. Five or more readings on azimuth mark.
2. The same number of readings on star, with level readings.
3. Reversal of telescope.
4. Five or more readings on star, with level readings.
5. The same number of readings on azimuth mark.

The zero-line of the horizontal circle was changed but once during the series of observations. Twice, however, the microscopes were turned 180° when the telescope was reversed, so that microscope A had, in all, the following positions for readings on mark: On July 10 and 11, 28°; on July 12, 208°; and on July 13, 358° and 178°. This disposition was not such as to secure a good elimination of periodic or accidental errors of graduation, though both are known to be small with the instrument used.

A determination of the value of one division of the striding level used was made on July 11 by comparing it with the well-determined level of Würdemann zenith-telescope No. 19, comparisons being made at temperatures varying from 50° to 82° Fahrenheit. These determinations, which indicate a nearly uniform increase in the value of one division of the level with the temperature-increase, were used in the reduction of the observations.

The azimuth mark was a light limited by a slit about one-fourth inch wide in the box contain ing the lantern, and 1.75 miles distant from the azimuth post.

The azimuth of the star for each observation was computed by the usual formula

$$\tan A = \frac{\sin t}{\cos \varphi \tan \delta - \sin \varphi \cos t},$$

wherein A, t, and δ are the azimuth, hour angle, and declination of star, respectively, and φ is the latitude of the place. In the reduction, the differences between the individual readings on the star and the mean of the readings on the mark for one position of the telescope were taken to obtain individual results for azimuth of the mark.

The following tables give the individual and mean results for azimuth of mark from each star for the separate nights. They are corrected for diurnal aberration, and give the azimuth of the mark from Ford River trigonometrical station, the instrument being centered vertically over the geodetic point of the latter. The individual results are affected by collimation, periodic and accidental errors of graduation, and by errors due to instability of the theodolite during the interval etween pointings to the star and mark.

Azimuth at Ford River.

AZIMUTH OF LINE FORD RIVER—AZIMUTH MARK.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms 14-inch theodolito No. 1.]

Star, date, &c.	Telescope—	Azimuth of mark.	Means.	Result for star.
		° ′ ″	° ′ ″	° ′ ″
32 Camelopardalis, near West Elongation, July 10, 1874.	Direct	218 38 63. 68		
$\alpha = 12^h\ 48^m\ 17^s.0$		59. 27		
$\delta = 84°\ 06'\ 02''.38$		62. 07		
		63. 89		
		61. 63	218 38 62. 108	
	Reversed ...	57. 26		
		55. 88		
		54. 54		
		55. 36		
		53. 78	55. 364	218 38 58. 736
Polaris, near East Elongation, July 10, 1874.	Reversed ...	60. 14		
$\alpha = 1^h\ 12^m\ 26^s.5$		58. 00		
$\delta = 88°\ 38'\ 03''.78$		58. 48		
		55. 96		
		56. 96		
		60. 97		
		62. 21	58. 960	
	Direct	62. 12		
		64. 56		
		63. 77		
		63. 67		
		62. 86		
		61. 85		
		62. 83	63. 094	61. 027
32 Camelopardalis, near West Elongation, July 11, 1874.	Direct	59. 41		
$\alpha = 12^h\ 48^m\ 16^s.8$		64. 99		
$\delta = 84°\ 06'\ 02''.25$		63. 12		
		64. 49		
		62. 23		
		62. 94	62. 863	
	Reversed ...	64. 31		
		63. 04		
		61. 91		
		59. 52		
		61. 43		
		63. 48	62. 282	62. 572

Azimuth at Ford River—Continued.

Star, date, &c.	Telescope—	Azimuth of mark.	Means.	Result for star.
		° ′ ″	° ′ ″	° ′ ″
Polaris, near East Elongation, July 11, 1874.	Reversed ...	218 38 63.79		
$\alpha=1^h\ 12^m\ 27^s.5$		63.74		
$\delta=88°\ 38'\ 30''.88$		64.46		
		66.43		
		65.98		
		67.31		
		65.71		
		64.64		
		62.25		
		63.55	218 38 64.786	
	Direct	56.43		
		56.19		
		56.83		
		58.57		
		57.84		
		55.94		
		57.42		
		59.37		
		59.27		
		59.62	57.748	218 38 61.267
λ Ursæ Minoris, near Upper Culmination, July 11, 1874.	Direct	59.53		
$\alpha=19^h\ 50^m\ 55^s.3$		61.68		
$\delta=88°\ 55'\ 35''.3$		61.01		
		62.18		
		62.33	61.346	
	Reversed ..	61.63		
		60.92		
		61.69		
		61.57		
		60.18	61.198	61.272
ε Ursæ Minoris, near West Elongation, July 11, 1874.	Reversed ...	59.28		
$\alpha=16^h\ 59^m\ 04^s.2$		61.38		
$\delta=82°\ 14'\ 30''.43$		60.75		
		58.75		
		61.18	60.268	
	Direct	61.42		
		63.38		
		63.28		
		60.75		
		61.69	62.104	61.186
Polaris, near East Elongation, July 12, 1874.	Direct	55.98		
$\alpha=1^h\ 12^m\ 28^s.4$		56.63		
$\delta=88°\ 38'\ 03''.97$		55.59		
		54.10		
		54.60		
		54.56		
		54.96		
		56.30		
		54.61		
		55.10	55.243	
	Reversed ...	58.62		
		58.20		
		59.37		
		59.24		
		58.57		
		59.33		
		59.08		
		57.26		
		57.43		
		56.30	58.340	56.791

Azimuth at Ford River—Continued.

Star, date, &c.	Telescope—	Azimuth of mark.	Means.	Result for star.
		° ′ ″	° ′ ″	° ′ ″
λ Ursæ Minoris, near Upper Culmination, July 12, 1874.	Reversed …	218 38 53.41		
$\alpha=19^h\ 50^m\ 55^s.1$		53.55		
$\delta=88°\ 55'\ 35''.7$		57.43		
		56.98		
		56.02	218 38 55.478	
	Direct ……	61.68		
		58.83		
		62.98		
		62.50		
		60.18	61.234	218 38 58.356
ε Ursæ Minoris, near West Elongation, July 12, 1874.	Direct ……	53.00		
$\alpha=16^h\ 59^m\ 04^s.1$		57.67		
$\delta=82°\ 14'\ 30''.66$		55.13		
		52.59		
		52.68	54.214	
	Reversed …	58.49		
		57.75		
		57.90		
		58.07		
		56.96	57.834	56.024
32 Camelopardalis, near West Elongation, July 13, 1874.	Direct ……	62.88		
$\alpha=12^h\ 48^m\ 16^s.5$		64.59		
$\delta=84°\ 06'\ 02''.0$		66.75		
		65.84		
		64.81		
		64.26	64.855	
	Reversed …	56.16		
		55.23		
		53.21		
		52.29		
		51.35		
		52.05	53.382	59.118
Polaris, near East Elongation, July 13, 1874.	Reversed ..	60.61		
$\alpha=1^h\ 12^m\ 29^s.5$		59.50		
$\delta=88°\ 38'\ 04''.1$		59.49		
		58.95		
		57.99		
		58.23		
		56.75		
		57.37	58.611	
	Direct ……	59.51		
		56.65		
		57.16		
		59.95		
		58.93		
		59.14		
		59.00		
		57.60	58.493	58.552

A summary of the preceding results is given in the table following. The individual results in column 6 are corrected for periodic error by the formula given in Chapter XIV, B, § 6, and are further corrected in the table for the errors in the declinations taken from the American Ephemeris in the manner explained in § 4. The mean result for azimuth of mark from a star on any date is weighted according to the number of pointings to the star. Hence, there results for weighted mean azimuth of mark,

$$218°\ 39'\ 00''.17 \pm 0''.50 \text{ west of south,}$$

the probable error being derived from the discrepancies between the individual results and the weighted mean.

Azimuth at Ford River.

SUMMARY OF RESULTS.

AZIMUTH OF LINE FORD RIVER—AZIMUTH MARK.

Date.	Star, &c.	Position of telescope.	Number of pointings— To mark.	Number of pointings— To star.	Azimuth. Declinations from American Ephemeris.	Reduction to Auwers' declinations. $\frac{dA}{d\delta}$	$\Delta\delta$	ΔA	Corrected azimuth of mark.	Weight.
1874.					° ′ 218 38				° ′ 218 38	
					″		″	″	″	
July 10	32 Camelopardalis, near West Elongation.	Direct	5	5						
		Reversed ...	5	5	59. 06	+1. 43	+1. 55	+2. 22	61. 28	0. 5
11	......do	Direct	6	6						
		Reversed ...	6	6	62. 89	+1. 43	+1. 55	+2. 22	65. 11	0. 6
13	......do	Direct	6	6						
		Reversed ...	6	6	59. 05	+1. 43	+1. 55	+2. 22	61. 27	0. 6
10	Polaris, near East Elongation	Reversed ...	7	7						
		Direct	7	7	61. 13	−1. 42	+0. 04	−0. 06	61. 07	0. 7
11	..... do	Reversed ...	10	10						
		Direct	10	10	61. 37	−1. 42	+0. 04	−0. 06	61. 31	1. 0
12	......do	Direct	10	10						
		Reversed ...	10	10	57. 13	−1. 42	+0. 04	−0. 06	57. 07	1. 0
13	......do	Reversed ...	8	8						
		Direct	8	8	58. 32	−1. 42	+0. 04	−0. 06	58. 26	0. 8
11	λ Ursæ Minoris, near Upper Culmination.	Direct	5	5						
		Reversed ...	5	5	61. 45	+0. 01	+0. 59	+0. 01	61. 46	0. 5
12	......do	Reversed ...	5	5						
		Direct	5	5	58. 79	+0. 01	+0. 59	+0. 01	58. 80	0. 5
11	ε Ursæ Minoris, near West Elongation.	Reversed ...	5	5						
		Direct	5	5	61. 52	+1. 45	−0. 05	−0. 07	61. 45	0. 5
12	......do	Direct	5	5						
		Reversed ..	5	5	56. 43	+1. 45	−0. 05	−0. 07	56. 36	0. 5

Weighted mean.. 218° 39′ 00″.17±0″.50.

The angles between Cedar River station and Azimuth Mark, and between Azimuth Mark and Pine Hill station, were observed with the angles of the triangles at Ford River station, 16 combined measures of each being obtained. The adjusted value of the angle Cedar River — Ford River — Azimuth Mark, is 187° 03′ 59″.89 (Chapter XV, C). The probable error of the *measured* value of this angle resulting from the adjustment is ±0″.44 (Chapter XV, C, § 7). Using this value for the probable error of the above adjusted angle, we have for the azimuth of the primary side Ford River — Cedar River,

31° 35′ 00″.28±0″.67 west of south.

AZIMUTH AT BRUCE.

§ **6.** Azimuth was observed at Bruce station on three nights in July, 1872, by Assistant Engineer A. R. Flint. The instrument used was theodolite No. 1, made by Repsold & Sons. It was mounted on a heavy oak post vertically over the geodetic point of Bruce triangulation station. The method of observation was the same as that followed at Ford River, and already explained, except that the numbers of pointings on the mark and on the star for one position of the telescope were not generally equal. Time was given by a chronometer. The stars observed were α, δ, and ε Ursæ Minoris. During the series of observations the horizontal circle of the theodolite had three different positions, as indicated by the following readings of Microscope A on the azimuth mark, viz: on July 12, 61°; on July 14, 175°; and on July 16, 270°. The light of Long Tail Point light-house, about 6 miles distant, was used as an azimuth mark. The position of the ball on the dome vertically over the lamp of this light-house was accurately determined by pointings from primary stations Bruce, East Depere, Oneida, Little Tail Point, and Red Banks (see Chapter XV, C).

The first of the following tables gives the individual and mean results for azimuth of the mark from each star for each night. The second table gives a summary of the results in the first table, corrected for periodic error according to the formula given in Chapter XV, B, § 4. These results are further corrected in the table for errors in the declinations taken from the American Ephemeris, as explained in § 4. Weights proportional to the number of pointings to the star are assigned to the means given in the last column of the first table, except for the result from δ Ursæ Minoris. The pointings to this star for telescope reversed were referred to station Fort Howard, and the above-named result is found by applying the adjusted value of the angle at Bruce between Fort Howard and the light-house, viz: 59° 49′ 05″.85 (Chapter XV, C), to the azimuth of Fort Howard computed from these pointings. For this reason the result from δ Ursæ Minoris is given a weight equal to three-fourths that indicated by the number of pointings.

Azimuth at Bruce.

AZIMUTH OF LINE BRUCE—LONG TAIL POINT LIGHT.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1.]

Star, date, &c.	Telescope—	Azimuth of mark.	Means.	Result for star.
		° ′ ″	° ′ ″	° ′ ″
Polaris, near East Elongation, July 12, 1872.	Direct	139 16 05.34		
$\alpha = 1^h\ 11^m\ 57^s.05$		05.47		
$\delta = 88°\ 37'\ 21''.7$		05.80		
		06.13		
		03.69		
		05.43		
		05.58		
		08.02		
		04.21		
		06.35		
		07.04		
		05.42	139 16 05.71	
	Reversed ...	06.50		
		05.69		
		07.33		
		08.28		
		08.40		
		10.18		
		06.04		
		06.82		
		06.91		
		06.49		
		07.13		
		07.02	07.23	139 16 06.47
Polaris, near East Elongation, July 14, 1872.	Direct	00.09		
$\alpha = 1^h\ 11^m\ 58^s.76$		00.61		
$\delta = 88°\ 37'\ 21''.8$		02.62		
		03.30		
		01.64		
		03.62		
		04.53		
		04.60	02.63	
	Reversed ...	06.24		
		06.12		
		06.54		
		05.79		
		04.66		
		04.57		
		02.81		
		03.84	05.07	03.85

Azimuth at Bruce—Continued.

Star, date, &c.	Telescope—	Azimuth of mark.	Means.	Result for star.
		° ′ ″	° ′ ″	° ′ ″
δ Ursæ Minoris, near West Elongation, July 14, 1872.	Direct	139 16 06.26		
$\alpha=18^h\ 13^m\ 54^s.53$		04.79		
$\delta=86°\ 36'\ 27''.7$		03.65		
		04.24		
		02.67		
		04.63		
		03.16		
		02.83	139 16 04.03	
	Reversed ...	05.59		
		07.54		
		06.15		
		07.29		
		06.49		
		07.76		
		09.21		
		09.64	07.46	139 16 054.74
ε Ursæ Minoris, near West Elongation, July 14, 1872.	Direct	06.90		
$\alpha=16^h\ 59^m\ 16^s.97$		06.20		
$\delta=82°\ 14'\ 47''.0$		04.40		
		04.00	05.38	
	Reversed ...	05.10		
		03.60		
		05.00		
		06.50	05.05	05.21
Polaris, near East Elongation, July 16, 1872.	Direct	04.89		
$\alpha=1^h\ 12^m\ 00^s.72$		04.63		
$\delta=88°\ 37'\ 22''.1$		05.06		
		05.17		
		04.51		
		03.25		
		04.10		
		03.01	04.33	
	Reversed ...	05.04		
		05.23		
		05.98		
		05.33		
		04.22		
		02.82		
		05.20		
		04.44	04.78	04.55

Azimuth at Bruce.

SUMMARY OF RESULTS.

AZIMUTH OF LINE BRUCE—LONG TAIL POINT LIGHT.

Date.	Star, &c.	Position of telescope.	Number of pointings— To mark.	Number of pointings— To star.	Azimuth. Declinations from American Ephemeris.	Reduction to Auwers' declinations. $\frac{dA}{d\delta}$	$\Delta\delta$	ΔA	Corrected azimuth.	Weight.
					° ′				° ′	
1872.					139 16				139 16	
					″					
July 12	Polaris, near East Elongation.	Direct	7	12						
		Reversed ...	7	12	05.37	−1.37	+0.04	−0.05	05.32	3
14	......do	Direct	6	8						
		Reversed ...	4	8	03.72	−1.37	+0.04	−0.05	03.67	2
16	......do	Direct	6	8						
		Reversed ...	8	8	03.87	−1.37	+0.04	−0.05	03.82	2
14	δ Ursæ Minoris, near West Elongation.	Direct	3	8						
		Reversed ...	4	8	05.96	+1.39	+0.33	+0.46	06.42	1.5
14	ε Ursæ Minoris, near West Elongation.	Direct	3	4						
		Reversed ...	4	4	05.68	+1.42	−0.04	−0.06	05.62	1

	° ′ ″ ″
Weighted mean	139 16 04.59±0.33.
Correction for diurnal aberration	+ 0.31
Azimuth of line Bruce—Long Tail Point Light	139 16 04.90±0.33 west of south.

The weighted mean of the results in the above table is 139° 16′ 04″.59±0″.33, the probable error being derived from the discrepancies between the separate results and the weighted mean. Applying to this mean the correction +0″.31 for diurnal aberration, there results for the azimuth of the line Bruce—Long Tail Point Light-house,

139° 16′ 04″.90±0″.33 west of south.

AZIMUTH AT MINNESOTA JUNCTION.

§ **7.** Observations for azimuth were made at Minnesota Junction station on September 3, 4, 6, 7, and 8, 1873, by Assistant Engineer A. R. Flint. The instrument used was the theodolite No. 1, by Repsold & Sons. It was mounted vertically over the geodetic point of the triangulation station on the post previously used in latitude determinations. The stars observed were α, δ, and λ Ursæ Minoris and 51 Cephei. The method of observation was substantially the same as that followed at Ford River and Bruce stations, §§ 5, 6, except that the telescope was usually reversed midway between each series of pointings to the azimuth mark. Time was given by a chronometer. The azimuth mark was a light limited by a slit about one-half inch wide in the box containing the lamp, and about 2 miles distant from the azimuth post.

In the reduction, the differences between the azimuths of the star at the times of observation and at elongation were computed, and these differences were applied to the circle-readings on the star to give the corresponding circle-readings for azimuth at elongation. The mean of these readings for one position of the telescope, subtracted from the mean of the corresponding readings on the mark, gives the horizontal angle between the star at elongation and the mark.

In the following tables the first column gives the date of observation, the star observed, its right ascension, declination, and azimuth at elongation, the latter being denoted by A_e. The second column gives the readings on the mark for each position of the telescope and their means. The third gives the readings on the star reduced to elongation and their means. The fourth gives the mean angles between the star at elongation and the mark for each position of the telescope and their means, and the fifth gives the result from each star for azimuth of mark. The individual results in the fourth column are affected by collimation, accidental and periodic errors of graduation, and by such errors as arise from lack of stability of the instrument or the post supporting it during the interval between the pointings to the star and to the mark. The position of the telescope is indicated in the tables by the letters D and R.

Azimuth at Minnesota Junction.

AZIMUTH OF LINE MINNESOTA JUNCTION—AZIMUTH MARK.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1.]

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Polaris, near East Elongation, September 3, 1873.	D. 291 23 23.00	D. 180 24 21.43		
$\alpha = 1^h\ 12^m\ 52^s.12$	21.25	22.68		
$\delta = 88°\ 37'\ 54''.7$	21.50	21.23		
$A_e = 181°\ 53'\ 07''.84$	21.25	21.46		
	21.00	22.50		
	21.60	21.80		
	Means..291 23 21.60	180 24 21.85	110 58 59.75	
	R. 111 23 19.95	R. 00 24 13.34		
	19.00	14.19		
	20.50	12.32		
	19.70	12.21		
	20.00	12.37		
	19.25	11.79		
	Means..111 23 19.73	00 24 12.70	110 59 07.03	
	Mean	....	110 59 03.39	292 52 11.23
Polaris, near East Elongation, September 4, 1873.	D. 111 54 42.40	R. 180 55 44.27		
$\alpha = 1^h\ 12^m\ 52^s.55$	39.00	43.11		
$\delta = 88°\ 37'\ 55''.1$	40.20	43.07		
$A_e = 181°\ 53'\ 07''.29$	40.50	41.98		
	R. 291 54 45.00	41.58		
	45.75	43.11		
	43.50	43.16		
	45.25	41.98		
	....	42.31		
	Means..291 54 42.70	180 55 42.73	110 58 59.97	
	D. 111 54 43.75	D. 00 55 44.19		
	44.50	44.87		
	45.25	45.81		
	45.00	46.52		
	R. 291 54 48.50	46.61		
	49.70	46.32		
	50.75	46.96		
	50.00	47.06		
	....	47.78		
	Means..111 54 47.18	00 55 46.24	110 59 00.94	
	Mean	....	110 59 00.46	292 52 07.75

Azimuth at Minnesota Junction—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
δ Ursæ Minoris, near West Elongation, September 4, 1873.	R. 232 28 05.50	D. 294 55 29.85		
	07.00	29.83		
	06.25	28.34		
$\alpha=18^h\ 13^m\ 17^s.20$	05.50	28.18		
$\delta=86^\circ\ 36'\ 36''.9$	D. 52 28 02.25	29.10		
$A_e=175^\circ\ 19'\ 34''.80$	02.00	29.23		
	03.10			
	03.60			
	Means .. 52 28 04.40	294 55 29.09	117 32 35.31	
	R. 232 28 05.25	R. 114 55 27.16		
	07.50	26.86		
	07.85	28.73		
	07.25	28.98		
	D. 52 28 05.25	29.28		
	06.50	26.76		
	05.75			
	05.75			
	Means .. 232 28 06.39	114 55 27.96	117 32 38.43	
	Mean	....	117 32 36.87	292 52 11.67
51 Cephei, near East Elongation, September 4, 1873.	D. 351 57 56.25	R. 62 54 34.26		
	56.00	33.36		
$\alpha=6^h\ 40^m\ 16^s.42$	57.50	33.85		
$\delta=87^\circ\ 14'\ 00''.4$	R. 171 58 00.75	33.49		
$A_e=183^\circ\ 48'\ 49''.52$	00.55	33.39		
	01.45	32.80		
		33.98		
		34.36		
	Means .. 171 57 58.75	62 54 33.69	109 03 25.06	
	D. 351 57 58.25	D. 242 54 33.96		
	59.50	33.97		
	59.50	35.15		
	R. 171 58 02.25	36.77		
	03.50	34.57		
	03.50	34.63		
		36.71		
		36.55		
	Means .. 351 58 01.08	242 54 35.29	109 03 25.79	
	Mean	....	109 03 25.43	292 52 14.95

Azimuth at Minnesota Junction—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
λ Ursæ Minoris, near West Elongation, September 4, 1873.	D. 351 57 58.25	D. 237 37 13.68		
	59.50	13.99		
	59.50	15.39		
$\alpha = 19^h\ 51^m\ 29^s.90$	R. 171 58 02.25	14.39		
$\delta = 88°\ 55'\ 43''.8$	03.50	14.48		
$A_e = 178°\ 31'\ 25''.73$	03.50	14.21		
		15.96		
		16.31		
	Means.. 351 58 01.08	237 37 14.80	114 20 46.28	
	R. 171 58 04.75	R. 57 37 16.42		
	05.50	14.59		
	05.50	15.13		
	D. 351 58 01.50	15.81		
	02.00	15.64		
	01.25	15.87		
		16.26		
		15.54		
	Means.. 171 58 03.42	57 37 15.66	114 20 47.76	
	Mean		114 20 47.02	292 52 12.75
Polaris, near East Elongation, September 6, 1873.	D. 88 11 02.75	R. 157 12 05.66		
$\alpha = 1^h\ 12^m\ 53^s.40$	03.65	05.77		
$\delta = 88°\ 37'\ 55''.7$	03.00	05.74		
$A_e = 181°\ 53'\ 06''.56$	03.75	05.77		
	R. 268 11 12.75	06.66		
	12.45	05.69		
	13.00	07.35		
	13.00	06.88		
		06.05		
		05.59		
		06.52		
	Means.. 268 11 08.04	157 12 06.15	110 59 01.89	
	D. 88 11 06.00	D. 337 12 03.38		
	06.55	03.82		
	06.35	04.89		
	07.25	05.42		
	R. 268 11 17.00	05.77		
	17.00	05.26		
	16.75	04.68		
	16.25	04.52		
		04.34		
		05.18		
		06.27		
	Means.. 88 11 11.64	337 12 04.87	110 59 06 77	
	Mean		110 59 04.33	292 52 10.89

Azimuth at Minnesota Junction—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
λ Ursæ Minoris, near West Elongation, September 6, 1873.	R. 208 35 17.75	D. 274 14 27.80		
	18.95	28.61		
$\alpha = 19^h\ 51^m\ 27^s.81$	19.00	27.03		
$\delta = 88°\ 55'\ 44''.2$	D. 28 35 12.50	27.05		
$A_e = 178°\ 31'\ 26''.28$	12.75	27.30		
	13.50	26.91		
	Means .. 28 35 15.74	274 14 27.45	114 20 48.29	
	R. 208 35 21.00	R. 94 14 29.76		
	22.00	31.66		
	21.50	31.10		
	D. 28 35 14.00	30.73		
	13.75	32.17		
	13.70	30.94		
	Means .. 208 35 17.66	94 14 31.06	114 20 46.60	
	Mean	....	114 20 47.45	292 52 13.73
Polaris, near East Elongation, September 7, 1873.	R. 176 29 39.75	D. 245 30 35.89		
	38.70	34.73		
$\alpha = 1^h\ 12^m\ 53^s.86$	39.20	36.59		
$\delta = 88°\ 37'\ 56''.0$	38.95	37.22		
$A_e = 181°\ 53'\ 06''.14$	D. 356 29 40.75	37.02		
	40.05	35.45		
	40.25	36.28		
	40.75	36.09		
	40.50	37.18		
	40.50	35.85		
	Means .. 356 29 39.94	245 30 36.23	110 59 03.71	
	R. 176 29 40.75	R. 65 30 30.50		
	41.10	30.91		
	41.25	30.60		
	40.75	30.53		
	41.40	30.35		
	41.60	30.13		
	D. 356 29 43.00	30.38		
	42.25	31.32		
	42.15	31.37		
	41.50	31.51		
	Means .. 176 29 41.56	65 30 30.76	110 59 10.80	
	Mean	....	110 59 07.26	292 52 13.40

Azimuth at Minnesota Junction—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
δ Ursæ Minoris, near West Elongation, September 7, 1873.	R. 262 38 43.75	D. 325 06 10.22		
$\alpha=18^h\,13^m\,16^s.04$	43.50	09.03		
$\delta=86°\,36'\,37''.1$	44.75	09.53		
$A_e=175°\,19'\,35''.08$	44.50	10.06		
	D. 82 38 45.60	08.93		
	45.20	09.06		
	45.10	09.07		
	45.80	09.07		
	Means .. 82 38 44.78	325 06 09.37	117 32 35.41*	
	R. 262 38 46.50	R. 145 06 07.42		
	46.90	08.10		
	48.20	07.81		
	47.00	09.02		
	D. 82 38 47.00	08.22		
	46.50	07.74		
	46.75	06.55		
	47.00	08.22		
	Means .. 262 38 46.98	145 06 07.89	117 32 39.09	
	Mean	....	117 32 37.25	292 52 12.33
51 Cephei, near East Elongation, September 7, 1873.	R. 262 38 46.50	D. 333 35 27.20		
$\alpha=6^h\,40^m\,17^s.81$	46.90	27.85		
$\delta=87°\,14'\,00''.0$	48.20	27.94		
$A_e=183°\,48'\,50''.08$	47.00	28.17		
	D. 82 38 47.00	28.38		
	46.50	29.92		
	46.75	28.96		
	47.00	28.29		
	47.75	28.58		
	47.75	28.16		
	48.20	28.50		
	Means .. 82 38 47.23	333 35 28.36	109 03 18.87	
	R. 262 38 47.75	R. 153 35 27.97		
	49.50	28.59		
	49.00	27.98		
	48.75	27.84		
	49.05	28.19		
	49.00	28.42		
	49.00	27.65		
	D. 82 38 48.75	27.25		
	49.25	27.27		
	48.95	26.54		
	48.75	27.33		
	Means .. 262 38 48.89	153 35 27.73	109 03 21.16	
	Mean	....	109 03 20.02	292 52 10.10

Azimuth at Minnesota Junction—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
λ Ursæ Minoris, near West Elongation, September 7, 1873.	D. 30 54 56.05	R. 96 34 05.17		
	56.25	05.94		
	57.50	07.03		
$\alpha=19^h\ 51^m\ 26^s.83$	57.00	07.04		
$\delta=88°\ 55'\ 44''.4$	R. 210 54 54.50	05.46		
$A_e=178°\ 31'\ 26''.55$	54.50	05.27		
	54.75	08.28		
	55.00	08.13		
	54.50	08.19		
	53.75	07.79		
		07.44		
	Means . . 210 54 55 38	96 34 06.89	114 20 48.49	
	D. 30 54 58.75	D. 276 34 13.42		
	58.75	13.42		
	59.00	13.91		
	59.50	13.81		
	59.25	13.30		
	60.00	13.39		
	R. 210 54 56.95	13.64		
	57.25	13.72		
	57.50	12.75		
	57.30	12.94		
		14.26		
	Means . . . 30 54 58.43	276 34 13.51	114 20 44.92	
	Mean	. . . .	114 20 46.70	292 52 13.25
Polaris, near East Elongation, September 8, 1873.	D. 29 50 12.00	D. 278 51 08.19		
	12.50	07.70		
$\alpha=1^h\ 12^m\ 54^s.36$	11.50	08.83		
$\delta=88°\ 37'\ 56''.3$	11.50	08.60		
$A_e=181°\ 53'\ 05''.73$	12.50	09.13		
	12.50	09.50		
	12.00	09.75		
		09.52		
	Means . . . 29 50 12.07	278 51 08.90	110 59 03.17	
	R. 209 50 04.50	R. 98 50 52.92		
	04.75	53.90		
	04.35	53.02		
	05.50	53.11		
	06.75	52.43		
	06.75	54.03		
	06.00	51.45		
	06.50	52.26		
	Means . . 209 50 05.63	98 50 52.89	110 59 12.74	
	Mean	. . . .	110 59 07.96	292 52 13.69

Azimuth at Minnesota Junction—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
δ Ursae Minoris, near West Elongation, September 8, 1873.	D. 252 49 50.00	D. 135 17 19.76		
	49.50	21.03		
	49.75	19.74		
$\alpha = 18^h\ 13^m\ 15^s.66$	49.60	20.62		
$\delta = 86^\circ\ 36'\ 37''.2$		21.33		
$A_e = 175^\circ\ 19'\ 35''.22$		19.67		
		19.50		
	Means..252 49 49.71	135 17 20.25	117 32 29.40	
	R. 72 49 44.65	R. 315 17 05.63		
	45.75	06.53		
	45.75	04.34		
	45.25	05.80		
		04.18		
		06.25		
		05.41		
	Means.. 72 49 45.35	315 17 05.45	117 32 39.90	
	Mean	....	117 32 34.68	292 52 09.90
51 Cephei, near East Elongation, September 8, 1873.	R. 72 49 46.25	R. 323 46 19.98		
	46.20	18.23		
$\alpha = 6^h\ 40^m\ 18^s.26$	45.75	17.98		
$\delta = 87^\circ\ 13'\ 59''.9$	46.00	20.25		
$A_e = 183^\circ\ 48'\ 50''.21$		19.93		
	Means.. 72 49 46.05	323 46 19.27	109 03 26.78	
	D. 252 49 54.00	D. 143 46 34.50		
	54.25	33.27		
	53.75	33.64		
	53.60	33.51		
		33.65		
	Means..252 49 53.90	143 46 33.71	109 03 20.19	
	Mean	....	109 03 23.49	292 52 13.70

The next table gives a summary of the preceding table, the results in the sixth column being corrected for periodic error by means of the formula given in Chapter XV, B, § 4. A correction is also applied in this table on account of errors in the declinations taken from the American Ephemeris, as explained in § 4. Weights proportional to the number of pointings on the star are assigned to the separate results.

Azimuth at Minnesota Junction.

SUMMARY OF RESULTS.

AZIMUTH OF LINE MINNESOTA JUNCTION — AZIMUTH MARK.

Date.	Star, &c.	Position of telescope.	Number of pointings— To mark.	Number of pointings— To star.	Azimuth. Declinations from American Ephemeris.	Reduction to Auwers' declinations. $\frac{dA}{d\delta}$	$\Delta\delta$	ΔA	Corrected azimuth of mark.	Weight.
1873.					° ′ 292 52				° ′ 292 52	
					″		″	″	″	
Sept. 3	Polaris, near East Elongation.	Direct	6	6						
		Reversed ...	6	6	12.93	−1.38	+0.04	−0.06	12.87	0.6
4	......do	Direct	8	9						
		Reversed ...	8	9	09.45	−1.38	+0.04	−0.06	09.39	0.9
6	......do	Direct	8	11						
		Reversed ...	8	11	10.90	−1.38	+0.04	−0.06	10.84	1.1
7	......do	Direct	10	10						
		Reversed ...	10	10	13.42	−1.38	+0.04	−0.06	13.36	1.0
8	......do	Direct	7	8						
		Reversed ...	8	8	12.57	−1.38	+0.04	−0.06	12.51	0.8
4	δ Ursæ Minoris, near West Elongation.	Direct	8	6						
		Reversed ...	8	6	10.45	+1.38	+0.32	+0.44	10.89	0.6
7	......do	Direct	8	8						
		Reversed ...	8	8	11.98	+1.38	+0.32	+0.44	12.42	0.8
8	......do	Direct	4	7						
		Reversed ...	4	7	09.02	+1.38	+0.32	+0.44	09.46	0.7
4	λ Ursæ Minoris, near West Elongation.	Direct	6	8						
		Reversed ...	6	8	12.46	+1.38	+0.59	+0.81	13.27	0.8
6	......do	Direct	6	6						
		Reversed ...	6	6	12.99	+1.38	+0.59	+0.81	13.80	0.6
7	......do	Direct	10	11						
		Reversed ...	10	11	12.36	+1.38	+0.59	+0.81	13.17	1.1
4	51 Cephei, near East Elongation.	Direct	6	8						
		Reversed ...	6	8	14.99	−1.38	+0.22	−0.30	14.69	0.8
7	......do	Direct	11	11						
		Reversed ...	11	11	09.87	−1.38	+0.22	−0.30	09.57	1.1
8	......do	Direct	4	5						
		Reversed ...	4	5	13.31	−1.38	+0.22	−0.30	13.01	0.5

		° ′ ″	″
Weighted mean		292 52 11.99	±0.32
Diurnal aberration	+	0.31	
Azimuth of mark		292 52 12.30	±0.32

From these results and weights the weighted mean 292° 52′ 11″.99 ± 0″.32 is derived, the probable error being computed from the differences between the separate results and weighted mean. Applying to this mean the correction +0″.31 for diurnal aberration, there results for azimuth of mark

292° 52′ 12″.30 ± 0″.32 west of south.

The angle between the primary line Minnesota Junction–Horicon and the line Minnesota Junction–Azimuth Mark was carefully measured with the instrument used in the azimuth work, twenty-two combined measures of it being made, giving the mean value

$$20^\circ\ 21'\ 27''.275.$$

In the adjustment of the triangulation between Fond du Lac Base and the line Minnesota Junction–Horicon, a weight unity was assigned to the mean value of an angle resulting from twenty combined measures; and the probable error of the *measured* value of an angle of weight unity, shown by this adjustment, is $\pm 0''.38$. (Chapter XV, C, § 7.) Assigning this probable error to the above angle between Horicon station and the azimuth mark, there results for the azimuth of the line Minnesota Junction–Horicon,

$$\mathbf{272^\circ\ 30'\ 45''.03 \pm 0''.50} \text{ west of south.}$$

AZIMUTH AT WILLOW SPRINGS.

§ **8.** Azimuth determinations were made at Willow Springs station on the nights of October 2, 3, 4, and 5, 1874, by Assistant Engineer A. R. Flint. The instrument used was the Repsold theodolite No. 1. It was mounted vertically over the geodetic point of the triangulation station on a heavy oak post. The stars observed were α, δ, and λ Ursæ Minoris and 51 Cephei. The method of observation was precisely the same as that followed at Minnesota Junction (§ 7). Time was given by a chronometer. The azimuth mark was a light limited by a vertical slit about one-fourth inch wide in the box containing the lamp, the mark being about 1 mile distant from the azimuth post. The observations were reduced in the same manner as those made at Minnesota Junction, and the following tables give the results arranged in the same form as those shown in § 7, to which reference may be made for a more detailed explanation. The mean result for a star given in the fifth column, however, is here corrected for diurnal aberration.

Azimuth at Willow Springs.

AZIMUTH OF LINE WILLOW SPRINGS—AZIMUTH MARK.

[Observer, A. R. Flint. Instrument, Repsold theodolite No. 1.]

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Polaris, near East Elongation, October 2, 1874.	D. 46 10 03.90	R. 112 56 54.51		
α=1^h 13^m 21^s.08	04.25	53.45		
δ=88° 38′ 26″.4	04.10	55.16		
A_e=181° 49′ 17″.46	04.25	54.65		
		52.98		
	R. 226 10 08.00	54.31		
	08.00	53.82		
	08.70	54.32		
	07.50			
	Means.. 10 06.09	56 54.17	113 13 11.92	
	D. 46 10 04.50	D. 292 56 53.99		
	05.15	54.07		
	04.33	54.19		
	04.50	53.67		
		54.51		
	R. 226 10 08.50	54.66		
	09.75	52.92		
	10.25	53.88		
	09.50			
	Means.. 10 07.06	56 53.99	13.07	
	Mean	....	113 13 12.50	295 02 30.27
δ Ursæ Minoris, near West Elongation, October 2, 1874.	R. 226 10 04.00	D. 286 34 58.73		
α=18^h 12^m 43^s.97	04.00	59.76		
δ=86° 36′ 37″.5	03.50	58.10		
A_e=175° 27′ 22″.17	05.00	35 01.92		
		34 59.59		
	D. 46 10 00.25	35 00.37		
	02.50	34 59.93		
	00.75	35 00.45		
	03.00			
	Means.. 10 02.88	34 59.86	119 35 03.02	
	R. 226 10 07.50	R. 106 34 55.03		
	03.30	55.39		
	04.00	55.04		
	05.25	56.25		
		56.03		
	D. 46 10 00.30	55.36		
	02.70	55.90		
	02.90	55.92		
	02.50			
	Means.. 10 03.56	34 55.62	07.94	
	Mean	....	119 35 05.48	295 02 27.96

Azimuth at Willow Springs—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
51 Cephei, near East Elongation, October 2, 1874.	D. 0 03 20.00	R. 68 43 30.72		
	20.50	29.03		
$\alpha = 6^h\ 41^m\ 04^s.33$	21.00	30.88		
$\delta = 87°\ 13'\ 55''.90$	20.50	28.86		
$A_e = 183°\ 42'\ 35''.11$		28.70		
	R. 180 03 23.50	29.41		
	21.75	29.45		
	23.00	29.32		
	23.50			
	Means.. 03 21.72	43 29.55	111 19 52.17	
	D. 0 03 19.00	D. 248 43 30.12		
	21.00	30.30		
	22.00	31.04		
	19.50	31.54		
		33.72		
	R. 180 03 25.30	29.94		
	25.50	32.99		
	24.75	30.89		
	24.50			
	Means.. 03 22.69	43 31.32	51.37	
	Mean	....	111 19 51.77	295 02 27.19
λ Ursæ Minoris, near West Elongation, October 2, 1874.	R. 180 03 23.20	D. 243 35 08.33		
	24.70	08.45		
$\alpha = 19^h\ 49^m\ 47^s.74$	24.50	07.73		
$\delta = 88°\ 55'\ 57''.70$	23.00	08.12		
$A_e = 178°\ 34'\ 11''.42$		09.17		
	D. 0 03 21.50	13.71		
	20.50	10.66		
	23.50	19.01		
	23.00			
	Means.. 03 22.99	09.40	116 28 13.59	
	R. 180 03 24.25	R. 63 35 08.53		
	23.30	08.54		
	22.50	05.84		
	26.00	03.68		
		03.12		
	D. 0 03 22.00	04.39		
	23.70	04.17		
	23.00	05.56		
	22.00			
	Means.. 23.34	05.48	17.86	
	Mean	....	116 28 15.73	295 02 27.46

Azimuth at Willow Springs—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Polaris, near East Elongation, October 3, 1874.	D. 60 04 38.00	R. 126 51 21.30		
$\alpha = 1^h\ 13^m\ 21^s.27$	37.50	25.35		
$\delta = 88°\ 38'\ 26''.80$	38.25	23.76		
$A_e = 181°\ 49'\ 16''.92$	38.00	22.76		
	R. 240 04 37.00	24.40		
	36.50	22.85		
	35.50	23.07		
	34.80	25.07		
	Means .. 04 36.94	23.57	113 13 13.37	
	D. 60 04 40.00	D. 306 51 27.48		
	38.50	29.58		
	39.70	29.26		
	35.00	28.61		
		27.33		
	R. 240 04 35.80	27.24		
	35.00	26.92		
	39.50	26.99		
	35.50			
	Means .. 04 37.38	51 27.89	09.49	
	Mean	....	113 13 11.43	295 02 28.66
δ Ursæ Minoris, near West Elongation, October 3, 1874.	R. 240 04 35.50	D. 300 29 33.53		
$\alpha = 18^h\ 12^m\ 43^s.51$	35.30	30.13		
$\delta = 86°\ 36'\ 37''.40$	34.90	30.31		
$A_e = 175°\ 27'\ 22''.09$	34.80	31.49		
		28.89		
	D. 60 04 36.75	32.63		
	35.50	31.55		
	37.80	31.66		
	37.30			
	Means .. 04 35.98	31.27	119 35 04.71	
	R. 240 04 35.00	R. 120 29 31.24		
	35.00	30.24		
	36.70	30.71		
	35.00	29.03		
		30.32		
	D. 60 04 35.50	27.00		
	36.00	29.87		
	36.00	30.43		
	35.90			
	Means .. 04 35.64	29.86	05.78	
	Mean	....	119 35 05.24	295 02 27.64

Azimuth at Willow Springs—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
51 Cephei, near East Elongation, October 3, 1874.	D. 90 01 15.50	R. 158 41 21.19		
	16.30	16.06		
$\alpha = 6^h\ 41^m\ 04^s.94$	15.90	14.01		
$\delta = 87°\ 13'\ 55''.90$	15.30	14.01		
$A_e = 183°\ 42'\ 35''.07$		21.31		
	R. 270 01 16.50	20.34		
	16.60	23.44		
	14.50	22.12		
	15.40			
	Means.. 01 15.75	41 19.07	111 19 56.68	
	D. 90 01 14.50	D. 338 41 22.68		
	15.00	21.06		
	12.00	19.69		
	13.50	21.32		
		19.75		
	R. 270 01 15.50	20.66		
	14.50	21.21		
	13.30	21.23		
	14.00			
	Means.. 01 14.04	41 20.96	53.08	
	Mean	....	111 19 54.88	295 02 30.26
λ Ursæ Minoris, near West Elongation, October 3, 1874.	R. 270 01 16.50	D. 333 32 57.84		
	14.00	57.88		
$\alpha = 19^h\ 49^m\ 46^s.30$	14.50	58.27		
$\delta = 88°\ 55'\ 57''.80$	14.75	59.82		
$A_e = 178°\ 34'\ 11''.55$		58.00		
	D. 90 01 11.50	57.44		
	13.50	58.56		
	14.00	59.16		
	14.00			
	Means.. 01 14.09	32 58.57	116 28 15.72	
	R. 270 01 14.50	R. 153 32 58.79		
	14.60	56.28		
	14.50	57.66		
	14.30	57.69		
		57.18		
	D. 90 01 13.50	57.42		
	13.00	57.55		
	13.50	58.34		
	14.50			
	Means.. 01 14.05	32 57.61	16.44	
	Mean	....	116 28 16.08	295 02 27.94

Azimuth at Willow Springs—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Polaris, near East Elongation, October 4, 1874.	D. 140 01 32. 50	R. 206 48 14. 08		
	30. 00	14. 54		
$\alpha=1^h\ 13^m\ 21^s.41$	29. 50	13. 27		
$\delta=88°\ 38'\ 27''.20$	29. 30	13. 92		
$A_e=181°\ 49'\ 16''.40$		15. 11		
	R. 320 01 27. 00	14. 55		
	28. 00	15. 52		
	28. 00	15. 94		
	27. 80			
	Means.. 01 29. 03	48 14. 62	113 13 14. 41	
	D. 140 01 31. 50	D. 26 48 18. 83		
	29. 00	17. 80		
	29. 50	17. 50		
	31. 50	18. 01		
		18. 32		
	R. 320 01 28. 50	18. 34		
	27. 00	17. 98		
	27. 20	17. 95		
	27. 00			
	Means.. 01 28. 90	48 18. 09	10. 81	
	Mean	....	113 13 12. 61	295 02 29. 32
δ Ursæ Minoris, near West Elongation October 4, 1874.	D. 140 01 32. 50	R. 200 26 23. 17		
$\alpha=18^h\ 12^m\ 43^s.05$	33. 00	22. 32		
$\delta=86°\ 36'\ 37''.4$	32. 00	24. 98		
$A_e=175°\ 27'\ 22''.00$	33. 30	22. 22		
		22. 37		
	R. 320 01 29. 00	24. 57		
	27. 20	22. 45		
	30. 00	20. 03		
	31. 15			
	Means.. 01 31. 02	26 22. 76	119 35 08. 26	
	D. 140 01 32. 50	D. 20 26 26. 87		
	32. 00	26. 96		
	33. 00	26. 34		
	34. 00	26. 61		
		26. 55		
	R. 320 01 27. 50	27. 87		
	27. 20	28. 29		
	28. 00	28. 67		
	29. 50			
	Means.. 01 30. 46	26 27. 27	03. 19	
	Mean	....	119 35 05. 72	295 02 28. 03

Azimuth at Willow Springs—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
51 Cephei, near East Elongation, October 4, 1874.	R. 339 59 45.20	D. 48 39 55.64		
$\alpha=6^h\ 41^m\ 05^s.52$	43.50	56.08		
$\delta=87°\ 13'\ 56''.00$	44.50	57.30		
$A_e=183°\ 42'\ 35''.02$	44.70	57.22		
		56.38		
	D. 159 59 47.20	55.70		
	46.00	55.60		
	48.50	55.93		
	47.20			
	Means.. 59 45.85	39 56.25	111 19 49.60	
	R. 339 59 41.50	R. 228 39 43.74		
	40.70	45.87		
	41.90	47.32		
	41.00	45.80		
		48.91		
	D. 159 59 45.70	45.74		
	44.50	47.35		
	44.80	47.50		
	45.50			
	Means.. 59 43.20	39 46.53	56.67	
	Mean ...		111 19 53.13	295 02 28.46
δ Ursæ Minoris, near West Elongation, October 5, 1874.	D. 159 59 45.00	R. 220 24 27.70		
$\alpha=18^h\ 12^m\ 42^s.61$	46.00	28.09		
$\delta=86°\ 36'\ 37''.3$	47.00	26.62		
$A_e=175°\ 27'\ 21''.91$	47.50	27.10		
		26.04		
	R. 339 59 37.00	26.82		
	37.50	26.86		
	36.00	26.46		
	35.00			
	Means.. 59 41.38	24 26.96	119 35 14.42	
	D. 159 59 46.00	D. 40 24 38.12		
	47.00	38.45		
	46 00	37.87		
	46.00	35.28		
		37.27		
	R. 339 59 35.00	37.59		
	34.50	38.62		
	34.00	38.71		
	35.50			
	Means.. 59 40.50	24 37.74	35 02.76	
	Mean ...		119 35 08.59	295 02 30.81

Azimuth at Willow Springs—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
51 Cephei, near East Elongation, October 5, 1874.	R. 339 59 38.50	D. 48 39 56.42		
$\alpha = 6^h\ 41^m\ 06^s.08$	38.00	57.03		
$\delta = 87°\ 13'\ 56''.0$	38.00	57.20		
$A_e = 183°\ 42'\ 34''.98$	39.00	40 00.27		
		39 58.79		
	D. 159 59 49.00	57.32		
	49.80	57.86		
	50.00	58.22		
	49.20			
	Means .. 59 43.94	39 57.90	111 19 46.04	
	R. 339 59 38.00	R. 228 39 40.19		
	38.80	42.98		
	39.20	43.00		
	39.20	43.21		
		43.21		
	D. 159 59 49.00	42.91		
	50.30	43.19		
	49.00	43.17		
	49.20			
	Means .. 59 44.09	39 42.73	111 20 01.36	
	Mean	....	111 19 53.70	295 02 28.99
λ Ursæ Minoris, near West Elongation, October 5, 1874.	D. 170 02 09.00	R. 233 33 39.02		
$\alpha = 19^h\ 49^m\ 43^s.45$	09.80	40.08		
$\delta = 88°\ 55'\ 58''.00$	09.00	39.35		
$A = 178°\ 34'\ 11''.82$	09.00	39.41		
		39.53		
	R. 350 01 58.50	40.16		
	58.50	40.32		
	58.50	40.79		
	58.00			
	Means .. 02 03.79	33 39.83	116 28 23.96	
	D. 170 02 09.20	D. 53 33 56.95		
	08.50	56.53		
	09.00	55.70		
	09.00	54.49		
		55.46		
	R. 350 01 58.30	53.85		
	56.90	56.14		
	58.70	56.64		
	58.50			
	Means .. 02 03.51	33 55.72	07.79	
	Mean	....	116 28 15.88	295 02 28.01

The next table gives a summary of the results in the preceding. The results for azimuth of mark in this table have each been corrected for periodic error by the formula given in Chapter XV, B, § 4. They are also corrected for the errors in the declinations taken from the American Ephemeris, as explained in § 4. They still involve such residual errors as are not eliminated by reversal of telescope, as may be due to inaccurate indications of the striding level, and such as arise from instability of the instrument or its support between the pointings to the star and to the mark. As may be seen from the readings on the azimuth mark, the collimation of the instrument remained very steady or changed very slowly during any night, and its effect must therefore have been well eliminated by the reversals. The corrections for inclination of telescope axis were usually small; the maximum correction was 6″.6, and the average, 2″.1.

Azimuth at Willow Springs.

SUMMARY OF RESULTS.

AZIMUTH OF LINE WILLOW SPRINGS—AZIMUTH MARK.

Date.		Star, &c.	Position of telescope.	Number of pointings— To mark.	To star.	Azimuth. Declinations from American Ephemeris.	Reduction to Auwers' declinations. $\frac{dA}{d\delta}$	$\Delta\delta$	ΔA	Corrected azimuth of mark.
						° ′				° ′
1874.						295 02				295 02
						″		″	″	″
Oct.	2	Polaris, near East Elongation	Direct	8	8					
			Reversed	8	8	29.01	−1.32	+0.04	−0.05	28.96
	3	......do	Direct	8	8					
			Reversed	8	8	27.68	−1.32	+0.04	−0.05	27.63
	4	......do	Direct	8	8					
			Reversed	8	8	29.93	−1.32	+0.04	−0.05	29.88
	2	δ Ursæ Minoris, near West Elongation.	Direct	8	8					
			Reversed	8	8	26.95	+1.35	+0.32	+0.43	27.38
	3	......do	Direct	8	8					
			Reversed	8	8	26.31	+1.35	+0.32	+0.43	26.74
	4	......do	Direct	8	8					
			Reversed	8	8	28.80	+1.35	+0.32	+0.43	29.23
	5	......do	Direct	8	8					
			Reversed	8	8	30.55	+1.35	+0.32	+0.43	30.98
	2	51 Cephei, near East Elongation	Direct	8	8					
			Reversed	8	8	27.28	−1.35	+0.21	−0.28	27.00
	3	......do	Direct	8	8					
			Reversed	8	8	30.31	−1.35	+0.21	−0.28	30.03
	4	......do	Direct	8	8					
			Reversed	8	8	27.96	−1.35	+0.21	−0.28	27.68
	5	......do	Direct	8	8					
			Reversed	8	8	28.50	−1.35	+0.21	−0.28	28.22
	2	λ Ursæ Minoris, near West Elongation.	Direct	8	8					
			Reversed	8	8	27.31	+1.35	+0.59	+0.80	28.11
	3	......do	Direct	8	8					
			Reversed	8	8	28.09	+1.35	+0.59	+0.80	28.89
	5	......do	Direct	8	8					
			Reversed	8	8	27.63	+1.35	+0.59	+0.80	28.43

Weighted mean........ 295° 02′ 28″.511 ± 0″.223

Assigning equal weights to the individual results in the last table, their mean is

$$295^\circ\ 02'\ 28''.51 \pm 0''.22,$$

the probable error being derived from the discrepancies between the individual results and their mean. This is the azimuth of the mark from the azimuth post or Willow Springs station. In the measurement of the angles of the primary triangulation at this station, the direction of the azimuth mark was determined by the following angles:

Military Academy — Willow Springs — Azimuth Mark.
Azimuth Mark — Willow Springs — Lombard.
Shot Tower — Willow Springs — Azimuth Mark.
Azimuth Mark — Willow Springs — Shot Tower.

The adjusted value of the last of these angles is

$$289^\circ\ 27'\ 12''.73$$

(see Chapter XVI, C). The weight assigned to the observed value of this angle in the adjustment is 1, and the probable error of an observed angle of weight 1 shown by this adjustment is $\pm 0''.44$ (Chapter XVI, C, § 11). The probable error of this angle is somewhat less after adjustment than before, but is not as small as the probable error of an adjusted angle of average weight, inasmuch as it does not enter directly into so many equations of condition. It will be sufficient if we assign $\pm 0''.44$ as its probable error. There results for the azimuth of the triangle-side Willow Springs — Shot Tower,

$$\mathbf{224^\circ\ 29'\ 41''.24 \pm 0''.49} \text{ west of south.}$$

AZIMUTH AT PARKERSBURG.

§ **9.** Two series of azimuth determinations were made at Parkersburg station in the year 1879. The first series was made during the nights of August 9, 10, 11, 12, 13, 16, and 17, and during the days of August 11, 12, and 17. The second series was made during the nights of November 20, 23, 24, 25, and 29, and during the days of November 20, 21, 24, and 25. The observations were made by Assistant Engineer A. R. Flint, with the Troughton & Simms theodolite No. 1. Time was given by a chronometer. The stars observed were α, δ, and λ Ursæ Minoris and 51 Cephei. The theodolite was mounted on a cut limestone post 5 feet long, 22 inches square, and set so as to project about 2 feet above the surface of the ground. For the night observations the azimuth mark was a light set vertically over a reference-point on the upper surface of a cut-stone post 3 feet long and 1 foot square. The latter stone was set so that its upper surface was about 15 inches below the ground surface, and it was about 2 miles distant from the theodolite, in a westerly direction. The mark to which the day observations were referred was a target nailed to a tree, about 11 miles distant from the theodolite, in an easterly direction. At the time of the first series of determinations the stations in that part of the triangulation had not been erected, and for this reason the azimuth could not then be referred to a triangulation-line. In order to check any possible movements of the observing-post and of the post over which the west mark was placed, stakes with small-headed tacks set in their tops were driven about them, and careful measurements of the relative positions of the points of reference on the posts and the tacks were made. For the observing-post seven such stakes were driven, four so that the heads of the tacks lay in the direction of the west azimuth mark, and three so that the heads of the tacks lay in a line at right angles to that direction. For the reference-post four stakes were set in such a manner that two lines defined by the tacks intersected vertically over the point fixed on the stone. The target constituting the east azimuth mark was referred to a spike driven into the base of the tree to which the target was nailed. Early in November, in erecting the triangulation-station at Parkersburg, the observing-post was disturbed so as to change the azimuth of the line from it to the west reference-stone by about 1″. As this disturbance made the connection of the azimuth observed in August, with a triangulation-line, depend wholly on the stability of the reference stakes, a second series of azimuth determinations was ordered and made. When the tacks were examined, however, their relative positions were found unchanged, and Assistant Engineer Flint, who made the examination, was of the opinion that the position of the instrument in August could be redetermined within 0.05 inch. In the November series the theodolite was set as nearly as possible in that position, which was taken as the center also of the trigonometrical station. As no change in the positions of the azimuth marks from August to November was indicated by their references above named, the directions of the lines from the theodolite to the marks are assumed to have been the same, respectively, during both series of determinations. In making the observations the following programme was adhered to as nearly as circumstances would permit:

1. Reading on mark.
2. Reading on star, with corresponding chronometer time.
3. Level readings, direct and reversed.
4. Reading on star as above.
5. Reading on mark.
6. Telescope transited and alidade revolved 180°.
7. Reading on mark.
8. Reading on star.
9. Level readings.
10. Reading on star.
11. Reading on mark.

This programme was then repeated in the reverse order. The readings on the mark for each star were the same during one night. On each night the circle reading was changed $\frac{120°}{n}$ where n represents the number of nights intended for a series, being five for the August series and four for the November series. Observations were made each night to determine chronometer error. The reduction of observations was performed in a manner similar to that followed at West Base, Sandusky, § 10. The cöordinates of the stars were taken from the American Ephemeris for 1879.

In the following table the first column gives the star, date, and cöordinates of star; the second, the corrected sidereal times of observation; the third, the readings on the mark; the fourth, the readings on the star; the fifth, the resulting angles between the star and the mark; the sixth, the computed azimuths of the star at the times of observation; the seventh, the correction for inclination of telescope axis; the eighth, the resulting azimuth of the mark for each observation and the means for each star; the ninth, the resulting azimuth of the mark for each star on correcting the mean for aberration, run of micrometers, and periodic error when it is not eliminated in the mean result for the several nights.

The first section of the table gives the observations on the west azimuth mark, made at night; and the second section of the table gives the observations on the east azimuth mark, made in the day time.

Azimuth at Parkersburg.

I.—AZIMUTH OF LINE PARKERSBURG—WEST AZIMUTH MARK.

[Observer, A. R. Flint. Instrument, Troughton & Simms theodolite No. 1.]

Star, date, &c.	Time of observation.	Reading on mark.	Reading on star.	Angle between mark and star.	Azimuth of star.	Level correction.	Azimuth of mark.	Result for star.
	h. m. s.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″	° ′ ″
Polaris, near East	18 59 01.58	D. 1 28 04.10	D. 71 37 35.33	70 09 31.23	181 42 07.59	+0.18	111 32 36.54	
Elongation, Au-	19 02 59.57	02.13	43.00	40.87	16.03	+0.18	35.34	
gust 9, 1879.	15 35.54	R. 181 27 59.13	R. 251 37 57.17	58.04	30.70	−0.34	32.32	
$\alpha = 1^h\,14^m\,59^s.24$	18 39.54	28 00.80	58.00	57.20	31.44	−0.34	33.90	
$\delta = 88^\circ\,39'\,51''.53$	28 57.52	R. 181 28 00.30	R. 251 37 52.97	52.67	25.96	−0.89	32.40	
	32 13.51	27 59.33	47.93	48.60	21.62	−0.89	32.13	
	42 42.49	D. 1 28 00.17	D. 71 37 26.23	26.06	41 59.21	−0.16	32.99	
	45 30.48	27 59.13	18.30	19.17	51.07	−0.16	31.74	
Mean							111 32 33.42	111 32 32.96
Polaris, near East	18 55 00.51	D. 25 55 06.67	D. 96 04 23.80	70 09 17.13	181 41 56.77	−0.92	111 32 38.72	
Elongation, Au-	57 32.50	04.03	31.07	27.04	42 03.54	−0.92	35.58	
gust 10, 1879.	19 06 20.49	R. 205 55 03.00	R. 276 04 44.80	41.80	21.34	−5.75	33.79	
$\alpha = 1^h\,15^m\,00^s.17$	08 58.48	04.23	49.30	45.07	24.92	−5.75	34.10	
$\delta = 88^\circ\,39'\,51''.82$	15 16.47	R. 205 55 06.50	R. 276 04 54.13	47.63	30.17	−4.60	37.94	
	18 56.47	03.40	55.03	51.63	31.10	−4.60	34.87	
	33 26.44	D. 25 55 08.37	D. 96 04 52.40	44.03	19.33	0.00	35.30	
	35 47.44	10.54	49.23	38.69	15.11	0.00	36.42	
Mean							111 32 35.84	111 32 36.38
Polaris, near East	19 09 44.02	D. 229 59 12.60	D. 300 09 05.13	70 09 52.53	181 42 25.50	+1.04	111 32 34.01	
Elongation, Au-	11 54.01	15.57	09.60	54.03	27.62	+1.04	34.63	
gust 11, 1879.	18 49.99	R. 49 59 16.63	R. 120 09 05.43	48.80	30.81	−0.41	41.60	
$\alpha = 1^h\,15^m\,01^s.09$	21 28.99	13.90	02.53	48.63	30.53	−0.41	41.49	
$\delta = 88^\circ\,39'\,52''.05$	26 36.98	R. 49 59 12.83	R. 120 09 02.30	49.47	27.67	−0.69	37.51	
	29 04.97	14.57	08 58.23	43.66	25.20	−0.69	40.85	
	35 49.95	D. 229 59 13.60	D. 300 08 58.73	45.13	14.77	+2.26	31.90	
	39 12.95	14.77	50.07	35.30	07.53	+2.26	34.49	
Mean							111 32 37.06	111 32 37.35
Polaris, near East	18 58 42.73	R. 253 27 18.40	R. 323 36 51.30	70 09 32.90	181 42 05.72	−0.58	111 32 32.24	
Elongation, Au-	19 01 08.73	16.90	58.27	41.37	11.22	−0.58	29.27	
gust 12, 1879.	08 56.71	D. 73 27 17.67	D. 143 37 01.80	44.13	24.20	−0.58	39.49	
$\alpha = 1^h\,15^m\,01^s.97$	12 19.70	12.73	02.83	50.10	27.62	−0.58	36.94	
$\delta = 88^\circ\,39'\,52''.32$	18 30.69	D. 73 27 14.23	D. 143 37 08.10	53.87	30.44	+0.46	37.03	
	22 03.68	14.60	08.27	53.67	30.02	+0.46	36.81	
	29 44.67	R. 253 27 21.77	R. 323 37 11.60	49.83	24.06	−1.84	32.39	
	32 17.66	20.90	06.63	45.73	20.57	−1.84	33.00	
Mean							111 32 34.65	111 32 34.96
Polaris, near East	19 03 45.90	R. 97 26 20.43	R. 167 36 04.00	70 09 43.57	181 42 16.05	−0.92	111 32 31.56	
Elongation, Au-	05 57.90	19.97	01.90	41.93	19.76	−0.92	36.91	
gust 13, 1879.	16 05.87	D. 277 26 16.55	D. 347 36 14.73	58.18	29.58	−0.35	31.05	
$\alpha = 1^h\,15^m\,02^s.79$	17 57.87	16.97	17.20	10 00.23	30.06	−0.35	29.48	
$\delta = 88^\circ\,39'\,52''.55$	24 57.86	D. 277 26 18.93	D. 347 36 12.60	09 53.67	28.31	−0.46	34.18	
	26 49.85	18.83	12.57	53.74	26.87	−0.46	32.67	
	33 27.84	R. 97 26 22.77	D. 167 36 01.07	38.30	18.45	−2.76	37.39	
	35 47.84	21.80	35 58.03	36.23	14.25	−2.76	35.26	
Mean							111 32 33.56	111 32 33.87

I.—*Azimuth at Parkersburg*—Continued.

Star, date, &c.	Time of observation.	Reading on mark.	Reading on star.	Angle between mark and star.	Azimuth of star.	Level correction.	Azimuth of mark.	Result for star.
	h. m. s.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″	° ′ ″
Polaris, near East	19 10 02.82	D. 181 34 03.67	D. 251 43 48.97	70 09 45.30	181 42 24.11	−3.42	111 32 35.39	
Elongation, Au-	12 03.81	04.53	50.93	46.40	26.07	−3.42	36.25	
gust 16, 1879.	18 54.80	R. 1 34 13.97	R. 71 44 07.53	53.56	29.05	−1.38	34.11	
$\alpha=1^h\ 15^m\ 04^s.97$	21 26.79	11.27	07.50	56.23	28.91	−1.38	31.30	
$\delta=88°\ 39'\ 53''.33$	27 16.78	R. 1 34 13.13	R. 71 44 04.83	51.70	25.48	−1.84	31.94	
	29 16.77	14.30	02.40	48.10	23.39	−1.84	33.45	
	36 24.76	D. 181 34 04.57	D. 251 43 40.60	36.03	12.11	−1.61	34.47	
	39 10.75	05.53	34.97	29.44	06.13	−1.61	35.08	
Mean							111 32 34.00	111 32 34.32
Polaris, near West	6 51 16.40	D. 326 55 24.40	D. 33 41 38.00	66 46 13.60	178 18 38.03	+3.09	111 32 27.52	
Elongation, No-	53 33.91	25.33	30.43	05.10	33.03	+3.09	31.02	
vember 23, 1879.	7 00 58.44	R. 146 55 23.60	R. 213 41 12.43	45 48.83	21.05	−0.23	31.99	
$\alpha=1^h\ 15^m\ 24^s.04$	03 05.45	26.67	08.40	41.73	18.79	−0.23	36.83	
$\delta=88°\ 40'\ 27''.85$	12 37.49	R. 146 55 26.30	R. 213 41 03.20	36.90	15.11	−1.25	36.96	
	15 34.00	24.73	03.50	38.77	16.11	−1.25	36.09	
	24 29.53	D. 326 55 26.37	D. 33 41 28.17	46 01.80	25.29	+6.84	30.33	
	26 50.55	22.20	29.00	06.80	29.24	+6.84	29.28	
Mean							111 32 32.50	111 32 33.01
51 Cephei, near	0 40 10.86	D. 1 27 58.73	D. 73 28 03.67	72 00 04.94	183 32 33.43	+3.22	111 32 31.71	
East Elongation,	42 33.83	28 01.97	09.67	07.70	39.71	+3.22	35.23	
August 9, 1879.	55 02.81	R. 181 28 01.47	R. 253 28 17.17	15.70	50.10	+1.04	35.44	
$\alpha=6^h\ 43^m\ 22^s.06$	58 33.80	03.20	14.23	11.03	46.13	+1.04	36.14	
$\delta=87°\ 13'\ 39''.08$								
Mean							111 32 34.63	111 32 34.11
51 Cephei, near	0 27 03.28	D. 229 59 12.07	D. 301 58 08.50	71 58 56.37*	183 31 35.19	−4.21	111 32 34.61	
East Elongation,	30 11.27	11.77	28.40	59 16.57	53.01	−4.21	32.23	
August 11, 1879.	38 31.25	R. 49 59 13.20	R. 121 58 54.73	41.51	32 29.04	−5.52	42.01	
$\alpha=6^h\ 43^m\ 22^s.86$	41 43.24	13.43	59 06.27	52.84	38.43	−5.52	40.07	
$\delta=87°\ 13'\ 38''.48$	47 31.23	R. 49 59 14.67	R. 121 59 25.27	72 00 10.61	49.12	−1.27	37.24	
	50 18.22	13.10	29.20	16.12	51.34	−1.27	33.95	
	1 00 45.20	D. 229 59 11.73	D. 301 59 28.00	16.28	42.98	+3.08	29.78	
	03 53.19	13.37	20.23	06.86	35.30	+3.08	31.52	
Mean							111 32 35.18	111 32 35.49
51 Cephei, near	0 32 05.05	D. 73 27 13.23	D. 145 26 37.90	71 59 24.67	183 32 03.07	−0.62	111 32 37.70	
East Elongation,	34 46.04	12.83	50.27	37.44	15.21	−0.62	37.07	
August 12, 1879.	44 48.02	R. 253 27 20.40	R. 325 27 33.10	72 00 12.70	45.46	−0.53	32.15	
$\alpha=6^h\ 43^m\ 23^s.28$	46 58.02	21.17	34.53	13.36	48.80	−0.53	34.83	
$\delta=87°\ 13'\ 38''.24$	52 38.01	R. 253 27 21.00	R. 325 27 39.40	18.40	52.23	−2.42	31.33	
	54 08.00	20.50	38.77	18.27	51.76	−2.42	30.99	
	1 04 23.98	D. 73 27 15.83	D. 145 27 09.80	71 59 53.97	34.23	−0.64	39.54	
	06 55.97	11.93	00.17	48.24	25.99	−0.64	37.03	
Mean							111 32 35.16	111 32 35.39†

* Includes correction for run of micrometers.

† Includes also corrections of $-0''.08$ for error in declination used in computation.

I.—*Azimuth at Parkersburg*—Continued.

Star, date, &c.	Time of observation.	Reading on mark.	Reading on star.	Angle between mark and star.	Azimuth of star.	Level corrections.	Azimuth of mark.	Result for star.
	h. m. s.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″	° ′ ″
51 Cephei, near	0 43 39.22	R. 97 26 25.57	R. 169 26 30.83	72 00 05.26	183 32 43.48	−0.35	111 32 37.87	
East Elongation,	46 01.22	26.97	35.17	08.20	47.66	−0.35	39.11	
August 13, 1879.	56 51.21	D. 277 26 19.67	D. 349 26 34.63	14.96	49.80	−3.45	31.39	
$\alpha=6^h\ 43^m\ 23^s.72$	58 39.19	20.23	32.90	12.67	47.40	−3.45	31.28	
$\delta=87°\ 13'\ 38''.05$	1 03 57.18	D. 277 26 20.77	D. 349 26 21.00	00.23	35.74	−1.73	33.78	
	05 27.18	18.27	17.03	71 59 58.76	31.14	−1.73	30.65	
	14 37.16	R. 97 26 25.20	R. 169 25 39.03	13.83	31 51.43	−2.53	35.07	
	17 22.15	29.40	23.33	58 53.93	35.50	−2.53	39.04	
Mean							111 32 34.77	111 32 35.08
51 Cephei, near	0 35 55.11	R. 1 34 12.67	R. 73 33 59.27	71 59 46.60	183 32 20.77	−2.07	111 32 32.10	
East Elongation,	38 48.11	12.33	34 10.50	58.17	31.14	−2.07	30.90	
August 16, 1879.	47 00.09	D. 181 34 04.03	D. 253 34 14.83	72 00 10.80	49.80	−3.34	35.66	
$\alpha=6^h\ 43^m\ 25^s.01$	49 51.08	04.00	16.70	12.70	52.45	−3.34	36.41	
$\delta=87°\ 13'\ 37''.43$	55 48.07	D. 181 34 05.10	D. 253 34 15.83	10.73	51.70	−3.54	37.43	
	58 53.06	04.53	14.50	09.97	47.88	−3.54	34.37	
	1 07 58.04	R. 1 34 12.57	R. 73 34 00.67	71 59 48.10	23.21	−0.92	34.19	
	11 12.03	12.00	33 48.73	36.73	09.53	−0.92	31.88	
Mean							111 32 34.12	111 32 34.43
51 Cephei, near	0 28 09.03	D. 25 27 09.40	D. 97 26 15.53	71 59 06.13	183 31 43.29	−2.71	111 32 34.45	
East Elongation,	30 39.02	06.70	29.80	23.10	56.87	−2.71	31.06	
August 17, 1879.	37 35.01	R. 205 27 14.40	R. 277 27 09.97	55.57	32 27.23	−2.30	29.36	
$\alpha=6^h\ 43^m\ 25^s.40$	40 12.00	13.60	17.90	72 00 04.30	35.69	−2.30	29.09	
$\delta=87°\ 13'\ 37''.25$	50 27.98	R. 205 27 12.30	R. 277 27 34.10	21.80	52.97	−2.88	28.29	
	53 34.97	13.13	36.87	23.74	53.17	−2.88	26.55	
	1 02 34.95	D. 25 27 09.77	D. 97 27 13.60	03.83	40.50	−2.07	34.60	
	05 11.95	09.40	05.40	71 59 56.00	33.12	−2.07	35.05	
Mean							111 32 31.06	111 32 31.37
51 Cephei, near	0 44 07.82	D. 117 22 10.67	D. 189 22 22.00	72 00 11.33	183 32 46.65	+1.49	111 32 36.81	
East Elongation,	46 27.83	10.47	24.53	14.06	51.02	+1.49	38.45	
November 20, 1879.	54 33.84	R. 297 22 07.27	R. 9 22 37.20	29.93	56.10	+2.75	28.92	
	56 43.84	10.10	30.33	29.23	54.67	+2.75	28.19	
$\alpha=6^h\ 44^m\ 12^s.88$	1 02 42.85	R. 297 22 10.00	R. 9 22 26.37	16.37	45.03	+1.37	30.03	
$\delta=87°\ 13'\ 34''.94$	04 43.86	09.33	18.87	09.54	39.83	+1.37	31.66	
	12 57.86	D. 117 22 08.40	D. 189 21 38.50	71 59 30.10	08.11	+1.42	39.43	
	15 12.87	10.93	29.60	18.67	31 56.60	+1.42	39.35	
Mean							111 32 34.10	111 32 33.63
51 Cephei, near	0 36 38.95	D. 177 02 02.90	D. 249 01 49.67	71 59 46.77	183 32 22.34	+0.92	111 32 36.49	
East Elongation,	39 46.95	01 55.97	02 03.50	72 00 07.53	33.68	+0.92	27.07	
November 24, 1879.	47 01.45	R. 357 02 02.00	R. 69 02 17.83	15.83	50.68	+2.86	37.71	
	53 05.95	00.50	28.03	27.53	55.14	+2.86	30.41	
$\alpha=6^h\ 44^m\ 14^s.30$	58 33.95	R. 357 01 57.83	R. 69 02 24.60	26.77	51.54	+2.76	27.53	
$\delta=87°\ 13'\ 35''.86$	1 00 39.45	02 00.07	17.30	17.23	48.17	+2.76	33.70	
	12 27.45	D. 177 01 56.90	D. 249 01 38.33	71 59 41.43	09.49	+4.35	32.41	
	14 37.45	58.77	26.90	28.13	31 58.66	+4.35	34.88	
Mean							111 32 32.53	111 32 32.67

I.—*Azimuth at Parkersburg*—Continued.

Star, date, &c.	Time of observation.	Reading on mark.	Reading on star.	Angle between mark and star.	Azimuth of star.	Level corrections.	Azimuth of mark.	Result for star.
	h. m. s.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″	° ′ ″
51 Cephei, near	0. 32 43. 86	D. 27 15 41. 37	D. 99 15 23. 63	71 59 42. 26	183 32 04. 78	+1. 60	111 32 24. 12	
East Elongation,	34 52. 86	43. 87	32. 23	48. 36	14. 76	+1. 60	28. 00	
November 25,	41 24. 86	R. 207 15 43. 13	R. 279 15 48. 03	72. 00 04. 90	38. 42	−1. 37	32. 15	
1879.	45 34. 86	44. 77	54. 53	09. 76	48. 09	−0. 69	37. 64	
$\alpha=6^h\ 44^m\ 14^s.64$	50 58. 86	R. 207 15 44. 13	R. 279 16 01. 70	17. 57	54. 41	−2. 45	34. 39	
$\delta=87^\circ\ 13'\ 35''.98$	53 43. 36	43. 50	02. 37	18. 87	54. 95	−2. 45	33. 63	
	1 03 17. 86	D. 27 15 47. 17	D. 99 16 04. 00	16. 83	42. 35	+3. 32	28. 84	
	10 24. 36	48. 27	15 39. 60	71 59 51. 33	18. 50	+2. 75	29. 92	
Mean	....	....	....	....	....	....	111 32 31. 09	111 32 31. 82
51 Cephei, at East	0 37 34. 90	D. 60 32 22. 53	D. 132 32 15. 07	71 59 52. 54	183 32 24. 77	−2. 29	111 32 29. 94	
Elongation, No-	39 22. 90	22. 80	22. 00	59. 20	31. 14	−2. 29	29. 65	
vember 29, 1879.	46 29. 40	R. 240 32 27. 37	R. 312 32 33. 57	72. 00 06. 20	48. 66	−6. 87	35. 59	
$\alpha=6^h\ 44^m\ 16^s.16$	48 48. 40	28. 27	35. 40	07. 13	51. 73	−6. 87	37. 73	
$\delta=87^\circ\ 13'\ 36''.76$	55 28. 40	R. 240 32 26. 07	R. 312 32 42. 07	16. 00	53. 39	+0. 80	38. 19	
	57 09. 90	27. 70	41. 70	14. 00	52. 08	+0. 80	38. 88	
	1 02 55. 40	D. 60 32 19. 90	D. 132 32 39. 00	19. 10	42. 36	+5. 50	38. 76	
	05 08. 90	20. 83	30. 27	09. 44	36. 43	+5. 50	32. 49	
Mean	....	....	....	....	....	....	111 32 35. 15	111 32 34. 63
δ Ursæ Minoris,	0 44 37. 93	R. 205 55 07. 87	R. 270 07 12. 00	64 12 04. 13	175 44 36. 57	+1. 12	111 32 33. 56	
near West Elon-	48 05. 92	08. 00	57. 93	49. 93	45 22. 87	+1. 12	34. 06	
gation, August	56 50. 91	D. 25 55 08. 87	D. 90 10 10. 67	15 01. 80	47 34. 57	+0. 78	33. 55	
10, 1879.	59 50. 90	09. 00	11 02. 33	53. 33	48 24. 55	+0. 78	32. 00	
$\alpha=18^h\ 11^m\ 19^s.26$	1 07 50. 89	D. 25 55 07. 93	D. 90 13 27. 60	18 19. 67	50 50. 31	+1. 46	32. 10	
$\delta=86^\circ\ 36'\ 42''.66$	10 13. 88	08. 50	14 12. 83	19 04. 33	51 37. 04	+1. 46	34. 17	
	20 05. 87	R. 205 55 03. 17	R. 270 17 43. 07	22 39. 90	55 07. 20	+0. 11	27. 41	
	23 05. 86	07. 20	18 50. 67	23 43. 47	56 16. 16	+0. 11	32. 80	
Mean	....	....	....	....	....	....	111 32 32. 45	111 32 32. 87
δ Ursæ Minoris,	23 41 17. 38	D. 229 59 15. 90	D. 294 07 33. 80	64 08 18. 01*	175 40 46. 09	+2. 65	111 32 30. 73	
near West Elon-	45 11. 37	14. 43	13. 50	07 59. 19	26. 12	+2. 65	29. 58	
gation, August	52 31. 36	R. 49 59 15. 63	R. 114 06 39. 37	23. 90	00. 66	+2. 76	39. 52	
11, 1879.	55 03. 35	15. 87	38. 63	22. 92	39 55. 64	+2. 76	35. 48	
$\alpha=18^h\ 11^m\ 18^s.92$	0 04 23. 33	R. 49 59 15. 57	R. 114 06 36. 43	21. 02	53. 45	+3. 15	35. 58	
$\delta=86^\circ\ 36'\ 42''.87$	07 18. 32	16. 20	39. 47	23. 43	58. 05	+3. 15	37. 77	
	15 36. 30	D. 229 59 16. 13	D. 294 07 13. 10	57. 10	40 25. 09	+5. 18	33. 07	
	13 13. 30	12. 83	26. 90	08 14. 18	37. 74	+5. 18	28. 74	
Mean	....	....	....	....	....	....	111 32 33. 81	111 32 34. 12
δ Ursæ Minoris,	23 47 58. 13	D. 73 27 13. 50	D. 137 34 51. 70	64 07 38. 20	175 40 15. 03	−1. 56	111 32 35. 27	
near West Elon-	50 36. 13	16. 20	44. 43	28. 23	06. 22	−1. 56	36. 43	
gation, August	00 24. 11	R. 253 27 20. 93	R. 317 34 39. 03	18. 10	39 51. 69	−1. 50	32. 09	
12, 1879.	02 55. 10	20. 77	40. 90	20. 13	52. 65	−1. 50	31. 02	
$\alpha=18^h\ 11^m\ 18^s.54$	10 06. 09	R. 253 27 21. 43	R. 317 34 54. 80	33. 37	40 05. 41	−2. 19	29. 85	
$\delta=86^\circ\ 36'\ 43''.15$	12 20. 08	21. 10	35 00. 03	38. 93	12. 53	−2. 19	31. 41	
	21 14. 07	D. 73 27 14. 93	D. 137 35 30. 60	08 15. 67	55. 26	−0. 46	39. 13	
	24 04. 06	15. 43	52. 23	36. 80	41 13. 76	−0. 46	36. 50	
Mean	....	....	....	....	....	....	111 32 33. 96	111 32 34. 27

* Includes correction for run of micrometers.

I.—*Azimuth at Parkersburg*—Continued.

Star, date, &c.	Time of observation.	Reading on mark.	Reading on star.	Angle between mark and star.	Azimuth of star.	Level corrections.	Azimuth of mark.	Result for star.
	h. m. s.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″	° ′ ″
δ Ursæ Minoris,	23 45 37.34	R. 97 26 23.57	R. 161 34 14.00	64 07 50.43	175 40 24.83	+0.46	111 32 34.86	
near West Elongation, August	48 27.33	26.23	01.97	35.74	13.41	+0.46	38.13	
13, 1879.	0 01 34.31	D. 277 26 22.43	D. 341 33 41.03	18.60	39 52.11	−1.84	31.67	
	05 54.30	23.33	45.77	22.44	56.33	−1.84	32.05	
$\alpha=18^h\ 11^m\ 18^s.17$	11 32.29	D. 277 26 23.73	D. 341 33 59.73	36.00	40 10.03	−2.88	31.15	
$\delta=86°\ 36'\ 43''.35$	13 57.28	24.33	34 09.77	45.44	18.81	−2.88	30.49	
	34 46.24	R. 97 26 28.33	R. 161 36 32.30	10 03.97	42 44.85	+0.12	41.00	
	36 50.23	26.67	53.00	26.33	43 06.20	+0.12	39.99	
Mean							111 32 34.92	111 32 35.23
δ Ursæ Minoris,	23 42 56.22	R. 1 34 14.27	R. 65 42 15.97	64 08 01.70	175 40 37.82	0.00	111 32 36.12	
near West Elongation, August	45 40.22	14.60	06.33	07 51.73	25.05	0.00	33.32	
16, 1879.	53 28.21	D. 181 34 05.50	D. 245 41 24.00	18.50	39 59.74	−3.97	37.27	
	56 58.20	05.50	21.73	16.23	54.41	−3.97	34.21	
$\alpha=18^h\ 11^m\ 17^s.28$	0 03 14.18	D. 181 34 04.80	D. 245 41 19.77	14.97	53.71	−3.91	34.83	
$\delta=86°\ 36'\ 43''.85$	06 02.17	06.00	23.13	17.13	57.15	−3.91	36.11	
	15 08.15	R. 1 34 12.33	R. 65 42 02.43	50.10	40 24.43	−1.15	33.18	
	17 52.15	13.07	16.43	08 03.36	37.32	−1.15	32.81	
Mean							111 32 34.73	111 32 35.13
δ Ursæ Minoris,	23 48 01.11	D. 25 27 07.67	D. 89 34 47.80	64 07 40.13	175 40 15.76	−2.76	111 32 32.87	
near West Elongation, August	50 18.11	08.60	42.23	33.63	08.10	−2.76	31.71	
17, 1879.	56 57.10	R. 205 27 11.37	R. 269 34 39.93	28.56	39 54.57	+0.12	26.13	
	58 47.09	14.87	39.17	24.30	53.18	+0.12	29.00	
$\alpha=18^h\ 11^m\ 16^s.74$	0 02 43.08	R. 205 27 14.90	R. 269 34 37.77	22.87	53.55	+0.46	31.14	
$\delta=86°\ 36'\ 44''.02$	04 25.08	15.43	40.20	24.77	55.09	+0.46	30.78	
	10 35.07	D. 25 27 10.77	D. 89 34 41.43	30.66	40 07.96	−1.50	35.80	
	13 31.06	05.57	52.40	46.83	18.05	−1.50	29.72	
Mean							111 32 30.89	111 32 31.09
δ Ursæ Minoris,	23 47 47.72	D. 117 22 48.80	D. 181 30 28.20	64 07 39.40	175 40 10.75	+3.80	111 32 33.15	
near West Elongation, November 20, 1879.	54 04.73	51.87	27.77	35.90	39 52.93	+3.80	20.83	
	0 05 00.75	R. 297 21 29.87	R. 1 28 40.67	10.80	53.54	+3.34	46.08	
	07 59.75	24.63	45.97	21.34	59.03	+3.34	41.03	
$\alpha=18^h\ 10^m\ 39^s.21$	14 51.76	R. 297 21 26.13	R. 1 29 07.27	41.14	40 22.75	+4.14	45.75	
$\delta=86°\ 36'\ 41''.22$	18 13.76	27.40	25.37	57.97	39.29	+4.14	45.47	
	27 23.77	D. 117 22 52.03	D. 181 32 12.90	09 20.87	41 41.26	+2.99	24.38	
	29 07.78	49.80	25.57	35.77	55.72	+2.99	22.94	
Mean							111 32 34.96	111 32 35.27
δ Ursæ Minoris,	1 18 21.40	D. 326 55 27.47	D. 31 17 43.90	64 22 16.43	175 54 40.59	+4.48	111 32 28.64	
near West Elongation, November 23, 1879.	21 17.41	25.27	18 47.33	23 22.06	55 47.23	+4.48	29.65	
	30 37.44	R. 146 55 24.00	R. 211 22 17.67	26 53.67	59 34.55	−1.57	39.31	
	32 38.95	23.50	23 15.40	27 51.90	176 00 26.79	−1.57	33.32	
$\alpha=18^h\ 10^m\ 38^s.43$	38 19.47	R. 146 55 23.70	R. 211 25 48.83	30 25.13	02 59.04	+0.11	34.02	
$\delta=86°\ 36'\ 40''.34$	40 45.98	23.93	26 56.80	31 32.87	04 07.17	+0.11	34.41	
	48 42.49	D. 326 55 23.23	D. 31 30 57.30	35 34.07	07 58.88	+5.04	29.85	
	51 44.50	24.60	32 28.53	37 03.93	09 31.56	+5.04	32.67	
Mean							111 32 32.73	111 32 32.94

I.—*Azimuth at Parkersburg*—Continued.

Star, date, &c.	Time of observation.	Reading on mark.	Reading on star.	Angle between mark and star.	Azimuth of star.	Level corrections.	Azimuth of mark.	Result for star.
	h. m. s.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″	° ′ ″
δ Ursæ Minoris,	23 55 26.95	D. 177 02 04.40	D. 241 09 21.20	64 07 16.80	175 39 50.66	+0.46	111 32 34.32	
near West Elon-	58 27.95	00.50	15.20	14.70	48.10	+0.46	33.86	
gation, Novem-	0 04 53.95	R. 357 02 01.43	R. 61 09 24.90	23.47	51.69	+2.28	30.50	
ber 24, 1879.	06 54.95	01.93	30.10	28.17	55.35	+2.28	29.46	
$\alpha=18^h\ 10^m\ 38^s.23$	12 32.95	R. 357 02 01.73	R. 61 09 46.00	44.27	40 11.86	+1.71	29.30	
$\delta=86°\ 36'\ 40''.12$	13 58.95	01.27	51.60	50.33	17.55	+1.71	28.93	
	23 48.95	D. 177 02 01.17	D. 241 10 39.83	08 38.66	41 12.83	+1.60	35.77	
	25 40.95	02.23	54.10	51.87	26.55	+1.60	36.28	
Mean	....	....	....	....	....	....	111 32 32.31	111 32 32.59
δ Ursæ Minoris,	23 49 59.88	D. 27 15 42.57	D. 91 23 19.47	64 07 36.90	175 40 01.74	+1.37	111 32 26.21	
near West Elon-	52 53.38	42.23	13.33	31 10	39 54.62	+1.37	24.89	
gation, Novem-	0 00 02.88	R. 207 15 42.20	R. 271 22 54.73	12.53	47.48	−0.91	34.04	
ber 25, 1879.	02 37.88	42.83	55.73	12.90	48.67	−0.91	34.86	
$\alpha=18^h\ 10^m\ 38^s.00$	08 28.87	R. 207 15 41.67	R. 271 23 04.17	22.50	58.70	−2.44	33.76	
$\delta=86°\ 36'\ 39''.84$	10 41.87	44.20	11.27	27.07	40 05.07	−2.44	35.56	
	17 53.37	D. 27 15 42.90	D. 91 23 55.50	08 12.60	35.77	+1.14	24.31	
	20 22.87	41.50	24 09.63	28.13	50.00	+1.14	23.01	
Mean	....	....	....	....	....	....	111 32 29.58	111 32 29.97
δ Ursæ Minoris,	23 58 46.40	D. 60 32 20.60	D. 124 39 41.43	64 07 20.83	175 39 46.35	+2.72	111 32 28.24	
near West Elon-	0 01 22.90	20.33	43.43	23.10	46.59	+2.72	26.21	
gation, Novem-	08 48.90	R. 240 32 26.00	R. 304 39 48.63	22.63	58.26	+1.36	36.99	
ber 29, 1879.	10 26.90	27.63	53.80	26.17	40 02.97	+1.36	38.16	
$\alpha=18^h\ 10^m\ 36^s.97$	15 19.90	R. 240 32 26.67	R. 304 40 12.23	45.56	21.89	+0.23	36.56	
$\delta=86°\ 36'\ 38''.84$	17 12.90	27.67	21.20	53.53	31.14	+0.23	37.84	
	23 20.40	D. 60 32 21.90	D. 124 41 04.13	08 42.23	41 08.05	+4.77	30.59	
	24 38.90	22.70	12.80	50.10	17.41	+4.77	32.08	
Mean	....	....	....	....	....	....	111 32 33.33	111 32 33.16
λ Ursæ Minoris,	1 25 19.94	D. 73 27 15.83	D. 140 33 49.33	67 06 33.50	178 39 10.62	−0.11	111 32 37.01	
near West Elon-	27 45.93	12.17	47.13	34.96	07.18	−0.11	32.11	
gation, August	36 29.92	R. 253 27 21.23	R. 320 33 50.17	28.94	38 59.33	+1.50	31.89	
12, 1879.	38 46.91	23.63	48.43	24.80	58.43	+1.50	35.13	
$\alpha=19^h\ 45^m\ 13^s.47$								
$\delta=88°\ 56'\ 39''.40$								
Mean	....	....	....	....	....	....	111 32 34.04	111 32 34.51
λ Ursæ Minoris,	1 28 01.00	R. 1 34 12.03	R. 68 40 48.03	67 06 36.00	178 39 08.32	−2.84	111 32 29.48	
near West Elon-	29 43.99	11.63	43.73	32 10	06.26	−2.84	31.33	
gation, August	40 52.97	D. 181 34 03.13	D. 248 40 26.40	23.27	38 59.73	−4.00	32.46	
16, 1879.	44 13.96	05.30	25.77	20.47	59.83	−4.00	35.36	
$\alpha=19^h\ 45^m\ 10^s.12$	55 17.94	D. 181 34 04.10	D. 248 40 35.33	31.23	39 07.94	−3.20	33.51	
$\delta=88°\ 56'\ 40''.58$	58 24.93	04.07	41.07	37.00	12.27	−3.20	32.07	
	2 09 41.91	R. 1 34 11.73	R. 68 41 15.57	07 03.84	35.42	−2.15	29.43	
	13 15.90	13.63	24.67	11.04	45.16	−2.15	31.97	
Mean	....	....	....	....	....	....	111 32 31.95	111 32 31.76

I.—*Azimuth at Parkersburg*—Continued.

Star, date, &c.	Time of observation.	Reading on mark.	Reading on star.	Angle between mark and star.	Azimuth of star.	Level corrections.	Azimuth of mark.	Result for star.
	h. m. s.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″	
λ Ursæ Minoris, near West Elongation, August 17, 1879.	1 28 36.90	D. 25 27 09.07	D. 92 33 40.73	67 06 31.66	178 39 07.92	−1.84	111 32 34.42	
	33 09.89	07.57	37.53	29.96	03.33	−1.84	31.53	
	41 04.87	R. 205 27 12.07	R. 272 33 43.30	31.23	38 59.91	−1.72	26.96	
	43 18.87	10.97	43.47	32.50	59.99	−1.72	25.77	
$\alpha=19^h\ 45^m\ 09^s.31$	48 28.85	R. 205 27 12.90	R. 272 33 45.23	32.33	39 01.97	−2.30	27.34	
$\delta=88°\ 56'\ 40''.88$	50 42.85	13.87	48.47	34.60	03.57	−2.30	26.67	
	2 01 44.82	D. 25 27 09.00	D. 92 33 50.50	41.50	18.28	−2.88	33.90	
	04 45.81	07.13	55.67	48.54	24.26	−2.88	32.84	
Mean							111 32 29.93	111 32 30.26
λ Ursæ Minoris, near West Elongation, November 20, 1879.	1 25 07.90	D. 117 22 08.67	D. 184 28 57.63	67 06 48.96	178 39 23.81	−0.11	111 32 34.74	
	28 13.91	10.03	51.80	41.77	20.04	−0.11	38.16	
	36 12.92	R. 297 22 09.93	R. 4 28 56.00	46.07	14.40	+2.62	30.95	
	40 29.93	11.37	55.50	44.13	13.82	+2.62	32.31	
$\alpha=19^h\ 43^m\ 13^s.33$	45 57.93	R. 297 22 12.00	R. 4 28 54.70	42.70	15.52	+1.60	34.42	
$\delta=88°\ 56'\ 51''.75$	48 27.94	12.33	56.43	44.10	17.11	+1.60	34.61	
	56 32.95	D. 117 22 10.80	D. 184 29 01.97	51.17	26.62	+1.60	37.05	
	58 29.95	08.83	08.77	59.94	29.79	+1.60	31.45	
Mean							111 32 34.21	111 32 33.89
λ Ursæ Minoris, near West Elongatien, November 25, 1879.	1 21 09.85	D. 27 15 43.97	D. 94 22 49.83	67 07 05.86	178 39 28.77	+1.82	111 32 24.73	
	25 19.35	47.17	45.00	06 57.83	22.43	+1.82	26.42	
	31 43.35	R. 207 15 44.03	R. 274 22 22.90	38.87	15.79	−2.37	34.55	
	35 01.85	45.33	19.40	34.07	13.84	−2.37	37.40	
$\alpha=19^h\ 43^m\ 07^s.71$	42 05.35	R. 207 15 43.77	R. 274 22 22.30	38.53	13.05	−3.82	30.70	
$\delta=88°\ 56'\ 50''.97$	45 30.35	43.53	21.17	37.64	14.31	−3.82	32.85	
	54 56.85	D. 27 15 46.33	D. 94 22 42.53	56.20	23.42	+2.68	29.90	
	59 24.85	46.67	50.67	07 04.00	30.57	+2.68	29.25	
Mean							111 32 30.73	111 32 31.51
λ Ursæ Minoris, near West Elongation, November 29, 1879.	1 27 46.90	D. 60 32 20.03	D. 127 39 16.00	67 06 55.97	178 39 18.68	+3.20	111 32 25.91	
	29 28.90	21.17	10.67	49.50	16.94	+3.20	30.64	
	35 26.90	R. 240 32 26.17	R. 307 39 04.73	38.56	12.96	+1.32	35.72	
	37 13.90	28.07	04.33	36.26	12.41	+1.32	37.47	
$\alpha=19^h\ 43^m\ 03^s.28$	41 37.40	R. 240 32 27.13	R. 307 39 02.93	35.80	12.26	+0.91	37.37	
$\delta=88°\ 56'\ 50''.44$	43 57.90	25.83	03.97	38.14	12.97	+0.91	35.74	
	50 03.90	D. 60 32 19.50	D. 127 39 14.43	54.93	17.05	+5.47	27.59	
	51 45.90	21.73	15.17	53.44	18.84	+5.47	30.87	
Mean							111 32 32.65	111 32 32.34

Azimuth at Parkersburg.

II.—AZIMUTH OF THE LINE PARKERSBURG—EAST AZIMUTH MARK.

Star, date, &c.	Time of observation.	Reading on mark.	Reading on star.	Angle between mark and star.	Azimuth of star.	Level corrections.	Azimuth of mark.	Result for star.
	h. m. s.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″	° ′ ″
Polaris, near Lower	15 15 20.54	R. 24 29 00.50	R. 275 13 06.47	109 15 54.03	180 50 34.36	−3.23	290 06 25.16	
Culmination, August 11, 1879.	17 17.53	00.27	51.20	09.07	51 19.04	−3.23	24.88	
	27 04.51	D. 204 28 58.37	D. 95 17 23.23	11 35.14	55 00.02	−4.15	31.01	
$\alpha=1^h\ 15^m\ 00^s.92$	29 46.51	58.10	18 22.50	10 35.60	56 00.00	−4.15	31.45	
$\delta=88°\ 39'\ 52''.00$	34 54.50	D. 204 28 58.37	D. 95 20 13.80	08 44.57	57 52.82	−5.24	32.15	
	37 35.50	57.60	21 12.50	07 45.10	58 51.13	−5.24	30.99	
	47 02.48	R. 24 28 59.60	R. 275 24 42.73	04 16.87	181 02 12.76	−4.35	25.28	
	50 05.48	58.73	25 46.17	03 12.56	03 16.56	−4.35	24.77	
Mean							290 06 28.21	290 06 28.67
Polaris, near Lower	15 28 16.22	D. 48 33 10.03	D. 299 22 08.57	109 11 01.46	180 55 26.07	−0.88	290 06 26.65	
Culmination, August 12, 1879.	30 23.22	08.90	56.13	10 12.77	56 12.98	−0.88	24.87	
	42 49.19	R. 228 33 10.53	R. 119 27 15.90	05 54.63	181 00 42.88	−0.77	36 74	
$\alpha=1^h\ 15^m\ 01^s.85$	44 39.18	09.27	56.17	13.10	01 21.78	−0.77	34.21	
$\delta=88°\ 39'\ 52''.30$	51 25.17	R. 228 33 10.17	R. 119 30 18.67	02 51.50	03 43.57	−1.98	33.09	
	55 41.15	07.20	31 46.50	01 20.70	05 11.34	−1.98	30.06	
	16 02 35.14	D. 48 33 08.73	D. 299 34 17.23	108 58 51.50	07 30.50	+0.33	22.33	
	05 37.14	09.31	35 16.97	57 52.34	08 30.56	+0.33	23.23	
Mean							290 06 28.90	290 06 29.04
Polaris, near Lower	14 46 06.29	D. 180 08 03.40	D. 70 40 36.80	109 27 26.60	180 38 58.27	+2.37	290 06 27.24	
Culmination, August 17, 1879.	51 44.28	04.27	42 53.50	25 10.77	41 15.01	+3.16	28.94	
	56 32.27	R. 0 08 07.87	R. 250 44 47.50	23 20.37	43 10.41	−1.13	29.65	
$\alpha=1^h\ 15^m\ 05^s.54$	15 01 34.25	07.97	46 46.77	21 21.20	45 10.07	+0.90	32.17	
$\delta=88°\ 39'\ 53''.52$	04 15.25	R. 0 08 08.67	R. 250 47 51.37	20 17.30	46 13.64	−0.11	30.83	
	09 51.24	08.60	50 01.20	18 07.40	48 24.75	−0.23	31.92	
	13 11.23	D. 180 08 04.93	D. 70 51 22.03	16 42.90	49 42.01	+2.94	27.85	
	19 39.22	04.20	53 50.50	14 13.70	52 10.17	+2.82	26.69	
Mean							290 06 29.41	290 06 29.29
Polaris, near Lower	15 41 39.17	D. 72 00 54.23	D. 322 54 44.63	109 06 09.60	181 00 15.79	+1.70	290 06 27.09	
Culmination, August 17, 1879.	42 59.17	53.53	55 11.43	05 42.10	44.23	+1.70	28.03	
	47 50.16	R. 252 00 59.43	R. 142 56 52.10	04 07.33	02 26.71	−0.46	33.58	
$\alpha=1^h\ 15^m\ 05^s.54$	49 45.15	59.03	57 33.47	03 25.56	03 06.75	−0.46	31.85	
$\delta=88°\ 39'\ 53''.47$	53 32.15	R. 252 00 59.93	R. 142 58 51.00	02 08.93	04 25.09	−0.11	34.01	
	55 36.14	01 01.93	59 33.77	01 28.16	05 07.41	−0.11	35.46	
	16 00 37.13	D. 72 00 57.13	R. 323 01 15.60	108 59 41.53	06 48.99	+0.46	30.98	
	02 23.12	54.80	53.53	01.27	07 24.22	+0.46	25.95	
Mean							290 06 30.87	290 06 31.00
Polaris, near East	19 46 05.35	R. 115 56 02.30	R. 7 30 38.23	108 25 24.07	181 41 05.57	−2.65	290 06 26.99	
Elongation, November 20, 1879.	51 32.36	01.53	19.53	42.00	40 47.82	−2.07	27.75	
	55 07.36	D. 295 55 56.90	D. 187 29 54.17	26 02.73	33.06	−4.95	30.84	
$\alpha=1^h\ 15^m\ 25^s.94$	20 00 42.37	57.77	31.17	26.60	08.19	−6.33	28.46	
$\delta=88°\ 40'\ 20''.86$	03 47.37	D. 295 55 54.20	D. 187 29 15.73	38.47	39 52.97	−0.46	30.98	
	11 11.38	53.83	28 32.87	27 20.96	11.92	+0.46	33.44	
	17 59.40	R. 115 56 02.23	R. 7 28 05.20	57.03	38 28.63	+4.14	29.80	
	24 49.41	00.97	27.17.50	28 43.47	37 39.82	+5.29	28.58	
Mean							290 06 29.61	290 06 29.92

II.—*Azimuth at Parkersburg*—Continued.

Star, date, &c.	Time of observation.	Reading on mark.	Reading on star.	Angle between mark and star.	Azimuth of star.	Level corrections.	Azimuth of mark.	Result for star.
	h. m. s.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″	° ′ ″
Polaris, near East	18 59 45.15	R. 145 48 53.20	R. 37 23 50.27	108 25 02.93	181 41 22.98	−0.46	290 06 25.45	
Elongation, November 21, 1879.	19 09 44.17	56.63	24 05.00	24 51.63	40.25	−2.28	29.60	
	14 43.18	D. 325 48 52.77	D. 217 24 02.37	50.40	44.53	−4.56	30.37	
$\alpha=1^h\ 15^m\ 25^s.40$	21 15.19	53.00	01.97	51.03	45.80	−5.36	31.47	
$\delta=88°\ 40'\ 27''.12$	25 11.20	D. 325 48 52.10	D. 217 23 59.97	52.13	44.17	−5.36	30.94	
	29 16.21	50.43	58.23	52.20	40.55	−4.90	27.85	
	33 22.22	R. 145 48 59.87	R. 37 24 03.30	56.57	35.00	−3.76	27.81	
	39 23.23	58.87	23 52.33	25 06.54	23.30	−2.28	27.56	
Mean							290 06 28.88	290 06 29.19
Polaris, near East	19 19 45.91	D. 325 29 21.57	D. 217 04 37.67	108 24 43.90	181 41 45.38	−0.68	290 06 28.60	
Elongation, November 23, 1879.	24 09.91	21.70	35.90	45.80	43.96	−1.14	28.62	
	28 33.91	R. 145 29 23.90	R. 37 04 35.47	48.43	40.53	+0.91	29.87	
$\alpha=1^h\ 15^m\ 24^s.30$	34 27.91	23.57	29.03	54.54	32.37	−0.34	26.57	
$\delta=88°\ 40'\ 27''.73$	37 33.92	R. 145 29 25.10	R. 37 04 19.83	25 05.27	26.47	+0.46	32.20	
	44 35.92	22.67	05.50	17.17	08.91	−0.23	25.85	
	47 29.92	D. 325 29 22.03	D. 217 03 49.57	32.46	00.02	−1.48	31.00	
	53 09.92	22.17	27.80	54.37	40 39.79	−3.08	31.08	
Mean							290 06 29.22	290 06 29.85
Polaris, near East	18 58 42.26	D. 175 36 05.80	D. 67 10 54.33	108 25 11.47	181 41 19.50	−0.57	290 06 30.40	
Elongation, November 24, 1879.	19 03 32.27	03.93	11 07.50	24 56.43	29.85	+0.11	26.39	
	07 17.77	R. 355 36 06.60	R. 247 11 05.67	25 00.93	36.06	−2.62	34.37	
$\alpha=1^h\ 15^m\ 23^s.80$	12 46.78	06.67	11.43	24 55.24	42.16	−2.96	34.44	
$\delta=88°\ 40'\ 27''.96$	16 24.78	R. 355 36 07.30	R. 247 11 17.37	49.93	44.28	+0.57	34.78	
	21 32.79	06.90	20.10	46.80	44.65	+0.91	32.36	
	25 27.29	D. 175 36 06.90	D. 67 11 24.33	42.57	42.89	+3.53	28.99	
	30 44.80	05.63	18.47	47.16	37.68	+2.74	27.58	
Mean							290 06 31.16	290 06 31.47
Polaris, near East	19 03 43.03	D. 205 49 26.83	D. 97 24 30.27	108 24 56.56	181 41 29.84	+0.68	290 06 27.08	
Elongation, November 25, 1879.	10 29.02	22.37	39.37	53.00	39.68	+1.60	34.28	
	13 59.02	R. 25 49 20.90	R. 277 24 29.70	51.20	42.67	−0.80	33.07	
$\alpha=1^h\ 15^m\ 23^s.33$	19 21.02	21.53	30.83	50.70	44.51	+0.11	35.32	
$\delta=88°\ 40'\ 28''.24$	21 07.02	R. 25 49 21.20	R. 277 24 32.57	48.63	44.39	−1.14	31.88	
	26 17.01	22.53	29.90	52.63	41.92	−0.75	33.80	
	29 10.01	D. 205 49 25.37	D. 97 24 39.87	45.50	39.22	+0.91	25.63	
	33 37.00	22.33	31.93	50.40	33.13	+1.71	25.24	
Mean							290 06 30.79	290 06 31.10

A summary of the data in the preceding tables is given in the following table. As already mentioned, the individual results for azimuth of mark have been corrected for periodic error when it is not eliminated by the shifting of the circle, for run of micrometers, and for diurnal aberration. They are further corrected in the table following for the errors in the declinations taken from the American Ephemeris (see § 4). They still involve such errors as are not eliminated by reversal of telescope, such as may be due to inaccurate indications of the striding level, and such as arise from instability of the theodolite or its support between pointings to the star and to the mark. The level corrections were usually quite small, the maximum being 6″.84 and the average 2″.06. The first section of the table gives the results for azimuth of the west mark; the second, for azimuth of the east mark.

Azimuth at Parkersburg.

SUMMARY OF RESULTS.

I.—WEST AZIMUTH MARK.

Date.	Star, &c.	Position of telescope.	No. of pointings— To mark.	To star.	Azimuth. Declinations from American Ephemeris.	Reduction to Auwers' Declinations. $\frac{dA}{d\delta}$	$\Delta\delta$	ΔA	Corrected azimuth of mark.	Weight.
					° ′ 111 32				° ′ 111 32	
1879.					″		″	″	″	
Aug. 9	Polaris, near East Elongation	Direct	8	8						
		Reversed ...	8	8	32. 96	−1. 27	+0. 03	−0. 04	32. 92	1
10	......do	Direct	8	8						
		Reversed ...	8	8	36. 38	−1. 27	+0. 03	−0. 04	36. 34	1
11	......do	Direct	8	8						
		Reversed ...	8	8	37. 35	−1. 27	+0. 03	−0. 04	37. 31	1
12	......do	Direct	8	8						
		Reversed ...	8	8	34. 96	−1. 27	+0. 03	−0. 04	34. 92	1
13	......do	Direct	8	8						
		Reversed ...	8	8	33. 87	−1. 27	+0. 03	−0. 04	33. 83	1
16	......do	Direct	8	8						
		Reversed ...	8	8	34. 32	−1. 27	+0. 03	−0. 04	34. 28	1
Nov. 23	Polaris, near West Elongation.	Direct	8	8						
		Reversed ...	8	8	33. 01	+1. 27	+0. 03	+0. 04	33. 05	1
Aug. 9	51 Cephei, near East Elongation.	Direct	4	4						
		Reversed ...	4	4	34. 11	−1. 28	+0. 17	−0. 22	33. 89	0. 5
11	......do	Direct	8	8						
		Reversed ...	8	8	35. 49	−1. 28	+0. 17	−0. 22	35. 27	1
12	......do	Direct	8	8						
		Reversed ...	8	8	35. 39	−1. 28	+0. 17	−0. 22	35. 17	1
13	......do	Direct	8	8						
		Reversed ...	8	8	35. 08	−1. 28	+0. 17	−0. 22	34. 86	1
16	......do	Direct	8	8						
		Reversed ...	8	8	34. 43	−1. 28	+0. 17	−0. 22	34. 21	1
17	......do	Direct	8	8						
		Reversed ...	8	8	31. 37	−1. 28	+0. 17	−0. 22	31. 15	1
Nov. 20	......do	Direct	8	8						
		Reversed ...	8	8	33. 63	−1. 28	+0. 17	−0. 22	33. 41	1
24	......do	Direct	8	8						
		Reversed ...	8	8	32. 67	−1. 28	+0. 17	−0. 22	32. 45	1
25	......do	Direct	8	8						
		Reversed ...	8	8	31. 82	−1. 28	+0. 17	−0. 22	31. 60	1
29	......do	Direct	8	8						
		Reversed ...	8	8	34. 63	−1. 28	+0. 17	−0. 22	34. 41	1
Aug. 10	δ Ursæ Minoris, near West Elongation.	Direct	8	8						
		Reversed ...	8	8	32. 87	+1. 27	+0. 29	+0. 37	33. 24	1
11	......do	Direct	8	8						
		Reversed ...	8	8	34. 12	+1. 27	+0. 29	+0. 37	34. 49	1
12	......do	Direct	8	8						
		Reversed ...	8	8	34. 27	+1. 27	+0. 29	+0. 37	34. 64	1
13	......do	Direct	8	8						
		Reversed ...	8	8	35. 23	+1. 27	+0. 29	+0. 37	35. 60	1
16	......do	Direct	8	8						
		Reversed ...	8	8	35. 13	+1. 27	+0. 29	+0. 37	35. 50	1
17	......do	Direct	8	8						
		Reversed ...	8	8	31. 09	+1. 27	+0. 29	+0. 37	31. 46	1
Nov. 20	......do	Direct	8	8						
		Reversed ...	8	8	35. 27	+1. 27	+0. 29	+0. 37	35. 64	1
23	......do	Direct	8	8						
		Reversed ...	8	8	32. 94	+1. 17	+0. 29	+0. 34	33. 28	1
24	......do	Direct	8	8						
		Reversed ...	8	8	32. 59	+1. 27	+0. 29	+0. 37	32. 96	1

Azimuth at Parkersburg—Summary of results—Continued.

I.—WEST AZIMUTH MARK—Continued.

Date.	Star, &c.	Position of telescope.	No. of pointings— To mark.	No. of pointings— To star.	Azimuth. Declinations from American Ephemeris.	Reduction to Auwers' Declinations. $\frac{dA}{d\delta}$	Reduction to Auwers' Declinations. $\Delta\delta$	Reduction to Auwers' Declinations. ΔA	Corrected azimuth of mark.	Weight.
1879.					″		″	″	″	
Nov. 25	δ Ursæ Minoris, near West Elongation.	Direct	8	8						
		Reversed ...	8	8	29.97	+1.27	+0.29	+0.37	30.34	1
29	......do	Direct	8	8						
		Reversed ...	8	8	33.16	+1.27	+0.29	+0.37	33.53	1
Aug. 12	λ Ursæ Minoris, near West Elongation.	Direct	4	4						
		Reversed ...	4	4	34.51	+1.29	+0.57	+0.74	35.25	0.5
16	......do	Direct	8	8						
		Reversed ..	8	8	31.76	+1.29	+0.57	+0.74	32.50	1
17	......do	Direct	8	8						
		Reversed ...	8	8	30.26	+1.29	+0.57	+0.74	31.00	1
Nov. 20	......do	Direct	8	8						
		Reversed ...	8	8	33.89	+1.29	+0.57	+0.74	34.63	1
25	......do	Direct	8	8						
		Reversed ...	8	8	31.51	+1.29	+0.57	+0.74	32.25	1
29	......do	Direct	8	8						
		Reversed ...	8	8	32.34	+1.29	+0.57	+0.74	33.08	1

Weighted mean.. 111° 32′ 33″.754 ± 0″.189

II.—EAST AZIMUTH MARK.

Date.	Star, &c.	Position of telescope.	No. of pointings— To mark.	No. of pointings— To star.	Azimuth. Declinations from American Ephemeris.	Reduction to Auwers' Declinations. $\frac{dA}{d\delta}$	Reduction to Auwers' Declinations. $\Delta\delta$	Reduction to Auwers' Declinations. ΔA	Corrected azimuth of mark.	Weight.
					° ′ 290 06				° ′ 290 06	
1879.					″		″	″	″	
Aug. 11	Polaris, near Lower Culmination.	Direct	8	8						
		Reversed ...	8	8	28.68	−0.70	+0.03	−0.02	28.66	1
12	......do	Direct	8	8						
		Reversed ...	8	8	29.04	−0.77	+0.03	−0.02	29.02	1
17	......do	Direct	8	8						
		Reversed ...	8	8	29.29	−0.56	+0.03	−0.02	29.27	1
17	......do	Direct	8	8						
		Reversed ...	8	8	31.00	−0.79	+0.03	−0.02	30.98	1
Nov. 20	Polaris, near East Elongation	Direct	8	8						
		Reversed ...	8	8	29.92	−1.27	+0.03	−0.04	29.88	1
21	......do	Direct	8	8						
		Reversed ...	8	8	29.19	−1.27	+0.03	−0.04	29.15	1
23	......do	Direct	8	8						
		Reversed ...	8	8	29.85	−1.27	+0.03	−0.04	29.81	1
24	......do	Direct	8	8						
		Reversed ...	8	8	31.47	−1.27	+0.03	−0.04	31.43	1
25	......do	Direct	8	8						
		Reversed ...	8	8	31.10	−1.27	+0.03	−0.04	31.06	1

Weighted mean.. 290° 06′ 29″.918 ± 0″.227

Assigning weights to the individual results for azimuth of mark proportional to the number of pointings made to the star, there results for azimuth of the west mark

111° 32′ 33″.75 west of south,

and for azimuth of the east mark

290° 06′ 29″.92 west of south.

The probable errors of these two results, as derived from the discrepancies between the individual results and their weighted means, are $\pm 0''.19$ and $\pm 0''.23$, respectively. The former of these may be considered the best value attainable for the probable error of the result for the west mark, but the latter ($\pm 0''.23$) is probably too small for the result for the east mark, since this result depends on observations to but one star, Polaris, which was observed at but one elongation, the eastern, and one culmination, the lower. It will, therefore, be better to assign relative weights to the mean results for the west and east marks proportional to the number of individual results to them respectively. This gives to the latter a weight $\frac{9}{33}$ times that of the former, so that the results, with their probable errors, are as follows:

(1) Azimuth of west mark, $111° 32' 33''.75 \pm 0''.19$.

(2) Azimuth of east mark, $290° 06' 29''.92 \pm 0''.36$.

These two azimuths were connected with the triangulation by the following angles, read by Assistant Engineer Flint, with the same theodolite used in the azimuth work. (Chapter XX, C.) The weights given are those of the observed angles.

West Azimuth Mark—Parkersburg—Denver,	weight 1.
Denver—Parkersburg—Claremont,	weight 1.
Claremont—Parkersburg—East Azimuth Mark,	weight 1.
East Azimuth Mark—Parkersburg—West Azimuth Mark,	weight 1.5.
West Azimuth Mark—Parkersburg—East Azimuth Mark,	weight 1.5.

The first and fourth of these angles will be used in deriving the azimuth of the line Parkersburg—Denver. Their adjusted values are:

(3) East Azimuth Mark—Parkersburg—West Azimuth Mark, $181° 26' 03''.51$.

(4) West Azimuth Mark—Parkersburg—Denver, $31° 43' 41''.74$.

The probable errors of these angles are less after adjustment than before, but are not as small as those of an average adjusted angle, since one of them (3) enters into but one equation of condition directly, and the other (4) into but two. In view of this, it will probably be best to assign to them the probable errors of their observed values.

The adjustment of that portion of the triangulation to which these angles belong shows a probable error of $\pm 0''.32$ for an observed angle of weight 1. Hence, there are obtained $\pm 0''.26$ and $\pm 0''.32$ for the probable errors of (3) and (4) respectively. Adding (3) to (2) gives

(5) $$111° 32' 33''.43 \pm 0''.44$$

for the azimuth of the west mark as derived from the east mark; and combining (5) and (1) with weights derived from their probable errors, the following weighted mean azimuth of the west mark is obtained: $111° 32' 33''.70 \pm 0''.17$. Adding to this the angle (4) whose probable error is $\pm 0''.32$, there results for the azimuth of the line Parkersburg—Denver,

143° 16′ 15″.44±0″.36 west of south.

AZIMUTH AT WEST BASE, SANDUSKY BASE-LINE.

§ **10.** Azimuth determinations were made at West Base station, on the Sandusky base-line, on the nights of September 13, 14, 17, 18, and 21, 1877, by Assistant Engineer G. Y. Wisner. The instrument used was the Troughton & Simms 14-inch theodolite No. 1. It was mounted on a heavy wooden post set about 5 feet in the ground, and nearly on the line between East and West Base stations, being 153.9 feet from West Base, and 0.447 inch northeast of the line joining the two Base stations. The stars observed were α, δ, and λ Ursæ Minoris and 51 Cephei. Time was given by Bond & Sons chronometer No. 255. The azimuth mark was a light set directly over the geodetic point at East Base, about 3.8 miles distant. The observations were made according to the following programme:

1. Two readings on the azimuth mark.
2. Four readings on the star with level readings.
3. Two readings on the mark.

4. Reversal of telescope, the pivots remaining on the same wyes respectively.
5. Two readings on the mark.
6. Four readings on the star with level readings.
7. Two readings on the mark.

A second set of readings was then taken in the reverse order of this programme. There were thus obtained 16 results for azimuth of mark from each star observed on any date. The circle remained in the same position throughout each night's work, but was set forward 24° on each succeeding night.

In the reduction the differences between the individual readings on the star and on the mark in the order of observation were taken to obtain individual results for the angle between the mark and the star. The azimuth of the star was then computed for each observation thereon by the formula

$$\tan A = \frac{\sin t}{\cos \varphi \tan \delta - \sin \varphi \cos t}$$

in which A, t, φ, δ, are respectively the azimuth of the star at the time of observation, the hour-angle, the latitude of the station, and the declination of the star. From these quantities and the corresponding level corrections are obtained the results for the azimuth of the mark.

In the following table the first column gives the star, date of observation, and the coördinates of the star as taken from the American Ephemeris; the second, the corrected time of observation; the third, the readings on the azimuth-light; the fourth, the readings on the star; the fifth, the observed angles between the mark and the star; the sixth, the computed azimuths of the star at the times of observation; the seventh, the correction for inclination of telescope axis; the eighth, the resulting azimuth of the mark for each observation; the ninth, the mean result for the star. The relative positions of the telescope are indicated by the letters D and R.

Azimuth at West Base, Sandusky.

AZIMUTH OF LINE AZIMUTH POST — EAST BASE, SANDUSKY.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1.]

Star, date, &c.	Time of observation.	Reading on mark.	Reading on star.	Angle between mark and star.	Azimuth of star.	Level corrections.	Azimuth of mark.	Result for star.
	h. m. s.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″	° ′ ″
Polaris, near East	19 05 24.8	R. 322 28 21.17	R. 184 38 57.53	137 49 23.64	181 47 26.26	— 7.33	319 36 42.57	
Elongation, Sep-	08 27.8	20.27	39 04.00	16.27	30.85	— 7.33	39.79	
tember 13, 1877.	09 15.3	21.10	04.17	16.93	31.85	— 7.12	41.66	
$\alpha=1^h\ 14^m\ 25^s.1$	10 30.8	20.30	04.77	15.53	33.29	— 7.12	41.70	
$\delta=88°\ 39'\ 22''.5$	18 30.8	D. 142 28 20.43	D. 4 39 18.50	01.93	37.88	+ 2.83	42.64	
	20 03.8	20.63	21.90	48 58.73	37.84	+ 2.83	39.40	
	21 13.8	21.50	21.50	49 00.00	37.63	+ 2.17	39.80	
	22 27.3	22.70	20.93	01.77	37.22	+ 2.17	41.16	
	28 37.8	D. 142 28 23.30	D. 4 39 12.53	10.77	32.40	+ 2.25	45.42	
	30 03.3	22.50	13.17	09.33	30.62	+ 2.25	42.20	
	31 04.3	21.03	12.00	09.03	29.18	+ 2.48	40.69	
	32 19.3	20.00	10.70	09.30	27.25	+ 2.48	39.03	
	41 18.8	R. 322 28 20.10	R. 184 38 35.83	44.27	07.74	— 8.62	43.39	
	42 54.8	19.20	31.73	47.47	03.20	— 8.62	42.07	
	44 00.3	18.87	27.33	51.54	46 59.95	— 8.45	43.04	
	45 22.8	18.60	23.63	55.57	55.62	— 8.45	42.74	319 36 41.70
Polaris, near East	18 58 25.6	R. 346 34 26.20	R. 208 44 54.67	137 49 31.53	181 47 10.96	— 2.13	319 36 40.46	
Elongation, Sep-	19 00 01.1	25.50	56.33	29.17	14.85	— 2.13	41.89	
tember 14, 1877.	56.6	23.17	45 00.50	22.67	16.96	— 1.63	38.00	
$\alpha=1^h\ 14^m\ 25^s.7$	01 57.1	23.60	02.10	21.50	19.16	— 1.63	39.03	
$\delta=88°\ 39'\ 22''.9$	08 51.1	D. 166 34 27.47	D. 28 45 25.83	01.64	30.82	+ 6.00	38.46	
	10 07.6	27.57	28.10	48 59.47	32.34	+ 6.00	37.81	
	57.6	28.43	28.77	59.66	33.22	+ 6.20	39.08	
	11 54.1	28.47	29.10	59.37	34.11	+ 6.20	39.68	
	17 17.6	D. 166 34 28.50	D. 28 45 29.53	58.97	37.15	+ 6.13	42.25	
	18 47.1	28.83	29.67	59.16	37.36	+ 6.13	42.65	
	19 38.1	27.93	31.60	56.33	37.34	+ 6.07	39.74	
	20 32.6	28.80	33.10	55.70	37.26	+ 6.07	39.03	
	29 27.1	R. 346 34 23.83	R. 208 45 10.93	49 12.90	30.88	— 3.10	40.68	
	30 55.6	21.93	06.10	15.83	28.87	— 3.10	41.60	
	31 55.6	21.43	05.37	16.06	27.37	— 2.70	40.73	
	32 59.6	20.77	00.77	20.00	25.63	— 2.70	42.93	319 36 40.25
Polaris, near East	19 02 32.5	D. 190 01 35.17	D. 52 12 15.93	137 49 19.24	181 47 18.73	+ 4.38	319 36 42.35	
Elongation, Sep-	03 53.0	35.30	21.23	14.07	21.36	+ 4.37	39.80	
tember 17, 1877.	04 53.0	34.63	21.93	12.70	23.16	+ 4.13	39.99	
$\alpha=1^h\ 14^m\ 27^s.0$	07 04.0	34.73	28.00	06.73	26.72	+ 4.12	37.57	
$\delta=88°\ 39'\ 24''.1$	14 20.0	R. 10 01 41.83	R. 232 12 32.43	09.40	34.30	— 8.25	35.45	
	15 41.5	41.27	34.50	06.77	34.99	— 8.25	33.51	
	16 29.5	43.17	34.60	08.57	35.30	— 7.45	36.42	
	17 28.0	41.60	35.40	06.20	35.56	— 7.45	34.31	
	21 53.0	R. 10 01 42.10	R. 232 12 33.50	08.60	35.31	— 7.38	36.53	
	23 14.0	42.27	29.45	12.82	34.75	— 7.37	40.20	
	24 11.0	41.17	29.60	11.57	34.22	— 7.20	38.59	
	25 02.0	42.07	29.60	12.47	33.66	— 7.20	38.93	
	32 33.0	D. 190 01 40.43	D. 52 12 26.20	14.23	24.80	+ 2.70	41.73	
	33 55.5	40.43	24.90	15.53	22.42	+ 2.70	40.65	
	34 51.0	40.53	19.67	20.86	20.69	+ 1.75	43.30	
	36 06.5	39.77	18.23	21.54	18.17	+ 1.75	41.46	319 36 38.80

Azimuth at West Base, Sandusky—Continued.

Star, date, &c.	Time of observation.	Reading on mark.	Reading on star.	Angle between mark and star.	Azimuth of star.	Level corrections.	Azimuth of mark.	Result for star.
	h. m. s.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″	° ′ ″
Polaris, near East	19 02 02.0	R. 214 11 35.43	R. 76 22 11.53	137 49 23.90	181 47 17.26	+ 3.33	319 36 44.49	
Elongation, Sep-	03 16.0	36.07	13.50	22.57	19.77	+ 3.32	45.66	
tember 18, 1877.	04 08.0	30.77	15.77	15.00	21.41	+ 3.88	46.29	
$\alpha=1^h\ 14^m\ 27^s.3$	51.0	30.90	16.53	14.37	22.68	+ 3.87	40.92	
$\delta=88°\ 39'\ 24''.4$	12 10.5	D. 34 11 26.40	D. 256 22 10.03	16.37	32.30	− 6.88	41.79	
	13 24.0	26.33	13.10	13.23	33.28	− 6.87	39.64	
	14 18.0	27.83	14.17	13.66	33.87	− 7.25	40.28	
	15 20.0	24.83	14.60	10.23	34.41	− 7.25	37.39	
	20 32.0	D. 34 11 26.60	D. 256 22 13.70	12.90	35.24	− 6.88	41.26	
	21 40.0	26.23	12.83	13.40	34.96	− 6.87	41.49	
	22 30.5	26.70	11.93	14.77	34.67	− 7.88	41.56	
	23 21.0	26.90	09.73	17.17	34.29	− 7.87	43.59	
	29 57.5	R. 214 11 30.83	R. 76 22 16.37	14.46	28.24	+ 1.50	44.20	
	31 06.5	31.43	13.73	17.70	26.63	+ 1.50	45.83	
	55.0	30.23	13.43	16.80	25.40	+ 1.50	43.70	
	32 45.0	29.50	11.67	17.83	24.06	+ 1.50	43.39	319 36 42.22
Polaris, near East	19 03 06.2	R. 238 16 55.67	R. 100 27 38.87	137 49 16.80	181 47 17.95	+ 6.25	319 36 41.00	
Elongation, Sep-	04 19.7	55.73	43.07	12.66	20.27	+ 6.25	39.18	
tember 21, 1877.	05 08.7	59.33	45.17	14.16	21.73	+ 6.75	42.64	
$\alpha=1^h\ 14^m\ 28^s.2$	06 08.2	55.17	47.93	07.24	23.37	+ 6.75	37.36	
$\delta=88°\ 39'\ 25''.5$	13 55.2	D. 58 17 06.17	D. 280 28 01.60	04.57	32.15	− 4.87	31.85	
	15 12.2	06.50	03.37	03.13	32.89	− 4.87	31.15	
	16 04.2	04.70	03.33	01.37	33.26	− 4.80	29.83	
	17 03.7	07.07	03.83	03.24	33.59	− 4.80	32.03	
	21 16.2	D. 58 17 05.00	D. 280 28 00.80	04.20	33.65	− 4.95	32.90	
	22 27.7	05.83	00.70	05.13	33.23	− 4.95	33.41	
	23 26.7	04.23	01.27	02.96	32.77	− 5.37	30.36	
	24 28.2	06.70	00.33	06.37	32.18	− 5.37	33.18	
	31 00.2	R. 238 17 02.00	R. 100 27 47.50	14.50	25.37	+ 5.05	44.92	
	32 14.2	01.00	45.60	15.40	23.46	+ 5.05	43.91	
	33 08.7	16 56.60	43.60	13.00	21.95	+ 5.75	40.70	
	34 04.7	58.33	41.67	16.66	20.29	+ 5.75	42.70	319 36 36.70
δ Ursæ Minoris,	23 41 51.5	R. 322 28 15.17	R. 178 20 42.73	144 07 32.44	175 29 18.42	−10.75	319 36 40.11	
near West Elon-	43 26.5	14.50	33.10	41.40	09.93	−10.75	40.52	
gation, Septem-	44 43.5	14.93	28.73	46.20	02.52	−10.38	38.34	
ber 13, 1877.	46 01.5	14.80	22.33	52.47	28 57.94	−10.38	40.03	
$\alpha=18^h\ 11^m\ 47^s.5$	56 37.5	D. 142 28 19.93	D. 358 20 15.40	08 04.53	30.06	0.00	34.59	
$\delta=86\ 36'\ 41''.0$	58 10.5	20.30	11.40	08.90	28.95	0.00	37.85	
	59 12.5	19.90	10.77	09.13	28.45	+ 0.50	38.08	
	0 00 25.5	20.20	11.77	08.43	28.61	+ 0.50	37.54	
	06 46.0	D. 142 28 20.73	D. 358 20 20.83	07 59.90	36.07	+ 1.68	37.65	
	08 04.5	19.93	24.37	55.56	39.17	+ 1.68	36.41	
	09 05.5	22.20	26.83	55.37	41.88	+ 0.98	38.23	
	55.0	20.93	27.80	53.13	44.38	+ 0.98	38.49	
	20 08.0	R. 322 28 17.80	R. 178 20 56.63	21.17	29 32.26	−12.60	41.43	
	21 38.0	15.50	21 07.23	08.27	41.94	−12.60	38.21	
	22 50.5	16.10	19.90	06 56.20	50.29	−11.13	35.36	
	23 55.5	16.33	24.17	52.16	58.10	−11.13	39.13	319 36 38.27

Azimuth at West Base, Sandusky—Continued.

Star, date, &c.	Time of observation.	Reading on mark.	Reading on star.	Angle between mark and star.	Azimuth of star.	Level corrections.	Azimuth of mark.	Result for star.
	h. m. s.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″	° ′ ″
δ Ursæ Minoris,	23 42 39.5	R. 346 34 16.77	R. 202 26 49.70	144 07 27.07	175 29 14.13	− 4.25	319 36 36.95	
near West Elon-	44 14.0	17.00	36.97	40.03	06.23	− 4.25	42.01	
gation, Septem-	45 36.0	15.20	33.50	41.70	28 59.89	− 3.85	37.74	
ber 14, 1877.	46 51.5	17.20	25.77	51.43	54.55	− 3.85	42.13	
$\alpha=18^h\ 11^m\ 47^s.0$	56 49.0	D. 160 34 23.33	D. 22 26 23.23	08 00.10	30.02	+ 6.50	36.62	
$\delta=86^\circ\ 36'\ 41''.1$	58 16.5	24.10	16.83	07.27	29.00	+ 6.50	42.77	
	59 15.5	25.37	15.40	09.97	28.65	+ 6.07	44.69	
	0 00 38.5	23.57	18.40	05.17	28.74	+ 6.07	39.98	
	08 10.5	D. 166 34 24.13	D. 22 26 30.53	07 53.60	39.56	+ 7.25	40.41	
	09 30.0	24.57	35.27	49.30	43.23	+ 7.25	39.78	
	10 32.5	25.20	37 33	47.87	46.51	+ 7.00	41.38	
	11 30.0	24.27	41.70	42.57	49.87	+ 7.00	39.44	
	20 03.5	R. 346 34 17.57	R. 202 27 06.20	11.37	29 31.97	− 4.38	38.96	
	21 23.5	15.60	14.83	00.77	40.60	− 4.38	36.99	
	22 13.0	17.70	21.10	06 56.60	46.08	− 4.37	38.31	
	23 37.5	15.23	27.33	47.90	56.12	− 4.37	39.65	319 36 39.87
δ Ursæ Minoris,	23 43 51.7	D. 190 01 38.57	D. 45 54 05.40	144 07 33.17	175 29 08.07	+ 2.00	319 36 43.24	
near West Elon-	45 22.7	39.10	00.57	38.53	00.85	+ 2.00	41.38	
gation, Septem-	46 31.2	38.83	53 56.47	42.36	28 55.98	+ 2.00	40.34	
ber 17, 1877.	47 59.2	37.77	51.77	46.00	50.26	+ 2.00	38.26	
$\alpha=18^h\ 11^m\ 45^s.7$	56 50.2	R. 10 01 45.43	R. 225 53 32.03	08 13.40	30.08	− 9.50	33.98	
$\delta=86^\circ\ 36'\ 41''.2$	58 30.7	45.60	30.57	15.03	28.97	− 9.50	34.50	
	0 00 00.2	46.43	31.00	15.43	28.73	− 8.13	36.03	
	02 46.7	46.27	32.60	13.67	30.12	− 8.12	35.67	
	08 48.2	R. 10 01 46.30	R. 225 53 42.87	03.43	41.34	− 8.98	35.79	
	10 23.7	46.43	47.00	07 59.43	46.30	− 8.97	36.76	
	11 24.2	45.93	51.40	54.53	49.75	− 9.45	34.83	
	12 41.7	46.60	55.23	51.37	54.60	− 9.45	36.52	
	21 37.2	D. 190 01 37.33	D. 45 54 38.83	06 58.50	29 42.34	+ 2.02	42.86	
	23 03.2	38.10	50.00	48.10	52.26	+ 2.03	42.39	
	24 04.7	36.63	58.77	37.86	59.80	+ 1.30	38.96	
	25 32.7	38.83	55 11.27	27.56	30 11.02	+ 1.30	39.88	319 36 38.22
δ Ursæ Minoris,	23 46 09.9	R. 214 11 30.60	R. 70 03 44.63	144 07 45.97	175 28 57.56	+ 1.40	319 36 44.93	
near West Elon-	47 36.9	30.00	37.37	52.63	51.68	+ 1.40	45.71	
gation, Septem-	48 31.9	26.57	32.43	54.14	48.43	+ 1.52	44.09	
ber 18, 1877.	49 49.4	28.67	27.93	08 00.74	44.17	+ 1.52	46.43	
$\alpha=18^h\ 11^m\ 45^s.3$	58 12.9	D. 34 11 19.27	D. 250 02 57.83	21.44	29.27	−11.93	38.78	
$\delta=86^\circ\ 36'\ 41''.3$	59 30.4	19.20	58.07	21.13	28.88	−11.92	38.09	
	0 00 33.9	19.17	57.23	21.94	29.03	−12.75	38.22	
	01 45.9	18.60	56.50	22.10	29.47	−12.75	38.82	
	07 43.4	D. 34 11 17.67	D. 250 03 13.57	04.10	38.65	− 6.62	36.13	
	09 09.9	18.03	15.10	02.93	42.56	− 6.63	38.86	
	10 16.9	18.63	17.40	01.23	46.00	− 6.65	40.58	
	11 25.9	18.10	23.17	07 54.93	50.00	− 6.65	38.28	
	19 02.9	R. 214 11 27.07	R. 70 04 15.57	11.50	29 26.30	+ 3.50	41.30	
	20 16.9	27.73	22.77	04.96	33.76	+ 3.50	42.22	
	21 05.9	29.67	29.43	00.24	39.07	+ 4.50	43.81	
	22 09.9	27.60	33.67	06 53.93	46.26	+ 4.50	44.69	319 36 41.31

Azimuth at West Base, Sandusky—Continued.

Star, date, &c.	Time of observation.	Reading on mark.	Reading on star.	Angle between mark and star.	Azimuth of star.	Level corrections.	Azimuth of mark.	Result for star.
	h. m. s.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″	° ′ ″
δ Ursæ Minoris,	23 42 14.0	R. 238 16 59.27	R. 94 09 38.60	144 07 20.67	175 29 16.33	+ 3.88	319 36 40.88	
near West Elon-	43 36.5	59.30	27.33	31.97	09.12	+ 3.87	44.96	
gation, Septem-	44 35.0	56.07	20.20	35.87	04.37	+ 4.13	44.37	
ber 21, 1877.	45 34.5	57.43	16.60	40.83	28 59.93	+ 4.12	44.88	
$\alpha=18^h\ 11^m\ 44^s.0$	52 24.0	D. 58 17 05.33	D. 274 08 58.67	08 06.66	37.11	− 7.93	35.84	
$\delta=86°\ 36'\ 41''.2$	53 50.5	05.63	57.80	07.83	34.08	− 7.92	33.99	
	54 46.0	05.27	56.90	08.37	32.57	− 8.13	32.81	
	55 43.0	04.90	56.07	08.83	31.19	− 8.12	31.90	
	0 00 52.5	D. 58 17 04.50	D. 274 08 50.57	13.93	28.93	− 7.08	35.78	
	02 16.5	04.27	52.07	12.20	29.80	− 7.07	34.93	
	03 04.0	04.77	56.83	07.94	30.54	− 7.08	31.40	
	04 37.0	05.27	57.73	07.54	32.41	− 7.07	32.88	
	12 15.0	R. 238 16 55.73	R. 94 09 09.03	07 46.70	52.97	+ 2.75	42.42	
	13 44.0	56.40	15.67	40.73	59.05	+ 2.75	42.53	
	14 39.5	57.90	22.07	35.83	29 03.08	+ 2.13	41.04	
	15 55.0	57.93	29.90	28.03	09.19	+ 2.13	39.35	319 36 38.12
51 Cephei, near	0 33 53.2	R. 322 28 16.07	R. 186 32 38.53	135 55 37.54	183 41 16.48	−11.38	319 36 42.64	
East Elongation,	35 23.7	15.67	47.47	28.20	23.17	−11.38	39.99	
September 13,	36 26.7	16.23	51.90	24.33	27.58	−10.50	41.41	
1877.	37 48.2	15.53	55.67	19.86	32.80	−10.50	42.16	
$\alpha=6^h\ 42^m\ 34^s.5$	48 49.7	D. 142 28 22.43	D. 6 33 44.70	54 37.73	58.07	+ 1.25	37.05	
$\delta=87°\ 13'\ 44''.8$	50 11.2	21.80	47.43	34.37	59.05	+ 1.25	34.67	
	51 10.2	19.50	47.40	32.10	59.44	+ 1.00	32.54	
	52 09.7	21.67	47.50	34.17	59.65	+ 1.00	34.82	
	56 48.2	D. 142 28 21.27	D. 6 33 41.70	39.57	57.18	+ 1.25	38.00	
	59 13.2	21.57	42.07	39.50	53.76	+ 1.25	34.51	
	1 00 13.2	22.07	38.00	44.07	51.83	+ 1.25	37.15	
	01 18.2	22.77	34.60	48.17	49.59	+ 1.25	39.01	
	10 20.7	R. 322 28 15.43	R. 186 32 40.27	55 35.16	18.71	−10.75	43.12	
	11 29.7	14.60	34.67	39.93	13.29	−10.75	42.47	
	12 22.2	14.27	30.90	43.37	08.94	−10.95	41.36	
	13 32.7	13.33	24.83	48.50	02.72	−10.95	40.27	319 36 38.82
51 Cephei, near	0 30 39.1	R. 346 34 15.90	R. 210 38 30.67	135 55 45.23	183 41 00.11	− 3.88	319 36 41.46	
East Elongation,	32 09.6	16.87	39.33	37.54	08.11	− 3.87	41.78	
September 14,	33 22.6	17.37	48.83	28.54	14.11	− 3.00	39.65	
1877.	34 34.1	17.17	53.73	23.44	19.60	− 3.00	40.04	
$\alpha=6^h\ 42^m\ 35^s.1$	43 49.1	D. 166 34 20.10	D. 30 39 38.70	54 41.40	50.46	+ 5.88	37.74	
$\delta=87°\ 13'\ 44''.7$	44 59.6	24.73	41.63	43.10	52.84	+ 5.87	41.81	
	46 02.6	21.27	42.00	39.27	54.61	+ 5.43	39.31	
	47 11.1	21.40	42.33	39.07	56.31	+ 5.42	40.80	
	52 32.1	D. 166 34 22.40	D. 30 39 47.87	34.53	59.69	+ 6.25	40.47	
	53 57.1	21.33	50.33	31.00	59.40	+ 6.25	36.65	
	54 49.1	20.17	47.23	32.94	58.96	+ 5.50	37.40	
	55 48.6	21.80	45.97	35.83	58.26	+ 5.50	39.59	
	1 03 51.6	R. 346 34 15.33	R. 210 39 18.10	57.23	43.05	− 3.37	36.91	
	04 58.1	15.77	10.53	55 05.24	39.68	− 3.38	41.54	
	05 51.6	16.93	06.50	10.43	36.68	− 3.25	43.86	
	06 59.6	15.23	01.83	13.40	32.67	− 3.25	42.82	319 36 40.11

Azimuth at West Base, Sandusky—Continued.

Star, date, &c.	Time of observation.	Reading on mark.	Reading on star.	Angle between mark and star.	Azimuth of star.	Level corrections.	Azimuth of mark.	Result for star.
	h. m. s.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″	° ′ ″
51 Cephei, near	0 32 52.5	D. 190 01 38.07	D. 54 06 08.43	135 55 29.64	183 41 11.99	+ 2.55	319 36 44.18	
East Elongation,	34 11.5	37.87	15.07	22.80	18.25	+ 2.55	43.60	
September 17,	35 13.5	37.17	22.17	15.00	22.79	+ 2.55	40.34	
1877.	36 27.0	36.70	27.10	09.60	27.96	+ 2.55	40.11	
$\alpha = 6^h\ 42^m\ 36^s.8$	43 59.5	R. 10 01 46.37	R. 234 06 51.10	54 55.27	51.19	− 9.15	37.31	
$\delta = 87°\ 13'\ 44''.4$	45 16.5	45.63	53.67	51.96	53.70	− 9.15	36.51	
	46 30.0	47.27	55.57	51.70	55.70	− 8.63	38.77	
	47 41.0	47.50	07 00.17	47.33	57.36	− 8.62	36.07	
	53 12.5	R. 10 01 47.70	R. 234 06 59.53	48.17	42 00.09	− 9.50	38.76	
	54 23.5	46.27	07 02.43	43.84	41 59.65	− 9.50	33.99	
	55 38.5	45.73	01.77	43.96	58.82	− 8.45	34.33	
	56 45.0	45.77	06 59.17	46.60	57.77	− 8.45	35.92	
	1 04 37.0	D. 190 01 37.27	D. 54 06 36.57	55 00.70	41.30	+ 1.93	43.93	
	06 50.0	36.17	32.70	03.47	33.83	+ 1.92	39.22	
	07 41.0	35.97	26.10	09.87	30.59	+ 0.75	41.21	
	08 55.0	35.50	22.40	13.10	25.66	+ 0.75	39.51	319 36 38.98
51 Cephei, near	0 32 44.2	R. 214 11 27.07	R. 78 15 57.73	135 55 29.34	183 41 11.20	+ 3.38	319 36 43.92	
East Elongation,	34 04.2	27.20	16 05.13	22.07	17.63	+ 3.37	43.07	
September 18,	35 00.2	25.83	08.27	17.56	21.84	+ 4.20	43.60	
1877.	36 05.7	25.07	12.87	12.20	26.44	+ 4.20	42.84	
$\alpha = 6^h\ 42^m\ 37^s.3$	43 24.7	D. 34 11 18.57	D. 258 16 23.07	54 55.50	49.89	− 5.25	40.14	
$\delta = 87°\ 13'\ 44''.4$	44 39.2	19.40	29.23	50.17	52.53	− 5.25	37.45	
	45 50.7	20.17	30.27	49.90	54.65	− 5.50	39.05	
	47 11.7	20.50	31.43	49.07	56.68	− 5.50	40.25	
	52 32.7	D. 34 11 20.00	D. 258 16 31.87	48.13	42 00.19	− 6.13	42.19	
	53 48.7	19.90	34.60	45.30	41 59.90	− 6.12	39.08	
	54 42.2	17.57	34.83	42.74	59.51	− 7.05	35.20	
	55 53.2	18.57	33.67	44.90	58.64	− 7.05	36.49	
	1 03 03.7	R. 214 11 23.93	R. 78 16 31.57	52.36	45.83	+ 4.58	42.77	
	04 22.2	22.90	28.03	54.87	42.11	+ 4.57	41.55	
	05 16.2	22.07	24.10	57.97	39.21	+ 2.83	40.01	
	06 26.2	23.33	19.50	55 03.83	35.26	+ 2.82	41.91	319 36 40.60
51 Cephei, near	0 32 16.0	R. 238 16 58.13	R. 102 21 23.73	135 55 34.40	183 41 09.00	+ 3.00	319 36 46.40	
East Elongation,	33 24.5	58.03	29.33	28.70	14.64	+ 3.00	46.34	
September 21,	34 23.0	56.90	37.67	19.23	19.17	+ 4.00	42.40	
1877.	35 26.0	57.80	41.47	16.33	23.88	+ 4.00	44.21	
$\alpha = 6^h\ 42^m\ 38^s.8$	42 49.0	D. 58 17 06.33	D. 282 22 12.40	54 53.93	48.68	− 6.00	36.61	
$\delta = 87°\ 13'\ 44''.2$	44 12.0	06.50	15.83	50.67	51.81	− 6.00	36.48	
	45 07.5	06.70	18.27	48.43	53.65	− 6.00	36.08	
	46 09.5	06.90	21.93	44.97	55.45	− 6.00	34.42	
	51 00.5	D. 58 17 06.27	D. 282 22 24.20	42.07	42 00.20	− 5.75	36.52	
	52 18.0	07.10	23.57	43.53	00.43	− 5.75	38.21	
	53 12.5	05.93	22.50	43.43	00.34	− 7.00	36.77	
	54 25.5	05.87	20.00	45.87	41 59.91	− 7.00	38.78	
	1 01 43.5	R. 238 16 58.00	R. 102 22 04.93	53.07	49.55	+ 1.75	44.37	
	02 49.0	55.93	00.17	55.76	46.79	+ 1.75	44.30	
	03 45.5	56.93	21 58.60	58.33	44.19	+ 3.00	45.52	
	04 46.5	56.17	53.30	55 02.87	41.21	+ 3.00	47.08	319 36 40.91

Azimuth at West Base, Sandusky—Continued.

Star, date, &c.	Time of observation.	Reading on mark.	Reading on star.	Angle between mark and star.	Azimuth of star.	Level corrections.	Azimuth of mark.	Result for star.
	h. m. s.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″	° ′ ″
λ Ursæ Minoris,	1 24 01.5	R. 322 28 13.83	R. 181 26 48.77	141 01 25.06	178 35 24.70	−11.65	319 36 38.11	
near West Elongation,	25 21.0	13.93	44.97	28.96	22.34	−11.65	39.65	
September	26 26.5	14.40	42.77	31.63	20.51	−11.25	40.89	
13, 1877.	27 44.0	13.43	42.27	31.16	18.50	−11.25	38.41	
$\alpha=19^h\ 46^m\ 48^s.3$	37 12.0	D. 142 28 19.93	D. 1 26 53.43	26.50	08.78	+ 0.13	35.41	
$\delta=88°\ 56'\ 24''.9$	38 34.5	21.27	53.07	28.20	08.09	+ 0.13	36.42	
	39 42.0	23.50	53.17	30.33	07.67	− 0.45	37.55	
	40 44.5	22.27	54.67	27.60	07.38	− 0.45	34.54	
	46 54.0	D. 142 28 23.00	D. 1 26 53.03	29.97	07.84	0.00	37.81	
	48 10.0	23.83	54.27	29.56	08.39	0.00	37.95	
	49 10.0	22.77	56.83	25.94	08.92	+ 0.15	35.01	
	50 25.0	21.63	58.27	23.36	09.73	+ 0.15	33.24	
	59 14.5	R. 322 28 12.67	R. 181 26 41.70	30.97	19.79	−11.25	39.51	
	2 00 45.0	13.33	44.43	28.90	22.27	−11.25	39.92	
	01 53.0	12.50	46.97	25.53	24.26	−10.70	39.09	
	02 45.0	12.60	49.50	23.10	25.89	−10.70	38.29	319 36 37.60
λ Ursæ Minoris,	1 29 49.6	R. 346 34 15.03	R. 205 32 46.03	141 01 29.00	178 35 15.84	− 4.80	319 36 40.04	
near West Elongation,	31 20.1	14.43	43.73	30.70	14.02	− 4.80	39.92	
September	32 27.6	15.10	43.57	31.53	12.80	− 3.83	40.50	
14, 1877.	33 36.6	15.13	43.00	32.13	11.68	− 3.82	39.98	
$\alpha=19^h\ 46^m\ 47^s.1$	42 36.1	D. 166 34 19.87	D. 25 32 52.67	27.20	07.39	+ 4.50	39.09	
$\delta=88°\ 56'\ 25''.1$	44 12.6	19.03	54.27	24.76	07.46	+ 4.50	36.72	
	45 22.1	21.13	54.63	26.50	07.65	+ 3.98	38.13	
	46 42.6	19.50	54.17	25.33	08.04	+ 3.97	37.34	
	51 59.6	D. 166 34 19.67	D. 25 32 55.53	24.14	11.27	+ 5.82	41.23	
	53 15.1	19.77	56.60	23.17	12.44	+ 5.82	41.43	
	54 15.6	19.50	58.87	20.63	13.48	+ 5.38	39.49	
	55 26.6	19.60	33 00.40	19.20	14.83	+ 5.37	39.40	
	2 05 44.1	R. 346 34 06.37	R. 205 32 58.53	07.84	32.29	− 4.33	35.80	
	08 04.1	06 07	33 03.03	03.04	37.67	− 4.32	36.39	
	09 11.1	06.93	05.83	01.10	40.44	− 3.68	37.86	
	10 30.1	06.07	08.83	00 57.24	43.83	− 3.67	37.40	319 36 38.79
λ Ursæ Minoris,	1 15 44.0	D. 190 01 36.67	D. 49 00 44.20	141 00 52.47	178 35 44.17	+ 1.50	319 36 38.14	
near West Elongation,	16 54.5	36.43	38.83	57.60	41.14	+ 1.50	40.24	
September	18 03.0	35.27	38.03	57.24	38.32	+ 1.50	37.06	
17, 1877.	19 20.5	35.83	33.47	01 02.36	35.27	+ 1.50	39.13	
$\alpha=19^h\ 46^m\ 43^s.2$	26 47.5	R. 10 01 45.00	R. 229 00 22.60	22.40	20.89	− 9.93	33.36	
$\delta=88°\ 56'\ 25''.7$	27 54.0	44.43	21.10	23.33	19.21	− 9.92	32.62	
	28 46.0	46.20	19.40	26.80	17.98	− 9.75	35.03	
	29 48.5	45.93	16.43	29.50	16.60	− 9.75	36.35	
	34 09.5	R. 10 01 46.47	R. 229 00 11.40	35.07	11.95	−10.25	36.77	
	35 19.0	46.37	11.20	35.17	11.05	−10.25	35.97	
	36 05.6	45.23	09.17	36.06	10.50	−10.13	36.43	
	37 04.0	45.37	08.50	36.87	09.87	−10.12	36.62	
	44 35.0	D. 190 01 38.00	D. 49 00 05.43	32.57	08.31	+ 2.30	43.18	
	46 03.5	35.53	06.10	29.43	08.65	+ 2.30	40.38	
	46 59.0	36.27	06.20	30.07	08.97	+ 1.60	40.64	
	48 18.0	35.77	06.77	29.00	09.57	+ 1.60	40.17	319 36 37.63

Azimuth at West Base, Sandusky—Continued.

Star, date, &c.	Time of observation.	Reading on mark.	Reading on star.	Angle between mark and star.	Azimuth of star.	Level corrections.	Azimuth of mark.	Result for star.
	h. m. s.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	″	° ′ ″	° ′ ″
λ Ursæ Minoris,	1 24 03.4	R. 214 11 21.73	R. 73 10 10.93	141 01 10.80	178 35 25.63	+ 4.50	319 36 40.93	
near West Elongation, September	25 26.9	22.83	07.90	14.93	23.17	+ 4.50	42.60	
18, 1877.	26 16.9	23.77	05.17	18.60	21.79	+ 5.58	45.97	
	27 27.9	24.60	02.70	21.90	19.95	+ 5.57	47.42	
$\alpha = 19^h\ 46^m\ 41^s.9$	35 44.4	D. 34 11 19.63	D. 253 09 47.50	32.13	10.83	− 6.70	36.26	
$\delta = 88°\ 56'\ 25''.8$	37 14.4	19.27	46.27	33.00	09.90	− 6.70	36.20	
	38 10.9	20.17	45.10	35.07	09.41	− 7.58	36.90	
	39 26.9	18.77	44.13	34.64	08.89	− 7.57	35.96	
	44 55.4	D. 34 11 18.73	D. 253 09 43.07	35.66	08.50	− 7.25	36.91	
	46 20.4	18.43	45.17	33.26	08.87	− 7.25	34.88	
	47 23.4	18.77	46.27	32.50	09.27	− 7.63	34.14	
	48 47.4	19.43	46.53	32.90	09.96	− 7.62	35.24	
	57 09.4	R. 214 11 22.60	R. 73 10 06.60	16.00	18.08	+ 5.30	39.38	
	59 49.9	24.17	09.37	14.80	22.09	+ 5.30	42.19	
	2 00 57.9	23.60	10.77	12.83	24.09	+ 4.88	41.71	
	02 36.4	23.77	13.83	09.94	27.00	+ 4.88	41.82	319 36 39.28
λ Ursæ Minoris,	1 18 28.8	R. 238 16 56.30	R. 97 15 52.20	141 01 04.10	178 35 37.88	+ 0.75	319 36 42.73	
near West Elongation, September	19 54.8	56.27	50.43	05.84	34.59	+ 0.75	41.18	
ber 21, 1877.	20 47.8	57.87	47.17	10.70	32.67	+ 1.50	44.87	
	21 57.8	57.57	44.83	12.74	30.23	+ 1.50	44.47	
$\alpha = 19^h\ 46^m\ 38^s.3$	29 03.8	D. 58 17 04.50	D. 277 15 39.00	25.50	18.26	− 7.00	36.76	
$\delta = 88°\ 56'\ 26''.3$	30 20.8	04.43	38.77	25.67	16.63	− 7.00	35.30	
	31 20.3	04.27	38.67	25.60	15.46	− 7.55	33.51	
	32 31.8	03.60	38.23	25.37	14.20	− 7.55	32.02	
	38 14.8	D. 58 17 03.60	D. 277 15 33.23	30.37	10.03	− 7.05	33.35	
	39 39.8	02.50	32.00	30.50	09.49	− 7.05	32.94	
	40 41.3	05.10	29.60	35.50	09.22	− 7.25	37.47	
	42 00.8	04.27	32.33	31.94	09.02	− 7.25	33.71	
	50 18.3	R. 238 16 56.83	R. 97 15 26.43	30.40	11.65	+ 0.83	42.88	
	51 40.3	56.47	28.23	28.24	12.73	+ 0.83	41.80	
	52 30.8	54.50	29.57	24.93	13.61	+ 1.03	39.57	
	53 59.8	54.90	31.40	23.50	14.96	+ 1.03	39.49	319 36 38.25

The next table gives a summary of the results in the preceding table. Periodic error is eliminated from the mean result by the shifting of the circle previously mentioned. The results are corrected in this table for errors in the declinations taken from the American Ephemeris, as explained in § 4. The individual results still involve such residual errors as are not eliminated by reversal of telescope, such as may be due to inaccurate indications of the striding-level, and such as arise from instability of the instrument or its support between pointings to star and to mark. The corrections for inclination of telescope axis are, in some cases, greater than is desirable in such work, but are not beyond the range of accurate determination with a good level. The maximum correction, as will be seen by the preceding table, was 12″.75, and the average 5″.27.

Azimuth at West Base, Sandusky.

SUMMARY OF RESULTS.

AZIMUTH OF LINE AZIMUTH POST — EAST BASE.

Date.	Star, &c.	Position of telescope.	Number of pointings— To mark.	Number of pointings— To star.	Azimuth. Declinations from American Ephemeris.	Reduction to Auwers' Declinations. $\frac{dA}{d\delta}$	$\Delta\delta$	ΔA	Corrected azimuth of mark.
					° ′ 319 36				° ′ 319 36
					″		″	″	″
1877. Sept. 13	Polaris, near East Elongation	Direct	8	8					
		Reversed ...	8	8	41.70	−1.31	+0.04	−0.05	41.65
14	......do	Direct	8	8					
		Reversed ...	8	8	40.25	−1.31	+0.04	−0.05	40.20
17	......do	Direct	8	8					
		Reversed ...	8	8	38.80	−1.31	+0.04	−0.05	38.75
18	......do	Direct	8	8					
		Reversed ...	8	8	42.22	−1.31	+0.04	−0.05	42.17
21	......do	Direct	8	8					
		Reversed ...	8	8	36.70	−1.31	+0.04	−0.05	36.65
13	δ Ursæ Minoris, near West Elongation.	Direct	8	8					
		Reversed ...	8	8	38.27	+1.32	+0.31	+0.41	38.68
14	......do	Direct	8	8					
		Reversed ...	8	8	39.87	+1.32	+0.31	+0.41	40.28
17	... do	Direct	8	8					
		Reversed ...	8	8	38.22	+1.32	+0.31	+0.41	38.63
18	......do	Direct	8	8					
		Reversed ...	8	8	41.31	+1.32	+0.31	+0.41	41.72
21	......do	Direct	8	8					
		Reversed ...	8	8	38.12	+1.32	+0.31	+0.41	38.53
13	51 Cephei, near East Elongation ..	Direct	8	8					
		Reversed ...	8	8	38.82	−1.32	+0.19	−0.25	38.57
14	......do	Direct	8	8					
		Reversed ...	8	8	40.11	−1.32	+0.19	−0.25	39.86
17	......do	Direct	8	8					
		Reversed ...	8	8	38.98	−1.32	+0.19	−0.25	38.73
18	......do	Direct	8	8					
		Reversed ...	8	8	40.60	−1.32	+0.19	−0.25	40.35
21	......do	Direct	8	8					
		Reversed ...	8	8	40.91	−1.32	+0.19	−0.25	40.66
13	λ Ursæ Minoris, near West Elongation.	Direct	8	8					
		Reversed ...	8	8	37.60	+1.32	+0.59	+0.78	38.38
14	......do	Direct	8	8					
		Reversed ...	8	8	38.79	+1.32	+0.59	+0.78	39.57
17	......do	Direct	8	8					
		Reversed ...	8	8	37.63	+1.32	+0.59	+0.78	38.41
18	.... do	Direct	8	8					
		Reversed ...	8	8	39.28	+1.32	+0.59	+0.78	40.06
21	......do	Direct	8	8					
		Reversed ...	8	8	38.25	+1.32	+0.59	+0.78	39.03

Weighted mean.. 319° 36′ 39″.544 ± 0″.209.

Assigning equal weights to the individual results in the last table, their mean is 319° 36′ 39″.54 ± 0″.21, the probable error being derived from the discrepancies between the individual results and the mean. This is the azimuth of East Base from the azimuth post. The correction to reduce this azimuth to the azimuth of East Base from West Base is −01″.25; the correction for diurnal aberration is +00″.31. There thus results as the azimuth of the line West Base — East Base,

319° 36′ 38″.60 ± 0.″21 west of south.

AZIMUTH AT TONAWANDA.

§ **11.** Azimuth determinations were made at Tonawanda station on the nights of August 24, 25, 26, 27, and 28, 1875, by Assistant Engineer G. Y. Wisner. The instrument used was Troughton & Simms 14-inch theodolite No. 1. It was mounted on a solid wooden post set $5\frac{1}{2}$ feet in the ground, and nearly on the line between Tonawanda station and the azimuth-mark, being 0.05 foot north of that line and 30.34 feet from the geodetic point. The stars observed were α, δ, and λ Ursæ Minoris, and 51 Cephei. On the first three nights observations were made in the following order for each star:

1. 8 readings to azimuth light.
2. 8 readings to star with 3 level readings.
3. Reversal of telescope.
4. 8 readings to star with 3 level readings.
5. 8 readings to azimuth light.

On the last two nights the programme was changed to the following, which gave two reversals during the observations on each star and diminished the interval between pointings to the star and mark: pointings to mark, to star, level readings, pointings to star, to mark, reversal, pointings to mark, to star, level readings, pointings to star, to mark—two consecutive pointings being made to each object—this comprising one-half the programme. The circle remained in the same position throughout each night's observations, but was changed in position on each succeeding night, the readings of microscope A on the azimuth-mark being in order 187°, 172°, 217°, 257°, 297°. Time was given by Bond & Sons' chronometer No. 206. The azimuth-mark was a light limited by a vertical slit $\frac{1}{8}$ inch wide, in a box fastened firmly on a post set in the ground 1.3 miles distant from the azimuth-post.

In the reductions for a part of the observations, the differences between the azimuth of the star at the times of observation and at elongation were computed, and these differences were applied to the circle readings on the star, to give the corresponding readings for azimuth of the star at elongation. The mean of these readings for one position of the telescope, subtracted from the mean of the corresponding readings on the mark, gives the horizontal angle between the star at elongation and the mark. In the reduction of the rest of the observations the circle readings on the star were reduced to the pole, the mean of these reduced readings for one position of the telescope, subtracted from the mean of the corresponding readings on the mark, giving the horizontal angle between the mark and the pole.

In the first part of the table following, the first column gives the date of observation, the star observed, and its right ascension, declination, and azimuth at elongation, the latter being denoted by A, and including the correction for diurnal aberration; the second gives the readings on the mark for each position of the telescope and their means; the third gives the readings on the star, reduced to elongation, for each position of the telescope and their means; the fourth gives the mean angles between the mark and the star at elongation for each position of the telescope and their means; the fifth gives the result from each star for azimuth of the mark. In the latter part of the table, the first column gives the date, star, and its coördinates; the second, the readings on the mark; the third, the readings on the star reduced to the pole; the fourth, the angles between the mark and the pole for each position of the telescope and their means; and the fifth, the resulting azimuth of the star, corrected for diurnal aberration. The position of the telescope is indicated by the letters D and R.

Azimuth at Tonawanda.

AZIMUTH OF LINE AZIMUTH POST — AZIMUTH MARK.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1.]

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Polaris, near East Elongation, August 24, 1875.	D. 187 24 09.60	D. 133 57 41.30		
$\alpha=1^h\ 13^m\ 23^s.57$	09.17	38.69		
$\delta=88°\ 38'\ 34''.03$	08.60	37.66		
$A_e=181°\ 51'\ 21''.82$	08.50	37.42		
	08.87	39.46		
	08.97	38.61		
	08.27	38.08		
	08.13	38.30		
	Means .. 08.76	38.69	53 26 30.07	
	R. 7 23 57.67	R. 313 57 31.88		
	58.17	28.47		
	57.20	26.87		
	56.77	24.35		
	56.70	22.47		
	57.17	23.54		
	56.87	23.96		
	57.53	22.61		
	Means .. 57.26	25.52	31.74	
	Mean	....	53 26 30.91	235 17 52.73
Polaris, near East Elongation, August 25, 1875.	D. 172 24 03.50	D. 118 57 34.53		
$\alpha=1^h\ 13^m\ 24^s.29$	04.30	34.68		
$\delta=88°\ 38'\ 34''.32$	02.90	33.75		
$A_e=181°\ 51'\ 21''.42$	03.70	32.66		
	04.13	32.71		
	03.60	32.84		
	04.67	32.73		
	03.63	32.31		
	Means .. 03.80	33.28	53 26 30.52	
	R. 352 23 59.50	R. 298 57 30.09		
	59.97	29.70		
	58.50	29.71		
	59.97	27.84		
	60.43	27.31		
	59.50	26.64		
	60.03	27.09		
	59.97	27.42		
	Means .. 59.73	28.22	31.51	
	Mean	....	53 26 31.01	235 17 52.43

Azimuth at Tonawanda—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Polaris, near East Elongation, August 26, 1875.	D. 217 24 04. 53	D. 163 57 32. 06		
α=1h 13m 25s.00	04. 20	33. 70		
δ=88° 38′ 34″.60	03. 63	33. 64		
A_e=181° 51′ 21″.04	03. 93	34. 05		
	04. 20	34. 88		
	04. 20	34. 13		
	04. 33	35. 29		
	04. 90	35. 37		
	Means .. 04. 24	34. 14	53 26 30. 10	
	R. 37 24 03. 57	R. 343 57 31. 34		
	02. 60	30. 64		
	02. 77	30. 28		
	03. 20	29. 04		
	03. 37	28. 70		
	01. 67	29. 01		
	03. 33	28. 07		
	02. 67	28. 99		
	Means .. 02. 90	29. 51	33. 39	
	Mean	....	53 26 31. 74	235 17 52. 78
Polaris, near Eastern Elongation, August 27, 1875.	D. 257 24 27. 50	D. 203 57 59. 57		
α=1h 13m 25s.62	28. 00	59. 17		
δ=88° 38′ 35″.02	28. 83	58. 81		
A_e=181° 51′ 20″.46	27. 33	61. 04		
	Means .. 27. 92	59. 65	53 26 28. 27	
	R. 77 24 24. 93	R. 23 57 57. 46		
	24. 70	55. 41		
	26. 60	56. 57		
	25. 27	57. 37		
	Means .. 25. 38	56. 70	28. 68	
	R. 77 24 26. 50	R. 23 57 57. 46		
	27. 13	56. 25		
	28. 17	54. 29		
	27. 60	54. 22		
	Means .. 27. 35	55. 55	31. 80	
	D. 257 24 28. 23	D. 203 57 58. 57		
	28. 43	57. 75		
	26. 67	58. 40		
	27. 47	57. 24		
	Means .. 27. 70	57. 99	29. 71	
	Mean	....	53 26 29. 61	235 17 50. 07

Azimuth at Tonawanda—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Polaris, near East Elongation, August 28, 1875.	D. 297 24 56.70	D. 243 58 25.22		
$\alpha=1^h\ 13^m\ 26^s.20$	57.27	25.64		
$\delta=88°\ 38'\ 35''.33$	56.37	26.04		
$A_e=181°\ 51'\ 20''.04$	55.97	26.59		
	Means .. 56.58	25.87	53 26 30.71	
	R. 117 24 55.83	R. 63 58 24.70		
	55.97	24.59		
	56.67	25.72		
	56.40	24.50		
	Means .. 56.22	24.88	31.34	
	R. 117 24 57.10	R. 63 58 23.82		
	56.37	24.28		
	55.87	25.46		
	56.37	24.49		
	Means . 56.43	24.51	31.92	
	D. 297 24 57.40	D. 243 58 24.78		
	57.33	24.59		
	58.03	24.47		
	56.97	21.99		
	Means .. 57.43	23.96	33.47	
	Mean	....	53 26 31.86	235 17 51.90
δ Ursæ Minoris, near West Elongation, August 25, 1875.	R. 352 23 58.93	R. 292 27 52.24		
$\alpha=18^h\ 12^m\ 38^s.52$	58.97	49.20		
$\delta=86°\ 36'\ 35''.43$	59.60	50.75		
$A_e=175°\ 21'\ 43''.56$	58.57	49.93		
	58.43	51.14		
	58.23	50.72		
	58.20	50.45		
	58.40	50.53		
	Means .. 58.67	50.62	59 56 08.05	
	D. 172 24 07.40	D. 112 27 59.97		
	06.70	57.27		
	07.97	57.20		
	06.37	56.51		
	05.90	56.50		
	06.57	56.79		
	05.73	59.72		
	05.77	61.26		
	Means .. 06.55	58.15	08.40	
	Mean	....	59 56 08.22	235 17 51.78

Azimuth at Tonawanda—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
δ Ursæ Minoris, near West Elongation, August 26, 1875.	R. 37 24 04.60	R. 337 27 57.64		
$\alpha=18^h\ 12^m\ 38^s.09$	03.43	55.77		
$\delta=86°\ 36'\ 35''.52$	04.67	54.90		
$A_e=175°\ 21'\ 43''.69$	03.13	55.98		
	03.90	55.87		
	03.07	56.41		
	04.40	56.35		
	04.67	56.21		
	Means .. 03.98	56.14	59 56 07.84	
	D. 217 24 06.67	D. 157 27 57.05		
	07.83	57.76		
	08.60	58.65		
	08.43	59.17		
	08.83	58.16		
	08.40	57.44		
	07.43	60.01		
	07.80	64.70		
	Means .. 08.00	59.12	08.88	
	Mean	....	59 56 08.36	235 17 52.05
51 Cephei, near East Elongation, August 24, 1875.	R. 187 24 02.93	R. 135 53 18.23		
$\alpha=6^h\ 41^m\ 17^s.73$	02.67	16.71		
$\delta=87°\ 13'\ 57''.15$	03.33	16.50		
$A_e=183°\ 47'\ 07''.91$	03.73	15.24		
	02.73	18.00		
	01.80	18.51		
	02.33	18.00		
	03.17	18.86		
	Means .. 02.84	17.51	51 30 45.33	
	D. 7 24 05.17	D. 315 53 27.27		
	06.07	23.19		
	06.03	24.58		
	06.70	21.04		
	06.03	22.46		
	05.03	25.08		
	05.27	26.90		
	05.17	27.07		
	Means .. 05.68	24.70	40.98	
	Mean	....	51 30 43.16	235 17 51.07

Azimuth at Tonawanda—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
λ Ursæ Minoris, near West Elongation, August 24, 1875.	D. 7 24 05.33	D. 310 38 46.74		
$a=19^h\ 49^m\ 23^s.66$	05.90	42.76		
$\delta=88°\ 55'\ 58''.67$	05.53	41.55		
$A_e=178°\ 32'\ 27''.50$	05.73	40.37		
	06.30	40.49		
	06.83	41.36		
	07.33	41.48		
	06.63	41.94		
	Means .. 06.20	42.09	56 45 24.11	
	R. 187 24 05.93	R. 130 38 41.99		
	05.30	41.67		
	06.00	40.86		
	06.07	41.04		
	06.33	40.90		
	06.03	40.79		
	06.23	42.89		
	06.70	43.73		
	Means .. 06.07	41.73	24.34	
	Mean	....	56 45 24.22	235 17 51.72
λ Ursæ Minoris, near West Elongation, August 25, 1875.	R. 357 23 59.33	R. 295 38 36.77		
$a=19^h\ 49^m\ 22^s.66$	59.23	39.08		
$\delta=88°\ 55'\ 58''.98$	59.80	38.82		
$A_e=178°\ 32'\ 27''.92$	60.23	36.98		
	58.90	37.45		
	59.90	37.35		
	60.10	37.51		
	59.63	38.21		
	Means .. 59.64	37.77	56 45 21.87	
	D. 172 24 02.37	D. 115 38 40.10		
	03.90	42.96		
	04.23	45.65		
	03.83	45.60		
	04.43	44.03		
	02.97	44.31		
	03.33	44.66		
	03.63	44.18		
	Means .. 03.59	43.94	19.65	
	Mean	....	56 45 20.76	235 17 48.68

Azimuth at Tonawanda—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
λ Ursæ Minoris, near West Elongation, August 26, 1875.	R. 37 24 05.67	R. 340 38 42.19		
	04.93	43.76		
	05.27	41.28		
$\alpha=19^h\ 49^m\ 21^s.61$	04.70	41.34		
$\delta=88°\ 55'\ 59''.27$	04.50	40.76		
$A_e=178°\ 32'\ 28''.32$	05.00	40.43		
	05.43	40.67		
	05.17	40.68		
	Means .. 05.08	41.39	56 45 23.69	
	D. 217 24 02.50	D. 160 38 40.99		
	05.00	45.34		
	05.20	44.07		
	05.13	45.88		
	05.03	47.83		
	05.67	47.17		
	05.97	47.79		
	05.43	47.05		
	Means .. 04.99	45.77	19.22	
	Mean	....	56 45 21.45	235 17 49.77
λ Ursæ Minoris, near West Elongation, August 27, 1875.	D. 257 24 34.10	D. 200 39 11.08		
	33.90	09.84		
	34.13	10.02		
$\alpha=19^h\ 49^m\ 20^s.55$	34.13	11.25		
$\delta=88°\ 55'\ 59''.55$	Means .. 34.06	10.55	56 45 23.51	
$A_e=178°\ 32'\ 28''.70$	R. 77 24 32.03	R. 20 39 07.88		
	32.33	08.64		
	32.23	09.31		
	31.93	07.70		
	Means .. 32.13	08.38	23.75	
	R. 77 24 31.90	R. 20 39 08.44		
	31.77	07.85		
	30.87	08.55		
	31.17	08.34		
	Means .. 31.43	08.29	23.14	
	D. 257 24 35.07	D. 200 39 10.94		
	33.17	13.60		
	33.63	14.57		
	33.17	14.16		
	Means .. 33.76	13.32	20.44	
	Mean	....	56 45 22.71	235 17 51.41

Azimuth at Tonawanda—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
λ Ursæ Minoris, near West Elongation, August 27, 1875.	D. 297 25 08.37	D. 240 39 46.63		
	08.80	47.30		
	09.10	45.65		
$\alpha=19^h\ 49^m\ 19^s.49$	08.23	45.74		
$\delta=88°\ 55'\ 59''.77$	Means.. 08.62	46.33	56 45 22.29	
$A_e=178°\ 32'\ 29''.00$				
	R. 117 25 06.60	R. 60 39 45.45		
	07.43	46.09		
	05.63	45.14		
	06.90	45.01		
	Means.. 06.64	45.42	21.22	
	R. 117 25 06.90	R. 60 39 46.18		
	06.87	45.53		
	08.37	43.93		
	08.23	44.33		
	Means.. 07.59	44.99	22.60	
	D. 297 25 09.83	D. 240 39 47.10		
	08.90	45.25		
	08.23	44.69		
	08.50	45.41		
	Means.. 08.87	45.61	23.26	
	Mean	....	56 45 22.34	235 17 51.34

Star, date, &c.	Readings on mark.	Readings on star reduced to pole.	Angle between mark and pole.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
δ Ursæ Minoris, near West Elongation, August 24, 1875.	D. 7 24 04.67	D. 312 06 15.61		
	04.23	12.03		
	04.93	14.05		
$\alpha=18^h\ 12^m\ 38^s.95$	03.67	13.75		
$\delta=86°\ 36'\ 35''.25$	03.97	15.00		
	03.83	13.12		
	03.43	13.01		
	03.67	13.43		
	Means.. 04.05	13.75	55 17 50.30	
	R. 187 24 02.93	R. 132 06 11.61		
	02.67	09.59		
	03.33	09.39		
	03.73	08.50		
	02.73	08.51		
	01.80	09.98		
	02.33	10.39		
	03.17	10.54		
	Means.. 02.84	09.81	53.03	
	Mean	....	55 17 51.66	235 17 51.97

Azimuth at Tonawanda—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to pole.	Angle between mark and pole.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
δ Ursæ Minoris, near West Elongation, August 27, 1875.	D. 257 24 32.00	D. 202 06 42.31		
$\alpha=18^h\ 12^m\ 37^s.67$	32.40	37.03		
$\delta=86°\ 36'\ 35''.63$	32.93	40.33		
	33.10	39.78		
	Means .. 32.61	39.86	55 17 52.75	
	R. 77 24 27.43	R. 22 06 38.67		
	28.97	38.73		
	29.83	37.93		
	31.17	38.03		
	Means .. 29.35	38.34	51.01	
	R. 77 24 31.00	R. 22 06 36.99		
	29.43	39.51		
	29.73	38.78		
	30.10	38.85		
	Means .. 30.06	38.53	51.53	
	D. 257 24 34.00	D. 202 06 42.22		
	34.27	39.88		
	34.23	40.19		
	33.47	37.67		
	Means .. 33.99	39.99	54.00	
	Mean	....	55 17 52.32	235 17 52.63
δ Ursæ Minoris, near West Elongation, August 28, 1875.	D. 297 25 00.50	D. 242 07 08.10		
$\alpha=18^h\ 12^m\ 37^s.25$	00.13	07.68		
$\delta=86°\ 36'\ 35''.72$	01.50	08.76		
	00.37	09.53		
	Means .. 00.62	08.52	55 17 52.10	
	R. 117 24 56.50	R. 62 07 05.25		
	57.93	08.72		
	59.10	09.99		
	58.93	09.82		
	Means .. 58.11	08.44	49.67	
	R. 117 24 58.33	R. 62 07 04.91		
	59.17	07.94		
	58.60	04.38		
	58.63	04.84		
	Means .. 58.68	05.52	53.16	
	D. 297 25 01.33	D. 242 07 07.70		
	01.30	11.42		
	00.17	09.67		
	01.27	10.70		
	Means .. 01.02	09.87	51.15	
	Mean	....	55 17 51.52	235 17 51.83

Azimuth at Tonawanda—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to pole.	Angle between mark and pole.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
51 Cephei, near East Elongation, August 25, 1875.	D. 172 24 06.03	D. 117 06 14.92		
$\alpha=6^h\ 41^m\ 18^s.24$	07.33	12.84		
$\delta=87^\circ\ 13'\ 56''.95$	07.60	12.76		
	07.70	13.41		
	08.17	15.41		
	06.83	15.97		
	07.30	15.57		
	06.17	15.54		
	Means .. 07.14	14.55	55 17 52.59	
	R. 352 24 01.63	R. 297 06 09.81		
	02.07	12.12		
	01.47	10 02		
	00.63	09.79		
	00.77	09.23		
	01.97	09.06		
	00.47	10.42		
	00.73	09.77		
	Means .. 01.22	10.03	51.19	
	Mean	....	55 17 51.89	235 17 52.20
51 Cephei, near East Elongation, August 26, 1875.	D. 217 24 07.17	D. 162 06 12.03		
$\alpha=6^h\ 41^m\ 18^s.77$	07.43	17.62		
$\delta=87^\circ\ 13'\ 56''.75$	08.70	20.17		
	08.37	20.10		
	08.40	18.00		
	08.73	17.90		
	07.10	16.98		
	07.47	15.16		
	Means .. 07.92	17.24	55 17 50.68	
	R. 37 24 03.50	R. 342 06 11.51		
	03.33	12.17		
	04.23	11.33		
	04.57	11.55		
	04.93	12.26		
	04.23	12.72		
	04.97	12.62		
	04.90	11.02		
	Means .. 04.33	11.90	52.43	
	Mean	....	55 17 51.55	235 17 51.86

Azimuth at Tonawanda—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to pole.	Angle between mark and pole.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
51 Cephei, near East Elongation, August 27, 1875.	D. 257 24 33.73	D. 202 06 45.33		
$\alpha = 6^h\ 41^m\ 19^s.30$	34.17	42.17		
$\delta = 87°\ 13'\ 56''.63$	35.53	43.04		
	33.17	45.12		
	Means .. 34.15	43.91	55 17 50.24	
	R. 77 24 29.17	R. 22 06 37.52		
	29.33	37.68		
	30.07	36.78		
	30.53	35.93		
	Means .. 29.39	36.98	52.41	
	R. 77 24 30.10	R. 22 06 38.32		
	30.50	37.27		
	29.33	37.43		
	29.17	37.82		
	Means .. 29.77	37.35	52.42	
	D. 257 24 34.93	D. 202 06 44.27		
	34.73	40.38		
	32.77	40.65		
	34.40	39.74		
	Means .. 34.21	41.26	52.95	
	Mean	....	55 17 52.01	235 17 52.32
51 Cephei, near East Elongation, August 28, 1875.	D. 297 25 00.77	D. 242 07 11.98		
$\alpha = 6^h\ 41^m\ 19^s.80$	01.83	11.04		
$\delta = 87°\ 13'\ 56''.45$	01.83	08.74		
	02.47	09.40		
	Means .. 01.72	10.29	55 17 51.43	
	R. 117 24 58.17	R. 62 07 08.74		
	59.03	07.88		
	58.67	07.61		
	59.30	05.95		
	Means .. 58.79	07.54	51.25	
	R. 117 24 59.93	R. 62 07 04.65		
	59.30	04.76		
	60.40	05.66		
	59.03	06.02		
	Means .. 59.66	05.27	54.39	
	D. 297 25 08.17	D. 242 07 12.68		
	05.10	14.03		
	08.17	12.90		
	08.17	12.51		
	Means .. 07.40	13.03	54.37	
	Mean	....	55 17 52.86	235 17 53.17

The next table gives a summary of the data in the preceding table, each result for azimuth of mark being corrected for periodic error, by the formula given in Chapter XIV, B, § 6. These results are further corrected in the table for errors in the declinations taken from the American Ephemeris, as explained in § 4. They still involve such residual errors as are not eliminated by reversal of telescope, such as may be due to inaccurate indications of the striding-level, and such as arise from instability of the instrument or its support between pointings to star and to mark. The greatest and least intervals of time between the first pointing to the mark and the last pointing to the star and *vice versa*, on the first three nights, were respectively 22 minutes and 12 minutes, the average interval being about 17 minutes, so that the stability of the instrument and its support was assumed for about 8 minutes. There being two reversals and fewer readings to mark and star in a set on the last two nights, the stability of the instrument was assumed for a much less time. The corrections for inclination of telescope axis were generally not large, the maximum being 9″.87 and the average 3″.73. The micrometers were thoroughly adjusted for error of run before any observations were made, and readings were made each night to test their stability.

Azimuth at Tonawanda.

SUMMARY OF RESULTS.

AZIMUTH OF LINE AZIMUTH POST — AZIMUTH MARK.

Date.	Star, &c.	Position of telescope.	Number of pointings— To mark.	Number of pointings— To star.	Azimuth. Declinations from American Ephemeris.	Reduction to Auwers' Declinations. $\frac{dA}{d\delta}$	Reduction to Auwers' Declinations. $\Delta\delta$	Reduction to Auwers' Declinations. ΔA	Corrected azimuth of mark.
					° ′ 235 17				° ′ 235 17
1875.					″		″	″	″
Aug. 24	Polaris, near East Elongation	Direct	8	8					
		Reversed	8	8	52.69	−1.37	+0.04	−0.05	52.64
25	......do	Direct	8	8					
		Reversed	8	8	52.14	−1.37	+0.04	−0.05	52.09
26	......do	Direct	8	8					
		Reversed	8	8	52.82	−1.37	+0.04	−0.05	52.77
27	......do	Direct	8	8					
		Reversed	8	8	50.30	−1.37	+0.04	−0.05	50.25
28	......do	Direct	8	8					
		Reversed	8	8	51.63	−1.37	+0.04	−0.05	51.58
24	δ Ursæ Minoris, near West Elongation.	Direct	8	8					
		Reversed	8	8	51.97	+1.38	+0.31	+0.43	52.40
25	......do	Direct	8	8					
		Reversed	8	8	51.78	+1.38	+0.31	+0.43	52.21
26	......do	Direct	8	8					
		Reversed	8	8	52.06	+1.38	+0.31	+0.43	52.49
27	......do	Direct	8	8					
		Reversed	8	8	52.63	+1.38	+0.31	+0.43	53.06
28	......do	Direct	8	8					
		Reversed	8	8	51.83	+1.38	+0.31	+0.43	52.26
24	51 Cephei, near East Elongation	Direct	8	8					
		Reversed	8	8	51.07	−1.37	+0.20	−0.27	50.80
25	......do	Direct	8	8					
		Reversed	8	8	51.83	−1.37	+0.20	−0.27	51.56
26	......do	Direct	8	8					
		Reversed	8	8	51.87	−1.37	+0.20	−0.27	51.60
27	......do	Direct	8	8					
		Reversed	8	8	52.63	−1.37	+0.20	−0.27	52.36
28	......do	Direct	8	8					
		Reversed	8	8	52.84	−1.37	+0.20	−0.27	52.57
24	λ Ursæ Minoris, near West Elongation.	Direct	8	8					
		Reversed	8	8	51.67	+1.39	+0.59	+0.82	52.49
25	......do	Direct	8	8					
		Reversed	8	8	48.54	+1.39	+0.59	+0.82	49.36
26	......do	Direct	8	8					
		Reversed	8	8	49.82	+1.39	+0.59	+0.82	50.64
27	......do	Direct	8	8					
		Reversed	8	8	51.51	+1.39	+0.59	+0.82	52.33
28	......do	Direct	8	8					
		Reversed	8	8	51.19	+1.39	+0.59	+0.82	52.01

Weighted mean .... 235° 17′ 51″.873 ± 0″.142

Assigning equal weights to the individual results in the last table, their mean is 235° 17′ 51″.873 ±0″.142, the probable error being derived from the discrepancies between the individual results and their mean. To reduce this azimuth to the azimuth of the line Tonawanda — Azimuth Light, the following corrections must be applied: For position of instrument, —1″.436; for convergence of meridians between the observing-post and station Tonawanda, —0″.229, making the azimuth of the line Tonawanda — Azimuth Light, 235° 17′ 50″.208±0″.142.

To refer this azimuth to the primary triangulation line Tonawanda — Buffalo Plains, the angle between the azimuth light and a similar light placed on Buffalo Plains station was read on two nights, 20 combined results being obtained, giving an angle of 79° 04′ 50″.403±0″.482.

The probable error assigned to this angle is the probable error of an observed angle of weight 1 in this section of the primary triangulation as shown by the adjustment. (See Chapter XVIII, C, § 5.) Combining this angle with the above azimuth, there results as the azimuth of the line Tonawanda — Buffalo Plains,

314° 22′ 40″.61 ± 0″.50 west of south.

AZIMUTH AT NORTH BASE, SANDY CREEK.

§ **12.** Observations for a primary azimuth at North Base (Sandy Creek base-line), were made by Assistant Engineer G. Y. Wisner on the nights of September 13, 14, 18, 20, and 21, 1874. The instrument used was the Troughton & Simms theodolite No. 1. It was mounted on a large pine post set firmly in the ground. The same post was subsequently used to support a zenith-telescope in latitude determinations. (See Chapter XXIII, § 19.)

The method of observation was substantially the same as that followed at Ford River station and described in § 5. The stars observed were α, δ, and λ Ursæ Minoris, and 51 Cephei. Time was taken from a chronometer. The reference-mark was a light placed at South Base station, about three miles distant. The light was limited by a narrow vertical slit in the box containing the lamp.

In the reduction the observed readings on the azimuth circle for pointings to the star, corrected for inclination of telescope axis, were increased or decreased by the small computed angle between the vertical planes through the star at the instant of observation and at elongation, giving thus the corresponding readings on the circle for star at elongation. The differences between the means of the readings on the mark and on the star at elongation for the direct and reversed positions of the telescope, give two results for the horizontal angle between the star at elongation and the mark; and the mean of these results gives this angle as nearly as may be, free from the error of collimation. Adding to the latter angle the azimuth of the star at elongation plus the correction for diurnal aberration, the azimuth of the mark results. The following table gives the individual and mean readings on the mark and star, the mean results from the different stars for each night, &c. The coördinates of the stars given in the first column are taken from the American Ephemeris. The azimuth of the star at elongation, denoted by A, is corrected for diurnal aberration. The relative positions of the telescope are indicated by the letters D and R. It should be observed that the means of the readings on the mark for telescope D and R do not give a reliable measure of the error of collimation. This arises from the fact that the telescope must be raised out of its wyes by hand in reversal, an operation which, on account of the weight of the telescope, is very likely to disturb the instrument in azimuth.

Azimuth at North Base, Sandy Creek.

AZIMUTH OF LINE AZIMUTH POST—AZIMUTH MARK.

[Observer, G. Y. Wisner. Instrument, Troughton & Simms theodolite No. 1.]

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
Polaris, near East Elongation, September 13, 1874.	R. 14 47 55.5	R. 199 31 15.98		
$\alpha=1^h\ 13^m\ 14^s.2$	56.1	15.76		
$\delta=88°\ 38'\ 19''.2$	56.2	19.23		
$A_e=181°\ 52'\ 57''.13$	55.7	18.67		
	55.7	17.30		
	56.1	17.78		
	Means.. 55.88	17.62	175 16 38.26	
	D. 14 47 58.7	D. 199 31 18.51		
	59.4	17.95		
	59.9	19.11		
	59.9	19.10		
	59.9	17.71		
	60.3	18.35		
	Means.. 59.68	18.45	41.23	
	Mean................	....	175 16 39.74	357 09 36.87
Polaris, near East Elongation, September 14, 1874.	D. 335 32 50.7	D. 160 16 09.55		
$\alpha=1^h\ 13^m\ 14^s.8$	51.4	09.51		
$\delta=88°\ 38'\ 19''.6$	51.8	10.70		
$A=181°\ 52'\ 56''.58$	50.9	11.95		
	50.1	11.80		
	50.1	12.05		
	Means.. 50.83	10.93	175 16 39.90	
	R. 335 32 40.7	R. 160 15 57.71		
	40.3	57.37		
	40.3	57.42		
	40.3	58.55		
	40.2	58.35		
	40.9	58.51		
	Means.. 40.45	57.98	42.47	
	Mean................	....	175 16 41.18	357 09 37.76
Polaris, near East Elongation, September 21, 1874.	D. 153 26 51.5	D. 338 10 12.80		
$\alpha=1^h\ 13^m\ 17^s.8$	51.4	14.00		
$\delta=88°\ 38'\ 22''.3$	51.4	12.57		
$A_e=181°\ 52'\ 52''.84$	51.7	12.57		
	51.0	11.23		
	51.0	10.71		
	Means.. 51.33	12.31	175 16 39.02	
	R. 333 26 47.3	R. 158 09 57.11		
	45.5	56.30		
	45.8	56.41		
	45.8	57.53		
	45.8	57.66		
	45.7	57.50		
	Means.. 45.98	57.08	48.90	
	Mean................	....	175 16 43.96	357 09 36.80

Azimuth at North Base, Sandy Creek—Continued.

Star, date, &c.	Reading on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
δ Ursæ Minoris, near West Elongation, September 14, 1874.	R. 335 32 39.0	R. 153 41 39.68		
	40.4	43.00		
	39.8	43.58		
$\alpha=18^h\ 12^m\ 51^s.87$	39.9	42.97		
$\delta=86°\ 36'\ 37''.1$	40.5	43.64		
$A_e=175°\ 18'\ 38''.60$	40.6	45.79		
	Means .. 40.03	43.11	181 50 56.92	
	D. 335 32 51.3	D. 153 41 52.24		
	50.5	52.43		
	52.0	52.44		
	51.2	53.19		
	52.0	52.23		
	51.1	51.75		
	Means .. 51.35	52.38	58.97	
	Mean	....	181 50 57.94	357 09 36.54
δ Ursæ Minoris, near West Elongation, September 18, 1874.	R. 215 28 21.2	R. 33 37 31.19		
	22.2	28.50		
	22.2	30.25		
$\alpha=18^h\ 12^m\ 50^s.07$	22.6	28.81		
$\delta=86°\ 36'\ 37''.4$	22.8	29.92		
$A_e=175°\ 18'\ 39''.01$	22.9	29.38		
	Means .. 22.32	29.67	181 50 52.65	
	D. 35 28 26.7	D. 213 37 29.33		
	27.2	30.19		
	27.0	32.17		
	27.7	33.00		
	27.4	33.04		
	27.7	32.23		
	Means .. 27.28	31.66	55.62	
	Mean	....	181 50 54.13	357 09 33.14
δ Ursæ Minoris, near West Elongation, September 20, 1874.	D. 154 17 32.2	D. 332 26 41.08		
	33.8	41.69		
	34.1	40.58		
$\alpha=18^h\ 12^m\ 49^s.15$	33.7	40.51		
$\delta=86°\ 36'\ 37''.5$	34.4	40.37		
$A_e=175°\ 18'\ 39''.15$	35.0	41.22		
	Means .. 34.03	40.91	181 50 53.12	
	R. 334 17 26.8	R. 152 26 29.95		
	27.3	32.59		
	28.1	33.14		
	28.5	32.34		
	28.0	31.87		
	28.5	32.51		
	Means .. 27.87	32.07	55.80	
	Mean	....	181 50 54.46	357 09 33.61

Azimuth at North Base, Sandy Creek—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
δ Ursæ Minoris, near West Elongation, September 21, 1874.	R. 333 26 44.8	R. 151 35 49.28		
	45.2	47.60		
	45.7	46.67		
$\alpha=18^h\ 12^m\ 48^s.7$	45.1	48.72		
$\delta=86°\ 36'\ 37''.4$	45.0	50.29		
$A_e=175°\ 18'\ 39''.01$	45.5	49.96		
	Means .. 45.22	48.75	181 50 56.47	
	D. 153 26 49.8	D. 331 35 51.90		
	49.6	52.39		
	50.3	52.68		
	49.4	52.47		
	49.8	52.05		
	49.6	51.90		
	Means .. 49.75	52.23	57.52	
	Mean		181 50 56.99	357 09 36.00
51 Cephei, near East Elongation, September 18, 1874.	D. 35 28 26.4	D. 222 08 34.94		
	27.2	34.15		
	26.9	33.30		
$\alpha=6^h\ 40^m\ 56^s.68$	26.6	33.95		
$\delta=87°\ 13'\ 56''.6$	26.7	35.63		
$A_e=183°\ 49'\ 41''.41$	27.2	36.75		
	Means .. 26.83	34.79	173 19 52.04	
	R. 215 28 26.9	R. 42 08 27.58		
	25.7	28.44		
	24.8	29.89		
	24.9	30.48		
	24.2	29.78		
	23.9	29.13		
	Means .. 25.07	29.22	55.85	
	Mean		173 19 53.94	357 09 35.35
51 Cephei, near East Elongation, September 20, 1874.	R. 334 17 28.8	R. 160 57 34.92		
	28.2	33.10		
	27.3	33.75		
$\alpha=6^h\ 40^m\ 57^s.85$	28.5	33.95		
$\delta=87°\ 13'\ 56''.5$	27.9	32.60		
$A_e=183°\ 49'\ 41''.54$	27.9	30.73		
	Means .. 28.10	33.18	173 19 54.92	
	D. 154 17 34.3	D. 340 57 41.39		
	34.8	40.70		
	33.9	39.21		
	33.3	39.27		
	33.2	40.40		
	32.9	39.56		
	Means .. 33.73	40.09	53.64	
	Mean		173 19 54.28	357 09 35.82

Azimuth at North Base, Sandy Creek—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
51 Cephei, near East Elongation, September 21, 1874.	D. 153 26 50.0	D. 340 06 56.11		
$\alpha=6^h\ 40^m\ 58^s.41$	49.6	54.94		
$\delta=87°\ 13'\ 56''.5$	49.5	54.51		
$A_e=183°\ 49'\ 41''.54$	49.7	56.28		
	49.3	55.67		
	49.3	54.36		
	Means .. 49.57	55.31	173 19 54.26	
	R. 333 26 47.0	R. 160 06 53.65		
	48.7	53.10		
	49.0	51.29		
	49.9	52.05		
	49.7	52.01		
	49.4	52.78		
	Means .. 48.95	52.48	56.47	
	Mean	....	173 19 55.36	357 09 36.90
λ Ursæ Minoris, near West Elongation, September 18, 1874.	R. 215 28 24.4	R. 36 50 14.96		
$\alpha=19^h\ 50^m\ 05^s.5$	24.6	14.14		
$\delta=88°\ 55'\ 55''.45$	25.2	15.22		
$A_e=178°\ 31'\ 24''.24$	24.9	15.17		
	24.2	13.88		
	24.2	13.68		
	Means .. 24.58	14.51	178 38 10.07	
	D. 35 28 28.8	D. 216 50 18.12		
	28.7	17.96		
	27.6	17.60		
	27.5	19.01		
	26.7	18.69		
	27.1	20.14		
	Means .. 27.73	18.59	09.14	
	Mean	....	178 38 09.60	357 09 33.84
λ Ursæ Minoris, near West Elongation, September 20, 1874.	D. 154 17 32.1	D. 335 39 26.22		
$\alpha=19^h\ 50^m\ 02^s.84$	32.6	26.41		
$\delta=88°\ 55'\ 55''.8$	33.2	25.95		
$A_e=178°\ 31'\ 24''.73$	33.5	25.47		
	31.3	24.47		
	30.8	24.86		
	Means .. 32.25	25.56	178 38 06.69	
	R. 334 17 28.8	R. 155 39 17.66		
	29.1	18.12		
	29.7	18.50		
	29.5	18.12		
	29.5	19.20		
	29.5	19.74		
	Means .. 29.35	18.56	10.79	
	Mean	....	178 38 08.74	357 09 33.47

Azimuth at North Base, Sandy Creek—Continued.

Star, date, &c.	Readings on mark.	Readings on star reduced to elongation.	Angle between mark and star at elongation.	Result for star.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″
λ Ursæ Minoris, near West Elongation, September 21, 1874.	R. 333 26 49.3	R. 154 48 40.55		
	49.6	39.47		
	48.3	40.30		
$\alpha=19^h\ 50^m\ 01^s.52$	49.0	40.63		
$\delta=88°\ 55'\ 55''.95$	48.5	41.14		
$A_e=178°\ 31'\ 24''.94$	48.9	42.02		
	Means.. 48.93	40.68	178 38 08.25	
	D. 153 26 49.7	D. 334 48 43.33		
	50.5	45.53		
	49.8	41.49		
	50.1	40.82		
	49.6	40.35		
	49.6	40.03		
	Means.. 49.88	41.92	07.95	
	Mean	....	178 38 08.10	357 09 33.04

The next table gives a summary of the results in the preceding table. The results for azimuth of mark in the sixth column have been corrected for periodic error by the formula given in Chapter XIV, B, § 6. The results are further corrected in the table for errors in the declinations given in the American Ephemeris in the manner explained in § 4. They still involve errors not wholly eliminated by reversal of telescope, those due to inaccurate indications of the striding-level, and those due to instability of the instrument or its support during the interval between the pointings to the star and to the mark. The corrections for inclination of telescope axis were generally greater than is desirable in such work, but not beyond the range of accurate determination with a good level. The maximum correction was $9''.3$, and the average $4''.8$. The values of one division of the striding-level (depending on the temperature) used in the computation were those determined earlier in the same season at Ford River station (see § 5). The greatest and least intervals of time between the first pointing to the mark and the last pointing to the star, and *vice versa*, were 17^m and 10^m, respectively; the average interval was 13^m, so that the stability of the instrument and its support was assumed for about 7^m.

Azimuth at North Base, Sandy Creek.

SUMMARY OF RESULTS.

AZIMUTH OF THE LINE AZIMUTH POST—AZIMUTH MARK.

Date.	Star, &c.	Position of telescope.	Number of pointings— To mark.	Number of pointings— To star.	Azimuth. Declinations from American Ephemeris.	Reduction to Auwers' Declinations. $\frac{dA}{d\delta}$	$\Delta\delta$	ΔA	Corrected azimuth of mark.
					° ′ 357 09				° ′ 357 09
					″		″	″	″
1874. Sept. 13	Polaris, near East Elongation	Direct	7	7					
		Reversed	7	7	37. 20	−1. 37	+0. 04	−0. 05	37. 15
14	......do	Direct	6	6					
		Reversed	6	6	37. 98	−1. 37	+0. 04	−0. 05	37. 93
21	......do	Direct	6	6					
		Reversed	6	6	36. 93	−1. 37	+0. 04	−0. 05	36. 88
14	δ Ursæ Minoris, near West Elongation.	Direct	6	6					
		Reversed	6	6	36. 56	+1. 39	+0. 32	+0. 44	37. 00
18	......do	Direct	6	6					
		Reversed	6	6	33. 09	+1. 39	+0. 32	+0. 44	33. 53
20	......do	Direct	6	6					
		Reversed	6	6	33. 54	+1. 39	+0. 32	+0. 44	33. 98
21	......do	Direct	6	6					
		Reversed	6	6	35. 93	+1. 39	+0. 32	+0. 44	36. 37
18	51 Cephei, near East Elongation	Direct	6	6					
		Reversed	6	6	35. 46	−1. 39	+0. 21	−0. 29	35. 17
20	......do	Direct	6	6					
		Reversed	6	6	35. 95	−1. 39	+0. 21	−0. 29	35. 66
21	......do	Direct	6	6					
		Reversed	6	6	37. 05	−1. 39	+0. 21	−0. 29	36. 76
18	λ Ursæ Minoris, near West Elongation.	Direct	6	6					
		Reversed	6	6	33. 88	+1. 40	+0. 59	+0. 83	34. 71
20	......do	Direct	6	6					
		Reversed	6	6	33. 52	+1. 40	+0. 59	+0. 83	34. 35
21	......do	Direct	6	6					
		Reversed	6	6	33. 09	+1. 40	+0. 59	+0. 83	33. 92

Weighted mean .... 357° 09′ 35″. 65 ± 0″. 27.

Giving equal weights to the results for azimuth of mark in the above table, their mean is 357° 09′ 35″.65 ± 0″.27, the probable error being derived from the discrepancies between the individual and mean results. This is the mean value of the azimuth of the mark from the position of the instrument. The distance from the geodetic point of North Base station to the center of the instrument, measured with the primary base-apparatus, was 81.09 feet. The angle at the instrument between North Base and the azimuth mark was 163° 45′ 09″, the instrument being on the west side of the base-line. These data and the length of the base-line (Chapter VI, § 9), give 4′ 51″.57, which must be added to the above azimuth to reduce it to North Base. The mark at South Base was 0.25 inch perpendicularly west of the line from the instrument to South Base, giving a correction of —0″.27 to the above azimuth. Applying these corrections, the resulting azimuth of South Base from North Base is

357° 14′ 26″.95 ± 0.27 west of south.

CHAPTER XXV.

LONGITUDES.

LONGITUDE OF DETROIT.

§ **1.** In 1871 a telegraphic determination of the difference of longitude of the east transit pier in the Lake-Survey Observatory at Detroit, built in 1871, and the Naval Observatory at Washington, was made.

The observer at Detroit was Assistant O. B. Wheeler, with a 43-inch Troughton & Simms transit and Bond & Sons sidereal clock 184, while Professor J. R. Eastman observed at the Naval Observatory with the old transit instrument and the mural clock. Professor Eastman's results are given in the Lake-Survey Report for 1872, and those of Assistant O. B. Wheeler, with the resulting longitude, in the Lake-Survey Report for 1871; but as this longitude was deduced from a preceding preliminary reduction of the Washington observations by Professor Eastman, it needs a small correction for the change in Professor Eastman's results, which correction will be given later. The wire-intervals and the inequality of pivots of the Detroit instrument were redetermined. On May 19 and 24 there were repeaters in the circuit at Detroit, Cleveland, Pittsburgh, and Philadelphia, with 829 miles of wire. On the 23d there were repeaters at Detroit, Cleveland, Cincinnati, Pittsburgh, and Philadelphia, the length of this route being about 1250 miles.

Clock-signals, which were registered on chronographs at both Detroit and Washington, were exchanged on three nights, and clock-errors were determined on the same nights.

After the observations Assistant Wheeler visited Washington and determined his personal equation with reference to Professor Eastman, on three nights, each observer noting the transit of stars over a portion of the wires of Professor Eastman's instrument, the observers alternating with reference to the wires. The not very concordant results were:

1871.	s. s.
August 9.	E. − W. = -0.036 ± 0.016
August 10.	E. − W. = $+0.002 \pm 0.006$
August 11.	E. − W. = $+0.057 \pm 0.008$

Combined with weights derived from their probable errors, these results give E.—W. = $+0^s.017$. The + sign signifies that W. observed the transit of a star earlier than E.

The following table, in which Assistant Wheeler's reduction of the Detroit observations, given in the Lake-Survey Report for 1871, and Professor Eastman's reduction of the Washington observations, given in the Lake-Survey Report for 1872, are used, shows the result of each night's work:

Date.	Signals from—	Washington sidereal time of mean of signals sent or received.	Detroit sidereal time of mean of signals sent or received.	Wave and armature time.	Final difference of longitude.
1871.		h. m. s.	h. m. s.	s.	m. s.
May 19	Washington to Detroit	18 49 58.24	18 25 58.09	0.16	24 00.31
19	Detroit to Washington	19 00 58.53	18 36 58.06		
23	Washington to Detroit	18 58 41.55	18 34 41.55	0.14	24 00.14
23	Detroit to Washington	19 04 47.80	18 40 47.53		
24	Washington to Detroit	18 01 14.45	17 37 14.55	0.16	24 00.06
24	Detroit to Washington	18 15 14.75	17 51 14.53		

	h. m. s.
Mean	0 24 00.17
Reduction to dome of Naval Observatory	− 00.04
Personal equation	+ 00.02
East pier of Lake-Survey Observatory of 1871 west of dome of Naval Observatory, Washington	0 24 00.15±0.06

If the discrepancies in the results of the different nights alone be considered, a probable error of not less than $0^s.05$ must be assigned, and if the uncertainties in the relative personal equation be included the total probable error may reach $0^s.06$.

In Washington Observations for 1872, a value less by $0^s.01$ is given for the difference of longitude between Washington and Detroit. The difference arises from two causes:

1. The value for relative personal equation of the two observers is neglected in the Washington Observations, while it is included in this reduction, since, although it is not a precise value, it is the best available.

2. The Washington Observations, 1872, give clock-errors depending on star-places from the American Nautical Almanac, with certain corrections applied. Professor Eastman's reduction, published in the Lake-Survey Report for 1872, used the places given in the American Ephemeris without corrections, and the reduction of the Detroit work used the same places. As many stars were observed in common at Detroit and Washington, common places must be used. Hence, the reduction by Professor Eastman of the Washington work in the Lake-Survey Report of 1872 has been adopted instead of that in Washington Observations of 1872. The correction for personal equation omitted in the Washington Observations for 1872, being $+0^s.02$, and that to reduce its results to those from star-places given in the American Ephemeris being $-0^s.03$, their sum, $-0^s.01$, applied to the difference of longitude of Detroit and Washington, given in Washington Observations for 1872, namely, $24^m\ 00^s.16$, gives the value which is adopted above, namely:

East pier of Lake-Survey Observatory of 1871, west of dome of Naval Observatory, $24^m 00^s.15 \pm 0^s.06$.

There are three not very precise checks on this value. In the Lake-Survey Reports for 1877 and 1878 are given the determinations of the longitude of Cairo, Ill., and Memphis, Tenn., from the Lake-Survey Observatory, there being two nights of exchange of signals for each place. The results are:

Lake-Survey transit post, Cairo, Ill., west of Detroit .... $24^m\ 25^s.87$
Lake-Survey transit post, Memphis, Tenn., west of Detroit .... $28^m\ 01^s.03$

The Coast Survey has also determined the longitudes of points in Cairo and Memphis, the results furnished by the Superintendent of the Coast Survey, by letters of March 11 and 16, 1880, being:

Cairo west of Greenwich .... $5^h\ 56^m\ 40^s.99$
Memphis west of Greenwich .... $6^h\ 00^m\ 13^s.2$

Subtracting from each the difference of longitude of Washington and Greenwich, namely, $5^h\ 08^m\ 12^s.09$, and from the first $3^s.09$, and from the second $-0^s.07$, to derive the longitudes of the Lake-Survey transit posts from those of the Coast Survey at Cairo and Memphis, there result from the Coast-Survey determinations:

Lake-Survey transit post, Cairo, west of Washington .... $48^m\ 25^s.81$
Lake-Survey transit post, Memphis, west of Washington .... $52^m\ 01^s.18$

Applying to these quantities the differences of longitude between these transit posts and Detroit, given above, there result other values of the longitude of east transit post of Lake-Survey Observatory of 1871, from dome of Naval Observatory, Washington, namely:

Detroit west of Washington (via Cairo) .... $23^m\ 59^s.94$
Detroit west of Washington (via Memphis) .... $24^m\ 00^s.15$

As the data furnished by the Superintendent of the Coast Survey are uncorrected for personal equation, the results may be slightly changed hereafter.

In 1861 (see Lake-Survey Report, 1861), the difference of longitude of Ann Arbor and Detroit was determined by six nights' exchange of telegraphic signals as $2^m\ 43^s.30$, the observers at Detroit being Lieutenant O. M. Poe and Assistant James Carr. In 1864 it was again determined (MS. Lake-Survey Report, 1865), by three nights' exchange of telegraphic signals as $2^m\ 43^s.17$, the observers at Detroit being Colonel W. F. Raynolds and Assistant S. W. Robinson. Personal equation was applied in both cases. Taking the mean of the two results, we have $2^m\ 43^s.23$. Applying the correction ($-0^s.127$), to reduce the old transit post to the east transit post of Lake-Survey Observatory of 1871, there results:

	m. s.
East transit post, Lake Survey Observatory, 1871, west of Ann Arbor	− 2 43.10
Ann Arbor west of Harvard Observatory (see Brünnow, Astronomical Notices, 27)	+50 24.21
Harvard Observatory west of Washington (see Coast-Survey Report for 1874)	−23 41.11
East transit pier, Lake-Survey Observatory, west of Washington	23 60.00

Collecting these results we have—

	m. s. s.
East transit pier, Lake-Survey Observatory, 1871, west of Washington (direct)	24 00.15±0.06
East transit pier, Lake-Survey Observatory, 1871, west of Washington (via Cairo)	23 59.94
East transit pier, Lake-Survey Observatory, 1871, west of Washington (via Memphis)	24 00.15
East transit pier, Lake-Survey Observatory, 1871, west of Washington (via Ann Arbor and Cambridge)	24 00.00

There were two other determinations of the longitude of Detroit. One was by the way of Quebec from Cambridge, in 1860, there being exchange of signals on but one night between Windsor and Quebec, personal equation not being determined. The resulting value of the longitude was $0^s.59$ larger than that by the direct route given above. The other was in 1859 by way of Hudson Observatory, Ohio, Philadelphia, and Seaton Station, Washington, to Naval Observatory. Personal equation was neglected for at least one of the steps making up the route. The result was $0^s.88$ less than the value by the direct route given above.

The value obtained by the direct route to Washington, namely, $24^m\ 00^s.15 \pm 0^s.06$ is adopted. Adding to this the longitude of Washington west of Greenwich, $5^h\ 08^m\ 12^s.09 \pm 0^s.05$ (see Coast-Survey Report, 1874) there results:

East transit pier of Lake-Survey Observatory of 1871, west of Greenwich,

$$5^h\ 32^m\ 12^s.24 \pm 0^s.08$$

LONGITUDE OF NORTH BASE STATION, MINNESOTA POINT.

§ **2.** In 1871 the longitude at North Base Station, near Duluth, Minnesota, was determined by telegraphic exchange of signals with the Lake-Survey Observatory at Detroit. (See Lake-Survey Report for 1872.)

At Detroit the observer was Assistant O. B. Wheeler, with a 43-inch Troughton & Simms transit instrument, sidereal clock No. 186, and Bond chronograph No. 216. At Duluth it had been intended that General Comstock should be the sole observer, but so much time was consumed by bad weather and bad insulation of telegraphic lines, that he had to be replaced on June 26, before the completion of the work, by Assistant G. Y. Wisner. The instruments at Duluth were Würdemann 32-inch transit No. 15, sidereal clock No. 256, and chronograph No. 245. At both stations the transits were mounted on stone piers. Wire intervals, pivot inequalities, and level values were redetermined. Signals were sent by placing the Duluth and the Detroit clocks alternately in the circuit, each clock recording its beats on the chronographs at both stations. On each night the exchange of signals was preceded and followed by time determinations. The length of the telegraphic line was 912 miles, repeaters being used at Chicago and Saint Paul.

The time determinations were reduced by least squares, the deviation, clock-error and clock-rate being the unknowns. The following results were obtained:

Detroit and North Base—Difference of Longitude.

Date.	From North Base record.	From Detroit record.	Mean.	Wave and armature time.
1871.	m. s.	m. s.	m. s.	s.
June 15	36 05.98	36 05.70	36 05.84	0.140
26	36 05.96	36 05.70	36 05.83	0.130
July 11	36 05.92	36 05.66	36 05.79	0.130

Differences of personal equation between Assistant O. B. Wheeler and General Comstock, and also between Assistant G. Y. Wisner and General Comstock, were determined. The method followed was for one observer to note the transit of a star over the first tally of five wires, and the other to note the transit over the last tally of five wires, the observers alternating in tallies used. The following results were obtained. The minus sign prefixed to a value for personal equation signifies that the first-named observer observed the transit of a star earlier than the second:

Date.	Observers.	Personal equation.
1871.		s. s.
July 7	C. B. C.—O. B. W.	—0.007±0.015
9	......do	—0.098±0.032
Aug. 3	......do	—0.071±0.010
4	......do	—0.056±0.012
June 21	C. B. C.—G. Y. W.	—0.091±0.029
22	......do	—0.181±0.022
24	......do	+0.005±0.026

In the Lake-Survey Report for 1872 mean results have been obtained by giving to the result of each night's work a weight depending on the probable error of that night's work. But as the discrepancies between results from different nights far exceed the probable errors derived from that night's work, it must be concluded that the personal equation varied from night to night, and the mean of the different night's results, attributing equal weights to each, will be a value more nearly correct, and a better value of the probable accuracy of the result will be obtained by considering the discrepancies between the results of each night's work and the mean of all nights' work than from attributing to each night's work the accuracy which results from its own work alone. Taking, then, the arithmetical mean of the results of different nights and computing its probable error from the discrepancies between this mean and the separate nights' results, we have for adopted personal equations,

$$\text{C. B. C.}-\text{O. B. W.}=-0^s.058\pm0^s.013$$
$$\text{C. B. C.}-\text{G. Y. W.}=-0.089\pm0.034.$$

From these values there results—

$$\text{G. Y. W.}-\text{O. B. W.}=+0^s.031\pm0.036.$$

Applying for June 15 the personal equation between C. B. C. and O. B. W., and for June 26 and July 11 that between G. Y. W. and O. B. W., there results for difference of longitude of the two instruments,

	m.	s.
June 15, 1871	36	05.90
June 26, 1871	36	05.80
July 11, 1871	36	05.76

The mean of these values is $36^m\ 05^s.82$.

The probable error of this mean, if derived from the discrepancies between it and the separate results, would be less than $\pm0^s.03$, but as the probable error in the personal equation of Assistant O. B. Wheeler and Assistant Wisner reaches $\pm0^s.036$, the probable error in the result may reach $\pm0^s.05$, giving $36^m\ 05^s.82\pm0^s.05$.

At Duluth the instrument was $0^s.143$ east of primary station North Base, being east 150.1 feet and north 110.2 feet from that station; so that we have station North Base west of east transit pier of Lake-Survey Observatory, built in 1871,

$$\mathbf{36^m\ 05^s.96\pm0^s.05.}$$

The details of this work can be found in Assistant O. B. Wheeler's report of the reduction, published in the Lake-Survey Report for 1872.

LONGITUDE OF STATION FORT HOWARD.

§ **3**. A telegraphic determination of the difference of longitude between Fort Howard, Wisconsin, and Detroit was made in 1864, on the nights of August 8, 10, and 13. The observer at

Detroit on the first two nights was Colonel W. F. Raynolds, and on the last night Assistant S. W. Robinson. The observations at Fort Howard were made by Professor C. A. Young. At Detroit the Troughton & Simms transit of 43-inch focal length and 3-inch objective, sidereal clock No. 184 and chronograph No. 216 by Bond & Sons were used. At Fort Howard Würdemann transit No. 15 of 31-inch focal length and 2½-inch objective, with Bond & Sons sidereal clock No. 256, and chronograph No. 245 were used. The telegraphic circuit was continuous between the two stations, the distance being 526 miles.

The plan of observation followed was that of recording the transits of the same stars at the two stations on both chronographs. In the reduction, the differences of the times of transit of the same stars at the two stations, corrected for instrumental errors and rate of clock, were computed from the Detroit chronographic record. These differences, when corrected for personal equation and wave and armature time, will give the difference of longitude. The following table gives these differences for each night and each star, with their means for the separate nights. The probable errors are deduced from the discrepancies between the means and the individual results.

Results for difference of longitude between Detroit and Fort Howard.

[Uncorrected for personal equation and wave and armature time.]

Stars observed.	August 8.		August 10.		August 13.	
	Individual results.	Residuals.	Individual results.	Residuals.	Individual results.	Residuals.
	m. s.	s.	m. s.	s.	m. s.	s.
α Vulpeculæ	....	....	....	....	19 52.057	0.447
β Sagittæ	19 51.598	0.017	....	....	51.970	.360
β Aquilæ	51.526	.055	....	....	51.837	.227
α² Capricorni	51.487	.094	19 51.752	0.054	51.537	.073
ρ Capricorni	51.345	.236	51.647	.051	51.566	.044
α Delphini	51.287	.294	51.673	.025	51.397	.213
μ Aquarii	51.493	.088	51.883	.185	....	....
12 Aquarii	51.584	.003	51.681	.017	51.588	.022
ζ Cygni	51.248	.333	51.552	.146	51.586	.024
ζ Capricorni	....	....	51.470	.228	51.643	.033
ξ Aquarii	51.796	.215	51.823	.125	51.838	.228
14 Pegasi	51.331	.250	....	....	51.126	.516
α Aquarii	51.560	.021	51.681	.017	51.470	.140
θ Aquarii	51.330	.251	....	....	51.587	.023
ζ Aquarii	51.441	.140	51.773	.075	....	....
ζ Pegasi	51.671	.090	51.767	.069	51.605	.005
λ Aquarii	51.671	.090	51.749	.051	51.432	.178
α Pegasi	51.649	.068	51.550	.148	51.332	.278
γ Piscium	51.820	.239	51.593	.105	51.591	.019
κ¹ Piscium	51.891	.310	51.941	.243	51.955	.345
ι Piscium	51.831	.250	51.806	.108	51.549	.061
δ Sculptoris	51.115	.466	51.655	.043	51.378	.232
29 Piscium	51.753	.172	....	....	51.305	.305
γ Pegasi	....	....	51.687	.011	....	....
10 Ceti	....	....	51.765	.067	51.440	.170
δ Andromedæ	51.550	.031	51.536	.162	51.411	.199
φ² Ceti	51.739	.158	51.757	.059	51.315	.295
ε Piscium	51.827	.246	51.723	.025	51.578	.032
θ Ceti	51.583	.002	51.455	.243	51.302	.308
η Piscium	....	....	51.829	.131	51.770	.160
ν Piscium	....	....	....	....	51.918	.308
β Arietis	51.828	.247	....	....	....	....
α Arietis	51.722	.141	....	....	....	....
Means and probable errors	19 51.581	± 0.227	19 51.698	± 0.017	19 51.610	± 0.030

At the time of making the above determinations, there were two intermediate stations in the circuit, viz: Ann Arbor and Chicago, observations being made at the former by Professor James C. Watson, and at the latter by Assistant O. B. Wheeler. The result for difference of longitude between Ann Arbor and Detroit has been given in § 1. Among the different observers the following relative personal equations affecting the longitude now sought were determined:

$$\begin{aligned}
&\text{Raynolds} - \text{Young} \ldots = + \overset{s.}{0.258} \pm \overset{s.}{0.022},\\
&\text{Young} - \text{Wheeler} \ldots = - 0.017 \pm 0.024,\\
&\text{Wheeler} - \text{Robinson.} = - 0.005 \pm 0.015,\\
&\text{Raynolds} - \text{Robinson} = + 0.093 \pm 0.016,
\end{aligned}$$

the probable error of each being derived from the discrepancies between the individual and mean values. The $\left\{ \begin{matrix} + \\ - \end{matrix} \right\}$ sign before the numerical value of any equation indicates that the first of the two observers observed a star's transit $\left\{ \begin{matrix} \text{later} \\ \text{earlier} \end{matrix} \right\}$ than the second. These equations were all determined at Detroit. The first two were determined partly by comparison of clock-errors obtained when the observers used separate instruments and the same stars, and partly by comparison of the mean times of transit over five or more wires reduced to the middle wire, obtained when the observers alternated in using the same instrument and in observing the same stars. The last two equations were determined entirely by the second method just mentioned. For the first two the Troughton & Simms and Würdemann No. 15 transits were used, for the third Würdemann No. 15, and for the fourth the Troughton & Simms transit. Professor Young observed only with Würdemann transit No. 15.

Table of results for personal equation.

Equation.	Date.	No. of stars observed.	Mean for date.	Total No. of stars observed.	Mean of all results.	Probable error.
	1864.		s.		s.	s.
Raynolds — Young..........	July 18	....	....			
	July 19	....	....	11	+0.258	±0.022
Young — Wheeler	July 18					
	July 19	....	....	24	−0.017	±0.024
Wheeler — Robinson	Mar. 3	12	+0.017			
	Mar. 24	24	+0.036			
	Apr. 18	30	−0.047	66	−0.005	±0.015
Raynolds — Robinson	July 27	12	+0.101			
	July 28	16	+0.140			
	Aug. 2	7	−0.004			
	Aug. 7	3	+0.043	38	+0.093	±0.016

The application of these equations requires the assumption that they were the same when observed as when the longitude determinations were made. As will be seen from the above and the preceding table, the first three equations were determined on dates somewhat remote from the dates when the difference of longitude was observed, and on this account they should have less weight as affecting that difference than the fourth equation. However, in view of the objections to assigning such weights by any arbitrary process and of the fact that the four equations are subject to a rigorous condition, it seems best to give each equation the weight indicated by its probable error derived as stated above. The observed differences of longitude shown in the first table, will also be given weights indicated by their probable errors.

In order, now, from the preceding data, to derive the most probable value of the difference of longitude, let it be denoted by $y+(y)$, and let the most probable values of the above personal equations be denoted in order by $x_1+(x_1)$, $x_2+(x_2)$, $x_3+(x_3)$, and $x_4+(x_4)$. Then substituting for y,

$19^m\ 51^s.0$ and for x_1, x_2, x_3, and x_4, their observed values, there result the following equations:

$$(y) \qquad -(x_1)=+0^s.839+\Delta_1, \text{ with weight } p_1=\left(\frac{1}{0.027}\right)^2$$

$$(y) \qquad -(x_1)=+0^s.956+\Delta_2, \text{ with weight } p_2=\left(\frac{1}{0.017}\right)^2$$

$$(y)+(x_2)+(x_3)=+0^s.632+\Delta_3, \text{ with weight } p_3=\left(\frac{1}{0.030}\right)^2$$

$$(x_1)=\ 0 \qquad +\Delta_4, \text{ with weight } p_4=\left(\frac{1}{0.022}\right)^2$$

$$(x_2)=\ 0 \qquad +\Delta_5, \text{ with weight } p_5=\left(\frac{1}{0.024}\right)^2$$

$$(x_3)=\ 0 \qquad +\Delta_6, \text{ with weight } p_6=\left(\frac{1}{0.015}\right)^2$$

$$(x_4)=\ 0 \qquad +\Delta_7, \text{ with weight } p_7=\left(\frac{1}{0.016}\right)^2$$

In addition we must have—

$$x_1+(x_1)+x_2+(x_2)+x_3+(x_3)=x_4+(x_4)$$

or

$$(x_1)+(x_2)+(x_3)-(x_4)=-0^s.143$$

Substituting the value of (x_4) from this last equation in the seventh of the above equations and solving so as to make $[p\Delta^2]$ a minimum, there results

$$(y)=+0^s.840\pm0^s.059,$$

and hence

$$y+(y)=+19^m\ 51^s.840\pm0^s.059.$$

The wave and armature time was computed from the differences between the two clock times of transit of those stars whose transits at the two stations were recorded on both chronographs. The difference between such differences for any star gives double the wave and armature time. The results obtained are given in the table below, the separate values being weighted inversely as the squares of their probable errors, which latter are derived from the discrepancies between the individual values and their mean for any date.

Table of results for wave and armature time.

Date.	Wave and armature time.	Probable error.	No. of stars observed.
1864.	*s.*	*s.*	
August 8	0.0462	±0.0017	29
10	0.0434	±0.0017	19
13	0.0461	±0.0020	15
Weighted mean	0.045	±0.001	

Subtracting the mean value of the wave and armature time from the above value of $y+(y)$, there results for the difference of longitude between the transit-post at Detroit and the transit-post at Fort Howard, $19^m\ 51^s.80\pm0^s.06$.

The Detroit transit-post of 1864 was $0^s.127$ east of the east pier of the present Lake-Survey Observatory, and the post at Fort Howard was 6910.8 feet or $6^s.360$ east, and north 4884.2 feet from Fort Howard triangulation station. Applying these corrections, we have Fort Howard station west of east pier of Lake-Survey Observatory at Detroit,

$$\mathbf{0^h\ 19^m\ 58^s.03\pm0^s.06.}$$

DIFFERENCE OF LONGITUDE BETWEEN FORT HOWARD AND ESCANABA, AND LONGITUDE OF STATION FORD RIVER.

§ 4. The differences of longitude between Fort Howard and Menomonee and Menomonee and Escanaba were determined telegraphically in 1865, and the results were published in the Report of the Chief of Engineers for 1866. Neither of these differences was corrected for personal equation, and no precise data exist for making such correction. Their sum, however, or the difference of longitude between Fort Howard and Escanaba, can be derived so as to give a good elimination of personal equation as well as of wave and armature time.

In these determinations the two extreme stations, Fort Howard and Escanaba, were each provided with a chronograph and sidereal clock made by Bond & Sons. Würdemann transit No. 1 was used at Fort Howard, and No. 15, by the same maker, at Escanaba. Each of these transits has a focal length of 31 inches and an objective 2.5 inches in diameter. At the intermediate station, Menomonee, Pistor & Martins transit No. 1 of 24 inches focal length and 2.5-inch objective was used. The transit-key of each observer was connected with the main circuit, so that the star transits at each station were recorded on both chronographs. The same list of stars was observed at the three stations. Observations were made on the nights of August 11, 12, 14, 16, 17, 18, and 22. On the first three nights the observers at Escanaba and Fort Howard were Assistant O. B. Wheeler and Professor C. A. Young, respectively. Wheeler and Young then exchanged stations and observed at Fort Howard and Escanaba, respectively, on the last four nights. The observer at Menomonee on each date was Assistant S. W. Robinson.

The results of the determinations are given in the following tables condensed from the account of the work given by Assistant O. B. Wheeler in the Report of 1866 previously referred to. The difference for each night is the mean of the results computed from the two chronographic records, and is given a weight equal to the number of stars observed:

Fort Howard — Menomonee.

Date.	Computed difference.	No. of stars observed.
1865.	m. s.	
August 11	1 34.421	39
12	34.348	47
14	34.437	39
Weighted mean	1 34.399	125
August 16	1 34.577	33
17	34.389	27
18	34.465	50
22	34.516	25
Weighted mean	1 34.487	135

Menomonee — Escanaba.

Date.	Computed difference.	No. of stars observed.
1865.	m. s.	
August 11	2 17.361	44
12	17.372	49
14	17.354	45
Weighted mean	2 17.363	138
August 16	2 17.395	33
17	17.406	22
18	17.395	49
22	17.294	27
Weighted mean	2 17.376	131

It will be seen from these tables that the number of stars observed on any date was about the same for each of the two differences Fort Howard—Menomonee and Menomonee—Escanaba. As those observed were also mostly the same stars at the three stations, the half sum of the four means in the table will give a result for the difference Fort Howard—Escanaba nearly free from the errors of observation at Menomonee, and free from the relative personal equations between the observers. The result thus derived makes the transit-post at Fort Howard west of the transit-post at Escanaba $3^m\ 51^s.81$.

The data from which this result is derived do not afford a measure of its probable error. Judging from similar work, however, such probable error may be safely estimated as not exceeding $\pm 0^s.1$.

The transit-post at Fort Howard was $6^s.36$ east of the triangulation station. Applying this correction and the longitude of Fort Howard triangulation station, given in § 3, to the above difference, there results for the longitude of the transit-post at Escanaba, $15^m\ 59^s.86 \pm 0^s.12$ west from Detroit. The transit-post at Escanaba was $11^s.99$ east of Ford River triangulation station. Hence the longitude of Ford River is west of Detroit,

$$\mathbf{16^m\ 11^s.85 \pm 0^s.12.}$$

DIFFERENCE OF LONGITUDE BETWEEN MARQUETTE AND ESCANABA.—LONGITUDE OF STATION TRILOBA.

§ **5**. The difference in longitude between Marquette and Escanaba was determined by the telegraphic method in July, 1865. Each station was provided with a chronograph and sidereal clock. Würdemann transit No. 1 was used at Marquette and No. 15 at Escanaba, these instruments being those described in § 4. The same list of stars was observed at both stations, the star transits being recorded on both chronographs. Observations were made on the nights of July 20, 21, 22, 27, 29, and 31. The observers were Assistants O. B. Wheeler and S. W. Robinson, Mr. Wheeler observing during the first three nights at Marquette and during the last three at Escanaba, Mr. Robinson observing on the same dates at Escanaba and Marquette, respectively. The results of these determinations were published in the Report of the Chief of Engineers for 1866, from which the following table of results is abridged. The result for each night is the mean computed from both chronographic records, the wave and armature time being thus eliminated:

Difference of longitude, Marquette—Escanaba.

Date.	Computed difference.	Number stars observed.
1865.	*m. s.*	
July 20	1 35.245	46
21	35.216	42
22	35.191	33
27	35.216	42
29	35.274	55
31	35.210	51

Putting now for the most probable value of the difference in longitude, Marquette — Escanaba, $1^m\ 35^s + y$, and for the relative personal equation, &c., eliminated by the interchange of observers, x, the following equations result:

$$\begin{aligned}
&\qquad\ \ ^{s.}\\
y-x-0.245&=\Delta_1, \text{ with weight } 46\\
y-x-0.216&=\Delta_2, \text{ with weight } 42\\
y-x-0.191&=\Delta_3, \text{ with weight } 33\\
y+x-0.216&=\Delta_4, \text{ with weight } 42\\
y+x-0.274&=\Delta_5, \text{ with weight } 55\\
y+x-0.210&=\Delta_6, \text{ with weight } 51
\end{aligned}$$

From these the normal equations are:

$$\begin{aligned}
269y+\ 27x&=+61.497\\
27y+269x&=+\ 8.207
\end{aligned}$$

whence

$$\begin{aligned}
y&=+0^s.228\pm 0^s.009\\
x&=+0^s.008\pm 0^s.009
\end{aligned}$$

Hence the difference in longitude between the transit-posts at the two stations is $1^m\ 35^s.23 \pm 0^s.01$. Adding to this the longitude of the Escanaba transit-post, § 4, the longitude of the Marquette transit-post is west of Detroit, $0^h\ 17^m\ 35^s.09 \pm 0^s.12$.

The Marquette transit-post was situated on Thone's Hill, 28422.8 feet distant, bearing south 19° 46′ 28″.65 east of triangulation station Triloba. This requires a correction of $+9^s.17$ to reduce the above longitude to Triloba. The longitude of Triloba is, therefore, west of Detroit,

$$\mathbf{0^h\ 17^m\ 44^s.26 \pm 0^s.12.}$$

LONGITUDE OF WILLOW SPRINGS.

§ **6.** The longitude of primary station Willow Springs was determined in 1876 by telegraphic exchange of signals with the Lake-Survey Observatory at Detroit.

The observer at Detroit was Captain H. M. Adams. The instruments used were Buff & Berger astronomical transit No. 2, with focal length of 39 inches, aperture of 3 inches, and eye-piece giving a magnifying power of 87 diameters; Bond & Sons sidereal clock No. 256; and Bond & Sons chronograph No. 216.

The observer at Willow Springs (Mount Forest) was Lieutenant D. W. Lockwood. The instruments used were Würdemann astronomical transit No. 1, with focal length of 31 inches, object-glass $2\frac{1}{2}$ inches in diameter, and eyepiece giving a magnifying power of 100 diameters; Negus sidereal break-circuit chronometer No. 1524; and Bond & Sons chronograph No. 245.

At both stations the transits were mounted on stone piers, that at Willow Springs being 5716.1 feet distant, bearing north 38° 09′ 01″ west from the trigonometrical station, near the railway station Mount Forest, Illinois, on the Chicago and Alton Railway; and that at Detroit being the east pier of Lake-Survey Observatory. The land within a radius of fifteen miles from Mount Forest does not vary from the general level more than about 100 feet at any point. About 16 miles to the northeast is Lake Michigan, with a low shore and a bottom gradually declining at a rate of about 75 feet in 10 miles.

Wire-intervals, inequalities of pivots, and level values for the Buff & Berger transit were determined in the summer of 1876. Wire-intervals of the Würdemann transit were determined in the winter of 1875–6, and inequalities of pivots and level values in the summer of 1875.

Observations for difference of time between Detroit and Mount Forest, Illinois, were made on four nights in August, 1876, automatic clock and chronometer signals being sent with complete time-determinations preceding and following according to a programme similar to that followed in determining the difference of longitude between Detroit and Tonawanda. (See § 9.) The length of the telegraphic line was about 300 miles.

The time-determinations were reduced by the method of least squares, the weights of the observation-equations being derived by the process explained in § 9, the value of ε_1 being taken as $\pm 0^s.049$ and ε from observations of equatorial stars as $\pm 0^s.066$, and weight unity being given to the mean of eleven wires for equatorial stars. The coördinates of the stars used were taken from the American Ephemeris, from the catalogue of "539 Sterne," Berlin, 1876, and from "General Bericht Europäische Gradmessung," 1874, preference being given to the catalogues in the order named. The following results for difference of time between Mount Forest and Detroit were obtained:

Detroit and Mount Forest.

Date.	Difference of observed local sidereal times.			
	By signals sent from Mount Forest to Detroit.	By signals sent from Detroit to Mount Forest.	Means.	Wave and armature time.
1876.	*h. m. s.*	*h. m. s.*	*h. m. s.*	*s.*
Aug. 17	0 19 14.944	0 19 15.128	0 19 15.036	0.092
23	15.023	15.216	15.120	0.096
25	15.036	15.234	15.135	0.099
26	14.963	15.141	15.052	0.089

Assigning equal weights to the results for the separate nights, their mean is $0^h\ 19^m\ 15^s.086 \pm 0^s.017$, the probable error being derived from the discrepancies between this mean and the four individual results.

The relative personal equation of Captain Adams and Lieutenant Lockwood was determined by observations on two nights before the departure of the field-party from Detroit and on one night after their return. The observations were made in the same manner as those in the longitude work, each observer using the same transit that he used in the longitude work, the transits being set on stone piers in meridians 5 feet apart, Lieutenant Lockwood's being the westerly one, and the observers working independently. The observations were reduced in the same manner as the field observations, and the following results were obtained:

Relative personal equation, Captain Adams and Lieutenant Lockwood.

Date.	Difference of observed local time.		
	By signals sent from Lockwood to Adams.	By signals sent from Adams to Lockwood.	Means.
1876.	s.	s.	s.
July 15	+0.243	+0.186	+0.215
18	+0.195	+0.160	+0.178
Oct. 23	+0.315	+0.245	+0.280

A result for difference of time in this table is Lieutenant Lockwood's observed local time of a given signal minus Captain Adams' observed local time of the same signal. Assigning equal weights to the results for the separate dates, their mean is $+0^s.224 \pm 0^s.020$, the probable error being derived from the discrepancies between the mean and the results for the separate dates. Applying the known difference of longitude of the two instruments, $+0^s.005$, to this quantity, there results for the relative personal equation between Captain Adams and Lieutenant Lockwood $+0^s.229 \pm 0^s.020$, which is to be added as a correction to the observed difference of longitude between Detroit and Mount Forest.

From a system of secondary triangulation measured by Lieutenant Lockwood, the position of the transit at Mount Forest was found to be $3^s.105$ west of the triangulation station Willow Springs.

The difference of longitude between Detroit and Willow Springs is then obtained by adding the following terms:

		h. m. s.	s.
Observed difference of time between Mount Forest and Detroit....		0 19 15.086±0.017	
Relative personal equation of observers	+	0.229±0.020	
Willow Springs triangulation station east of Mount Forest	—	3.105	

There results, Williow Springs west of Detroit,

$$\mathbf{0^h\ 19^m\ 12^s.210 \pm 0^s.026}$$

Further details may be found in the reports of Captain Adams and Lieutenant Lockwood, published in the Report of the Chief of Engineers for 1877.

LONGITUDE OF PARKERSBURG.

§ **7.** The longitude of primary station Parkersburg was determined in 1879 by telegraphic exchange of signals with the Lake-Survey Observatory at Detroit.

The observing-pier for Parkersburg was set about 10 miles north of the trigonometrical station in the city of Olney, on the Ohio and Mississippi and the Grayville and Mattoon Railroads, this being the nearest available telegraph station. The transit was 124.2 feet distant, bearing south 56° 00′ east from the southeast corner of the High School building, Olney, and 55861.2 feet distant, bearing north 16° 38′ 35″.4 west from station Parkersburg. The land in this vicinity is slightly undulating, but there is no variation from the general level to exceed one or two hundred feet within a radius of perhaps 50 miles.

The observer at Detroit was Captain D. W. Lockwood. The instruments used were Buff & Berger astronomical transit No. 2 (mounted on the east pier of the Lake-Survey Observatory) with focal length of 39 inches, aperture of 3 inches, and eye-piece giving a magnifying power of 87 diameters; Bond and Sons sidereal clock No. 256; and Bond and Sons chronograph No. 216.

The observer at Olney was Lieutenant P. M. Price. The instruments used were Würdemann astronomical transit No. 1, with focal length of 31 inches, aperture of 2½ inches, and eye-piece giving a magnifying power of about 100 diameters; Negus break-circuit sidereal chronometer No. 1524; and Bond and Sons chronograph No. 245. At both stations the instruments were mounted on stone piers.

Wire-intervals, inequalities of pivots, and level values are those used in the reduction of the observations for longitude of Willow Springs, excepting the wire-intervals of the Buff & Berger transit. The spider-thread diaphragm of that instrument was replaced by one of glass, in March, 1877, and the values of the intervals graduated thereon were determined at that time.

Observations for difference of time between Detroit and Olney were made on four nights in July, 1879, automatic clock and chronometer signals being sent, with complete time-determinations preceding and following, according to a programme in all respects similar to that followed in determining the longitude of Tonawanda. (See § 9.) Stars were selected from the American Ephemeris, from "Mittlere und Scheinbare Oerter von 539 Sternen" for 1879, and from the "General Bericht Europäische Gradmessung" for 1874, preference being given to the catalogues in the order named.

The time-determinations were reduced by the method of least squares, weights being assigned to the observation-equations as in the reduction of observations for longitude of Willow Springs. (See § 6.) The following results were obtained:

Detroit and Olney.

Date.	Difference of observed local sidereal times.			
	From signals sent from Olney to Detroit.	From signals sent from Detroit to Olney.	Means.	Wave and armature time.
1879.	h. m. s.	h. m. s.	h. m. s.	s.
July 26	0 20 08.582	0 20 08.404	0 20 08.493	0.089
28	08.602	08.409	08.505	0.097
29	08.645	08.410	08.527	0.117
30	08.498	08.308	08.403	0.095

Assigning equal weights to the results for the separate nights their mean is $0^h\,20^m\,08^s.482 \pm 0^s.018$, the probable error being derived from the discrepancies between the mean and individual results.

The relative personal equation of Captain Lockwood and Lieutenant Price was determined by observations on two nights before the departure of the field-party from Detroit, and on two nights after their return. The observations were made in the same manner as those in the field, Lieutenant Price's transit being set on a stone post outside of the Lake-Survey Observatory, a few feet to the south and west, and the observers working independently. The observations were reduced in the same manner as the field observations, and the following results were obtained:

Relative personal equation, Captain Lockwood and Lieutenant Price.

Date.	Difference of observed local times.		
	By signals sent from Lockwood to Price.	By signals sent from Price to Lockwood.	Means.
1879.	s.	s.	s.
July 5	−0.009	−0.037	−0.023
8	−0.104	−0.176	−0.140
Aug. 26	+0.143	+0.078	+0.110
27	+0.054	+0.032	+0.043

A result in this table for difference of time is Lieutenant Price's observed local time of a given signal minus Captain Lockwood's observed local time of the same signal. Assigning equal weights to the results for the separate dates their mean is— $0^s.002 \pm 0^s.036$, the probable error being derived from the discrepancies between this mean and the results for the separate nights. Applying the known difference of longitude between the two instruments, $+0^s.005$, there results for the personal equation between Captain Lockwood and Lieutenant Price, $+0^s.003 \pm 0^s.036$, which is to be added as a correction to the observed difference of time between Detroit and Olney.

The position of the transit at Olney with reference to the spire of the "Immanuels Kirche der Ev. Gemeinschaft," was determined by Lieutenant Price (1442.8 feet distant, bearing south 81° 49′ 04″ west from the spire), and the spire was located with reference to the main triangulation by observations from stations Parkersburg, Claremont, Check Base, and Denver. The reduction of these observations places the observing-post at Olney $13^s.461$ west of triangulation station Parkersburg.

The difference of longitude between Detroit and Parkersburg is then obtained by adding the following terms:

		h.	m.	s.	s.
Observed difference of time between Detroit and Olney		0	20	08.482	±0.018
Relative personal equation of observers	+			0.003	±0.036
Parkersburg east of Olney observing-post	−			13.461	

There results, Parkersburg west of Detroit,

$$\mathbf{0^h\ 19^m\ 55^s.024 \pm 0^s.040}$$

Further details of this work may be found in the reports of Captain Lockwood and Lieutenant Price, published in the Report of the Chief of Engineers for 1880.

LONGITUDE OF TOLEDO.

§ **8.** The longitude of Toledo was determined in June and July, 1881, by telegraphic exchange of signals with the Lake Survey Observatory at Detroit. The stone observing-post used at Toledo was set near the north corner of Monroe and Michigan streets, and was 233.0 feet north and 239.7 feet west from a stone which marks the intersection of Monroe and Ontario streets. The land in the vicinity of Toledo is low, flat, and marshy, a large part of the ground on which the city is built being filled in. The observing-post, however, stands in the natural bed-clay. The instrument at Detroit was set on the west stone pier in the Lake-Survey Observatory.

The observations were made by Assistant Engineers O. B. Wheeler and Thomas Russell. Bond & Sons sidereal clock No. 256 and spring-governor chronograph No. 216 were used at Detroit, and Negus sidereal break-circuit chronometer No. 1524 and Bond & Sons electro-motor chronograph No. 316 were used at Toledo. The transit used by Assistant Engineer O. B. Wheeler was Würdemann No. 15, of 31-inch focal length and 2½-inch objective; that used by Assistant Engineer T. Russell was Troughton & Simms transit No. 2, of 29-inch focal length and 2½-inch objective. Wire-intervals were determined for a new set of wires inserted in Würdemann transit No. 15 by Buff & Berger, in April, 1881, and were redetermined for Troughton & Simms transit No. 2. Pivot corrections and level values were redetermined for Würdemann transit No. 15, and for Troughton & Simms transit No. 2 the values found in 1875 were used, after testing their accuracy by a few additional observations.

Observations for difference of time were made on four nights in June, 1881, by Mr. Wheeler at Detroit, and Mr. Russell at Toledo. The observers then changed places, taking with them their transits and break-circuit keys, and observations were made on four nights in July, 1881, personal equation and kindred instrumental errors being thus eliminated from the mean result for the eight nights. Automatic clock and chronometer signals were exchanged between the observers on each night of observation, preceded and followed by complete time-determinations, according to a programme quite similar to that followed in determining the longitude of Tonawanda (§ 9). Stars were selected and star-places were taken from "Mittlere und Scheinbare Oerter von 539 Sternen" for 1881, with the exception of two stars taken from the American Ephemeris by Mr. Russell. The length of the telegraph line was about 60 miles.

The time-determinations were reduced by the method of least squares, the weights of the observation-equations being derived by the process explained in § 9, the value of ε_1 being taken as $\pm 0^s.056$ and ε as $\pm 0^s.08$, and weight unity being given to the mean of eleven wires for equatorial stars.

The following results for difference of time between the observing-piers at Toledo and Detroit were obtained:

Detroit and Toledo.

O. B. WHEELER, AT DETROIT. T. RUSSELL, AT TOLEDO.

Date.	Difference of observed local sidereal times.		
	By signals sent from Detroit.	By signals sent from Toledo.	Means.
1881.	*h. m. s*	*h. m. s.*	*h. m. s.*
June 23	0 1 57.968	0 1 57.966	0 1 57.967
24	57.792	57.848	57.820
28	57.760	57.761	57.760
29	57.789	57.810	57.800

T. RUSSELL, AT DETROIT. O. B. WHEELER, AT TOLEDO.

July 1	0 1 57.748	0 1 57.717	0 1 57.732
2	57.724	57.722	57.723
4	57.773	57.763	57.768
5	57.824	57.772	57.798

Putting now $1^m\ 57^s \pm y$ for the value of the difference in longitude (Toledo minus Detroit), and x for the quantity eliminated by the exchange of position of observers, the following observation-equations result from the above column of means:

$$
\begin{aligned}
&\overset{s.}{}\\
y-x-0.967&=\Delta_1\\
y-x-0.820&=\Delta_2\\
y-x-0.760&=\Delta_3\\
y-x-0.800&=\Delta_4\\
y+x-0.732&=\Delta_5\\
y+x-0.723&=\Delta_6\\
y+x-0.768&=\Delta_7\\
y+x-0.798&=\Delta_8
\end{aligned}
$$

Assigning equal weights to these equations, the normal equations are

$$
\begin{aligned}
8y&=+6.368\\
8x&=-0.326
\end{aligned}
$$

whence

$$
\begin{aligned}
y&=+0.796^s\pm0.016^s\\
x&=-0.041\pm0.016
\end{aligned}
$$

Hence, Toledo observing-post is west of the west post in Detroit Observatory $1^m\ 57^s.796\pm0^s.016$. The reduction to the east post in the Detroit Observatory is $+0^s.004$.

There results, Toledo west of Detroit,

$0^h\ 01^m\ 57^s.800\pm0^s.016$.

The longitude of Toledo was also determined in 1868, Assistant Engineer O. B. Wheeler making the observations at Toledo on a wooden post $16^m.6$ east and $13^m.6$ south of the stone post occupied in 1881. Lieutenant E. H. Ruffner made the observations at Detroit. Chronographs were used and automatic signals were exchanged. No correction for personal equation was applied to the result of 1868, nor was there any attempt at its elimination. The results of the two determinations agree quite well, however. The result obtained in 1868, reduced to the post occupied in 1881, is, Toledo west of Detroit, $0^h\ 01^m\ 57^s.693$. The result obtained in 1881 is adopted.

Further details may be found in the reports of Assistant Engineer A. R. Flint on longitude work in the Report of the Chief of Engineers for 1881.

The longitude of Toledo was referred to station Cedar Point of the primary triangulation in the following manner: By a secondary triangulation the longitude-post of 1881 was found to be 879.0 metres south and 650.5 metres west of the spire of Saint Mary's Church, situated on the corner of Michigan and Cherry streets, Toledo. From angles observed at stations Bedford and Cedar Point, the spire of Saint Mary's Church was found to be 17535.95 metres distant in azimuth 71° 05′ 56″.27 from Cedar Point station. These coördinates give longitude-post of 1881 28″.11 west of church spire, and church spire 11′ 57″.05 west of station Cedar Point. Therefore, Cedar Point is west of Detroit,

$0^h\ 01^m\ 08^s.123$.

LONGITUDES OF TONAWANDA AND MANNSVILLE.

§ **9.** The longitudes of primary stations Tonawanda and Mannsville, New York, were determined, in 1875, by telegraphic exchange of signals with the Lake-Survey Observatory at Detroit.

The observer at Detroit was Lieutenant D. W. Lockwood. The instruments used were Würdemann astronomical transit No. 1, with focal length of 31 inches, object-glass 2½ inches in diameter, and eye-piece giving a magnifying power of 100 diameters; Bond & Sons sidereal clock No. 256, with break-circuit attachment; and Bond & Sons chronograph No. 216. The transit was mounted on the east pier of the Lake-Survey Observatory.

The field observer was Assistant Engineer A. R. Flint. The instruments used by him were Troughton & Simms astronomical transit No. 2, with focal length of 29 inches, object-glass 2½ inches in diameter, and eye-piece giving a magnifying power of about 80 diameters; Negus sidereal break-circuit chronometer No. 1524; and Bond & Sons chronograph No. 245.

Wire-intervals, pivot inequalities, and level values for both instruments were redetermined. Observations were commenced on August 30, but owing to unfavorable weather were not completed until December 27, 1875. The programme included two nights' observations to determine the relative personal equation between Lieutenant Lockwood and Assistant Engineer Flint; observations to determine the difference of time between Detroit and Tonawanda; observations to determine the difference of time between Detroit and Mannsville; and two nights' observations to determine relative personal equation on the return of Assistant Engineer Flint from the field. The following programme was followed as closely as the weather would permit on each night of observations for difference of time, the same stars being observed at both stations when possible:

Circumpolar star, reversed on, with level readings.
Equatorial stars, with level readings.
Reversal of telescope.
Equatorial stars, with level readings.
Circumpolar star, reversed on, with level readings.
Clock and chronometer comparisons in duplicate.
Circumpolar star, reversed on, with level readings.
Equatorial stars, with level readings.
Reversal of telescope.
Equatorial stars, with level readings.
Circumpolar star, reversed on, with level readings.

Clock and chronometer signals were sent by placing the Detroit clock and the field chronometer alternately in circuit, each for $2^m\ 20^s$, two sets of signals being sent from each station.

The time-determinations were reduced by the method of least squares. For observations on stars south of the zenith, weight unity was assigned to each equation resulting from a complete transit over seven wires. The weight resulting from a transit over any other number of wires was computed from the formula

$$p=\frac{\varepsilon_1^2+\frac{\varepsilon^2}{7}}{\varepsilon_1^2+\frac{\varepsilon^2}{n}}$$

where ε_1 is the probable error of culmination; ε, the probable error of an observed transit over a single wire; and n, the number of wires over which the star's transit was observed. ε_1 was taken as $\pm 0^s.056$ from Appendix 12, United States Coast-Survey Report for 1872, and ε from observations of equatorial stars was found to be $\pm 0^s.07$. For equations resulting from transits of polar stars, weights were computed by the formula

$$p' = p \frac{\varepsilon^2}{\varepsilon'^2},$$

in which ε' is the probable error of an observed transit of a polar star over a single wire, and ε and p are the same as defined above, p being computed for n, the number of wires over which the polar star was observed. The coördinates of the stars were taken from the American Ephemeris for 1875, and from the Catalogue of "529 Sterne," Berlin, 1875.

The relative personal equation of Lieutenant Lockwood and Assistant Engineer Flint was determined by observations on two nights before the departure of the field party and on two nights after their return. The method followed was that of observing the known difference in position of adjacent meridians, each observer using the same instruments employed by him in the longitude work. The observations were made on the same programme and reduced by the same method as the observations for difference of longitude. The field transit was set on a stone pier 5 feet west of the transit of Detroit Observatory. The following results were obtained:

Relative personal equation, Lieutenant Lockwood and Assistant Engineer Flint.

Date.	Difference of observed local times.		
	By signals sent from Flint to Lockwood.	By signals sent from Lockwood to Flint.	Means.
1875.	s.	s.	s.
Aug. 30	+0.139	+0.081	+0.110
31	+0.208	+0.106	+0.157
Dec. 22	+0.148	+0.084	+0.116
27	+0.213	+0.139	+0.176

A result for difference of time in this table is Lieutenant Lockwood's observed local time of a given signal minus Mr. Flint's observed local time of the same signal. Assigning equal weights to the results for the separate dates, the mean difference of observed local sidereal times is $+0^s.140 \pm 0^s.011$, the probable error being derived from the discrepancies between the mean and the individual results. The known difference of longitude of the two instruments was $-0^s.004$. Applying this quantity there results, as the relative personal equation between Lieutenant Lockwood and Assistant Engineer Flint, $+0^s.136 \pm 0^s.011$. This quantity is to be added as a correction to the observed differences of longitude between Detroit and the two field-stations.

For the difference of longitude between Detroit and Tonawanda, five nights' observations were obtained. The transit at Tonawanda was mounted on a stone pier 31.9 feet distant, bearing north 86° 30′ east from trigonometric station Tonawanda. The length of the telegraphic line was about 250 miles, being through Canada and by way of Buffalo to Tonawanda, with a repeater at Buffalo.

The land eastward and westward from the station has no change of elevation to exceed a hundred feet or so for many miles in either direction. The changes in elevation to the northward and southward are indicated in the description of the latitude determination at this station. (See Chap. XXIII, § 18.)

The following results for the difference of time between Tonawanda and Detroit were obtained:

Detroit and Tonawanda.

Date.	Difference of observed local sidereal times.			
	By signals sent from Tonawanda to Detroit.	By signals sent from Detroit to Tonawanda.	Means.	Wave and armature time.
1875.	h. m. s.	h. m. s.	h. m. s.	s.
Sept. 23	0 16 38.703	0 16 38.790	0 16 38.747	0.043
24	38.663	38.753	38.708	0.045
Oct. 2	38.662	38.724	38.693	0.031
8	38.687	38.800	38.743	0.057
13	38.687	38.738	38.713	0.025

Giving equal weights to the results on separate nights, their mean is $0^h\,16^m\,38^s.721 \pm 0^s.007$, the probable error being derived from the discrepancies between the mean and the results for the separate nights.

The difference of longitude between Detroit and Tonawanda is then obtained as follows:

		h. m. s.	s.
Observed difference of longitude		0 16 38.721	± 0.007
Relative personal equation of observers	+	0.136	± 0.011
Station Tonawanda west of observing-post	−	0.028	

There results, station Tonawanda east of Detroit,

$$\mathbf{0^h\ 16^m\ 38^s.829 \pm 0^s.013.}$$

Four nights' observations for the difference of longitude between Detroit and Mannsville were obtained. The transit at Mannsville was mounted on a stone pier distant 2070.2 feet from the trigonometrical station on a true azimuth 68° 28′ 24″ west of south. West of the station the ground declines with considerable irregularity until Lake Ontario is reached at a distance of about eight miles, where it is about 400 feet below the station. The lake bottom declines at the rate of about 18 feet in one mile until a depth of 225 to 250 feet is reached, and continues farther west at about the same level. East of the station the ground is quite uneven, and at a distance of 80 to 100 miles the Adirondack Mountains are reached. The length of the telegraphic line was about 450 miles, being through Canada by way of Toronto and Kingston, with a repeater at Toronto. The following results for the difference of time between Detroit and and Mannsville were obtained:

Detroit and Mannsville.

Date.	Difference of observed local sidereal times.			
	By signals sent from Mannsville to Detroit.	By signals sent from Detroit to Mannsville.	Means.	Wave and armature time.
1875.	h. m. s.	h. m. s.	h. m. s.	s.
Nov. 6	0 27 56.415	0 27 56.577	0 27 56.496	0.081
8	56.470	56.643	56.557	0.086
11	56.546	56.695	56.620	0.075
12	56.536	56.704	56.620	0.084

Giving equal weights to the results for the several nights their mean is $0^h\,27^m\,56^s.573 \pm 0^s.020$, the probable error being derived from the discrepancies between the mean and the individual results.

The difference of longitude is then obtained by adding the following terms:

		h. m. s.	s.
Observed difference of longitude		0 27 56.573	± 0.020
Relative personal equation of observers	+	0.136	± 0.011
Station Mannsville east of observing-post	+	1.748	

There results, station Mannsville east of Detroit,

$$\mathbf{0^h\ 27^m\ 58^s.457 \pm 0^s.023.}$$

Further details of this work can be found in the reports of Lieutenant Lockwood and Assistant Engineer Flint, published in the Report of the Chief of Engineers for 1876.

CHAPTER XXVI.

ASTRONOMICAL DETERMINATIONS OF OTHER THAN THE FUNDAMENTAL POSITIONS FOR THE PRIMARY TRIANGULATION.

§ 1. Prior to the adoption of the plan of obtaining a continuous chain of triangulation from Duluth, Minn., to St. Regis, on the Saint Lawrence, many determinations of latitude and longitude were made along the lakes which have not been given in Chapters XXIII and XXV. The positions of many points were needed to check the accumulation of errors in the ordinary topographical work, and to serve as fixed points in the projection of the charts of the lakes.

Latitudes were determined with the zenith-telescope, and the results have probable errors of but a few tenths of a second. In the following table are given the principal data as to this latitude work for all stations where the latitude-post was either permanently marked or was referred to a permanent mark. Where this permanent mark was the marking stone of a primary triangulation station, or was directly connected with one, the seconds of the geodetic latitudes derived from Chapter XXVII, and the differences between the geodetic and astronomical latitudes are given. This difference is due in part or mainly to deviations of the vertical from the normal at the point to Clarke's spheroid, which has been adopted for the geodetic computations. In the table are given (1) the name of the station, (2) the dates, (3) the number of pairs of stars observed, (4) the seconds of the resulting latitudes for each night, and the seconds of the mean latitude (a result marked with a (†) is the mean of the results obtained by giving to each night's result a weight proportional to the number of pairs on that night, and a result marked with an (*) is derived by weighting the result from each pair as in Chapter XXIII, § 3), (5) probable error of mean, (6) reduction to permanent fixed point, (7) astronomical latitude of fixed point, (8) seconds of geodetic latitude of fixed point, (9) the difference of the geodetic and astronomical latitudes, (10) observer, (11) instrument, (12) data as to instrumental constants, catalogue of star declinations used, and description of permanent fixed point, &c. In some cases the results for latitude differ from those originally published in the reports of the Chief of Engineers for the various years, the differences being due to more precise values of star-places and instrumental contants used in the later reductions.

Table of Astronomical Latitudes, not fundamental.

Station.	Date.	No. of pairs of stars observed.	Seconds of observed latitude.	Probable error of mean.	Reduction to fixed point.	Astronomical latitude of fixed point.	Seconds of geodetic latitude.	Geodetic *minus* astronomical latitude.	Observer.	Instrument.	Remarks.
	1867.		″	″	″	° ′ ″	″	″			
Tip Top, Ontario, (North Post).	July 21	22	26.86	....	....	....	....	....	O. B. Wheeler	Pistor & Martins Berlin instrument No. 1.	Fixed point is primary triangulation station.
	22	3	26.73								1 turn of micrometer........... =85″.22
	23	28	26.42								1 division of level............... = 2″.29
	25	7	25.97								Safford's Catalogue of 2018 Stars.
	26	10	26.09								
			*26.53	± 0.11	— 0.65	48 16 25.88					
(South Post)..........	Aug. 4	26	25.66								
	6	23	25.68								
	12	6	26.44								
	13	7	25.73								
			*25.74	± 0.11	— 0.06	48 16 25.68					
Weighted mean....	....	......	....	± 0.08	....	48 16 25.78	24.64	— 1.14			
	1866.										
Isle Royale, Mich.....	Aug. 26	3	54.85	....	....	....	....	....	S. W. Robinson ...	Pistor & Martins Berlin instrument No. 1.	Fixed point is primary triangulation station.
	Sept. 1	19	55.34								1 turn of micrometer-screw ..... =85″.22
	2	25	55.08								1 division of level............... = 2″.29
	3	10	55.44								Safford's Catalogue of 2018 Stars.
	16	15	55.29								Published in 1867.
			*55.25	± 0.12	— 0.03	48 07 55.22	43.59	—11.63			
	1869.										
Farquhar's Knob, Minn.	Oct. 4	25	33.85	....	....	....	....	....	Lieut. E. H. Ruffner.	Würdemann zenith-telescope No. 1.	Fixed point is primary triangulation station.
	5	22	35.44								1 turn of micrometer-screw ..... =69″.20
	6	32	35.63								1 division of level............... = 1″.08
			*35.21	± 0.24	+ 0.06	47 52 35.27	40.18	+ 4.91			Safford's Catalogue of 2018 Stars.
	1869.										
Michipicoten, Ontario.	July 5	10	19.47	....	....	....	....	....	O. B. Wheeler	Pistor & Martins Berlin instrument No. 1.	Fixed point is primary triangulation station.
	7	7	20.12								1 turn of micrometer-screw ..... =85″.22
			*19.73	± 0.15	+ 0.85	47 45 20.58	21.67	+ 1.09			1 division of level............... = 2″.29
											Safford's Catalogue of 2018 Stars.
											Published in 1870.

Table of Astronomical Latitudes, not fundamental—Continued.

Station.	Date.	No. of pairs of stars observed.	Seconds of observed latitude.	Probable error of mean.	Reduction to fixed point.	Astronomical latitude of fixed point.	Seconds of geodetic latitude.	Geodetic *minus* astronomical latitude.	Observer.	Instrument.	Remarks.
			″	″	″	° ′ ″	″	″			
Gargantua, Ontario	1869. Sept. 14	20	38.52						A. R. Flint	Troughton & Simms No. 3, U. S. C. S.	Fixed point is primary triangulation station.
	15	5	38.85								1 turn of micrometer-screw ..... =46″.66
	17	3	38.95								1 division of level .... = 0″.89
	19	24	39.46								Safford's Catalogue of 2018 Stars.
	20	22	39.07								Published in 1870.
	21	13	39.72								
			†39.12		− 0.32	47 34 38.80	42.72	+ 3.92			
Copper Harbor (Fort Wilkins), Mich.	1864. June 14	15	06.29						O. B. Wheeler	Würdemann zenith-telescope No. 1.	Fixed point is stone post marked S.
	15	21	05.91								1 revolution of micrometer...... =69″.2
	16	29	05.44								1 division of level............... = 1″.08
	17	30	04.83								British Association Catalogue and Safford's Manuscript Catalogue.
			†05.46		− 2.31	47 28 03.15					
Mt. Houghton, Mich	1865. July 4	7	15.84						S. W. Robinson	Würdemann zenith-telescope No. 12.	Fixed point is triangulation station.
	9	7	16.14								1 revolution of micrometer...... =63″.5
	12	10	15.74								1 division of level............... = 1″.00
			†15.89		− 0.06	47 24 15.83	26.00	+10.17			British Association Catalogue and Safford's Manuscript Catalogue.
Sawteeth East, Minn.	1870. July 18	3	09.13						A. R. Flint	Würdemann zenith-telescope No. 18.	Fixed point is primary triangulation station.
	20	20	10.25								1 turn of micrometer-screw .... =62″.63
	21	2	08.28								1 division of level............... = 0″.86
	24	18	08.75								Safford's Catalogue of 2018 Stars.
			*09.45	± 0.21	− 0.45	47 23 09.00	19.02	+10.02			Published in 1871.
Wheal Kate, Mich.	1866. Sept. 18	3	19.23						O. B. Wheeler	Würdemann zenith-telescope No. 12.	Fixed point is primary triangulation station.
	21	29	18.48								1 turn of micrometer-screw ..... =63″.47
	24	25	18.68								1 division of level .... = 1″.00
	25	46	18.24								Safford's Manuscript Catalogue and British Association Catalogue.
			†18.44		−0.32	47 04 18.12	20.27	+2.15			Published in 1867.

Outer Island, Wis	1870. Aug. 13 14 26	15 5 16	13.97 13.26 14.84 *14.16	.... ±0.19	.... +0.35	.... 47 04 14.51	.... 17.09	.... +2.58	G. Y. Wisner	Würdemann zenith-telescope No. 12.	Fixed point is primary triangulation station. 1 turn of micrometer at 00...... =63″.07 1 turn of micrometer at 40...... =63″.87 1 division of level.............. = 1″.03 Safford's Catalogue of 2018 Stars. Published in 1871.
Portage Entry, Mich..	1864. June 9 10 11	2 33 29	48.78 47.06 48.03 †47.55	....	.... +0.17	.... 46 58 47.72	....	....	O. B. Wheeler.....	Würdemann zenith-telescope No. 1.	Fixed point is stone post marked S. 1 revolution of micrometer ..... = 69″.2 1 division of level .... = 1″.08 British Association Catalogue and Safford's Manuscript Catalogue.
Crebassa, Mich	1871. Sept. 22 23 24 28	32 43 48 23	41.11 40.93 41.06 41.15 *41.01	.... ±0.07	.... +0.20	.... 46 58 41.21	.... 41.07	.... −0.14	G. A. Marr........	Würdemann zenith-telescope No. 12.	Fixed point is light-house. Variable values of micrometer-screw, viz; 1 turn at 00=63″.07, with uniform increase to 63″.87 at 40. 1 division of level................ =1″.03 Safford's Catalogue of 981 Stars. Published in 1873.
Buchanan, Minn	1871. July 20 21 26	32 26 18	22.05 22.08 22.49 *22.15	.... ±0.25	.... +2.30	.... 46 56 24.45	.... 28.07	.... +3.62	A. R. Flint........	Würdemann zenith-telescope No. 1.	Fixed point is primary triangulation station. 1 turn of micrometer-screw..... =69″.20 1 division of level............... = 1″.39 Safford's Catalogue of 981 stars.
Porcupine Mts., Mich.	1870. June 20 22 July 1	16 18 10	04.24 04.00 04.55 *04.24	.... ±0.17	.... −0.44	.... 46 47 03.80	.... 01.05	.... −2.75	G. Y. Wisner	Würdemann zenith-telescope No. 12.	Fixed point is primary triangulation station. 1 turn of micrometer-screw at 00=63″.07, with uniform increase to 63″.87 at 40. 1 division of level.............. = 1″.03 Safford's Catalogue of 2018 Stars. Published in 1871.

Table of Astronomical Latitudes, not fundamental—Continued.

Station.	Date.	No. of pairs of stars observed.	Seconds of observed latitude.	Probable error of mean.	Reduction to fixed point.	Astronomical latitude of fixed point.	Seconds of geodetic latitude.	Geodetic *minus* astronomical latitude.	Observer.	Instrument.	Remarks.
			"	"	"	° ′ "	"	"			
Whitefish Point, Mich.	1867. Sept. 16	2	07.50	....	....	....	....	....	O. B. Wheeler....	Pistor & Martins Berlin instrument No. 1.	Fixed point is light-house. 1 turn of micrometer-screw..... =85″.22 1 division of level............... = 2″.29 Safford's Manuscript Catalogue and British Association Catalogue.
	18	59	06.99								
	20	35	07.22								
	21	27	07.62								
			†07.20	....	+10.42	46 46 17.62					
Brulé River, Wis	1871. July 13	18	*19.77	....	....	....	....	....	G. A. Marr........	Pistor & Martins Berlin instrument No. 1.	Fixed point is primary triangulation station. 1 turn of micrometer-screw..... =85″.23 1 division of level............... = 2″.29 Safford's Catalogue of 981 Stars. Published in 1873.
	14	24	20.13								
	16	27	19.69								
	17	36	19.95								
	20	2	20.62								
			*19.99	±0.14	+0.12	46 45 20.11	17.89	−2.22			
South Base, Minnesota Point, Minn.	1871. July 14	39	51.81	....	....	....	....	....	G. Y. Wisner.	Würdemann zenith-telescope No. 12.	Fixed point is primary triangulation station. 1 turn of micrometer-screw ..... =63″.47 1 division of level............... = 1″.00 Safford's Catalogue of 981 Stars. Published in 1873.
	16	38	51.69								
			*51.67	±0.14	−0.25	46 42 51.42	49.44	−1.98			
Aminicon, Wis	1871. July 20	35	36.22	....	....	....	....	....	G. Y. Wisner	Würdemann zenith-telescope No. 12.	Fixed point is primary triangulation station. 1 turn of micrometer-screw..... =63″.47 1 division of level............... = 1″.00 Safford's Catalogue of 981 Stars. Published in 1873.
	21	30	35.95								
			*36.13	±0.14	+0.19	46 41 36.32	32.15	−4.17			
Grand Island, Mich ...	1867. Aug. 26	71	40.95	....	....	....	....	....	O. B. Wheeler ...	Pistor & Martins Berlin instrument No. 1.	Fixed point is light-house. 1 turn of micrometer-screw..... =85″.22 1 division of level .... = 2″.29 Safford's Manuscript Catalogue and British Association Catalogue.
	30	84	40.98								
			†40.96	....	−1.19	46 33 39.77					

Thone's Hill, Marquette, Mich.	1865. July 10	8	00. 78						S. W. Robinson	Würdemann zenith-telescope No. 1.	Fixed point is Marquette light-house. 1 revolution of micrometer-screw =69″. 20 1 division of level = 1″. 08 Safford's Catalogue of 2018 Stars. Published in 1866 and 1867.
	25	10	59. 99						O. B. Wheeler		
	26	13	59. 99						S. W. Robinson		
	30	13	00. 18						...do		
			*00. 19	±0. 20							
	1867. Aug. 31	17	00. 75						Lt. J. F. Gregory		
	Sept. 1	18	01. 81								
			*01. 30	±0. 26							
Weighted mean			00. 60	±0. 16	+53. 93	46 32 54. 53	47. 80	−6. 73			
Sault Ste. Marie, Mich.	1867. Sept. 27	80	‡07. 18						Lt. J. F. Gregory	Würdemann zen.-tel. No. 1.	Fixed point is beacon light (1867) on west end of canal pier. For No. 1: 1 turn of micrometer-screw =69″. 20 1 division of level = 1″. 08 For No. 12: 1 turn of micrometer-screw =63″. 50 1 division of level = 1″. 00 Safford's Manuscript and British Association Catalogues.
	27	76	‖08. 18						G. Y. Wisner	Würdemann zen.-tel. No. 12.	
			†07. 67		+ 2. 62	46 30 10. 29					
Iroquois Point, Mich.	1867. Sept. 23	76	05. 01						O. B. Wheeler	Pistor & Martins Berlin instrument No. 1.	Fixed poiot is light-house. 1 turn of micrometer-screw =85″. 22 1 division of level = 2″. 29 Safford's Manuscript Catalogue.
	25	20	05. 62								
	26	3	05. 73								
			†05. 16		− 0. 32	46 29 04. 84					
Escanaba, Mich.	1865. July 25	62	22. 83						S. W. Robinson	Würdemann zenith-telescope No. 12.	Fixed point is light-house tower built over station, 1867. 1 turn of micrometer-screw =63″. 50 1 division of level = 1″. 00 Safford's Manuscript Catalogue. Published in 1866 and 1870.
	26	22	22. 82						O. B. Wheeler		
	Aug. 8	38	22. 82						...do		
	13	47	22. 70						...do		
			†22. 79		+12. 26	45 44 35. 05	41. 24	+6. 19			
Burnt Bluff (Bay de Noq Bluff), Mich.	1863. Sept. 29	20	33. 74						O. B. Wheeler	Würdemann zenith-telescope No. 1.	Fixed point is triangulation station. 1 turn of micrometer-screw =69″. 20 1 division of level = 1″. 08 Safford's Catalogue of 2018 Stars. Published in 1870.
	30	18	33. 20								
	Oct. 1	13	33. 43								
			*33. 47	±0. 15	+30. 41	45 41 03. 88	09. 73	+5. 85			

‡ Reduced to north post.

‖ At north post.

Table of Astronomical Latitudes, not fundamental—Continued.

Station.	Date.	No. of pairs of stars observed.	Seconds of observed latitude.	Probable error of mean.	Reduction to fixed point.	Astronomical latitude of fixed point.	Seconds of geodetic latitude.	Geodetic *minus* astronomical latitude.	Observer.	Instrument.	Remarks.
	1863.		″	″	″	° ′ ″	″	″			
Beaver Island, Mich ..	Nov. 2	6	30.47	....	....	....	....	....	O. B. Wheeler and S. W. Robinson.	Würdemann zenith-telescope No. 1.	Fixed point is light-house. 1 turn of micrometer-screw .... =69″.14 1 division of level .... = 1″.08 Safford's Manuscript Catalogue.
	3	26	30.51								
			†30.50	....	−1.72	45 34 28.78	32.45	+3.67			
	1865.										
Cedar River, Mich	Sept. 14	46	42.97	....	....	....	....	....	S. W. Robinson ...	Würdemann zenith-telescope No. 1.	Fixed point is primary triangulation station. 1 turn of micrometer-screw ..... =69″.20 1 division of level .... = 1″.08 Safford's Catalogue of 2018 Stars. Published in 1866 and 1870.
	15	46	43.08								
	16	8	43.67								
			*43.06	±0.17	+0.07	45 25 43.13	48.61	+5.48			
	1862.										
Rock Island, Wis ...	Sept. 3	22	31.23	....	....	....	....	....	James Carr	Würdemann zenith-telescope No. 1.	Fixed point is light-house. 1 turn of micrometer-screw ..... =69″.14 1 division of level .... = 1″.08 Safford's Manuscript Catalogue. Published in 1870.
	8	52	30.22								
	9	36	30.53								
	10	11	31.62								
			†30.61	....	+4.88	45 25 35.49					
	1865.										
Boyer's Bluff, Wis	Aug. 26	16	09.11	....	....	....	....	....	Prof. C. A. Young.	Pistor & Martins Berlin instrument No. 1.	Fixed point is primary triangulation station. 1 turn of micrometer-screw ..... =85″.22 1 division of lovel .... = 2″.29 Safford's Catalogue of 2018 Stars. Published in 1866 and 1870.
	29	1	09.52								
	Sept. 3	11	08.76								
	4	20	08.92								
			*08.95	±0.09	+0.35	45 25 09.30	12.74	+3.44			
	1865.										
Door Bluff, Wis	Aug. 31	4	43.75	....	....	....	....	....	S. W. Robinson ...	Würdemann zenith-telescope No. 1.	Fixed point is primary triangulation station. 1 turn of micrometer-screw .... =69″.20 1 division of level .... = 1″.08 Safford's Catalogue of 2018 Stars. Published in 1866 and 1870.
	Sept. 3	17	45.63								
	4	28	44.78								
	5	32	44.91								
	6	36	44.95								
			*44.96	±0.15	+0.73	45 17 45.69	46.97	+1.28			

	1865.										
Marinette, Wis. (near Menomonee).	Aug. 13	21	55. 81	....	....	....	....	....	S. W. Robinson ...	Pistor & Martins Berlin instrument No. 1.	Fixed point is new primary triangulation station Menomonee (1874).
	15	21	56. 45								1 turn of micrometer-screw..... =85″. 22
			*56. 13	±0. 10	−43. 35	45 05 12. 78	12. 76	−0. 02			1 division of level........ ..... = 2″. 29
											Safford's Catalogue of 2018 Stars.
											Published in 1866 and 1870.
	1862.										
Green Island, Wis....	Sept. 24	16	33. 68	......	....	....	....	....	James Carr	Würdemann zenith-telescope No. 1.	Fixed point is dome of light-house.
	25	35	33. 94								1 turn of micrometer-screw ..... =69″. 14
	26	31	33. 80								1 division of level .... = 1″. 08
	27	36	33. 92								Safford's Manuscript Catalogue.
	Oct. 2	44	33. 99								
			†33. 90	....	− 9. 10	45 03 24. 80					
	1858.										
Thunder Bay Island, Mich.	Aug. 13	14	12. 62	....	....	....	....	....	Lieut. C. N. Turnbull.	Würdemann zenith-telescope No. 12.	Fixed point is light-house.
	15	14	13. 85								1 turn of micrometer-screw ..... =63″. 70
	16	14	12. 81								1 division of level .... = 1″. 00
	18	13	12. 26								British Association Catalogue.
	20	13	12. 60								Published in 1859.
	23	27	12. 30								
	24	27	12. 70								
			†12. 69	....	+ 4. 58	45 02 17. 27					
	1871.										
Saint Paul, Minn	June 9	7	42. 98	....	....	....	....	....	A. R. Flint	Würdemann zenith-telescope No. 1.	Fixed point is southeast corner of custom-house on Wabasha street.
	10	23	41. 30								1 turn of micrometer-screw ..... =69″. 20
	11	25	42. 12								1 division of level .... = 1″. 08
	12	10	42. 95								Safford's Catalogue of 2018 Stars.
			*42. 48	±0. 28	+ 0. 48	44 56 42. 96					
	1868.										
Ogdensburg, N. Y	July 19	30	25. 11	....	....	....	....	....	Lieut. E. H. Ruffner.	Würdemann zenith-telescope No. 1.	Fixed point is light-house dome.
	20	27	25. 96								1 turn of micrometer-screw ..... =69″. 14
			*25. 60	±0. 26	+27. 59	44 41 53. 19					1 division of level .... = 1″. 08
											Safford's Catalogue of 981 Stars.
											Published in 1870.

Table of Astronomical Latitudes, not fundamental—Continued.

Station.	Date.	No. of pairs of stars observed.	Seconds of observed latitude.	Probable error of mean.	Reduction to fixed point.	Astronomical latitude of fixed point.	Seconds of geodetic latitude.	Geodetic *minus* astronomical latitude.	Observer.	Instrument.	Remarks.
			″	″	′ ″	° ′ ″	″	″			
Clay Banks, Door County, Wis.	1866. Oct. 4	20	15.72	......	....	....	....	....	S. W. Robinson...	Pistor & Martins Berlin instrument No. 1.	Fixed point is azimuth station. 1 turn of micrometer-screw ..... =85″.22 1 division of level ... . = 2″.29 Safford's Manuscript Catalogue.
	5	127	15.25								
			†15.31	......	+ 3.88	44 41 19.19					
	1872. Aug. 11	5	33.84	....	....	....	....	....	Lieut. C. F. Powell.	Würdemann zenith-telescope No. 1.	1 turn of micrometer-screw ..... =69″.02 1 division of level .... = 1″.36 Safford's Catalogue of 981 Stars. Published in 1867 and 1874.
	12	1	36.63								
	14	4	33.57								
	15	13	34.98								
	22	9	35.29								
	23	11	36.24								
	25	4	36.50								
			†35.26	....	− 15.28	44 41 19.98					
Mean, giving double weight to result of 1866.	....	....	....	....	....	44 41 19.45					
Frankfort, or Pointe aux Bec Scies, Mich.	1871. July 9	21	53.61	....	....	....	....	....	Lt. J. H. Weeden.	Würdemann zenith-telescope No. 18.	Fixed point is azimuth station. 1 turn of micrometer-screw .... =62″.70 1 division of level .... = 0″.86 Safford's Catalogue of 981 Stars.
	10	19	52.95								
			†53.30	....	− 0.24	44 38 53.06					
Red Wing, Minn......	1878. Oct. 18	21	44.17	....	....	....	....	....	Lieut. C. F. Powell	Würdemann zenith-telescope No. 12.	Fixed point is Catholic church spire. 1 turn of micrometer-screw ..... =63″.47 1 division of level .... = 1″.03 Safford's Catalogue of 981 Stars, and Catalogue of "539 Sterne." Published in 1879.
	19	20	44.15								
			†44.16	....	+ 0.48	44 33 44.64					
Kewaunee, Wis	1873. June 30	24	41.74	....	....	....	....	....	Lieut. C. F. Powell	Würdemann zenith-telescope No. 1.	Fixed point is azimuth station. 1 turn of micrometer-screw ..... =69″.02 1 division of level .... = 0″.92 Safford's Catalogue of 981 Stars. Published in 1874.
	July 2	18	40.75								
	4	23	41.24								
	5	23	41.49								
			†41.12	....	0.00	44 25 41.12					

Manistee, Mich	1871. July 22	17	46.28						Lt. J. H. Weeden.	Würdemann zenith-telescope No. 18.	Fixed point is azimuth station. 1 turn of micrometer-screw ..... =62″.70 1 division of level ..... = 0″.86 Safford's Catalogue of 981 Stars.
	23	30	46.51								
			†46.43		− 0.90	44 16 45.53					
Valley Junction, Wis	1873. Aug. 16	23	10.92						S. W. Robinson.	Pistor & Martins Berlin instrument No. 1.	Fixed point is southwest corner of section 31, township 21 north, range 3 east, on correction line. 1 turn of micrometer-screw ..... =85″.22 1 division of level ..... = 2″.73 Safford's Catalogue of 981 Stars. Published in 1874.
	18	29	11.06								
			†11.00		+11 46.24	44 14 57.24					
Rawley's Point (or Two Rivers Point), Wis.	1866. Oct. 8	96	34.87						S. W. Robinson	Pistor & Martins Berlin instrument No. 1.	Fixed point is azimuth station. For P. & M. inst., 1866: 1 turn of micrometer-screw ..... =85″.22 1 division of level ..... = 2″.29 Safford's Manuscript Catalogue.
	9	51	34.59								
			†34.79		+ 29.61	44 12 04.40					
	1872. Sept. 6	8	04.29						Lieut. C. F. Powell.	Würdemann zenith-telescope No. 1.	For Würd. inst., 1872: 1 turn of micrometer-screw ..... =69″.02 1 division of level ..... = 1″.36 Safford's Catalogue of 981 Stars. Published in 1867 and 1874.
	7	4	03.62								
	9	15	05.00								
			†04.58		− 0.01	44 12 04.57					
Mean, giving double weight to result of 1866.						44 12 04.46					
Big Pointe au Sable, Mich.	1866. Aug. 4	25	30.36						O. B. Wheeler	Würdemann zenith-telescope No. 12.	Fixed point is light-house. For No. 12. 1866: 1 turn of micrometer-screw ..... =63″.47 1 division of level ..... = 1″.00 Safford's Manuscript Catalogue. Published in 1867.
	5	68	30.70								
	6	48	30.28								
			†30.47								
	1872. Aug. 5	11	30.00						Lieut. E. Maguire.	Würdemann zenith-telescope No. 18.	For No. 18, 1872: 1 turn of micrometer-screw ..... =62″.63 1 division of level ..... = 0″.86 Safford's Catalogue of 981 Stars.
	7	12	28.68						do		
	11	10	29.35						do		
	14	13	29.73						do		
	15	14	29.03						do		
	18	7	28.98						Gen. C. B. Comstock.		
			†29.31								
Mean, giving double weight to result of 1866.			30.09		− 0.80	44 03 29.29					

Table of Astronomical Latitudes, not fundamental—Continued.

Station.	Date.	No. of pairs of stars observed.	Seconds of observed latitude.	Probable error of mean.	Reduction to fixed point.	Astronomical latitude of fixed point.	Seconds of geodetic latitude.	Geodetic *minus* astronomical latitude.	Observer.	Instrument.	Remarks.
	1868.		″	″	″	° ′ ″	″	″			
Watertown, N. Y	July 2	7	43. 32	....	....	....	....	....	G. Y. Wisner	Würdemann zenith-telescope No. 12.	Fixed point is northeast corner of cupola of court-house.
	3	31	42. 80								1 turn of micrometer-screw at 0. =63″. 07 with uniform increase to 63″. 87 at 40.
	5	3	42. 48								1 division of level.............. = 0″. 03
	7	31	43. 10								Safford's Catalogue of 981 Stars.
	14	25	43. 26								Published in 1870.
			*43. 02	± 0. 10	−10. 14	43 58 32. 88	32. 20	− 0. 68			
	1873.										
Point Creek, Wis	Aug. 2	23	57. 79	....	....	....	....	....	Lieut. C. F. Powell	Würdemann zenith-telescope No. 1.	Fixed point is azimuth station.
	3	22	58. 17								1 turn of micrometer-screw ..... =69″. 02
	4	28	58. 02								1 division of level..... .... = 0″. 92
	5	28	58. 69								Safford's Catalogue of 981 Stars.
			†58. 29	....	+ 0. 56	43 57 58. 85					Published in 1874.
	1866.										
North Sheboygan, Wis.	Oct. 13	44	54. 00	....	....	....	....	....	Lieut. M. R. Brown	Pistor & Martins Berlin instrument No. 1.	Fixed point is Sheboygan light-house.
	15	38	54. 39								1 turn of micrometer-screw ..... =85″. 22
			†54. 13	....	− 3. 77	43 45 50. 36					1 division of level............... = 2″. 29
											Safford's Manuscript Catalogue.
											Published in 1867.
	1860.										
Goderich, Ont.........	Mar. 26	29	21. 93	....	....	....	....	....	Lieut. O. M. Poe...	Würdemann zenith-telescope No. 1.	Fixed point is court-house.
	29	11	21. 62								1 turn of micrometer-screw ..... =69″. 12
	Apr. 1	14	21. 99								1 division of level............... = 1″. 08
	6	30	21. 63								British Association Catalogue.
	23	11	21. 47								
	26	18	21. 78								
	July 1	13	22. 02								
	10	26	21. 53								
			†21. 74	....	+14. 59	43 44 36. 33					

	1872.										
South Sheboygan, Wis.	S pt. 13	3	01. 07	....	....	....	....	....	Liout. C. F. Powell	Würdemann zenith-telescope No. 1.	Fixed point is azimuth station.
	14	10	02. 77								1 turn of micrometer-screw ..... =69′. 02
	16	15	01. 08								1 division of level .... = 1″. 36
	Oct. 12	8	00. 25								Safford's Catalogue of 981 Stars.
			†01. 37	....	— 0. 09	43 41 01. 28					Pnblished in 1874.
	1873.										
Clay Banks, Mich	June 12	5	23. 73	....	....	....	....	....	Lieut. E. Maguire.	Würdemann zenith-telescope No. 18.	Fixed point is azimuth station.
	16	16	21. 96								1 turn of micrometer-screw ..... =62″. 63
	17	17	23. 36								1 division of level .... = 0″. 86
	20	4	24. 85								Safford's Catalogue of 981 Stars.
			†23. 01	....	0. 00	43 30 23. 01					Published in 1874.
Ada 1, Wis............	Sept. 4	18	18. 69	....	....	....	....	....	Lieut. C. F. Powell	Würdemann zenith-telescope No. 1.	Fixed point is azimuth station.
	8	21	18. 77								1 turn of micrometer-screw ..... =69″. 02
	10	19	18. 16								1 division of level .... = 0″. 92
	11	21	19. 05								Safford's Catalogue of 981 Stars.
			†18. 68	....	+ 0. 02	43 29 18. 70					Published in 1874.
	1868.										
Oswego, N. Y	July 24	2	47. 32	....	....	....	....	....	O. B. Wheeler.....	Pistor & Martins Berlin instrument No. 1.	Fixed point is primary triangulation station.
	Aug. 1	5	49. 02								1 turn of micrometer-screw..... =85″. 22
	6	29	49. 26								1 division of level .... = 2″. 29
	7	24	49. 29								Safford's Catalogue of 981 Stars.
	12	5	48. 40								Published in 1870.
			*49. 05	±0. 10	—69. 48	43 26 39. 57	37. 28	—2. 29			
	1875.										
Newaygo, Mich	Oct. 8	3	15. 64	....	....	....	....	....	Lieut. T. N. Bailey.	Pistor & Martins Berlin instrument No. 2.	Fixed point is intersection of axes of Wood and State streets.
	18	7	13. 97								1 turn of micrometer-screw at 0=82″. 90 with uniform increase to 83″. 38 at 30.
	19	27	14. 04								1 division of level A............ = 0″. 98
			†14. 16	....	+ 1. 29	43 25 15. 45					1 division of level B . ... = 1″. 04
											Safford's Catalogue of 981 Stars, and American Nautical Almanac.
											Published in 1876.
	1876.										
Saginaw, Mich	Sept. 19	26	07. 61	....	....	....	....	....	Lient. D. W. Lockwood.	Würdemann zenith-telescope No. 19.	Fixed point is east corner of court-house.
	21	26	07. 57								1 turn of micrometer-screw..... =62″. 22
			†07. 59	....	— 1. 87	43 25 05. 72					1 division of level .... = 1″. 29
											Safford's Catalogue of 981 Stars.
											Published in 1877.

Table of Astronomical Latitudes, not fundamental—Continued.

Stations.	Date.	No. of pairs of stars observed.	Seconds of observed latitude.	Probable error of mean.	Reduction to fixed point.	Astronomical latitude of fixed point.	Seconds of geodetic latitude.	Geodetic *minus* astronomical latitude.	Observer.	Instrument.	Remarks.
	1876.		″	″		° ′ ″	″	″			
Saint Louis, Mich.	Oct. 2	30	*33. 81	±0. 11	−0. 83	43 24 32. 98			Lieut. D. W. Lockwood.	Würdemann zenith-telescope No. 19.	Fixed point is intersection of axes of Washington and Mill streets. 1 turn of micrometer-screw = 62″. 22 1 division of level = 1″. 29 Safford's Catalogue of 981 Stars. Published in 1877.
	1871.										
Duck Lake, Mich.	Sept. 20	20	03. 39						Lt. J. H. Weeden.	Würdemann zenith-telescope No. 18.	Fixed point is azimuth station. 1 turn of micrometer-screw = 62″. 70 1 division of level = 0″. 86 Safford's Manuscript Catalogue and British Association Catalogue.
	21	18	03. 74								
			†03. 56		−0. 49	43 20 03. 07					
	1875.										
Stanton, Mich.	Oct. 21	29	29. 85						Lieut. P. M. Price.	Würdemann zenith-telescope No. 12.	Fixed point is intersection of axes of Main and Camburn streets. 1 turn of micrometer-screw at 0 = 63″. 07 with uniform increase to 63″. 87 at 40. 1 division of level = 1″. 03 Safford's Catalogue of 981 Stars. Published in 1876.
	22	30	29. 65								
			†29. 76		+0. 93	43 17 30. 69					
	1873.										
Fox Point, Wis.	Sept. 22	19	13. 44						Lieut. C. F. Powell	Würdemann zenith-telescope No. 1.	Fixed point is azimuth station. 1 turn of micrometer-screw = 69″. 02 1 division of level = 0″. 92 Safford's Catalogue of 981 Stars. Published in 1874.
	23	8	13. 22								
	25	22	13. 32								
	26	26	13. 73								
	27	20	13. 07								
			†13. 35		−0. 09	43 10 13. 26					
	1868.										
Rochester, N. Y.	Aug. 1	30	24. 94						G. Y. Wisner.	Wurdemann zenith-telescope No. 12.	Fixed point is City Hall (statue). 1 turn of micrometer-screw at 0 = 63″. 07 with uniform increase to 63″. 87 at 40. 1 division of level = 1″. 03 Safford's Catalogue of 981 Stars. Published in 1870.
	5	33	25. 00								
	6	14	26. 19								
			*25. 31	±0. 16	−2. 87	43 09 22. 44	18. 11	− 4. 33			

Lapeer, Mich	1874. June 27	10	07.42						Lieut. C. F. Powell.	Pistor & Martins Berlin instrument No. 1.	Fixed point is intersection of axes of Calhoun and Franklin streets. 1 turn of micrometer-screw ..... =85″.27 1 division of level .... = 2″.16 Safford's Catalogue of 981 Stars. Published in 1875.
	29	27	07.62								
	30	25	07.67								
			†07.61		− 0.35	43 03 07.26					
Flint, Mich	1874. July 17	29	13.20						Lieut. C. F. Powell.	Pistor & Martins Berlin instrument No. 1.	Fixed point is intersection of axes of Keasley and East streets. 1 turn of micrometer-screw ..... =85″.27 1 division of level .... = 2″.16 Safford's Catalogue of 981 Stars. Published in 1875.
	21	10	12.95								
	22	22	12.82								
			†13.02		− 1.31	43 01 11.71					
Fort Gratiot, Mich	1858. Oct. 17	34	51.87						Lieut. C. N. Turnbull.	Würdemann zenith-telescope No. 12.	Fixed point is light-house. 1 turn of micrometer-screw ..... =63″.70 1 division of level....... .... = 1″.00 British Association Catalogue. Published in 1859.
	18	20	51.66								
	21	31	52.44								
	25	33	52.21								
	26	4	51.98								
	27	12	52.61								
			†52.12		+29.68	43 00 21.80					
Saint Johns, Mich	1874. July 27	6	00.52						Lieut. C. F. Powell.	Pistor & Martins Berlin instrument No. 1.	Fixed point is intersection of axes of Clinton avenue and State street. 1 turn of micrometer-screw .... =85″.27 1 division of level....... . .. = 2″.16 Safford's Catalogue of 981 Stars. Published in 1875.
	28	27	00.00								
	29	18	59.66								
			†59.94		+ 2.45	43 00 02.39					
Ionia, Mich	1874. Aug. 24	21	52.63						Lieut. C. F. Powell.	Pistor & Martins Berlin instrument No. 1.	Fixed point is intersection of axes of Main and First streets. 1 turn of micrometer-screw ..... =85″.27 1 division of level................ = 2″.16 Safford's Catalogue of 981 Stars. Published in 1875.
	27	26	52.45								
			†52.53		− 1.14	42 58 51.39					
Corunna, Mich	1874. July 28	32	22.34						Capt. H. M. Adams.	Würdemann zenith-telescope No. 12.	Fixed point is intersection of axes of Shiawassee avenue and Flint road. 1 turn of micrometer-screw at 0 =63″.07 with uniform increase to 63″.87 at 40. 1 division of level................ = 1″.00 Safford's Catalogue of 981 Stars. Published in 1875.
	29	38	22.14								
			†22.23		+15.69	42 58 37.92					

Table of Astronomical Latitudes, not fundamental—Continued.

Station.	Date.	No. of pairs of stars observed.	Seconds of observed latitude.	Probable error of mean.	Reduction to fixed point.	Astronomical latitude of fixed point.	Seconds of geodetic latitude.	Geodetic *minus* astronomical latitude.	Observer.	Instrument.	Remarks.
	1871.		"		"	° ′ "	′	"			
South Twin Sisters, or Grand Haven, Mich.	Sept. 7	22	34.62	....	....	....	....	....	Lieut. J. H. Weeden	Würdemann zenith-telescope No. 18.	Fixed point is azimuth station. 1 turn of micrometer-screw ..... =62″.70 1 division of level .... = 0″.86 Safford's Manuscript Catalogue and British Association Catalogue.
	8	10	35.40								
			†34.88	....	0.00	42 58 34.88					
	1874.										
Grand Rapids, Mich ..	Sept. 1	22	49.25	....	....	....	....	....	Lieut. C. F. Powell	Pistor & Martins Berlin instrument No. 1.	Fixed point is quarter-section corner on west side of section 30, township 7 north, range 11 west, at intersection of axes of Fulton and Division streets. 1 turn of micrometer-screw ..... =85″.27 1 division of level .. = 2″.16 Safford's Catalogue of 981 Stars. Published in 1875.
	3	25	48.82								
			†49.02	....	− 1.98	42 57 47.04					
	1868.										
Buffalo, N. Y	Sept. 14	34	41.80	....	....	....	....	....	Lieut. E. H. Ruffner	Würdemann zenith-telescope No. 1.	Fixed point is intersection of axes of Exchange and Michigan streets. 1 turn of micrometer-screw ..... =69″.14 1 division of level .... = 1″.08 Safford's Catalogue of 981 Stars and American Ephemeris for 1868. Published in 1870.
	15	7	41.96								
	16	25	42.07								
			*41.99	±0.19	+ 2.67	42 52 44.66	41.09	−3.57			
	1875.										
Lansing, Mich	June 29	7	54.13	....	....	....	....	....	Lieut. T. N. Bailey	Pistor & Martins Berlin instrument No. 2.	Fixed point is intersection of axes of Michigan and Capitol avenues. 1 turn of micrometer-screw at 0 =82″.90 with uniform increase to 83″.38 at 30. 1 division of level A . ... = 0″.98 1 division of level B .... = 1″.04 Safford's Catalogue of 981 Stars. Published in 1876.
	July 7	19	53.27								
	10	21	52.85								
			*53.11	±0.17	+ 2.68	42 43 55.79					

Hastings, Mich	1875. July 29	17	53. 18	....	....	....	....	....	Lieut. T. N. Bailey	Pistor & Martins Berlin instrument No. 2.	Fixed point is intersection of axes of State and Church streets. 1 turn of micrometer-screw at 0 =82″.90 with uniform increase to 83″.38 at 30. 1 division of level A .... = 0″.98 1 division of level B....... ..... = 1″.04 Safford's Catalogue of 981 Stars and American Nautical Almanac. Published in 1876.
	Aug. 11	22	52. 60								
			†52. 85	....	+ 1. 28	42 38 54. 13					
Douglas, Mich........	1873. July 26	18	11. 34	....	....	....	....	....	Lieut. E. Maguire.	Würdemann zenith-telescope No. 18.	Fixed point is azimuth station. 1 turn of micrometer-screw ..... =62″.63 1 division of level .... = 0″.86 Safford's Catalogue of 981 Stars. Published in 1874.
	27	21	11. 12								
	29	19	10. 91								
			†11. 12	....	+ 0. 50	42 38 11. 62					
Pontiac, Mich	1874. June 7	33	†07. 20	....	— 0. 25	42 38 06. 95	....	....	Lieut. C. F. Powell	Würdemann zenith-telescope No. 12.	Fixed point is southeast corner of section 29, township 3 north, range 10 east. 1 turn of micrometer-screw at 0 =63″.07 with uniform increase to 63″.87 at 40. 1 division of level .... = 1″.00 Safford's Catalogue of 981 Stars. Published in 1875.
Howell, Mich..... ..	1875. June 3	9	00. 38	....	....	....	....	....	Lieut. T. N. Bailey	Pistor & Martins Berlin instrument No. 2.	Fixed point is intersection of axes of Higgins and Monroe streets. 1 turn of micrometer-screw at 0 =82″.90 with uniform increase to 83″.38 at 30. 1 division of level A............. = 0″.98 1 division of level B...... = 1″.04 Safford's Catalogue of 981 Stars and American Nautical Almanac. Published in 1876.
	5	15	00. 49								
	7	20	59. 95								
	14	1	01. 94								
	16	1	00. 88								
	18	3	59. 76								
	20	16	00. 18								
	22	9	00. 89								
			*00. 24	± 0. 13	+10. 94	42 36 11. 18					
Charlotte, Mich.......	1875. July 7	17	04. 42	....	....	....	....	....	Lieut. P. M. Price.	Würdemann zenith-telescope No. 12.	Fixed point is intersection of axes of Oliver and Stoddard streets. 1 turn of micrometer-screw at 0 =63″.07 with uniform increase to 63″.87 at 40. 1 division of level .... = 1″.03 Safford's Catalogue of 981 Stars. Published in 1876.
	9	25	03. 65								
	10	30	04. 36								
			†04. 12	....	— 5. 16	42 33 58. 96					

Table of Astronomical Latitudes, not fundamental—Continued.

Station.	Date.	No. of pairs of stars observed.	Seconds of observed latitude.	Probable error of mean.	Reduction to fixed point.	Astronomical latitude of fixed point.	Seconds of geodetic latitude.	Geodetic *minus* astronomical latitude.	Observer.	Instrument.	Remarks.
			″	″	′ ″	° ′ ″	″				
Allegan, Mich.	1875. Sept. 10	24	43. 33						Lieut. T. N. Bailey	Pistor & Martins Berlin instrument No. 2.	Fixed point is northwest corner of county building in public square.
	12	24	42. 68								1 turn of micrometer-screw at 0 =82″.90 with uniform increase to 83″.38 at 30.
			†43. 01		+ 1. 73	42 31 44. 74					1 division of level A = 0″.98 1 division of level B = 1″.04 Safford's Catalogue of 981 Stars. Published in 1876.
Galena, Ill	1873. July 14	27	10. 04						S. W. Robinson	Pistor & Martins Berlin instrument No. 1.	Fixed point is intersection of 4th principal meridian with State line between Wisconsin and Illinois.
	17	32	09. 54								1 turn of micrometer-screw =85″.22
	23	30	09. 04								1 division of level = 2″.73
	26	18	09. 40								
			†09. 50		+5 16. 60	42 30 26. 10					Safford's Catalogue of 981 Stars. Published in 1874.
Dunkirk, N. Y.	1868. Aug. 26	36	20. 23						G. Y. Wisner	Würdemann zenith-telescope No. 12.	Fixed point is new light-house (1876).
	27	35	19. 83								1 turn of micrometer-screw at 0 =63″.07 with uniform increase to 63″.87 at 40.
	Sept. 3	3	18. 68								1 division of level = 1″.03
	4	3	22. 47								
			*20. 03	± 0. 12	+ 24. 27	42 29 44. 30	37. 84	— 6. 46			Safford's Catalogue of 981 Stars. Published in 1870.
Detroit, Mich.	1872. Aug. 1	26	58. 60						Maj. C. B. Comstock	Pistor & Martins No. 1	Fixed point is intersection of center lines of Grand River and Washington avenues.
	2	28	58. 48						do	do	
	3	30	58. 89						do	do	
	Oct. 28	7	59. 20						do	Würdemann No. 12	For Pistor & Martins No. 1:
	29	17	57. 88						do	do	1 turn of micrometer-screw =85″.22
	31	3	57. 60						do	do	1 division of level = 2″.29
	Nov. 1	21	59. 03						do	do	For Würdemann No. 12:
	9	22	58. 78						do	do	1 turn of micrometer-screw at 0 =63″.07
			*58. 65	± . 09	+ 0. 39	42 19 59. 04	01. 19	+ 2. 15			with uniform increase to 63″.87 at 40. 1 division of level = 1″.03 Safford's Catalogue of 981 Stars. Published in 1873.

Kalamazoo, Mich	1875. Sept. 3	25	26.03	....	....	....	....	....	Lieut. P. M. Price	Würdemann zenith-telescope No. 12.	Fixed point is intersection of axes of South and Park streets.
	6	26	25.64								1 turn of micrometer-screw at 0=63″.07 with uniform increase to 63″.87 at 40.
			†25.83	....	− 1.37	42 17 24.46					1 division of level .... = 1″.03
											Safford's Catalogue of 981 Stars.
											Published in 1876.
Marshall, Mich	1875. Aug. 8	28	21.04	....	....	....	....	....	Lieut. P. M. Price	Würdemann zenith-telescope No. 12.	Fixed point is intersection of axes of Kalamazoo avenue and Mansion street.
	17	24	21.40								1 turn of micrometer-screw at 0=63″.07 with uniform increase to 63″.87 at 40.
			†21.21	....	+ 3.46	42 16 24.67					1 division of level .... = 1″.03
											Safford's Catalogue of 981 Stars.
											Published in 1876.
South Haven, Mich	1873. Sept. 26	7	13.66	....	....	....	....	....	Lieut. E. Maguiro.	Würdemann zenith-telescope No. 18.	Fixed point is azimuth station.
	Oct. 1	21	10.48								1 turn of micrometer-screw .. =62″.63
	2	22	13.11								1 division of level.......... = 0″.86
	7	9	15.32								Safford's Catalogue of 981 Stars.
			†13.24	....	+ 0.24	42 16 13.48					Published in 1874.
Jackson, Mich	1875. Aug. 17	25	50.99	....	....	....	....	....	Lieut. T. N. Bailey	Pistor & Martins Berlin instrument No. 2.	Fixed point is intersection of axes of Main and Jackson streets.
	22	25	51.06								1 turn of micrometer-screw at 0=82″.90 with uniform increase to 83″.38 at 30.
			†51.02	....	− 1.62	42 14 49.40					1 division of level A.......... = 0″.98
											1 division of level B.......... = 1″.04
											Safford's Catalogue of 981 Stars and American Nautical Almanac.
											Published in 1876.
Paw Paw, Mich	1875. Sept. 22	25	02.30	....	....	....	....	....	Lieut. P. M. Price.	Würdemann zenith-telescope No. 12.	Fixed point is intersection of axes of Main and Van Buren streets.
	27	29	02.51								1 turn of micrometer-screw at 0=63″.07 with uniform increase to 63″.87 at 40.
			†02.41	....	− 0.75	42 13 01.66					1 division of level.......... = 1″.03
											Safford's Catalogue of 981 Stars.
											Published in 1876.

Table of Astronomical Latitudes, not fundamental—Continued.

Station.	Date.	No. of pairs of stars observed.	Seconds of observed latitude.	Probable error of mean.	Reduction to fixed point.	Astronomical latitude of fixed point.	Seconds of geodetic latitude.	Geodetic minus astronomical latitude.	Observer.	Instrument.	Remarks.
	1868.		″	″	″	° ′ ″	″	″			
Erie, Pa	Sept. 2	3	11.39						O. B. Wheeler	Pistor & Martins Berlin instrument No. 1.	Fixed point is beacon light-house.
	3	3	12.41								1 turn of micrometer-screw = 85″.22
	4	5	11.27								1 division of level = 2″.29
	7	6	11.21								Safford's Catalogue of 2018 Stars.
	14	18	09.55								Published in 1870.
			*10.38	±0.22	+67.11	42 09 17.49	11.85	−5.64			
	1868.										
Monroe, Mich	Oct. 29	19	50.68						G. Y. Wisner	Würdemann zenith-telescope No. 12.	Fixed point is steeple of court-house.‡
	Nov. 2	11	49.09								1 turn of micrometer-screw at 0=63″.07 with uniform increase to 63″.87 at 40.
	4	9	50.02								1 division of level = 1″.03
	5	11	48.93								Safford's Catalogue of 981 Stars.
			*49.88	±0.18	−1.23	41 54 48.65	52.60	+3.95			Published in 1870.
	1864.										
Chicago, Ill	Aug. 15	4	01.44						O. B. Wheeler	Würdemann zenith-telescope No. 1.	Fixed point is light-house.
	17	16	00.92								1 turn of micrometer-screw = 69″.20
	18	15	01.22								1 division of level = 1″.08
	19	7	01.65								Safford's Catalogue of 2018 Stars.
	21	14	01.63								
	23	15	01.28								
			*01.30	±0.08	+21.19	41 53 22.49	20.52	−1.97			
	1868.										
Ashtabula, Ohio	Sept. 17	11	34.07						G. Y. Wisner	Würdemann zenith-telescope No. 12.	Fixed point is southwest corner of passenger depot.
	18	8	33.38								1 turn of micrometer-screw at 0=63″.07 with uniform increase to 63″.87 at 40.
	21	1	34.25								1 division of level = 1″.03
	23	5	33.37								Safford's Catalogue of 981 Stars.
	26	9	33.65								Published in 1870.
	29	4	34.30								
	Oct. 3	18	33.99								
			*33.75	±0.16	+2.38	41 52 36.13					

Rook Island, Ill.	1878. Oct. 2	14	03. 52						Lieut. C. F. Powell	Würdemann zenith-telescope No. 12.	Fixed point is southeast corner of guard-house. 1 turn of micrometer-screw at 20=63″.47 with varying value as above. 1 division of level = 1″.03 Safford's Catalogue of 981 Stars and Catalogue of "539 Sterne." Published in 1879.
	5	22	03. 34								
			† 03. 41		− 1. 90	41 31 01. 51					
Cleveland, Ohio	1868. Oct. 4	18	17. 16						O. B. Wheeler	Pistor & Martins Berlin instrument No. 1.	Fixed point is light-house on high bank in the city. 1 turn of micrometer-screw =85″.22. 1 division of level = 2″.29. Safford's Catalogue of 2018 Stars. Published in 1870.
	6	13	16. 14								
	7	10	16. 53								
	9	3	17. 62								
	10	16	15. 81								
			* 16. 51	± 0. 19	−10. 38	41 30 06. 13	01. 63	−4. 50			
Louisiana, Mo	1878. Nov. 21	10	14. 49						Lieut. C. F. Powell	Würdemann zenith-telescope No. 12.	Fixed point in northeast corner of High-School building. 1 turn of micrometer-screw at 0=63″.07 with uniform increase to 63″.87 at 40. 1 division of level = 1″.03. Safford's Catalogue of 981 Stars. Catalogue of "539 Sterne." Coast-Survey Catalogue of Latitude Stars, and American Nautical Almanac. Published in 1879.
	22	21	14. 13								
	23	20	14. 38								
			† 14. 30		− 1. 65	39 27 12. 65					
Cairo, Ill.	1876. Dec. 2	20	48. 42						Lieut. D. W. Lockwood.	Würdemann zenith-telescope No. 19.	Fixed point is triangulation-station Defiance. 1 turn of micrometer-screw =62″.22. 1 division of level = 1″.29. Safford's Catalogue of 981 Stars and American Nautical Almanac. Published in 1877.
	3	25	48. 18								
			† 48. 29		− 0. 30	36 59 47. 99					
Memphis, Tenn.	1877. June 28	22	39. 52						Lieut. C. F. Powell	Würdemann zenith telescope, No. 19.	Fixed point is triangulation station. 1 turn of micrometer-screw =62″.22. 1 division of level = 1″.29. Safford's Catalogue of 981 Stars, American Nautical Almanac, Catalogue of "539 Sterne," and Coast-Survey Catalogue. Published in 1878.
	29	21	39. 25								
	30	24	39. 48								
			† 39. 42		+ 0. 43	35 08 39. 85					

‡ The reduction from this court-house (which has since been destroyed by fire) to steeple of First Presbyterian Church, is +0″.81.

§ **2.** Longitudes other than those given in Chapter XXV were determined telegraphically for the Lake Survey at many points along the lakes between 1863 and 1870. In this work personal equation was only occasionally eliminated. The methods were essentially the same as those described in Chapter XXV, save that in some cases telegraphic signals were only sent when the same star crossed each wire of the instrument at the first and second station.

In the following table the first column gives the name of the station whose longitude was determined; the second gives the date of observation; the third, the number of stars observed in common at both stations; the fourth, the observed difference of time between the two stations occupied; the fifth, the reduction to fixed point at each station; the sixth, the difference of time between the station and Detroit; the seventh, the longitude of the station west of Greenwich, in arc, derived by applying to the result in the sixth column, the longitude of Detroit west of Greenwich ($5^h\ 32^m\ 12^s.24$); the eighth column gives the seconds of geodetic longitude computed through the triangulation from the longitude observed at Fort Howard; the ninth, the discrepancy between the geodetic and observed longitudes; the tenth, the names of the observers; the eleventh, the instruments used; the twelfth, data as to personal equation, catalogues used, points of reference, &c.:

Astronomical Longitudes, not fundamental.

Station.	Date.	Number of stars observed in common at both stations.	Observed difference of time between stations.	Reductions to fixed points.	Difference of time between fixed point and Detroit.	Longitude of point west of Greenwich.	Seconds of geodetic longitude.	Geodetic *minus* astronomical longitude.	Observers.	Instruments.	Remarks.
	1865.		m. s.	s.	m. s.	° ′ ″	″	″			
(Menomonee from Fort Howard.)	Aug. 11	39	− 1 34.42	....	....	....	....	....	S. W. Robinson, at Menomonee.	Pistor & Martins astronomical transit No. 1, clocks and chronographs at Fort Howard and Escanaba.	Fixed point at Menomonee is primary triangulation station (1874).
	12	47	1 34.35								Personal equation applied.
	14	39	1 34.44								Fixed point at Detroit in this table is east pier of Lake-Survey Observatory of 1871–1882, which is 224 feet west and 39 feet south of the intersection of the axes of Washington and Grand River avenues.
	16	33	1 34.58								Published in 1866 and 1870.
	17	27	1 34.39						C. A. Young and O. B. Wheeler, at Fort Howard.	Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	
	18	50	1 34.46								
	22	25	1 34.52								
Mean (corrected for personal equation).	....	....	− 1 34.52								
(Fort Howard from Detroit, see Chapter XXV, § 3.)	....	....	+19 51.80	−0.127*a*							
Menomonee, Mich	....	....	+18 17.28	−7.539*b*	+18 09.61	87 35 27.75	34.60	+6.85			
	1871.										
(Saint Paul from Duluth.)	June 14	....	+ 4 03.86	....	....	....	....	....	A. R. Flint, at Saint Paul.	Würdemann astronomical transit No. 1, sidereal chronometer, mean time chronometer.	Fixed point at Saint Paul is southeast corner of United States custom-house.
	15	....	4 03.66								Personal equation applied.
Weighted mean..	....	....	+ 4 03.79								Published in 1872.
(Duluth from Detroit, see Chapter XXV, § 2.)	....	....	+36 05.82	0.000*a*					Gen. C. B. Comstock, at Duluth.	Würdemann astronomical transit No. 15, sidereal clock, and chronograph.	
Saint Paul, Minn......	....	....	+40 09.61	+0.418*c*	+40 10.03	93 05 34.05					
	1868.										
(Ogdensburg from Oswego.)	July 10	4	− 4 01.81	....	....	....	....	....	Lieut. E. H. Ruffner, at Ogdensburg.	Würdemann astronomical transit No. 15, sidereal clock, and chronograph.	Fixed point at Ogdensburg is light-house.
	11	25	4 02.31								Personal equation is eliminated.
	13	23	4 02.12								Published in 1870.
Mean	....	....	− 4 02.19						O. B. Wheeler, at Oswego.	Pistor & Martins astronomical transit No. 1, sidereal clock, and chronograph.	
(Oswego from Detroit, see below.)	....	....	−26 10.52	−0.127*a*							
Ogdensburg, N. Y.....	....	....	−30 12.71	+1.857*d*	−30 10.98	75 30 18.90					

a At Detroit. *b* At Menomonee. *c* At Saint Paul. *d* At Ogdensburg.

Astronomical Longitudes, not fundamental—Continued.

Station.	Date.	Number of stars observed in common at both stations.	Observed difference of time between Stations.	Reductions to fixed points.	Difference of time between fixed point and Detroit.	Longitude of point west of Greenwich.	Seconds of geodetic longitude.	Geodetic *minus* astronomical longitude.	Observers.	Instruments.	Remarks.
	1868.		*m. s.*	*s.*	*m. s.*	° ′ ″	″	″			
(Watertown from Oswego.)	July 10	5	— 2 21.89	....	....	....	....	....	G. Y. Wisner, at Watertown.	Würdemann astronomical transit No. 1, clocks and chronographs at Oswego and Ogdensburg.	Fixed point at Watertown is north-east corner of cupola of court-house. Personal equation not eliminated. Published in 1870.
	11	28	2 21.92								
	13	25	2 21.99								
	14	14	2 21.95								
	16	11	2 21.93						O. B. Wheeler, at Oswego.	Pistor & Martins astronomical transit No. 1, sidereal clock, and chronograph.	
	17	3	2 21.89								
Mean	....	....	— 2 21.94								
(Oswego from Detroit, see below.)	....	....	—26 10.52	—0.127*a*							
Watertown, N. Y	....	....	—28 32.46	+0.476*b*	—28 32.11	75 55 01.95	54.02	—7.93			
	1860.										
Goderich, Ontario	Apr. 17	22	— 5 20.99	....	....	....	....	....	Lieut. O. M. Poe, at Goderich.	Würdemann astronomical transit No. 15, sidereal clock, and chronograph.	Fixed point at Goderich is the court-house. Personal equation found to be small, and not applied. Published in 1860.
	18	28	5 20.94								
	27	27	5 20.98						Jas. Carr, at Detroit.	Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	
	28	22	5 20.95	—0.127*a*							
Mean	....	....	— 5 20.97	—0.150*c*	— 5 21.25	81 42 44.85					
	1868.										
(Oswego from Buffalo)	Aug. 12	15	— 9 26.53	....	....	....	....	....	O. B. Wheeler, at Oswego.	Pistor & Martins astronomical transit No. 1, sidereal clock, and chronograph.	Fixed point at Oswego is primary triangulation station. Personal equation not eliminated. Published in 1870.
	13	20	9 26.62								
	16	5	9 26.67								
	17	3	9 26.47						Lieut. E. H. Ruffner, at Buffalo.	Würdemann astronomical transit No. 15, sidereal clock, and chronograph.	
	20	14	9 26.53								
	21	33	9 26.45								
Mean	....	....	— 9 26.53								
(Buffalo from Detroit, see below.)	....	....	—16 43.99	—0.127*a*							
Oswego, N. Y	....	....	—26 10.52	+1.973*d*	—26 08.67	76 30 53.55	50.40	— 3.15			

	1868.										
(Rochester from Buffalo).	Aug. 12	16	− 4 57.45	....	....	....	....	....	G. Y. Wisner, at Rochester.	Würdemann astronomical transit No. 1, clocks and chronographs at Buffalo and Oswego.	Fixed point is City Hall (statue). Personal equation not eliminated. Published in 1870.
	31	19	4 57.44								
	16	7	4 57.46								
	17	4	4 57.47								
	20	17	4 57.41						Lieut. E. H. Ruffner, at Buffalo.	Würdemann astronomical transit No. 15, sidereal clock, and chronograph.	
	21	31	4 57.50								
Mean	....	....	− 4 57.46								
(Buffalo from Detroit, see below.)	....	....	−16 43.99	− 0.127a							
Rochester, New York.	....	....	−21 41.45	− 3.261e	−21 44.84	77 36 51.00	50.69	− 0.31			
	1860.										
Grand Haven, Mich...	Nov. 12	21	+12 45.86	....	....	....	....	....	Lieut. O. M. Poe, at Detroit.	Würdemann astronomical transit No. 15, sidereal clock, and chronograph.	Fixed point at Grand Haven is light-house. "No personal equation." (J. Carr, 1861.) Published in 1861.
	13	30	12 45.84								
	1861.										
	Jan. 22	16	12 45.84	− 0.127a					James Carr, at Grand Haven.	Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	
Mean	....	....	+12 45.85	+ 1.453f	+12 47.18	86 14 51.15					
	1868.										
(Buffalo from Cleveland.)	Sept. 18	34	−11 19.81	....	....	....	....	....	Lieut. E. H. Ruffner, at Buffalo.	Würdemann astronomical transit No. 15, sidereal clock, and chronograph.	Fixed point at Buffalo is intersection of axes of Exchange and Michigan streets. Personal equation is eliminated. Published in 1870.
	21	21	11 19.42								
	23	7	11 19.73								
	29	16	11 19.60						O. B. Wheeler, at Cleveland.	Pistor & Martins astronomical transit No. 1, sidereal clock, and chronograph.	
	Oct. 3	30	11 19.93								
Mean	....	....	−11 19.73								
(Cleveland from Detroit, see below.)	....	....	− 5 24.26	− 0.127a							
Buffalo, N. Y	....	....	−16 43.99	+ 1.081g	−16 43.04	78 52 18.00	14.28	− 3.72			
	1868.										
(Dunkirk from Buffalo.)	Sept. 2	12	+ 1 49.68	....	....	....	....	....	G. Y. Wisner, at Dunkirk.	Würdemann astronomical transit No. 1, clocks and chronographs at Buffalo and Erie.	Fixed point at Dunkirk is light-house (1876). Personal equation not eliminated. Published in 1870.
	4	17	1 49.92								
	7	38	1 49.87								
	13	30	1 49.99								
Mean	....	....	+ 1 49.90						Lieut. E. H. Ruffner, at Buffalo.	Würdemann astronomical transit No. 15, sidereal clock, and chronograph.	
(Buffalo from Detroit, see above.)	....	....	−16 43.99	− 0.127a							
Dunkirk, N. Y	....	....	−14 54.09	+ 7.701h	−14 46.52	79 21 25.80	15.81	− 9.99			

a At Detroit. b At Watertown. c At Goderich. d At Oswego. e At Rochester. At Grand Haven. g At Buffalo. h At Dunkirk.

Astronomical Longitudes, not fundamental—Continued.

Station.	Date.	Number of stars observed in common at both stations.	Observed difference of time between stations.	Reductions to fixed points.	Difference of time between fixed point and Detroit.	Longitude of point west of Greenwich.	Seconds of geodetic longitude.	Geodetic *minus* astronomical longitude.	Observers.	Instruments.	Remarks.
	1868.		*m. s.*	*s*	*m. s.*	° ′ ″	″	″			
(Erie from Buffalo)....	Sept. 2	9	+ 4 53. 19	....	....	....	....	....	O. B. Wheeler, at Erie.	Pistor & Martins astronomical transit No. 1, sidereal clock, and chronograph.	Fixed point at Erie is beacon light-house.
	4	13	4 53. 13						Lieut. E. H. Ruffner, at Buffalo.	Würdemann astronomical transit No. 15, sidereal clock, and chronograph.	Personal equation not eliminated.
	7	36	4 53. 12								Published in 1870.
	13	26	4 53. 25								
Mean	....	....	+ 4 53. 17								
(Buffalo from Detroit, see above.)	....	....	−16 43. 99	−0. 127*a*							
Erie, Pa..............	....	....	−11 50. 82	−2. 160*b*	−11 53. 11	80 04 46. 95	40. 21	− 6. 74			
	1868.										
Monroe, Mich.........	Nov. 2	40	+ 1 21. 90	....	....	....	....	....	G. Y. Wisner, at Monroe.	Würdemann astronomical transit No. 1, clocks and chronographs at Detroit and Toledo.	Fixed point at Monroe is steeple of court-house. This court-house has since been destroyed by fire. The reduction from the court-house to steeple of First Presbyterian Church is +0ˢ. 109.
	13	41	1 21. 74						Lieut. E. H. Ruffner, at Detroit.	Würdemann astronomical transit No. 15, sidereal clock, and chronograph.	Personal equation not eliminated.
	14	10	1 21. 62	−0. 127*a*							
Mean	....	....	+ 1 21. 80	+1. 174*c*	+ 1 22. 85	83 23 46. 35	49. 77	+3. 42			
	1864.										
Chicago, Ill..........	Aug. 8	27	+18 17. 34	....	....	....	....	....	Gen. W. F. Raynolds and S. W. Robinson, at Detroit.	Troughton & Simms astronomical transit, 43-inch focal length, sidereal clock and chronograph.	Fixed point at Chicago is iron light-house. On August 8 and 10 General Raynolds observed at Detroit, and on August 13 Assistant Robinson.
	10	23	18 17. 50						O. B. Wheeler, at Chicago.	Würdemann astronomical transit No. 1, sidereal clocks and chronographs at Detroit and Fort Howard.	Personal equation as obtained from data in Chapter XXV, § 3, is applied in obtaining the mean.
	13	23	18 17. 45	−0. 127*a*							Gen. R. — Assistant W. =+0ˢ. 12.
Mean (corrected for personal equation).	....	....	+18 17. 52	−2. 370*d*	+18 15. 02	87 36 48. 90	52. 20	+3. 30			Assist. R. — Assistant W. =+0ˢ. 03.

(Ashtabula from Buffalo.)	1868. Sept. 18	31	+ 7 42.60	....	....	....	....	....	G. Y. Wisner, at Ashtabula.	Würdemann astronomical transit No. 1, clocks and chronographs at Buffalo and Cleveland.	Fixed point at Ashtabula is southwest corner of passenger depot. Personal equation not eliminated. Published in 1870.
	21	23	7 42.41								
	23	8	7 42.48								
	29	21	7 42.31								
	Oct. 3	28	7 42.48						Lieut. E. H. Ruffner, at Buffalo.	Würdemann astronomical transit No. 15, sidereal clock, and chronograph.	
Mean	....	....	+ 7 42.47								
(Buffalo from Detroit, see above.)	....	....	−16 43.99	−0.127*a*							
Ashtabula, Ohio	....	....	− 9 01.52	−0.026*e*	− 9 01.67	80 47 38.55					
Cleveland, Ohio	1868. Oct. 11	26	− 5 24.35	....	....	....	....	....	O. B. Wheeler, at Cleveland.	Pistor & Martins astronomical transit No. 1 sidereal clock and chronograph.	Fixed point at Cleveland is lighthouse on high bank in the city. Personal equation not eliminated. Published in 1870.
	12	30	5 24.08								
	17	33	5 24.20								
	25	19	5 24.55	−0.127*a*					Lieut. E. H. Ruffner, at Detroit.	Würdemann astronomical transit No. 15, sidereal clock, and chronograph.	
Mean	....	....	− 5 24.26	+1.023*f*	− 5 23.36	81 42 13.20	09.58	−3.62			
Sandusky, Ohio	1868. Oct. 11	26	− 1 18.71	....	....	....	....	....	G. Y. Wisner, at Sandusky.	Würdemann astronomical transit No. 1, clocks and chronographs at Detroit and Cleveland.	Fixed point at Sandusky is northeast corner of United States custom-house. Personal equation not eliminated.
	12	28	1 18.75								
	17	34	1 18.51								
	25	23	1 18.65	−0.127*a*							
Mean	....	....	− 1 18.65	−2.277*g*	− 1 21.05	82 42 47.85	43.61	−4.24	Lieut. E. H. Ruffner, at Detroit.	Würdemann astronomical transit No. 15, sidereal clock, and chronograph.	
Hudson, Ohio	1859. Jan. 18	6	− 6 27.06	....	....	....	....	....	Prof. C. A. Young, at Hudson.	Würdemann astronomical transit No. 1, sidereal clock, and Morse register.	Fixed point at Hudson is Western Reserve College Observatory. Personal equation found to be small, and not applied. Published in 1859.
	19	9	6 26.97								
	24	15	6 26.97								
	26	11	6 26.99						Lieut. C. N. Turnbull, at Detroit.	Würdemann astronomical transit No. 15, sidereal clock, and chronograph.	
	31	9	6 26.91								
	Feb. 7	18	6 27.04	−0.127*a*							
Mean	....	....	− 6 26.99	0.000*h*	− 6 27.12	81 26 16.80					
Cairo, Ill	1876. Dec. 5	....	+24 25.65	....	....	....	....	....	Lieut. D. W. Lockwood, at Cairo.	Würdemann astronomical transit No. 1, sidereal break-circuit chronometer, and chronograph.	Fixed point at Cairo is triangulation station Defiance. Personal equation applied. Published in 1877.
	19	....	24 25.79	0.000*a*							
Mean, with personal equation applied.	....	....	+24 25.87	−0.035*i*	+24 25.84	89 09 31.20			Capt. H. M. Adams, at Detroit.	Buff & Berger astronomical transit No. 2, sidereal clock, and chronograph.	

a At Detroit. *b* At Erie. *c* At Monroe. *d* At Chicago. *e* At Ashtabula. *f* At Cleveland. *g* At Sandusky. *h* At Hudson. *i* At Cairo.

Astronomical Longitudes, not fundamental—Continued.

Station.	Date.	Number of stars observed in common at both stations.	Observed difference of time between stations.	Reductions to fixed points.	Difference of time between fixed point and Detroit.	Longitude of point west of Greenwich.	Seconds of geodetic longitude	Geodetic *minus* astronomical longitude.	Observers.	Instruments.	Remarks.
	1877.		*m. s.*	*s*	*m. s.*	° ′ ″	″	″			
Memphis, Tenn	June 22	....	+28 00.91	0.000*a*	....	....	....	....	Lieut. C. F. Powell, at Memphis.	Würdemann astronomical transit No. 1, break-circuit chronometer, and chronograph.	Fixed point is triangulation station, Memphis. Personal equation applied. Published in 1878.
	23	....	28 00.90								
Mean, with personal equation applied.	....	....	+28 01.03	+0.083*b*	+28 01.11	90 03 20.25			Capt. H. M. Adams, at Detroit.	Buff & Berger astronomical transit No. 2, sidereal clock, and chronograph.	

a At Detroit. *b* At Memphis.

§ **3.** At the request of the Geological Survey of Wisconsin and of the Governor of Michigan, the latitudes and longitudes of many points in these States were determined between 1873 and 1878. The latitudes (given in the table of non-fundamental latitudes) were obtained by two or three nights' work with the zenith-telescope, and the longitudes by two or three nights' exchange of telegraphic signals with Detroit personal equation being also determined.

The following table gives the longitudes of places so determined, the first column giving the name of the station; the second, the dates of observations; the third, the observed difference of time at the station from Detroit for each night of observation; the fourth, the mean difference with personal equation applied; the fifth, the reductions to fixed points at each station; the sixth, the difference of time between the fixed points at Detroit and the station at which observations were made; the seventh, the longitude of the station west of Greenwich, in arc, obtained by applying the longitude of Detroit ($5^h\ 32^m\ 12^s.24$) to the result in the sixth column; the eighth, the names of the observers; the ninth, the instruments used; and the tenth, data concerning location of fixed point:

Astronomical Longitudes determined in aid of State Surveys.

Station.	Date.	Observed difference of time from Detroit for each night.	Mean observed difference, with personal equation applied.	Reductions to fixed points.	Difference of time between Detroit and fixed point.	Longitude of fixed point west of Greenwich.	Observers.	Instruments.	Remarks.
		m. s.	m. s.	s.	m. s.	° ′ ″			
Red Wing, Minn......	1878. Oct. 24 31	+37 55.57 37 55.64	.... +37 55.71	0.000*a* −0.640*b*	.... +37 55.07	.... 92 31 49.65	Capt. H. M. Adams and A. R. Flint, at Detroit. Lieut. C. F. Powell, at Red Wing.	Buff & Berger astronomical transit No. 2, sidereal clock, and chronograph. Würdemann astronomical transit No. 1, a sidereal and a mean-time chronometer.	Fixed point at Detroit for this table is east pier of Lake-Survey Observatory of 1871–'82. Fixed point at Red Wing is spire of Catholic Church. Published in 1879.
Valley Junction, Wis..	1873. Aug. 6 8	+29 26.04 29 26.16	.... +29 26.38	0.000*a* +5.16 *c*	.... +29 31.54	.... 90 25 56.70	S. W. Robinson, at Valley Junction. O. B. Wheeler, at Detroit.	Pistor & Martins astronomical transit No. 1, and mean-time chronometer. Troughton & Simms astronomical transit with 43-inch telescope, sidereal clock, and chronograph.	Fixed point at Valley Junction is section corner on 4th principal meridian at northeast corner of section 1, township 18 north, range 1 west. Published in 1874.
Newaygo, Mich.......	1875. Nov. 8 18	+11 00.20 11 00.08	.... +11 00.01	0.000*a* +0.037*d*	.... +11 00.05	.... 85 48 04.35	Lieut. T. N. Bailey, at Newaygo. Lieut. D. W. Lockwood, at Detroit.	Pistor & Martins astronomical transit No. 2, a sidereal break-circuit and a mean-time chronometer. Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	Fixed point at Newaygo is intersection of axes of Wood and State streets. Published in 1876.
Saginaw, Mich........	1876. Sept. 12 16	+ 3 38.73 3 38.75	.... + 3 38.97	0.000*a* +0.149*e*	.... + 3 39.12	.... 83 57 50.4	Lieut. D. W. Lockwood, at Saginaw. Capt. H. M. Adams, at Detroit.	Würdemann astronomical transit No. 1, sidereal break-circuit chronometer, and chronograph. Buff & Berger astronomical transit No. 2, sidereal clock, and chronograph.	Fixed point at Saginaw is east corner of court-house. Published in 1877.
Saint Louis, Mich.....	1876. Oct. 11 12	+ 6 12.40 6 12.29	.... + 6 12.57	0.000*a* +0.833*f*	.... + 6 13.40	.... 84 36 24.6	Lieut. D. W. Lockwood, at Saint Louis. Capt. H. M. Adams, at Detroit.	Würdemann astronomical transit No. 1, sidereal break-circuit chronometer, and chronograph. Buff & Berger astronomical transit No. 2, sidereal clock, and chronograph.	Fixed point at Saint Louis is intersection of axes of Washington and Mill streets. Published in 1877.

a At Detroit. *b* At Red Wing. *c* At Valley Junction. *d* At Newaygo. *e* At Saginaw. *f* At Saint Louis.

Astronomical Longitudes determined in aid of State Surveys—Continued.

Station.	Date.	Observed difference of time from Detroit for each night.	Mean observed difference, with personal equation applied.	Reductions to fixed points.	Difference of time between Detroit and fixed point.	Longitude of fixed point west of Greenwich.	Observers.	Instruments.	Remarks.
		m. s.	m. s.	s	m. s.	° ′ ″			
	1875.								
Stanton, Mich	Oct. 13	+ 8 07.64	....	0.000*a*	....	....	Lieut. P. M. Price, at Stanton.	Würdemann astronomical transit No. 1, a sidereal break-circuit and a mean-time chronometer.	Fixed point at Stanton is intersection of axes of Main and Camburn streets. Published in 1876.
	20	8 07.70	+ 8 07.68	−0.423*b*	+ 8 07.26	85 04 52.50	Lieut. D. W. Lockwood, at Detroit.	Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	
	1874.								
Lapeer, Mich	June 24	+ 1 03.32	....	0.000*a*	....	....	Lieut. C. F. Powell, at Lapeer.	Pistor & Martins astronomical transit No. 1, and mean-time chronometer.	Fixed point at Lapeer is intersection of axes of Calhoun and Franklin streets. Published in 1875.
	26	1 03.52	+ 1 03.42	+0.101*c*	+ 1 03.32	83 18 53.40	Lieut. E. Maguire, at Detroit.	Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	
	1874.								
Flint, Mich	July 14	+ 2 32.18	....	0.000*a*	....	....	Lieut. C. F. Powell, at Flint.	Pistor & Martins astronomical transit No. 1, and mean-time chronometer.	Fixed point at Flint is intersection of axes of Keasley and East streets. Published in 1875.
	16	2 32.24	+ 2 32.21	−0.024*d*	+ 2 32.19	83 41 06.45	Lieut. E. Maguire, at Detroit.	Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	
	1874.								
Saint Johns, Mich	Aug. 1	+ 6 02.24	....		....	....	Lieut. C. F. Powell, at Saint Johns.	Pistor & Martins astronomical transit No. 1, and mean-time chronometer.	Fixed point at Saint Johns is intersection of axes of Clinton avenue and State street. Published in 1875.
	4	6 02.24		0.000*a*					
	8	6 02.26	+ 6 02.25	−0.055*e*	+ 6 02.20	84 33 36.60	Lieut. E. Maguire, at Detroit.	Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	
	1874.								
Ionia, Mich	Aug. 15	+ 8 03.10	....	0.000*a*	....	....	Lieut. C. F. Powell, at Ionia.	Pistor & Martins astronomical transit No. 1, and mean-time chronometer.	Fixed point at Ionia is intersection of axes of Main and First streets. Published in 1875.
	25	8 02.98	+ 8 03.04	+0.266*f*	+ 8 03.31	85 03 53.25	Lieut. E. Maguire, at Detroit.	Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	
	1874.								
Corunna, Mich	Aug. 4	+ 4 15.69	....	0.000*a*	....	....	Capt. H. M. Adams, at Corunna.	Würdemann astronomical transit No. 15, sidereal break-circuit and mean-time chronometers.	Fixed point at Corunna is intersection of axes of Shiawassee avenue and Flint road. Published in 1875.
	8	4 15.50	+ 4 16.06	−0.110*g*	+ 4 15.95	84 07 02.85	Lieut. E. Maguire, at Detroit.	Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	

Grand Rapids, Mich ..	1874. Sept. 9 12 29	+10 28.04 10 27.85 10 27.72	.... +10 27.87	.... .000*a* +0.306*h*	.... +10 28.17	.... 85 40 06.15	Lieut. C. F. Powell, at Grand Rapids. Lieut. E. Maguire, at Detroit.	Pistor & Martins astronomical transit No. 1, and mean-time chronometer. Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	Fixed point at Grand Rapids is quarter-section corner on west side of section 30, township 7 north, range 11 west, being at intersection of Fulton and Division streets. Published in 1875.
Lansing, Mich	1875. July 13 19 26	+ 6 01.22 6 01.36 6 01.14	.... + 6 01.07	.... 0.000*a* −0.180*i*	.... + 6 00.89	.... 84 33 16.95	Lieut. T. N. Bailey, at Lansing. Lieut. D. W. Lockwood, at Detroit.	Pistor & Martins astronomical transit No. 2, a sidereal break-circuit and a mean-time chronometer. Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	Fixed point at Lansing is intersection of axes of Michigan and Capitol avenues. Published in 1876.
Hastings, Mich	1875. July 30 Aug. 7 12	+8 57.25 8 56.94 8 57.12	.... +8 56.98	.... 0.000*a* −0.047*j*	.... +8 56.93	.... 85 17 17.55	Lieut. T. N. Bailey, at Hastings. Lieut. D. W. Lockwood, at Detroit.	Pistor and Martins astronomical transit No. 2, a sidereal break-circuit and a mean-time chronometer. Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	Fixed point at Hastings is intersection of axes of State and Church streets. Published in 1876.
Pontiac, Mich	1874. May 26 June 11 17	+0 56.96 0 57.23 0 57.25	.... +0 57.15	.... 0.000*a* −0.354*k*	.... +0 56.80	.... 83 17 15.60	Lieut. C. F. Powell, at Pontiac. Lieut. E. Maguire, at Detroit.	Würdemann astronomical transit No. 15, mean-time chronometer. Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	Fixed point at Pontiac is southeast corner of section 29, township 3 north, range 10 east. Published in 1875.
Howell, Mich	1875. June 18 19	+3 30.83 3 30.84	.... +3 30.71	0.000*a* +0.435*l*	.... +3 31.15	.... 83 55 50.85	Lieut. T. N. Bailey, at Howell. Lieut. D. W. Lockwood, at Detroit.	Pistor & Martins astronomical transit No. 2, a sidereal break-circuit and a mean-time chronometer Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	Fixed point at Howell is intersection of axes of Higgins and Monroe streets. Published in 1876.
Charlotte, Mich.......	1875. July 20 24	+7 07.82 7 07.88	.... +7 07.86	0.000*a* −0.017*m*	.... +7 07.84	.... 84 50 01.20	Lieut. P. M. Price, at Charlotte. Lieut. D. W. Lockwood, at Detroit.	Würdemann astronomical transit No. 15, a sidereal break-circuit and a mean-time chronometer. Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	Fixed point at Charlotte is intersection of axes of Oliver and Stoddard streets. Published in 1876.

a At Detroit, *b* At Stanton. *c* At Lapeer. *d* At Flint. *e* At Saint Johns. *f* At Ionia. *g* At Corunna. *h* At Grand Rapids. *i* At Lansing. *j* At Hastings. *k* At Pontiac. *l* At Howell. *m* At Charlotte.

Astronomical Longitudes determined in aid of State Surveys—Continued.

Stations.	Date.	Observed difference of time from Detroit for each night.	Mean observed difference, with personal equation applied.	Reductions to fixed points.	Difference of time between Detroit and fixed point.	Longitude of fixed point west of Greenwich.	Observers.	Instruments.	Remarks.
		m. s.	m. s.	s.	m. s.	° ′ ″			
Allegan, Mich	1875. Sept. 23	+11 12.43	....	0.000*a*	....	....	Lieut. T. N. Bailey, at Allegan.	Pistor & Martins astronomical transit No. 2, a sidereal break-circuit and a mean-time chronometer.	Fixed point at Allegan is northwest corner of county building in public square.
	24	11 12 52	+11 12.35	—0.019*b*	+11 12.33	85 51 08.55	Lieut. D. W. Lockwood, at Detroit.	Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	Published in 1876.
Galena, Ill	1873. July 11	+29 31.09	....	....	....	....	S. W. Robinson, at Galena.	Pistor & Martins astronomical transit No. 1 and mean-time chronometer.	Fixed point at Galena is section corner at intersection of 4th principal meridian with State line between Wisconsin and Illinois.
	12	29 31.00							
	21	29 30.91		0.000*a*			O. B. Wheeler, at Detroit.	Troughton & Simms astronomical transit with 43-inch telescope, sidereal clock, and chronograph.	Published in 1874.
	22	29 30.83	+29 31.23	—1.228*c*	+29 30.00	90 25 33.60			
Kalamazoo, Mich	1875. Aug. 30	+10 08.12	....	0.000*a*	....	....	Lieut. P. M. Price, at Kalamazoo.	Würdemann astronomical transit No. 15, a sidereal break-circuit and a mean-time chronometer.	Fixed point at Kalamazoo is intersection of axes of South and Park streets.
	Aug. 31	10 08.14	+10 08.13	+0.413*d*	+10 08.54	85 35 11.70	Lieut. D. W. Lockwood, at Detroit.	Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	Published in 1876.
Marshall, Mich	1875. Aug. 13	+7 39.05	....	0.000*a*	....	....	Lieut. P. M. Price, at Marshall.	Würdemann astronomical transit No. 15, a sidereal break-circuit and a mean-time chronometer.	Fixed point at Marshall is intersection of axes of Kalamazoo avenue and Mansion street.
	24	7 39.00	+7 39.03	+0.003*e*	+7 39.03	84 57 49.05	Lieut. D. W. Lockwood, at Detroit.	Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	Published in 1876.
Jackson, Mich	1875. Aug. 28	+5 25.92	....	0.000*a*	....	....	Lieut. T. N. Bailey, at Jackson.	Pistor & Martins astronomical transit No. 2, a sidereal break-circuit and a mean-time chronometer.	Fixed point at Jackson is intersection of axes of Main and Jackson streets.
	30	5 26.10	+5 25.88	—0.127*f*	+5 25.75	84 24 29.85	Lieut. D. W. Lockwood, at Detroit.	Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	Published in 1876.

Paw Paw, Mich.......	1875. Sept. 23	+11 20.06	....	0.000a	....	....	Lieut. P. M. Price, at Paw Paw.	Würdemann astronomical transit No. 15, a sidereal break-circuit and a mean-time chronometer.	Fixed point at Paw Paw is intersection of axes of Main and Van Buren streets.
	24	11 20.11	+11 20.09	−0.106g	+11 10.98	85 53 03.30	Lieut. D. W. Lockwood, at Detroit.	Würdemann astronomical transit No. 1, sidereal clock, and chronograph.	Published in 1876.
Rock Island, Ill.......	1878. Sept. 26	+30 02.34	....	0.000a	....	....	Capt. H. M. Adams, at Detroit.	Buff & Berger astronomical transit No. 2, sidereal clock, and chronograph.	Fixed point at Rock Island is south-east corner of guard-house.
	30	30 02.30	+30 02.43	+0.393h	+30 02.82	90 33 45.90	Lieut. C. F. Powell, at Rock Island.	Würdemann astronomical transit No. 1, a sidereal and a mean-time chronometer.	Published in 1879.
Louisiana, Mo	1878. Nov. 13	+32 00.12	....	0.000a	....	....	Capt. H. M. Adams, at Detroit.	Buff & Berger astronomical transit No. 2, sidereal clock, and chronograph.	Fixed point at Louisiana is north-east corner of high school building.
	19	32 00.02	+32 00.18	+0.125i	+32 00.31	91 03 08.25	Lieut. C. F. Powell, at Louisiana.	Würdemann astronomical transit No. 1, a sidereal and a mean-time chronometer.	Published in 1879.

a At Detroit. b At Allegan. c At Galena. d At Kalamazoo. e At Marshall. f At Jackson. g At Paw Paw. h At Rock Island. i At Louisiana.

§ 4. The Lake Survey has also co-operated from time to time in the telegraphic determination of longitudes of points in the Western Territories.

The method followed in this work embraced a comparison of local times by exchange of signals made usually by hand at both ends of the line, the signals of the field observer being recorded on the Detroit chronograph, and those of the Detroit observer being recorded by means of the sounds of the field-relay ticks. Signals were exchanged usually on two nights. Personal equation was not generally eliminated. Details as to observers, instruments, &c., are given in the following table of results, arranged in the same order as the preceding table:

Astronomical Longitudes of points in Western States and Territories.

Station.	Date.	Observed difference of time from Detroit for each night.	Mean observed difference of time from Detroit.	Reductions to fixed points.	Difference of time between Detroit and fixed point.	Longitude of fixed point west of Greenwich.	Observers.	Instruments.	Remarks.
		h. m. s.	h. m. s.	s.	h. m. s	° ′ ″			
Deadwood, Dak.	1877. Aug. 12	+1 22 44.9	+1 22 44.9	0.00a	....	....	Capt. W. S. Stanton, at Deadwood.	Sextant and chronometer	Fixed point at Detroit in this table is east pier of Lake-Survey Observatory of 1871–'82.
				−3.90b	+1 22 41.0	103 43 18.6	Lieut. P. M. Price, at Detroit.	Buff & Berger astronomical transit No. 2, sidereal clock, and chronograph.	Fixed point at Deadwood is post-office. Personal eqnation not eliminated. Published in 1878.
Fort Fetterman. Wyo.	1876. July 22	+1 29 45.59	....	0.00a	....	....	Capt. W. S. Stanton, at Fort Fetterman.	Sextant and chronometer	At military post, Fort Fetterman. Personal equation not eliminated. Published in 1877.
	28	1 29 43.65	+1 29 44.6	0.00c	+1 29 44.6	105 29 12.6	Capt. H. M. Adams, at Detroit.	Buff & Berger astronomical transit No. 2, sidereal clock, and chronograph.	
Camp Robinson, Nebr.	1877. Oct. 26	+1 21 39.7	+1 21 39.7	0.00a	....	....	Capt. W. S. Stanton, at Camp Robinson.	Sextant and chronometer	Fixed point at military post, Camp Robinson, is flag-staff.
				−0.98d	+1 21 38.7	103 27 44.1	Lieut. P. M. Price, at Detroit.	Buff & Berger astronomical transit No. 2, sidereal clock, and chronograph.	Personal equation not eliminated. Published in 1878.
Fort Laramie, Wyo.	1877. July 27	+1 26 02.8	....	0.00a	....	....	Capt. W. S. Stanton, at Fort Laramie.	Sextant July 27, astronomical transit September 27, and chronometer.	Fixed point at military post, Fort Laramie, is flag-staff.
	Sept. 27	1 26 03.6	+1 26 03.2	−2.00e	+1 26 01.2	104 33 21.6	Lieut. P. M. Price. at Detroit.	Buff & Berger astronomical transit No. 2, sidereal clock, and chronograph.	Personal equation not applied. Published in 1878.
Fort McDermit, Nev.	1880. Nov. 22	+2 18 17.65	....	....	....	....	Capt. W. A. Jones, at Fort McDermit.	Würdemann transit No. 22, and sidereal chronometer.	At military post, Fort McDermit. Personal equation not eliminated. Published in 1881.
	24	2 18 18.31		0.00a					
	25	2 18 17.87	+2 18 17.94	0.00f	+2 18 17.94	117 37 32.7	A. R. Flint, at Detroit.	Buff & Berger astronomical transit No. 2, sidereal clock, and chronograph.	
Ogden, Utah	1873. Sept. 29	+1 55 47.25g	....	....	....	....	Dr. F. Kampf, at Ogden.	Würdemann astronomical transit No. 28, sidereal break-circuit chronometer, and chronograph.	Fixed point at Ogden is east pier in transit room of observatory of Lieut. G. M. Wheeler.
	30	1 55 47.43							
	Oct. 1	1 55 47.29							
	Oct. 9	1 55 47.52		0.00a			O. B. Wheeler, at Detroit.	Troughton & Simms astronomical transit with 43-inch telescope, sidereal clock, and chronograph.	Personal equation applied. Published in 1874.
	10	1 55 47.41	+1 55 47.50h	0.00i	+1 55 47.50	111 59 56.10			

Fort Leavenworth, Kans.	1871. Sept. 1j	+ 47 27.29	....		....	....	Lieut. E. H. Ruffner, at Fort Leavenworth.	Würdemann astronomical transit No. 12, stars recorded at Detroit by clock and chronograph.	Fixed point at Fort Leavenworth is observatory. Personal equation observed for and found to be zero. Wave-time not eliminated. Published in Lieut. E. H. Ruffner's report to the Chief of Engineers for 1872.
	2	47 27.19							
	7	47 26.94		0.00a					
	20	47 27.09 +	47 27.11	0.00k +	47 27.11	94 54 50.25	O. B. Wheeler, at Detroit.	Troughton & Simms astronomical transit with 43-inch telescope, sidereal clock, and chronograph.	
Fort Richardson, Tex.	1876. May 10	+1 00 26.55	....	0.00a	....	....	Lieut. Wm. Hoffman, at Fort Richardson.	Sextant and chronometer	At military post, Fort Richardson. Personal equation not applied. Published in 1877.
	12	1 00 26.48	+1 00 26.5	0.00l	+1 00 26.5	98 09 41.1	O. B. Wheeler, at Detroit.	Buff & Berger astronomical transit No. 2, sidereal clock, and chronograph.	
Fort Griffin, Tex......	1876. June 5	+1 04 43.21	....	0.00a	....	....	Lieut. Wm. Hoffman, at Fort Griffin.	Sextant and chronometer	At military post, Fort Griffin. Personal equation not eliminated. Published in 1877.
	6	1 04 42.84	+1 04 43.0	0.00m	+1 04 43.0	99 13 48.6	A. R. Flint, at Detroit.	Buff & Berger astronomical transit No. 2, sidereal clock, and chronograph.	
Fort Concho, Tex.....	1876. July 31	+1 09 30.6	+1 09 30.6	0.00a	....	....	Lieut. Wm. Hoffman, at Fort Concho.	Sextant and chronometer	At military post, Fort Concho. Personal equation not eliminated. Published in 1877.
				0.00n	+1 09 30.6	100 25 42.6	Capt. H. M. Adams, at Detroit.	Buff & Berger astronomical transit No. 2, sidereal clock, and chronograph.	
Fort Stockton, Tex ...	1876. July 17	+1 19 18.08	....	0.00a	....	....	Lieut. Wm. Hoffman, at Fort Stockton.	Sextant and chronometer	At military post, Fort Stockton. Personal equation not eliminated. Published in 1877.
	22	1 19 16.98	+1 19 17.5	0.00o	+1 19 17.5	102 52 26.1	Capt. H. M. Adams, at Detroit.	Buff & Berger astronomical transit No. 2, sidereal clock, and chronograph.	
Fort McKavett, Tex..	1869. Aug. 29	+1 08 13.3	+1 08 13.3	0.00a	....	....	Lieut. Wm. Hoffman, at Fort McKavett.	Sextant and chronometer	At military post, Fort McKavett. Personal equation not applied. Published in 1877.
				0.00p	+1 08 13.3	100 06 23.1	Capt. H. M. Adams, at Detroit.	Buff & Berger astronomical transit No. 2, sidereal clock, and chronograph.	

a At Detroit.
b At Deadwood.
c At Fort Fetterman.
d At Camp Robinson.
e At Fort Laramie.
f At Fort McDermit.
g Result for September 29 is given half weight in obtaining mean.
h Personal equation is applied in mean.
i At Ogden.
j The number of stars observed in common at the two stations was 12, 24, 21, 19, for the four nights respectively.
k At Fort Leavenworth.
l At Fort Richardson.
m At Fort Griffin.
n At Fort Concho.
o At Fort Stockton.
p At Fort McKavett.

§ **5.** For convenience of reference, the results for astronomical latitudes and longitudes given in this chapter and in Chapters XXIII and XXV will be collected in a single table arranged in the order of latitudes. All longitudes depend on the longitude of the east pier of the Lake-Survey Observatory of 1871 at Detroit, given in Chapter XXV, § 1.

Table of Astronomical Latitudes and Longitudes.

Station.	Latitude.	Longitude.	Details given on page—	Point of reference.
	° ′ ″	° ′ ″		
St. Ignace, Ontario	48 47 28.65	....	622	Triangulation station.
Tip Top, Ontario	48 16 25.78	....	733	Do.
Isle Royale, Mich	48 07 55.22	....	733	Do.
Farquhar's Knob, Minn	47 52 35.27	....	733	Do.
Michipicoten, Ontario	47 45 20.58	....	733	Do.
Gargantua, Ontario	47 34 38.89	....	734	Do.
Copper Harbor (Fort Wilkins), Mich	47 28 03.15	....	734	Stone post marked S.
Vulcan, Mich	47 26 44.58	....	625	Triangulation station.
Mount Houghton, Mich	47 24 15.83	....	734	Do.
Sawteeth East, Minn	47 23 09.00	....	734	Do.
Wheal Kate, Mich	47 04 18.12	....	734	Do.
Outer Island, Wis	47 04 14.51	....	735	Do.
Portage Entry, Mich	46 58 47.72	....	735	Stone post marked S.
Crebassa, Mich	46 58 41.21	....	735	Light-house.
Buchanan, Minn	46 56 24.45	....	735	Triangulation station.
Huron Mountains, Mich	46 52 53.07	....	626	Do.
South Base, Keweenaw Point, Mich	46 52 22.35	....	623	Do.
Porcupine Mountains, Mich	46 47 03.80	....	735	Do.
Whitefish Point, Mich	46 46 17.62	....	736	Light-house.
North Base, Minnesota Point, Minn	46 45 28.32	92 04 33.00	625, 717	Triangulation station.
Brulé River, Wis	46 45 20.11	....	736	Do.
South Base, Minnesota Point, Minn	46 42 51.42	....	736	Do.
Aminicon, Wis	46 41 36.32	....	736	Do.
Grand Island, Mich	46 33 39.77	....	736	Light-house.
Thone's Hill, Mich	....	87 26 49.95	723	Stone post.
Marquette, Mich	46 32 54.53	....	737	Light-house.
Sault Ste. Marie, Mich	46 30 10.29	....	737	Do.
Iroquois Point, Mich	46 29 04.84	....	737	Do.
Escanaba, Mich	45 44 35.05	....	737	Do.
Ford River, Mich	45 41 05.34	87 06 01.35	626, 722	Triangulation station.
Burnt Bluff, Mich	45 41 03.88	....	737	Do.
Beaver Island, Mich	45 34 28.78	....	738	Light-house.
Cedar River, Mich	45 25 43.13	....	738	Triangulation station.
Rock Island, Wis	45 25 35.49	....	738	Light-house.
Boyer's Bluff, Wis	45 25 09.30	....	738	Triangulation station.
Door Bluff, Wis	45 17 45.69	....	738	Do.
Menomonee, Mich. (near Marinette, Wis.)	45 05 12.78	87 35 27.75	739, 753	Do.
Green Island, Wis	45 03 24.80	....	739	Light-house.
Thunder Bay Island, Mich	45 02 17.27	....	739	Do.
Saint Paul, Minn	44 56 42.96	93 05 34.05	739, 753	Custom-house.
Ogdensburg, N. Y	44 41 53.19	75 30 18.90	739, 753	Light-house.
Clay Banks, Door County, Wis	44 41 19.45	....	740	Azimuth station.
Frankfort, Mich	44 38 53.06	....	740	Do.
Red Wing, Minn	44 33 44.64	92 31 49.65	740, 760	Spire of Catholic church.
Fort Howard, Wis	44 30 30.28	88 02 34.05	627, 718	Triangulation station.
Kewaunee, Wis	44 25 41.12	....	740	Azimuth station.
Deadwood, Dak	....	103 43 18.6	766	Post-office.
Manistee, Mich	44 16 45.53	....	741	Azimuth station.
Valley Junction, Wis	44 14 57.24	....	741	Southwest corner of section 31, township 21 north, range 3 east.
Valley Junction, Wis	....	90 25 56.70	760	Northeast corner of section 1, township 18 north, range 1 west.

Table of Astronomical Latitudes and Longitudes—Continued.

Station.	Latitude.	Longitude.	Details given on page—	Point of reference.
	° ′ ″	° ′ ″		
Rawley's Point, Wis	44 12 04.46		741	Azimuth station.
Big Pointe au Sable, Mich	44 03 29.29		741	Light-house.
Watertown, N. Y	43 58 32.88	75 55 01.95	742, 754	Court-house.
Point Creek, Wis	43 57 58.85		742	Azimuth station.
North Sheboygan, Wis	43 45 50.36		742	Light-house.
Goderich, Ontario	43 44 36.33	81 42 44.85	742, 754	Court-house.
Mannsville, N. Y		76 02 26.70	729	Triangulation station.
South Sheboygan, Wis	43 41 01.28		743	Azimuth station.
North Base, Sandy Creek, N. Y	43 40 41.52		638	Triangulation station.
Clay Banks, Mich	43 30 23.01		743	Azimuth station.
Ada 1, Wis	43 29 18.70		743	Do.
Minnesota Junction, Wis	43 28 31.82		628	Triangulation station.
Oswego, N. Y	43 26 39.57	76 30 53.55	743, 754	Do.
Newaygo, Mich	43 25 15.45	85 48 04.35	743, 760	Intersection of Wood and State streets.
Saginaw, Mich	43 25 05.72	83 57 50.4	743, 760	Court-house.
Saint Louis, Mich	43 24 32.98	84 36 24.6	744, 760	Intersection of Washington and Mill streets.
Duck Lake, Mich	43 20 03.07		744	Azimuth station.
Stanton, Mich	43 17 30.69	85 04 52.50	744, 761	Intersection of Main and Camburn streets.
Fox Point, Wis	43 10 13.26		744	Azimuth station.
Rochester, N. Y	43 09 22.44	77 36 51.00	744, 755	City Hall.
Lapeer, Mich	43 03 07.26	83 18 53.40	745, 761	Intersection of Calhoun and Franklin streets.
Flint, Mich	43 01 11.71	83 41 06.45	745, 761	Intersection of Keasley and East streets.
Fort Gratiot, Mich	43 00 21.80		745	Light-house
Tonawanda, N. Y	43 00 07.83	78 53 21.15	637, 729	Triangulation station.
Saint Johns, Mich	43 00 02.39	84 33 36.60	745, 761	Intersection of Clinton and State streets.
Grand Haven, Mich		86 14 51.15	755	Light-house.
Ionia, Mich	42 58 51.39	85 03 53.25	745, 761	Intersection of Main and First streets.
Corunna, Mich	42 58 37.92	84 07 02.85	745, 761	Intersection of Shiawassee avenue and Flint road.
South Twin Sisters, Mich	42 58 34.88		746	Azimuth station.
Grand Rapids, Mich	42 57 47.04	85 40 06.15	746, 761	¼-section corner on west side of section 30, township 7 north, range 11 west.
Buffalo, N. Y	42 52 44.66	78 52 18.00	746, 755	Intersection of Exchange and Michigan streets.
Fort Fetterman, Wyo		105 29 12.6	766	At military post.
Lansing, Mich	42 43 55.79	84 33 16.95	746, 762	Intersection of Michigan and Capitol avenues.
Camp Robinson, Nebr		103 27 44.1	766	Flag-staff at military post.
Hastings, Mich	42 38 54.13	85 17 17.55	747, 762	Intersection of State and Church streets.
Douglas, Mich	42 38 11.62		747	Azimuth station.
Pontiac, Mich	42 38 06.95	83 17 15.60	747, 762	Southeast corner section 29, township 3 north, range 10 east.
Howell, Mich	42 36 11.18	83 55 50.85	747, 762	Intersection of Higgins and Monroe streets.
Charlotte, Mich	42 33 58.96	84 50 01.20	747, 762	Intersection of Oliver and Stoddard streets.
Allegan, Mich	42 31 44.74	85 51 08.55	748, 763	Northwest corner of county building.
Galena, Ill	42 30 26.10	90 25 33.60	748, 763	Intersection of 4th principal meridian with State line.

Table of Astronomical Latitudes and Longitudes—Continued.

Station.	Latitude.	Longitude.	Details given on page—	Point of reference.
	° ′ ″	° ′ ″		
Dunkirk, N. Y	42 29 44.30	79 21 25.80	748, 755	Light-house (1876).
Detroit, Mich	42 19 58.65	83 03 03.60	748, 715	East pier of Lake-Survey Observatory.
Kalamazoo, Mich	42 17 24.46	85 35 11.70	749, 763	Intersection of South and Park streets.
Marshall, Mich	42 16 24.67	84 57 49.05	749, 763	Intersection of Kalamazoo avenue and Mansion street.
South Haven, Mich	42 16 13.48	....	749	Azimuth station.
Jackson, Mich	42 14 49.40	84 24 29.85	749, 763	Intersection of Main and Jackson streets.
Paw Paw, Mich	42 13 01.66	85 53 03.30	749, 763	Intersection of Main and Van Buren streets.
Fort Laramie, Wyo	....	104 33 21.6	766	Flag-staff at military post.
Erie, Pa	42 09 17.49	80 04 46.95	750, 756	Light-house.
Fort McDermit, Nev	....	117 37 32.7	766	At military post.
Monroe, Mich	41 54 48.65	83 23 46.35	750, 756	Court-house.
Chicago, Ill	41 53 22.49	87 36 48.90	750, 756	Light-house.
Ashtabula, Ohio	41 52 36.13	80 47 38.55	750, 757	Southwest corner of passenger depot.
Willow Springs, Ill	41 43 38.63	87 51 06.75	630, 724	Triangulation station.
Toledo, Ohio	41 39 03.65	83 32 30.60	635, 727	Stone longitude-post of 1881.
Rock Island, Ill	41 31 01.51	90 33 45.90	751, 764	Southeast corner of guard-house.
Cleveland, Ohio	41 30 06.13	81 42 13.20	751, 757	Light-house.
West Base, Sandusky, Ohio	41 29 04.59	....	636	Triangulation station.
Sandusky, Ohio	....	82 42 47.85	757	Northeast corner of custom-house.
Ogden, Utah	....	111 59 56.10	766	Stone post in observatory.
Hudson, Ohio	....	81 26 16.80	757	Do.
Fairmount, Ill	40 01 36.70	....	631	Triangulation station.
Louisiana, Mo	39 27 12.65	91 03 08.25	751, 764	Northeast corner of high school building.
Fort Leavenworth, Kans	....	94 54 50.25	767	Observatory.
West Base, Olney, Ill	38 51 41.23	....	632	Triangulation station.
Parkersburg, Ill	38 34 53.20	88 01 48.96	633, 725	Do.
Cairo, Ill	36 59 47.99	89 09 31.20	751, 757	Triangulation station Defiance.
Memphis, Tenn	35 08 39.85	90 03 20.25	751, 758	Triangulation station.
Fort Richardson, Tex	....	98 09 41.1	767	At military post.
Fort Griffin, Tex	....	99 13 48.6	767	Do.
Fort Concho, Tex	....	100 25 42.6	767	Do.
Fort Stockton, Tex	....	102 52 26.1	767	Do.
Fort McKavett, Tex	....	100 06 23.1	767	Do.

§ 6. The following are descriptions of such of the stations in the foregoing tables as are not sufficiently described therein. The years indicated are those in which the stations were occupied:

COPPER HARBOR (*Fort Wilkins*), 1864.—Observations for latitude were made on a pine post situated just north of Fort Wilkins, about one mile east of Copper Harbor, Keweenaw County, Michigan. A stone post marked S, just outside the fort fence, is 233.9 feet south of the observatory-post, and a cross on the rock near the water's edge is 515.9 feet north.

MOUNT HOUGHTON, 1865.—This station was situated on Mount Houghton, a mountain on the south side of Keweenaw Point, about $1\frac{1}{5}$ miles north of Bête Grise Bay, Keweenaw County, Michigan. Observations for latitude were made in 1865 from a wooden post set about 100 feet northeast from the cliff on the south side of the summit. Observations for longitude were made from a wooden post distant 38.4 feet in a southeasterly direction from the latitude-post. A secondary triangulation-station, with tripod 20 feet high for support of instrument, and marked by a stone buried in the ground, was distant 76 feet, bearing south 85° 30′ west. The height of the mountain above Lake Superior is 877 feet.

PORTAGE ENTRY, 1864.—Observations for latitude were made on a pine observatory-post set near the west end of the island at the mouth of Portage River, Houghton County, Michigan, about 1100 feet west of the canal. A stone marked S stands 17.0 feet north of the observatory-post.

CLAY BANKS (on west shore of Lake Michigan), 1866, '72.—This station is situated near the shore of the lake, 8½ miles northward from Ahnepee village, Door County, Wisconsin. As described in 1873, it is about 15 feet back from the edge of the bank, about 120 feet from the edge of the lake, and about 330 feet from the wagon-road running from Ahnepee to Clay Banks. The height of station used was 48 feet. This station was occupied in making azimuth observations and in reading a horizontal angle for carrying the azimuth southward. The geodetic point is marked by a stone post of the usual form, set 3½ feet below the ground surface. Three stone reference-posts, with square tops rising 10 inches above ground, with holes in their centers and the letters U. S. cut on them, were set as follows: One bearing south 39° 38′ west, distant $154^m.66$; one bearing north 27° 15′ east, distant $145^m.42$; and one bearing north 53° 16′ west, distant $46^m.12$. A latitude-post occupied by Mr. Robinson in 1866 bears south 39° 38′ west, distant $155^m.50$, and one occupied by Lieutenant Powell in 1872 bears north 27° 15′ east, distant $530^m.60$. The height of ground at the station above Lake Michigan is 60.7 feet.

FRANKFORT, 1871, '73.—This station, on the east shore of Lake Michigan, is in Benzie County, Michigan, about $1\frac{1}{5}$ miles north of the village of Frankfort, 3 miles south of the Pointe aux Becs Scies light-house, and about 170 metres back from the shore. It is on a high and steep hill, the highest land in the vicinity. The height of station used was about 10 feet. There is no record of stone markings, but it is probable that the geodetic point was marked by stone posts in the usual manner. Horizontal angles were measured at this station in 1871 to carry azimuth southward. Latitude observations were made in 1871, and azimuth observations in 1873 from a post bearing north 74° 55′ east, distant $27^m.75$ from the station. The old triangulation-station, Pointe aux Becs Scies, bears north 1° 16′ 32″ east, distant $5570^m.8$. The height of ground at the station above Lake Michigan is 323 feet.

KEWAUNEE, 1873.—This station is situated on the west shore of Lake Michigan, in Kewaunee County, Wisconsin, near the mouth of the Kewaunee River. It was occupied for measuring a horizontal angle to carry azimuth southward. Latitude was observed at a post near by, and referred to the station. The height of station used was 8.1 feet. The geodetic point was marked in the usual manner. Two stone reference-posts, 2 feet long and 6 inches square, projecting 10 inches above the surface of the ground, with a small hole and the letters U. S. cut in the top of each, are set as follows: One bearing south 26° 13′ west, distant $36^m.76$; the other bearing north 32° 29′ west, distant $34^m.78$ from the geodetic point. The latitude-post bears south 89° 31′ west, and is distant $13^m.08$. The station was about 6 metres from the edge of the bank, and about 40 metres from the water's edge. The height of ground at the station above Lake Michigan was 64.9 feet.

MANISTEE, 1871.—This station, on the east shore of Lake Michigan, is in Manistee County, Michigan, 2½ miles north along the shore from the mouth of Manistee River, and about 260 metres north of a cross-road leading down to the beach from the main road. A large pine post was occupied as the station in 1871 for measuring horizontal angles for carrying azimuth. No stone markings nor references are recorded. Latitude observations were made in 1871 at a post which bears north 23° 14′ east, distant $30^m.3$ from the station.

Hydrographical station, Rush Lake, 30 feet high, built in 1866, and used for locating off-shore soundings, on ground 74 feet above the lake, and probably having a stone marking, bears north 15° 59′ 50″ east, distant $475^m.5$ from the station. The height of ground at the station above Lake Michigan is about 65 feet.

RAWLEY'S POINT (*or* TWO RIVERS), 1866, '72, '73.—This station is situated on the west shore of Lake Michigan, in Manitowoc County, Wisconsin, about 5 miles north of the village of Two Rivers, and about 1000 metres south of the Twin River Point light-house. A horizontal angle was measured in 1873 for carrying azimuth southward, and latitude observations were made near by in 1866 and in 1872. As described in 1873, the station was about 36 metres distant from the edge of the lake, and the height of its center-post was 65.4 feet. The geodetic point was marked by a stone post set 2 feet below the ground surface. Three stone reference-posts, rising 10 inches above ground, and having the letters U. S. cut on them, were set as follows: One bearing north 15° 49′ east, distant 43^{m}.34; one bearing north 82° 23′ west, distant 56^{m}.02; and one bearing south 17° 49′ west, distant 44^{m}.93. The latitude-post of 1866 bears south 14° 12′ 30″ west, distant 941^{m}.80, and the latitude-post of 1872 bears north 89° 15′ west, distant 12^{m}.04 from the geodetic point. The height of ground at the station above Lake Michigan is 14.7 feet.

BIG POINTE AU SABLE, 1866, '71, '72.—This station, on the east shore of Lake Michigan, is situated in Mason County, Michigan, on the end of the point nearly southwest from the Big Pointe au Sable light-house. The height of station used was 20 feet. The geodetic point was marked by a stone post set 3 feet below the ground surface, with another stone post set directly over it for a surface-mark. The station was occupied in 1871 and 1872 for measuring horizontal angles, and latitude observations were made on a post near by in 1866 and 1872. The light-house bears north 49° 28′ east, distant 109^{m}.5, and the latitude-post bears north 4° 31′ east, distant 96^{m}.3 from the geodetic point.

POINT CREEK, 1873.—This station, on the west shore of Lake Michigan, is in Manitowoc County, Wisconsin, about 15 miles north of Sheboygan, 9 miles south of Manitowoc village, and about 151 metres north of the mouth of Point Creek. It was occupied in 1873 for reading horizontal angles, and latitude and azimuth observations were made on posts near by. The height of station used was 26.8 feet. The geodetic point is marked by two stone posts set one above the other, the upper rising 1 foot above the ground surface. Three stone reference-posts, rising about 10 inches above the surface, and having the letters U. S. cut on them, are set as follows: One bearing north 5° 12′ east, distant 48^{m}.65; one bearing north 78° 28′ west, distant 35^{m}.87; and one bearing south 37° 05′ west, distant 37^{m}.12. The latitude-post bears south 22° 32′ west, distant 18^{m}.62; and the azimuth-post bears north 0° 19′ west, distant 4^{m}.91 from the geodetic point. The station is about 36 metres distant from the edge of the lake, and 64 feet above it.

NORTH SHEBOYGAN, 1871, '73.—This station, on the west shore of Lake Michigan, is situated to the northeast of Sheboygan, Wis., near the light-house. A station 75 feet high was used in 1871 in locating off-shore soundings. This station having been destroyed, one of 9 feet height, with 20-foot pyramid, was built and occupied in 1873 to read horizontal angles for carrying azimuth southward. Latitude was observed in 1866 from a post near by. As described in 1873, the geodetic point was marked by two stones set one above the other, the upper one as a surface-mark. Three stone reference-posts were set as follows, the bearings being approximate: one south, distant 19^{m}.35; one west, 8^{m}.44; and one north, 10^{m}.88. The latitude-post bears north 52° 26′ west, distant 53^{m}.98, and the center of the light-house dome bears south 36° 08′ west, distant 103^{m}.39 from the geodetic point.

SOUTH SHEBOYGAN, 1872, '73.—This station, on the west shore of Lake Michigan, is about 5 miles south of Sheboygan, Wis., and about 23 metres back from the beach. It was occupied in 1873 for measuring horizontal angles to carry azimuth southward. Observations for latitude and azimuth were made in 1872 near the station. The height of station used was 35 feet. The geodetic point is marked by a stone post set 3 feet below the ground surface and inclosed in a barrel. Four stone reference-posts were set as follows, the bearings being approximate: one north, distant 8^{m}.60; one south, 10^{m}.03; one east, 17^{m}.98; and one west, 25^{m}.06 distant from the geodetic point.

CLAY BANKS (east shore of Lake Michigan), 1871, '73.—This station is situated in Oceana County, Michigan, about 9½ miles northward along the shore from the White River lights, about 10½ miles southward from Little Pointe au Sable, and about 4 miles southward from Stony Creek, on land belonging to Mr. Harvey Tower in 1873. It is situated on the highest ground nearest the lake, northwest from Tower's barn, and almost directly south of Dr. Phillip's house. In 1873 horizontal angles were measured at this station, and latitude and azimuth observations were made at a post bearing south 89° 09′ east, distant 14^{m}.12. The height of station used was 15 feet. The geodetic point was marked with a stone sunk in the ground, having a hole and the letters U. S. cut in the top. Three piles of stones were made around the station, one north, one south, and one west. The height of ground at the station above Lake Michigan was 261 feet.

ADA 1, 1873.—This station, on the west shore of Lake Michigan, is in Ozaukee County, Wisconsin, 8 miles north of Port Washington or Ozaukee. A station 19 feet high was built on this site in 1871 and used for locating off-shore soundings. It was replaced in 1873 by one 49.7 feet high used for measuring a horizontal angle for carrying azimuth. The geodetic point is marked by a stone post set 2 feet below the ground surface. Three cedar posts, each 4 inches in diameter, rising 2.5 feet above ground, faced on the side towards the station and marked U. S., and surrounded by a pyramid of loose stones, were set as references; one bearing north 15° 15′ east, distant $19^m.81$; one bearing north 73° 19′ west, distant $19^m.75$; and one bearing south 16° 04′ west, distant $19^m.66$. A latitude-post, occupied in 1873, bears south 88° 00′ west, distant $17^m.59$. A lime-kiln is approximately 27 metres distant to the southwest. The station was about 7 metres from the water's edge and 7 feet above it.

DUCK LAKE, 1871, '73.—This station, on the east shore of Lake Michigan, is in Muskegon County, Michigan, about 3 miles south of White River lights and one-half mile south of Duck Lake, on the highest bluff close to the shore. The station was occupied in 1873 for measuring a horizontal angle for carrying azimuth, and latitude was observed in 1871 from a post bearing north 29° 34′ west, distant 17.3 metres from the station. The height of station used was 20 feet. The geodetic point is marked by a stone post set about 3 feet below the ground surface. A stone post is set directly over this as a surface-mark. The height of ground at the station above Lake Michigan is 146 feet.

FOX POINT, 1873.—This station, on the west shore of Lake Michigan, is in Milwaukee County, Wisconsin, about 8 miles north of the city of Milwaukee. It was occupied in 1873 for measuring horizontal angles, and latitude and azimuth were observed from posts near by. The height of station used was 33 feet. The geodetic point was marked by a stone post. The latitude-post bears north 82° 20′ west, distant $20^m.33$; the azimuth-post bears north 7° 38′ east, distant $4^m.48$; and the northeast corner of Vruwink's house bears south 35° 13′ west, distant 160.9 metres from the geodetic point. The height of ground at the station above Lake Michigan is 98 feet.

SOUTH TWIN SISTER, 1871.—This station, on the east shore of Lake Michigan, is in Ottawa County, Michigan, about 6 miles south of Grand Haven, nearly 1 mile north of Little Pigeon River, and about 950 metres south of a landing pier. It is about 100 metres back from the edge of the lake, on the summit of a hill about 134 feet in height above the lake. Another hill, about 120 feet high, is about 300 metres northward from the station. The station was occupied in 1871 for measuring horizontal angles for carrying azimuth, and latitude observations were made from a post $4^m.2$ directly west of the station. The geodetic point is marked by a stone post set 3 feet below the ground surface, with another stone post set directly over it for a surface-mark.

DOUGLAS, 1873.—This station, on the east shore of Lake Michigan, is situated in Allegan County, Michigan, on the edge of the bank about 2½ miles south of the mouth of Kalamazoo River, and about 1½ miles southwest of the village of Douglas. The station was occupied in 1873 for measuring horizontal angles, and latitude and azimuth observations were made from posts near by. The height of station used was 20 feet. The geodetic point is marked by a stone post set 3 feet below the ground surface, with a stone post set directly over it as a surface-mark. The stone latitude-post standing near the intersection of two roads bears south 45° 49′ east, distant $25^m.5$ from the station, and the azimuth-post is 10^m directly north. The height of ground at the station above Lake Michigan is 70 feet.

SOUTH HAVEN, 1873.—This station, on the east shore of Lake Michigan, is in Van Buren County, Michigan, 9½ miles southward along the shore from South Haven, and 3¼ miles south of Little Brandywine Creek, on a sand-dune nearly 100 feet above the lake. The station was occupied in 1873 for measuring horizontal angles, and latitude was observed from a post bearing south 59° 01′ east, distant $14^m.25$ from the station. The height of station used was 10 feet. The geodetic point was marked by a stone post set 3 feet below the ground surface, with a stone post set directly over it as a surface-mark. A blaze on a large oak tree bears north 70° east, distant 18^m, and a blaze on an oak stump bears south 20° east, distant 14^m from the station, these coördinates being roughly estimated.

CAIRO (*Defiance*), 1876, '77.—This station, a triangulation-station of the Mississippi River Survey, is situated on the site of Fort Defiance, in the city of Cairo, Ill. The height of station used was 10 feet. The geodetic point is marked by a cross cut on the top of a stone post 1½ feet long set 3 feet below the ground surface. Another stone post 5 inches square at top and 3½ feet long, and having a half-inch hole in the top, is set directly over the geodetic point, rising 6 inches above the ground, as a surface-mark. The posts are freestone. Latitude and longitude observations were made in the winter of 1876–'77 from a post bearing north 54° 13′ west, distant 16 metres from the station.

MEMPHIS, 1877.—This station, a triangulation-station of the Mississippi River Survey, is situated on a bluff near the corner of Front and Monroe streets, in the city of Memphis, Tenn. The geodetic point is marked by a quarter-inch hole in the top of a stone post 2½ feet long, set 3 feet below the ground surface. Another stone post, rising 6 inches above the ground surface, is set directly above the geodetic point as a surface-mark. Latitude and longitude observations were made in 1877 from a limestone post bearing south 67° 32′ east, distant $34^{m}.1$ from the station. The following bearings and distances are from the latitude-post: Gas-lamp at northwest corner of Front and Monroe streets, bearing north 52° 31′ east, distance $75^{m}.8$; southwest corner of Vaccaro & Co.'s store, bearing south 68° 06′ east, distance $68^{m}.1$; southwest corner of Magnolia Block, at northeast corner of Front and Union streets, bearing south 33° 16′ east, distance $71^{m}.6$.

PART V.

PRINCIPAL RESULTS OF THE GEODETIC WORK.

CHAPTER XXVII.

GEOGRAPHICAL COÖRDINATES DERIVED GEODETICALLY.

§ **1.** The adopted mean values of the triangle-sides and of their angles, for the principal chains given in Chapters XIV–XX under the heading D, and for the lateral chains in Chapter XXI, are the data for computing the latitudes and longitudes of all the primary stations in these chains, as soon as the latitude and longitude of one station and the azimuth of a triangle-side radiating from it are known. The computation has to be made for an assumed spheroid, which should differ little from the actual geoid, in order that this difference may introduce little error into the computed latitudes, longitudes, and azimuths. As Clarke's spheroids represent our present knowledge of the form of the earth with greater accuracy than Bessel's does, that one of Clarke's spheroids has been adopted for our geodetic computations which is given in Colonel Clarke's Comparisons of Standards of Length, namely:

Equatorial semi-axis $=a=20926062$ English feet $=6378206.4$ metres.
Polar semi-axis $=b=20855121$ English feet $=6356583.8$ metres.

The metre here used must be defined by Clarke's value for it, namely, 3.28086933 English feet. With this assumed spheroid, when the latitude and longitude of a station and the length and azimuth of a triangle-side of ordinary length radiating from it are known, the differences of latitude, longitude, and azimuth at the two ends of the line can be computed with all needed accuracy by formulæ given, with convenient tables for their application, in the United States Coast-Survey Report for 1875. The tables have been computed for the metre; but that metre is the one whose ratio to the English yard is that adopted by Colonel Clarke, namely, 1.09362311. The international metre to be adopted by the International Commission for the Metre will probably be somewhat shorter.

§ **2.** The following are the formulæ for the difference in seconds of arc of latitude, longitude, and azimuth at the ends of a triangle-side. Let L, M, Z, N, and R be the latitude, longitude, azimuth, normal drawn to the polar axis, and meridian radius of curvature at the first station, while the same letters primed represent corresponding quantities for the second station, respectively, K the length of the triangle-side, and e the eccentricity; then

$$-dL=\frac{K\cos Z}{R\text{ arc }1''}+\frac{K^2\sin^2 Z\tan L}{2\,N{\cdot}R\text{ arc }1''}-h\,K^2\sin^2 Z\,\frac{(1+3\tan^2 L)}{6\,N^2}+\frac{l^2\,\frac{3}{2}\,e^2\sin L\cos L\text{ arc }1''}{(1-e^2\sin^2 L)^{\frac{3}{2}}}$$

in which h stands for the first term of the second member, and l for the first three terms.

For the difference of longitude the formula is

$$dM = \frac{K \sin Z}{N' \cos L' \operatorname{arc} 1''}$$

where dM is yet to be corrected for the assumption that small sines are proportional to their arcs. This correction is derived from a table.

For the difference of azimuth the formula is

$$-dZ = dM \frac{\sin \frac{1}{2} (L+L')}{\cos \frac{1}{2} (L'-L)}$$

where it is assumed that tan $\frac{1}{2}\, dZ$ and tan $\frac{1}{2}\, dM$ may be replaced by dZ and dM. When dM is large a correction

$$+\tfrac{1}{12}\, dM^3 \sin \lambda \cos^2 \lambda \sin^2 1'',$$

in which $\lambda = \frac{1}{2} (L+L')$, is to be applied to $-dZ$ on account of the error in this assumption. The tabulation of several of the factors entering these formulæ makes the computation easy.

§ **3.** A preliminary computation having indicated that great deviation of the vertical from the normal to the spheroid did not exist at station Fort Howard, that point was taken as the fundamental one for the triangulation stretching from North Base, Minnesota Point, Minn., to Parkersburg, Ill., and from Willow Springs, Ill., east to Mannsville, N. Y.

The observed latitude and longitude from Detroit of the station Fort Howard, given in Chapter XXIII, § 10, and Chapter XXV, § 3, as $44° 30' 30''.28$ and $19^m 58^s.03 = 4° 59' 30''.45$ west, were adopted for it, and the fundamental azimuth, which was that of the line Fort Howard—Bruce, namely, $259° 20' 57''.32$, was derived from the azimuth of the line Bruce—Long Tail Point, given in Chapter XXIV, § 6. To refer the longitudes to Greenwich, the longitude of the east transit-post in the Lake-Survey Observatory of 1870 to 1882, at Detroit, given in Chapter XXV, § 1, as $5^h 32^m 12^s.24$, has been applied to the longitude of Fort Howard trigonometrical station.

The computations have been carried to thousandths of a second of arc for latitude and longitude, and to hundredths for azimuth, and the results are given to hundredths. As the differences of latitude, longitude, and azimuth are computed for each of the three sides of a triangle, the work is self-checking throughout. In the following tables, which include all of the main triangulation. the first column gives the names of stations; the second, their latitudes; the third, their longitudes west from Greenwich; the fourth, the azimuth of the line joining the station named in the first column and the corresponding station named in the fifth column; the fifth gives the names of stations; the sixth gives the reverse azimuths; the seventh gives the logarithms of the lengths of the lines in feet; and the eighth gives the lengths of the lines in English miles. It will be remembered that these stations are permanently marked. The descriptions of the markings are given in Chapters XIV to XX, under the heading A.

Geographical Coördinates of Primary Triangulation Stations.

I.—MINNESOTA POINT BASE TO KEWEENAW BASE.

Stations.	Latitudes.	Longitudes.	Azimuths.	Stations.	Reverse azimuths.	Logarithms of distances in feet.	Distances in English miles.
	° ′ ″	° ′ ″	° ′ ″		° ′ ″		
Oneota	46 45 18.54	92 07 59.85					
North Base	46 45 27.89	92 04 43.26	86 03 29.20	Oneota	266 01 05.98	4.1374120	2.5988
South Base	46 42 49.44	92 01 55.13	120 46 11.22	Oneota	300 41 45.64	4.4706648	5.5980
Do			143 54 16.18	North Base	323 52 13.75	4.2982175	3.7634
Lester	46 51 55.11	91 59 14.32	11 27 32.99	South Base	191 25 35.79	4.7513085	10.6825
Do			30 17 08.82	North Base	210 13 09.00	4.6571959	8.6013
Do			42 21 04.17	Oneota	222 14 41.01	4.7349569	10.2878
Aminicon	46 41 32.14	91 51 43.59	100 28 20.45	South Base	280 20 55.37	4.6368589	8.2078
Do			113 48 48.23	North Base	293 39 20.58	4.7733389	11.2384
Do			153 36 54.59	Lester	333 31 26.14	4.8480696	13.3486
Buchanan	46 56 28.07	91 47 19.82	11 27 12.62	Aminicon	191 24 00.28	4.9666298	17.5386
Do			60 56 17.74	Lester	240 47 35.94	4.7543694	10.7581

Geographical Coördinates of Primary Triangulation Stations—Continued.

Stations.	Latitudes.	Longitudes.	Azimuths.	Stations.	Reverse azimuths.	Logarithms of distances in feet.	Distances in English miles.
	° ′ ″	° ′ ″	° ′ ″		° ′ ″		
Burlington	47 01 49.46	91 38 29.36	48 32 04.34	Buchanan	228 25 36.47	4.6912740	9.3034
Brulé	46 45 17.89	91 35 18.79	71 39 55.63	Aminicon	251 27 58.64	4.8592869	13.6979
Do			112 05 42.71	Lester	291 48 16.08	5.0320605	20.3904
Do			143 38 22.63	Buchanan	323 29 36.61	4.9263069	15.9835
Do			172 30 49.53	Burlington	352 28 30.40	5.0057294	19.1910
Split Rock	47 11 16.03	91 23 11.01	47 59 30.75	Burlington	227 48 17.92	4.9325734	16.2158
Clay Banks	46 49 05.61	91 18 40.63	71 43 41.10	Brulé	251 31 33.64	4.8644787	13.8626
Do			133 17 04.78	Burlington	313 02 36.47	5.0535632	21.4254
Do			172 06 43.44	Split Rock	352 03 25.68	5.1338226	25.7744
West Sawteeth	47 22 19.08	91 11 43.63	35 16 27.88	Split Rock	215 08 02.87	4.9149615	15.5714
East Sawteeth	47 23 19.02	91 10 15.79					
Detour	46 56 36.68	90 58 09.32	62 00 42.87	Clay Banks	241 45 44.10	4.9866863	18.3676
Do			130 44 52.83	Split Rock	310 26 33.38	5.1364556	25.9311
Do			160 16 38.99	West Sawteeth	340 06 41.92	5.2203645	31.4580
Outer Island	47 04 17.09	90 26 27.25	70 42 24.36	Detour	250 19 13.10	5.1456152	26.4839
Do			120 35 20.87	West Sawteeth	300 02 07.07	5.3368632	41.1367
Do			122 47 43.96	East Sawteeth	302 15 34.37	5.3327798	40.7517
Farquhar's Knob	47 52 40.18	89 59 23.77	20 55 53.10	Outer Island	200 35 56.60	5.4977391	59.5807
Do			58 37 08.96	West Sawteeth	237 43 42.82	5.5439449	66.2691
Do			58 56 21.84	East Sawteeth	238 04 00.17	5.5334571	64.6879
Porcupine Mountains	46 47 01.05	89 43 50.84	122 06 58.56	East Sawteeth	301 03 41.06	5.6245226	79.7790
Do			170 56 51.75	Farquhar's Knob	350 45 25.75	5.6066591	76.5641
Wheal Kate	47 04 20.27	88 39 45.21	68 51 50.11	Porcupine Mountains	248 05 00.87	5.4577812	54.3436
Do			132 19 04.05	Farquhar's Knob	311 20 22.18	5.6440078	83.4399
Isle Royale	48 07 43.58	88 33 37.91	3 46 30.11	Wheal Kate	183 41 58.86	5.5868087	73.1433
Do			75 52 20.78	Farquhar's Knob	254 48 36.35	5.5583825	68.5092
Saint Ignace	48 47 18.90	87 51 07.36	35 48 02.54	Isle Royale	215 16 13.46	5.4710078	56.0240
Vulcan	47 26 46.65	87 47 38.17	2 35 14.27	Ives Hill	182 33 26.16	5.3564610	43.0355
Do			8 45 39.99	Huron Mountains	188 40 03.10	5.3212858	39.6873
Do			42 12 10.87	Crebassa	221 45 16.25	5.3561762	43.0073
Do			44 56 10.35	Traverse Point	224 36 48.00	5.1897764	29.3186
Do			58 00 33.29	Wheal Kate	237 22 16.66	5.4069097	48.3366
Do			143 09 25.99	Isle Royale	322 35 21.91	5.4944975	59.1376
Do			178 21 39.93	Saint Ignace	358 19 04.17	5.6900927	92.7810
South Base	46 52 17.76	88 29 16.75					
North Base	46 56 48.01	88 26 59.46	19 12 37.95	South Base	199 10 57.68	4.4622750	5.4909

II.—KEWEENAW BASE TO FOND DU LAC BASE.

Quaquaming	46 52 20.23	88 22 21.79	89 32 40.59	South Base	269 27 37.74	4.4598903	5.4608
Do			135 20 15.85	Wheal Kate	315 07 33.08		
Do			144 37 45.32	North Base	324 34 22.55	4.5222086	6.3034
Do			168 44 51.12	Crebassa	348 43 26.10	4.6164988	7.8319
Middle	46 55 55.09	88 14 22.75	56 50 56.51	Quaquaming	236 45 06.72	4.5993969	7.5294
Do			70 33 45.51	South Base	250 22 52.72	4.8186960	12.4756
Do			95 54 21.82	North Base	275 45 08.94	4.7225054	9.9970
Do			114 31 06.97	Crebassa	294 23 51.80	4.6568328	8.5941
Crebassa	46 59 00.52	88 24 18.17	26 57 19.70	South Base	206 53 41.58	4.6605542	8.6680
Do			39 49 07.18	North Base	219 47 09.29	4.2424468	3.3099
Traverse Point	47 08 42.06	88 13 59.86	1 10 14.95	Middle	181 09 58.21	4.8905454	14.7201
Do			36 03 46.35	Crebassa	215 56 13.68	4.8622959	13.7931
Huron Mountains	46 52 42.30	87 55 17.59	107 46 31.98	Crebassa	287 25 20.42	5.1029038	24.0032
Do			111 11 24.33	Wheal Kate	290 38 54.12	5.2967696	37.5090
Do			141 27 35.54	Traverse Point	321 13 54.59	5.0952241	23.5825
Ives Hill	46 49 26.19	87 50 05.66	132 32 01.12	Huron Mountains	312 28 13.55	4.4684643	5.5697
Granite Island	46 43 15.47	87 24 41.29	24 51 13.75	Triloba	204 47 49.71	4.6684399	8.8268
Do			109 38 45.11	Ives Hill	289 20 14.39	5.0513905	21.3185
Do			114 23 21.83	Huron Mountains	294 01 03.23	5.1463514	26.5288
Do			160 19 53.12	Vulcan	340 03 04.74	5.4490065	53.2565

Geographical Coördinates of Primary Triangulation Stations—Continued.

Stations.	Latitudes.	Longitudes.	Azimuths.	Stations.	Reverse azimuths.	Logarithms of distances in feet.	Distances in English miles.
	° ′ ″	° ′ ″	° ′ ″		° ′ ″		
Triloba	46 36 17.94	87 29 21.83	132 46 52.28	Ives Hill	312 31 46.83	5.0713485	22.3211
Do			166 12 29.06	Vulcan	345 59 06.89	5.4998473	59.8706
Mount Mesnard	46 30 48.34	87 23 59.23	146 00 24.03	Triloba	325 56 29.79	4.6051923	7.6306
Do			177 46 57.15	Granite Island	357 46 26.57	4.8793673	14.3461
Shelter Bay	46 28 17.93	86 59 16.95	98 30 26.34	Mount Mesnard	278 12 31.26	5.0204753	19.8537
Do			111 15 29.09	Triloba	290 53 39.03	5.1398139	26.1324
Do			130 39 21.37	Granite Island	310 20 53.88	5.1461317	26.5154
Grand Island	46 31 43.34	86 41 35.49	20 43 06.43	Mud Lake	200 38 10.16	4.9092249	15.3671
Do			74 27 09.73	Shelter Bay	254 14 19.77	4.8871678	14.6061
Do			98 11 48.43	Triloba	277 37 06.95	5.3057922	38.2964
Do			111 29 29.74	Granite Island	290 58 10.18	5.2869707	36.6722
Do			112 55 37.48	Huron Mountains	292 01 58.94	5.5231691	63.1735
Divide	46 21 41.12	86 39 37.71	68 05 29.51	Mud Lake	247 59 08.44	4.6004196	7.5472
Do			172 18 45.25	Grand Island	352 17 19.90	4.7893244	11.6598
Mud Lake	46 19 14.00	86 48 24.45	140 22 19.54	Shelter Bay	320 14 27.05	4.8549746	13.5625
Monistique	46 08 31.57	86 35 55.05	31 03 08.14	Sturgeon	210 59 20.56	4.6355206	8.1825
Do			141 05 00.09	Mud Lake	320 55 58.93	4.9228926	15.8584
Do			168 57 09.94	Divide	348 54 29.10	4.9111521	15.4354
Sturgeon	46 02 26.07	86 41 10.95	2 47 40.09	Burnt Bluff	182 46 36.30	5.1120680	24.5151
Do			39 29 00.76	Pine Hill	219 19 51.26	4.9299228	16.1172
Do			163 24 33.91	Mud Lake	343 19 21.13	5.0276101	20.1826
Fishdam	45 57 37.33	86 29 34.19	29 09 40.60	Burnt Bluff	209 00 17.14	5.0586973	21.6801
Do			120 48 18.93	Sturgeon	300 39 57.72	4.7576122	10.8387
Do			157 58 22.20	Monistique	337 53 47.99	4.8543707	13.5437
Pine Hill	45 51 36.96	86 53 55.48	39 29 39.09	Ford River	219 20 53.18	4.9134232	15.5163
Burnt Bluff	45 41 09.73	86 42 39.83	30 52 14.07	Boyer's Bluff	210 42 34.64	5.0524484	21.3705
Do			90 16 53.26	Ford River	270 00 04.68	5.0003213	18.9534
Do			143 03 26.49	Pine Hill	322 55 22.33	4.9007204	15.0691
Ford River	45 41 12.16	87 06 09.40	31 34 53.74	Cedar River	211 25 18.53	5.0402654	20.7793
Boyer's Bluff	45 25 12.74	86 56 11.48	36 16 04.40	Door Bluff	216 10 34.88	4.7479328	10.5998
Do			40 14 13.81	Eagle Bluff	220 01 24.67	5.0793057	22.7338
Do			92 13 02.08	Cedar River	271 56 22.26	5.0007686	18.9730
Do			156 24 45.31	Ford River	336 17 38.45	5.0256587	20.0921
Cedar River	45 25 48.62	87 19 35.05	29 56 03.72	Rochereau	209 51 20.90	4.7554282	10.7844
Door Bluff	45 17 46.97	87 03 54.60	43 35 55.82	Eagle Bluff	223 28 36.31	4.8083346	12.1815
Do			89 47 02.12	Rochereau	269 31 11.21	4.9806721	18.1150
Do			126 05 24.45	Cedar River	305 54 15.25	4.9190836	15.7199
Rochereau	45 17 41.23	87 26 12.51	28 01 33.00	Menomonee	207 54 54.21	4.9336585	16.2564
Eagle Bluff	45 10 06.62	87 14 13.66	24 22 08.25	South Egg	204 17 16.43	4.8560595	13.5965
Do			72 10 13.78	Menomonee	251 55 06.00	4.9847317	18.2851
Do			131 53 55.11	Rochereau	311 45 24.75	4.8391206	13.0764
Do			166 29 33.22	Cedar River	346 25 44.78	4.9918303	18.5864
Menomonee	45 05 12.76	87 35 34.60	2 28 25.26	Débroux	182 27 46.93	4.9569314	17.1513
Do			14 00 18.58	Peshtigo	193 58 47.56	4.5821187	7.2358
South Egg	44 59 20.75	87 21 05.79	50 33 28.76	Débroux	230 22 37.04	4.9351581	16.3127
Do			88 57 47.63	Peshtigo	268 46 02.49	4.8555568	13.5807
Do			119 49 46.37	Menomonee	299 39 31.61	4.8564798	13.6096
Do			168 52 16.56	Rochereau	348 48 39.13	5.0554037	21.5164
Peshtigo	44 59 06.73	87 37 43.23	10 12 19.91	Red River	190 09 56.10	4.9191391	15.7219
Do			51 10 39.18	Gales	231 00 57.60	4.8817257	14.4242
Débroux	44 50 19.43	87 36 28.84	95 00 44.34	Gales	274 50 11.05	4.8125340	12.2999
Do			174 16 55.67	Peshtigo	354 16 03.14	4.7297432	10.1651
Gales	44 51 14.48	87 51 26.86	1 16 45 38	Red Banks	181 16 26.14	4.9472261	16.7723
Do			29 32 30.49	Little Tail	209 26 55.11	4.8437319	13.2159
Red River	44 45 40.00	87 41 07.07	40 36 20.66	Red Banks	220 28 45.58	4.8569088	13.6231
Do			71 21 27.62	Little Tail	251 08 36.37	4.9220499	15.8276
Do			127 13 26.92	Gales	307 06 10.14	4.7487696	10.6203
Little Tail	44 41 14.76	87 59 23.07	11 58 24.12	Fort Howard	191 56 10.03	4.8242219	12.6354
Red Banks	44 36 40.26	87 51 54.19	51 05 56.78	Fort Howard	230 58 27.82	4.7751393	11.2851
Do			130 37 39.57	Little Tail	310 32 24.11	4.6307451	8.0930

Geographical Coördinates of Primary Triangulation Stations—Continued.

Stations.	Latitudes.	Longitudes.	Azimuths.	Stations.	Reverse azimuths.	Logarithms of distances in feet.	Distances in English miles.
	° ′ ″	° ′ ″	° ′ ″		° ′ ″		
Bruce	44 31 39.37	87 53 58.14	32 24 07.51	East Depere	212 19 34.85	4.7221002	9.9877
Do			79 26 59.04	Fort Howard	259 20 57.32	4.5801468	7.2030
Do			89 51 38.28	Oneida	269 40 16.99	4.8474917	13.3309
Do			158 03 39.65	Little Tail	337 59 51.47	4.7982004	11.9004
Fort Howard	44 30 30.28	88 02 34.05					
East Depere	44 24 19.52	88 00 27.38	39 47 03.25	Calumet	219 35 40.07	5.0470683	21.1074
Do			74 52 46.14	Freedom	254 37 41.58	4.9885292	18.4457
Do			89 31 15.07	West Depere	269 20 11.29	4.8380578	13.0444
Do			136 23 53.42	Oneida	316 17 05.53	4.7865523	11.5856
Do			166 15 42.57	Fort Howard	346 14 13.86	4.5872049	7.3210
Oneida	44 31 36.53	88 10 09.68	30 37 32.42	West Depere	210 33 15.81	4.7177418	9.8880
West Depere	44 24 12.73	88 16 16.02	1 27 38.57	Calumet	181 27 17.79	4.9299369	16.1177
Do			45 10 58.97	Freedom	225 06 57.79	4.5482776	6.6933
Freedom	44 20 06.61	88 22 00.92	43 41 38.85	Clayton	223 34 11.99	4.8293865	12.7865
Clayton	44 12 04.07	88 32 41.11					
Calumet	44 10 12.64	88 16 45.78	51 39 18.22	Oshkosh	231 28 22.48	4.9434966	16.6289
Do			99 17 59.85	Clayton	279 06 54.00	4.8483345	13.3567
Do			159 09 21.34	Freedom	339 05 41.43	4.8087159	12.1922
Stockbridge	44 06 16 17	88 19 26.12	7 21 44.04	Taycheedah	187 19 43.90	4.9955863	18.7479
Do			38 50 52.70	Eldorado	218 40 07.60	5.0353374	20.5449
Do			61 51 47.53	Oshkosh	241 42 43.68	4.8115917	12.2732
Do			121 22 05.50	Clayton	301 12 51.73	4.8314203	12.8466
Oshkosh	44 01 13.64	88 32 28.14	179 10 40.30	Clayton	359 10 31.27	4.8186989	12.4757
Eldorado	43 52 20.81	88 34 54.97	46 09 44.43	Springvale	226 03 31.17	4.7391321	10.3872
Taycheedah	43 50 06.63	88 22 19.17	31 52 01.81	East Base	211 47 36.93	4.7264783	10.0889
Do			37 11 01.22	Oakfield	217 02 31.47	4.9528877	16.9924
Do			103 51 23.09	Eldorado	283 42 39.46	4.7560692	10.8003
Do			146 38 21.14	Oshkosh	326 31 18.67	4.9080614	15.3260
Springvale	43 46 05.32	88 43 54.06					
East Base	43 42 39.67	88 28 42.05	44 46 26.81	Oakfield	224 42 22.04	4.5683186	7.0094
Do			78 15 21.60	West Base	258 11 37.35	4.3865918	4.6127

III.—FOND DU LAC BASE TO CHICAGO BASE.

East Base	43 42 39.67	88 28 42.05	107 21 46.62	Springvale	287 11 16.08	4.8458427	13.2803
West Base	43 41 50.58	88 34 06.62	5 53 12.72	Oakfield	185 52 52.08	4.3310453	4.0589
Oakfield	43 38 20.04	88 34 36.51	1 15 20.53	Horicon	181 15 07.90	4.7901086	11.6809
Do			34 10 01.76	Minnesota Junction	214 03 42.06	4.8594459	13.7029
Do			85 05 05.61	Waupun	264 58 43.81	4.6111590	7.7362
Do			139 03 00.36	Springvale	318 56 35.13	4.7953707	11.8232
Do			179 05 23.35	Eldorado	359 05 10.58	4.9301562	16.1258
Waupun	43 37 45.11	88 43 49.81	179 38 51.50	Springvale	359 38 48.57	4.7045790	9.5928
Minnesota Junction	43 28 28.45	88 43 47.54	179 49 47.56	Waupun	359 49 45.99	4.7510004	10.6749
Horicon	43 28 11.07	88 34 54.84	2 23 06.49	Lebanon	182 22 39.25	4.8483432	13.3570
Do			92 36 53.73	Minnesota Junction	272 30 47.23	4.5946414	7.4474
Do			145 55 05.98	Waupun	325 48 57.39	4.8464525	13.2990
Woodland	43 22 14.71	88 29 50.48	36 31 23.74	Lebanon	216 27 27.69	4.6310909	8.0995
Do			148 07 27.60	Horicon	328 03 58.40	4.6284275	8.0500
Lebanon	43 16 35.13	88 35 34.51	153 17 22.12	Minnesota Junction	333 11 43.52	4.9078643	15.3190
Erin	43 14 39.00	88 19 39.84	12 42 17.93	Delafield	192 39 37.82	4.8980795	14.9777
Do			99 32 24.55	Lebanon	279 21 30.30	4.8550205	13.5640
Do			135 40 38.04	Woodland	315 33 39.20	4.8099423	12.2267
New Lisbon	43 08 16.85	88 11 57.32	53 25 13.55	Delafield	233 17 17.66	4.8091430	12.2042
Do			138 30 59.99	Erin	318 25 43.43	4.7133469	9.7884
Delafield	43 01 50.97	88 23 34 00	149 04 28.19	Lebanon	328 56 15.38	5.0158600	19.6439
New Berlin	42 58 17.60	88 11 01.80	10 57 24.41	Waterford	190 55 29.45	4.8213249	12.5514
Do			111 44 27.72	Delafield	291 35 54.70	4.7792218	11.3917
Do			176 07 05.40	New Lisbon	356 06 27.50	4.7839812	11.5172
Waterford	42 47 34.91	88 13 50.73	153 36 43.49	Delafield	333 30 06.35	4.9888921	18.4611

Geographical Coördinates of Primary Triangulation Stations—Continued.

Stations.	Latitudes.	Longitudes.	Azimuths.	Stations.	Reverse azimuths.	Logarithms of distances in feet.	Distances in English miles.
	° ′ ″	° ′ ″	° ′ ″		° ′ ″		
Caledonia	42 45 32.89	87 56 48.43	49 56 47.63	Dover	229 51 11.66	4.6842779	9.1547
Do			99 17 57.82	Waterford	279 06 23.53	4.8878535	14.6292
Do			140 41 48.58	New Berlin	320 32 08.05	5.0007224	18.9709
Dover	42 40 25.35	88 05 03.68	137 55 10.41	Waterford	317 49 12.76	4.7681904	11.1060
Somers	42 37 10.07	87 52 33.60	0 54 50.98	Benton	180 54 43.88	4.6930965	9.3425
Do			58 45 07.68	Bristol	238 38 38.47	4.7021064	9.5383
Do			109 29 43.22	Dover	289 21 15.05	4.7741096	11.2584
Do			159 31 26.26	Caledonia	339 28 33.48	4.7351665	10.2928
Bristol	42 32 51.61	88 02 08.78	6 36 02.63	Antioch	186 35 19.60	4.6188035	7.8735
Do			164 07 20.71	Dover	344 05 22.30	4.6790725	9.0456
Benton	42 29 02.87	87 52 44.10	3 37 51.67	Warren	183 37 25.34	4.6654302	8.7659
Do			68 59 49.94	Antioch	248 52 45.73	4.7031098	9.5604
Do			118 45 19.85	Bristol	298 38 58.24	4.6831925	9.1318
Antioch	42 26 03.67	88 03 12.50					
Warren	42 21 26.58	87 53 23.14	55 14 40.54	Fremont	235 08 22.94	4.7103081	9.7202
Do			122 26 27.44	Antioch	302 19 50.07	4.7190847	9.9186
Fremont	42 16 37.18	88 02 44.01	177 51 59.35	Antioch	357 51 40.16	4.7588102	10.8687
Deerfield	42 10 20.55	87 50 45.94	69 18 41.94	Palatine	249 10 51.77	4.7516642	10.6913
Do			125 16 34.44	Fremont	305 08 31.86	4.8203822	12.5241
Do			170 04 13.03	Warren	350 02 27.30	4.8353826	12.9643
Palatine	42 07 02.94	88 02 26.62	178 42 40.63	Fremont	358 42 28.95	4.7645096	11.0122
Park Ridge	42 00 58.27	87 49 40.50	46 47 40.18	Lombard	226 41 09.36	4.7827887	11.4857
Do			122 38 29.09	Palatine	302 29 55.79	4.8361459	12.9871
Do			175 03 07.17	Deerfield	355 02 23.30	4.7568717	10.8202
Lombard	41 54 07.70	87 59 25.02	170 06 24.66	Palatine	350 04 23.12	4.9012564	15.0877
Shot Tower	41 53 11.42	87 38 30.13	6 58 58.50	Morgan Park	186 57 41.97	4.8553452	13.5741
Do			7 23 34.69	Military Academy	187 22 12.84	4.8598821	13.7167
Do			26 45 27.39	East Base	206 42 02.97	4.7127050	9.7740
Do			44 38 03.37	Willow Springs	224 29 39.26	4.9117967	15.4583
Do			51 10 21.10	West Base	231 03 34.11	4.7733648	11.2391
Do			93 33 05.27	Lombard	273 19 07.31	4.9780974	18.0080
Do			133 04 24.62	Park Ridge	312 56 56.48	4.8405906	13.1207
West Base	41 47 03.40	87 48 40.33	27 55 09.62	Willow Springs	207 53 32.37	4.3738197	4.4791
East Base	41 45 36.06	87 43 36.69	70 33 36.33	Willow Springs	250 28 36.94	4.5582259	6.8484
Do			111 02 41.31	West Base	290 59 19.04	4.3917929	4.6683

IV.—CHICAGO BASE TO OLNEY BASE.

Stations.	Latitudes.	Longitudes.	Azimuths.	Stations.	Reverse azimuths.	Logarithms of distances in feet.	Distances in English miles.
Willow Springs	41 43 36.93	87 51 06.36	0 58 46.93	Orland	180 58 38.50	4.7510692	10.6767
Do			149 26 40.11	Lombard	329 21 07.64	4.8702855	14.0492
Morgan Park	41 41 28.60	87 40 24.98	48 55 53.06	Orland	228 48 38.54	4.8191711	12.4893
Do			105 00 32.48	Willow Springs	284 53 25.74	4.7020318	9.5367
Do			132 06 27.16	West Base	312 00 57.41	4.7040315	9.5807
Do			149 53 17.07	East Base	329 51 09.47	4.4618328	5.4853
Orland	41 34 20.08	87 51 19.04					
Garden	41 26 24.48	87 49 35.93	1 58 28.43	Manteno	181 58 07.62	4.8440394	13.2253
Do			170 45 12.00	Orland	350 44 03.66	4.6881846	9.2374
Crete	41 24 53.57	87 38 40.50	0 38 28.85	Grant	180 38 22.94	4.7859646	11.5699
Do			40 55 22.93	Manteno	220 47 49.27	4.9036798	15.1721
Do			100 30 04.33	Garden	280 22 50.65	4.7056295	9.6160
Do			134 52 35.01	Orland	314 44 12.45	4.9104405	15.4101
Do			175 29 58.39	Morgan Park	355 28 49.07	5.0044381	19.1339
Manteno	41 14 54.96	87 50 07.43	8 32 48.87	Kankakee	188 31 47.53	4.6810324	9.0865
Grant	41 14 50.04	87 38 49.45	13 59 20.00	Saint Anne	193 56 24.62	4.9270502	16.0109
Do			51 32 41.84	Kankakee	231 24 14.07	4.8772127	14.2751
Do			90 36 48.13	Manteno	270 29 21.13	4.7142784	9.8095
Kankakee	41 07 06.19	87 51 40.58	17 48 44.64	Clifton	197 45 43.36	4.8406673	13.1230
Saint Anne	41 01 19.46	87 43 16.04	10 42 26.22	Watseka	190 39 48.24	4.9999159	18.9357
Do			62 47 16.47	Clifton	242 38 44.47	4.8285119	12.7608
Do			132 17 12.06	Kankakee	312 11 40.58	4.7176842	9.8867
Do			159 09 38.73	Manteno	339 05 08.11	4.9461423	16.7305

Geographical Coördinates of Primary Triangulation Stations—Continued.

Stations.	Latitudes.	Longitudes.	Azimuths.	Stations.	Reverse azimuths.	Logarithms of distances in feet.	Distances in English miles.
	° ′ ″	° ′ ″	° ′ ″		° ′ ″		
Clifton	40 56 14.29	87 56 16.74	8 35 46.03	Spring Creek	188 33 39.67	4.9986957	18.8826
Watseka	40 45 08.69	87 47 17.39	2 24 35.70	Ash Grove	182 24 14.46	4.7766646	11.3248
Do			61 06 38.54	Spring Creek	240 58 40.64	4.8095049	12.2144
Do			148 26 29.56	Clifton	328 20 36.82	4.8981496	14.9801
Spring Creek	40 40 00.18	87 59 30.11	8 35 20.17	Paxton	188 33 37.39	4.9129914	15.5009
Ash Grove	40 35 18.36	87 47 49.98	51 44 41.67	Paxton	231 35 24.12	4.9268686	16.0042
Do			117 54 34.51	Spring Creek	297 46 58.63	4.7857315	11.5637
Paxton	40 26 40.47	88 02 08.20					
Butler	40 24 17.70	87 46 54.55	11 41 19.70	Pilot Grove	191 39 11.60	4.8799893	14.3667
Do			66 28 48.77	Rantoul	246 19 00.00	4.8853728	14.5458
Do			101 38 16.09	Paxton	281 28 23.63	4.8580853	13.6600
Do			148 36 54.41	Spring Creek	328 28 43.35	5.0484638	21.1753
Do			176 20 23.97	Ash Grove	356 19 47.97	4.8260451	12.6885
Rantoul	40 19 13.84	88 02 03.67	9 25 59.98	Lynn Grove	189 23 04.04	5.1127321	24.5526
Do			18 13 53.09	Mayview	198 10 27.89	4.8972036	14.9475
Do			179 33 23.72	Paxton	359 33 20.79	4.6551275	8.5604
Pilot Grove	40 12 03.60	87 50 12.60	2 32 14.37	Fairmount	182 31 51.06	4.8033644	12.0429
Do			4 10 00.09	Palermo	184 08 48.94	5.0743857	22.4777
Do			42 17 08.49	Lynn Grove	222 06 34.89	5.0563949	21.5656
Do			68 36 46.62	Mayview	248 25 43.10	4.9337432	16.2596
Do			128 21 48.97	Rantoul	308 14 09.43	4.8466332	13.3045
Do			148 04 23.14	Paxton	327 56 40.08	5.0196348	19.8153
Mayview	40 06 52.95	88 07 21.47					
Fairmount	40 01 35.84	87 50 48.78	6 02 19.89	Palermo	186 01 32.10	4.7415547	10.4453
Do			74 18 52.22	Lynn Grove	254 08 43.02	4.8845077	14.5169
Do			112 39 55.18	Mayview	278 18 09.67	4.9221146	15.8300
Do			153 59 08.37	Rantoul	333 51 52.98	5.0762526	22.5745
Lynn Grove	39 58 09.97	88 06 36.56	176 13 40.79	Mayview	356 13 11.90	4.7245673	10.0446
Palermo	39 52 33.82	87 52 03.20	39 04 06.14	Oakland	218 57 21.60	4.8942312	14.8456
Do			116 38 11.71	Lynn Grove	296 28 51.22	4.8812397	14.4081
Oakland	39 42 31.90	88 02 35.30	0 18 30.82	Westfield	180 18 26.77	4.9669909	17.5532
Do			168 48 22.28	Lynn Grove	348 45 47.72	4.9857317	18.3273
Kansas	39 30 51.85	87 51 48.11	22 43 45.05	Casey	202 37 50.42	5.0564159	21.5666
Do			66 58 05.67	Westfield	246 51 10.09	4.7459001	10.5503
Do			144 29 21.81	Oakland	324 22 29.18	4.9399211	16.4925
Do			179 29 18.03	Palermo	359 29 08.39	5.1197336	24.9517
Westfield	39 27 15.94	88 02 41.66					
Martinsville	39 18 56.71	87 51 15.48	4 47 17.23	Belle Air	184 46 43.16	4.7058952	9.6219
Do			54 56 56.26	Casey	234 50 41.73	4.7549629	10.7728
Do			133 12 47.75	Westfield	313 05 32.35	4.8683663	13.9873
Do			177 58 33.90	Kansas	357 58 13.19	4.8597546	13.7126
Casey	39 13 33.42	88 01 07.18	174 54 39.48	Westfield	354 53 39.58	4.9219506	15.8240
Belle Air	39 10 36.33	87 52 09.33	1 28 39.10	Oblong	181 28 25.73	4.8125787	12.3011
Do			46 00 46.73	Hunt City	225 55 13.66	4.7624492	10.9601
Do			112 59 01.48	Casey	292 53 21.54	4.6625386	8.7077
Hunt City	39 03 58.76	88 00 57.21	11 00 22.74	Mound	190 58 50.30	4.7842392	11.5241
Do			179 13 38.79	Casey	359 13 32.50	4.7645248	11.0126
Oblong	38 59 54.58	87 52 30.54	21 17 58.78	Claremont	201 13 28.25	4.9730990	17.8019
Do			55 54 29.82	Mound	235 47 38.93	4.7951572	11.8174
Do			121 45 19.80	Hunt City	301 40 00.72	4.6720931	8.9014
Do			153 52 07.44	Casey	333 46 41.53	4.9652666	17.4837
Mound	38 54 08.38	88 03 24.15	40 45 24.36	West Base	220 43 40.86	4.3005614	3.7838
Do			43 01 46.29	Denver	222 55 54.97	4.8130817	12.3154
Do			46 53 10.37	Onion Hill	226 48 44.46	4.6623231	8.7034
East Base	38 51 44.29	88 01 35.77	4 56 52.21	Check Base	184 56 38.08	4.3153521	3.9149
Do			68 14 13.93	Onion Hill	248 08 40.17	4.6566510	8.5905
Do			88 32 32.91	West Base	268 29 41.45	4.3349191	4.0953
Do			149 33 24.86	Mound	329 32 16.82	4.2281571	3.2028
West Base	38 51 38.76	88 06 09.04	44 00 28.92	Denver	223 56 21.11	4.6538821	8.5359
Do			51 31 54.44	Onion Hill	231 29 12.06	4.4179165	4.9577

Geographical Coördinates of Primary Triangulation Stations—Continued.

Stations.	Latitudes.	Longitudes.	Azimuths.	Stations.	Reverse azimuths.	Logarithms of distances in feet.	Distances in English miles.
	° ′ ″	° ′ ″	° ′ ″		° ′ ″		
Check Base	38 48 20.73	88 01 58.29	95 20 38.42	Onion Hill	275 15 19.00	4.6076530	7.6740
Do			135 17 52.92	West Base	315 15 15.69	4.4502122	5.3495
Do			169 04 36.45	Mound	349 03 42.60	4.5541401	6.7843
Onion Hill	38 48 57.72	88 10 27.97	33 47 55.67	Denver	213 46 30.20	4.2883121	3.6786
Denver	38 46 18.16	88 12 44.38					
Claremont	38 45 28.75	87 59 41.54	8 58 17.62	Parkersburg	188 56 57.57	4.8145247	12.3564
Do			94 40 36.96	Denver	274 32 26.80	4.7938582	11.7821
Do			112 29 51.91	Onion Hill	292 23 06.97	4.7433627	10.4889
Do			140 42 05.28	West Base	320 38 02.44	4.6848081	9.1659
Do			148 06 17.53	Check Base	328 05 21.86	4.3116199	3.8814
Do			161 29 34.65	Mound	341 27 15.07	4.7438494	10.5006
Do			166 37 20.99	East Base	346 36 09.39	4.5916612	7.3965
Parkersburg	38 34 51.73	88 01 49.66	143 16 14.68	Denver	323 09 25.53	4.9380695	16.4224

V.—WILLOW SPRINGS—SHOT TOWER TO SANDUSKY BASE.

Military Academy	41 41 21.85	87 40 32.96	105 56 36.49	Willow Springs	285 49 35.07	4.6985558	9.4607
Millers	41 36 55.42	87 14 18.63	102 51 36.95	Military Academy	282 34 10.64	5.0882205	23.2053
Do			132 03 15.59	Shot Tower	311 47 09.04	5.1698804	28.0057
Otis	41 35 18.65	86 52 35.94	95 46 18.44	Millers	275 31 53.51	4.9976421	18.8369
Michigan City	41 44 07.79	86 52 20.79	1 13 51.27	Otis	181 13 41.20	4.7289293	10.1460
Do			66 29 17.74	Millers	246 14 41.49	5.0380948	20.6758
Do			104 57 29.06	Shot Tower	284 26 42.81	5.3361079	41.0652
Springville	41 39 52.82	86 44 30.61	53 04 10.33	Otis	232 58 47.94	4.6640602	8.7383
Do			125 55 50.61	Michigan City	305 50 37.83	4.6437121	8.3383
Galena	41 41 45.88	86 40 32.13	105 01 39.31	Michigan City	284 53 47.74	4.7453335	10.5366
Bald Tom	41 54 19.34	86 35 59.45	15 10 42.92	Galena	195 07 41.16	4.8977018	14.9647
Do			23 52 12.41	Springville	203 46 31.80	4.9817274	18.1591
Do			50 17 40.54	Michigan City	230 06 46.18	4.9854893	18.3170
Carlisle	41 40 05.16	86 29 38.80	101 40 52.77	Galena	281 33 38.31	4.7042716	9.5860
Do			161 35 25.10	Bald Tom	341 31 11.46	4.9597282	17.2621
Bertrand	41 46 44.96	86 22 40.33	38 08 29.34	Carlisle	218 03 50.83	4.7111924	9.7400
Do			69 40 01.31	Galena	249 28 07.79	4.9381048	16.4237
Do			127 19 20.33	Bald Tom	307 10 27.26	4.8807476	14.3918
Penn	41 38 53.13	86 12 26.72	95 24 40.72	Carlisle	275 13 14.71	4.8958941	14.9025
Do			135 47 48.35	Bertrand	315 41 00.04	4.8240273	12.6297
Milton	41 48 09.36	86 10 29.74	8 57 51.23	Penn	188 56 33.38	4.7558420	10.7946
Do			60 44 46.19	Carlisle	240 32 01.27	4.9998338	18.9322
Do			81 17 35.41	Bertrand	261 09 28.54	4.7481981	10.6063
Calvin	41 50 10.38	85 55 27.18	48 31 28.74	Penn	228 20 09.96	5.0141538	19.5668
Do			79 55 18.39	Milton	259 45 16.58	4.8415624	13.1501
Jefferson	41 41 00.04	85 48 30.08	83 24 50.31	Penn	263 08 55.26	5.0405924	20.7950
Do			113 36 00.34	Milton	293 21 21.73	5.0377260	20.6582
Do			150 27 50.00	Calvin	330 23 12.21	4.8065216	12.1308
Porter	41 49 22.48	85 46 00.10	12 36 52.59	Jefferson	192 35 12.71	4.7169430	9.8698
Do			96 29 42.08	Calvin	276 23 23.88	4.6355470	8.1830
Van Buren	41 42 29.79	85 36 46.74	80 24 13.99	Jefferson	260 16 26.14	4.7334239	10.2516
Do			118 52 31.08	Calvin	298 40 04.67	4.9861527	18.3450
Do			134 56 23.20	Porter	314 50 14.61	4.7722632	11.2106
Sherman	41 50 48.58	85 27 12.45	40 48 35.34	Van Buren	220 42 12.72	4.8238213	12.6237
Do			58 30 57.97	Jefferson	238 16 46.95	5.0557017	21.5312
Do			84 16 30.41	Porter	264 03 58.28	4.9335412	16.2520
Do			88 25 48.77	Calvin	268 06 58.25	5.1083675	24.3071
Mongo	41 41 34.56	85 16 08.70	93 31 13.29	Van Buren	273 17 29.70	4.9735237	17.8193
Do			138 10 16.80	Sherman	318 02 54.65	4.8769764	14.2673
Bronson	41 51 22.46	85 14 38.98	6 31 30.52	Mongo	186 30 30.76	4.7773846	11.3436
Do			86 37 45.73	Sherman	266 29 23.02	4.7568475	10.8196
Fremont	41 42 32.09	84 57 58.97	86 04 18.54	Mongo	265 52 13.61	4.9184075	15.6954
Do			125 24 44.71	Bronson	305 13 38.40	4.9678157	17.5866

Geographical Coördinates of Primary Triangulation Stations—Continued.

Stations.	Latitudes.	Longitudes.	Azimuths.	Stations.	Reverse azimuths.	Logarithms of distances in feet.	Distances in English miles.
	° ′ ″	° ′ ″	° ′ ″		° ′ ″		
Quincy	41 57 40.01	84 55 20.05	7 28 23.17	Fremont	187 26 37.17	4.9670178	17.5543
Do			66 32 43.05	Bronson	246 19 48.91	4.9864828	18.1071
Reading	41 50 06.06	84 44 31.56	53 09 59.21	Fremont	233 01 01.33	4.8837651	14.4921
Do			133 11 41.17	Quincy	313 04 28.10	4.8274159	12.7286
Hillsdale	41 55 14.25	84 39 01.08	38 44 23.90	Reading	218 40 43.30	4.6018220	7.5716
Do			101 22 07.43	Quincy	281 11 13.13	4.8776303	14.2888
Pittsford	41 49 33.25	84 29 00.51	92 47 01.36	Reading	272 36 40.42	4.8485968	13.3648
Do			127 16 39.90	Hillsdale	307 09 59.03	4.7563460	10.8072
Bunday	42 03 16.04	84 27 42.10	46 30 00.89	Hillsdale	226 22 26.67	4.8498289	13.4628
Wheatland	41 55 06.32	84 27 19.48	12 47 03.73	Pittsford	192 45 56.27	4.5387016	6.5474
Do			68 49 37.26	Reading	248 38 08.30	4.9231806	15.8689
Do			90 55 55.36	Hillsdale	270 48 06.62	4.7246861	10.0474
Do			178 01 40.03	Bunday	358 01 24.90	4.6954906	9.3941
Woodstock	41 59 12.55	84 15 35.76	46 08 13.95	Pittsford	225 59 16.43	4.9269027	16.0055
Do			64 57 08.89	Wheatland	244 49 18.44	4.7688406	11.1226
Do			114 16 25.12	Bunday	294 08 18.91	4.7789667	11.3856
Fairfield	41 48 56.12	84 02 37.00	91 56 31.09	Pittsford	271 38 55.20	5.0790430	22.7200
Do			168 36 36.23	Wheatland	288 20 06.82	5.0728575	22.3987
Do			136 43 36.66	Woodstock	316 34 56.57	4.9334911	16.2501
Raisin	41 57 11.50	83 55 30.14	32 49 09.86	Fairfield	212 41 24.87	4.7755426	11.2956
Do			97 46 28.44	Woodstock	277 33 02.19	4.9633038	17.4048
Blissfield	41 51 08.39	83 48 27.86	78 18 50.59	Fairfield	258 09 21.24	4.8173199	12.4361
Do			111 51 49.25	Woodstock	291 33 41.68	5.1221269	25.0896
Do			139 03 15.82	Raisin	318 58 33.80	4.6874393	9.2216
Dundee	41 55 28.08	83 40 13.93	54 54 55.21	Blissfield	234 49 25.42	4.6597457	8.6519
Do			98 40 52.51	Raisin	278 30 40.18	4.8453080	13.2640
Bedford	41 49 11.15	83 37 06.14	103 00 46.68	Blissfield	282 53 11.97	4.7238998	10.0292
Do			120 18 43.36	Raisin	300 06 26.27	4.9851221	18.3016
Do			159 35 34.15	Dundee	339 33 28.81	4.6097416	7.7110
Monroe	41 55 04.51	83 28 16.17	48 18 59.61	Bedford	228 13 05.89	4.7302472	10.1768
Do			92 35 03.74	Dundee	272 27 04.20	4.7349291	10.2872
Cedar Point	41 42 37.77	83 20 06.66	117 21 31.15	Bedford	297 10 15.09	4.9391141	16.4619
Do			153 55 15.19	Monroe	333 49 48.83	4.9252458	15.9446
Stony Point	41 55 53.86	83 15 51.33	13 30 56.11	Cedar Point	193 28 05.85	4.9183879	15.6947
Do			85 00 00.58	Monroe	264 51 42.96	4.7522738	10.7063
Locust Point	41 36 54.83	83 06 21.38	119 04 05.39	Cedar Point	298 54 56.78	4.8550244	13.5641
Middle Sister	41 50 53.03	83 00 07.80	18 29 49.64	Locust Point	198 25 40.98	4.9515644	16.9407
Do			61 13 03.79	Cedar Point	240 59 45.03	5.0159921	19.6498
Do			113 11 39.98	Stony Point	293 01 09.99	4.8898156	14.6954
Danbury	41 31 11.94	82 50 20.30	115 29 53.56	Locust Point	295 19 15.88	4.9078562	15.3187
Middle Bass	41 41 47.87	82 47 15.12	12 20 44.16	Danbury	192 18 41.20	4.8187959	12.4785
Do			71 16 59.53	Locust Point	251 04 17.67	4.9634700	17.4115
Do			133 22 28.80	Middle Sister	313 13 54.06	4.9055607	15.2379
Sandusky	41 24 34.43	82 45 02.76	149 01 39.36	Danbury	328 58 09.10	4.6715463	8.8902
Kelley's	41 36 17.77	82 41 45.29	11 56 08.41	Sandusky	191 53 57.54	4.8618824	13.7800
Do			51 43 01.11	Danbury	231 37 19.43	4.6982066	9.4529
Do			143 10 49.92	Middle Bass	323 07 10.72	4.6207005	7.9080
Pointe Peléo	41 48 50.72	82 41 02.55	33 27 03.06	Middle Bass	213 22 54.94	4.7099326	9.7118
Do			98 13 56.40	Middle Sister	278 01 12.59	4.9424226	16.5878
West Base	41 29 02.32	82 40 58.43	34 28 58.21	Sandusky	214 26 16.49	4.5170020	6.2283
Do			107 06 41.98	Danbury	287 00 29.66	4.6505325	8.4703
Do			159 45 34.37	Middle Bass	339 41 24.32	4.9170111	15.6451
Do			175 22 54.84	Kelley's	355 22 23.76	4.6456118	8.3749
East Base	41 26 28.58	82 38 04.66	70 06 05.74	Sandusky	250 01 29.11	4.5299777	6.4172
Do			117 11 21.90	Danbury	297 03 14.64	4.7987504	11.9156
Do			139 38 36.26	West Base	319 36 41.20	4.3101715	3.8685
Do			164 18 08.83	Kelley's	344 15 42.56	4.7920637	11.7336

Geographical Coördinates of Primary Triangulation Stations—Continued.

VI.—SANDUSKY BASE TO BUFFALO BASE.

Stations.	Latitudes.	Longitudes.	Azimuths.	Stations.	Reverse azimuths.	Logarithms of distances in feet.	Distances in English miles.
	° ′ ″	° ′ ″	° ′ ″		° ′ ″		
Townsend	41 15 43.48	82 29 00.63	126 17 53.13	Sandusky	306 07 17.67	4.9588920	17.2289
Do			145 58 21.28	West Base	325 50 26.83	4.9896039	18.4914
Do			147 36 51.39	East Base	327 30 51.95	4.8885251	14.6518
Do			155 04 23.97	Kelley's	334 55 57.94	5.1393775	26.1062
Brownhelm	41 23 18.46	82 18 28.76	46 22 17.80	Townsend	226 15 20.56	4.8239377	12.6271
Do			93 46 04.23	Sandusky	273 28 30.12	5.0853920	23.0546
Do			102 13 43.72	East Base	282 00 45.85	4.9620930	17.3564
Do			108 49 36.64	West Base	288 34 43.45	5.0355767	20.5562
Do			126 42 41.27	Kelley's	306 27 15.96	5.1217230	25.0662
Camden	41 16 26.04	82 17 46.73	85 16 38.78	Townsend	265 09 14.28	4.7130185	9.7810
Do			175 36 40.20	Brownhelm	355 36 12.44	4.6218624	7.9292
Elyria	41 24 55.41	82 08 44.06	38 48 40.36	Camden	218 42 41.87	4.8203119	12.5221
Do			77 38 08.31	Brownhelm	257 31 41.62	4.6592378	8.6418
Do			94 11 01.05	East Base	273 51 36.07	5.1285745	25.4648
Do			99 48 22.39	West Base	279 27 01.89	5.1744007	28.2987
Grafton	41 12 44.21	81 59 37.85	105 12 13.40	Camden	285 00 15.55	4.9353236	16.3189
Do			150 39 37.23	Elyria	330 33 36.63	4.9291049	16.0869
Olmsted	41 22 52.18	81 57 22.96	9 30 42.65	Grafton	189 29 13.63	4.7951209	11.8164
Do			67 23 58.15	Camden	247 10 29.01	5.0052732	19.1708
Do			103 34 20.02	Elyria	283 26 49.61	4.7274411	10.1113
Rockport	41 27 45.93	81 47 32.30	56 36 07.11	Olmsted	236 29 36.33	4.7318781	10.2151
Royalton	41 19 29.89	81 44 10.65	59 58 33.20	Grafton	239 48 21.62	4.9130266	15.5022
Do			83 17 23.91	Camden	262 55 13.30	5.1903610	29.3580
Do			106 28 46.47	Elyria	286 12 32.66	5.0684691	22.1735
Do			108 47 25.49	Olmsted	288 38 42.01	4.8048380	12.0838
Do			162 59 50.87	Rockport	342 57 37.53	4.7202118	9.9444
Warrensville	41 28 39.24	81 30 12.88	49 01 34.35	Royalton	228 52 20.31	4.9277023	16.0350
Do			74 21 17.74	Olmsted	254 03 19.12	5.1107599	24.4414
Do			86 11 42.79	Rockport	266 00 14.46	4.8993612	15.0220
Willoughby	41 40 28.71	81 26 25.77	13 22 12.61	Warrensville	193 29 41.90	4.8683930	13.9881
Do			51 23 22.21	Rockport	231 09 21.87	5.0913492	23.3730
Chester	41 32 02.88	81 21 17.20	63 13 22.66	Warrensville	243 07 27.67	4.6596760	8.6505
Do			155 25 34.47	Willoughby	335 22 09.59	4.7505951	10.6650
Little Mountain	41 38 26.34	81 16 28.60	29 29 45.84	Chester	209 26 34.28	4.6491472	8.4433
Do			105 20 15.20	Willoughby	285 13 38.27	4.6720413	8.9004
Claridon	41 30 39.61	81 07 06.27	97 29 49.54	Chester	277 20 25.44	4.8147613	12.3631
Do			137 54 52.13	Little Mountain	317 48 38.96	4.8041887	12.0657
Thompson	41 41 14.35	81 02 37.47	17 39 35.29	Claridon	197 36 36.83	4.8287604	12.7681
Do			74 59 25.40	Little Mountain	254 50 12.89	4.8151713	12.3748
Mesopotamia	41 26 59.98	80 59 54.09	124 05 18.90	Claridon	304 00 32.64	4.5988306	7.5196
Do			171 50 33.82	Thompson	351 48 45.42	4.9413361	16.5464
Conneaut	41 53 52.58	80 36 51.56	56 54 11.37	Thompson	236 37 01.12	5.1461772	26.5182
Do			178 11 19.79	Houghton	358 10 08.35	5.4065795	47.6371
Andover	41 37 05.72	80 36 30.62	60 15 26.08	Mesopotamia	239 59 55.48	5.0902526	23.3141
Do			74 31 26.60	Claridon	254 11 08.71	5.1611187	27.4464
Do			102 05 25.45	Thompson	281 48 04.10	5.0848769	23.0273
Do			179 06 34.17	Conneaut	359 06 20.22	5.0082948	19.3046
Houghton	42 35 16.01	80 38 37.82					
Erie	42 03 41.44	80 11 11.64	63 00 33.62	Conneaut	242 43 23.62	5.1162670	24.7533
Do			147 20 22.79	Houghton	327 01 54.36	5.3583198	43.2201
Long Point	42 33 44.47	80 07 29.77	5 14 22.62	Erie	185 11 53.26	5.2631194	34.7125
Do			93 58 07.65	Houghton	273 37 03.80	5.1463866	26.5309
Edinboro	41 52 28.41	80 10 46.88	51 33 02.44	Andover	231 15 54.56	5.1752957	28.3571
Do			94 15 46.45	Conneaut	273 58 21.78	5.0743146	22.4740
Do			154 20 39.93	Houghton	334 01 56.83	5.4604794	54.6821
Do			178 25 45.97	Erie	358 25 29.42	4.8334910	12.9079
Grand River	42 50 49.96	79 37 23.82	0 08 31.66	Westfield	180 08 26.99	5.3201818	39.5866
Do			52 34 41.43	Long Point	232 14 16.59	5.2309175	32.2317
Westfield	42 16 25.30	79 37 30.71	63 15 19.75	Erie	242 52 43.09	5.2322631	32.3318
Do			128 06 20.66	Long Point	307 46 07.14	5.2332720	32.4070

Geographical Coördinates of Primary Triangulation Stations—Continued.

Stations.	Latitudes.	Longitudes.	Azimuths.	Stations.	Reverse azimuths.	Logarithms of distances in feet.	Distances in English miles.
	° ′ ″	° ′ ″	° ′ ″		° ′ ″		
Font Hill	43 03 00.21	79 18 30.22	48 51 36.53	Grand River	228 38 44.17	5.0497488	21.2381
Sugar Loaf	42 52 23.38	79 16 50.62	23 07 44.26	Westfield	202 53 45.28	5.3754142	44.9552
Do			84 14 13.91	Grand River	264 00 15.08	4.9653814	17.4883
Do			173 27 20.99	Font Hill	353 26 13.11	4.8122352	12.2914
Silver Creek	42 31 13.94	79 13 27.81	50 24 47.49	Westfield	230 08 34.59	5.1484992	26.6603
Do			138 07 06.93	Grand River	317 50 53.38	5.2047947	30.3502
Do			173 17 52.59	Sugar Loaf	353 15 35.06	5.1119524	24.5086
Ridgeway	42 52 33.99	79 03 35 31	18 54 26.78	Silver Creek	198 47 44.98	5.1365153	25.9347
Do			89 02 10.78	Sugar Loaf	268 53 09.66	4.7725610	11.2183
Do			133 41 54.43	Font Hill	313 31 44.52	4.9633744	17.4077
Do			172 50 23.78	Drummondville	352 48 53.97	4.8944527	14.8532
Sturgeon Point	42 41 24.88	79 02 53.93	37 31 44.58	Silver Creek	217 24 35.48	4.8916954	14.7592
Do			136 58 23.30	Sugar Loaf	316 48 55.02	4.9605257	17.2939
Do			177 23 44.53	Ridgeway	357 23 16.43	4.8312950	12.8428
Buffalo	42 53 03.18	78 52 41.83	32 54 22.43	Sturgeon	212 47 26.64	4.9250530	15.9375
Do			86 35 10.95	Ridgeway	266 27 46.28	4.6879460	9.2324
Do			142 07 52.04	Drummondville	321 58 56.66	4.9773578	17.9773
Do			176 00 00.46	Tonawanda	355 59 33.16	4.6303573	8.0858
Do			179 33 28.41	West Base	359 33 26.57	4.4159612	4.9354
Tonawanda	43 00 03.84	78 53 21.91	45 06 44.24	Ridgeway	224 59 46.38	4.8093563	12.2102
Do			120 19 07.78	Drummondville	300 10 39.17	4.8065472	12.1315
West Base	42 57 20.56	78 52 44.54	59 08 07.65	Ridgeway	239 00 44.52	4.7516934	10.6920
Do			170 27 48.74	Tonawanda	350 27 23.27	4.2243328	3.1747
Hamburg	42 46 50.61	78 51 31.92	57 07 35.95	Sturgeon	236 59 53.12	4.7828231	11.4865
Do			122 53 06.02	Ridgeway	302 44 54.26	4.8071615	12.1486
Do			172 08 32.89	Buffalo	352 07 45.36	4.5806595	7.2115
Buffalo Plains	42 56 58.22	78 49 03.79	34 18 35.67	Buffalo	214 16 07.19	4.4594101	5.4548
Do			97 52 06.39	West Base	277 49 35.96	4.2193925	3.1387
Do			134 25 35.14	Tonawanda	314 22 39.18	4.4290971	5.0870
East Base	42 59 19.57	78 48 32.98	9 05 43.91	Buffalo Plains	189 05 22.91	4.1611579	2.7449
Do			25 56 02.59	Buffalo	205 53 13.07	4.6270087	8.0237
Do			57 14 03.51	West Base	237 11 12.04	4.3472757	4.2135
Do			101 49 05.70	Tonawanda	281 45 48.67	4.3411538	4.1545

VII.—BUFFALO BASE TO SANDY CREEK BASE.

Stations.	Latitudes.	Longitudes.	Azimuths.	Stations.	Reverse azimuths.	Logarithms of distances in feet.	Distances in English miles.
Drummondville	43 05 22.54	79 05 47.03	32 04 53.91	Sugar Loaf	211 57 21.50	4.9686615	17.6209
Do			133 41 54.43	Font Hill	255 39 22.58	4.7668287	11.0712
Pekin	43 10 14.70	78 52 31.28	3 28 50.67	Tonawanda	183 28 16.09	4.7921160	11.7349
Do			63 27 10.73	Drummondville	243 18 06.71	4.8196052	12.5018
Gasport	43 11 56.77	78 32 31.90	83 28 50.25	Pekin	263 15 09.44	4.9516449	16.9438
Falkirk	43 01 22.85	78 28 58.22	85 55 55.07	Tonawanda	265 39 16.61	5.0376081	20.6526
Do			117 19 06.30	Pekin	297 03 00.84	5.0713860	22.3230
Do			166 08 53.08	Gasport	346 06 27.06	4.8202687	12.5209
Batavia	43 01 18.16	78 15 46.90	90 32 14.12	Falkirk	270 23 14.21	4.7692870	11.1340
Do			131 01 44.84	Gasport	310 50 18.01	4.9942453	18.6901
Albion	43 14 22.65	78 09 19.67	19 54 48.45	Batavia	199 50 23.72	4.9266383	15.9957
Do			81 58 45.80	Gasport	261 42 52.42	5.0176437	19.7247
Morganville	43 00 40.21	78 02 59.86	93 55 50.50	Batavia	273 47 07.22	4.7567571	10.8174
Do			161 20 47.87	Albion	341 16 28.23	4.9440103	16.6485
Brockport	43 11 28.81	77 56 03.68	25 13 21.99	Morganville	205 08 37.62	4.8607270	13.7434
Do			106 42 04.28	Albion	286 32 59.24	4.7890042	11.6512
Scottsville	42 59 59.29	77 42 37.59	92 43 38.13	Morganville	272 29 44.45	4.9586559	17.2196
Do			139 29 14.32	Brockport	319 20 03.58	4.9634519	17.4108
Pinnacle Hill	43 08 06.47	77 35 10.67	33 58 04.37	Scottsville	213 52 59.19	4.7741052	11.2583
Do			102 33 30.88	Brockport	282 19 13.73	4.9781945	18.0119
Turk's Hill	43 02 10.49	77 25 24.52	80 16 44.31	Scottsville	260 04 59.52	4.8914867	14.7521
Do			129 41 56.05	Pinnacle Hill	309 35 15.65	4.7519722	10.6989
Walworth	43 09 22.43	77 20 01.25	28 46 28.31	Turk's Hill	208 42 47.44	4.6979053	9.4465
Do			83 34 44.88	Pinnacle Hill	263 24 22.96	4.8316163	12.8523

Geographical Coördinates of Primary Triangulation Stations—Continued.

Stations.	Latitudes.	Longitudes.	Azimuths.	Stations.	Reverse Azimuths.	Logarithms of distances in feet.	Distances in English miles.
	° ′ ″	° ′ ″	° ′ ″		° ′ ″		
Palmyra	43 04 03.79	77 09 39.04	80 48 35.05	Turk's Hill	260 37 49.61	4.8520820	13.4725
Do			125 00 38.47	Walworth	304 53 33.23	4.7506038	10.6652
Sodus	43 14 13.74	77 04 44.74	19 29 04.41	Palmyra	199 25 43.13	4.8162110	12.4044
Do			51 34 42.88	Turk's Hill	231 20 35.19	5.0701434	22.2592
Do			66 36 19.30	Walworth	246 25 51.95	4.8693621	14.0194
Do			74 48 15.35	Pinnacle Hill	254 27 25.73	5.1470576	26.5720
Clyde	43 03 05.08	76 51 50.60	94 23 12.00	Palmyra	274 11 02.52	4.9006195	15.0656
Do			139 46 55.56	Sodus	319 38 06.17	4.9481924	16.8097
Victory	43 13 06.81	76 36 23.36	48 32 42.52	Clyde	228 22 08.54	4.9631454	17.3985
Do			93 14 28.15	Sodus	272 55 02.87	5.1008931	23.8924
Oswego	43 26 37.28	76 30 50.41	16 43 33.26	Victory	196 39 44.78	4.9328313	16.2255
Do			63 35 42.81	Sodus	243 12 26.58	5.2256678	31.8445
Do			144 05 29.21	Vanderlip	323 43 59.20	5.3671089	44.1037
Do			170 54 46.87	Duck Island	350 50 16.55	5.2584620	34.3422
Vanderlip	43 57 35.79	77 01 57.52	2 41 24.94	Sodus	182 39 29.62	5.4212020	49.9538
Duck Island	43 56 05.40	76 37 21.74	94 59 16.21	Vanderlip	274 42 12.02	5.0348647	20.5225
Do			165 25 47.44	Amherst	345 22 57.45	4.8507813	13.4322
Stony Point	43 50 47.56	76 17 28.19	21 57 45.02	Oswego	201 48 31.33	5.1993518	29.9722
Do			110 19 45.40	Duck Island	290 05 57.93	4.9691551	17.6409
Do			162 12 59.02	Grenadier	342 09 27.92	4.8619243	13.7813
North Base	43 40 43.61	76 12 02.03	44 13 25.37	Oswego	224 06 27.78	5.0768595	22.6061
Do			158 39 20.78	Stony Point	338 35 35.19	4.8173813	12.4379
South Base	43 38 05.19	76 11 51.52	50 24 27.19	Oswego	230 11 22.66	5.0375782	20.6511
Do			162 16 41.76	Stony Point	342 12 48.99	4.9087847	15.3515
Do			177 14 36.06	North Base	357 14 28.81	4.2057301	3.0416
Sandy Creek	43 39 06.93	76 06 08.40	55 19 36.88	Oswego	235 02 35.84	5.1236857	25.1798
Do			76 06 54.37	South Base	256 02 57.56	4.4148795	4.9232
Do			110 40 05.13	North Base	290 36 06.97	4.4436766	5.2607
Do			144 56 36.67	Stony Point	324 48 46.00	4.9381931	16.4270
Mannsville	43 42 54.41	76 03 14.61	29 01 23 36	Sandy Creek	208 59 23.34	4.4205585	4.9880
Do			51 05 31.07	Oswego	230 46 29.63	5.1958649	29.7325
Do			52 25 20.85	South Base	232 19 23.89	4.6809512	9.0848
Do			71 10 47.73	North Base	251 04 43.36	4.6122529	7.7556
Do			127 29 58.81	Stony Point	307 20 08.21	4.8967942	14.9335
Amherst	44 07 23.19	76 41 26.32	56 38 21.83	Vanderlip	236 24 05.92	5.0327199	20.4214
Grenadier	44 02 11.71	76 22 32.40	60 23 01.73	Duck Island	240 12 44.10	4.8742517	14.1781
Do			110 58 06.57	Amherst	290 44 57.75	4.9473459	16.7769
Wolfe	44 07 31.12	76 23 19.98	41 36 45.38	Duck Island	221 27 00.32	4.9673435	17.5675
Do			89 31 28.14	Amherst	269 18 51.81	4.8989881	15.0091
Do			145 30 49.16	Kingston	325 26 38.52	4.6651390	8.7600
Kingston	44 13 47.44	76 29 19.64	53 45 54.53	Amherst	233 37 28.13	4.8176931	12.4468
Carlton	44 10 58.97	76 17 38.75	49 48 01.60	Wolfe	229 44 03.90	4.5130557	6.1719
Do			108 32 41.86	Kingston	288 24 33.17	4.7310300	10.1952
Do			170 36 69.48	Sir John	350 35 30.41	4.3975651	4.7308
Sir John	44 15 02.32	76 18 34.78	24 29 05.65	Wolfe	204 25 46.86	4.7006608	9.5066
Do			80 53 16.28	Kingston	260 45 46.38	4.6772028	9.0067

§ **4.** In Chapter XXI, § 3, are given the angles and sides of a triangulation stretching from the primary side, Cedar River — Door Bluff, in Green Bay, eastward to Mackinac, Michigan. Although the triangulation in some of its parts is not good, it yet will give the geodetic positions of its vertices with reference to Fort Howard with a probable error which, reaching a maximum in the vicinity of Mackinac, will not then exceed a few tenths of a second of arc.

The results of the geodetic computation are given in the following table, which has the same form as the preceding one. They are based on the length and azimuth of the primary side, Cedar River — Door Bluff, given in § 3 as logarithm 4.9190836, and 305° 54′ 15″.25, respectively.

Geographical Coördinates of Stations at north ends of Lakes Michigan and Huron.

Stations.	Latitudes.	Longitudes.	Azimuths.	Stations.	Reverse azimuths.	Logarithms of distances in feet.	Distances in English miles.
	° ′ ″	° ′ ″	° ′ ″		° ′ ″		
Door Bluff	45 17 46.97	87 03 54.60	126 05 24.45	Cedar River	305 54 15.25	4.9190836	15.7199
Cedar River	45 25 48.62	87 19 35.05					
Boyer's Bluff*	45 25 12.81	86 56 11.49	36 15 47.36	Door Bluff	216 10 17.84	4.7479703	10.601
Do			92 12 48.50	Cedar River	271 56 08.69	5.0007638	18.973
Do			126 02 49.13	Bark River	305 49 48.54	4.9835935	18.237
Bark River	45 34 30.78	87 14 25.94	22 37 54.62	Cedar River	202 34 14.14	4.7580733	10.850
Burnt Bluff	45 41 09.73	86 42 39.84	15 20 04.95	Rock Island	195 15 33.56	5.0109514	19.423
Do			30 52 18.36	Boyer's Bluff	210 42 38.93	5.0524311	21.370
Do			73 34 41.96	Bark River	253 11 59.39	5 1503232	26.772
Rock Island	45 24 53.11	86 49 00.00					
N. W. Manitou	45 09 23.00	86 02 46.52	115 40 59.60	Rock Island	295 08 08.68	5.3415242	41.581
Pointe aux Becs Scies.	44 41 48.58	86 14 59.36					
South Manitou	45 00 35.24	86 09 01.74	12 45 30.58	Pointe aux Becs Scies.	192 41 18.36	5.0680981	22.155
Sleeping Bear	44 52 22.69	86 04 08.05	36 14 58.72	Pointe aux Becs Scies.	216 07 19.91	4.9007324	15.069
Do			157 04 36.72	South Manitou	337 01 09.26	4.7337897	10.260
North Manitou	45 03 51.52	85 59 44.32	63 38 43.55	South Manitou	243 32 09.14	4.6502522	8.465
Pyramid Point	44 58 04.40	85 55 28.16	47 16 45.96	Sleeping Bear	227 10 38.85	4.7072630	9.652
Do			104 43 16.33	South Manitou	284 33 41.15	4.7813048	11.446
Do			152 23 42.71	North Manitou	332 20 41.52	4.5985820	7.515
Cathead	45 10 58.09	85 37 05.65	45 23 09.57	Pyramid Point	225 10 08.95	5.0467182	21.090
Do			66 13 06.13	North Manitou	245 57 03.33	5.0276476	20.184
Do			85 09 59.28	N. W. Manitou	264 51 46.49	5.0446113	20.988
Do			143 51 39.72	South Fox	323 41 20.02	5.0230893	19.974
Do			159 05 07.80	North Fox	338 58 29.15	5.0489246	21.198
South Fox	45 24 58.05	85 51 37.49	15 14 29.77	North Manitou	195 08 44.09	5.1236067	25.175
Do			26 51 53.42	N. W. Manitou	206 43 57.99	5.0257201	20.095
Do			90 13 25.85	Rock Island	269 32 34.00	5.3901967	46.512
Do			114 36 25.03	Burnt Bluff	293 59 58.87	5.3786337	45.290
North Fox	45 28 09.94	85 46 26.25	48 49 08.49	South Fox	228 45 26.71	4.4698301	5.587
Pine River	45 19 55.73	85 14 50.43	60 26 40.18	Cathead	240 10 51.80	5.0411919	20.824
Do			147 30 43.59	Beaver Island	327 20 02.82	5.0754617	22.533
Middle Village	45 34 42.39	85 06 13.43	22 21 49.40	Pine River	202 15 40.93	4.9871130	18.386
Do			96 03 57.48	Beaver Island	275 47 06.02	5.0053005	19.172
Do			150 13 23.06	Hat Island	330 04 59.41	5.0016108	19.010
Beaver Island	45 36 25.56	85 29 49.27	11 25 26.33	Cathead	191 20 15.64	5.1981495	29.889
Do			53 21 13.37	South Fox	233 05 40.10	5.0656396	22.030
Hat Island	45 49 01.87	85 17 57.18	33 28 51.38	Beaver Island	213 20 21.63	4.9627166	17.381
Do			91 53 18.99	Garden Island	271 43 47.68	4.7516827	10.692
Do			121 25 06.31	Point Patterson	301 09 48.66	5.0244333	20.035
Do			168 00 09.80	Biddle Point	347 56 45.68	4.9837783	18.245
Garden Island	45 49 19.46	85 31 13.82	51 43 47.50	High Island	231 36 46.35	4.7251043	10.057
Do			110 00 57.48	Seul Choix	289 44 07.86	5.0246374	20.045
Do			138 24 48.28	Scott's Point	318 17 15.47	4.8268900	12.713
Do			147 24 38.37	Point Patterson	327 18 52.72	4.8002380	11.956
High Island	45 43 54.35	85 41 01.50	74 29 32.63	Gull Island	254 23 06.89	4.5986558	7.517
Do			139 59 59.58	Seul Choix	319 50 12.24	4.9545771	17.059
Do			177 54 29.17	Scott's Point	357 53 58.35	4.9198396	15.747
Gull Island	45 42 09.24	85 50 00.32	166 01 31.64	Seul Choix	345 58 10.80	4.9138056	15.530
Seul Choix	45 55 14.69	85 54 40.40					
Scott's Point	45 57 34.63	85 41 44.46					
Point Patterson	45 58 04.29	85 39 15.18	5 00 25.21	High Island	184 59 08.92	4.9366262	16.368
Biddle Point	46 04 32.01	85 22 41.20	60 51 16.64	Point Patterson	240 30 21.37	4.9052163	15.226
Manitou Payment	46 01 47.76	85 05 15.61	34 50 08.17	Hat Island	214 41 01.05	4.9751235	17.885
Do			102 49 05.28	Biddle Point	282 36 32.48	4.8785246	14.318
Pointe aux Chênes	45 55 39.82	84 54 48.49	5 45 35.65	*d*	185 44 33.02	4.7900886	11.680
Do			20 56 30.30	Middle Village	200 48 19.65	5.1345446	25.817
Do			67 50 01.37	Hat Island	247 33 24.58	5.0261687	20.116
Gros Cap	45 52 57.81	84 50 06.98	30 11 50.54	*d*	210 07 26.05	4.7159120	9.846
Do			78 44 18.56	Hat Island	258 24 20.16	5.0814289	22.845
d	45 45 34.05	84 56 15.78	102 59 14.14	Hat Island	282 43 41.34	4.9758813	17.916

* The position of this station is perhaps not identical with the one of the same name in the principal chain of triangles.

Geographical Coördinates of Stations at north ends of Lakes Michigan and Huron—Continued.

Stations.	Latitudes.	Longitudes.	Azimuths.	Stations.	Reverse azimuths.	Logarithms of distances in feet.	Distances in English miles.
	° ′ ″	° ′ ″	° ′ ″		° ′ ″		
A (West Base)	45 47 13.80	84 46 22.40	76 33 05.88	*d*	256 26 00.67	4.6360837	8.193
Do			155 29 32.20	Gros Cap	335 26 51.09	4.5832411	7.255
B′ (East Base)	45 45 22.14	84 42 02.99	121 37 25.32	West Base	301 34 19.44	4.3342326	4.089
Do			176 11 10.07	St. Ignace	356 10 46.34	4.5468605	6.672
D (St. Ignace)	45 51 09.12	84 43 36.09	33 56 26.56	West Base	213 53 44.26	4.4582410	5.440
Do			173 08 21.37	Rabbit's Back	353 07 50.59	4.4044164	4.806
E (Rabbit's Back)	45 55 17.84	84 43 18.97					
C (Mackinac Island)	45 51 28.73	84 36 58.03	17 42 14.47	*B*	197 39 58.42	4.6459962	8.382
Do			30 13 18.89	East Base	210 09 40.23	4.6330767	8.137
Do			57 11 30.79	West Base	237 04 46.04	4.6774798	9.013
Do			85 17 22.39	St. Ignace	265 13 19.81	4.3804089	4.548
Do			99 15 36.97	Gros Cap	279 06 10.69	4.7524569	10.711
Do			130 46 28.04	Rabbit's Back	310 41 54.53	4.5509998	6.735
Do			176 07 22.07	*H*	356 06 36.13	4.8240836	12.631
B	45 44 32.44	84 40 07.80	121 38 45.25	West Base	301 34 16.86	4.4939361	5.006
Do			165 21 34.39	St. Ignace	345 19 48.08	4.6183846	7.866
Round Island (astronomical station).	45 50 06.81	84 36 44.60	104 16 02.04	St. Ignace	284 11 49.86	4.4094734	4.862
Do			173 27 40.94	Mackinac Island	353 27 31.29	3.9217867	1.582
H	46 02 25.61	84 38 01.96	27 21 18.86	Rabbit's Back	207 17 30.89	4.6881917	9.238
I (Pointe St. Martin)	45 58 10.21	84 31 55.29	27 47 26.36	Mackinac Island	207 43 48.90	4.6623647	8.704
Do			135 01 23.58	*H*	314 56 59.80	4.5634411	6.931
L (Pointe Fuyard)	45 55 58.07	84 22 48.77	65 39 26.48	Mackinac Island	245 29 16.65	4.8193836	12.495
Do			109 10 16.03	Pointe St. Martin	289 03 43.22	4.6114235	7.741
O (Bois Blanc Island)	45 46 19.12	84 21 21.47	115 23 42.24	Mackinac Island	295 12 30.64	4.8655733	13.897
Do			173 59 37.73	Pointe Fuyard	353 58 35.09	4.7706277	11.168
N (Beaver Tail)	45 57 56.72	84 10 15.24	33 46 36.96	Bois Blanc Island	213 38 38.78	4.9291512	16.089
Do			77 21 18.91	Pointe Fuyard	257 12 17.34	4.7370795	10.338
Spectacle Reef	45 46 05.02	84 08 10.73	0 52 38.73	Mast	180 52 27.86	4.8485743	13.364
Do			91 32 17.50	Bois Blanc Island	271 22 50.90	4.7487619	10.620
Do			173 02 38.31	Beaver Tail	353 01 08.94	4.8611081	13.755
Mast	45 34 28.44	84 08 25.89	142 39 28.25	Bois Blanc Island	322 30 13.45	4.9572988	17.166

The following are descriptions of the markings of such of these stations as have been permanently marked. The dates are years in which the stations were occupied.

DOOR BLUFF.—See Chapter XV, A, § 2.

CEDAR RIVER.—See Chapter XV, A, § 2.

BOYER'S BLUFF, 1864.—This station was situated on the northwest end of Washington Island, Door County, Wisconsin. In 1871 the station was found to have been burned. The center probably had nothing to mark it, but it bears north 66° 39′ west, and is distant 90.2 feet from a transit-post used in 1863. Three stone reference-posts, with nails secured in their ends, are set, each 10 feet distant from the transit-post, bearing north, south, and east, respectively. The station was probably in the same, or nearly the same position, as the station of the same name described in Chapter XV, A, § 2.

BARK RIVER, 1864.—This station was situated on the west shore of Green Bay, near the mouth of Bark River, Delta County, Michigan. In 1871 it was found to be partially burned and without a center-mark. A stone post, with a nail leaded in its top, was then set 3 feet below the ground surface. Three reference-stakes of cedar were set, each 10 feet distant from the center-mark, and bearing north, south, and east, respectively, each being surrounded by a pile of stones.

BURNT BLUFF.—See Chapter XV, A, § 2.

ROCK ISLAND, 1864.—This station was situated on Rock Island, Door County, Wisconsin. The center was marked by a stone post of the usual form, set 4 feet below the surface of the ground. Four wooden reference-stakes were set 10 feet distant to the north, south, east, and west, respectively.

POINTE AUX BECS SCIES, 1860.—See description of Frankfort, Chapter XXVI, § 6.

Gros Cap, 1851, '53.—This station was situated on the north shore of the Straits of Mackinac, about two miles northeast of the island of Saint Helena. The height of the center-post used was 40 feet. The geodetic point was marked by a spike driven into a crevice in the limestone. The height of ground at the station above Lake Michigan is 143.5 feet.

A (West Base), 1851, '52, '53.—This station marking the northwest end of the Mackinac base-line, was situated on the south shore of the Straits of Mackinac, on McGulpin's Point. The height of center-post used was 27 feet. The geodetic point was marked by the intersection of two lines at the center of a square cut in the upper surface of a limestone block, the figure 7 being cut in one of the small squares. The height of ground at the station above Lake Michigan is 63.6 feet.

B′ (East Base), 1852, '53.—This station marked the southeast end of the Mackinac base-line, and was situated near the south shore of the south channel of the Straits of Mackinac, about 4 miles southeasterly from A (West Base). The geodetic point was marked by a fine line on a silver ten-cent piece set in the top of a limestone post 4½ feet long, dressed at its upper end. A side-stone was set 22 feet 4 inches north of the line, the top being marked by a square divided into four small squares, the number 14 being cut in one of the small squares. A similar stone was set at about the same distance south of the geodetic point, on line with the other two stones, this line not being at right angles to the base-line.

D (St. Ignace), 1851, '53.—This station was situated on Pointe St. Ignace, at the summit of the hill on the southeast end of a crest of a rocky ridge. The height of the center-post was 12 feet. The geodetic point was marked by a hole 2 inches in depth drilled in the solid limestone rock, 18 inches below the surface of the ground, and filled with broken nails. The height of the rock at the geodetic point above Lake Huron is 103 feet.

E (Rabbit's Back), 1851.—This station was situated on the summit of Rabbit's Back Peak, about 5 miles north of Point St. Ignace. The height of the center-post was 10 feet. The geodetic point was marked by three thirty-penny nails driven into a cleft of the rock.

C (Mackinac Island), 1851, '52, '53.—This station was situated at Fort Holmes, summit of the island of Mackinac. The height of the center-post was 22 feet. The geodetic point was marked by the center of a circle, as indicated by the intersection of two diameters, cut in a small granite bowlder.

B, 1852.—This station was situated on the south shore of the south channel of the Straits of Mackinac, about 6 miles southeast of A (West Base). It marked the southeast end of the Mackinac base-line as first laid out. The height of the center-post was 17 feet. The geodetic point was marked by the center of a square divided into four small squares cut in a square stone, the number 12 being cut in one of the small squares.

H, 1851.—This station was situated at Boiling Spring Point, four miles north of Great St. Martin's Island. The height of center-post was 12 feet. A great stone heap was made around the station.

I (Pointe St. Martin), 1851, '53.—This station was situated on Pointe St. Martin, on the north shore of the Straits of Mackinac, 1½ miles east by north from St. Martin's Island the less. The height of the center-post was 12 feet. The geodetic point was marked by a hole about one inch deep, drilled into a rectangular block of limestone about 9 inches thick, and weighing about 400 pounds. Piles of stones were made as supports to the center-post and its braces.

L (Pointe Fuyard), 1849, '51.—This station was situated on Pointe Fuyard, the southeast point of Marquette Island. The height of center-post was 14.5 feet. The geodetic point was marked by a hole drilled in a block of limestone which was set in the pebbles.

N (Beaver Tail), 1849, '51.—This station was situated on Beaver Tail Point, on the north shore of the Straits of Mackinac, about 10½ miles east of Pointe Fuyard. The height of the center-post was 26 feet. The geodetic point was marked by a hole drilled in the solid rock under the center-post.

§ **5.** In Chapter XXI, § 4, are also given the angles and sides of a triangulation in Lake Superior, stretching from the primary side, Vulcan — St. Ignace, eastward to Mamainse. The angles of some of these triangles were not well read, as will be seen by referring to Chapter XXI, § 4, but they will give geodetic positions with little error.

The results of the geodetic computation are given in the following table in the same form as in the preceding ones. The length and azimuth of the base-side, Vulcan — St. Ignace, namely logarithm 5.6900927, and 178° 21′ 39″.93, are taken from § 3.

Geographical Coördinates of Stations in east end of Lake Superior.

Stations.	Latitudes.	Longitudes.	Azimuths.	Stations.	Reverse azimuths.	Logarithms of distances in feet.	Distances in English miles.
	° ′ ″	° ′ ″	° ′ ″		° ′ ″		
Vulcan	47 26 46.65	87 47 38.17	178 21 39.93	St. Ignace	358 19 04.17	5.6900927	92.7819
St. Ignace	48 47 18.90	87 51 07.37					
Tip Top	48 16 24.64	86 00 12.83	56 11 44.06	Vulcan	234 52 04.50	5.7268419	100.974
Do			113 26 59.65	St. Ignace	292 03 53.07	5.6863956	91.995
Michipicoten	47 45 21.66	85 52 58.76	77 13 52.44	Vulcan	255 49 11.99	5.6855744	91.821
Do			171 09 33.74	Tip Top	351 04 11.09	5.2812042	36.188
Paugon	47 57 25.05	85 34 33.69	45 54 35.89	Michipicoten	225 40 56.53	5.0217410	19.912
Do			138 01 53.09	Tip Top	317 42 47.22	5.1923103	29.490
Gargantua	47 34 42.72	84 58 55.18	106 35 23.78	Michipicoten	285 55 25.95	5.3641071	43.800
Do			133 35 46.15	Paugon	313 09 22.72	5.3031891	38.068
Mamainse	47 01 36.54	84 36 28.71	130 34 15.14	Michipicoten	309 37 56.75	5.6158495	78.202
Do			155 22 49.00	Gargantua	335 06 19.42	5.3455870	41.971

The following are descriptions of such of these stations as have been permanently marked:

VULCAN.—See Chapter XIV, A, § 2.

ST. IGNACE.—See Chapter XIV, A, § 2.

TIP TOP.—See Chapter XIV, A, § 2.

MICHIPICOTEN.—See Chapter XIV, A, § 2.

PAUGON, 1869.—This station was situated on the northeast coast of Lake Superior, nearly due north of the east end of Michipicoten Island. The height of the station used was 27 feet. The geodetic point was marked by a brass pin set in the surface rock. The approximate height of the hill on which the station is located above Lake Superior is 850 feet.

GARGANTUA, 1869.—This station was located on the east coast of Lake Superior, four or five miles southeast of Cape Gargantua. The height of the station was 3.5 feet. The geodetic point was marked by a brass point about one-quarter inch in diameter, set in the rock. An astronomical stone post, used in 1869, bears north 25° 14′ 10″ east, and is 36.4 feet distant from the geodetic point. The front of the hill facing the lake rises nearly perpendicularly, and there are no large hills in the immediate vicinity. The height of the hill at the station above Lake Superior is 431 feet.

MAMAINSE, 1869.—This station was situated on the east coast of Lake Superior, and is approached from the north side of Bachewauaung Bay. The trail starts from a dock used by some mining company, and for the first four miles follows an old railway grade, whence it continues by a blazed trail the remaining five miles. The height of station used was 8 feet. The geodetic point was marked by a nail driven into a stake about 2½ inches in diameter, set about 1 foot below the ground surface. An astronomical stone post, used in 1869, bears south 74° 19′ 07″ east, and is 68.6 feet distant from the geodetic point. The approximate height of the hill on which the station was situated above Lake Superior is 1250 feet.

§ **6.** In Chapter XXI, § 5, are given the angles and sides of a detached triangulation in Saginaw Bay, Lake Huron. For this triangulation the fundamental latitude, longitude, and azimuth are those observed at the transit-post at Sand Point, Michigan, taken as the origin for the geodetic positions. Its latitude was determined by fifteen nights' work, with a zenith telescope, by Lieutenant C. N. Turnbull, in 1857, giving as a mean result 43° 54′ 39″.79. (See Report of Chief Topographical Engineer, U. S. A., 1858.) Its longitude was determined by Lieutenant Turnbull, in 1858–'59, by chronometer-transfers between Detroit, Fort Gratiot, Forestville, and Sand Point, as $0^h\ 1^m\ 19^s.288$ west of Detroit, (see reports of Chief Topographical Engineer, U. S. A., for 1859 and 1860). This value needs a correction of $-0^s.127$ to reduce it from the position of the transit-post in 1859 to the position of the east transit-post in the Lake-Survey Observatory at Detroit of 1871–1882, so that the longitude adopted for Sand Point is $0^h\ 1^m\ 19^s.16$ west from Detroit. Applying to this the longitude of east transit-post of 1871–1882, given in Chapter XXV, § 1, as $5^h\ 32^m\ 12^s.24$, there results, longitude of Sand Point trigonometrical station west from Greenwich, $5^h\ 33^m\ 31^s.40$. The azimuths of several stations from the transit-post appear to have been determined by reading at the transit-post the angles between the meridian mark and those stations. The results are given below as they were published in the Report of the Chief of Topographical Engineers for 1859, with the exception of the longitudes, which have been corrected as stated.

The only stations of this triangulation known to have been permanently marked are those of the Sand Point base-line. The following description of the markings of these base stations is taken from Captain George G. Meade's report to the Chief Topographical Engineer for 1857:

It has been previously reported that triangulation stations had been erected at the termini of the proposed base. The measurement accordingly began 100 yards to the east of the western terminus. This point was marked by a limestone post sunk to the depth of 3 feet, and rising some 6 inches above the ground. The top of this post, 10 inches square, has cut in it two lines at right angles, one of which is in the general direction of the base, and the intersection is the point of beginning. To further identify this point, two reference-stone posts were sunk level with the ground and at right angles to the measured line, the northern one being distant 70.75 feet and the southern one 73.7 feet. These posts have on them lines cut, the intersection of which are the points of the perpendicular to the base at the distances above mentioned. From the point of beginning an offset was measured to the northwest, at an angle of 60°, and 22 tubes measured, being about the distance of the point of beginning from the station at the end of the base. The termination of this offset was marked likewise by a stone. It will be as well to remark here that the eastern end of the measured line was marked in a similar manner. The reference-stones there being distant, the northern one 37.55 feet, and the southern 25.45 feet, the termination of this offset of 24 tubes was also marked by setting a stone in the ground, the offset here being to the southeast. In addition to the stones above described as left in the ground, there were two intermediate stones left on the measured line at the distances respectively of 330 and 693 tubes from point of beginning.

The tube-length here referred to is approximately 15 feet.

Geographical Coördinates of Stations on Saginaw Bay, Lake Huron.

Stations.	Latitudes.	Longitudes.	Azimuths.	Stations.	Reverse azimuths.	Logarithms of distances in feet.	Distances in English miles.
	° ′ ″	° ′ ″	° ′ ″		° ′ ″		
Pointe aux Barques	44 04 05.58	82 57 32.52	66 51 14.12	Oak Point	246 43 38.38	4.9366361	16.368
Do			119 09 18.18	Tawas Point	298 49 08.23	5.1604846	27.406
Do			135 08 22.21	Pointe au Sable	314 52 41.07	5.1436388	26.364
Oak Point	43 58 29.77	83 15 40.03	35 26 43.64	East Base	215 24 17.39	4.4248571	5.038
Do			57 19 15.92	West Base	237 13 31.25	4.6354105	8.180
Do			100 32 15.56	Little Charity	280 23 39.75	4.7420190	10.456
Do			155 36 32.76	Tawas Point	335 29 00.85	5.0583804	21.664
Do			171 46 11.61	Pointe au Sable	351 43 07.96	5.1265567	25.347
East Base	43 54 55.71	83 19 10.78	25 14 48.55	Wild Fowl	205 13 13.16	4.3736028	4.477
Do			85 26 47.11	West Base	265 23 28.80	4.3221344	3.976
Do			129 13 36.91	Little Charity	309 07 27.67	4.7005855	9.505
Wild Fowl	43 51 24.53	83 21 28.39	91 51 55.10	Sebouin	271 48 06.96	4.3826456	4.571
Do			151 08 59.71	West Base	331 07 16.89	4.3521900	4.261
Do			162 59 38.87	Observatory	342 48 41.60	4.3154513	3.916
Observatory	43 54 39.79	83 22 51.00	55 30 17.29	C	235 28 09.48	4.2142388	3.102
C	43 53 08.15	83 25 55.32					
West Base	43 54 39.15	83 23 56.71	35 00 11.68	Sebouin	214 58 06.25	4.3637724	4.377
Do			151 41 49.15	Little Charity	331 38 58.39	4.5788666	7.182
Sebouin	43 51 32.16	83 26 57.65	21 20 47.30	Fish Point	201 17 32.28	4.7548150	10.769
Do			128 47 18.70	Pointe aux Gres	308 37 48.95	4.8866178	14.588
Do			174 48 22.77	Little Charity	354 47 37.62	4.7203892	9.948
Little Charity	44 00 08.81	83 28 02.72	85 47 07.25	Pointe aux Gres	265 38 21.96	4.7437459	10.498
Do			120 27 06.25	Gravelly Point	300 22 27.33	4.5314505	6.439
Do			146 01 56.61	Whitestone Point	325 57 52.12	4.6619249	8.695
Do			161 41 50.81	Mason's Creek	341 38 09.78	4.8673528	13.955
Big Charity	44 02 17.46	83 26 27.97	27 59 47.50	Little Charity	207 58 41.66	4.1688279	2.794
Do			96 38 09.33	Gravelly Point	276 32 24.48	4.5619598	6.907
Pointe au Sable	44 20 17.79	83 20 03.64	44 45 15.11	Tawas Point	224 40 45.83	4.6004103	7.547
Tawas Point	44 15 38.14	83 26 29.19	4 09 18.25	Little Charity	184 08 13.12	4.9747189	17.868
Do			30 06 19.81	Whitestone Point	210 01 09.57	4.8112103	12.262
Do			51 08 16.69	Mason's Creek	231 03 29.91	4.5850753	7.285
Mason's Creek	44 11 39.56	83 33 20.34					
Whitestone Point	44 06 24.72	83 33 54.33					
Gravelly Point	44 02 58.79	83 34 44.06	50 33 38.87	Pointe aux Gres	230 29 32.24	4.5262332	6.362
Pointe aux Gres	43 59 27.88	83 40 38.96	41 52 36.85	Nyaquonk	221 42 17.20	4.9921419	18.600
Do			84 25 41.60	Pine River	264 18 21.85	4.6686010	8.830
Pine River	43 58 42.70	83 51 13.62	15 28 42.97	Nyaquonk	195 25 43.33	4.8526021	13.489
Nyaquonk	43 47 24.74	83 55 32.76					
Fish Point	43 42 49.02	83 31 39.49	57 42 52.72	Quannakissee	237 34 14.47	4.8149650	12.369
Do			105 00 02.63	Nyaquonk	284 43 31.47	5.0367569	20.612
Do			158 42 19.63	Pointe aux Gres	338 36 05.89	5.0357773	20.566
Quannakissee	43 37 03.81	83 44 10.09	141 29 26.00	Nyaquonk	321 21 34.32	4.9053464	15.230

§ **7.** Besides the points used as vertices of large triangles, the positions of many section-corners of the land surveys and of many light-houses and public buildings have been determined from the positions of the main triangulation and the astronomical stations. The positions of light-houses and public buildings have usually been determined by pointings from the main stations, while the positions of section-corners in many cases have been determined topographically. The connections of such points with the main triangulation stations rarely have a probable error exceeding 2 feet, and usually much less, so that the latitudes and longitudes of these points are known with an accuracy essentially the same as that of the coördinates of the main stations. These positions being all permanently marked and well-known points, the knowledge of their latitudes and longitudes is of great value for future maps of the counties and States in which they lie. On the St. Lawrence River some of the secondary triangulation points were well marked. Such have been included in the list of positions for that section, and a description of the markings is given. The positions of points marked with an (*) in the following table, depend on astronomical determinations in their immediate vicinity. The positions in Saginaw Bay, Lake Huron, marked with a (†) in the table, depend on that of the transit instrument at Sand Point, given in § 6. All others depend on the position of Fort Howard, given in § 3.

The points are arranged in the table by States and counties, following the order of their longitudes as far as possible, beginning with the most western points. Following the table is a description of such stations as are not sufficiently described in the table itself. In addition to the data given in these descriptions, there are in the records of the Lake-Survey, topographical sketches of the localities of nearly all stations described, although these sketches are less precise and embrace less area than the sketches of primary stations.

Table of Geographical Positions of Secondary Points.

State and county.	Description of point.	Latitude.	Longitude.
MINNESOTA.		° ′ ″	° ′ ″
Ramsey County	Southeast corner of custom-house, Saint Paul	*44 56 43.0	*93 05 34.0
Goodhue County	Astronomical post near northwest corner of court-house yard, Red Wing.	*44 33 44.2	*92 31 59.2
	Spire of Catholic church at Red Wing	*44 33 44.6	*92 31 49.7
Saint Louis County	Light-house on Minnesota Point	46 42 36.2	92 01 33.0
MISSOURI.			
Pike County	Northeast corner of high school building, Louisiana	*39 27 12.6	*91 03 08.2
	Southeast corner of brick residence on northwest corner of Noyles and Fourth streets, Louisiana.	*39 27 15.2	*91 03 07.4
	Stone observing-post in northwest corner of high school yard, Louisiana.	*39 27 14.3	*91 03 06.3
WISCONSIN.			
Saint Croix County	Southwest corner of section 33, township 31 north, range 19 west.	*45 07 22.1	*92 43 52.1
	Southeast corner of section 34, township 31 north, range 19 west.	*45 07 21.8	*92 41 24.6
Bayfield County	Light-house on Raspberry Island	46 58 12.8	90 48 16.7
	Light-house on Michigan Island	46 52 16.3	90 29 48.8
Grant County	Southwest corner of section 31, township 1 north, range 1 west.	*42 30 26.4	*90 26 44.2
Monroe County	Northeast corner of section 1, township 18 north, range 1 west.	*44 04 26.6	*90 25 56.7
	Southeast corner of section 13, township 18 north, range 1 west.	*44 01 49.6	*90 25 56.0
Wood County	Southwest corner of section 31, township 21 north, range 3 east.	*44 14 57.2	*90 11 48.9
	Southeast corner of section 31, township 21 north, range 3 east.	*44 14 57.3	*90 10 45.8
Oconto County	Northeast corner of section 3, township 30 north, range 23 east.	45 06 24.2	87 39 42.9

* Astronomical.

Table of Geographical Positions of Secondary Points—Continued.

State and county.	Description of point.	Latitude.	Longitude.
WISCONSIN—Continued.		° ′ ″	° ′ ″
Oconto County—Continued	Light-house on Green Island	45 03 23.2	87 29 34.6
Door County	Light-house on Chambers Island	45 12 08.2	87 21 53.6
	Light-house on Eagle Bluff	45 10 07.5	87 14 12.3
	Light-house on Rock Island (Pottawatomie)	45 25 39.5	86 49 41.6
Outagamie County	Center of dome of Saint Lawrence University, Appleton.	44 15 39.2	88 23 58.8
	Southeast corner of section 35, township 21 north, range 17 east.	44 14 38.7	88 23 43.7
Brown County	Dome of court-house, Green Bay	44 30 50.9	88 00 48.0
	Light-house on Long Tail Point	44 35 49.1	87 58 50.2
	Northwest corner of section 25, township 24 north, range 21 east.	44 31 53.7	87 54 25.2
Winnebago County	Southwest corner of section 13, township 20 north, range 16 east.	44 11 58.9	88 32 39.3
Calumet County	Northwest corner of section 31, township 20 north, range 19 east.	44 10 12.8	88 17 02.2
Fond du Lac County	Southwest corner of section 10, township 16 north, range 16 east.	43 51 54.3	88 35 03.8
	Northwest corner of southwest quarter of section 34, township 14 north, range 16 east.	43 38 22.5	88 34 54.9
	Southwest corner of section 34, township 14 north, range 16 east.	43 37 56.6	88 34 54.3
	Northeast corner of section 10, township 14 north, range 16 east.	43 42 19.0	88 33 47.3
	Northwest corner of section 4, township 14 north, range 17 east.	43 43 08.6	88 28 56.1
	Spire of Catholic church in Byron township	43 40 29.0	88 27 02.5
	Southwest corner of section 20, township 16 north, range 18 east.	43 50 01.7	88 22 55.4
Dodge County	Southwest corner of southeast quarter of section 29, township 12 north, range 15 east.	43 28 24.5	88 44 01.9
	Northwest corner of southwest quarter of section 3, township 9 north, range 16 east.	43 16 36.0	88 35 48.6
	Southwest corner of section 3, township 9 north, range 16 east.	43 16 09.7	88 35 48.3
	Northwest corner of section 3, township 13 north, range 16 east.	43 37 56.7	88 34 55.1
	Southeast corner of section 33, township 12 north, range 16 east.	43 27 28.6	88 34 52.6
	Southeast corner of section 32, township 11 north, range 17 east.	43 22 14.3	88 30 04.7
	Southeast corner of southwest quarter of section 33, township 11 north, range 17 east.	43 22 14.3	88 29 29.0
Waukesha County	Northwest corner of section 19, township 6 north, range 20 east.	42 58 25.8	88 11 16.3
Milwaukee County	Axis of figure of Justice on court-house, Milwaukee	43 02 31.8	87 54 18.4
	Spire of Catholic cathedral, Milwaukee	43 02 30.5	87 54 15.0
Racine County	Spire of Methodist Episcopal church, Racine	42 43 28.7	87 46 56.9
Kenosha County	Spire of Catholic church, Kenosha	42 35 32.9	87 49 17.7
ILLINOIS.			
Jo Daviess County	Northeast corner of Methodist Episcopal church, Galena	*42 25 11.5	*90 25 55.7
	Intersection of fourth principal meridian with State line between Illinois and Wisconsin.	*42 30 26.1	*90 25 33.6
Lake County	Southwest corner of section 27, township 46 north, range 10 east.	42 25 47.8	88 03 45.8
	Pinnacle of Congregational church, Dean's Corners	42 16 38.5	88 02 41.3
	Southeast corner of section 24, township 45 north, range 11 east.	42 21 21.6	87 53 06.4
	Northwest corner of section 6, township 46 north, range 12 east.	42 29 37.2	87 52 56.5
	Dome of court-house, Waukegan	42 21 33.9	87 50 58.6

*Astronomical.

Table of Geographical Positions of Secondary Points—Continued.

State and county.	Description of point.	Latitude.	Longitude.
ILLINOIS—Continued.		° ′ ″	° ′ ″
Lake County—Continued	Southwest corner of section 28, township 43 north, range 12 east.	42 10 02.3	87 50 51.6
	Spire of Presbyterian church, Waukegan	42 21 41.4	87 50 03.3
Cook County	Spire of Methodist Episcopal church on corner of Plum Grove and Wood streets, Palatine.	42 06 46.2	88 02 37.4
	Southwest corner of section 14, township 42 north, range 10 east.	42 06 37.5	88 02 38.4
	Flag-staff on high-school building on corner of Wood and Benton streets, Palatine.	42 06 48.8	88 02 31.1
	Southwest corner of section 34, township 38 north, range 13 west, Lyons township.	41 43 56.4	87 51 27.1
	Northwest corner of section 3, township 37 north, range 13 west, Palos township.	41 43 56.4	87 51 25.9
	Corner of sections 27, 28, 33, 34, township 36 north, range 12 east.	41 34 20.1	87 51 06.7
	Southwest corner of section 25, township 41 north, range 13 west, Maine township.	42 00 40.6	87 49 34.4
	Center of section 13, township 38 north, range 13 west, Palos township.	41 47 03.6	87 48 41.9
	Northeast corner of section 13, township 38 north, range 13 west, Lyons township.	41 47 30.8	87 48 05.9
	Northeast corner of section 27, township 38 north, range 12 west.	41 45 50.8	87 43 21.0
	Southeast corner of section 13, township 37 north, range 12 west.	41 41 30.5	87 40 52.4
	Spire of Baptist church at Morgan Park	41 41 37.2	87 40 39.5
	Center of tower of Northwestern University at Evanston.	42 03 06.5	87 40 34.2
	Tower of water works, Chicago	41 53 49.8	87 37 28.3
	Chicago light-house	41 53 20.5	87 36 52.2
	Tall tower of Chicago University	41 49 58.6	87 36 45.2
	Prominent brick chimney at Dalton	41 38 59.2	87 17 08.4
Du Page County	Northwest corner of section 4, township 39 north, range 11 east.	41 54 18.4	87 59 54.0
	Center of section 4, township 39 north, range 11 east	41 53 49.5	87 59 19.4
Rock Island County	Southeast corner of guard-house, Rock Island	*41 31 01.5	*90 33 46.0
	Astronomical post on United States arsenal grounds at Rock Island.	*41 31 03.4	*90 33 40.1
Will County	Flag-staff of elevator between Ash, Kansas, and Oak streets and Michigan Central Railroad, Frankfort.	41 29 55.5	87 51 00.8
	German Lutheran church spire at southeast corner of Ash street and main road running east from Frankfort.	41 29 45.0	87 50 58.8
	Corner of sections 10, 11, 14, 15, township 34 north, range 13 west.	41 26 27.0	87 49 33.0
	Southwest corner of section 20, township 34 north, range 11 west.	41 24 47.5	87 38 05.6
Kankakee County	Tower of high-school building at southeast corner of Merchant street and Indiana avenue, Kankakee.	41 07 06.2	87 51 40.6
	Spire of First Baptist church at northeast corner of Court street and Indiana avenue, Kankakee.	41 07 12.8	87 51 39.1
	Southwest corner of water-table of court-house between Merchant and Court streets, Harrison and Indiana avenues, Kankakee.	41 07 09.0	87 51 38.8
	Center of section 8, township 30 north, range 13 west	41 05 55.8	87 51 34.4
	Corner of section 15, 16, 21, 22, township 32 north, range 12 east.	41 15 01.9	87 50 19.5
	Spire of Methodist church on north side of North Second street, between Hickory street and section-line road, Manteno.	41 15 09.0	87 50 16.6
	Northeast corner of Catholic church on north side of South Third street, near Chestnut street, Manteno.	41 14 48.8	87 50 12.5

* Astronomical.

Table of Geographical Positions of Secondary Points—Continued.

State and county.	Description of point.	Latitude.	Longitude.
ILLINOIS—Continued.		° ′ ″	° ′ ″
Kankakee County—Continued	Southwest corner of section 4, township 29 north, range 12 west.	41 01 15.4	87 43 56.8
	Spire of Catholic church, St. Anne	41 01 36.2	87 43 19.6
	Northeast corner of section 20, township 32 north, range 11 west.	41 15 07.3	87 37 33.3
Ford County	Spire of Congregational church, a white frame building on Center street, between Taft and American streets, Paxton.	40 27 37.7	88 06 00.8
	Spire of Swedish Evangelical Lutheran church, a large white frame building in extreme southeast corner of Paxton.	40 27 23.0	88 05 24.4
	Corner of sections 14, 15, 22, 23, township 23 north, range 10 east.	40 26 39.8	88 02 27.7
	Land-survey corner, ¼ mile east of preceding section-corner.	40 26 39.9	88 02 10.6
Iroquois County	Southwest corner of southeast quarter of section 31, township 26 north, range 14 west.	40 40 46.5	87 59 00.5
	Corner of sections 33, 34, township 29 north, range 14 west, and sections 3, 4, township 28 north, range 14 west.	40 56 35.8	87 56 36.5
	Tower of school-house on north side of First avenue, between Forest and Locust avenues, Clifton.	40 56 15.9	87 56 11.0
	Northeast corner of Congregational church, between Second, Third, Locust, and Elliott streets, Clifton.	40 56 08.6	87 56 08.9
	Southeast corner of section 35, township 27 north, range 13 west.	40 46 19.0	87 47 12.5
	Northeast corner of section 2, township 26 north, range 13 west.	40 46 19.0	87 47 11.5
	Northeast corner of section 35, township 25 north, range 13 west.	40 35 19.7	87 47 07.3
	Tower of court-house, Watseka	40 46 28.9	87 44 09.0
Vermillion County	Spire of Embury Methodist church, near corner of sections 8, 9, 16, 17, township 20 north, range 13 west.	40 12 05.3	87 50 45.9
	Corner of sections 8, 9, 16, 17, township 20 north, range 13 west.	40 12 07.1	87 50 45.0
	Southeast corner of section 8, township 18 north, range 13 west.	40 01 35.3	87 50 37.1
	Spire of Baptist church, southwest corner of main and and South streets, Fairmount.	40 02 43.2	87 49 51.1
	Spire of Methodist church, northeast corner of High and South streets, Fairmount.	40 02 45.1	87 49 44.8
	Southeast corner of section 35, township 23 north, range 13 west.	40 24 05.0	87 46 46.8
	Southeast corner of section 36, township 23 north, range 13 west.	40 24 05.7	87 45 39.0
	Spire of Methodist church on south side of Main street, nearly south of school-house in eastern part of Hoopeston.	40 28 02.0	87 40 00.2
Champaign County	Center of east tower of Illinois Industrial University, Champaign.	40 06 33.1	88 13 38.5
	White steeple of church facing west, the westerly of two churches in Rantoul.	40 18 41.7	88 09 33.6
	Steeple (with cross) of easterly of two churches in Rantoul.	40 18 37.3	88 09 25.8
	Corner of sections 7, 18, township 19 north, range 10 east, and sections 12, 13, township 19 north, range 9 east.	40 06 47.8	88 07 28.9
	Corner of township 18 north, range 10 east; township 17 north, range 10 east; township 17 north, range 9 east; and township 18 north, range 9 east.	39 58 03.4	88 07 24.0
	Tall chimney at Sydney	40 01 25.6	88 04 10.7
	Center of section 35, township 22 north, range 10 east	40 19 12.2	88 01 52.2
Douglas County	Land-survey mark, one-fourth mile west of corner of sections 25, 26, 35, 36, township 15 north, range 10 east.	39 42 38.2	88 02 59.0
Edgar County	Spire of Methodist church on east side of Front street, between Buena Vista and Water streets, Kansas.	39 33 08.6	87 56 22.2

Table of Geographical Positions of Secondary Points—Continued.

State and county.	Description of point.	Latitude.	Longitude.
ILLINOIS—Continued.		° ′ ″	° ′ ″
Edgar County—Continued	Spire of Presbyterian church on southwest corner of North and Franklin streets, Kansas.	39 33 22.7	87 56 11.6
	Center of section 5, township 16 north, range 13 west	39 52 21.6	87 52 16.2
	Southwest corner of section 4, township 12 north, range 13 west.	39 30 43.3	87 52 06.4
	Corner of sections 5, 6, 7, 8, township 16 north, range 13 west.	39 51 55.5	87 51 42.0
	Spire of brick Methodist church, with clock in base of spire on east side of Main street, 200 feet north of Wood street, Paris.	39 36 45.6	87 41 41.6
	White steeple of brick Christian church on south side of Washington street, 200 feet east of Main street, Paris.	39 36 36.2	87 41 38.9
Coles County	Corner of sections 25, 26, 35, 36, township 12 north, range 10 east.	39 26 58.7	88 02 43.0
Clark County	High steeple of Methodist church in Westfield	39 27 21.4	88 00 01.8
	Steeple of Methodist church in Casey	39 17 58.4	87 59 27.9
	Tower of School-house at Casey	39 18 03.8	87 59 02.1
	Corner of sections 4, 5, township 8 north, range 13 west, Clark County, and section 32, township 9 north, range 13 west, Crawford County.	39 10 35.2	87 52 32.6
	Center of section 16, township 10 north, range 13 west	39 19 03.4	87 51 10.6
	Brown steeple of Catholic church, Marshall	39 23 15.4	87 41 40.9
Cumberland County	Northeast corner of section 18, township 9 north, range 11 east.	39 13 57.4	88 00 33.6
Jasper County	Northeast corner of section 21, township 5 north, range 10 east.	38 51 47.9	88 05 41.2
	Northeast corner of section 23, township 5 north, range 10 east.	38 51 50.8	88 03 25.5
	Corner of sections 1, 2, 11, 12, township 5 north, range 10 east.	38 53 37.1	88 03 25.2
	Northwest corner of section 19, township 5 north, fractional range 11 east.	38 51 51.9	88 02 18.4
	White spire, with black cross, of Catholic church at Saint Marie.	38 55 56.7	88 01 30.9
	Northwest corner of section 7, township 7 north, range 14 west.	39 04 08.3	88 01 25.9
	Northeast corner of section 7, township 7 north, fractional range 11 east.	39 04 08.2	88 01 25.8
Crawford County	Section corner. (See Clark County.)		
	Southeast corner of section 32, township 7 north, range 13 west.	38 59 51.6	87 52 17.6
Richland County	Corner of sections 15, 16, 21, 22, township 4 north, range 9 east.	38 46 28.7	88 12 22.1
	Northeast corner of section 2, township 4 north, range 9 east.	38 49 03.5	88 10 08.4
	Tower of school-house at corner of Kitchell avenue and Church street, Olney.	38 43 42.0	88 05 12.6
	Small spire at center of dome of court-house at Olney	38 43 50.6	88 05 08.7
	Tower of white frame Baptist church at Dundas	38 50 06.1	88 05 03.6
	Spire of Immanuels-Kirche der Ev. Gemeinschaft, on southeast corner of Church and Morgan streets, Olney.	38 43 42.8	88 04 53.5
	Southwest corner of northwest quarter of section 6, township 4 north, fractional range 11 east.	38 48 46.8	88 02 18.9
	Corner of sections 19, 30, township 2 north, fractional range 11 east, and sections 24, 25, township 2 north, range 10 east.	38 35 07.2	88 02 18.6
	Southeast corner of German Reformed church in section 6, township 4 north, range 11 east.	38 48 21.1	88 01 59.0
	Northwest corner of section 29, township 4 north, range 14 west.	38 45 42.5	88 00 11.9

Table of Geographical Positions of Secondary Points—Continued.

State and county.	Description of point.	Latitude.	Longitude.
TENNESSEE.		° ′ ″	° ′ ″
Shelby County	Southwest corner of Magnolia block on northeast corner of Front and Union streets, Memphis.	*35 08 37.5	*90 03 17.4
	Gas-lamp at northwest corner of Front and Monroe streets Memphis.	*35 08 40.9	*90 03 16.6
	Southwest corner of Vaccaro & Co.'s store, Memphis	*35 08 38.6	*90 03 16.5
MICHIGAN.			
Ontonagon County	Light-house, Ontonagon	46 52 27.6	89 19 33.4
Keweenaw County	Light-house, Isle Royale	48 05 23.6	88 34 45.6
Houghton County	Franklin shaft-house	47 08 33.4	88 34 10.2
	Light-house (Portage River)	46 58 41.1	88 24 50.7
Marquette County	Light-house, Huron Island	46 57 47.7	87 59 56.4
	Light-house, Granite Island	46 43 15.0	87 24 43.7
	Dome of City Hall at Marquette	46 32 30.0	87 23 47.6
	Spire of Methodist Episcopal church at Marquette	46 32 43.4	87 23 31.7
	Light-house, Marquette	46 32 47.9	87 22 34.7
	Day beacon, Stanard's Rock ‡	47 10 37.7	87 13 23.1
Delta County	Southeast corner of section 12, township 38 north, range 23 west.	45 41 12.9	87 06 03.4
	Light-house, Peninsula Point	45 40 04.6	86 58 02.4
	Northwest corner of northeast quarter of section 16, township 42 north, range 19 west.	46 02 31.4	86 41 17.4
Schoolcraft County	Southeast corner of section 4, township 45 north, range 20 west.	46 19 06.5	86 48 13.9
	Northeast corner of section 27, township 46 north, range 19 west.	46 21 44.0	86 39 27.7
Berrien County	Southeast corner of section 35, township 6 south, range 20 west.	41 53 58.6	86 35 23.9
	Spire of Congregational church at Saint Joseph	42 06 19.0	86 29 00.1
	Corner of sections 14, 15, 22, 23, township 8 south, range 18 west.	41 46 34.9	86 22 39.7
Van Buren County	Southwest corner of section 36, township 1 north, range 17 west.	*42 25 08.8	*86 15 56.3
	Intersection of axes of Main and Van Buren streets, Paw Paw.	*42 13 01.7	*85 53 03.4
	Southwest corner of section 7, township 3 south, range 13 west.	*42 12 55.0	*85 52 40.4
Allegan County	Northwest corner of section 28, township 2 north, range 13 west.	*42 32 10.5	*85 51 32.1
	Intersection of axes of Walnut and Trowbridge streets, Allegan.	*42 31 45.6	*85 51 09.7
	Northwest corner of county record building in Allegan	*42 31 44.7	*85 51 08.5
Cass County	Corner of sections 4, 5, 8, 9, township 8 south, range 16 west.	41 47 49.8	86 11 10.3
	Southwest corner of northwest quarter of section 27, township 7 south, range 14 west.	41 49 57.7	85 56 07.3
	Southwest corner of northwest quarter of section 36, township 7 south, range 13 west.	41 48 59.4	85 46 47.4
Newaygo County	Southwest corner of section 18, township 12 north, range 12 west.	*43 25 26.1	*85 48 10.4
	Intersection of axes of Wood and State streets, Newaygo	*43 25 15.4	*85 48 04.3
Kent County	Southwest corner of northwest quarter of section 30, township 7 north, range 11 west, at intersection of Fulton and Division streets, Grand Rapids.	*42 57 47.0	*85 40 06.2
	Intersection of axes of Fulton street and Jefferson avenue, Grand Rapids.	*42 57 47.1	*85 39 52.9
Kalamazoo County	Southwest corner of section 15, township 2 south, range 11 west.	*42 17 21.0	*85 35 20.1
	Intersection of axes of South and Church streets, Kalamazoo.	*42 17 24.6	*85 35 07.3
	Southwest corner of jail, Kalamazoo	*42 17 28.4	*85 35 05.7
Saint Joseph County	North cupola of school-house at Constantine	41 50 07.1	85 40 01.1
	First Baptist church at White Pigeon	41 47 47.4	85 38 35.4

* Astronomical. ‡ The light-house (1882) is approximately 2100 feet north and 550 feet west of the beacon.

Table of Geographical Positions of Secondary Points—Continued.

State and county.	Description of point.	Latitude.	Longitude.
MICHIGAN—Continued.		° ′ ″	° ′ ″
Saint Joseph County—Continued.	Woolen-mill at White Pigeon	41 47 41.9	85 38 28.9
	Church at Three Rivers	41 56 59.0	85 38 06.3
	Tall steeple at Three Rivers	41 56 54.2	85 38 04.5
	Southwest corner of southeast quarter of section 22, township 7 south, range 10 west.	41 50 21.4	85 27 28.8
	School-house at Sturgis	41 48 05.6	85 25 11.2
	Spire of Methodist Episcopal church at Sturgis	41 48 01.2	85 25 08.8
	Spire of Presbyterian church at Sturgis	41 48 03.5	85 25 08.5
Montcalm County	Southwest corner of section 31, township 11 north, range 6 west.	43 17 30.7	*85 05 03.2
	Northwest corner of county record office at Stanton	*43 17 29.0	*85 04 58.4
	Intersection of axes of Main and Camburn streets, Stanton.	*43 17 30.7	*85 04 52.5
Barry County	Northwest corner of section 17, township 3 north, range 8 west.	42 39 11.6	*85 17 23.3
	Intersection of axes of Broadway and Court streets, Hastings.	*42 38 50.7	*85 17 23.0
	Southwest corner of court-house at Hastings	*42 38 51.6	*85 17 20.4
Ionia County	Southeast corner of corner-stone of projected soldiers' monument at Ionia.	*42 58 52.9	*85 03 49.7
	Intersection of axes of Main and Kidd streets, Ionia	*42 58 51.9	*85 03 47.0
	Southeast corner of northeast quarter of section 19, township 7 north, range 6 west.	*42 58 42.1	*85 03 18.6
Calhoun County	Northwest corner of section 25, township 2 south, range 6 west.	*42 16 27.6	*84 57 49.2
	Intersection of axes of Kalamazoo avenue and Mansion street, Marshall.	42 16 24.7	*84 57 49.2
	Northwest corner of court-house at Marshall	42 16 16.7	*84 57 47.0
Branch County	Southwest corner of southeast quarter of section 16, township 7 south, range 8 west.	41 51 16.1	85 14 36.0
	Southwest corner of southeast quarter of section 7, township 6 south, range 5 west.	41 57 22.7	84 55 55.7
Eaton County	Northwest corner of jail at Charlotte	*42 33 52.4	*84 50 13.9
	Northeast corner of section 13, township 2 north, range 5 west.	*42 34 04.3	*84 50 10.1
	Intersection of axes of Oliver and Stoddard streets, Charlotte.	*42 33 59.0	*84 50 01.2
Gratiot County	Intersection of Washington and Mill streets, Saint Louis.	*43 24 33.0	*84 36 24.7
	Southwest corner of section 19, township 12 north, range 2 west.	*43 24 33.0	*84 36 19.0
Clinton County	Northwest corner of section 16, township 7 north, range 2 west.	*43 00 02.8	*84 33 51.1
	Intersection of axes of State and Ottawa streets, Saint Johns.	*43 00 02.6	*84 33 47.0
	Corner-stone at northwest corner of court-house at Saint Johns.	*43 00 00.1	*84 33 37.2
Hillsdale County	Corner of sections 22, 23, 26, 27, township 7 south, range 4 west.	41 50 22.7	84 44 53.2
	Spire of First Baptist church on east side of Chestnut street, between Michigan and Maple streets, Reading.	41 50 18.5	84 44 46.8
	Cupola of school-house on east side of Chestnut street, between Michigan and Maple streets, Reading.	41 50 16.2	84 44 45.4
	Southeast corner of factory of Colby Wringer Company, Reading.	41 50 07.1	84 44 33.2
	Northeast corner of section 26, township 7 south, range 4 west.	41 50 22.4	84 43 42.6
	Southwest corner of northwest quarter of section 27, township 6 south, range 3 west.	41 55 11.9	84 39 05.1
	Northwest corner of section 27, township 6 south, range 3 west.	41 55 38.1	84 39 05.0
	Clock-tower of college at Hillsdale	41 55 57.0	84 38 02.0
	Dome of court-house at Hillsdale	41 55 15.4	84 37 54.6

*Astronomical.

Table of Geographical Positions of Secondary Points—Continued.

State and county.	Description of point.	Latitude.	Longitude.
MICHIGAN—Continued.		° ′ ″	° ′ ″
Hillsdale County—Continued.	Southeast corner of section 8, township 5 south, range 1 west.	42 03 55.0	84 29 00.4
	Southeast corner of section 36, township 7 south, range 2 west.	41 48 41.0	84 28 34.8
	Corner of sections 19, 20, 29, 30, township 6 south, range 1 west.	41 55 38.6	84 27 34.2
	Southwest corner of northwest quarter of section 29, township 6 south, range 1 west.	41 55 12.7	84 27 34.2
	Church at Prattville	41 46 50.1	84 24 00.1
Jackson County	First Methodist Episcopal church on Main street, between Blackstone and Jackson streets, Jackson.	42 14 50.0	84 24 39.6
	Main tower of Michigan State Prison at Jackson	42 14 34.7	84 24 36.6
	Intersection of axes of Main and Jackson streets, Jackson.	*42 14 49.4	*84 24 30.0
	Brick chimney of Cornwell & Emerson's Chemical Works at Jackson.	42 15 45.5	84 24 23.9
	Intersection of axes of Main and Mechanic streets, Jackson.	*42 14 49.4	*84 24 19.2
	Northeast corner of Hibbard House at Jackson	*42 14 48.9	*84 24 13.7
	Northwest corner of section 2, township 3 south, range 1 west.	*42 14 49.3	*84 24 12.4
Ingham County	Northeast corner of section 16, township 4 north, range 2 west.	*42 44 21.9	*84 32 37.4
	Dome of State Capitol at Lansing	*42 43 56.1	*84 33 23.2
	Intersection of Washtenaw and Washington avenues, Lansing.	*42 43 47.6	*84 33 11.2
Saginaw County	East corner of court-house at Saginaw	*43 25 05.7	*83 57 50.3
	Intersection of Adams and Washington streets, Saginaw.	*43 25 02.0	*83 57 50.2
	South corner of jail at Saginaw	*43 25 08.4	*83 57 48.4
	Northeast corner of section 26, township 12 north, range 4 east.	*43 25 27.1	*83 57 25.3
Shiawassee County	Southwest corner of section 28, township 7 north, range 3 east.	*42 58 11.2	*84 07 40.3
	Northeast corner of school-house on corner of State street and Shiawassee avenue, Corunna.	*42 58 47.5	*84 07 05.0
	Intersection of axes of McArthur street and Shiawassee avenue, Corunna.	*42 58 52.0	*84 07 02.9
	Southwest corner of court-house at Corunna	*42 58 54.3	*84 07 00.8
Lenawee County	Spire of Congregational church with cross and clock-dial at Hudson.	41 51 12.6	84 21 25.8
	Northeast corner of section 1, township 6 south, range 1 east.	41 59 11.9	84 14 46.6
	Northeast corner of section 3, township 8 south, range 3 east.	41 49 00.2	84 02 30.1
	Flagstaff on engine-house No. 2, on southeast corner of Church and Tecumseh streets, Adrian.	41 53 45.4	84 01 36.8
	Presbyterian church on northeast corner of Chicago street and Maiden Lane, Tecumseh.	42 00 15.0	83 56 54.6
	Corner of sections 14, 15, 22, 23, township 6 south, range 4 east.	41 56 57.1	83 55 34.3
	Corner of sections 22, 23, 26, 27, township 7 south, range 5 east.	41 50 57.0	83 48 26.9
Bay County	Saginaw River light-house	†43 38 37.8	†83 51 17.5
	Charity light-house	†44 02 16.3	†83 26 27.7
Livingston County	Northwest corner of section 1, township 2 north, range 4 east.	*42 35 59.6	*83 56 01.6
	Intersection of axes of Grand River and Higgins streets, Howell.	*42 36 23.2	*83 55 42.6
	Southwest corner of court-house at Howell	*42 36 24.5	*83 55 41.4
Iosco County	Tawas light-house	†44 15 35.4	†83 26 37.7
Genesee County	Intersection of axes of Keasley and Saginaw streets, Flint.	*43 01 01.2	*83 41 29.3
	Intersection of axes of Keasley and East streets, Flint	*43 01 11.7	*83 41 06.4

* Astronomical. † Geodetic, depending on Sand Point astronomical.

Table of Geographical Positions of Secondary Points—Continued.

State and county.	Description of point.	Latitude.	Longitude.
MICHIGAN—Continued.		° ′ ″	° ′ ″
Monroe County	Corner of sections 25, 26, 35, 36, township 6 south, range 6 east.	41 55 28.2	83 40 18.7
	Northeast corner of section 5, township 8 south, range 7 east.	41 49 19.8	83 36 32.4
	Southwest corner of private claim No. 432	41 54 42.6	83 28 29.9
	Spire of Methodist Episcopal church on southeast corner of Monroe and Second streets, Monroe.	41 54 52.4	83 23 56.6
	Spire of Presbyterian church, Monroe	41 54 53.4	83 23 51.4
	Spire of German Lutheran church, southeast corner of Scott and Third streets, Monroe.	41 54 43.4	83 23 49.1
	Light-house, Monroe	41 53 27.7	83 19 53.2
	Pointe Mouillée	42 00 49.6	83 10 53.7
Lapeer County	Southwest corner of southeast quarter of section 32, township 8 north, range 10 east.	*43 03 26.3	*83 18 55.6
	Intersection of axes of Calhoun and Liberty streets, Lapeer.	*43 03 08.6	*83 18 53.5
	Intersection of axes of Calhoun and Franklin streets, Lapeer.	*43 03 07.3	*83 18 53.4
Oakland County	Cupola of court-house at Pontiac	*42 38 22.0	*83 17 30.0
	Intersection of axes of Saginaw and Church streets, Pontiac.	*42 38 06.7	*83 17 22.6
	Spire of Methodist Episcopal church on southeast corner of Thomas and Saginaw streets, Pontiac.	*42 38 03.2	*83 17 20.5
	Southeast corner of section 29, township 3 north, range 10 east.	*42 38 07.0	83 17 15.5
Wayne County	North Wyandotte	42 12 49.2	83 08 48.5
	Sugar Island	42 05 25.4	83 08 35.4
	Barker	42 07 39.5	83 08 26.3
	Biddle	42 08 02.9	83 08 17.8
	Brodhead	42 09 30.5	83 08 15.9
	Ecorse	42 15 21.4	83 08 09.5
	River Rouge	42 16 15.4	83 07 34.9
	Field	42 17 34.2	83 06 23.1
	Astronomical post (east pier) in Lake-Survey Observatory on south side of Grand River avenue, between Park Place and Washington avenue, Detroit.	42 20 00.8	83 03 07.4
	Intersection of axes of Washington and Grand River avenues, Detroit.	42 20 01.4	83 03 04.5
	Spire of Saint Paul's Episcopal church, on northeast corner of Congress and Shelby streets, Detroit.	42 19 45.9	83 02 53.6
	Tower of fire-engine house on Russell street, Detroit	42 20 41.0	80 02 21.5
INDIANA.			
Lake County	Southeast corner of northeast quarter of section 6, township 36 north, range 7 west.	41 36 05.1	87 15 40.5
La Porte County	Light-house, Michigan City	41 43 21.1	86 54 21.8
	Spire of Deutsche Ver. Ev. Luth. St. Joh. Kirche, southwest corner of Franklin and Ninth streets, Michigan City.	41 42 45.4	86 53 59.6
	Spire of high school between Eighth, Cedar, Ninth, and Pine streets, Michigan City.	41 42 51.8	86 53 47.6
	Northwest corner of section 10, township 36 north, range 4 west.	41 35 32.8	86 52 39.0
	Southeast corner of section 10, township 37 north, range 3 west.	41 39 52.2	86 44 19.7
	Southwest corner of southeast quarter of southwest quarter of section 32, township 38 north, range 2 west.	41 41 39.0	86 40 33.4
Saint Joseph County	Spire of Christian church at New Carlisle	41 42 22.3	86 30 41.9
	Spire of Methodist Episcopal church at New Carlisle	41 42 25.7	86 30 40.8
	Corner of sections 2, 3, 10, 11, township 37 north, range 1 west.	41 40 46.4	86 30 23.6

* Astronomical.

Table of Geographical Positions of Secondary Points—Continued.

State and county.	Description of point.	Latitude.	Longitude.
INDIANA—Continued.		° ′ ″	° ′ ″
Saint Joseph County—Continued.	Stand-pipe of waterworks at South Bend	41 40 31.4	86 14 52.6
	Dome of University of Notre Dame at South Bend in 1877.	41 42 11.0	86 14 20.9
	Corner of sections 17, 18, 19, 20, township 37 north, range 3 east.	41 39 04.4	86 12 57.3
Elkhart County	Corner of sections 2, 3, 10, 11, township 37 north, range 6 east.	41 40 54.8	85 48 43.5
La Grange County	Southwest corner of northwest quarter of section 33, township 38 north, range 8 east.	41 42 16.1	85 37 13.6
	Tower of large brick school-house at Lima	41 43 30.7	85 25 13.6
	Corner of sections 4, 5, 8, 9, township 37 north, range 11 east.	41 41 08.4	85 16 23.3
Steuben County	Northwest corner of section 31, township 38 north, range 14 east.	41 42 55.9	84 58 16.0
OHIO.			
Lucas County	Spire of Saint Mary's Catholic church, corner of Michigan and Cherry streets, Toledo.	41 39 32.1	83 32 02.5
	Turtle Island light-house	41 45 08.8	83 23 28.8
Ottawa County	West Sister Island light-house	41 44 13.1	83 06 39.0
	Green Island light-house	41 38 43.9	82 52 04.1
	Marblehead light-house	41 32 11.0	82 42 43.4
Erie County	Court-house at Sandusky	41 27 13.8	82 42 41.9
	Cedar Point light-house	41 29 17.4	82 41 37.4
	Spire of Presbyterian church on north corner of William and Homan streets, Huron.	41 23 42.6	82 33 21.0
	Huron light-house	41 24 03.1	82 32 51.1
	German Lutheran church at Vermillion	41 25 12.7	82 21 59.4
Loraine County	Spire of brick Second Congregational church on West College street, Oberlin.	41 17 28.9	82 13 10.5
	Pointed tower of Union school building on South Main street, Oberlin.	41 17 26.2	82 13 01.8
	Black River light-house	41 28 23.9	82 11 08.3
Cuyahoga County	Spire of stone Methodist church at Berea	41 21 46.7	81 51 12.5
	Spire of German Wallace College chapel, stone, at Berea	41 22 06.1	81 51 06.3
	Spire of stone Baptist church at North Royalton	41 18 53.6	81 44 10.7
	Spire of frame Methodist church in northern part of town of North Royalton.	41 19 29.2	81 44 10.4
	Cleveland light-house	41 30 01.6	81 42 09.6
	Euclid	41 35 37.3	81 33 05.2
Lake County	Light-house on pier, Fairport	41 45 41.6	81 16 48.8
	Light-house on bank, Fairport	41 45 24.8	81 16 39.7
	Spire of brick Congregational church on southwest corner of Main and Liberty streets, Painesville.	41 43 26.0	81 14 44.3
	Spire of Episcopal church on west side of State street, Painesville.	41 43 41.2	81 14 36.1
	Perry	41 48 04.7	81 08 55.6
Ashtabula County	Geneva	41 51 14.2	80 59 47.0
	Saybrook	41 53 33.0	80 49 55.0
	Baptist church, Ashtabula	41 52 02.7	80 47 01.6
PENNSYLVANIA.			
Erie County	State line	41 59 17.7	80 29 08.2
	Miles Grove	42 03 38.8	80 21 49.9
	Presqu'ile light-house	42 09 56.7	80 06 56.7
	Stand-pipe of waterworks at Erie	42 07 53.3	80 05 50.8
	Erie beacon light-house No. 2	42 09 11.9	80 04 40.2
	Harbor Creek	42 12 03.7	79 56 35.0
NEW YORK.			
Chautauqua County	Barcelona	42 20 32.0	79 35 24.4
	Methodist Episcopal church at Westfield	42 19 26.5	79 34 33.2
	Westfield High School	42 19 29.7	79 34 23.2
	Prospect	42 19 29.5	79 29 59.6
	Brocton	42 24 07.0	79 28 41.8

Table of Geographical Positions of Secondary Points—Continued.

State and county.	Description of point.	Latitude.	Longitude.
NEW YORK—Continued.		° ′ ″	° ′ ″
Chautauqua County—Cont'd	Dunkirk light-house	42 29 37.8	79 21 15.8
	Chimney of Brooks' locomotive works, Dunkirk	42 29 06.4	79 19 26.8
Erie County	Horse-shoe light-house	42 52 52.5	78 54 56.2
	Buffalo light-house	42 52 40.1	78 53 24.0
	Intersection of Michigan and Exchange streets, Buffalo	42 52 41.1	78 52 14.3
Niagara County	Fort Niagara light-house	43 15 42.3	79 03 40.0
	Northeast tower of railroad suspension bridge at Niagara Falls.	43 06 34.1	79 03 26.3
	Thirty-mile Point light	43 22 29.6	78 29 11.8
Orleans County	Spire of church at Yates	43 20 15.8	78 23 17.5
Monroe County	Corner of lots 5, 8, 11, 14, Brockport	43 11 54.0	77 56 39.3
	Baptist church on northwest corner of Main and Halloy streets, Brockport.	43 12 49.7	77 56 23.5
	Tower of waterworks at Rochester	43 12 45.8	77 37 36.7
	Court-house at Rochester	43 09 18.1	77 36 50.7
	Genesee light-house	43 15 09.8	77 36 40.2
Wayne County	Dome of academy at Sodus	43 14 08.5	77 04 09.3
	Big Sodus light-house	43 16 25.7	76 59 11.8
Oswego County	Oswego light-house	43 27 53.9	76 30 50.9
	Oswego court-house	43 27 26.0	76 30 22.8
	Greene	43 31 30.8	76 22 24.2
	Houston	43 31 30.3	76 15 19.2
Jefferson County	Galloo light-house	43 53 18.6	76 26 41.9
	Galloo	43 55 07.6	76 24 36.1
	Tibbett's Point light-house	44 06 02.3	76 22 15.4
	Vincent	44 07 31.4	76 20 32.1
	West Base, Cape Vincent	44 07 54.0	76 18 55.3
	Daly	44 10 03.6	76 18 35.1
	Stony Point light-house	43 50 21.6	76 17 56.8
	Ellis	44 08 34.2	76 17 44.0
	East Base, Cape Vincent	44 07 41.6	76 17 34.7
	Cooper	43 57 48.2	76 16 41.5
	Gleason	43 52 48.9	76 15 32.8
	Deuel	43 54 23.9	76 12 29.3
	Fox	43 58 05.6	76 12 04.2
	Hogsback	44 12 02.2	76 09 08.1
	Grindstone	44 16 04.0	76 08 57.8
	Sacket's Harbor, or Horse Island light-house	43 56 34.7	76 08 43.0
	Roman Catholic church at Clayton	44 14 18.8	76 05 12.2
	South Base, Clayton	44 14 04.1	76 04 34.1
	Dorr Farm	44 14 37.9	76 03 21.7
	Rock Island light-house	44 16 49	76 01 02
	Alexandria	44 19 38.9	75 55 23.4
	Thousand Island House, at Alexandria Bay	44 20 15.2	75 55 16.4
	Sunken Rock light-house	44 20 44.6	75 54 57.4
	Dingman	44 21 19.0	75 53 04.5
	Sister Island light-house	44 24 50.8	75 50 41.9
Saint Lawrence County	Lyons	44 25 01.5	75 49 00.7
	Cross-over Island light-house	44 29 48.6	75 46 44.1
	Peach Island	44 29 27.8	75 46 07.9
	Oak Point	44 30 56.9	75 45 13.1
	Hall's Dock	44 32 46.7	75 42 45.9
	Taylor	44 33 05.2	75 42 23.7
	Chapman	44 33 32.6	75 41 46.9
	Morristown Point	44 35 55.5	75 38 33.5
	Brooks Point	44 36 54.4	75 36 38.0
	I	44 37 39.9	75 35 44.6
	Ogdensburg light-house, or A	44 41 52.3	75 30 14.5
	Peak of Ogdensburg light-house	44 41 52.3	75 30 14.0
	West Base, Ogdensburg	44 42 19.0	75 28 48.5
	East Base, Ogdensburg	44 42 29.6	75 27 56.3

Table of Geographical Positions of Secondary Points—Continued.

State and county.	Description of point.	Latitude.	Longitude.
NEW YORK—Continued.		° ′ ″	° ′ ″
Saint Lawrence County—Continued.	Chimney Point	44 43 55.7	75 26 56.4
	Wagner	44 46 25.0	75 21 43.2
	Sparrowhawk	44 48 02.8	75 20 43.3
	Sharps	44 49 07.0	75 17 40.8
Franklin County	Boundary post No. 2	44 59 58.2	74 39 32.2
CANADA.			
Province of Ontario	Bois Blanc	42 06 09.7	83 07 28.3
	Hackett	42 05 15.5	83 07 17.9
	Bar Point, 1874	42 03 24.6	83 06 59.3
	Fighting Island	42 14 38.4	83 06 58.5
	Clark	42 08 19.0	83 06 57.4
	Turkey Island	42 11 09.4	83 06 50.7
	Bar Point, 1878	42 02 58.0	83 06 44.3
	Sandwich Spring	42 17 25.7	83 05 27.8
	Sandwich court-house	42 18 01.0	83 04 36.2
	East Sister	41 48 51.0	82 51 03.6
	Hen Island	41 47 20.2	82 47 48.8
	Kingsville	42 01 36.1	82 44 21.4
	Light-house, Middle Island	41 41 00.2	82 40 48.0
	Light-house, Pelée Island	41 49 54.8	82 38 21.7
	Light-house, Pelée Spit	41 52 17.9	82 30 25.3
	Light-house, Long Point, east extremity	42 33 00.4	80 03 21.4
	Mohawk light-house	42 50 02.3	79 31 23.2
	Port Dalhousie light-house	43 12 36.4	79 15 50.0
	Port Colborne light-house	42 52 30.5	79 15 03.2
	Court-house, Niagara	43 15 17.5	79 04 21.3
	Brock's Monument	43 09 36.3	79 03 12.7
	Timber Island	43 57 42.8	76 49 56.9
	False Duck light-house	43 56 53.4	76 47 55.4
	Nine-Mile, or Gage Point light-house, Simcoe Island	44 09 05.6	76 33 23.5
	Pigeon Island light-house	44 03 59.7	76 33 00.8
	Snake Island light-house	44 11 16.7	76 32 15.7
	Transit Pier in Observatory at Kingston	44 13 30.4	76 29 24.2
	Bear Point	44 05 43.4	76 26 33.2
	Brown Point light-house	44 13 56	76 23 55
	Rainy	44 09 41.0	76 21 36.0
	Burnt Island light-house	44 17 47	76 11 31
	Wolfe Island light-house	44 14 19.6	76 11 05.0
	Wesleyan Methodist church spire at Gananoque	44 19 41	76 10 08
	Jack-Straw light-house	44 19 31.1	76 07 11.8
	Gananoque light-house	44 19 28	76 04 54
	Smoke Island	44 20 28.6	76 04 13.7
	Lyndoc light-house	44 26 59	76 00 16
	Wells	44 19 07.6	75 59 07.3
	Darling	44 22 16.0	75 57 52.6
	Grenadier Island light-house	44 22 58.3	75 54 20.0
	Grenadier	44 23 40.3	75 53 32.9
	Chimney	44 28 10.7	75 50 41.7
	Slide	44 30 52.1	75 47 06.8
	Cole's Ferry light-house	44 31 58	75 45 26
	Molly's Gut Island	44 32 51.4	75 44 02.3
	McDonald's Point	44 33 21.9	75 43 02.3
	Moulson	44 33 42.3	75 42 32.6
	A. C. L. XXI	44 34 38.3	75 41 36.7
	Brockville Rock	44 34 48.2	75 40 38.1
	Campbell	44 35 47.6	75 40 13.5
	Eaton	44 36 32.7	75 39 13.3
	Southwest Base, Maitland	44 37 54.5	75 38 10.6
	K	44 37 21.0	75 37 50.6
	Northeast Base, Maitland	44 38 06.6	75 37 39.4
	Brennan	44 37 58.6	75 37 30.5

Table of Geographical Positions of Secondary Points—Continued.

State and county.	Description of point.	Latitude.	Longitude.
CANADA—Continued.		° ′ ″	° ′ ″
Province of Ontario—Cont'd..	Maitland	44 37 55. 9	75 36 52. 1
	G	41 39 48. 0	75 35 10. 8
	Railroad	44 41 17. 5	75 33 31. 2
	Windmill light-house	44 43 16	75 29 16
	Windmill Point	44 43 15. 6	75 29 15. 2
	Fraser Point	44 43 33. 9	75 28 44. 2
	Johnstown	44 44 30. 2	75 27 59. 9
	Edwardsburg	44 47 28. 9	75 22 44. 7
	Stethem	44 48 05. 2	75 22 02. 6
	Wort	44 48 52. 4	75 20 53. 4
	Binion	44 49 15. 6	75 19 52. 0

POINTE MOUILLÉE, 1873.—This station is situated on a point of land of the same name, which juts into the Detroit River at its mouth, about 100 feet from the shore. The soil is sand, and is liable to shift considerably from time to time. The height of station used was 10 feet. The geodetic point is marked by a ½-inch hole in a stone post 2 feet long, marked with the letters U. S. and set 1½ feet below the ground surface. Three stone reference-posts were set as follows: one bearing south 35° 30′ west, distant 7.6 metres; one bearing north 63° 32′ west, distant 6.1 metres; and one bearing north 35° 20′ east, distant 8.3 metres from the geodetic point. The height of ground at the station above the river is about 3 feet.

NORTH WYANDOTTE, 1873.—This station is situated in the northeast part of the city of Wyandotte, near the edge of the swamp, about 250 metres from the edge of the river and the same distance from the main road running parallel to the river, and about 100 metres southeast of the cemetery. The geodetic point is marked by a stone post 2 feet long, marked U. S., and 1½ feet below the ground surface.

SUGAR ISLAND, 1873.—This station is situated on the southeast side of Sugar Island, near the mouth of the Detroit River, about 40 feet from the bank, 10 feet above the water's level. The height of station used was 8 feet. The geodetic point is marked by a ½-inch hole drilled in the top of a stone post 2 feet long, marked U. S., and set 2 feet under ground. Two stone reference-posts were set, one bearing north 63° 06′ west, distant 39.17 feet; and one bearing south 22° 15′ west, distant 29.72 feet from the geodetic point.

BARKER, 1873.—This station is situated on Grosse Isle, Detroit River, about 500 feet above the Canada Southern Railroad bridge connecting Stony Island with Grosse Isle. It is 5 feet from the water's edge. The geodetic point is marked by a hole in a stone post 2 feet in length, marked U. S., and set 1½ feet below the ground surface.

BIDDLE, 1873.—This station is situated on the east side of Grosse Isle, Detroit River, about 2400 feet above station Barker, and 6 feet from the water's edge. The geodetic point is marked by a hole in the top of a stone post 2 feet long, marked U. S., and set 1½ feet below the ground surface.

BRODHEAD, 1873.—This station is situated on Grosse Isle, Detroit River, about 1½ miles from the north end of the island. The height of station used was 10 feet. The geodetic point is marked by a ½-inch hole in the top of a stone post 2 feet long, marked U. S., and set 1½ feet below the ground surface. Two stone reference-posts were set, one bearing south 42° 06′ west, distant 53.5 feet, and one bearing north 26° 56′ west, distant 26.5 feet from the geodetic point. The ground at the station is about 2 feet above the river.

ECORSE, 1873.—This station is situated about one mile above the present village of Ecorse, Wayne County, Michigan, about 850 feet east of the river road, in an open field, near the edge of the swamp bordering on the river, which is 3000 feet distant. The geodetic point is marked by a stone post 2 feet long, with the letters U. S. cut on it, set 1 foot below the ground surface.

RIVER ROUGE, 1873.—This station is situated on a point of land about 1½ miles below the mouth of the river Rouge, in Wayne County, Michigan, about half a mile east of the main traveled wagon-road running down the river from Detroit to Wyandotte. The height of station used was 6 feet. The geodetic point is marked by a hole in the top of a stone post 2 feet long, marked U. S., and set 1 foot below the ground surface. Two stone reference-posts were set, one bearing south 12° 45′ west, distant 48.05 feet, and one bearing north 74° 38′ west, distant 51.50 feet.

FIELD, 1873.—This station is situated near the northeast side of an open field about a fourth of a mile north of the river Rouge, about 850 feet southeast of the river road, and about 1500 feet from the bank of the Detroit River. The geodetic point is marked by a stone post 2 feet long, with the letters U. S. cut on it, and set 1 foot below the ground surface.

EUCLID, 1877.—This station is situated about 2 miles northwest of the village of Euclid, Cuyahoga County, Ohio, about 350 metres back from the shore of Lake Erie, and about 100 metres west of a north-and-south road leading to the lake. The geodetic point is marked by a single stone post. Two stone reference-posts are set on the west side of the road east of the geodetic point, one bearing north 64° 55′ east, distant 318 feet, and one bearing south 89° 55′ east, distant 293 feet.

PERRY, 1876.—This station is situated in a piece of woods, near the shore of Lake Erie, in Lake County, Ohio, about 2½ miles nearly due north of railroad station Perry, on the Lake Shore and Michigan Southern Railway, and 7½ miles east of Fairport. No station was erected. The geodetic point is marked by a hole drilled in the top of a stone post set 27 inches below the ground surface. Another stone post stands over the geodetic point as a surface-mark. Three stone reference-posts were set as follows: one bearing north 78° east, distant 52.0 metres; one bearing south 42° east, distant 37.1 metres; and one bearing south 76° west, distant 17.6 metres from the geodetic point. A rail fence running north and south is 123.5 metres distant to the east of the station.

GENEVA, 1876.—This station is situated on the shore of Lake Erie, 4½ miles northwesterly from the village of Geneva, Ashtabula County, Ohio, 10¾ miles from Ashtabula Harbor, and 16 miles from Fairport, on the farm of Jeremiah Seamens. Observations were made from a wooden post about 5 feet high. The geodetic point is marked by a hole drilled in the top of a stone post set 3 feet below the ground surface. A stone post is set directly over the geodetic point as a surface reference-mark. Three stone reference-posts were set as follows: one bearing south 46° 30′ east, distant 137.0 metres; one bearing south 1° 31′ west, distant 96.4 metres; and one bearing south 48° 15′ west, distant 136.0 metres from the geodetic point. The station was 32 metres from the lake, and 12 feet above it.

SAYBROOK, 1876.—This station is situated on the shore of Lake Erie, about 1½ miles west of Ashtabula Harbor, Ohio. It is in a small clump of woods between the highway and the lake. No station was erected. The geodetic point is marked by a hole drilled in the top of a stone post set 20 inches below the ground surface. A stone post is set directly over the geodetic point, rising 1 foot above the ground surface. Three stone reference-posts are set with their tops about 1 foot above ground, along the road fences, two being on the north side and one on the south side of the road, one bearing north 82° east, distant 76.5 metres; one bearing south 63° east, distant 19.5 metres; and one bearing south 51° west, distant 75.8 metres from the geodetic point. In 1876 the station was 42 metres back from the shore of the lake, which was a clay bluff rapidly wearing away, so that the station can exist but a few years.

STATE LINE, 1876.—This station is situated in Springfield Township, Erie County, Pennsylvania, 3150 metres easterly along the shore of Lake Erie from the State line between Ohio and Pennsylvania, 4150 metres easterly from the mouth of Turkey Creek, 5775 metres easterly from the mouth of the Conneaut River, 130 metres from the lake shore, 25 metres north of the wagon-road running parallel to the lake shore, and 340 metres west of bridge crossing Coon Creek. The geodetic point is marked by two stone posts in the usual manner. Three stone reference-posts were set as follows: one near the intersection of two fences on the north side of the road, bearing south 35° east, distant 26.5 metres; one on the south side of the road, bearing south 5° east, distant 36.4 metres; and one near a fence, bearing south 73° west, distant 12.1 metres from the geodetic point. The height of ground at the station above Lake Erie is about 55 feet.

MILES GROVE, 1876.—This station is situated in Girard Township, Erie County, Pennsylvania, about 60 metres from edge of Lake Erie, and about 700 metres east of mouth of Elk Creek. The station consisted of a wooden post 4 or 5 feet high. Three stone reference-posts were set as follows: one bearing south 88° 16′ east, distant 62.42 metres; one bearing south 24° 24′ west, distant

133.73 metres; and one bearing south 38° 49′ west, distant 48.81 metres. The height of the ground at the station above Lake Erie is about 124 feet.

HARBOR CREEK, 1876.—This station is situated about 3 miles northeast of Harbor Creek, Erie County, Pennsylvania, within 20 metres of the shore of Lake Erie. The geodetic point is marked by a single cut stone post. Three similar stone posts are set for reference-marks as follows: one bearing north 31° 56′ east, distant 10.51 metres; one bearing south 89° 11′ west, distant 19.66 metres; and one bearing north 11° 46′ west, distant 10.24 metres from the geodetic point. An azimuth station, used in shore-line work, occupied a position north 12° 32′ west, 17.60 metres distant from the geodetic point.

BARCELONA, 1876.—This station is situated a short distance east of the old light-house, Barcelona, Chautauqua County, New York, close to the shore of Lake Erie. The geodetic point is marked by a cut stone post set so that its upper surface is about 2 feet below the ground surface. A second post is set directly over the former as a surface-reference. Two stone reference-posts are set in the road south of the station, one bearing south 23° 24′ east, distant 22.75 metres, and one bearing south 36° 43′ west, distant 34.63 metres from the geodetic point.

PROSPECT, 1876.—This station is situated a short distance southeast of Prospect Railway station, Chautauqua County, New York. The geodetic point is marked by a cut stone post set so that its upper surface is about 2½ feet below the ground surface. Above this stone a second stone post is set as a surface-reference. Two stone reference-posts are set on the south side of the road north of the station, one bearing north 00° 54′ west, distant 211.5 metres, and one bearing north 35° 00′ west, distant 258.9 metres from the geodetic point.

BROCTON, 1876.—This station is situated about 1½ miles northwest of Brocton Junction, Chautauqua County, New York, close to the shore of Lake Erie. The geodetic point is marked by a cut stone post set so that its upper surface is about 2½ feet below the ground surface. Directly above this stone is a second stone rising above the ground surface as a reference-mark. Northeast 10 metres from the geodetic point is set a second reference-post, and southwest 9.94 metres a third post, the line joining these two posts passing through the geodetic point.

GREENE, 1874.—This station is situated on Nine-Mile Point, about nine miles east of Oswego, Oswego County, New York. The height of station used was 4.5 feet. The geodetic point is marked by a ⅜-inch hole drilled in the top of a stone post set 20 inches below the ground-surface. A stone post is set directly over it as a surface-mark. Three stone reference-posts were set on the west side of the road east of the station. The relative bearings of these stones and of other objects from Hammond's house, and their distances from the geodetic point are as follows, the bearings being read from a circle whose graduation is numbered in a direction opposite to that on a clock dial:

	°	*Metres.*
Middle of front of Hammond's house	00	161.6
Most southerly stone post	347	134.2
Most northerly stone post	20	112.9
Middle stone post	3	117.9
Stone heap	264	100.0
Rocky cliff	140	20.0

The height of ground at the station above Lake Ontario is about 40 feet.

HOUSTON, 1874.—This station is situated about one-fifth mile east of Little Salmon Creek, Oswego County, New York. The height of station used was 50 feet. The geodetic point is marked by a ⅜-inch hole drilled in a sandstone post, 6 inches by 9 inches by 2½ feet, set 1½ feet below the ground surface in a coarse, stony soil. Three stone reference-posts were set as follows: One bearing south 51° 17′ west, distant 36.61 metres; one bearing south 31° 38′ east, distant 11.72 metres; and one bearing north 86° 12′ east, distant 39.95 metres from the geodetic point. The height of ground at the station above Lake Ontario is about 20 feet.

GALLOO, 1874.—This station is situated on the north side of Galloo Island, near the west end of North Pond. The height of station used was 23 feet. The geodetic point is marked by a ½-inch hole drilled in the bed-rock and filled with lead, 20 inches below the ground surface. Three stone reference-posts were set as follows: one bearing north 78° west, distant 18.0 metres; one bearing south 11° west, distant 18.2 metres; and one bearing south 68° east, distant 11.5 metres. Tibbett's Point light-house bears north 8° 46′ east from the station.

VINCENT, 1874.—This station is situated near the corner of Broadway and Kannaday streets, in the town of Cape Vincent, Jefferson County, New York. The geodetic point is marked by a stone post 2 decimeters square having in its center a triangular piece of brass, and buried 7 decimeters under ground. Three marking-stones projecting 2 decimeters above the ground, dressed 2 decimeters square, marked U. S., and numbered, are situated as follows: No. 1 bears south 88° 14′ east, distant 44.62 metres; No. 2 bears south 13° 41′ east, distant 16.94 metres; and No. 3 bears south 46° 43′ east, distant 18.28 metres from the geodetic point.

WEST BASE, CAPE VINCENT, 1874.—This station is situated about 1 mile northeast of the town of Cape Vincent, Jefferson County, New York, on the north side of the Watertown and Cape Vincent Railroad. The height of station used was 7 feet. The geodetic point is marked by a piece of brass set in the top of a stone post set 2 feet below the surface of the ground. Three stone reference-posts are set as follows: No. 1 bearing north 78° west, distant 30.3 metres; No. 2 bearing south 14° west, distant 21.7 metres; and No. 3 bearing south 83° east, distant 30.2 metres.

DALY, 1874.—This station is situated on the west end of Carlton Island, about 20 feet back from the river, and 7 feet above it. The height of station used was 10 feet. The geodetic point is marked by a limestone boulder set about 2½ feet below the surface of the sandy soil, and having a ⅜-inch hole drilled 1 inch deep in its top. Reference-marks were made as follows: triangle on boulder marked No. 1 bearing north 45° west, distant 25.8 metres; mark No. 2 on bed-rock, bearing south 43° west, distant 10.5 metres; mark No. 3 on bed-rock bearing south 10° west, distant 19.0 metres; mark No. 4 on bed-rock bearing north 64° east, distant 48.0 metres; and mark No. 5 on bed-rock bearing north 30° east, distant 36.0 metres.

ELLIS, 1874.—This station is situated about 2½ miles northeast of Cape Vincent, Jefferson County, New York. The height of station used was 15 feet. The geodetic point is marked by a ⅜-inch hole drilled in a boulder 2½ feet below the surface of the ground. Three stone reference-posts are set as follows: No. 1 bearing south 51° west, distant 165.7 metres; No. 2 bearing south 27° west, distant 155.1 metres; No. 3 bearing south 3° west, distant 172.8 metres.

EAST BASE, CAPE VINCENT, 1874.—This station is situated about 2 miles east of Cape Vincent, Jefferson County, New York, on the south side of the Watertown and Cape Vincent Railroad. The height of station used was 7 feet. The geodetic point is marked by a piece of brass in the top of a stone post set 2½ feet below the surface of the ground. Three stone reference-posts are set as follows: one bearing south 72° 50′ east, distant 51.75 metres; one bearing north 74° 59′ east, distant 33.83 metres; and one bearing north 60° 51′ west, distant 52.62 metres.

COOPER, 1874.—This station is about one-fourth of a mile from the end of Peninsula Point, Jefferson County, New York, nearly west of Sacket's Harbor, New York. The height of station used was 9 feet. The geodetic point is marked by a ½-inch hole drilled in the top of a stone post set 1 foot below the surface of the ground. Three stone reference-posts were set as follows: one bearing south 68° west, distant 36.0 metres; one bearing north 73° east, distant 52.0 metres; and one bearing north 24° east, distant 62.5 metres.

GLEASON, 1874.—This station is situated about 4 miles northeast of Stony Point light-house, and about 1⅝ miles southwest of the west end of Snow Shoe Bay, in Jefferson County, New York. The height of station used was 34.4 feet. The geodetic point is marked by a triangular hole drilled in the bed-rock one foot beneath the ground surface. A surface reference-stone, rising flush with the ground, is set directly over the geodetic point. Three stone reference-posts are set as follows: No. 1 bearing north 77° east, distant 44.0 metres; No. 2 bearing south 8° west, distant 47.5 metres; No. 3 bearing south 31° east, distant 46.6 metres.

DEUEL, 1874.—This station is situated about 4 miles southwest of Sacket's Harbor light-house, on the north side of Six Town Point, Jefferson County, New York. The height of station used was 10 feet. The geodetic point is marked by a ½-inch hole 1 inch deep, drilled in the top of a stone post set 20 inches below the ground surface. Three stone reference-posts are set as follows: one bearing north 89° east, distant 15 metres; one bearing south 73° west, distant 15.25 metres; and one bearing south 3° east, distant 12.3 metres. The height of ground at the station above the river is about 10 feet.

FOX, 1874.—This station is situated on Pillar Point, Jefferson County, New York, northwest of Sacket's Harbor. The height of station used was 10 feet. The geodetic point is marked by a ½-inch hole, drilled in the top of a stone post set 1 foot below the ground surface. A stone reference-post bears north 64° east, distant 24.0 metres; a cross on a large boulder, with a hole filled with lead in its center, bears north 3° east, distant 26.2 metres; a similar cross on the bed-rock (limestone) bears south 39° west, and is distant 18.0 metres.

HOGSBACK, 1873.—This station is situated on the northeast extreme point of the hill known as Hogsback, about 4½ miles southwest of Clayton, Jefferson County, New York, about 1 mile from the Saint Lawrence River, and near French Creek. It is the highest point in the vicinity, being at least 100 feet above the general level of the land towards the river, and 200 feet above the river. The height of station used was 20 feet. The geodetic point is marked by a triangle drilled in a small boulder set about 2 feet below the ground surface. A stone surface-reference is set directly over the point. Two triangles cut on the upper surfaces of boulders are situated as follows: one bearing north 59° 46′ east, distant 24.3 metres, and one bearing south 41° 14′ east, distant 11.3 metres.

GRINDSTONE, 1873.—This station is situated on the northwest side of Grindstone Island. The hill on which it stands is about 15 feet higher a little west of the station than at the station. It is about one-third of a mile from the river, on the northwest side of the island. The height of station used was 20 feet. The geodetic point is marked by a cross cut in the surface-rock, and referred to a hole 1 inch in diameter drilled into the rock, which projects about 1 foot above the general level, distant 0.136 metre from the geodetic point. A prominent rock bears south 46° 43′ east, distant 132.0 metres.

SOUTH BASE, CLAYTON, 1873.—This station is situated about two-thirds of a mile southeast of the village of Clayton, Jefferson County, New York, near to and on the east side of a railroad cutting. The height of station used was 8 feet. The geodetic point is marked by two stone reference-posts set centrally one above the other, the upper serving as a surface-reference. Three stone reference-posts are set as follows: one bearing south 27° west, distant 60 feet 5 inches; one bearing north 58° west, distant 114 feet 7 inches; and one bearing north 27° east, distant 114 feet 3.5 inches. The perpendicular distance to the west rail of the railroad was 96 feet.

DORR FARM, 1873.—This station is situated about 2500 metres east of Clayton, Jefferson County, New York, on a bluff about 40 feet above the adjoining land, and 90 feet above the Saint Lawrence River. The height of station used was 20 feet. The geodetic point is marked by a cross cut in the solid rock. A triangular hole about 1¼ inches deep is cut in the rock 12 centimeters west of the cross.

ALEXANDRIA, 1872.—This station is situated about 1000 metres southwest of Alexandria Bay, Jefferson County, New York, on a hill about 100 feet high. The height of station used was 20 feet. The geodetic point is marked by a hole drilled in the surface-rock.

DINGMAN, 1873.—This station is situated about 2 miles below Alexandria Bay, Jefferson County, New York, about 1200 metres nearly due west of the point where two small streams unite and flow into the Saint Lawrence. The height of station used was 7 feet. The geodetic point is defined by four boulders each having a triangle cut on its surface. Two of these stones bear south 40° 14′ west, and are distant 2.06 metres and 1.12 metres, respectively. The other two bear south 43° 11′ east, and are distant 2.17 metres and 1.41 metres, respectively. They are all buried beneath the ground surface. A triangle cut on the surface of a large boulder bears south 31° 12′ west, and is 29.33 metres distant. A stone fence bears south 9° 56′ west, and is 101.00 metres distant.

LYONS, 1872, '73.—This station is situated on a small, almost bare island, about 80 metres wide and 120 metres long, about 400 metres west of the southwest end of Oak Island. The station is about one-third the length of the island from the east end. The island rises from 4 to 6 feet above the water. The height of station used was 20 feet. The geodetic point is marked by a hole 1½ inches deep, drilled in the bare, solid rock.

PEACH ISLAND, 1872, '73.—This station is situated on Peach Island, about 2 miles above Oak Point, Saint Lawrence County, New York. The island is about 200 feet long and 45 feet wide, and the station is near the center of the island. The height of station used was 3 feet. The geodetic point is marked by a hole drilled in the rock. The height of ground above the river is 4.5 feet.

OAK POINT, 1872.—This station is situated on a small rocky island near Oak Point, Hammond Township, Saint Lawrence County, New York. The island is about 300 feet from the shore. The station is 12 feet from the water's edge, on the southwest side of the island. The height of station used was 4 feet. The geodetic point is marked by a hole drilled in the rock.

HALL'S DOCK, 1872.—This station is situated on the bank of the Saint Lawrence River, in Saint Lawrence County, New York, nearly due east of Molly's Gut Island, on a low, flat, rocky point about 2 feet above the water. The height of station used was 3 feet. The geodetic point is marked by a hole drilled in the rock.

TAYLOR, 1872.—This station is situated in Morristown Township, Saint Lawrence County, New York, opposite Brock's group of islands, and about 3¾ miles above Morristown. The height of station used was 8 feet. The geodetic point is marked by a hole drilled in the surface-rock.

CHAPMAN, 1872.—This station is situated on the river bank about 2½ miles above Morristown, New York. It stands on a flat rock 9 feet from the water's edge and 2 feet from the edge of the rock. The height of station used was 3 feet. The geodetic point is marked by a hole drilled in the rock.

MORRISTOWN POINT, 1873.—This station is situated about 1 mile northeast of the village of Morristown, Saint Lawrence County, New York, on the bank of the Saint Lawrence River. The geodetic point is marked by a hole drilled in the rock.

BROOKS POINT, 1873.—This station is situated in Morristown Township, Saint Lawrence County, New York, about 15 feet from the bank of the Saint Lawrence River, at Brooks Point, which is nearly due south from Maitland, Ontario. The height of station used was 4 feet. The geodetic point is marked by a stone post of the usual form, set 2½ feet below the ground surface. The coördinates of two reference-marks, consisting of triangles cut on boulders, are as follows: One bearing north 75° 43′ west, distant 3.67 metres, and one bearing north 2° 49′ east, distant 4.92 metres.

I, 1873.—This station is situated on the south side of, and within 10 feet of the Saint Lawrence River, above Ogdensburg, N. Y., and nearly opposite Maitland, Ontario. The height of station used was 4 feet. The geodetic point is marked by a triangle cut in the rock with a small hole drilled in its center.

LIGHT-HOUSE, or A, 1871.—This station is situated a short distance west of the light-house near Ogdensburg, N. Y. It stands west of the light-house building, and is 35 feet from the southwest corner of the light-house and 36 feet from the southwest corner of the light-house tower. The height of station used was 3 feet. The geodetic point is marked by a stone in the usual manner.

WEST BASE, OGDENSBURG, 1871–'73.—This station is situated in the eastern part of the city of Ogdensburg, N. Y., near to and on the south side of the line of the Ogdensburg and Lake Champlain Railroad. It is on a high bank above the railroad track, directly on line between the elevator and south corner of Washington and Tate streets, which corner is 160 feet distant. The height of station used was 8 feet. The geodetic point is (probably) marked by a single stone buried 2 feet below the ground surface.

EAST BASE, OGDENSBURG, 1871.—This station is situated to the east of Ogdensburg, N. Y., on the north side of the line of the Ogdensburg and Lake Champlain Railroad, and just west of the second bridge over the track. It stands on a high bank 25 feet above the railroad track. The height of station used was 10 feet. The geodetic point is marked by a stone in the usual manner.

CHIMNEY POINT, 1871.—This station is situated on Chimney Point, about 3 miles below Ogdensburg, Saint Lawrence County, New York. The height of station used was 10 feet. The geodetic point is marked by a stone with a small triangle cut on it. A surface-stone is set directly over it.

WAGNER, 1873.—This station is situated on the south side of the Saint Lawrence River, nearly due south from the east side of Sheldon's Island, about one-third of a mile from the river, and about 100 metres south of the highway running approximately parallel to the river. The height of station used was 40 feet. The geodetic point is marked by a cut stone post set about 2 feet below the ground surface, with a stone post set directly over it as a surface-reference. North-northwest 5.3 feet is a triangular hole drilled in the surface of a large boulder.

SPARROWHAWK, 1872, '73.—This station is situated on Long Point nearly due south from the west end of Presqu' Isle, and about 250 metres from the shore of the Saint Lawrence River. The geodetic point is marked by two stone posts set centrally one above the other, the upper one serving as a surface-mark. 68.2 feet northwest is a triangle engraved on the surface of a deeply buried boulder. The most easterly corner of Mr. Sparrowhawk's house bears north 8° west, and is about 350 feet distant.

SHARPS, 1872, '73.—This station is situated on the south shore of the Saint Lawrence River, in a line bearing south 41° east from Iroquois Point, about 500 metres from the shore, and about 130 metres south of the highway. The height of station used was 20 feet. The geodetic point is marked by two stones set one above the other, the lower one being about 4 feet below the ground surface, and the upper one serving as a surface-mark. Two boulders having holes drilled in their upper surfaces are distant 30.9 feet and 35.7 feet, respectively, from the geodetic point, in a northwesterly direction.

BOUNDARY POST NO. 2, 1871, '72.—This station is situated on the boundary line between the United States and Canada, about 170 metres east of the bank of the Saint Lawrence River and 1000 metres west of the town of Saint Regis. It is of cast iron, square and tapering, projecting 4 feet above ground. The post is marked with the date of the boundary survey. The height of ground above the Saint Lawrence River is 25 feet.

BOIS BLANC, 1873.—This station is situated on the northwest part of the upper end of Bois Blanc Island, Detroit River, about 160 feet from the shore and 8 feet above the level of the water. The geodetic point is marked by a hole in the top of a stone post set 2 feet long, marked U. S., and set 1½ feet below the ground surface.

HACKETT, 1873.—This station is situated on the south end of Bois Blanc Island, Detroit River, about 500 feet northwest of Bois Blanc light-house, and about 10 feet from the edge of the bank, which is nearly perpendicular. The geodetic point is marked by a ½-inch hole in the top of a stone post 2 feet long, marked U. S., and set 1½ feet below the ground surface.

BAR POINT, 1873, '74.—This station was situated about 880 metres northward along the shore of the Detroit River, from station Bar Point of 1877, '78, about 3 metres from the water's edge, on ground about 2 feet above the water-level. The height of station used was 10 feet. The geodetic point is marked by a single stone post 2 feet long, marked U. S., and sunk 1 foot below the ground surface. The distance to station Sugar Island is 4332 metres.

FIGHTING ISLAND, 1873.—This station is situated at the north end of Fighting Island, Detroit River, about 35 feet from the water's edge, and 3 feet above it. The height of station used was 10 feet. The geodetic point is marked by a stone post 2 feet long, marked with the letters U. S., and sunk 1½ feet beneath the surface. Two stone reference-posts were set, one bearing south 31° 32′ west, distant 40.8 feet, and one bearing north 60° 38′ west, distant 24.85 feet from the geodetic point.

CLARK, 1873.—This station is located about three-fourths of a mile above Texas Dock, Essex County, Ontario, about 8 feet from the edge of the river bank, which is nearly perpendicular, and about 15 feet above the level of the water. The geodetic point is marked by a stone post 2 feet long, with the letters U. S. cut on it, set 1½ feet below the ground surface.

TURKEY ISLAND, 1873.—This station is situated on Turkey Island, Detroit River, about one-fourth of a mile from the north end and one-fourth of a mile from the east side of the island. The height of station used was 6 feet. The geodetic point is marked by a stone post 2 feet long set 1 foot below the ground surface. Two stone reference-posts were set, one bearing south 9° 34′ west, distant 14.35 feet, and one bearing south 86° 02′ east, distant 15.65 feet from the geodetic point. The ground at the station is about 3½ feet above the river.

BAR POINT, 1877, '78.—This station is situated at the mouth of the Detroit River, in Essex County, Ontario, on a point of land known as Bar Point, about 5 metres from the water's edge and about 2 feet above the water-level. The height of station used was about 15 feet. The geodetic point is marked by a stone post set below the ground surface. Another stone post stands over the geodetic point for a surface-mark. Three stone reference-posts were set as follows: one bearing south 68° east, distant 41 feet; one bearing north 33° east, distant 49 feet; and one bearing north 15° west, distant 58.2 feet from the geodetic point. The distance to station Sugar Island is 5216 metres. (See Bar Point of 1873, '74.)

SANDWICH SPRING, 1873.—This station is situated near the mineral springs west of the village of Sandwich, Essex County, Ontario, on the north side of the canal running up towards the springs, and about 30 feet from the edge of the Detroit River. The geodetic point is marked by a stone post 2 feet long, with the letters U. S. cut on it, set 2 feet below the ground surface.

EAST SISTER, 1877.—This station is situated on the northeast point of East Sister Island, Essex County, Ontario. The height of station used was 5 feet. The geodetic point is marked by a 1¼-inch hole, 3½ inches deep, drilled in the solid limestone rock 3 feet below the ground surface, the lower part of the hole being filled with lead. A stone post with a hole drilled in its top and the letters U. S. cut on one side is set directly over the geodetic point as a surface-reference. Three stone reference-posts, with the letters U. S. cut on their tops, were set as follows: one bearing south 64° 43′ west, distant 6.7 metres; one bearing south 75° 54′ west, distant 17.2 metres; and one bearing south 83° 16′ west, distant 29.2 metres.

HEN ISLAND, 1877.—This station is situated on the east side of Hen Island, Essex County, Ontario. The height of station used was 5 feet. The geodetic point is marked by a 1-inch hole filled with lead in the limestone rock 20 inches below the surface of the soil. An irregular excavation from which the loose rock was picked leads down to the geodetic point, over which a stone post, marked with the letters U. S., was set as a surface-mark. Three stone reference-posts, having the letters U. S. cut on them, were set as follows: one bearing south 19° 14′ west, distant 27.4 metres; one bearing south 45° 33′ west, distant 12.9 metres; and one bearing north 89° 39′ west, distant 7.1 metres from the geodetic point.

KINGSVILLE, 1877.—This station is situated on the shore of Lake Erie about three-fourths of a mile south of the village of Kingsville, Grosfield Township, Essex County, Ontario. The height of station used was 29 feet. The geodetic point is marked by a stone post set 2 feet below the ground surface. Directly above it is set a similar post as a surface-reference. Three stone reference-posts were set as follows: one bearing north 78° 00′ east, distant 20.1 metres; one bearing south 78° 32′ west, distant 36.3 metres; and one bearing north 10° 27′ west, distant 33.3 metres from the geodetic point. The southeast corner of John Herrington's brick house bears north 7° 47′ east, and is 143.7 metres distant. The height of ground at the station above Lake Erie was 24 feet. The ground at the station is liable to be washed away by the lake within a few years. In 1877 the station was about 17 metres back from the bank, which is of clay, and gradually wearing away.

TIMBER ISLAND, 1874.—This station is situated on the northeast end of Timber Island, Lake Ontario. The height of station used was 20 feet. The geodetic point is marked by a hole drilled in the bed-rock and filled with lead, 1 foot below the ground surface. Three stone reference-posts were set as follows: one bearing north 62° 36′ west, distant 53.5 metres; one bearing south 61° 35′ west, distant 32.5 metres; and one bearing south 27° 53′ east, distant 33.9 metres from the geodetic point.

BEAR POINT, 1874.—This station is situated on Bear Point, at the southwest end of Wolfe or Long Island. The height of station used was 15 feet. The geodetic point is marked by a ⅜-inch hole 1 inch deep, drilled in the solid limestone rock 20 inches below the ground surface. A stone post 20 inches long is set over the geodetic point as a surface-mark. Three stone reference-posts are set as follows: one bearing north 57° east, distant 75.0 metres; one bearing north 18° east, distant 37.5 metres; and one bearing south 60° west, distant 34.2 metres from the geodetic point.

RAINY, 1874.—This station is situated on the south side of Wolfe Island, close to the shore of Button Bay, and a short distance east of a road leading back from the bay, the road being about two miles southwest of the entrance to Big Bay. The geodetic point is marked by a triangular piece of brass 4 millimeters on a side set in the top of a stone dressed 2 decimeters square and set 6 decimeters below the ground surface. A similar stone is set above this as a surface-reference. Three stone reference-posts are set as follows: No. 1 bearing south 74° west, distant 52.09 metres; No. 2 bearing south 46° west, distant 34.31 metres; and No. 3 bearing north 63° west, distant 68.50 metres from the geodetic point.

SMOKE ISLAND, 1873.—This station is situated on the west side of Smoke Island, Leeds County, Ontario, about 170 metres from the north end of the island, and near its highest point. The height of station used was 20 feet. The geodetic point is marked by a hole drilled into the rock about 2 feet below the ground surface. A cut sandstone post is set directly above as a surface-mark. A triangle cut in the solid rock bears south 89° 27′ west, and is 3.05 metres distant from the geodetic point.

WELLS, 1872, '73.—This station is situated near the center of Wellesley Island, Saint Lawrence River, on a rocky hill about 140 feet high and about 400 metres south of the shore of Lake Waterloo. The height of station used was 35 feet. The geodetic point is marked by a triangle drilled in the solid rock 1.5 feet below the ground surface. A triangle cut on white smooth rock bears south 39° 19′ west, distant 4.3 metres from the geodetic point.

DARLING, 1872, '73.—This station is situated on the highest part of the ridge northeast of Darling's dock, on the Ontario side of the river, and about one-third of a mile from the dock. It is on a bare granite rock. The height of station used was 35 feet. The geodetic point is marked by a hole 1 inch in diameter and 2 inches deep drilled in the rock.

GRENADIER, 1872, '73.—This station is situated on a gravelly knoll on Grenadier Island, about 1 mile from the upper end. The knoll is at about equal distances from the water on both sides. The height of station used was 20 feet. The geodetic point is marked by two stone posts of the usual kind set one above the other.

CHIMNEY, 1872, '73.—This station is situated on the second bluff above Chimney Island, on the highest point of the bluff, 64 feet from its edge. The height of station used was 3 feet. The geodetic point is marked by a round hole drilled in the rock.

SLIDE, 1872.—This station is situated in Leeds County, Ontario, nearly due west of Oak Point, Saint Lawrence County, New York, and but a short distance from the Saint Lawrence River. The height of station used was 3 feet. The geodetic point is marked by a hole drilled in the rock. The rock at the station is about 1.5 feet above the river.

MOLLY'S GUT ISLAND, 1872.—This station is situated near the southeast side of Molly's Gut Island, Leeds County, Ontario, 20 feet from the water's edge. The height of station used was 3 feet. The geodetic point is marked by a hole drilled in the solid rock.

MCDONALD'S POINT, 1872.—This station is situated on an island directly south of McDonald's Point, about 80 feet from the southeast side, on a flat portion of the rock about 15 feet above the water. The height of tripod used was 3 feet. The geodetic point is marked by a $\frac{3}{4}$-inch hole 1 inch deep drilled in the rock.

MOULSON, 1872.—This station is situated on the east side of Moulson's Island, east of McDonald's Point, on a flat rock 20 feet from the shore and 10 feet above the water's level. The height of station used was 3 feet. The geodetic point is marked by a hole drilled in the rock.

A. C. L. XXI, 1872, '73.—This station is situated on the highest point on Big Island, near Brockville, Ontario. The geodetic point is referred to two small crosses cut in the rock in the prolongation of the lines from Brockville rock and Chapman stations, and distant $16\frac{1}{2}$ inches and $13\frac{1}{4}$ inches, respectively. The height of ground at the station above the Saint Lawrence River is 40 feet.

BROCKVILLE ROCK, 1872, '73.—This station is situated on a rock in the Saint Lawrence River nearly south of Brockville, Leeds County, Ontario, between the town and Old Man's Island. The height of station used was 3 feet. The distance from the geodetic point to the edge of the rock towards the English Church in Brockville was 6.75 feet, to the opposite edge was 38 feet, and to the edge towards Morristown, 50.8 feet. The rock is about $2\frac{1}{2}$ feet above the water.

CAMPBELL, 1873.—This station is situated about three-fourths of a mile northeast of Brockville, Ontario, on a cliff 28 feet above the Saint Lawrence River. The height of station used was 4 feet. The center is marked by a triangle drilled in the rock with a small hole in its center. This mark is 80 feet up stream from a staircase cut in the cliff and leading to a boat-house.

EATON, 1873.—This station is situated in Grenville County, Ontario, opposite Morristown Point, New York, 198 metres from the edge of the Saint Lawrence River, on line to Morristown Point, and 76 metres from a road-fence in the opposite direction. The geodetic point is marked by a triangle cut in the solid rock under the station.

SOUTHWEST BASE (MAITLAND), 1873.—This station is situated about 1700 metres west of Maitland Railway depot on the Grand Trunk Railway, on the south side of the track, and 12.25 metres from the south rail. The height of station used was 10 feet. The geodetic point is marked by a sandstone post 6 inches square and 15 inches long, marked with a triangle and the letters U. S. on top, set 2 feet under ground.

K, 1873.—This station is situated on the left bank of the Saint Lawrence, about 1¼ miles above Maitland, Grenville County, Ontario. It stands on a bare ledge 37 metres from the water's edge. The height of station used was 4 feet. The geodetic point is marked by a triangle cut in the surface-rock.

NORTHEAST BASE (MAITLAND), 1873.—This station is situated about 1000 metres west of Maitland Railway depot, on the Grand Trunk Railway, 13.3 metres south of the south rail of the track. The height of station used was 10 feet. The geodetic point is marked by a stone post of the usual form set 3 feet under ground.

BRENNAN, 1873.—This station is situated about 1000 metres west of Maitland, Grenville County, Ontario, about 330 metres from the Grand Trunk Railway, and 500 metres from the Saint Lawrence River. The height of station used was 7 feet. The geodetic point is marked by a stone post of the usual form set 2½ feet under ground. A triangle cut in the surface of a boulder bears north 51° 45′ west, and is distant 4.8 metres. A triangle cut on another boulder bears south 36° 29′ east, and is distant 16.5 metres. The height of ground at the station above the river is about 100 feet.

MAITLAND, 1873.—This station is situated in Grenville County, Ontario, on Langley's Point, near Maitland, Ontario, and about 10 metres from the Saint Lawrence River. The height of station used was 4 feet. The geodetic point is marked by a stone post of the usual form set 2½ feet below the ground surface. A triangle cut in the surface of a boulder bears south 17° east, and is 11.5 metres distant. The southwest corner of Langley's house bears north 44° 30′ west, and is 67 metres distant.

G, 1873.—This station is situated about 4½ miles above Prescott, Grenville County, Ontario, on the highest land between the railroad and the river, the railroad being about 270 metres and the river about 800 metres distant. The height of station used was 6 feet. The geodetic point is marked by a triangle cut in stone, with a cut Sandusky stone, marked U. S., set as a surface-mark.

RAILROAD, 1871, '73.—This station is situated about 2½ miles above Prescott, Grenville County, Ontario, on the north side of the railroad, and about 100 metres distant. The height of station used was 10 feet. The geodetic point is marked by a triangle cut in stone, a cut Sandusky stone being set as a surface-mark.

WINDMILL POINT, 1871.—This station is situated about 1½ miles below Prescott, Grenville County, Ontario, near a stone tower upon which a windmill was formerly placed. This tower was converted into a light-house in 1873. The height of station used was 10 feet. The geodetic point is marked with a stone post set directly on the line between the center of the old windmill and station West Base, Ogdensburg. The height of ground at the station above the Saint Lawrence River is 25 feet.

FRASER POINT, 1871.—This station is situated about 2 miles below Prescott, Grenville County, Ontario, and about midway between the road and the river. The height of station used was 10 feet. The geodetic point is marked by a hole drilled in the solid rock, 1 foot below the surface of the ground, with a stone post set directly over it as a surface-mark. A triangle cut on the surface of the ledge-rock next to the river bears south 83° east, and is 20.75 metres distant. The northeast corner of a stone house bears north 8° 57′ east, and is about 200 metres distant.

JOHNSTOWN, 1871.—This station is situated about 3¼ miles below Prescott, Grenville County, Ontario, and close to the river bank. The height of station used was 10 feet. The geodetic point is marked with a stone post set 3 feet below the ground surface, with another stone set directly over it as a surface-mark. A reference-stone, marked U. S. on its upper surface, is set on the east side of the road, bearing north 88° 14′ west, distant 107.5 metres.

EDWARDSBURG, 1873.—This station is situated about one-third of a mile north of Edwardsburg, Ont., about 120 metres northwest of the highway leading northeast from Edwardsburg. The height of station used was 7 feet. The geodetic point is marked by a bottle set neck upwards about 2½ feet below the ground surface. The northeast corner of a stone Methodist Episcopal church bears south 24° 12′ east, and is 122 metres distant. The tombstone of Thomas Louden bears south 12° 46′ east, and 106.2 metres distant.

STETHEM, 1873.—This station is situated about 1 mile northeast of Edwardsburg, Grenville County, Ontario, and about 130 metres north of the highway running approximately parallel to the Saint Lawrence River. The height of station used was 7 feet. The geodetic point is marked by a small irregular stone sunk about 2 feet in the ground and having a triangle on the top, with a hole in its center marking the geodetic point. The southeast corner of a dwelling bears south 46° 50′ west, and is 78.8 metres distant.

WORT, 1873.—This station is situated about $2\frac{1}{2}$ miles below Edwardsburg, Ont., about 300 metres back from the Saint Lawrence River. It bears north 38° west, and is about 320 metres from the most westerly end of Presqu' Isle. The height of station used was 20 feet. The geodetic point is marked by a cut stone post set about $2\frac{1}{2}$ feet below the ground surface. A stone surface reference-post is set directly over the geodetic point.

BINION, 1873.—This station is situated on the north side of the Saint Lawrence River, about $3\frac{1}{5}$ miles below Edwardsburg, Ontario, and about 250 metres from the river bank. The extreme east end of Toussaints Island bears south 27° east, and the extreme eastern point of Presqu' Isle bears south 37° 30′ west from the station. The height of station used was 20 feet. The geodetic point is marked by a hole drilled in a rock about 2.5 feet below the ground surface. A cut stone post is set as a surface-mark directly over the rock.

CHAPTER XXVIII.

LOCAL DEFLECTIONS OF THE PLUMB-LINE.

§ **1.** In the tables of Chapter XXVII, the geographical coördinates of many points have been given as derived by geodetic computation for Clarke's spheroid from Fort Howard, Wis., with the observed latitude, longitude, and azimuth, at or near that station; while the observed latitudes and longitudes of the same points may be found in Chapters XXIII, XXV, and XXVI.

If the observed latitudes and longitudes were exact; if the geoid or sea-level surface on which these stations are supposed to be projected coincided perfectly in the area containing the stations with Clarke's spheroid with its axis parallel to that of the earth; and if the geodetic computations were rigidly accurate; then the latitudes and longitudes of these stations when derived from Fort Howard should coincide with those obtained by direct astronomical determination.

As to the magnitude of the various sources of error which give rise to non-coincidences in these values, it may be said that the probable errors in the astronomical determinations amount only to some tenths of a second of arc; that the probable errors in the computed latitudes and longitudes arising from errors in the triangulation or computations cannot exceed in any case 0″.2 or 0″.3, and hence that these non-coincidences when large must be mainly due to the fact that Clarke's spheroid having its axis parallel to that of the earth, does not, in the area covered by the triangulation, coincide perfectly with the geoid. These non-coincidences arise from two causes, first, that the actual geoid or sea-level within this triangulation is a very irregular surface, with which it is impossible to make any spheroid coincide, but with which one spheroid can be made to coincide more closely than any other; and second, that Clarke's spheroid is not, in all probability, that spheroid.

So far as the results contained in this report are concerned, that best spheroid would be obtained by so changing the elements of Clarke's spheroid as to make the sum of the squares of the deviations of the verticals at the several astronomical points from the normals to the spheroid at those points a minimum. This determination has not been undertaken because the astronomical determinations are not distributed with sufficient uniformity, and are not in sufficient number to make it certain that the spheroid resulting would better represent the form of this part of the earth than Clarke's, which is based on a much greater number of astronomically determined points. The tables following will show large discrepancies between the astronomical determinations and the computed values of the same quantities, the discrepancies reaching 11″.6 in latitude and 14″.3 in longitude. A part of the discrepancy must be attributed to the difference of Clarke's spheroid from the best possible spheroid for this portion of the earth.

An idea of the effect of possible changes in the elements of Clarke's spheroid on computed latitudes and longitudes, may be obtained as follows: Bessel's spheroid was long accepted as well representing the geoid, and Clarke's was based on additional data. For Bessel's spheroid the semi-axis major is $\frac{1}{7882}$ less than for Clarke's, while the eccentricity is $\frac{1}{144}$ less. By substituting these changes in a and e in the values for $d\,L$ and $d\,M$, the differences in the latitudes and longitudes of the ends of a triangle-side, in Chapter XXVII, § 2, it will be found that for a meridional line 63 kilometers long in latitude 45° the computed latitude would be changed by about 0″.3 and for a similar side in the direction of a parallel the change in longitude would be about 0″.4.

The most distant points from the origin, Fort Howard, are Parkersburg and Mannsville, distant, respectively, about 400 and 600 miles. For Parkersburg the computed latitude will be changed about 3″, and for Mannsville the longitude will be changed by about 6″ by such changes in a and e. It does not seem probable that Clarke's spheroid can need corrections so large as these to make it fit the portion of the geoid under consideration, and hence a part of the discrepancies must be assigned to local irregularities in the form of the geoid.

In the following table the latitudes derived geodetically from Fort Howard, Wis., as an origin, together with their observed values, and the resulting excesses of computed over observed values, are given for many stations. The geodetic latitudes are taken from Chapter XXVII, and the observed from Chapters XXIII and XXVI. Since when the plumb-bob is deflected to the north the observed zenith moves to the south, thus diminishing the observed latitude, a plus sign before these excesses corresponds to a northerly deflection of the plumb-bob.

Computed and Observed Latitudes of Primary Stations.

Stations.	Latitudes. Computed from latitude of Fort Howard.	Latitudes. Observed.	Computed *minus* observed values.	Point of reference.
	° ′ ″	″	″	
Saint Ignace	48 47 18.90	28.65	− 9.75	Geodetic point.
Tip Top	48 16 24.64	25.78	− 1.14	Do.
Isle Royale East	48 07 43.58	55.22	−11.64	Do.
Farquhar's Knob	47 52 40.18	35.27	+ 4.91	Do.
Michipicoten	47 45 21.66	20.58	+ 1.08	Do.
Gargantua	47 34 42.72	38.80	+ 3.92	Do.
Vulcan	47 26 46.65	44.58	+ 2.07	Do.
Mount Houghton	47 24 26.00	15.83	+10.17	Do.
Sawteeth East	47 23 19.02	09.00	+10.02	Do.
Wheal Kate	47 04 20.27	18.12	+ 2.15	Do.
Outer Island	47 04 17.09	14.51	+ 2.58	Do.
Crebassa	46 58 41.07	41.21	− 0.14	Do.
Buchanan	46 56 28.07	24.45	+ 3.62	Do.
Huron Mountains	46 52 42.30	53.07	−10.77	Do.
South Base, Keweenaw	46 52 17.76	22.34	− 4.58	Do.
Porcupine Mountains	46 47 01 05	03.80	− 2.75	Do.
North Base, Minnesota Point	46 45 27.89	28.32	− 0.43	Do.
Brulé River	46 45 17.89	20.11	− 2.22	Do.
South Base, Minnesota Point	46 42 49.44	51.50	− 2.06	Do.
Aminicon	46 41 32.15	36.32	− 4.17	Do.
Thone's Hill	46 32 47.80	54.53	− 6.73	Transit-post.
Escanaba	45 44 41.24	35.05	+ 6.19	Light-house.
Ford River	45 41 12.16	05.34	+ 6.82	Geodetic point.
Burnt Bluff	45 41 09.73	03.88	+ 5.85	Do.
Beaver Island	45 34 32.45	28.78	+ 3.67	Light-house.
Cedar River	45 25 48.61	43.13	+ 5.48	Geodetic point.
Boyer's Bluff	45 25 12.74	09.30	+ 3.44	Do.
Door Bluff	45 17 46.97	45.69	+ 1.28	Do.
Menomonee	45 05 12.76	12.78	− 0.02	Do.
Fort Howard	Initial latitude	44 30 30.28	....	Do.
Watertown	43 58 32.20	32.88	− 0.68	Court-house.
North Base, Sandy Creek	43 40 43.61	41.52	+ 2.09	Geodetic point.
Minnesota Junction	43 28 28.45	31.82	− 3.37	Do.
Oswego	43 26 37.28	39.57	− 2.29	Do.
Rochester	43 09 18.11	22.44	− 4.33	Court-house.
Tonawanda	43 00 3.84	07.83	− 3.99	Geodetic point.
Buffalo	42 52 41.09	44.66	− 3.57	Intersection of streets.
Dunkirk	42 29 37.84	44.30	− 6.46	New light-house of 1876.
Detroit	42 20 01.18	19 59.04	+ 2.14	Intersection of Grand River and Washington avenues.
Erie	42 09 11.85	17.49	− 5.64	Beacon light.
Monroe	41 54 52.60	48.65	+ 3.95	Court-house dome.
Chicago	41 53 20.52	22.49	− 1.97	Light-house.
Willow Springs	41 43 36.93	38.63	− 1.70	Geodetic point.
Toledo	41 39 04.54	03.65	+ 0.89	Post of 1881.
Cleveland	41 30 01.63	06.13	− 4.50	Light-house on bank.
Sandusky City	41 29 02.32	04.59	− 2.27	Geodetic point, West Base.
Fairmount	40 01 35.84	36.70	− 0.86	Geodetic point.
West Base, Olney	38 51 38.76	41.23	− 2.47	Do.
Parkersburg	38 34 51.73	53.20	− 1.47	Do.

An examination of this table shows five cases in which the difference exceeds 7″, reaching −11″.64 at Isle Royale East. At Sawteeth East and Huron Mountains the large differences have the signs which would have been expected from the contours of the earth in their vicinity, since they have high land on one side of them and deep water on the other. At Saint Ignace, where the difference is −9″.75, its sign is the opposite of what would have been anticipated (see Chapter XXIII, § 4); at Isle Royale East, where a small positive difference would have been expected, the actual difference is −11″.64; and at Mount Houghton the difference is +10″.2, which is not accounted for by the form of the surface. But these three stations are on eruptive rocks, where great variations in density may be looked for.

If, in the table, the mean algebraical difference for all the stations in Lake Superior (Saint Ignace to Thone's Hill inclusive) be taken, it is found to be −0″.76. If latitudes had been observed at points differently distributed in Lake Superior, this mean value would be somewhat changed. But its smallness, taken in connection with the fact that the mean difference for the three southern latitude stations of the meridian chain, namely, Parkersburg, West Base, Olney and Fairmount, is only −1″.6, indicates that the choice of the observed latitude at Fort Howard as fundamental was judicious, and that Clarke's spheroid satisfies pretty well this arc of the meridian.

The stations Escanaba, Ford River, Burnt Bluff, and Cedar River have land to the north of them and Green Bay to the south. But Green Bay is shallow, not exceeding 140 feet in depth, while going north from these stations the ground rises gradually to the hills of Lake Superior, which are from 20 to 50 miles distant and 500 to 1000 feet high. The mean difference for these stations is +6′.08, indicating, as would be anticipated, a deflection of the plumb-bob to the north but to an unexpectedly large amount. The strata in this vicinity are Silurian.

If the differences at Chicago and Willow Springs, near the south end of Lake Michigan, at Sandusky, Cleveland, Erie, Dunkirk and Buffalo, along the shore of Lake Erie; and at Tonawanda, Rochester and Oswego, south of Lake Ontario, be examined, they will be found to vary from −1″.70 to −6″.46, giving a mean of −3″.67, thus showing very clearly the influence of the deficiency of matter in the lake basins to the northward of these stations upon the plumb-bob.

But if the stations along Lakes Erie and Ontario be examined, where no such marked differences in the topography of the country immediately to the north or south of them exist, such as Detroit, Monroe, Toledo, North Base, Sandy Creek and Watertown, it will be seen that the differences vary between +3″.95 and −0″.68, giving, in algebraical mean +1″.68. Although the number of stations in this last group is not sufficient to give a very reliable average value, it shows no large discrepancy between the computed and observed latitudes for these stations where there are no marked disturbing causes apparent, while its contrast with the −3″.67 of the preceding group is sufficiently marked.

§ **2.** In the following table, derived from Chapters XXIV, XXV, and XXVII, are given the longitudes of all stations at which the determinations were good, and for which personal equation was eliminated in determining the longitudes telegraphically; all azimuths observed at primary triangulation stations; the longitudes and azimuths derived geodetically from Fort Howard, and the differences between the geodetic and observed values. The observed latitudes of the stations are also given.

TABLE I.—*Excesses of computed azimuths and longitudes over observed values.*

[Azimuths are computed from the initial azimuth of the line Bruce—Long Tail Point Light. Longitudes are computed from the initial longitude of Fort Howard.]

Station.	Observed latitude.	Station whose azimuth is observed.	Longitudes. Observed.	Longitudes. Computed.	Longitudes. Discrepancy.	Azimuths. Observed.	Azimuths. Computed.	Azimuths. Discrepancy.
	° ′ ″		° ′ ″	″	″	° ′ ″	″	″
North Base, Minnesota Point.	46 45 28.32	To South Base	92 04 33.00	43.26	+10.26	323 52 24.32	13.75	−10.58
Aminicon		To Lester				153 36 66.15	54.59	−11.50
South Base, Koweenaw Point.	46 52 22.35	To North Base				199 10 67.29	57.68	− 9.61
Ford River	45 41 05.34	To Cedar River	87 06 01.35	09.40	+ 8.05	31 34 60.28	53.74	− 6.54
Bruce		To Long Tail Point Light.				139 16 04.90	Initial	azimuth.
Fort Howard	44 30 30.28		88 02 34.05	Initial	longitude.			
Minnesota Junction	43 28 31.82	To Horicon				272 30 45.03	47.23	+ 2.20
Willow Springs	41 43 38.63	To Shot Tower	87 51 06.75	06.36	− 0.39	224 29 41.24	39.26	− 1.98
Parkersburg	38 34 53.20	To Denver	88 01 48.96	49.66	+ 0.70	143 16 15.44	14.68	− 0.76
Toledo, longitude-post of 1881.	41 39 03.65		83 32 30.60	31.85	+ 1.25			
West Base, Sandusky	41 29 04.59	To East Base				319 36 38.60	41.20	+ 2.60
Tonawanda	43 00 07.83	To Buffalo Plains	78 53 21.16	21.91	+ 0.75	314 22 40.61	39.18	− 1.43
North Base, Sandy Creek.	43 40 41.52	To South Base				357 14 26.95	28.81	+ 1.86
Mannsville			76 03 26.74	14.61	−12.13			

In the following table are given some longitudes also obtained telegraphically, but in which either personal equation was not eliminated, or for other reasons the precision is less than for those given in the preceding table. They are derived from Chapter XXVI. Detroit, however, was well determined, and was the base for all the other longitude stations, but it is connected with the main triangulation in Lake Erie by a chain of small triangles running down the Detroit River, so that its geodetic position with reference to Fort Howard is less accurately known than that of a station of the primary triangulation.

TABLE II.—*Comparison of computed and observed longitudes.*

Stations.	Longitudes computed from longitude of Fort Howard.	Observed values.	Computed *minus* observed values.	Point of comparison.
	° ′ ″	″	″	
Chicago	87 36 52.20	48.90	+ 3.30	Light-house.
Menomonee	87 35 34.60	*27.75	+ 6.85	Geodetic point, Menomonee.
Thone's Hill	87 29 21.83	*07.50	+14.33	Geodetic point, Triloba.
Monroe	83 23 49.77	46.35	+ 3.42	Court-house.
Detroit	83 03 07.42	03.60	+ 3.83	Observatory-post east.
Sandusky	82 40 58.43	41 02.67	− 4.24	Geodetic point, West Base.
Cleveland	81 42 09.58	13.20	− 3.62	Light-house.
Erie	80 04 40.21	46.95	− 6.74	Beacon light.
Dunkirk	79 21 15.81	25.80	− 9.99	Light-house, 1876.
Buffalo	78 52 14.28	*18.00	− 3.72	Intersection of streets.
Rochester	77 36 50.69	51.00	− 0.31	Court-house.
Oswego	76 30 50.40	53.55	− 3.15	Geodetic point, Oswego.
Watertown	75 55 54.02	56 01.95	− 7.93	Court-house.

* Personal equation applied or eliminated.

An examination of Table I shows large discrepancies between the computed and observed longitudes at North Base, Minnesota Point, and at Mannsville.

At Minnesota Point the longitude station is on a low sand point at the west end of Lake Superior. To the east-northeast the lake deepens to 300 feet within twenty miles, while to the west, hills whose rocks contain iron rise to a height of about 500 feet within a few miles. A considerable deflection of the plumb-bob to the west was therefore to be expected, and hence too small a value for the observed longitude. The lack of knowledge of the heights of the country about Duluth, and still more the uncertainties as to the distribution of densities below the surface, make an attempt to compute the deflection of the plumb-line at this station of little value.

The longitude station at Mannsville is eight miles east of Lake Ontario and 400 feet above the lake. West of it Lake Ontario reaches a depth of 400 feet within 25 miles, while to the east the country reaches a height of 1000 feet in eleven miles, and 1500 feet in fifty-five miles. The effects of the deficiency of matter in the basin of Lake Ontario for 170 miles west, and of the high land east of Mannsville within a distance of about 60 miles, in producing easterly deviation of the plumb-bob at Mannsville have been computed. East of Mannsville approximate contours were used which Verplanck Colvin, esq., Superintendent of the Adirondack Survey, has kindly furnished. The mean depth of Lake Ontario was taken as 300 feet, the depth of its surface below that of the surrounding country as 400 feet, and the density of the earth in the vicinity as 2.8. The computation gives but about 4″ as the easterly deflection of the plumb-bob due to these causes, but, owing to the lack of accurate contours near Mannsville, this result can only be regarded as a rough approximation to the correct computed value. The discrepancy between the computed and observed values of the longitudes is $-12''.1$, the deflection of the plumb-bob being to the east, as was to be expected from the form of the surface of the earth in the vicinity.

But the form of the earth's surface, so far as is known, does not account fully for the large seemingly eastern deflection of the plumb-bob, nor does the form of the earth near North Base, Minnesota Point, account for the large seemingly western deflection there. A part of these discrepancies of $-12''.2$ and $+10''.3$ must be accounted for by supposing either irregular distributions of density in the earth in the vicinity of the stations, or that the actual arc of the parallel of 42° is longer than results from Clarke's spheroid. In § 3 some evidence is given that at North Base, Sandy Creek, the deviation in longitude is small. When a connection is made between the Lake Ontario triangulation and that in the New England States, this arc of the parallel will be extended to Massachusetts Bay, and a better judgment can be formed as to whether Clarke's spheroid needs a modification for this area or whether a large local deviation of the vertical exists at Mannsville. In Table II it will be noticed that, for nearly all the stations along Lakes Erie and Ontario, the discrepancies have the minus sign as at Mannsville, but the most of them have a lake basin to the west of them. It may also be noticed that the discrepancy of $+10''.3$ for North Base, Minnesota Point, is for a station only 200 miles west of Fort Howard, while that of $-12''.2$ at Mannsville is for a station 600 miles east of Fort Howard.

If the somewhat less precise longitudes in Table II be examined, having perhaps a probable error of $1''.5$, it will be seen that in passing from Mannsville to Watertown, 18 miles farther north, the difference between computed and observed longitudes has changed from $-12''.1$ to $-7''.9$, although possibly a part of this change may be due to non-elimination of personal equation. At Erie, Dunkirk, and Buffalo the effect of the basin of Lake Erie seems to show itself in an easterly deflection of the plumb-bob. At Menomonee, on the west shore of Green Bay, a westerly deflection might be expected. At Thone's Hill the discrepancy is $+14''.3$. This station is about four miles west of Marquette, among hills from 500 to 700 feet in height, while to the west the hills of the Marquette iron region rise to 1000 feet in height in about ten miles. To the northeast of the station the lake deepens to 300 feet in 15 miles. A westerly deflection of the plumb-bob would therefore be expected at this place, and its large amount may be due to the great density of the iron-bearing rocks to the west of it.

§ **3.** For an ellipsoid of revolution coinciding approximately with the geoid over the area covered by a triangulation, and having its axis parallel to that of the earth, the normal to the ellipsoid at a given point will not in general coincide with the vertical to the geoid through that point, and the angle between the two lines is the deviation of the plumb-line. If the normal be prolonged till it intersects the celestial sphere, we shall have the ellipsoidal zenith; and if the vertical be similarly prolonged the observed zenith results. The difference between the polar distances of these two zeniths is the difference between the observed and the ellipsoidal latitude which may be represented by $d\varphi$, φ being the observed latitude.

The angle at the pole between the two zeniths may be represented by $d\lambda$ where λ is the longitude.

The observed azimuth will be the angle between two planes intersecting in the vertical, one plane passing through the pole and the other through the object. The ellipsoidal azimuth will be the angle between two planes which pass through the same points, but which intersect in the normal to the ellipsoid. The difference between the observed and the ellipsoidal azimuth may be represented by $d\alpha$ where α is the azimuth.

In practice these differentials are all small, and Laplace has shown (see Zachariae, Geodätische Hauptpunkte, 317, or Helmert, Geodäsie I, 537), that $d\alpha + d\lambda \sin \varphi = 0$. This equation should be satisfied for each point of the primary triangulation, for which the azimuths and longitudes were computed geodetically and were also observed. It should be satisfied almost exactly if the astronomical observations were exact, and if there were no errors in the angles between the successive sides of the triangulation by which the geodetic computation is carried from one astronomical station to the next. But both these errors exist. Substituting for different points at which both longitudes and azimuths were observed the values for the symbols in the above equation, the results will indicate the effects of these errors. Taking the values of $d\alpha$ and $d\lambda$ from the table in § 2, and the corresponding values of φ from § 1, the above equation takes the following form for the various stations where longitude and azimuth have been both determined by observation.

	$''$	$''$	$^\circ\ \ '$	$''$
North Base, Minnesota Point	-10.6	$+\ 10.3$	sin 46 45 =	-3.1
Ford River	$-\ 6.5$	$+\ 8.0$	sin 45 41 =	-0.8
Willow Springs	$-\ 2.0$	$-\ 0.4$	sin 41 44 =	-2.3
Parkersburg	$-\ 0.8$	$+\ 0.7$	sin 38 35 =	-0.4
Tonawanda	$-\ 1.4$	$+\ 0.7$	sin 43 00 =	-0.9

From these equations, in which the second members should be zero, and which give the discrepancies in azimuth at the station, arising not only from the errors in the determination of the azimuths at the several stations and their longitudes with reference to Fort Howard, but also from the errors in the angles of the triangles by which the azimuths have been carried from Fort Howard to the several stations, an idea of the accuracy of these operations can be formed. The smallness of the errors is satisfactory when it is remembered that North Base, Minnesota Point, and Tonawanda are respectively 450 and 750 miles from Fort Howard, in opposite directions, these distances being measured along the axis of the main chain of the triangulation. It will be noticed that these errors all have the same sign.

It is to be regretted that an azimuth was not determined at Mannsville. North Base, Sandy Creek, was taken as the latitude and azimuth station, but to reach a telegraph line for longitude it was necessary to go to Mannsville, and even there the longitude station was nearly one-third of a mile from the triangulation station. It will be noticed that at North Base, Sandy Creek, the azimuth discrepancy was only $+1''.9$, which would require, to satisfy Laplace's theorem, a longitude discrepancy of only $-2''.8$. This would indicate that in passing from North Base, Sandy Creek, to Mannsville, a distance of but 8 miles, there is a large change in the deviation of the plumb-line.

CHAPTER XXIX.

ARC OF MERIDIAN AND OF PARALLEL OF 42°.

§ **1.** In order to determine the relation between the observed differences of latitude of certain points along the chain of primary triangulation stretching from Parkersburg, Ill., to Saint Ignace, Ont., through 10° 13′ of latitude, and the distances on the meridian of Fort Howard, between the parallels of latitude through these points, it is necessary to compute the latter distances from the adjusted triangulation.

Two methods have been followed in this computation in different parts of the chain: First, that of polar triangles; second, that of projecting each triangle-side on the meridian by parallels of latitude through the ends of the side.

In the first method, some station of the chain is selected as a pole, and radii-vectores, which are geodetic lines on the spheroid, are drawn to the other stations of the part of the chain selected. In the first polar triangle there will be two sides and the included angle known; from these the radius-vector and the other angles of the triangle can be computed by the aid of Legendre's theorem with all necesssary precision, so long as the polar triangle does not have a spherical excess greater than 30 seconds. From the first polar triangle, and the adjusted triangulation, a second triangle can be computed, giving a second radius-vector, and so on until the polar triangles become too large to be computed by Legendre's theorem, when it becomes necessary to choose a new pole. The azimuth of the first radius-vector is derived from that of the first triangle-side, supposed to be known. The reverse azimuth of any radius-vector is derived from the angle at its farther end between it and a triangle-side, and the azimuth of the triangle-side as given by the geodetic computation of latitudes, longitudes, and azimuths of the different stations and their connecting lines, given in Chapter XXVII. This method was used from Fort Howard northward to Vulcan. The initial azimuth was that of the line Bruce—Long Tail Point, given in Chapter XXIV, § 6, as 139° 16′ 04″.90 from which the azimuth Fort Howard—Bruce results from the latitude, longitude, and azimuth computation as 259° 20′ 57″.32. (Chapter XXVII, § 3.) The difference between the astronomical and geodetic azimuth of Bruce—Long Tail Point is so small that it may be neglected. The adjusted angles and sides of the triangulation are given in Chapters XIV, D—XX, D.

The first set of polar triangles covered the triangulation from Fort Howard to Ford River. At the latter station a new pole was taken, and with the azimuth derived from Bruce, a new set of polar triangles was computed covering the triangulation from Ford River to Vulcan. From Vulcan, with the azimuth derived from Bruce, a single triangle side reached Saint Ignace, the most northern point of the triangulation, and a second side reached Huron Mountains station, about 40 miles south. As these polar triangles had in no case a spherical excess exceeding 15″ they could be computed as plane triangles by Legendre's theorem without introducing any error approaching that of the errors in the data. For a triangle having excesses less than 10″ the excesses were computed by the formula given in the United States Coast-Survey Report for 1875, for the Clarke spheroid, namely:

$$\varepsilon = \frac{a' \, b' \sin C}{2a^2(1 - \frac{1}{2}e^2 \cos 2L)^2 \sin 1''}$$

For larger triangles the formula given by Zachariæ "Die Geodätischen Hauptpunkte," page 133,

$$\varepsilon = \frac{1}{\sin 1''} s\mu(1 + \tfrac{1}{8}\mu m^2)$$

was used.

In the first formula a is the equatorial radius, e the eccentricity, a', b' triangle-sides, C the included angle of the plane triangle, L the mean of the latitudes of the vertices of the triangle, and ϵ the spherical excess.

In the second, s is the area of the plane triangle having the same sides, μ the mean measure of curvature for the vertices of the triangle, and m^2 the mean of the squares of the triangle sides.

The radii-vectores were carried along both flanks of the triangulation as a check. Computations were made with eight place logarithms.

The following table gives for Fort Howard, Ford River, and Vulcan, as poles, the direct and reverse azimuths and radii-vectores to the stations connected with them:

POLE.—FORT HOWARD.

Stations.	Azimuths. Direct.	Azimuths. Reverse.	Logarithm of radius-vector in feet.
BY WEST ROUTE.	° ′ ″	° ′ ″	
Little Tail	191 56 10.033	11 58 24.124	4.82422194
Gales	200 52 17.640	21 00 06.785	5.13004929
Peshtigo	211 36 40.480	31 54 09.990	5.31056447
Menomonee	208 48 56.976	29 07 58.068	5.38215585
Rochereau	208 29 45.936	28 55 25.914	5.51441037
Door Bluff	221 03 41.807	41 45 06.276	5.58323132
Cedar River	208 38 03.650	29 08 26.370	5.58414070
Ford River	209 08 29.157	29 48 26.691	5.69326851
BY EAST ROUTE.			
Little Tail	191 56 10.033	11 58 24.124	4.82422194
Gales	200 52 17.640	21 00 06.785	5.13004929
Débroux	223 03 09.537	43 21 29.994	5.21806056
South Egg	225 27 23.560	45 56 35.327	5.39954451
Eagle Bluff	220 41 20.212	41 15 25.366	5.50345091
Door Bluff	221 03 41.802	41 45 06.277	5.58323128
Boyer's Bluff	220 21 05.362	41 07 59.849	5.64223167
Ford River	209 08 29.155	29 48 26.693	5.69326848

MEAN VALUES BY THE TWO ROUTES FOR THE LINE FORT HOWARD - FORD RIVER.

Logarithm of length in feet	5.69326850
	° ′ ″
Direct azimuth	209 08 29.156
Reverse azimuth	29 48 26.692
Mean azimuth	209 28 27.924

POLE.—FORD RIVER.

Stations.	Azimuths. Direct.	Azimuths. Reverse.	Logarithm of radius-vector in feet.
BY WEST ROUTE.	° ′ ″	° ′ ″	
Pine Hill	219 20 53.175	39 29 39.092	4.91342324
Sturgeon	219 15 53.678	39 33 49.094	5.22278094
Mud Lake	197 54 38.378	18 07 24.494	5.38571810
Shelter Bay	185 45 32.078	5 50 29.164	5.45896922
Mount Mesnard	166 04 00.750	345 51 09.855	5.49243521
Triloba	163 49 12.768	343 32 28.638	5.54274088
Ives Hill	156 11 38.024	335 39 53.399	5.65726978
Vulcan	165 05 16.672	344 35 09.163	5.82273613
BY EAST ROUTE.			
Burnt Bluff	270 00 04.683	90 16 53.265	5.00032134
Fishdam	237 05 11.201	57 31 25.604	5.26656269
Monistique	217 30 20.943	37 52 04.195	5.32189746
Divide	204 22 41.065	24 41 46.508	5.43212161
Grand Island	198 32 22.061	18 50 04.249	5.51073554
Granite Island	168 23 48.041	348 10 25.429	5.58568270
Vulcan	165 05 16.682	344 35 09.171	5.82273615

MEAN VALUES BY THE TWO ROUTES FOR THE LINE FORD RIVER-VULCAN.

Logarithm of length in feet	5.8227361 4
	° ′ ″
Direct azimuth	165 05 16.677
Reverse azimuth	344 35 09.167
Mean azimuth	164 50 12.922

POLE.—VULCAN.

Stations.	Azimuths.			Logarithm of radius-vector in feet.
	Direct.	Reverse.	Mean.	
	° ′ ″	° ′ ″	° ′ ″	
Huron Mountains	8 45 39.987	188 40 03.101	8 42 51.544	5.3212858 4
Saint Ignace	178 21 39.931	358 19 04.172	178 20 22.051	5.6900927 6

§ **2.** It remains to project the lines Fort Howard—Ford River, Ford River—Vulcan, Vulcan—Saint Ignace, and Vulcan—Huron Mountains upon the meridian of Fort Howard by parallels of latitude, and to determine the lengths of the projections. This has been done in two ways.

First, following Bessel's method in the convenient form for computation given to it by Helmert, Höhere Geodäsie, erster Theil, Seite 308.

$$(1)\qquad M=-s\frac{\cos \alpha}{\cos\frac{\Delta\alpha}{2}}\left\{\begin{array}{l}1+\frac{1}{12}\frac{s^2}{a_0^2}\sin^2\alpha\,(1+e^2\cos 2B)\\ -\frac{1}{240}\frac{s^4}{a_0^4}\sin^2\alpha\,(-2+[5\tan^2 B+3]\cos^2\alpha)+Gl_6\end{array}\right\}$$

where

$$B=\frac{B_1+B_2}{2};\ \alpha=\frac{\alpha_{1\cdot2}+\alpha_{2\cdot1}-180^\circ}{2}\text{ and }\Delta\alpha=\alpha_{2\cdot1}-\alpha_{1\cdot2}-180^\circ.$$

In these formulæ M is the projection of the line 1·2 on the meridian; s is its length; a_0 is the semi-axis-major of Clarke's spheroid=20926062 English feet; e is its eccentricity=0.082272; B_1 and B_2 are the latitudes of 1 and 2; and $\alpha_{1\cdot2}$ $\alpha_{2\cdot1}$ are the azimuths of the line 1·2. Gl_6 represents terms of the 6th order.

Using this formula and the radii-vectores and azimuths given in § 1, the lines Fort Howard—Ford River, Ford River—Vulcan, Vulcan—Saint Ignace, and Vulcan—Huron Mountains were projected on the meridian of Fort Howard, giving the following results:

	English feet.
Projection of line Fort Howard—Ford River	=429622.52
Projection of line Ford River—Vulcan	=641731.58
Projection of line Vulcan—Saint Ignace	=489677.82
Projection of line Vulcan—Huron Mountains	=207130.21

These lines were also projected on the Fort Howard meridian by Andræ's methods. (See Zachariæ Die Geodätischen Hauptpunkte, p. 197.) The projections thus found were substantially the same as those given above, the greatest difference between the two results for the same projection being less than $\frac{1}{3000000}$ of the projection.

§ **3.** The second method used was to project by Helmert's formula each triangle-side separately on the meridian of Fort Howard, using the latitudes and azimuths obtained by the geodetic computation. Between Ford River and Vulcan the sides of the triangulation were separately projected on the meridian, using Helmert's formula for M. The sum of these projections was $0^{ft}.06$ larger than the value of the projection of the radius-vector from Ford River to Vulcan given above, a difference that is insignificant in comparison with the errors arising from the triangles. The computation of the spherical excesses of the polar triangles, and of the triangles themselves, involves more work than does the projection of the separate triangle-sides. Accordingly, from Fort Howard southward to the end of the meridian chain at Parkersburg the separate triangle-sides were projected by Helmert's formula (1) given above. The lengths of the triangle-sides were taken from Chapters XV, D, § 8; XVI, D, § 12; XVII, D, § 6; and XX, D, § 6; and the latitudes and azimuths at their ends from Chapter XXVII, § 3.

Summing the values found for the projections between the parallels of Fort Howard, Minnesota Junction, Willow Springs, Fairmount, West Base (Olney), Parkersburg, and including in the table the values previously given for Ford River, Huron Mountains, Vulcan, and Saint Ignace, the following table results, in which the first column gives the name of the station, the second its observed latitude taken from Chapter XXIII, the third the interval on the meridian of Fort Howard between the parallels of the stations, derived as just explained, and the fourth the sums of these intervals, counting from Parkersburg.

Distances between parallels along the meridian passing through Fort Howard.

Stations.	Observed latitudes.	Intervals between parallels passing through adjacent stations, in English feet.	Intervals from Parkersburg to the several parallels, in English feet.
	° ′ ″		
Parkersburg	38 34 53. 20		00. 00
		101878. 83	
West Base (Olnsy)	38 51 41. 23		101878. 83
		424660. 27	
Fairmount	40 01 36. 70		526539. 10
		619488. 24	
Willow Springs	41 43 38. 63		1146027. 34
		636930. 58	
Minnesota Junction	43 28 31. 82		1782957. 92
		376877. 73	
Fort Howard	44 30 30. 28		2159835. 65
		429622. 52	
Ford River	45 41 05. 34		2589458. 17
		434601. 37	
Huron Mountains	46 52 53. 07		3024059. 54
		207130. 21	
Vulcan	47 26 44. 58		3231189. 75
		489677. 82	
St. Ignace	48 47 28. 65		3720867. 57

The preceding table gives the intervals on the meridian of Fort Howard in English feet. As it is expected ultimately to obtain the length of R 1876 in terms of the International metre with great precision, it will be well to give these intervals in terms of R 1876 at some defined temperature.

The bases measured with the Bache-Würdemann base-apparatus depend for their length on that of the 15-feet brass bar, which depends on brass yard No. 6, which again depends on the mean of the values assigned by Colonel Clarke to the lengths of the Clarke yards A and B. (See Chapter II, § 9.) A value of the length of the 15-feet brass bar has been used for the Buffalo and Sandy Creek Bases, differing slightly from that used for the Minnesota Point, Keweenaw, and Fond du Lac Bases; but as the change in length at 32° F. or 62° F. did not exceed $\frac{1}{4500000}$ part, a small quantity compared with the probable errors in these bases, this change may be neglected. (See Chapter II, § 19.) The lengths in feet of the Chicago, Sandusky, and Olney Bases depend on Clarke yard A alone, instead of on the mean of A and B, but it is shown in Chapter II, § 2, that the discrepancy between the two values is insignificant. It may then be said that the intervals on the meridian given in § 3, which depend on the bases measured with the Bache-Würdemann base-apparatus, depend on Colonel Clarke's value of Clarke yard A, and on its expansion given in Chapter II, § 2.

For the portions of these intervals which depend on bases measured with the Repsold base-apparatus it may be said that their lengths depend on that of Clarke yard A through metre R1876. The mean temperature of the comparisons of R1876 and A is given in Chapter IX, § 12, as 57°.92 F., and there is also given

$$R1876 \text{ at } 57^\circ.92=\frac{1000}{919}A+5^{\text{mm}}.2966\pm 0^{\mu}.40; \text{ or,}$$

substituting for A its value at 57°.92, as given by Colonel Clarke, R1876 at 57°.92=1$^{\text{y}}$.09388063± 0$^{\text{y}}$.00000045, this last expression gives

$$1^{\text{y}}.000000=0.91417653\ R1876 \text{ at } 57^\circ.92,$$

and enables us to transform the feet in the table of § 3 into multiples of R1876 at 57°92 F.

In the following table, then, the intervals depending on bases measured with the Repsold apparatus have, for errors depending on the standard, only errors of comparison of R1876 with S_2 and of S_2 with S_1, while those depending on bases measured with the Bache-Würdemann apparatus have in addition, for errors depending on standards, the errors of comparison of Clarke yard A with R1876.

Distances between parallels along the meridian of Fort Howard.

Stations.	Observed latitudes.	Intervals between parallels passing through adjacent stations, in multiples of R1876 at 57°.92 F.	Intervals from Parkersburg to the several parallels, in multiples of R1876 at 57°.92 F.
	° ′ ″		
Parkersburg	38 34 53.20		0.00
		31045.08 R1876	
West Base (Olney)	38 51 41.23		31045.08 R1876
		129404.82 R1876	
Fairmount	40 01 36.70		160449.90 R1876
		188773.87 R1876	
Willow Springs	41 43 38.63		349223.77 R1876
		194089.00 R1876	
Minnesota Junction	43 28 31.82		543312.77 R1876
		114844.26 R1876	
Fort Howard	44 30 30.28		658157.03 R1876
		130916.94 R1876	
Ford River	45 41 05.34		789073.97 R1876
		132434.12 R1876	
Huron Mountains	46 52 53.07		921508.09 R1876
		63117.86 R1876	
Vulcan	47 26 44.58		984625.95 R1876
		149217.32 R1876	
St. Ignace	48 47 28.65		1133843.27 R1876

§ **4.** The chain of triangles running from Chicago east to the eastern end of Lake Ontario does not deviate widely from the parallel of 42°, and hence may be used to obtain the length of the arc of the parallel of 42° which lies between the meridians of Willow Springs and Mannsville, these meridians differing in longitude by 11° 48′.

The method followed has been to project, by meridians through its ends, each triangle-side on the parallel whose latitude is the mean of the latitudes of the ends of the side, and then to project chosen ones of these projections on the parallel of 42°. In selecting these projections to be projected on the parallel of 42°, those were taken corresponding to a continuous broken line of triangle-sides connecting the end stations, this broken line being made up of sides so chosen as to run as nearly east and west, and to be as near the parallel of 42° as was practicable. The three sides of each triangle were projected on the parallel of 42° also, in order to check the computation, since the projection of one side should be nearly equal to the sum of the projections of the other two.

Helmert, Höhere Geodäsie, erster Theil, 311, gives the following formula for the difference of longitude in seconds of arc, of the ends of a geodetic line on a spheroid, the length of the line being s the latitudes of its ends B_1 and B_2, and the azimuths at its ends $\alpha_{1\cdot2}$ and $\alpha_{2\cdot1}$. The equatorial radius and the eccentricity of the spheroid are a_0 and e. The formula is—

$$(1)\qquad L_{1\cdot2}=\rho''\frac{s}{a_0}W\sec B\sin\alpha\Big[1-\frac{1}{24}\frac{s^2}{a_0^2}(W^2[1-\sec^2 B\sin^2\alpha]-e^2[10\sin^2 B-1]\cos^2\alpha)$$
$$+\frac{1}{1920}\cdot\frac{s^4}{a_0^4}(1-\sec^2 B\sin^2\alpha)(1-9\sec^2 B\sin^2\alpha)+Gl_6\Big]$$

In this equation $W^2=1-e^2\sin^2 B$; $B=\frac{B_1+B_2}{2}$; $\alpha=\frac{\alpha_{1\cdot2}+\alpha_{2\cdot1}-180^\circ}{2}$; ρ'' is the number of seconds in radius; and Gl_6 represents terms of the 6th order, $\frac{s}{a_0}$ being of the first.

Representing the arc of the parallel of B included between two meridians by $P_{1\cdot2}$ its value will be

$$P_{1\cdot2}=a_0\cdot\frac{L_{1\cdot2}}{\rho''}\cdot\frac{\cos B}{W}$$

and (1) may be written—

$$(2)\qquad P_{1\cdot2}=s\sin\alpha\Big[1-\frac{1}{24}\cdot\frac{s^2}{a_0^2}(W^2[1-\sec^2 B\sin^2\alpha]-e^2[10\sin^2 B-1]\cos^2\alpha)$$
$$+\frac{1}{1920}\cdot\frac{s^4}{a_0^4}(1-\sec^2 B\sin^2\alpha)(1-9\sec^2 B\sin^2\alpha)+Gl_6\Big]$$

In the chain between Willow Springs and Mannsville there is but a single side having an approximately eastern and western direction whose length exceeds 63 kilometers, and that one does not exceed 70 kilometers. Hence $\frac{s}{a_0}$ may be taken as always less than $\frac{1}{100}$ or of the second order, $\frac{1}{10}$ being the maximum value of small quantities of the first order, and $P_{1\cdot2}$ will be accurate to quantities of the 10th or 11th order.

The value of $P_{1\cdot2}$ would be little changed, B and α being observed quantities, by small changes in a_0 and e, which determine the spheroid. A change of $\frac{1}{5000}a_0$ in a_0, and $\frac{1}{100}e$ in e, both of which are greater than the differences between Clarke's and Bessel's spheroids, would only change the logarithm of $P_{1\cdot2}$ in the 8th place.

Since in (2) terms multiplied by $\frac{s^4}{a_0^4}$ are at least of the 8th order, they may be omitted in the computation of projections.

Having referred the triangle-sides to the parallels of $\frac{1}{2}(B_1+B_2)$ for each, it remains to refer these projections to the parallel of 42°. To do this, it is only necessary to multiply the first projection by the ratio of the radius of the parallel of 42° to the radius of the parallel of B, that is by

$$\frac{\cos 42^\circ}{\cos B}\sqrt{\frac{1-e^2\sin^2 B}{1-e^2\sin^2 42^\circ}}$$

Effecting the division under the radical it becomes

$$[1+e^2\sin^2 42^\circ-\sin^2 B+e^4\sin^2 42^\circ(\sin^2 42^\circ-\sin^2 B)+\text{etc.},]^{\frac{1}{2}}$$

or developing by the binomial formula

$$(3)\qquad \frac{P^{42}_{1\cdot2}}{P^{B}_{1\cdot2}}=\frac{\cos 42^\circ}{\cos B}[1+\tfrac{1}{2}e^2(\sin^2 42^\circ-\sin^2 B)+\tfrac{1}{2}e^4\sin^2 42^\circ(\sin^2 42^\circ-\sin^2 B)$$
$$-\tfrac{1}{8}e^4(\sin^2 42^\circ-\sin^2 B)^2+Gl_6]$$

To find the change in this ratio caused by a change in e for the reference-spheroid, differentiate with reference to e, and there results

$$(4)\qquad d\frac{P^{42}_{1\cdot2}}{P^{B}_{1\cdot2}}=\frac{\cos 42^\circ}{\cos B}[e\,de(\sin^2 42^\circ-\sin^2 B)+2e^3de\sin^2 42^\circ(\sin^2 42^\circ-\sin^2 B)-\tfrac{1}{2}e^3de(\sin^2 42^\circ-\sin^2 B)^2]$$

To get the maximum change for this chain take $de=\frac{e}{100}$; $B=44^\circ$; and there results

$$(5)\qquad \frac{\cos 42^\circ}{\cos 44^\circ}\Big[\frac{e^2}{100}(.034)+2\frac{e^4}{100}\cdot\sin^2 42(.034)-\tfrac{1}{2}\frac{e^4}{100}(.034)^2\Big]$$

Since $e^2 = .0068$, the second and third terms in the brackets do not exceed 10^{-8} and may be neglected. But the first term becomes $.000068\ (.034) = 0.0000023$, or the ratio in the extreme case is changed about $\frac{1}{400,000}$ part. As this would be an extreme value, and would apply to but few sides, no serious error would be produced in the length of the parallel by neglecting it. But it can be easily taken into account by computing to three significant places the coefficient of de in the first term of (4) for each of the selected sides forming the continuous broken line between two longitude stations. The projection of one of these triangle-sides on the parallel of the mean latitude of its ends will then be projected on the parallel of 42° by multiplying it by $c + a\ \delta e$, where c is the value of the ratio in (3) for Clarke's spheroid, a is the coefficient of δe computed as just stated, and δe is a symbol for the correction to $e = 0.082272$ required when a more accurate value of e is found and used.

The following is the list of stations determining the broken line of triangle-sides between Willow Springs and Mannsville, which have been projected on the parallel of 42°:

List of the stations determining the lines projected on the parallel of 42°.

Willow Springs to Cedar Point.	Cedar Point to Tonawanda.	Tonawanda to Mannsville.
Willow Springs.	Cedar Point.	Tonawanda.
Shot Tower.	Middle Sister.	Falkirk.
Michigan City.	Middle Bass.	Batavia.
Bald Tom.	Kelley's.	Morganville.
Bertrand.	Brownhelm.	Scottsville.
Milton.	Elyria.	Turk's Hill.
Calvin.	Olmsted.	Palmyra.
Porter.	Rockport.	Clyde.
Sherman.	Willoughby.	Victory.
Bronson.	Little Mountain.	Oswego.
Quincy.	Thompson.	Sandy Creek.
Hillsdale.	Conneaut.	Mannsville.
Bunday.	Erie.	
Woodstock.	Westfield.	
Raisin.	Silver Creek.	
Dundee.	Sturgeon Point.	
Monroe.	Ridgeway.	
Cedar Point.	Tonawanda.	

The lengths of the sides are taken from Chapters XVII, D, to XIX, D, and the latitudes and azimuths at their ends from Chapter XXVII.

The following table contains the summation of these projections on the parallel of 42° for the intervals between the principal longitude stations. The first column gives the name of the longitude station, the second its observed longitude from Detroit derived from Chapter XXV, the third the included arc of the parallel of 42°, and the fourth the sum of these arcs, counted from Willow Springs as an origin.

Distances between meridians along parallel of 42°.

Stations.	Observed longitudes referred to Detroit. + west. — east.	Intervals between meridians of adjacent stations, in English feet.	Intervals from Willow Springs to the several meridians, in English feet.
	° ′ ″		
Willow Springs	+4 48 03.15		0.00
		1227742.62+ 224.2 δe	
Cedar Point	+ 17 01.84		1227742.62+224.2 δe
		1208492.33+ 204.5 δe	
Tonawanda	—4 09 42.44		2436234.95+428.7 δe
		770735.68—1248.5 δe	
Mannsville	—6 59 36.86		3206970.63—819.8 δe

By means of the relation given in § 3, namely, 1 yard = 0.91417653 R1876 at 57°.92 F., the distances expressed in feet in the above table may be converted into multiples of the metre R1876 at the temperature 57°.92 F. The following table gives those distances expressed as multiples of R1876:

Distances between meridians along parallel of 42°.

Stations.	Observed longitudes referred to Detroit. +west. —east.	Intervals between meridians of adjacent stations, expressed in multiples of R1876 at 57°.92 F.	Interval from Willow Springs to the several meridians, expressed in multiples of R1876 at 57°.92 F.
	° ′ ″		
Willow Springs.....	+4 48 03.15		00.0
		374124.50 R1876+ 68.3 R1876 δe	
Cedar Point.........	+ 17 01.84		374124.50 R1876+ 68.3 R1876 δe
		368258.44 R1876+ 62.3 R1876 δe	
Tonawanda	−4 09 42.44		742382.94 R1876+130.6 R1876 δe
		234862.82 R1876−380.4 R1876 δe	
Mannsville	−6 59 36.86		977245.76 R1876−249.8 R1876 δe

§ **5.** It is of interest to form an approximate idea of the precision of the triangulation and of the arcs of the meridian and of the parallel of 42°. In computing the weighted mean sides of the triangulation in Chapters XIV, D, to XX, D, the quantities $\frac{1}{p}$ are given for each triangle side. These quantities are the squares of the probable errors of the logarithms of the sides when computed from the first base with the observed angles. South and east of the Chicago Base the $\frac{1}{p}$ include only errors arising from the observed angles, and do not include errors in standards nor in measurement of bases. North of Chicago Base they exclude errors of standard, but include errors in measurement of base. South and east of Chicago the errors in measurement of the bases did not sensibly affect the approximate weights for triangle-sides when computed from two such bases, and hence those errors were neglected in computing the weighted mean logarithms of the sides.

In the chapters referred to, if $\frac{1}{p}$ and $\frac{1}{p'}$ represent the squares of such probable errors in the logarithm of a side when computed from the the two adjacent bases, $\frac{1}{p}+\frac{1}{p'}=$ a constant whose value is given in each of the chapters referred to, and hence from the tabulated values of $\frac{1}{p}$ and the value of this constant, the values of $\frac{1}{p'}$ follow at once. The weight of the logarithm of a weighted mean side in Chapters XIV, D, to XX, D, is then $p+p'$, and the square of the probable error in its logarithm is $\frac{1}{p+p'}$. But this square of the probable error in the logarithm of a side has been obtained on the assumption that the side has been computed from the bases with observed angles; in fact, the sides have been computed with adjusted angles, and the question at once arises as to the reduction in the probable error of the logarithm of a side in consequence. In Chapters XIV, C, to XX, C, it may be seen that the probable errors of adjusted angles are approximately 0.6 of the probable errors of the observed angles. If, then, the adjusted angles and their errors were independent of each other, in order to obtain the probable error squared in the logarithm of a side computed from a base, it would only be necessary to replace the $\rho^2[\alpha^2+\beta^2]$ of Chapters XIV, D, to XX, D, by $(0.6)^2\rho^2[\alpha^2+\beta^2]$. But, in fact, the errors of the adjusted angles depend on each other in consequence of the rigid equations of condition of the triangulation, and hence this value is not exact, and is probably too small. On the other hand, it is evident that the computation of a side with the adjusted angles must give their values much more accurately than when the observed angles are used, and hence that $\rho^2[\alpha^2+\beta^2]$ is much too large. The factor by which $\rho^2[\alpha^2+\beta^2]$ should be multiplied in order to give the proper result for adjusted angles, lies then between $(0.6)^2$ and the square of a quantity considerably less than unity. It seems probable that its value is in the vicinity of $(0.7)^2$, but to avoid overestimating the accuracy of the work it will be taken as $(0.8)^2$, so that $(0.8)^2\rho^2[\alpha^2+\beta^2]$ will be taken as the square of the probable error arising from angle errors in the logarithm of a side when computed from one base with the adjusted angles.

To get the reduced values of the probable errors of the weighted mean logarithms of sides given in Chapters XIV, D—XX, D, we must multiply the values of $\frac{1}{p}$ and $\frac{1}{p'}$ by $(0.8)^2$ after subtracting the square of the probable error of measurement of the base where such error is included in $\frac{1}{p}$ or $\frac{1}{p'}$, and add to the resulting values the squares of the probable errors of the bases due to measurement. The reciprocal of the sum of p and p', thus modified, plus the square of the probable error of the logarithm of the standard, will give the square of the probable error required. Where consecutive bases depend on different standards, a mean value of the probable errors in the logarithms of the lengths of the standards has been used for the intermediate sides, since the errors of the two standards are not independent. In this way the total probable error in the seventh place of the weighted mean logarithms of all the sides of the principal chains of triangulation have been computed, and the means of these have been taken for the sides included between each of the pairs of parallels of latitude given in the table of § 3. From the probable errors in the mean logarithms, the probable errors in the mean sides may be obtained, and from these an estimate of the probable error in the interval between two parallels, along the meridional arc, or between two meridians, along the parallel of 42°, may be obtained.

The following tables give the results. The first column gives the names of the stations limiting the sections of the chain; the second the number of the included triangle-sides of the principal chain; the third and fourth give the average probable error of a triangle-side in the section:

Average probable errors of triangle-sides projected on the meridian to obtain the intervals between parallels.

Sections between stations.	Number of triangle-sides projected on meridian in the respective sections.	Average probable error of a triangle-side, Expressed in units of seventh place of logarithms.	Expressed as a fractional part of length of side.
Parkersburg to West Base (Olney)	4	10.0	$\frac{1}{434000}$
West Base (Olney) to Fairmount	8	15.2	$\frac{1}{286000}$
Fairmount to Willow Springs	9	20.0	$\frac{1}{217000}$
Willow Springs to Minnesota Junction	10	23.6	$\frac{1}{184000}$
Minnesota Junction to Fort Howard	8	24.4	$\frac{1}{178000}$
Fort Howard to Ford River	8	37.0	$\frac{1}{117000}$
Ford River to Huron Mountains	8	33.9	$\frac{1}{128000}$
Huron Mountains to Vulcan	1	22.2	$\frac{1}{196000}$
Vulcan to St. Ignace	1	25.2	$\frac{1}{172000}$

Average probable errors of triangle-sides projected on parallel of 42°.

Sections between stations.	Number of triangle-sides projected on parallel in section.	Average probable error of a triangle-side,	
		Expressed in units of seventh place of logarithms.	Expressed as a fractional part of length of side.
Willow Springs to Cedar Point ...	17	26.1	$\frac{1}{166000}$
Cedar Point to Tonawanda	17	20.8	$\frac{1}{209000}$
Tonawanda to Mannsville	11	24.1	180000

§ **6.** In the chain of triangles connecting two bases the probable error of a triangle-side is a minimum at the bases and increases to a maximum for a side about half way between the bases. The tables which have just been given show the average probable errors of triangle-sides for certain sections. It is also of interest to know the maximum and minimum probable errors of sides in the principal chain; accordingly, the following table gives in order the names of the bases, the names of the mean sides of maximum probable error lying between each pair of consecutive bases, and the probable errors of such bases and lines, expressed in units of the seventh place of logarithms and in fractions of their lengths. The probable errors of the bases are taken from Chapters III—VII and X—XIII, and the probable errors of the sides of maximum probable error are derived from the $\frac{1}{p}$ of Chapters XIV, D—XX, D, by applying the changes for probable error of standard and of measurement of base, when necessary, as in §5.

Table of probable errors of base-lines and of lines of maximum probable error in each section of the chain of principal triangles.

I.—OLNEY BASE TO MINNESOTA POINT BASE.

Base.	Line of maximum probable error.	Probable error of logarithm of line in units of seventh place.	Probable error as fraction of length of line.
Olney Base	....	4.2	$\frac{1}{1013000}$
	Ash Grove—Spring Creek............	22.2	$\frac{1}{196000}$
Chicago Base	....	4.1	$\frac{1}{1052000}$
	New Berlin—Delafield	28.2	$\frac{1}{154000}$
Fond du Lac Base	....	6.7	$\frac{1}{649000}$
	Burnt Bluff—Sturgeon	38.7	$\frac{1}{112000}$
Keweenaw Base	....	5.2	$\frac{1}{830000}$
	Vulcan—Wheal Kate	22.3	$\frac{1}{195000}$
Minnesota Point Base	....	8.2	$\frac{1}{530000}$

Table of probable errors of base-lines, &c.—Continued.

II.—CHICAGO BASE TO SANDY CREEK BASE.

Base.	Line of maximum probable error.	Probable error of logarithm of line in units of seventh place.	Probable error as fraction of length of line.
Chicago Base	....	4.1	$\frac{1}{1052000}$
	Jefferson—Milton	30.3	$\frac{1}{143000}$
Sandusky Base	....	3.8	$\frac{1}{1149000}$
	Claridon—Little Mountain	24.2	$\frac{1}{179000}$
Buffalo Base	....	4.9	$\frac{1}{890000}$
	Turk's Hill—Pinnacle Hill	26.4	$\frac{1}{164000}$
Sandy Creek Base	....	4.8	$\frac{1}{913000}$

§ 7. As the methods used in estimating probable errors of triangle-sides can only give rough approximations to the true values, any check on the results is valuable. Such a check is afforded by the comparison of the difference between the measured length of a base and its length computed from the next base by means of the adjusted angles, with the estimated probable error in that computed length.

In the following table the data for such comparisons are given. The first column gives the name of a base and the second that from which its length has been computed with the adjusted angles given in Chapters XIV, C—XX, C; the third gives the distance between the bases measured along the axis of the chain of triangles connecting them; the fourth gives the excess of the length of the base computed with adjusted angles over its measured length expressed in parts of the length of the base; the fifth gives the same excess in feet; the sixth gives the probable error in the computed length of the base derived from the probable errors of the observed angles between the two bases; the seventh gives this probable error when derived from the probable errors of the adjusted angles between the two bases by the method given in §5; the eighth gives references.

Comparison of computed lengths of bases with measured lengths.

Base.	Name of the base from which computed.	Distance between bases along axis of chain.	Computed length *minus* measured length.		Probable error of computed length expressed in parts of base.		Reference to chapter and §.
			Expressed in parts of the base.	Expressed in feet.	Using probable errors of observed angles.	Using 0.8 of probable errors of observed angles as probable error of adjusted angles.	
		Miles.					
Minnesota Point..	Keweenaw	240	$-\frac{1}{94000}$	0.21	$\frac{1}{80000}$	$\frac{1}{100000}$	IV, § 14
Fond du Lac......	Keweenaw	320	$-\frac{1}{252000}$	.10	$\frac{1}{44000}$	$\frac{1}{55000}$	V, § 12
Chicago	Fond du Lac......	150	$+\frac{1}{54000}$	.46	$\frac{1}{62000}$	$\frac{1}{77000}$	X, § 35
Sandusky..........	Chicago	280	$-\frac{1}{45000}$	.46	$\frac{1}{58000}$	$\frac{1}{72000}$	XI, § 9
Sandusky..........	Buffalo	250	$+\frac{1}{161000}$	.13	$\frac{1}{71000}$	$\frac{1}{90000}$	XI, § 10
Buffalo	Sandy Creek......	210	$-\frac{1}{185000}$	.12	$\frac{1}{67000}$	$\frac{1}{84000}$	VII, § 8
Olney.............	Chicago	200	$+\frac{1}{109000}$	.20	$\frac{1}{78000}$	$\frac{1}{97000}$	XII, § 9

An examination of this table shows that on comparing the measured length of a base with its length computed from the next base with the adjusted angles of the intermediate triangulation, the differences are in three cases greater than the probable error in this computed length, estimated from the probable errors of the adjusted angles, and in four cases less, and hence that the discrepancies between the observed and computed lengths of the bases are such as are to be expected if the probable errors assigned to the adjusted angles and sides are nearly correct. In other words, these discrepancies indicate that those probable errors are nearly the true ones, and, so far as the number of bases is sufficient to justify a conclusion, they indicate that the estimated probable errors are rather too large than too small.

§ **8.** It is desirable to know approximately the accuracy of the values which have been assigned in §§ 3 and 4 to the intervals on the meridian of Fort Howard, between the parallels through certain astronomical stations; and to similar intervals on the parallel of 42°. In §§ 5 and 6 data have been given as to the maxima, minima, and average probable errors of triangle sides in these intervals.

From § 5 it will be seen that for intervals containing eight or more triangles, either along the meridian of Fort Howard or on the parallel of 42°, the average probable error in the length of a triangle-side varied from $\frac{1}{117000}$ to $\frac{1}{286000}$ part. These errors enter by equal amounts in the projections of a side on the meridian or parallel. If in the projections the errors were all of the same sign for each interval, the sum of the projections would be in error by an equal part of its value. On the other hand if the projections were n in number, of equal lengths, and their errors were independent of each other, the resulting probable error in the sum of the projections would be the probable error of a side, expressed in parts of its length, divided by $\sqrt{n}$. Neglecting the fact that the sides of the triangles in each of the intervals are of unequal length, and treating them as if of equal length, an approximation to the probable error in the interval arising from the triangulation errors would be obtained by multiplying the average probable errors in § 5 by $\frac{1}{2}+\frac{1}{2\sqrt{n}}$ (except when there is but a single triangle-side, where it will remain as given).

But an error in the projection of the triangle-sides on the meridian and parallel also arises from the errors in the azimuths of the sides. The amount of error from this cause will be essentially the same as if the lines joining the astronomical stations in the tables of §§ 2 and 4 were projected directly, and the errors would mainly arise from the errors in the values of the azimuths of these lines. An examination of the discrepancies given in Chapter XXVIII, § 2, between computed azimuths and those observed at Bruce, Minnesota Junction, Willow Springs, and Parkersburg, where there are no inequalities in the earth's surface, which would indicate local deviations in azimuth, seems to indicate that the azimuth observed at Bruce and adopted as the fundamental one for the whole triangulation was not subject to local error of any great amount. The question then arises as to the amount of error which accumulates in carrying the azimuth through the triangles from Fort Howard to distant points, such as North Base, Minnesota Point, Parkersburg, or Tonawanda. Some information on this point is given by the second members of the numerical equations in Chapter XXVIII, § 3. Those second members are mainly the accumulated errors in azimuth, provided the observed longitudes are exact; if inexact, the values of the second members will be in part due to the longitude errors. From the table it will be seen that in the part of the triangulation belonging either to the arc of the meridian or of the parallel the greatest discrepancy is $-3''.1$. So far as these results can furnish the basis for a conclusion, it would seem safe to assume that the probable error in the azimuth of any line connecting adjacent astronomical stations in the tables of §§ 3 and 4 does not exceed $\pm 2''$. It may be noticed that at North Base, Sandy Creek, but 8 miles from Mannsville, the discrepancy between the observed and computed azimuths is but $+1''.9$. The effect of an error of $2''$ in azimuth on the projection of the line on the meridian or parallel will depend essentially on the cosine of the angle of projection.

In the following table are given the principal astronomical stations for the meridian of Fort Howard and the parallel of 42°, the approximate probable error in their intervals obtained as explained in the first part of this section, expressed as a fraction of the interval; the approximate angles made by the lines joining the astronomical stations with the meridians or parallel; the probable errors in the projection due to a probable error of $\pm 2''$ in this angle expressed in parts of the length of the line; and the combined probable error arising from both linear and azimuth errors.

Intervals between stations.	Lineal probable error.	Angle of projection.	Azimuthal probable error.	Combined probable error.
		° ′		
Parkersburg to West Base	$\frac{1}{579000}$	11 24	$\frac{1}{517000}$	$\frac{1}{385000}$
West Base to Fairmount	$\frac{1}{423000}$	9 39	$\frac{1}{603000}$	$\frac{1}{346000}$
Fairmount to Willow Springs	$\frac{1}{326000}$	0 07	0	$\frac{1}{326000}$
Willow Springs to Minnesota Junction	$\frac{1}{280000}$	20 22	$\frac{1}{275000}$	$\frac{1}{196000}$
Minnesota Junction to Fort Howard	$\frac{1}{263000}$	25 38	$\frac{1}{215000}$	$\frac{1}{167000}$
Fort Howard to Ford River	$\frac{1}{173000}$	29 28	$\frac{1}{182000}$	$\frac{1}{126000}$
Ford River to Huron Mountains	$\frac{1}{189000}$	25 29	$\frac{1}{215000}$	$\frac{1}{142000}$
Huron Mountains to Vulcan	$\frac{1}{196000}$	8 43	$\frac{1}{678000}$	$\frac{1}{188000}$
Vulcan to St. Ignace	$\frac{1}{172000}$	1 40	$\frac{1}{3619000}$	$\frac{1}{172000}$
Willow Springs to Cedar Point	$\frac{1}{267000}$	0 17	0	$\frac{1}{267000}$
Cedar Point to Tonawanda	$\frac{1}{336000}$	21 22	$\frac{1}{265000}$	$\frac{1}{208000}$
Tonawanda to Mannsville	$\frac{1}{277000}$	19 02	$\frac{1}{297000}$	$\frac{1}{203000}$

§ **9.** The principal intervals on the arc of the meridian through Fort Howard and on the parallel of 42°, with their approximate probable errors, may now be given. It will be remembered that the probable errors are only rough approximations.

Arc of meridian through Fort Howard.

Stations.	Observed latitude.	Intervals between parallels through adjacent stations, in English feet.	Approximate probable error— Expressed in feet.	Expressed as fractional part of interval.
	° ′ ″			
Parkersburg	38 34 53.20			
		101878.83	±0.26	$\frac{1}{385000}$
West Base (Olney)	38 51 41.23			
		424660.27	±1.23	$\frac{1}{346000}$
Fairmount	40 01 36.70			
		619488.24	±1.90	$\frac{1}{326000}$
Willow Springs	41 43 38.63			
		636930.58	±3.25	$\frac{1}{196000}$
Minnesota Junction	43 28 31.82			
		376877.73	±2.26	$\frac{1}{167000}$
Fort Howard	44 30 30.28			
		429622.52	±3.41	$\frac{1}{126000}$
Ford River	45 41 05.34			
		434601.37	±3.06	$\frac{1}{142000}$
Huron Mountains	46 52 53.07			
		207130.21	±1.10	$\frac{1}{188000}$
Vulcan	47 26 44.58			
		489677.82	±2.85	$\frac{1}{172000}$
St. Ignace	48 47 28.65			

Arc of Parallel of 42°.

Stations.	Observed longitude referred to Detroit. +west / −east } of Detroit.	Intervals between meridians of adjacent stations, in English feet.	Approximate probable error— Expressed in feet.	Expressed as fractional part of interval.
	° ′ ″			
Willow Springs	+4 48 03.15			
		1227742.62 + 224.2 δe	±4.60	$\frac{1}{267000}$
Cedar Point	+0 17 01.84			
		1208492.33 + 204.5 δe	±5.81	$\frac{1}{208000}$
Tonawanda	−4 09 42.44			
		770735.68 − 1248.5 δe	±3.80	$\frac{1}{203000}$
Mannsville	−6 59 36.86			

APPENDICES.

APPENDIX I.

ADDITIONAL DATA RELATIVE TO METRE R1876.

§ **1.** The data given in Chapter IX, § 57, showing a considerable difference between the expansion of R1876, given by the Kaiserliche Normal-Eichungs-Kommission, Chapter IX, § 67, and those obtained by the Lake Survey, the following letter was addressed to the Kommission:

In your esteemed letter of June 20, 1879, you gave me as a preliminary value for the Lake-Survey metre, R1876,

$$R1876=1 \text{ metre}+248^{\mu}.89\pm0^{\mu}.25+(10^{\mu}.31\pm0^{\mu}.034)(t-15)$$

and in your letter of September 15, 1880, you advised me that the error in this value, with reference to the true metric measure, would not exceed 1^{μ} or 2^{μ}. By true metric measure, I understand the metre which will be fixed by the International Standard Metre, which is to be, or has been, derived from the Mètre des Archives.

Since the receipt of your letter a long series of comparisons at high and low temperatures, in this office, of R1876 with steel Clarke yard A, whose coëfficient of expansion was carefully determined in 1874 by Colonel Clarke at the Ordnance-Survey office, Southampton, England, have given a value for the expansion of R1876 as depending on that found by Colonel Clarke for the Clarke yard. That value is

Expansion of $R1876=10^{\mu}.59$ for 1° C. derived from Clarke yard A.

R1876 has been also carefully compared with each metre of a steel bar 4 metres long, belonging to a base-measuring apparatus by Repsold, at both high and low temperatures. This 4-metre steel bar has been well compared at high and low temperatures with four metres of a brass bar 15 feet long, whose absolute expansion has been carefully determined directly in this office. Combining the two sets of comparisons there results

Expansion of $R1876=10^{\mu}.64$ for 1° C. derived from absolute expansion of brass bar.

These two values derived by entirely independent methods agree well, and it is difficult to suppose them more than $0^{\mu}.1$ in error, while their probable error is much less. Their mean differs by $0^{\mu}.3$ from the value, $10^{\mu}.31$, assigned in your letter. This leads me to ask whether any error of copying may have slipped into the value $10^{\mu}.31$, and, if not, whether you now think it possible that the true value may be $0^{\mu}.1$ or $0^{\mu}.2$ greater than $10^{\mu}.31$. I would also ask whether in the value assigned in your letter, namely,

$$R1876=1 \text{ metre}+248^{\mu}.89\pm0^{\mu}.25+(10^{\mu}.31\pm0^{\mu}.0.34)(t-15)$$

the value of R1876 is free at 15° C. from any small error that may exist in this value of its expansion.

FEBRUARY 26, 1881.

As this letter was not immediately answered, and the preparation of this report could not be further delayed, the value of the expansion of R1876, derived from the adjustment of different values, Chapter IX, § 58, namely,

$$E_{R1876}=1^{m}.000092[5773(10)^{-9}+2\times1852(10)^{-12}(t-32)] \text{ for } 1^{\circ}\text{ F.}=5^{\mu}.885\pm0^{\mu}.043 \text{ at } 62^{\circ}\text{ F.}$$

was adopted.

§ **2.** On September 12, 1881, the following letter was received, but too late to incorporate its results in the main report. It is, therefore, given in this appendix:

BERLIN, *den* 10. *August* 1881.

Die Kommission ist nunmehr in der Lage Ihnen nähere Details über die hier angestellten Bestimmungen der Länge und der Ausdehnung sowie der Theilungsfehler Ihres von Repsold in Hamburg verfertigten biegungsfreien Stahlmeters mit der Theilung auf Platin, welches hier mit R1876 bezeichnet wurde, mitzutheilen und dadurch die in Ihrem Schreiben vom 20. Juni 1879 angegebenen Resultate zu erläutern und theilweise zu modificiren. Es dürften damit gleichzeitig die in Ihren gefälligen Schreiben vom 16. October 1880, 16. Februar und 27. April 1881 angeregten Fragen, soweit sich dieselben auf dieses Meter beziehen, ihre Beantwortung finden. Die Hauptursache der bisherigen Verzögerung besteht darin, dass wir die genaueren Resultate der in Breteuil angestellten Vergleichungen abwarten wollten, um Ihnen möglichst definitive Resultate mittheilen zu können, und dass uns diese Vergleichungen erst seit kurzer Zeit bekannt sind.

Die ersten Bestimmungen der Gesammtlänge des Stabes wurden im Jahre 1876 angestellt, indem 15 Vergleichungen mit dem Messingmeter 1605 bei Temperaturen zwischen 11° und 20° ausgeführt wurden. Diese Vergleichungen verdienen aber nur ein sehr geringes Vertrauen, weil sie zu den ersten mit dem betreffenden Komparator angestellten Beobachtungen gehören, und die Methode der Beobachtung noch nicht genügend durchgebildet war. Namentlich die

Bestimmung der Temperatur der Stäbe liess in dieser Versuchsreihe noch viel zu wünschen übrig. Es herrscht daher auch einige Unsicherheit darüber, wie die Vergleichungen am besten zu berechnen sind, und die Resultate der Rechnungen sind innerhalb weiter Grenzen unsicher.

In der Beilage A sind die einzelnen Resultate dieser Vergleichungen aufgeführt: nämlich die Temperaturen der beiden Stäbe t_{1605} und t_R nach den Angaben der auf ihnen liegenden Thermometer und die unmittelbar gefundene Längendifferenz. Die Berechnung ist nur beiläufig mit den später gefundenen Ausdehnungskoefficienten ausgeführt und zwar einmal unter der Annahme, dass die Temperaturen der Stäbe der Angabe der aufliegenden Thermometer entsprechen (I), bei welcher Annahme im Mittel

$$R-1605=+21^{\mu}.9 \text{ bei } +15^{\circ}.00$$

folgt, zweitens unter der Annahme, dass die Temperaturen *beider* Stäbe durch t_{1605} allein gegeben sind, weil die Berührungsfläche des anderen Thermometers mit $R1876$ eine verhältnissmässig zu kleine war, um sichere Resultate erwarten zu lassen (II). Aus dieser zweiten Annahme folgt im Mittel

$$R1876-1605=+25^{\mu}.7 \text{ bei } +15^{\circ}.00.$$

Endlich würde noch die Annahme in Frage kommen und den vorerwähnten vielleicht vorzuziehen sein, dass als Temperatur der beiden Stäbe das Mittel aus den beiden Thermometerangaben anzusehen ist. Aus dieser Annahme würde folgen

$$R1876-1605=+27^{\mu}.2 \text{ bei } 15^{\circ}.00.$$

Das Messingmeter 1605 ist mehrfach mit dem deutschen Urnormal (Pt) verglichen worden. Aus den neuesten besten Messungen folgt

$$1605-Pt=-63^{\mu}.14\pm0^{\mu}.79+t(9^{\mu}.653\pm0^{\mu}.044),$$

also bei Anwendung der gesetzmässigen Gleichung des Urnormals

$$1605=1^{\mathrm{m}}-60^{\mu}.13+18^{\mu}.253t,$$

d. h.

$$1605=1^{\mathrm{m}}+213^{\mu}.7 \text{ bei}+15^{\circ}.00.$$

Die Verbindung dieses Werthes mit den obigen Differenzen von $R1876-1605$ ergiebt viel kleinere Werthe für die Länge von $R1876$ als die späteren Vergleichungen, doch liegen, wie schon erwähnt, grössere Bedenken gegen die Genauigkeit dieser ersten Messungsreihe vor.

Eine Bestimmung der Länge von $R1876$, welche sehr viel grössere Garantieen für ihre Richtigkeit bietet, beruht auf der Vergleichung von $R1876$ mit einem diesem Stabe ganz gleichartigen, mit $R1878$ bezeichneten Meterstabe von Repsold. Es sind im März 1878 drei Reihen bei den Temperaturen von 8°.60, 15°.75, und 21°.49 angestellt worden, deren Details sich in der Beilage befinden, und welche, wenn man die sehr kleinen Differenzen zwischen den Temperaturen der beiden Stäbe berücksichtigt, auf das Resultat führen:

$$\begin{aligned} R1876-R1878&=+21^{\mu}.86 \text{ bei } \ 8^{\circ}.602\\ &=+22\ .88 \text{ bei } 15\ .751\\ &=+22\ .81 \text{ bei } 21\ .489.\end{aligned}$$

Das Mittel sämmtlicher Reihen giebt

$$R1876-R1878=+22^{\mu}.52\pm0^{\mu}.21$$

Vorzuziehen ist aber dasjenige Resultat, welches man erhält, wenn die Ausdehnungsdifferenz der beiden Stäbe berücksichtigt wird:

$$R1876-R1878=22^{\mu}.52\pm0^{\mu}.17+(t-15^{\circ}.281)\,(0^{\mu}.076\pm0^{\mu}.029)$$

oder in einer anderen Form

$$R1876-R1878=21^{\mu}.36\pm0^{\mu}.48+t\,(0^{\mu}.076\pm0^{\mu}.029).$$

Der Stab $R1878$ ist in zwei Reihen in den Jahren 1878 und 1879 mit dem deutschen Urmeter verglichen worden. Die erste dieser Reihen liess nur die Berechnung der relativen Ausdehnung zu. Als Resultat ergab sich:

$$\begin{aligned} &1.\quad R1878-Pt=\text{const}\quad \pm1^{\mu}.68+t\,(1^{\mu}.676\pm0^{\mu}.074)\\ &2.\quad R1878-Pt=+68^{\mu}.79\pm0^{\mu}.49+t\,(1^{\mu}.643\pm0^{\mu}.032).\end{aligned}$$

Anzuwenden ist die aus der Verbindung der beiden Beobachtungsreihen sich ergebende Gleichung

$$R1878-Pt=+68^{\mu}.71\pm0^{\mu}.49+t\,(1^{\mu}.648\pm0^{\mu}.029)$$

aus deren Verbindung mit der obigen sich ergiebt

$$R1876-Pt=+90^{\mu}.07\pm0^{\mu}.68+t\,(1^{\mu}.724\pm0^{\mu}.041).$$

Setzt man nun für Pt seinen legalen Werth, wie er aus der Vergleichung mit dem Mètre des Archives unter der Annahme folgt, dass die Ausdehnung der *beiden* Meter $8^{\mu}.6\,t$ beträgt, so wird

$$R1876=1^{\mathrm{m}}+93^{\mu}.08+10^{\mu}.324\,t.$$

Der $-1^{\mu}.16+0^{\mu}.014\,t$ betragende Unterschied gegen die Ihnen unter dem 20. Juni 1879 gemachte Angabe

$$R1876=1^{\mathrm{m}}+248^{\mu}.89\pm0^{\mu}.25+(10^{\mu}.31\pm0^{\mu}.034)\,(t-15^{\circ})$$

rührt von nachträglichen kleinen Verbesserungen her, die sich bei einer Superrevision der thermometrischen Rechnungen ergaben. Bei den damals angegebenen wahrscheinlichen Fehlern ist ausserdem nur die Unsicherheit der Vergleichung der beiden Repsoldmeter unter einander berücksichtigt worden.

Die Ihnen unterm 15. September 1880 gemachte Angabe, dass die gegebene Gleichung von $R1876$ nur um 1^{μ} bis 2^{μ} von dem wahren metrischen Maasse abweichen dürfte, hat sich leider, soweit es sich um niedere Temperaturen handelt, nicht bestätigt. Dieselbe stützte sich auf eine in Breteuil ausgeführte Vergleichung von $R1878$ mit dem Platin-Iridiummeter type I bei 30°, welche ein mit den bisherigen Annahmen sehr gut übereinstimmendes Resultat ergeben hatte. Vergleichungen bei 0° und 15° haben dies Resultat nich bestätigt. Als Resultat der *drei* Vergleichungsreihen ergab sich vielmehr

$$R\,1878 = 1^{\mathrm{m}} + 64^{\mu}.82 + 10^{\mu}.371\,t + 0^{\mu}.0969\,t^2.$$

Dabei ist angenommen, dass die Gleichung von type I gegeben ist durch

$$\mathrm{I} = 1^{\mathrm{m}} + 67^{\mu}.6 + 8^{\mu}.525\,t + 0^{\mu}.0039\,t^2.$$

Der Fehler von type I bei 0° ist durch indirekte Vergleichung mit dem Mètre des Archives, die Ausdehnung des Stabes als Mittelwerth aus verschiedenen absoluten Bestimmungen abgeleitet worden. In der Gleichung für R 1878 ist der Koefficient von t^2 nach den Fizeau'schen Bestimmungen angenommen, und die Grösse der anderen Glieder aus den Bestimmungen bei 0° und 30° berechnet. Da die Vergleichung bei 15° ein nur um $0^{\mu}.05$ von der Formel abweichendes Resultat ergab, so wird man bei derselben stehen bleiben können.

Unter Fortlassung des in t^2 multiplicirten Gliedes wird in der Nähe von 15° nach den Bestimmungen in Breteuil zu setzen sein

$$R\,1878 = 1^{\mathrm{m}} + 64^{\mu}.82 + 10^{\mu}.578\,t.$$

Die Verbesserung welche hiernach an die aus der Vergleichung mit dem deutschen Meter gefolgerte Gleichung anzubringen wäre, ist

$$-6^{\mu}.9 + 0^{\mu}.330\,t.$$

Mit dieser Verbesserung geht die Gleichung für R 1876 über in

$$R\,1876 = 1^{\mathrm{m}} + 86^{\mu}.18 + 10^{\mu}.654\,t.$$

Dies dürfte augenblicklich der wahrscheinlichste Werth von R 1876 in wahrem metrischen Maasse sein. Unter wahrem metrischen Maasse müssen wir das durch das Mètre des Archives als Einheit dargestellte Maasssystem verstehen, da eine von diesem unabhängige Definition bisher nicht existirt, und auch bei einer späteren Festsetzung einer solchen jedenfalls ein möglichst enger Anschluss an das bisherige System erstrebt werden wird.

Die grosse Differenz des in Breteuil gefundenen und auch durch die von Ihnen mitgetheilten Versuche bestätigten Ausdehnungskoefficienten von R 1878, gegen den von uns mitgetheilten Werth muss wohl grössteutheils dem Umstande zur Last gelegt werden, dass die nach Borda für Platin angenommene Ausdehnung zu klein ist. In der That ergiebt sich nach Fizeau die Ausdehnung des Platinmeters zu

$$8^{\mu}.68\,t + 0^{\mu}.0030\,t^2,$$

also in der Nähe von 15° zu $8^{\mu}.80\,t$.

Eine kleine Differenz der Ausdehnungen ist auch zwischen dem Mètre des Archives und dem deutschen Urmeter angedeutet, da die Vergleichungen dieser Meter sich auch durch die Gleichung darstellen lassen

$$Pt - A = 1^{\mu}.14 + 0^{\mu}.094\,t,$$

oder, wenn man nur die Vergleichungen durch Brix berücksichtigt,

$$Pt - A = 0^{\mu}.39 + 0^{\mu}.136\,t.$$

Doch sind diese Vergleichungen nur bei wenig von einander verschiedenen Temperaturen angestellt.

Wir hoffen, auch unsererseits in wenigen Monaten durch hier angestellte absolute Ausdehnungsbestimmungen zur Entscheidung der noch bestehenden Unsicherheiten beitragen zu können.

Ein Anschluss an das aus der Toise abgeleitete System, insbesondere an die für geodätische Operationen wichtig gewordene Bessel'sche Kopie der Toise ist für Ihren Stab durch zwei in den Jahren 1878 und 1879 ausgeführte Vergleichungen von R 1878 mit der Toise Lenoir gegeben. Es ergab sich

1. $L = 1.948\ 570\ 76\ R\ 1878$ bei 18°.255
2. $L = 1.948\ 575\ 53\ R\ 1878$ bei 15°.74.

Verbindet man die beiden Gleichungen, indem man dieselben durch Interpolation nach der Temperatur auf die Normaltemperatur reducirt, so erhält man

$$L = 1.948\ 574\ 57\ R\,1878 \text{ bei } 16°.25.$$

Die Toise Lenoir, im Jahre 1852 von Baumann umgearbeitet, ist in den Jahren 1852 und 1872 und auf unserem Repsold'schen Komparator in den Jahren 1877 und 1878 mit der Bessel'schen Toise verglichen worden. Die Resultate dieser Vergleichungen sind

1. $L - B = -0'''.0043$
2. $L - B = -0'''.0066$
3. $L - B = -0'''.00372 \pm 0.00013$
4. $L - B = -0'''.00336 \pm 0.00022.$

Als bestes Resultat dürfte das Mittel aus den beiden letzten Vergleichungen gelten können; also

$$L - B = -0'''.00354.$$

Die Bessel'sche Toise wird allgemein nach Bessel um $0'''.00080$ kürzer angenommen als "die Toise." Nach ihrem eigenen Certifikate würde sie um $0'''.00078$ zu kurz sein; aus einer von Bessel ausgeführten Vergleichung mit zwei anderen mit Certifikaten versehenen Toisen ergiebt sie sich um $0'''.00333$ zu kurz und um $0'''.00411$ zu lang. Im Mittel aus den drei Certifikaten würde sich daher ihre völlige Identität mit der Toise du Pérou ergeben.

Bleibt man bei der stets benutzten Gleichung stehen

$$B = 1^{\mathrm{T}} - 0'''.00080 \text{ bei } 16°.25,$$

so folgt daraus

$$L = 1^{\mathrm{T}} - 0'''.00434 \text{ bei } 16°.25.$$

und durch Verbindung mit der früher abgeleiteten Gleichung

$$L = 1.948\ 574\ 57\ R\,1878 \text{ bei } 16^\circ.25,$$

aus welcher

$$R\,1878 = \frac{443.296}{864} 1.000\ 236\ 96\ L \text{ bei } 16^\circ.25$$

folgt,

$$R\,1878 = 443'''.296 + 231^\mu.94 \text{ bei } 16^\circ.25$$

oder mit dem früher gefundenen Ausdehnungskoefficienten

$$R\,1878 = 443'''.296 + 65^\mu.41 + 10^\mu.248\,t$$

und demnach

$$R\,1876 = 443'''.296 + 86^\mu.77 + 10^\mu.324\,t.$$

um $-4^\mu.2 + 0^\mu.014\,t$ von der vorläufigen Angabe vom 20. Juni 1879

$$R\,1876 = 443'''.296 + 245^\mu.6 + 10^\mu.31\ (t - 15^\circ)$$

verschieden. Mit dem Pariser Ausdehnungskoeffieienten hätte man

$$R\,1876 = 443'''.296 + 240^\mu.1 + 10^\mu.55\ (t - 15^\circ).$$

Die wichtigeren Messungen zur Bestimmung der Theilungsfehler von R 1876 gründen sich ebenfalls vorzugsweise auf Vergleichungen mit R 1878. Es sind daher in der Beilage B auch die Messungen und Rechnungen zur Bestimmung der Theilungsfehler dieses letzteren Stabes ausführlicher mitgetheilt.

Die Theilungsfehler sind sämmtlich auf einer Längentheilmaschine von Repsold untersucht worden, bei welcher zwei mit einander fest verbundene Mikrometermikroskope in der Längsrichtung eines Stabes verschoben werden können, während entweder jedes der beiden Mikroskope auf denselben Stab oder je ein Mikroskop auf einen von zwei nebeneinander liegenden Stäben pointirt. Die erstere Einrichtung wurde bei der Untersuchung der Fehler der Decimeterstriche von R 1878 angewandt, indem man den beiden Mikroskopen Abstände von resp. 5, 2 und 1 Decimeter gab und mit diesen festen Abständen die Intervalle des Stabes verglich. Die Beobachtungsresultate und der Gang der Rechnung sind in den Beilagen vollständig mitgetheilt—abgesehen davon, dass die Ablesungen der Mikrometer schon wegen der periodischen Ungleichheit der Schrauben verbessert sind. Zum vollen Verständniss ist nur noch zu bemerken, dass bei beiden Mikroskopen einer grösseren Ablesung ein kleinerer Werth des zwischen den Mikroskopen eingestellten Intervalles entsprach, und dass die Einheit, $= 1^p$, in welcher sämmtliche Ablesungen gegeben sind, und die Rechnung durchgeführt wurde, dem tausendsten Theile eines Umlaufs der Mikrometerschrauben entspricht und gleich $0^\mu.05$ gesetzt werden kann.

Gegen die Art und Weise, in welcher aus den drei Beobachtungsreihen die definitiven Fehler der Decimeterstriche abgeleitet sind, würden sich Einwendungen erheben lassen. Eine strenge Ausgleichung nach der Methode der kleinsten Quadrate, welche nachträglich ausgeführt wurde, führte jedoch zu den folgenden ganz unerheblich abweichenden Resultaten:

Decimeterstrich.	Negativer Fehler— Nach angenäherter Rechnung.	Nach strenger Rechnung.
	p.	p.
0	0	0
1	+12	+12
2	+10	+10
3	+ 5	+ 4
4	+27	+25
5	+ 9	+10
6	−25	−23
7	−26	−25
8	−23	−23
9	− 4	− 3
10	0	0

Von einer Berücksichtigung dieser Abweichungen, welche nur in wenigen Fällen $0^\mu.1$ betragen, wurde Abstand genommen.

Es ist aber noch ein anderer Umstand in Betracht zu ziehen. Auch bei der zweiten strengeren Rechnung ist angenommen worden, dass die Fehler der Messung nur aus den zufälligen Pointirungsfehlern bestehen und unabhängig sind von der Länge des gemessenen Intervalls und von der Zeit der Pointirung. Dies ist nicht strenge richtig, da durch die Temperaturvariationen während der Messung Aenderungen des Unterschiedes zwischen der Entfernung der Mikroskopachsen und den entsprechenden Intervallen des Stabes eintreten können. Diesem Umstande ist leicht Rechnung zu tragen, wenn die Aenderungen gleichmässig zwischen den einzelnen Messungen erfolgen. Ist nämlich die relative Verlängerung des zu bestimmenden Stabes zwischen je zwei Messungen gleich y, die Zahl der Theilintervalle, welche direkt bestimmt werden, n, so muss dem auf die gewöhnliche Weise berechneten Fehler des Striches, welcher das $i$te Theilintervall abschliest, noch die Korrektion

$$+\frac{i\,(n-i)}{2\,n}\,y$$

hinzugefügt werden. Diese regelmässigen Aenderungen sind nun bei den vorliegenden Messungen stets dadurch vollständig eliminirt worden, dass an jede Messungsreihe sich eine Zweite unmittelbar anschloss, by welcher die Einstellungen aber in der entgegengesetzten Richtung erfolgten. Die hierbei noch zurückbleibenden Fehler, welche davon herrühren, dass die Voraussetzung einer gleichmässigen Aenderung der Entfernung der Mikroskope und der Länge des Stabes nicht strenge erfüllt ist, können als zufällige mit den Pointirungsfehlern vereinigt werden, und bleibt daher die gewählte Berechnung gerechtfertigt.

Die Centimeterstriche von R 1878 sind in ganz ähnlicher Weise wie die Decimeterstriche bestimmt, nur treten hier Hülfsintervalle von resp. 5, 2 und 1 cm. Länge an Stelle des konstanten Abstandes der Mikroskope. Für die Berechnung gelten daher auch hier dieselben Bemerkungen wie für die Decimeter.

Die in Beilage C gegebenen Resultate für die Bestimmung der Theilungsfehler von R 1876 durch Vergleichung der Decimeter- und Centimeterstriche mit R 1878, der Millimeterstriche mit einer ihrerseits absolut bestimmten Silbertheilung auf dem Messingstabe Nr. 6, und der Zehntelmillimeter durch direkte mikrometrische Vergleichung werden ohne weiteres verständlich sein. Als Einheit gilt hier $1^{\mu}=0.001$ mm. Die zum Theil stattfindenden Abweichungen der Resultate von den Ihnen unterm 16. April 1879 gemachten Mittheilungen rühren von der genaueren Bestimmung der Vergleichsstäbe her.

* * * * *

Kaiserliche Normal-Aichung-Kommission.
FOERSTER.

BEILAGE A.

Bestimmung der Gesammtlänge und der Ausdehnung des Stahlmeters R 1876.

a. Resultate der Vergleichung des Stahlmeters R 1876 mit dem Messingmeter 1605.

Datum der Vergleichung.	t_{1605}.	t_R.	R—1605.	18.25 ($t_{1605}-t_R$).	7.921 ($t_{1605}-15$).	R—1605 bei 15°. I.	R—1605 bei 15°. II.
1876.	°	°	μ		μ	μ	μ
Mai 8	12.28	12.51	+57.7	−4.20	−21.55	+32.0	+36.2
8	12.26	12.51	+66.6	−4.56	−21.70	+40.3	+44.9
9	11.36	11.39	+56.0	−0.55	−28.83	+26.6	+27.2
Juli 18	18.72	18.96	− 4.2	−4.38	+29.47	+20.9	+25.3
18	18.82	19.06	− 7.3	−4.38	+30.26	+18.6	+23.0
19	18.80	19.10	− 1.8	−5.47	+30.10	+22.8	+28.3
19	18.90	19.15	− 7.2	−4.56	+30.89	+19.1	+23.7
28	19.59	19.71	−16.3	−2.19	+36.36	+17.9	+20.1
28	19.61	19.77	−11.6	−2.92	+36.52	+22.0	+24.9
28	19.69	19.87	−14.5	−3.28	+37.15	+19.4	+22.7
28	19.69	19.91	−15.3	−4.01	+37.15	+17.8	+21.9
Sept. 4	17.63	17.85	− 0.8	−4.01	+20.83	+16.0	+20.0
4	17.73	17.92	+ 2.7	−3.65	+21.63	+20.7	+24.3
4	17.71	17.96	− 0.7	−4.56	+21.47	+16.2	+20.8
4	17.77	17.96	− 0.5	−3.46	+21.94	+18.0	+21.4

Die Stäbe liegen auf je zwei Rollen, welche 0.23 der Stablängen von den Enden entfernt sind. Auf jedem der Stäbe liegt ein Thermometer mit seiner Metallskale auf.

b. Vergleichung der Repsold'schen Stahlmeter R 1876 und R 1878.

I Reihe. t_{1876}.	I Reihe. t_{1878}.	I Reihe. R 1876—R 1878.	II Reihe. t_{1876}.	II Reihe. t_{1878}.	II Reihe. R 1876—R 1878.	III Reihe. t_{1876}.	III Reihe. t_{1878}.	III Reihe. R 1876—R 1878.
°	°	μ μ	°	°	μ μ	°	°	μ μ
8.592	8.608	+22.7	15.740	15.758	+23.2	21.489	21.487	22.1
8.596	8.607	20.9	15.749	15.760	22.0	21.489	21.486	22.9
8.605	8.619	21.9	15.758	15.766	23.3	21.490	21.486	22.7
8.614	8.628	21.4	15.757	15.775	22.4	21.487	21.487	23.6
Mittel . . 8.602	8.615	21.73±0.2	15.751	15.765	22.73±0.2	21.489	21.487	22.83±0.2

Auf jedem der Stäbe liegen zwei Thermometer Fuess'scher Konstruktion, deren Gefässe in mit Quecksilber gefüllten Kupferklötzen stecken und mittelst der letzteren auf den Stäben aufliegen. Jede Beobachtung besteht aus einer doppelten Einstellung auf die Striche der Stäbe und einer doppelten Ablesung der Thermometer.

Der Direktor der Kaiserlichen Normal-Aichungs-Kommission.

Berlin, den 10. August 1881. [L. S.] FOERSTER.

BEILAGE B.

Bestimmung der Theilungsfehler des Stahlmeters R 1878.

I. DIE DECIMETER.

a. Der Mittelstrich

Cm.	A	A	A	A	A	A	A	A	A	A	B	B	B	B	B	B	B	B	B	B
0																				
	−6 890	−3 895	−10 901	+6 895	− 6 898	−5 905	−9 903	−6 905	−1 907	−5 908	884	892	891	901	892	900	894	899	906	903
50																				
	−6 874	−7 879	+ 5 868	0 892	+10 876	+6 881	0 889	+1 888	+8 892	+1 897	868	872	873	892	886	887	889	889	900	898
100											+ 8	+10	+ 9	+4.5	+ 3	+6.5	+2.5	+ 5	+ 3	+2.5

D. Negativer innerer Fehler des Mittelstriches; Generalmittel, +5.4.

b. Die Decimeterstriche mit geradem Index.

Cm	A_1	A_2	A_3	A_4	A_5	B_2	B_3	B_4	B_5	C_2	C_3	C_4	C_5	D_2	D_3	D_4	D_5	M
0	Vom Beobachter gestrichene Messungsreihe.										0	0	0	....	0	0	0	0
		−2 430	0 425	−8 447	− 4 441	....	425	439	437									
20											25	39	37	....	− 1	+24	+19	+14
		−5 464	−6 467	−7 452	− 2 426	....	461	445	424									
40											86	84	61	....	+34	+54	+26	+38
		+2 410	−3 399	−1 372	− 2 371	....	396	371	369									
60											82	55	30	....	+ 5	+11	−23	− 2
		−7 338	−2 433	+4 390	+ 3 414	....	431	394	417									
80											113	49	47	....	+10	−10	−23	− 8
		−8 325	+1 415	+4 421	−11 452	....	416	425	441									
100											129	74	88	....	0	0	0	0

Reihe 2 ist ausgeschlossen; die beiden letzten Beobachtungen der Reihe enthalten Ablesungsfehler von 100.

Cm.	A_6	A_7	A_8	A_9	A_{10}	B_6	B_7	B_8	B_9	B_{10}	C_6	C_7	C_8	C_9	C_{10}	D_6	D_7	D_8	D_9	D_{10}	M	Generalmittel.
0											0	0	0	0	0	0	0	0	0	0	0	0
	+13 402	+ 5 396	−5 427	+3 413	0 410	415	401	422	416	410												
20											15	1	22	16	10	+ 2	−11	+16	+ 6	+ 5	+ 4	+ 3
	− 3 439	− 6 419	−4 442	+8 421	−2 422	436	413	438	429	420												
40											51	14	60	45	30	+25	−10	+47	+25	+21	+21	+20
	− 6 375	+10 362	+4 335	0 364	+7 343	369	372	339	364	350												
60											20	−14	− 1	9	−20	−18	−50	−20	−20	−34	−28	−25
	− 9 421	− 7 423	+7 394	−1 399	+2 419	412	416	401	398	421												
80											32	2	0	7	+1	−19	−46	−26	−32	−17	−28	−25
	− 7 439	− 6 464	−4 436	+1 441	−5 427	432	458	432	442	422												
100											64	60	32	49	23	0	0	0	0	0	0	0

Cm.	A_{11}	A_{12}	A_{13}	A_{14}	B_{11}	B_{12}	B_{13}	B_{14}	C_{11}	C_{12}	C_{13}	C_{14}	D_{11}	D_{12}	D_{13}	D_{14}	M
0									0	0	0	0	0	0	0	0	0
	+4 429	−11 400	−1 408	−7 422	433	389	407	415									
20									33	−11	7	15	+16	−10	− 5	− 2	0
	−4 438	− 1 417	−8 431	+1 432	434	416	423	433									
40									67	+ 5	30	48	+33	+ 7	+ 5	+14	+15
	0 363	+ 2 366	−2 375	+5 369	363	368	373	374									
60									30	−27	3	22	−20	−24	−34	−30	−27
	+1 405	−10 406	−3 427	+2 428	406	396	424	430									
80									36	−31	27	52	−31	−27	−23	−17	−25
	−3 451	− 9 435	+1 434	+1 433	448	426	435	434									
100									84	− 5	62	86	0	0	0	0	0

Bei der Bildung des Generalmittels ist dem Mittel aus den Reihen 6 bis 10 das Gewicht 5, dem Mittel aus den Reihen 11 bis 14 das Gewicht 4, dem Mittel aus den Reihen 3 bis 5 aber nur das Gewicht 1 ertheilt worden, weil die Messungen nach einer Bemerkung des Beobachters unsicher sind.

A. Korrigirte Ablesungen der beiden Mikroskope.
B. Summe derselben d. h. Unterschiede der gemessenen Intervalle gegen den konstanten Abstand der Achsen der Mikroskope I und II von einander.
C. Negative Theilungsfehler der 2-Decimeterstriche mit willkürlichem Schlusswerth.
D. Negative innere Theilungsfehler.
M. Mittelwerthe.

c. Die einzelnen Decimeterstriche.

Cm.	A_1	A_2	A_3	A_4	A_6	A_7	A_8	A_9	A_6	A_{10}	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
0																				
	− 1 246	− 8 237	+ 7 236	− 5 220	−1 240	+ 8 234	0 331	−4 344	− 2 231	− 6 337	245	229	243	215	239	229	242	331	340	331
10																				
	0 228	+ 3 212	−10 227	− 6 231	0 222	− 8 223	+ 3 307	−5 324	− 1 223	− 2 326	228	215	217	225	222	222	215	310	319	324
20																				
	+ 4 242	+10 185	− 3 208	− 9 202	−2 201	− 8 215	− 9 321	−5 303	− 1 216	−10 331	246	195	205	193	199	215	207	312	298	321
30																				
	−12 267	+ 6 217	+ 6 211	− 1 234	−5 239	−11 249	− 3 336	−3 347	− 8 243	− 2 342	255	223	217	233	234	235	238	333	344	340
40																				
	+ 3 197	+ 6 185	− 9 200	− 4 191	−6 201	+ 6 186	+12 299	+4 299	+ 3 195	+ 7 298	200	191	191	187	195	198	192	311	303	305
50																				
	− 3 169	+ 4 169	+ 9 155	− 4 171	+5 158	+ 7 164	− 3 279	−9 281	− 5 173	− 8 281	166	173	164	168	163	168	171	276	272	273
60																				
	− 3 214	+ 1 205	0 201	0 201	+3 183	− 5 208	−11 316	+7 305	+ 4 204	− 8 317	211	206	201	201	186	208	203	305	312	309
70																				
	− 7 218	− 7 202	0 211	+ 4 210	+2 215	+ 4 195	− 1 328	−6 307	+ 4 212	− 2 310	211	195	211	214	217	216	199	327	301	308
80																				
	+10 211	− 6 217	− 9 236	+10 217	−2 231	− 8 229	+ 6 310	−9 324	−13 231	+ 1 334	221	211	227	227	229	218	221	316	315	335
90																				
	+ 5 212	+ 9 203	−12 212	+ 1 202	+6 196	− 3 209	− 1 306	−9 326	+ 4 196	− 2 315	217	212	200	203	211	200	206	305	317	313
100																				

Beilage B—*Bestimmung der Theilungsfehler des Stahlmeters R* 1878—Continued.

C. Die einzelnen Decimeterstriche—Continued.

Cm.	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	M	M′	M″
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	45	29	43	15	39	29	42	31	40	31	+25	+24	+35	+ 8	+29	+18	+33	+18	+28	+15	+23	+17	+12
20	73	44	60	40	61	51	57	41	59	55	+33	+34	+45	+27	+42	+29	+38	+16	+35	+23	+32	+20	+10
30	119	39	65	33	60	66	64	53	57	76	+59	+24	+42	+13	+31	+33	+36	+15	+21	+28	+30	+12	+ 5
40	174	62	82	66	94	101	102	86	101	116	+94	+42	+52	+40	+56	+57	+64	+36	+53	+52	+55	+31	+27
50	174	53	73	53	89	99	94	97	104	121	+74	+28	+35	+20	+41	+44	+47	+34	+44	+41	+41	+11	+ 9
60	140	26	37	21	52	67	65	73	76	94	+20	− 4	− 9	−19	− 5	+ 2	+ 9	− 3	+ 3	− 1	− 1	−25	−25
70	151	32	38	22	38	75	68	78	88	103	+11	− 3	−15	−24	−29	− 1	+ 2	−10	+ 3	− 8	− 7	−25	−26
80	162	27	49	36	55	91	67	105	89	111	+ 2	−13	−12	−17	−21	+ 4	− 8	+ 4	− 8	−16	− 8	−20	−23
90	183	38	76	63	84	109	88	121	104	146	+ 3	− 7	+ 8	+ 4	− 2	+11	+ 4	+ 8	− 5	+ 3	+ 3	− 3	− 4
100	200	50	76	66	95	109	94	126	121	159	0	0	0	0	0	0	0	0	0	0	0	0	0

Ausgleichung.

1. *Der Mittelstrich.*—Giebt man dem vorseitig erhaltenen Mittel der 10 Vergleichreihen der halben Meter unter einander das Gewicht 10, dan kommt dem Mittel aus den 10 Vergleichreihen der einzelnen Decimeter das Gewicht $\frac{10}{5}=2$ zu, es wird also der negative Fehler des Mittelstriches definitiv: $\frac{10\times5.4+2\times41}{12}=+11.$

2. *Die Decimeterstriche mit geradem Index.*—Nach Verbesserung der Decimeterstriche für diesen Werth (M′) erhalten die hieraus resultirenden negativen Fehler der Striche 20 und 80 das Gewicht $10\times\frac{5}{12}=\frac{50}{12}$, die der Striche 40 und 60, $10\times\frac{5}{8}=\frac{75}{12}$. Aus den direkten Messungsreihen der Doppeldecimeter erhalten dagegen die vorseitig ermittelten negativen Fehler für die Striche 20 und 80 das Gewicht $10\times\frac{5}{8}=\frac{75}{12}$, für die Striche 40 und 60, $10\times\frac{5}{12}=\frac{50}{12}$.

Damit wird für Strich...
$$\left\{\begin{array}{ll} 20 \text{ der negative Fehler} & \frac{50\times20+75\times3}{50+75}=+10 \\ 80 \text{ der negative Fehler} & -\frac{50\times20+75\times25}{50+75}=-23 \\ 40 \text{ der negative Fehler} & \frac{75\times31+50\times20}{75+50}=+27 \\ 60 \text{ der negative Fehler} & -\frac{75\times25+50\times25}{75+50}=-25 \end{array}\right.$$

3. *Die Decimeterstriche mit ungeradem Index.*—Die negativen Fehler derselben werden unmittelbar aus der Reihe M′ durch Verbesserung für die Decimeterstriche mit geradem Index erhalten. Der Mittelstrich erfährt dabei noch eine Verbesserung um −2, die auch mit Rücksicht auf die starke Differenz zwischen dem aus der Vergleichreihe der Decimeter erhaltenen Werth und den aus der Vergleichung der halben Meter resultirenden ganz gerechtfertigt erscheint.

II.—DIE CENTIMETER.

a.—Der Strich 5 cm.

Cm.	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
0	+3 396	+4 202	+9 201	− 2 115	+ 4 168	+1 222	0 220	−2 259	+1 196	+1 221	399	206	210	113	172	223	220	257	197	222
5	+5 364	+7 148	−2 169	+ 4 74	+ 4 122	−1 196	−5 173	−2 210	+3 293	−5 268	369	155	167	78	126	195	168	208	296	263
	−3 149	−7 161	−3 180	+12 173	−12 197	+5 182	+1 164	+6 154	0 290	0 294	146	154	177	185	185	187	165	160	290	294
10	−1 118	−3 135	−1 161	+ 5 160	0 128	+6 133	0 117	+1 126	+1 319	0 334	117	132	160	165	128	139	117	127	320	334

Cm.	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}	A_{17}	A_{18}	A_{19}	A_{20}	B_{11}	B_{12}	B_{13}	B_{14}	B_{15}	B_{16}	B_{17}	B_{18}	B_{19}	B_{20}
0	0 220	− 9 227	+ 2 217	+8 94	−8 97	−9 110	+8 74	+4 135	− 4 189	+ 9 188	220	218	219	102	89	101	82	139	185	197
5	+3 271	−11 298	+12 252	+1 159	+1 149	−4 154	+2 126	−8 196	+ 5 249	+12 237	274	287	264	160	150	150	128	188	254	249
	+3 276	+ 2 290	− 3 162	−5 171	−2 171	+1 151	+3 152	+8 201	−10 257	+ 5 237	279	292	159	166	169	152	155	209	247	242
10	+5 326	+ 3 331	− 9 196	−3 202	−9 206	+6 197	+8 183	+1 202	+ 2 301	− 7 305	331	334	187	199	197	203	191	203	303	298

Cm.	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}	C_{19}	C_{20}
0																				
	−30	−51	−43	−35	−46	−28	−52	−49	−99	−41	−54	−69	−45	−58	−61	−49	−46	−49	−69	−52
5																				
	−29	−22	−17	−20	−57	−48	−48	−33	−30	−40	−52	−42	−28	−33	−28	−51	−36	+ 6	−56	−56
10																				
D.	− 0.5	−14.5	−13	− 7.5	+ 5.5	+10	− 2	− 8	(−34.5)	− 0.5	− 1	−13.5	− 8.5	−12.5	−16.5	+ 1	− 5	(−27.5)	− 6.5	+ 2

Schliesst man die beiden stark herausfallenden Werthe C_9 und C_{18} aus, so erhält man als Mittel aus den übrigen 18 : — 5 mit dem mittleren Fehler $\pm$ 1.4. Hätte man die beiden ausgeschlossenen Werthe mitgenommen, so hätte man als Mittel erhalten — 7.5 mit dem mittleren Fehler $\pm$ 2.1.

P. N. A_9 bis A_{20} sind unter Vertauschung der Stäbe in Bezug auf die messenden Mikroskope gegen ihre Lage bei A_0 bis A_8 gemessen worden.

b.—Die Centimeterstriche mit geradem Index.

Cm.	A_1	A_2	A_3	A_4	A_5	B_1	B_2	B_3	B_4	B_5	C_1	C_2	C_3	C_4	C_5	D_1	D_2	D_3	D_4	D_5	M
0	+ 5 180	+ 7 99	−1 97	−3 100	−8 107	185	106	96	97	99						0	0	0	0	0	0
2	+ 1 197	+ 9 111	−1 115	0 111	+3 111	198	120	114	111	114	−13	−14	−18	−14	−15	−23	−14	−21	−20	−18	−19
	+ 4 207	+ 1 111	−3 105	−7 118	+7 107	211	112	102	111	114											
4	+13 189	− 9 127	−3 107	+1 80	−1 118	202	118	104	81	117	+ 9	− 6	− 2	+30	− 3	−24	−19	−27	+ 3	−24	−18
	+ 1 177	−11 125	+5 108	−4 76	−3 121	178	114	113	72	118											
6	+ 2 152	+ 1 108	−2 93	−1 59	−7 106	154	109	91	58	99	+24	+ 5	+22	+14	+19	− 9	−14	− 8	+11	− 9	− 6
	− 8 142	− 7 109	−1 96	−6 66	−9 122	134	102	95	60	113											
8	0 101	−12 118	+8 69	−9 50	−2 96	101	106	77	41	94	+33	− 4	+18	+19	+19	+14	−17	+ 6	+23	+ 7	+ 7
	− 8 112	+ 3 121	−4 65	−5 54	−6 94	104	124	61	49	88											
10	+12 96	0 107	+1 63	0 66	−1 93	108	107	64	66	92	− 4	+17	− 3	−17	− 4	0	0	0	0	0	0

BEILAGE B—*Bestimmung der Theilungsfehler des Stahlmeters R 1878*—Continued.

c. DIE EINZELNEN CENTIMETERSTRICHE.

Cm.	A_1	A_2	A_3	B_1	B_2	B_3	C_1	C_2	C_3	D_1	D_2	D_3	M_0	M_1		M
0	−3 210	−3 209	+ 1 172	207	206	173				0	0	0	0	0	0	0
							−42	−60	−51							
1	+ 3 246	−5 271	− 2 226	249	266	224				− 6	− 6	− 1	− 4	− 6	......	− 8
	− 6 250	−3 266	− 9 236	244	263	227										
							−40	−51	−50							
2	+ 4 280	+1 313	− 2 279	284	314	277				−10	− 4	− 1	− 5	− 9	−19	−14
	− 6 286	+3 314	+ 2 292	280	317	294										
							−38	−58	−63							
3	+ 3 315	+5 370	+14 343	318	375	357				−11	− 8	−14	−11	−16	......	−18
	0 320	−2 370	− 7 351	320	368	344										
							−27	−58	−50							
4	− 8 355	−2 428	−11 405	347	426	394				− 2	−13	−14	−10	−17	−18	−17
	+ 1 378	0 430	−11 415	379	430	404										
							−25	−34	−40							
5	+ 7 397	+2 462	− 1 445	404	464	444				+ 9	+ 7	− 4	+ 4	− 5	......	− 5
	− 1 400	−3 474	+ 7 448	399	471	455										
							−43	−58	−57							
6	+ 3 439	+9 520	− 7 519	442	529	512				+ 2	+ 2	−11	− 2	− 9	− 6	− 7
	+ 5 449	−6 519	+ 4 529	454	513	533										
							−25	−65	−64							
7	− 1 480	−5 583	0 597	479	578	597				+13	− 9	−25	− 7	−13	......	−16
	− 4 475	−7 296	− 5 598	471	289	593										
							−26	−20	−20							
8	+13 484	−2 311	+ 3 610	497	309	613				+24	+25	+ 5	+18	+14	+ 7	+10
	− 5 490	−9 326	+ 2 619	485	317	621										
							−51	−71	−36							
9	− 5 541	−3 391	− 4 661	536	388	657				+ 9	+ 7	+19	+12	+10	......	+ 9
	− 1 540	−9 385	+ 6 669	539	376	675										
							−45	−61	−69							
10	+ 4 580	−1 438	+ 6 738	584	437	744				0	0	0	0	0	0	0

A. Für periodische Fehler der Schrauben korrigirte Ablesungen beider Mikroskope.
B. Summe der beiden Ablesungen.
C. Relative negative Fehler der einzelnen Intervalle auf das Hülfsintervall.
D. Negative innere Fehler der Theilstriche innerhalb des Decimeters.

AUSGLEICHUNG.

a. Der 5 cm-Strich.—Giebt mandem Mittel aus den 18 wirklich benutzten Vergleichungen der beiden halben Decimeter unter einander das Gewicht 18, dann kommt dem Mittel M_0 aus den auf der vorhergehenden Seite berechneten 3 Vergleichreihen der einzelnen Centimeter das Gewicht $\frac{3}{5}$ zu, es wird also der negative Fehler des Mittelstriches des ersten Decimeters $\frac{-18\times5+\frac{3}{5}\times4}{18+\frac{3}{5}}=-5$.

b. Die übrigen Centimeterstriche.—Nach Verbesserung der Mittel M_0 für diesen Werth des 5 cm-Strichs erhalten die Fehler M_1 für die Centimeterstriche mit geradem Index Werthe, die von denen aus der direkten Vergleichreihe der Doppelcentimeter nur um wenige Einheiten abweichen. Nimmt man deshalb ohne Rücksicht auf die nicht sehr verschieden ausfallenden Gewichte die Mittel aus beiden und verbessert dafür die Centimeterstriche mit ungeradem Index, so wird ein wesentlicher Fehler nicht begangen.

Der Direktor der Kaiserlichen, Normal-Aichungs-Kommission.
Berlin, den 10. August 1881. [L. S.] FOERSTER.

BEILAGE C.

Bestimmung der Theilungsfehler von Repsold's Meter aus Stahl für Amerika, R 1876.

Für die Bestimmung der Theilungsfehler der Decimeter sind diese 20 mal mit Doppeleinstellungen beider Mikrometer mit denen auf dem der Kommission gehörigen Stabe von Repsold *R* 1878 verglichen worden. Die Ergebnisse der einzelnen Messungsreihen, vom periodischen Fehler der Schrauben und den Konstanten der Anfangs- und Schlusseinstellung befreit, sind, ausgedrückt in Tausendtelmillimetern, im Sinne relativer Fehler gegen den Vergleichstab in den Kolumnen 1 bis 20 zusammengestellt. Kolumne 21 enthält das Mittel der Kolumnen 1 bis 20, Kolumne 22 die inneren Fehler des Vergleichstabes, Kolumne 23=21+22 die absoluten inneren Fehler des amerikanischen Stabes. Zur allgemeinen Vergleichung ist unter 24 noch das Resultat einer älteren, aber unvollkommeneren Vergleichungsreihe mit einem anderen Stabe mitgetheilt, deren relatives Gewicht aber so gering ist, dass sie keinen Einfluss mehr auf die Werthe der Kolumne 23 zu äussern vermag.

Beilage C—*Bestimmung der Theilungsfehler von Repsold's Meter*, *R* 1876—Continued.

Decimeter-strich.	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0
1	−0.1	+0.7	+0.6	+0.7	−0.2	+0.3	−0.5	+0.7	+0.5	+1.1	−1.1	+0.1
2	−2.1	−0.4	−0.7	−0.7	−0.6	−0.4	−1.3	−1.3	−0.8	−0.7	−4.1	−1.6
3	−1.0	−0.1	+0.1	+0.6	0.0	−0.9	−1.5	−0.7	0	+0.5	−2.4	−1.9
4	−1.8	+0.4	+0.4	+0.1	0.0	+0.3	+0.2	+0.9	0	+0.9	−1.3	−0.7
5	−1.3	−0.4	−0.4	−0.3	−0.5	−0.4	−1.8	−1.4	−0.7	−0.5	−1.8	−1.3
6	−2.4	−1.1	−1.9	−1.7	−1.5	−0.4	−1.6	−1.6	−1.3	−1.3	−0.3	+0.5
7	−3.2	−2.2	−2.7	−3.1	−2.4	−2.7	−2.9	−2.0	−2.1	−2.7	−1.8	−1.3
8	−1.9	−1.9	−1.8	−1.4	−1.0	−1.4	−2.4	−1.3	−0.8	−1.1	−1.8	−0.5
9	−0.6	−0.1	+0.8	−0.4	−0.6	−0.7	−1.9	−1.1	−0.8	−0.7	−1.0	−0.4
10	0	0	0	0	0	0	0	0	0	0	0	0

Decimeter-strich.	13	14	15	16	17	18	19	20	21	22	23	24
0	0	0	0	0	0	0	0	0	0	0	0	0
1	−0.5	−0.5	−0.6	−0.3	+0.4	−0.5	−0.4	−0.6	0	−0.6	−0.6	−1.2
2	−3.1	−3.3	−3.5	−2.4	−2.0	−2.9	−3.3	−3.2	−1.9	−0.5	−2.4	−2.0
3	−1.7	−2.4	−2.6	−1.5	−1.4	−1.4	−2.3	−2.3	−1.1	−0.2	−1.3	−0.5
4	−1.3	+0.1	−1.2	0	+0.5	−0.6	−1.0	−0.8	−0.2	−1.3	−1.5	−1.0
5	−1.7	−0.7	−1.1	−1.0	−1.1	−1.9	−1.1	−1.5	−1.0	−0.4	−1.4	−0.8
6	−0.5	+0.1	−0.3	+0.7	+0.1	+0.4	+0.1	+0.2	−0.7	+1.2	+0.5	−0.8
7	−1.6	−1.6	−1.7	−0.8	−0.9	−0.8	−1.4	−1.1	−2.0	+1.3	−0.7	−2.0
8	−1.0	−1.3	−0.9	−0.4	−1.2	−0.6	−0.9	−0.3	−1.2	+1.1	−0.1	−0.1
9	+0.2	−1.1	−0.9	−0.2	+0.4	−0.3	−0.3	−0.6	−0.5	+0.2	−0.3	−1.3
10	0	0	0	0	0	0	0	0	0	0	0	0

Für die Bestimmung der Theilungsfehler der Centimeter des ersten Decimeters sind diese ebenfalls 20 mal unter Doppeleinstellung beider Mikroskope mit denen auf dem der Kommission gehörigen Stabe von Repsold verglichen worden. Die Ergebnisse sind analog denen für die Decimeter zusammengestellt. Kolumne 23 enthält hier die inneren Theilungsfehler der Centimeter innerhalb des Decimeters, Kolumne 24 dieselben innerhalb des ganzen Meters.

Centimeter-strich.	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0
1	+1.5	+2.5	+2.3	+1.5	+1.4	+2.3	+1.9	+2.3	+2.2	+3.2	+1.6	+3.0
2	+1.0	+2.1	+2.1	+0.4	+1.7	+1.6	+0.9	+1.0	+1.0	+0.9	+2.0	+2.6
3	+2.4	+2.3	+2.0	+2.4	+2.2	+2.3	+2.1	+2.3	+2.1	+2.3	+1.8	+1.1
4	+1.4	+1.6	+1.8	+1.5	+1.7	+1.0	+1.4	+1.8	+1.9	+2.0	+1.3	+0.9
5	+1.4	+1.6	+1.6	+2.8	+2.0	+1.3	+1.2	+1.4	+1.3	+1.5	+1.3	+0.6
6	+0.4	+0.7	+0.9	−0.2	+0.5	+0.6	+0.8	0	+0.5	+1.0	+1.1	+0.4
7	+0.4	+1.0	+1.4	+0.4	+1.6	+0.8	+0.6	+0.3	−0.4	+1.2	+1.3	+1.0
8	−0.2	+0.3	−0.3	+0.8	−0.2	+0.8	+0.4	−0.5	−0.4	−0.2	+0.7	+0.7
9	+1.7	+2.2	+1.5	+2.1	+2.4	+2.1	+2.0	+1.8	+2.4	+2.2	+1.4	+1.2
10	0	0	0	0	0	0	0	0	0	0	0	0

Centimeter-strich.	13	14	15	16	17	18	19	20	21	22	23	24
0	0	0	0	0	0	0	0	0	0	0	0	0
1	+2.4	+2.0	+2.5	+2.4	+2.6	+1.4	+2.0	+2.3	+2.2	+0.4	+2.6	+2.5
2	+2.5	+1.6	+2.2	+2.5	+2.4	+1.5	+2.4	+2.3	+1.7	+0.7	+2.4	+2.3
3	+1.8	+1.2	+1.6	+2.1	+2.7	+2.5	+2.4	+2.3	+2.1	+0.9	+3.0	+2.8
4	+1.4	+1.0	+1.2	+1.2	+1.9	+0.8	+2.0	+1.9	+1.5	+0.8	+2.3	+2.1
5	+1.3	+1.3	+0.7	+1.0	+0.5	+0.5	+0.8	+1.8	+1.2	−0.2	+1.4	+1.1
6	+1.4	+1.9	+0.7	+0.5	+0.9	−0.4	+0.2	+1.8	+0.7	+0.3	+0.1	+0.6
7	+0.7	+2.2	+0.9	+1.3	+1.3	+0.9	+1.3	+2.4	+1.0	+0.8	+1.8	+1.4
8	+0.8	+1.5	+1.0	+0.9	+1.2	+0.7	+0.5	+1.6	+0.5	−0.5	0.0	−0.5
9	+2.4	+2.4	+1.7	+2.5	+1.5	+1.7	+2.3	+2.9	+2.0	−0.5	+1.5	+1.0
10	0	0	0	0	0	0	0	0	0	0	0	−0.6

Beilage C—*Bestimmung der Theilungsfehler von Repsold's Meter aus Stahl für Amerika, R1876*—Continued.

Die Millimeter sind zweimal mit denen auf Hauptnormal Nr. 6 verglichen worden und zwar in Gruppen von je zehn. Die folgende Zusammenstellung enthält in den Kolumnen 1 und 2 die vom periodischen Fehler der Schraube, den Konstanten und den Theilungsfehlern des Vergleichstabes befreiten Resultate, Kolumne 3, das Mittel aus 1 und 2, stellt die inneren Theilungsfehler der Millimeter innerhalb der einzelnen Centimeter, Kolumne 4 innerhalb des ganzen Meters dar.

Millimeter-strich.	1.	2.	3.	4.	Millimeter-strich.	1.	2.	3.	4.	Millimeter-strich.	1.	2.	3.	4.
0	0	0	0	0	20	0	0	0	+2.3	40	0	0	0	+2.1
1	−1.1	−1.2	−1.2	−0.9	21	−0.8	−0.6	−0.7	+1.6	41	−0.5	−1.3	−0.9	+1.1
2	−1.0	−1.0	−1.0	−0.5	22	−0.1	−0.8	−0.4	+2.0	42	+0.8	−0.5	+0.2	+2.1
3	−1.4	−0.5	−1.0	−0.2	23	+0.1	+0.1	+0.1	+2.5	43	−0.6	−1.7	−1.1	+0.7
4	−0.7	−1.8	−1.2	−0.2	24	−0.9	−1.5	−1.2	+1.3	44	+0.1	−1.6	−0.8	+0.9
5	+1.0	−1.7	−0.3	+1.0	25	−0.8	−1.5	−1.1	+1.4	45	−0.2	−0.8	−0.5	+1.1
6	−1.5	−1.8	−1.6	0.0	26	+0.8	−0.5	+0.2	+2.8	46	+0.7	−1.1	−0.2	+1.3
7	−1.2	−1.8	−1.5	+0.3	27	−0.3	−1.0	−0.7	+1.9	47	+0.1	−1.3	−0.6	+0.8
8	−0.7	−1.0	−0.9	+1.1	28	+0.2	−1.5	−0.7	+2.0	48	+0.4	−2.5	−1.0	+0.3
9	−1.6	−2.0	−1.8	+0.4	29	+0.5	−0.1	+0.2	+2.9	49	−1.2	−2.0	−1.6	−0.4
10	0	0	0	+2.5	30	0	0	0	+2.8	50	0	0	0	+1.1
10	0	0	0	+2.5	30	0	0	0	+2.8	50	0	0	0	+1.1
11	−0.6	0.0	−0.3	+2.2	31	+1.8	0	+0.9	+3.6	51	−0.2	−0.5	−0.3	+0.8
12	−1.1	−0.2	−0.6	+1.2	32	+0.1	−0.3	−0.1	+2.5	52	+1.2	+1.2	+1.2	+2.2
13	−0.5	−0.5	−0.5	+2.0	33	−0.1	+0.1	0.0	+2.6	53	−0.1	−0.2	−0.2	+0.8
14	+0.1	−1.3	−0.6	+1.8	34	−0.3	−0.6	−0.4	+2.1	54	−1.6	−1.9	−1.8	−0.9
15	+0.6	+1.1	+0.8	+3.2	35	0.0	+0.7	+0.3	+2.7	55	+0.6	−0.4	+0.1	+1.0
16	+0.3	−0.8	−0.2	+2.2	36	−1.5	−0.1	−0.8	+1.6	56	−0.3	−1.2	−0.7	+0.1
17	+0.6	+1.4	+1.0	+3.3	37	−0.2	+0.3	+0.1	+2.4	57	−0.4	−1.1	−0.7	+0.1
18	+0.4	+0.9	+0.7	+3.0	38	−0.6	−0.6	−0.6	+1.6	58	−0.1	−1.2	−0.7	0.0
19	+1.1	+0.9	+1.0	+3.3	39	−1.1	−1.1	−1.1	+1.1	59	−0.4	+0.6	+0.1	+0.8
20	0	0	0	+2.3	40	0	0	0	+2.1	60	0	0	0	+0.6

Millimeter-strich.	1.	2.	3.	4.	Millimeter-strich.	1.	2.	3.	4.
60	0	0	0	+0.6	80	0	0	0	−0.5
61	+0.6	+1.7	+1.1	+1.8	81	+1.1	+0.1	+0.6	+0.3
62	0	+0.4	+0.2	+1.0	82	+0.2	+0.4	+0.3	+0.1
63	−0.1	+0.5	+0.2	+1.0	83	+1.6	+1.1	+1.3	+1.2
64	+0.2	+0.7	+0.4	+1.3	84	+0.3	+0.6	+0.4	+0.5
65	+2.5	+1.5	+2.0	+3.0	85	+0.4	+1.7	+1.1	+1.3
66	+0.4	+1.1	+0.8	+1.9	86	−0.7	+1.2	+0.3	+0.7
67	+1.3	+1.7	+1.5	+2.6	87	−1.1	+1.9	+0.4	+1.0
68	+1.1	+0.9	+1.0	+2.2	88	+1.3	+2.8	+2.1	+2.8
69	+1.4	+1.3	+1.4	+2.7	89	−3.1	−0.1	−1.6	−0.7
70	0	0	0	+1.4	90	0	0	0	+1.0
70	0	0	0	+1.4	90	0	0	0	+1.0
71	+1.6	+1.4	+1.5	+2.7	91	−1.4	−0.2	−0.8	0.0
72	+1.0	+0.8	+0.9	+1.9	92	−1.6	−1.0	−1.3	−0.6
73	+1.0	+0.9	+1.0	+1.8	93	0.0	+0.8	+0.4	+0.9
74	−0.7	−0.2	−0.5	+0.1	94	−0.7	+0.2	−0.2	+0.2
75	−0.2	−0.6	−0.4	0.0	95	+0.3	+1.5	+0.9	+1.1
76	−0.2	+0.4	+0.1	+0.3	96	−0.2	+1.0	+0.4	+0.4
77	−0.1	+0.2	+0.1	+0.1	97	−0.1	+0.8	+0.4	+0.3
78	+0.4	+0.4	+0.4	+0.2	98	+0.4	+1.5	+1.0	+0.7
79	−0.2	+0.4	+0.1	−0.3	99	+0.4	+1.2	+0.8	+0.4
80	0	0	0	−0.5	100	0	0	0	−0.6

Die Zehntelmillimeter des ersten Millimeters nebst den beiden Zusatzzehnteln sind zweimal mit einem Hülfsintervall im Gesichtsfeld des einen Mikroskopes verglichen, die Resultate sind in den Reihen I und II enthalten; III, das Mittel aus I und II, enthält die inneren Theilungsfehler innerhalb des ganzen Millimeters, IV innerhalb des Meters

Millimeterstrich.	I.	II.	III.	IV.
−0.1	+0.7	−0.2	+0.2	+0.4
0.0	0	0	0	0
+0.1	−0.8	−0.1	−0.4	−0.6
0.2	+0.4	+0.9	+0.6	+0.5
0.3	+0.9	+1.6	+1.2	+0.9
0.4	+2.4	+2.3	+2.3	+2.0
0.5	−0.3	+0.3	0	−0.5
0.6	+2.0	+0.6	+1.3	+0.7
0.7	+1.0	+0.5	+0.7	+0.1
0.8	+0.4	+0.3	+0.3	−0.3
0.9	−0.9	−0.6	−0.7	−1.6
1.0	0	0	0	−0.9
1.1	+1.5	+1.8	+1.7	+0.7

Der Director der Kaiserlichen Normal-Aichungs-Kommission.
Berlin, den 10. August 1881.

[L. S.] FOERSTER.

[TRANSLATION.]

BERLIN, *August* 10, 1881.

The Commission is now enabled to communicate to you more complete details concerning the determinations made here of the length and the expansion, and also the errors of graduation of the steel metre, graduated on the neutral axis on platinum, made for you by Repsold, Hamburg, which is known here as R 1876, and to explain the results given in letter of June 20, 1879, and partially modify them. This will also answer the questions raised in your letters of October 16, 1880, February 16, and April 27, 1881, so far as they relate to this metre. The chief cause of delay has been that we waited for the more exact results of the comparisons made at Breteuil, in order to be able to send you as definitive results as possible, and that these comparisons have only lately been made known to us.

The first determinations of the total length of the bar were made in 1876 by 15 comparisons with brass metre 1605 at temperatures between 11° and 20°. These comparisons, however, are worthy of but little confidence, because they were among the first made on the comparator, and the method of observation was not at that time sufficiently elaborated. Especially did the determination of the temperature of the bar in this series of comparisons leave much to be desired. There is also on this account some uncertainty as to the best method of reducing the comparisons, and the results of the computation are unreliable within wide limits. The individual results of these comparisons are given in Appendix A, viz: the temperature of the two bars t_{1605} and t_R as given by the readings of the thermometers lying upon them, and the difference of length found directly. The computation was made incidentally with the coëfficient of expansion found later, and, in fact, under the assumption (I) that the temperatures of the bars corresponded to the readings of the thermometers lying upon them, from which assumption results in the mean—

$$R-1605=+21^{\mu}.9 \text{ at } +15^{\circ}.00$$

In the second place it is assumed that the temperatures of both bars are given by t_{1605} alone, because the surface of the other thermometer in contact with R 1876 was proportionally too small to expect reliable results. (II) From this second assumption there results in the mean—

$$R1876-1605=+25^{\mu}.7 \text{ at } +15^{\circ}.00$$

Finally, the assumption would come under consideration, and would perhaps be preferable to the foregoing ones, that the mean of the temperature-indications of both thermometers be taken as the temperature of both bars. From this assumption would follow—

$$R1876-1605=+27^{\mu}.3 \text{ at } 15^{\circ}.00$$

Brass metre 1605 has been often compared with the German original standard (Pt). From the latest best measurement results—

$$1605-Pt=-53^{\mu}.14\pm 0^{\mu}.79+t\,(9^{\mu}.653\pm 0^{\mu}.044)$$

and therefore, by applying the legal equation of the original standard,—

$$1605=1^{m}-60^{\mu}.13+18^{\mu}.253\,t$$

that is—

$$1605=1^{m}+213^{\mu}.7 \text{ at } +15^{\circ}.00$$

The combination of this value with the above differences of R 1876—1605 gives much smaller values for the length of R 1876 than the later comparisons, but there is, as already remarked, greater doubt as to the accuracy of this first series of measurements.

A determination of the length of R 1876 which affords very much better guarantees for its correctness, rests on the comparison of R 1876 with a Repsold metre exactly similar to it, designated R 1878. In March, 1878, three series were made at temperatures 8°.60, 15°.75 and 21°.49, the details of which are given in the appendix, and which, if we take into account the very small differences between the temperatures of the two bars, lead to the result—

$$\begin{aligned} R1876 - R1878 &= +\overset{\mu}{21.86} \text{ at } \overset{\circ}{8.602} \\ &+22.28 \text{ at } 15.751 \\ &+22.81 \text{ at } 21.489 \end{aligned}$$

The mean of all the series gives

$$R1876 - R1878=+22^{\mu}.52\pm 0^{\mu}.21.$$

That result, however, is to be preferred which is obtained by taking into account the difference of expansion of both bars,

$$R1876 - R1878=22^{\mu}.52\pm 0^{\mu}.17+(t-15.281)\,(0^{\mu}.076\pm 0^{\mu}.029),$$

or, in another form,

$$R1876 - R1878=21^{\mu}.36\pm 0^{\mu}.48+t\,(0^{\mu}.076\pm 0^{\mu}.029)$$

The bar R 1878 was compared with the German original metre in two series, in the years 1878 and 1879. The first of these series permits only of the computation of the relative expansion. The result was—

1. $R1878-Pt=\text{Const.}\ \pm 1^{\mu}.68+t\,(1^{\mu}.676\pm 0^{\mu}.074)$
2. $R1878-Pt=+68^{\mu}.79\pm 0^{\mu}.49+t\,(1^{\mu}.643\pm 0^{\mu}.032)$

The following equation, resulting from the combination of both series of observations, is to be used:

$$R1878 - Pt=+68^{\mu}.71\pm 0^{\mu}.49+t\,(1^{\mu}.648\pm 0^{\mu}.029)$$

Combining this with the above we have

$$R1876 - Pt=+90^{\mu}.07\pm 0^{\mu}.68+t\,(1^{\mu}.724\pm 0^{\mu}.041)$$

If we place for Pt its legal value, resulting from the comparison with the Mètre des Archives, on the assumption that the expansion of both metres is $8^{\mu}.6t$, then

$$R1876=1^{m}+93^{\mu}.08+10^{\mu}.324t$$

The considerable difference, $-1^{\mu}.16+0^{\mu}.014t$, from the result given to you by letter of June 20, 1879,

$$R1876=1^{m}+248^{\mu}.89\pm 0^{\mu}.25+(10^{\mu}.31\pm 0^{\mu}.034)\,(t-15)$$

is caused by small supplementary corrections which resulted from a further revision of the thermometer computations. Moreover, in the probable errors there given, only the uncertainty of the comparison of the two Repsold metres with each other was taken into account.

The statement made to you in letter of September 15, 1880, that the given equation of R 1876 ought to differ only from 1^{μ} to 2^{μ} from the true metric measure, has, unfortunately, not been confirmed, so far as low temperatures are concerned. It was founded on a comparison made at Breteuil of R 1878 with the platinum-iridium metre, type I, at 30° C., which had given a result very consistent with assumptions previously made. Comparisons at 0° and 15° C. have not confirmed this result. As the result of the three series of comparisons we now have

$$R\,1878=1^{m}+64^{\mu}.82+10^{\mu}.371t+0^{\mu}.0069t^2$$

In this it is assumed that the equation of type I is

$$I=1^{m}+67^{\mu}.6+8^{\mu}.525t+0^{\mu}.0039t^2$$

The error of type I at 0° has been derived by indirect comparison with the Mètre des Archives, and the expansion of the bar is the mean value of various absolute determinations. In the equation of R 1878, the coëfficient of t^2 is assumed from Fizeau's determinations, and the amounts of the other terms are computed from the determinations at 0° and 30°. As the comparison at 15° gave a result differing only about $0^{\mu}.05$ from the formula the latter is retained. Neglecting the term involving t^2 we can place, when the temperature is near 15°, according to the determinations at Breteuil,

$$R\,1878=1^{m}+64^{\mu}.82+10^{\mu}.578t$$

The correction which according to this should be applied to the equation derived from the comparison with the German metre is

$$-6^{\mu}.9+0^{\mu}.330\,t$$

With this correction, the equation for R 1876 becomes

$$R\,1876=1^{m}+86^{\mu}.18+10^{\mu}.651\,t$$

This may, for the present, be considered the most probable value of R 1876 in true metric measure. By true metric measure must be understood that system of measure represented by the Mètre des Archives as unit, as a definition independent of this does not as yet exist, and in the future establishment of such a definition, as close a junction as possible with this system will certainly be adopted.

The great difference in the coëfficient of expansion of R 1878, found at Breteuil, and also confirmed by the investigations communicated by you, from the value sent by us, must be attributed mainly to the circumstance that the assumed expansion as given by Borda for platinum is too small. In fact, according to Fizeau, the expansion of the platinum metre is

$$8^{\mu}.68\,t+0^{\mu}.0039\,t^2$$

and, therefore, $8^{\mu}.80t$ in the vicinity of 15°.

A small difference of expansion is also indicated between the Mètre des Archives and the German original metre, as the comparison of these metres may also be represented by the equation

$$Pt-A=1^{\mu}.14+0^{\mu}.094t$$

or, if we consider only the comparisons by Brix,

$$Pt-A=0^{\mu}.39+0^{\mu}.136t$$

These comparisons, however, were made at only slightly different temperatures.

From absolute expansion determinations which are being carried on here, we are in hopes of being able in a few months to contribute to a decision of the uncertainties yet existing.

A union with the system derived from the toise, especially with Bessel's copy of the toise, which has become so important in geodetic work, has been effected for your bar through two comparisons in 1878 and 1879 of R 1878 and the Lenoir toise. The results were:

1. $L=1.948\ 570\ 76\ R\,1878$ at 18°.255
2. $L=1.948\ 575\ 53\ R\,1878$ at 15°.74

Combining the two equations, that is, reducing them by interpolation according to temperature to the standard temperature, we have

$$L=1.948\ 574\ 57\ R\,1878 \text{ at } 16^{\circ}.25$$

The Lenoir toise, as repaired (*umgearbeitet*) in 1852, by Baumann, was compared with the Bessel toise in the years 1852 and 1872, and on our Repsold comparator in the years 1877 and 1878. The results of these comparisons are:

$$\begin{aligned}
&1.\ L-B=-0'''.0043\\
&2.\ L-B=-0'''.0066\\
&3.\ L-B=-0'''.00372\pm0'''.00013\\
&4.\ L-B=-0'''.00336\pm0'''.00022
\end{aligned}$$

As the best result, the mean of the last two comparisons may be taken; hence

$$L-B=-0'''.00354$$

The Bessel toise is generally taken, according to Bessel, as 0'''.00080 shorter than "the toise." According to its own certificate it would be 0'''.00078 too short. From a comparison made by Bessel with two other certified toises it was found to be 0'''.00333 too short and 0'''.00411 too long. In the mean from the three certificates, its complete identity with the toise of Peru would result.

If we adhere to the equation always used

$$B=1^{T}-0'''.00080 \text{ at } 16^{\circ}.25$$

then there follows

$$L=1^{T}-0'''.00434 \text{ at } 16^{\circ}.25$$

and, in combination with the equation previously derived,

$$L=1.948\ 574\ 57\ R\,1878 \text{ at } 16^{\circ}.25$$

from which

$$R\,1878=\frac{443.296}{864}\,1.000\ 236\ 96\ L \text{ at } 16^{\circ}.25$$

whence follows

$$R\,1878=443'''.296+231^{\mu}.94 \text{ at } 16^{\circ}25$$

or, with the earlier-found coëfficient of expansion

$$R\,1878=443'''.296+65\ .41+10^{\mu}.248\,t$$

and, consequently

$$R\,1876=443'''.296+86^{\mu}.77+10^{\mu}.324\,t$$

differing by $-4^{\mu}.2+0^{\mu}.014\,t$ from the preliminary result of June 20, 1879, which is

$$R\,1876=443'''.296+245^{\mu}.6+10^{\mu}.31\ (t-15)$$

With the Paris coëfficient of expansion we would have had

$$R\,1876=443'''.296+240^{\mu}.1+10^{\mu}.55\ (t-15)$$

The more important measurements for the determination of the graduation errors of R1876 are also based preferably on the comparisons with R1878. We have therefore also given in Appendix B the measurements and computations for the determination of the graduation errors of the last-named bar in more detail. The errors of graduation have all been examined on a Repsold graduating machine, which was so arranged that two micrometer-microscopes rigidly connected with each other could be moved in the direction of the length of a bar, while each of the two microscopes points at the same bar, or a single microscope points at one of two bars lying side by side. The first arrangement was adopted in the investigation of the errors of the decimeter marks of R1878 by making the distances between the microscopes respectively 5, 2, and 1 decimeters, and then comparing the intervals on the bar with these fixed distances. The results of the observations and the process of computation are given in full in the appendices, except that the micrometer readings are first corrected for periodic inequality of screws. For the complete understanding of the subject, it only remains to be stated that for both microscopes a greater reading corresponded to a smaller value of the interval at which the microscopes were placed, and that the unit, $=1^{p}$, in which all the readings are given, and the computation carried out, corresponds to the thousandth part of a revolution of the micrometer-screws, and may be placed equal to $0^{\mu}.05$.

Objections may possibly be raised against the manner in which the definitive errors of the decimeter marks have been derived from the three series of observations. A rigid adjustment according to the method of least squares, which was made in addition, led to the following only slightly differing results:

Decimeter mark.	Negative error—	
	By approximate computation.	By rigid computation.
	μ.	μ.
0	0	0
1	+12	+12
2	+10	+10
3	+ 5	+ 4
4	+27	+25
5	+ 9	+10
6	−25	−23
7	−26	−25
8	−23	−23
9	− 4	− 3
10	0	0

These differences, which in only a few cases amounted to $0^{\mu}.1$, were neglected.

Another circumstance must be taken into account. In the second rigorous computation it has been assumed that the errors of measurement consist only of accidental errors of pointing, and are independent of the length of the measured interval and of the time of pointing. This is not strictly correct, since from temperature variations during measurement, changes may occur in the difference between the distance of the microscope-axes and the corresponding interval of the bar. It is easy to take account of these changes if they occur uniformly between the single readings, for if the relative lengthening of the bar between any two measurements equals y, and the number of graduation intervals, which are directly observed, is n, then there must still be applied to the error of graduation which closes the ith interval, derived in the ordinary way, the further correction

$$+\frac{i\,(n-i)\,y}{2n}.$$

These regular changes were completely eliminated from the measurements in question by combining directly with each series of measurements a second immediately following, in which the pointings were made in the reverse order. The errors yet remaining, which result from the fact that the supposition of a regular change of distance of microscopes and of the length of the bar is not strictly fulfilled, may be combined with the errors of pointing as accidental errors, and so the method of computation chosen is justified.

The centimeter marks on R 1878 were determined in a way quite similar to that of the decimeter marks, except that auxiliary intervals of 5, 2, and 1^{cm} in length were used instead of the constant distance between the microscopes. The same remarks therefore apply to the computation as to that of the decimeter.

The results given in Appendix C for the determination of the graduation errors of R 1876 by comparison of the decimeter and centimeter marks with R 1878, of the millimeter marks with an absolutely determined silver scale on brass bar No. 6, and of the tenths of a millimeter by direct micrometric comparison, will be understood without further explanation. The unit here is $1^{\mu}=0^{mm}.001$. The differences of these results, in some instances, from those communicated to you on April 16, 1879, arise from the more accurate determination of the comparison bars.

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Kaiserliche Normal-Aichung-Kommission.

FOERSTER.

APPENDIX A.

Determination of the total length and expansion of steel metre R1876.

a. RESULTS OF THE COMPARISON OF STEEL METRE *R*1876 WITH BRASS METRE 1605.

Date of comparison.	t_{1605}	t_R	$R-1605$	$18.25\,(t_{1605}-t_R)$	$7.921\,(t_{1605}-15)$	$R-1605$ at 15° I.	$R-1605$ at 15° II.
1876.	°	°	μ	μ	μ	μ	μ
May 8	12.28	12.51	+57.7	−4.20	−21.55	+32.0	+36.2
8	12.26	12.51	+66.6	−4.56	−21.70	+40.3	+44.9
9	11.36	11.39	+56.0	−0.55	−28.83	+26.6	+27.2
July 18	18.72	18.96	− 4.2	−4.38	+29.47	+20.9	+25.3
18	18.82	19.06	− 7.3	−4.38	+30.26	+18.6	+23.0
19	18.80	19.10	− 1.8	−5.47	+30.10	+22.8	+28.3
19	18.90	19.15	− 7.2	−4.56	+30.89	+19.1	+23.7
28	19.59	19.71	−16.3	−2.19	+36.36	+17.9	+20.1
28	19.61	19.77	−11.6	−2.92	+36.52	+22.0	+24.9
28	19.69	19.87	−14.5	−3.28	+37.15	+19.4	+22.7
28	19.69	19.91	−15.3	−4.01	+37.15	+17.8	+21.9
Sept. 4	17.63	17.85	− 0.8	−4.01	+20.83	+16.0	+20.0
4	17.73	17.92	+ 2.7	−3.65	+21.63	+20.7	+24.3
4	17.71	17.96	− 0.7	−4.56	+21.47	+16.2	+20.8
4	17.77	17.96	− 0.5	−3.46	+21.94	+18.0	+21.4

The bars lie each on two rollers, which are distant 0.23 of the bar-lengths from the ends. A thermometer lies on each of the bars with its metal scale in contact.

b. COMPARISON OF THE REPSOLD STEEL METRES *R*1876 AND *R*1878.

	t_{76}	t_{78}	$R76-R78$
	°	°	μ.
I series	8.592	8.608	+22.7
	8.596	8.607	20.9
	8.605	8.619	21.9
	8.614	8.628	21.4
Means	8.602	8.615	21.73±0.2
II series	15.740	15.758	+23.2
	15.749	15.760	22.0
	15.758	15.766	23.3
	15.757	15.775	22.4
Means	15.751	15.765	22.73±0.2
III series	21.489	21.487	22.1
	21.489	21.486	22.9
	21.490	21.486	22.7
	21.487	21.487	23.6
Means	21.489	21.487	22.83±0.2

On each of the bars lie two Fuess thermometers, whose cups are inserted in copper blocks filled with mercury, and, by means of the latter, lie on the bars. Each observation consists of a double pointing to the marks on the bars and a double reading of the thermometers.

Berlin, August 10, 1881.

Director of the Kaiserliche Normal-Aichungs-Kommission:

[L. S.] FOERSTER.

APPENDIX B.

Determination of the graduation-errors of steel metre R 1878.

I.—THE DECIMETERS.

a. THE MIDDLE-MARK.

Cm.	A	A	A	A	A	A	A	A	A	A	B	B	B	B	B	B	B	B	B	B
0																				
50	−6 890	−3 895	−10 901	+6 895	− 6 898	−5 905	−9 903	−6 905	−1 907	−5 908	884	892	801	901	892	900	894	899	906	903
100	−6 874	−7 879	+ 5 868	0 892	+10 876	+6 881	0 889	+1 888	+8 892	+1 897	868	872	873	892	886	887	889	889	900	898
D. Negative relative error of middle-mark											+8	+10	+9	+4.5	+3	+6.5	+2.5	+5	+3	+2.5

General mean: +5.4

b. THE EVEN DECIMETER-MARKS.

Cm.	A_1	A_2	A_3	A_4	A_5	B_2	B_3	B_4	B_5	C_2	C_3	C_4	C_5	D_2	D_3	D_4	D_5	M
0	Series of readings stricken out by observer.										0	0	0		0	0	0	0
0–20		−2 430	0 425	−8 447	− 4 441		425	439	437									
20											25	39	37		− 1	+24	+19	+14
20–40		−5 464	−6 467	−7 452	− 2 426		461	445	424									
40											86	84	61		+34	+54	+26	+38
40–60		+2 410	−3 399	−1 372	− 2 371		396	371	369									
60											82	55	30		+ 5	+11	−23	− 2
60–80		−7 338	−2 433	+4 390	+ 3 414		431	394	417									
80											113	49	47		+10	−10	−23	− 8
80–100		−8 325	+1 415	+4 421	−11 452		416	425	441									
100											129	74	88		0	0	0	0

Series 2 is rejected; the last two observations of the series contain errors of reading of 100.

Cm.	A_6	A_7	A_8	A_9	A_{10}	B_6	B_7	B_8	B_9	B_{10}	C_6	C_7	C_8	C_9	C_{10}	D_6	D_7	D_8	D_9	D_{10}	M	General mean.
0											0	0	0	0	0	0	0	0	0	0	0	0
0–20	+13 402	+ 5 396	−5 427	+3 413	0 410	415	401	422	416	410												
20											15	1	22	16	10	+ 2	−11	+16	+ 6	+ 5	+ 4	+ 3
20–40	− 3 439	− 6 419	−4 442	+8 421	−2 422	436	413	438	429	420												
40											51	14	60	45	30	+25	−10	+47	+25	+21	+21	+20
40–60	− 6 375	+10 362	+4 335	0 364	+7 343	369	372	339	364	350												
60											20	−14	− 1	9	−20	−18	−50	−20	−20	−34	−28	−25
60–80	− 9 421	− 7 423	+7 394	−1 399	+2 419	412	416	401	398	421												
80											32	2	0	7	+ 1	−19	−46	−26	−32	−17	−28	−25
80–100	− 7 439	− 6 464	−4 436	+1 441	−5 427	432	458	432	442	422												
100											64	60	32	49	23	0	0	0	0	0	0	0

Determination of the graduation-errors of steel metre R 1878—Continued.

b. THE EVEN DECIMETER-MARKS—Continued.

Cm.	A_{11}	A_{12}	A_{13}	A_{14}	B_{11}	B_{12}	B_{13}	B_{14}	C_{11}	C_{12}	C_{13}	C_{14}	D_{11}	D_{12}	D_{13}	D_{14}	M
0									0	0	0	0	0	0	0	0	0
	+4 429	−11 400	−1 408	−7 422	433	389	407	415									
20									33	−11	7	15	+16	−10	− 5	− 2	0
	−4 438	− 1 417	−8 431	+1 432	434	416	423	433									
40									67	+ 5	30	48	+33	+ 7	+ 5	+14	+15
	0 363	+ 2 366	−2 375	+5 369	363	368	373	374									
60									30	−27	3	22	−20	−24	−34	−30	−27
	+1 405	−10 406	−3 427	+2 428	406	396	424	430									
80									36	−31	27	52	−31	−27	−23	−17	−25
	−3 451	− 9 435	+1 434	+1 433	448	426	435	434									
100									84	− 5	62	86	0	0	0	0	0

In the formation of the general mean the weight 5 is given to the mean of series 6 to 10, the weight 4 to the mean of series 11 to 14, but only the weight 1 to the mean of series 3 to 5, because the measurements, according to a note of the observer, are unreliable.

A. Corrected readings of the two microscopes.

B. Sum of the same, that is, differences of the measured intervals in comparison with the constant distance of the axes of microscopes I and II from each other.

C. Negative graduation errors of the 2-decimeter marks with arbitrary terminal value.

D. Negative relative errors of graduation.

M. Mean values.

c. THE SINGLE DECIMETER-MARKS.

Cm.	A_1	A_2	A_3	A_4	A_5	A_7	A	A_9	A_8	A_{10}	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
0																				
	− 1 246	− 8 237	+ 7 236	− 5 220	− 1 240	+ 8 234	0 331	− 4 344	− 2 231	− 6 337	245	229	243	215	239	229	242	331	340	331
10																				
	0 228	+ 3 212	−10 227	− 6 231	0 222	− 8 223	+ 3 307	− 5 324	− 1 223	− 2 326	228	215	217	225	222	222	215	310	319	324
20																				
	+ 4 242	+10 185	− 3 208	− 9 202	− 2 201	− 8 215	+ 9 321	− 5 303	− 1 216	−10 331	246	195	205	193	199	215	207	312	298	321
30																				
	−12 267	+ 6 217	+ 6 211	− 1 234	− 5 239	−11 240	− 3 336	− 3 347	− 8 243	− 2 342	255	223	217	233	234	235	238	333	344	340
40																				
	+ 3 197	+ 6 185	− 9 200	− 4 191	− 6 201	+ 6 186	+12 299	+ 4 299	+ 3 195	+ 7 298	200	191	191	187	195	198	192	311	303	305
50																				
	− 3 169	+ 4 169	+ 9 155	− 4 171	+ 5 158	+ 7 164	− 3 279	− 9 281	− 5 173	− 8 281	166	173	164	168	163	168	171	276	272	273
60																				
	− 3 214	+ 1 205	0 201	0 201	+ 3 183	− 5 208	−11 316	+ 7 305	+ 4 204	− 8 317	211	206	201	201	186	208	203	305	312	309
70																				
	− 7 218	− 7 202	0 211	+ 4 210	+ 2 215	+ 4 195	− 1 328	− 6 307	+ 4 212	− 2 310	211	195	211	214	217	216	199	327	301	308
80																				
	+10 211	− 6 217	− 9 236	+10 217	− 2 231	− 8 229	+ 6 310	− 9 324	−13 231	+ 1 334	221	211	227	227	229	218	221	316	315	335
90																				
	+ 5 212	+ 9 203	−12 212	+ 1 202	+ 6 196	− 3 209	− 1 306	− 9 326	+ 4 196	− 2 315	217	212	200	203	211	200	206	305	317	313
100																				

Cm	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	M	M′	M″
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	45	29	43	15	39	29	42	31	40	31	+25	+24	+35	+ 8	+29	+18	+33	+18	+28	+15	+23	+17	+12
20	73	44	60	40	61	51	57	41	59	55	+33	+34	+45	+27	+42	+29	+38	+16	+35	+23	+32	+20	+10
30	119	39	65	33	60	66	64	53	57	76	+59	+24	+42	+13	+31	+33	+36	+15	+21	+28	+30	+12	+ 5
40	174	62	82	66	94	101	102	86	101	116	+94	+42	+52	+40	+56	+57	+64	+36	+53	+52	+55	+31	+27
50	174	53	73	53	89	99	94	97	104	121	+74	+28	+35	+20	+41	+44	+47	+34	+44	+41	+41	+11	+ 9
60	140	26	37	21	52	67	65	73	76	94	+20	− 4	− 9	−19	− 5	+ 2	+ 9	− 3	+ 3	− 1	− 1	−25	−25
70	151	32	38	22	38	75	68	78	88	103	+11	− 3	−15	−24	−29	− 1	+ 2	−10	+ 3	− 8	− 7	−25	−26
80	162	27	49	36	55	91	67	105	89	111	+ 2	−13	−12	−17	−21	+ 4	− 8	+ 4	− 8	−16	− 8	−20	−23
90	183	38	76	63	84	109	88	121	104	146	+ 3	− 7	+ 8	+ 4	− 2	+11	+ 4	+ 8	− 5	+ 3	+ 3	− 3	− 4
100	200	50	76	66	95	109	94	126	121	159	0	0	0	0	0	0	0	0	0	0	0	0	0

ADJUSTMENT.

1. *The middle graduation.*

If the weight 10 is given to the series of ten comparisons of the half metres with each other, found on a preceding page, then the weight $\frac{10}{5}=2$ must be given to the mean of the series of ten comparisons of the single decimeters. Hence the negative error of the middle graduation will be

$$\frac{10\times 5.4+2\times 41}{12}=+11$$

2. *The even decimeter graduations.*

After correction of the decimeter graduations for this value (M′) the resulting negative errors of the graduations 20 and 80 have the weight $10\times\frac{5}{12}=\frac{50}{12}$; those of the graduations 40 and 60, $10\times\frac{5}{8}=\frac{75}{12}$. From the direct series of measurements of the double-decimeters are obtained, on the other hand, for the negative errors, deduced on a previous page, for the graduations 20 and 80, the weight $10\times\frac{5}{8}=\frac{75}{12}$; for the graduations 40 and 60, $10\times\frac{5}{12}=\frac{50}{12}$.

Hence there results for the graduation ..

$$\begin{cases} 20, \text{ the negative error, } \dfrac{50\times 20+75\times 3}{50+75}=+10 \\ 80, \text{ the negative error, } -\dfrac{50\times 20+75\times 25}{50+75}=-23 \end{cases}$$

$$40:\ \frac{75\times 31+50\times 20}{75+50}=+27$$

$$60:\ -\frac{75\times 25+50\times 25}{75+50}=-25$$

3. *The odd decimeter graduations.*

The negative errors of these are found directly from the series M′ by correction of the even decimeter-graduations. The middle graduation receives from this a still further correction of about −2, which seems entirely justifiable considering the marked difference between the value obtained from the series of comparisons of the decimeter and that resulting from the comparison of the half metres.

*Determination of the graduation errors of the steel metre R*1878—Continued.

II.—THE CENTIMETERS.

a. THE FIVE-CENTIMETER GRADUATIONS.

Cm.	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
0	+3 396	+4 202	+9 201	− 2 115	+ 4 168	+1 222	0 220	−2 259	+1 196	+1 221	399	206	210	113	172	223	220	257	197	222
5 {	+5 364	+7 148	−2 169	+ 4 74	+ 4 122	−1 196	−5 173	−2 210	+3 293	−5 268	369	155	167	78	126	195	168	208	296	263
	−3 149	−7 161	−3 180	+12 173	−12 197	+5 182	+1 164	+6 154	0 290	0 294	146	154	177	185	185	187	165	160	290	294
10	−1 118	−3 135	−1 161	+ 5 160	0 128	+6 133	0 117	+1 126	+1 319	0 334	117	132	160	165	128	139	117	127	320	334

Cm.	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}	A_{17}	A_{18}	A_{19}	A_{20}	B_{11}	B_{12}	B_{13}	B_{14}	B_{15}	B_{16}	B_{17}	B_{18}	B_{19}	B_{20}
0	0 220	− 9 227	+ 2 217	+8 94	−8 97	−9 110	+8 74	+4 135	− 4 189	+ 9 188	220	218	219	102	89	101	82	139	185	197
5 {	+3 271	−11 298	+12 252	+1 159	+1 149	−4 154	+2 126	−8 196	+ 5 249	+12 237	274	287	264	160	150	150	128	168	254	249
	+3 276	+ 2 290	− 3 162	−5 171	−2 171	+1 151	+3 152	+8 201	−10 257	+ 5 237	279	292	159	166	169	152	155	209	247	242
10	+5 326	+ 3 331	− 9 196	−3 202	−9 206	+6 197	+8 183	+1 202	+ 2 301	− 7 305	331	334	187	199	197	203	191	203	303	298

Cm.	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}	C_{19}	C_{20}
0 5	−30	−51	−43	−35	−46	−28	−52	−49	−99	−41	−54	−69	−45	−58	−61	−49	−46	−49	−69	−52
5 10	−29	−22	−17	−20	−57	−48	−48	−33	−30	−41	−52	− 42	−28	−33	−28	−51	−36	+ 6	−56	−56
D:	− 0.5	−14.5	−13	− 7.5	+ 5.5	+10	− 2	− 8	(−34.5)	−0.5	− 1	−13.5	− 8.5	−12.5	−16.5	+ 1	− 5	(−27.5)	− 6.5	+ 2

If the very discordant values C_9 and C_{18} are rejected, then there is obtained as the mean of the remaining 18, —5, with the mean errror ±1.4. If the rejected values had been included, then we would have obtained as the mean, —7.5, with the mean error ±2.1.

P. N.—A_9 to A_{20} were measured after interchange of the bars with reference to the measuring microscopes from their positions at A_0 to A_8.

b. THE EVEN CENTIMETER GRADUATIONS.

Cm.	A_1	A_2	A_3	A_4	A_5	B_1	B_2	B_3	B_4	B_5	C_1	C_2	C_3	C_4	C_5	D_1	D_2	D_3	D_4	D_5	M
0	+ 5 180	+ 7 99	−1 97	−3 100	−8 107	185	106	96	97	99						0	0	0	0	0	0
2 {	+ 1 197	+ 9 111	−1 115	0 111	+3 111	198	120	114	111	114 }	−13	−14	−18	−14	−15						
	+ 4 207	+ 1 111	−3 105	−7 118	+7 107	211	112	102	111	114						−23	−14	−21	−20	−18	−19
4 {	+13 189	− 9 127	−3 107	+1 80	−1 118	202	118	104	81	117 }	+ 9	− 6	− 2	+30	− 3						
	+ 1 177	−11 125	+5 108	−4 76	−3 121	178	114	113	72	118						−24	−19	−27	+ 3	−24	−18
6 {	+ 2 152	+ 1 108	−2 93	−1 59	−7 106	154	109	91	58	99 }	+24	+ 5	+22	+14	+19						
	− 8 142	− 7 109	−1 96	−6 66	−9 122	134	102	95	60	113						− 9	−14	− 8	+11	− 9	− 6
8 {	0 101	−12 118	+8 69	−9 50	−2 96	101	106	77	41	94 }	+33	− 4	+18	+19	+19						
	− 8 112	+ 3 121	−4 65	−5 54	−6 94	104	124	61	49	88						+14	−17	+ 6	+23	+ 7	+ 7
10	+12 96	0 107	+1 63	0 66	−1 93	108	107	64	66	92	− 4	+17	− 3	−17	− 4	0	0	0	0	0	0

c. THE SINGLE CENTIMETER GRADUATIONS.

Cm.	A_1	A_2	A_3	B_1	B_2	B_3	C_1	C_2	C_3	D_1	D_2	D_3	M_0	M_1		M
0	− 3 210	−3 209	+ 1 172	207	206	173				0	0	0	0	0	0	0
							−42	−60	−51							
1	+ 3 246	−5 271	− 2 226	240	266	224				− 6	− 6	− 1	− 4	− 6		− 8
	− 6 250	−3 266	− 9 236	244	263	227										
							−40	−51	−50							
2	+ 4 280	+1 313	− 2 279	284	314	277				−10	− 4	− 1	− 5	− 9	−19	−14
	− 6 286	+3 314	+ 2 292	280	317	294										
							−38	−58	−63							
3	+ 3 315	+5 370	+14 343	318	375	357				−11	− 8	−14	−11	−16		−18
	0 320	−2 370	− 7 351	320	368	344										
							−27	−58	−50							
4	− 8 355	−2 428	−11 405	347	426	394				− 2	−13	−14	−10	−17	−18	−17
	+ 1 378	0 430	−11 415	379	430	404										
							−25	−34	−40							
5	+ 7 397	+2 462	− 1 445	404	464	444				+ 9	+ 7	− 4	+ 4	− 5		− 5
	− 1 400	−3 474	+ 7 448	399	471	455										
							−43	−58	−57							
6	+ 3 439	+9 520	− 7 519	442	529	512				+ 2	+ 2	−11	− 2	− 9	− 6	− 7
	+ 5 449	−6 519	+ 4 529	454	513	533										
							−25	−65	−64							
7	− 1 480	−5 583	0 597	479	578	597				+13	− 9	−25	− 7	−13		−16
	− 4 475	−7 296	− 5 598	471	289	593										
							−26	−20	−20							
8	+13 484	−2 311	+ 3 610	497	309	613				+24	+25	+ 5	+18	+14	+ 7	+10
	− 5 490	−9 326	+ 2 619	485	317	621										
							−51	−71	−36							
9	− 5 541	−3 391	− 4 661	536	388	657				+ 9	+ 7	+19	+12	+10		+ 9
	− 1 540	−9 385	+ 6 669	539	376	675										
							−45	−61	−69							
10	+ 4 580	−1 438	+ 6 738	584	437	744				0	0	0	0	0	0	0

A. Readings of both microscopes corrected for periodic errors of the screws.
B. Sum of both readings.
C. Relative negative errors of the single intervals to the auxiliary interval.
D. Negative relative errors of the graduations within the decimeter.

ADJUSTMENT.

a. The 5^{cm} *graduation.*

If the weight 18 is given to the mean of the 18 comparisons actually used, of the two half decimeters with each other, then to the mean M_0 of the three series of comparisons of the single centimeters computed on the preceding page the weight $\frac{3}{5}$ must be assigned. The negative error of the middle graduation of the first decimeter will then be

$$\frac{-18\times5+\frac{3}{5}\times4}{18+\frac{3}{5}}=-5$$

b. The remaining centimeter graduations.

By correction of the means M_0 for this value of the 5^{cm} graduation, the errors M_I of the even centimeter graduations obtain values which differ by only a few units from those of the direct series of comparisons of the double centimeters. If, therefore, the mean of both is taken without reference to the weights, which do not come out very different, and the odd-centimeter graduations are corrected for this, no essential error will be committed.

Director of the Kaiserliche Normal-Aichungs-Kommission.

Berlin, August 10, 1881.

[L. S.] FOERSTER.

APPENDIX C.

DETERMINATION OF THE GRADUATION ERRORS OF REPSOLD STEEL METRE, *R* 1876.

For the determination of the graduation errors of the decimeters they have been compared 20 times by double pointings of both micrometers with those on the bar belonging to the Commission, made by Repsold, *R* 1878. The results of the single series of measurements, freed from periodic errors of the screws and the constants of the initial and final pointings, are collected together in columns 1 to 20, and are expressed in thousandths of millimeters in the sense of errors in relation to the comparison-bar. Column 21 contains the mean of the columns 1 to 20, column 22 the relative errors of the comparison-bar, column 23 = 21 + 22 the absolute relative (*inneren*) errors of the American bar. For general comparison there is besides, given under 24, the result of an older but more imperfect series of comparisons with another bar, of which, however, the relative weight is so small that it exerts no influence on the values of column 23.

Decimeter mark.	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0
1	−0.1	+0.7	+0.6	+0.7	−0.2	+0.3	−0.5	+0.7	+0.5	+1.1	−1.1	+0.1
2	−2.1	−0.4	−0.7	−0.7	−0.6	−0.4	−1.3	−1.3	−0.8	−0.7	−4.1	−1.6
3	−1.0	−0.1	+0.1	+0.6	0.0	−0.9	−1.5	−0.7	0	+0.5	−2.4	−1.9
4	−1.8	+0.4	+0.4	+0.1	0.0	+0.3	+0.2	+0.9	0	+0.9	−1.3	−0.7
5	−1.3	−0.4	−0.4	−0.3	−0.5	−0.4	−1.8	−1.4	−0.7	−0.5	−1.8	−1.3
6	−2.4	−1.1	−1.9	−1.7	−1.5	−0.4	−1.6	−1.6	−1.3	−1.3	−0.3	+0.5
7	−3.2	−2.2	−2.7	−3.1	−2.4	−2.7	−2.9	−2.0	−2.1	−2.7	−1.8	−1.3
8	−1.9	−1.9	−1.8	−1.4	−1.0	−1.4	−2.4	−1.3	−0.8	−1.1	−1.8	−0.5
9	−0.6	−0.1	+0.8	−0.4	−0.6	−0.7	−1.9	−1.1	−0.8	−0.7	−1.0	−0.4
10	0	0	0	0	0	0	0	0	0	0	0	0

Decimeter mark.	13	14	15	16	17	18	19	20	21	22	23	24
0	0	0	0	0	0	0	0	0	0	0	0	0
1	−0.5	−0.5	−0.6	−0.3	+0.4	−0.5	−0.4	−0.6	0	−0.6	−0.6	−1.2
2	−3.1	−3.3	−3.5	−2.4	−2.0	−2.9	−3.3	−3.2	−1.9	−0.5	−2.4	−2.0
3	−1.7	−2.4	−2.6	−1.5	−1.4	−1.4	−2.3	−2.3	−1.1	−0.2	−1.3	−0.5
4	−1.3	+0.1	−1.2	0	+0.5	−0.6	−1.0	−0.8	−0.2	−1.3	−1.5	−1.0
5	−1.7	−0.7	−1.1	−1.0	−1.1	−1.9	−1.1	−1.5	−1.0	−0.4	−1.4	−0.8
6	−0.5	+0.1	−0.3	+0.7	+0.1	+0.4	+0.1	+0.2	−0.7	+0.2	+0.5	−0.8
7	−1.6	−1.6	−1.7	−0.8	−0.9	−0.8	−1.4	−1.1	−2.0	+1.3	−0.7	−2.0
8	−1.0	−1.3	−0.9	−0.4	−1.2	−0.6	−0.9	−0.3	−1.2	+1.1	−0.1	−0.1
9	+0.2	−1.1	−0.9	−0.2	+0.4	−0.3	−0.3	−0.6	−0.5	+0.2	−0.3	−1.3
10	0	0	0	0	0	0	0	0	0	0	0	0

For the determination of the graduation errors of the centimeters of the first decimeter they have also been compared 20 times by double pointings of both microscopes with those on the Repsold bar belonging to the Commission. The results are collected analogously to those for the decimeter. Column 23 contains here the relative graduation errors of the centimeters with respect to the decimeter, column 24 the same with respect to the whole metre.

Centimeter mark.	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0
1	+1.5	+2.5	+2.3	+1.5	+1.4	+2.3	+1.9	+2.3	+2.2	+3.2	+1.6	+3.0
2	+1.0	+2.1	+2.1	+0.4	+1.7	+1.6	+0.9	+1.0	+1.0	+0.9	+2.0	+2.6
3	+2.4	+2.3	+2.0	+2.4	+2.2	+2.3	+2.1	+2.3	+2.1	+2.3	+1.8	+1.1
4	+1.4	+1.6	+1.8	+1.5	+1.7	+1.0	+1.4	+1.8	+1.9	+2.0	+1.3	+0.9
5	+1.4	+1.6	+1.6	+2.8	+2.0	+1.3	+1.2	+1.4	+1.3	+1.5	+1.3	+0.6
6	+0.4	+0.7	+0.9	—0.2	+0.5	+0.6	+0.8	0	+0.5	+1.0	+1.1	+0.4
7	+0.4	+1.0	+1.4	+0.4	+1.6	+0.8	+0.6	+0.3	—0.4	+1.2	+1.3	+1.0
8	—0.2	+0.3	—0.3	+0.8	—0.2	+0.8	+0.4	—0.5	—0.4	—0.2	+0.7	+0.7
9	+1.7	+2.2	+1.5	+2.1	+2.4	+2.1	+2.0	+1.8	+2.4	+2.2	+1.4	+1.2
10	0	0	0	0	0	0	0	0	0	0	0	0

Centimeter mark.	13	14	15	16	17	18	19	20	21	22	23	24
0	0	0	0	0	0	0	0	0	0	0	0	0
1	+2.4	+2.0	+2.5	+2.4	+2.6	+1.4	+2.0	+2.3	+2.2	+0.4	+2.6	+2.5
2	+2.5	+1.6	+2.2	+2.5	+2.4	+1.5	+2.4	+2.3	+1.7	+0.7	+2.4	+2.3
3	+1.8	+1.2	+1.6	+2.1	+2.7	+2.5	+2.4	+2.3	+2.1	+0.9	+3.0	+2.8
4	+1.4	+1.0	+1.2	+1.2	+1.9	+0.8	+2.0	+1.9	+1.5	+0.8	+2.3	+2.1
5	+1.3	+1.3	+0.7	+1.0	+0.5	+0.5	+0.8	+1.8	+1.2	—0.2	+1.4	+1.1
6	+1.4	+1.9	+0.7	+0.5	+0.9	—0.4	+0.2	+1.8	+0.7	+0.3	+1.0	+0.6
7	+0.7	+2.2	+0.9	+1.3	+1.3	+0.9	+1.3	+2.4	+1.0	+0.8	+1.8	+1.4
8	+0.8	+1.5	+1.0	+0.0	+1.2	+0.7	+0.5	+1.6	+0.5	—0.5	0.0	—0.5
9	+2.4	+2.4	+1.7	+2.5	+1.5	+1.7	+2.3	+2.9	+2.0	—0.5	+1.5	+1.0
10	0	0	0	0	0	0	0	0	0	0	0	0.6

The millimeters have been compared twice in groups of ten each with those on the principal standard No. 6. The following collection contains in columns 1 and 2 the results freed from the periodic error of the screw, the constants and graduation errors of the comparison-bar; column 3, the mean of 1 and 2, presents the relative graduation errors of the millimeters with respect to the single centimeter, column 4 with respect to the whole metre.

Millimeter mark.	1	2	3	4	Millimeter mark.	1	2	3	4	Millimeter mark.	1	2	3	4
0	0	0	0	0	20	0	0	0	+2.3	40	0	0	0	+2.1
1	−1.1	−1.2	−1.2	−0.9	21	−0.8	−0.6	−0.7	+1.6	41	−0.5	−1.3	−0.9	+1.1
2	−1.0	−1.0	−1.0	−0.5	22	−0.1	−0.8	−0.4	+2.0	42	+0.8	−0.5	+0.2	+2.1
3	−1.4	−0.5	−1.0	−0.2	23	+0.1	+0.1	+0.1	+2.5	43	−0.6	−1.7	−1.1	+0.7
4	−0.7	−1.8	−1.2	−0.2	24	−0.9	−1.5	−1.2	+1.3	44	+0.1	−1.6	−0.8	+0.9
5	+1.0	−1.7	−0.3	+1.0	25	−0.8	−1.5	−1.1	+1.4	45	−0.2	−0.8	−0.5	+1.1
6	−1.5	−1.8	−1.6	0.0	26	+0.8	−0.5	+0.2	+2.8	40	+0.7	−1.1	−0.2	+1.3
7	−1.2	−1.8	−1.5	+0.3	27	−0.3	−1.0	−0.7	+1.9	47	+0.1	−1.3	−0.6	+0.8
8	−0.7	−1.0	−0.9	+1.1	28	+0.2	−1.5	−0.7	+2.0	48	+0.4	−2.5	−1.0	+0.3
9	−1.6	−2.0	−1.8	+0.4	29	+0.5	−0.1	+0.2	+2.9	49	−1.2	−2.0	−1.6	−0.4
10	0	0	0	+2.5	30	0	0	0	+2.8	50	0	0	0	+1.1
10	0	0	0	+2.5	30	0	0	0	+2.8	50	0	0	0	+1.1
11	−0.6	0.0	−0.3	+2.2	31	+1.8	0	+0.9	+3.6	51	−0.2	−0.5	−0.3	+0.8
12	−1.1	−0.2	−0.6	+1.2	32	+0.1	−0.3	−0.1	+2.5	52	+1.2	+1.2	+1.2	+2.2
13	−0.5	−0.5	−0.5	+2.0	33	−0.1	+0.1	0.0	+2.6	53	−0.1	−0.2	−0.2	+0.8
14	+0.1	−1.3	−0.6	+1.8	34	−0.3	−0.6	−0.4	+2.1	54	−1.6	−1.9	−1.8	−0.9
15	+0.6	+1.1	+0.8	+3.2	35	0.0	+0.7	+0.3	+2.7	55	+0.6	−0.4	+0.1	+1.0
16	+0.3	−0.8	−0.2	+2.2	36	−1.5	−0.1	−0.8	+1.6	56	−0.3	−1.2	−0.7	+0.1
17	+0.6	+1.4	+1.0	+3.3	37	−0.2	+0.6	+0.1	+2.4	57	−0.4	−1.1	−0.7	+0.1
18	+0.4	+0.9	+0.7	+3.0	38	−0.6	−0.6	−0.6	+1.6	58	−0.1	−1.2	−0.7	0.0
19	+1.1	+0.9	+1.0	+3.3	39	−1.1	−1.1	−1.1	+1.1	59	−0.4	+0.6	+0.1	+0.8
20	0	0	0	+2.3	40	0	0	0	+2.1	60	0	0	0	+0.6

Millimeter mark.	1	2	3	4	Millimeter mark.	1	2	3	4
60	0	0	0	+0.6	80	0	0	0	−0.5
61	+0.6	+1.7	+1.1	+1.8	81	+1.1	+0.1	+0.6	+0.3
62	0	+0.4	+0.2	+1.0	82	+0.2	+0.4	+0.3	+0.1
63	−0.1	+0.5	+0.2	+1.0	83	+1.6	+1.1	+1.3	+1.2
64	+0.2	+0.7	+0.4	+1.3	84	+0.3	+0.6	+0.4	+0.5
65	+2.5	+1.5	+2.0	+3.0	85	+0.4	+1.7	+1.1	+1.3
66	+0.4	+1.1	+0.8	+1.9	86	−0.7	+1.2	+0.3	+0.7
67	+1.3	+1.7	+1.5	+2.6	87	−1.1	+1.9	+0.4	+1.0
68	+1.1	+0.9	+1.0	+2.2	88	+1.3	+2.8	+2.1	+2.8
69	+1.4	+1.3	+1.4	+2.7	89	−3.1	−0.1	−1.6	−0.7
70	0	0	0	+1.4	90	0	0	0	+1.0
70	0	0	0	+1.4	90	0	0	0	+1.0
71	+1.6	+1.4	+1.5	+2.7	91	−1.4	−0.2	−0.8	0.0
72	+1.0	+0.8	+0.9	+1.9	92	−1.6	−1.0	−1.3	−0.6
73	+1.0	+0.9	+1.0	+1.8	93	0.0	+0.8	+0.4	+0.9
74	−0.7	−0.2	−0.5	+0.1	94	−0.7	+0.2	−0.2	+0.2
75	−0.2	−0.6	−0.4	0.0	95	+0.3	+1.5	+0.9	+1.1
76	−0.2	+0.4	+0.1	+0.3	96	−0.2	+1.0	+0.4	+0.4
77	−0.1	+0.2	+0.1	+0.1	97	−0.1	+0.8	+0.4	+0.3
78	+0.4	+0.4	+0.4	+0.2	98	+0.4	+1.5	+1.0	+0.7
79	−0.2	+0.4	+0.1	−0.3	99	+0.4	+1.2	+0.8	+0.4
80	0	0	0	−0.5	100	0	0	0	−0.6

The tenths of millimeters of the first millimeter, besides the two additional tenths, were compared twice with an auxiliary interval in the field of view of a microscope. The results are contained in columns I and II; III, the mean of I and II, contains the relative graduation errors with respect to the whole millimeter, IV with respect to the metre.

Millimeter mark.	I	II	III	IV
−0.1	+0.7	−0.2	+0.2	+0.4
0.0	0	0	0	0
+0.1	−0.8	−0.1	−0.4	−0.6
0.2	+0.4	+0.9	+0.6	+0.5
0.3	+0.9	+1.6	+1.2	+0.9
0.4	+2.4	+2.3	+2.3	+2.0
0.5	−0.3	+0.3	0	−0.5
0.6	+2.0	+0.6	+1.3	+0.7
0.7	+1.0	+0.5	+0.7	+0.1
0.8	+0.4	+0.3	+0.3	−0.3
0.9	−0.9	−0.6	−0.7	−1.6
1.0	0	0	0	−0.9
1.1	+1.5	+1.8	+1.7	+0.7

Director of the Kaiserliche Normal-Aichungs-Kommission:
Berlin, August 10, 1881.

[L. S.] FOERSTER.

§ **3**. From Professor Foerster's letter of August 10, 1881, it will be seen that he gives as at present the most probable value of R1876

$$R1876 = 1^{m} + 86^{\mu}.18 + 10^{\mu}.654t$$

where t is the temperature in centigrade degrees. A more general value for R1876 may be obtained by substituting in the value given by Professor Foerster

$$R1876 - R1878 = 21^{\mu}.36 \pm 0^{\mu}.48 + t(0^{\mu}.076 \pm 0^{\mu}.29)$$

the value of R1878, given by him and derived from the platinum-iridium metre, type I, of the International Metre Committee, namely,

$$R1878 = 1^{m} + 64^{\mu}.82 + 10^{\mu}.371t + 0^{\mu}.0069t^2$$

There results

$$R1876 = 1^{m} + 86^{\mu}.18 + 10^{\mu}.447t + 0^{\mu}.0069t^2$$

t being in degrees centigrade. This expression gives to R1876, at 15° C., a length $1^{\mu}.6$ less than that derived from Professor Foerster's approximate formula above. The later expression involving t^2 should be more accurate, and, until the equation of the platinum-iridium metre, type I, is more accurately known in terms of the prototype metre yet to be adopted by the International Metre Committee, is the best that can be obtained. The Procès Verbaux of the International Committee of Weights and Measures of 1880, p. 103, states that the metre type I will probably differ by but one or two microns from the prototype yet to be adopted. The value for the expansion of R1876, derived from the adjustment in Chapter IX, § 58, is for centigrade degrees, at 15° C., $10^{\mu}.572$, while the new value given by the Eichungs Amt is $10^{\mu}.654$. Between 0° C. and 30° C. the greatest difference of length of R1876, computed from 15° C., with one or the other expansion, would be $1^{\mu}.2$.

The letter of Professor Foerster makes some slight changes in the values for the errors of the subordinate graduations of R1876, previously furnished by him, and given in Chapter IX, § 67. For the 80^{mm} graduation, which is a very important one in determining the length of R1876 from Clarke Yard A, the correction is changed by $0^{\mu}.5$, bringing it into closer agreement with the correction found for this graduation in the Lake-Survey office.

A new adjustment of E_B, E_A, E_{S_2}, and $E_{R\,1876}$, similar to that given in Chapter IX, § 57, can now be made by substituting the new value assigned by Professor Foerster in his letter August 10, 1881, for the expansion of R1876. Differentiating the value

$$R\,1876 = 1^m + 86^\mu.18 + 10^\mu.447t + 0^\mu.0069t^2$$

there results $10^\mu.447 + 0^\mu.0138t$ as the rate of expansion, and at 62° F. (=16°.67 C.), which is the temperature adopted in Chapter IX, § 57, it becomes

$$10^\mu.677 \text{ for } 1^\circ \text{ C., or for } 1^\circ \text{ F., } E_{R\,1876} = 5^\mu.932.$$

The weight of this value is unknown. It is undoubtedly great, and on the scale of weights used in Chapter IX, § 57, a weight of 16 will be assigned to it. The equations of condition of Chapter IX, § 57, for a temperature of 62° F., may now be written with a column added for their residuals.

	μ	μ	*Weights.*
E_B	$-39.945 = v_1 =$	-0.032	1
$E_{R\,1876}$	$-\ 5.932 = v_2 =$	-0.000	16
E_A	$-\ 5.371 = v_3 =$	$+0.034$	4
$E_{R\,1876} - 1.0937E_A$	$-\ 0.006 = v_4 =$	$+0.015$	8
$4E_{R\,1876} - E_{S_2}$	$+\ 1.335 = v_5 =$	-0.012	3
$E_B\ \ - E_{S_2}$	$-14.833 = v_6 =$	$+0.005$	6

The following are the values resulting from solving these equations by least squares. The residuals result from substituting these values in the equations of condition:

$$\begin{aligned} E_B &= 39^\mu.913 \pm 0^\mu.033 \\ E_R &= 5.932 \pm 0.008 \\ E_A &= 5.405 \pm 0.013 \\ E_{S_2} &= 25.075 \pm 0.033 \end{aligned}$$

These values are the best that can be obtained at present. The residuals in the preceding are very small, except in the case of E_A, where it is $\frac{1}{157}$ part of the expansion, a quantity that is not large.

The length of Clarke Yard A in terms of the Ordnance-Survey standard, Y_{55}, is best known at the mean temperature of their comparisons, which was 57°.71 F. A change of $\frac{1}{157}$ in the expansion of Clarke Yard A would only change its length, as assigned by Colonel Clarke in Chapter II, § 2, by $\frac{1}{10000000}$ at a distance of 25° F. from the mean temperature. The change in the relative expansion of R1876 and Clarke Yard A, in passing from the adjustment of Chapter IX, § 57, to this new adjustment, is but $0^\mu.016$, a quantity so small that it practically leaves the relation between these standards, given in Chapter IX, § 12, unchanged.

In April, 1882, the metre R1876 was sent to the International Bureau of Weights and Measures, at Sevres, France, for comparison with the standards of that Bureau.

APPENDIX II.

SLOW RETURN OF ZINC BAR Z_1 TO ITS ORIGINAL LENGTH, AFTER BEING HEATED.

By E. S. WHEELER, *Assistant Engineer.*

§ **1.** A series of comparisons of S_1, Z_1, S_2, and Z_2 was made from August 26 to December 15, 1881, for the especial purpose of finding the time required for Z_1 to return to its normal length after it had been largely disturbed by a rapid change in temperature. These comparisons were made in the comparing-room, which was visited but once a day.

The thermometers used were Geissler 1, 2, 3, and 4, and Casella 21474 and 21476. The microscopes were Repsold Nos. 5 and 6. The observations were made and reduced in the same manner as previous tube-comparisons, described in § 21, Chapter IX. The old graduations on the bars were used throughout, but from time to time observations were made on the neutral-axis graduations to ascertain if there was any bending of the bars. None was observed.

The difference in the length of a zinc bar at different times though at the same temperature, as described in Chapter IX, § 33, will be called the set. The set of Z_1 and of Z_2 at the time of each observation has been computed in the following manner: The differences at the observed temperature between Z_1 and S_1, Z_2 and S_2, have been computed with the constants given in Chapter IX, §§ 24, 26. These computed differences were then subtracted from the observed differences, and the remainders taken as values of the set. The set is, therefore, called plus when the zinc bar is too long, and *vice versa*.

The outline of the work is as follows: Tubes 1 and 2 were first compared for ten days, then tube 1 was taken to a warm room and its temperature kept between 115° F. and 120° F. for twenty-four hours. It was then returned to the comparing-room and allowed to cool for twenty-four hours, when observations were resumed and continued for one hundred days. This heating and cooling left Z_1 with a set of $+73^\mu$; that is, Z_1 was 73^μ longer than its temperature and expansion indicated. The subsequent observations have been studied to ascertain at what rate this excess of 73^μ would disappear provided no new disturbances occurred.

The results of the observations are given in the following table: The first column contains the date, the second the temperature, the third and fourth the sets of Z_2 and Z_1, respectively, the fifth ordinates of the logarithmic curve defined in § 2 following, with date for abscissa, and the sixth residuals obtained by subtracting the fifth column from the fourth.

Date.	Temperature.	Set.		Ordinates of logarithmic curve with date for abscissa.	v.
		Z_2.	Z_1.		
1881.	° F.	μ	μ		
Aug. 26	68.5	+ 1	+18	....	....
27	68.9	+ 3	+17	....	....
29	70.4	− 1	+15	....	....
31	73.2	− 6	+13	....	....
Sept. 2	75.3	−10	+11	....	....
5	75.0	− 7	+10	....	....
	Tube 1 now heated to 115° F. for 24 hours.				
Sept. 7	77.2	− 8	+73	66	+ 7
8	76.6	− 9	+72	66	+ 6
9	76.3	−12	+67	65	+ 2

Date.	Temperature.	Set.		Ordinates of logarithmic curve with date for abscissa.	v.
		Z_2.	Z_1.		
1881.	° F.	μ	μ		
Sept. 10	75.2	−2	+68	65	+3
12	72.5	0	+65	64	+1
14	69.6	+6	+69	63	+6
16	68.5	+6	+63	62	+1
19	68.1	+6	+59	61	−2
21	67.6	+7	+58	60	−2
24	69.0	+3	+54	59	−5
27	71.5	−3	+50	58	−8
30	71.6	−2	+50	57	−7
Oct. 3	71.0	−1	+52	55	3
4	70.8	+1	+50	55	−5
8	63.6	+12	+56	54	+2
12	61.4	+17	+53	52	+1
17	61.3	+11	+48	50	−2
20	60.3	+12	+52	49	+3
24	58.2	+14	+47	47	0
28	57.5	+13	+44	46	−2
Nov. 1	59.3	+9	+43	45	−2
5	55.7	+12	+50	44	+6
9	55.2	+13	+44	43	+1
10	55.5	....	+42	42	0
11	54.4	....	+47	42	+5
15	51.3	+17	+52	41	+11
19	50.7	+7	+41	40	+1
23	47.0	+17	+46	39	+7
28	44.0	+14	+42	37	+5
Dec. 1	47.2	+11	+28	37	−9
2	48.3	+6	+30	36	−6
3	48.5	+4	+30	30	−6
10	46.0	+8	+37	34	+3
15	46.6	+7	+32	33	−1

§ **2.** The heating of tube 1 occurred between September 5 and 7. The table shows that tube 1, after being heated to 120° F., fell, in twenty-four hours, to 77° F., and that Z_1 had a set of $+10^{\mu}$ before heating and $+73^{\mu}$ after cooling. Therefore, a fall of 43° F. in twenty-four hours produced a set of $+63^{\mu}$. The table also shows that the set of Z_1 at the end of the observations was about half as large as at the beginning, and that the rate of decrease was most rapid at first, thus suggesting that the rate of decrease of the set at any time was proportional to the amount of the set at the time. This law, when expressed graphically (time being the abscissa and set the ordinate), is the characteristic of the logarithmic curve. The following discussion of the observations was therefore made.

The set of each observation was plotted as ordinate, with the date or time as the abscissa. The most probable logarithmic curve was then passed among the plotted points (the most probable when the errors are those of observation and are wholly in the ordinates). The method of computation was that given in Jordan's Vermessungskunde, volume 1, page 44. An approximate logarithmic curve was first chosen, from which the values of X and Y in the equation

$$B = X\, 10^{-\frac{h}{Y}}$$

were computed, in which B, the observed quantity, is the ordinate or set expressed in microns, and h is the abscissa expressed in days. The most probable corrections to the approximate values X and Y, called ΔX and ΔY, respectively, were computed as follows: From the above equation is derived

$$\Delta B = \frac{dB}{dX}\,\Delta X + \frac{dB}{dY}\,\Delta Y$$

or, substituting a for $\frac{dB}{dX}$, b for $\frac{dB}{dY}$, and putting l for $(X\,10^{-\frac{h}{Y}} - B)$, there results

$$a\,\Delta X + b\,\Delta Y + l = v$$

in which

$$\log a = -\frac{h}{Y}$$

$$\log b = \log a + \log \frac{X\,h}{Y^2\,M}$$

M being the modulus of common logarithms. Each observation gives an equation of the above form. There were 34 observation-equations of this kind, from which the values ΔX and ΔY were derived and added to the first values of X and Y. A second approximation was not thought necessary for this work. The corrected values are

$$X = 66.4$$
$$Y = 328.6$$

With these values the ordinates corresponding to the dates of each observation were computed, and are given in the fifth column of the preceding table. These computed ordinates, subtracted from the observed ordinates given in the column headed "Set", leave the residuals, which are given in the column headed v. These residuals are much larger than the errors of observation. This is probably due in part to the changing temperature of the comparing-room; that is, a set at any time is not alone a function of the first large set and the time which has elapsed, but is also a function of all the small sets which have occurred since the large one.

The table shows that in 100 days the set had diminished from $+65^{\mu}$ to $+33^{\mu}$, or one-half. Therefore, at the end of 200 days the set would be $+16^{\mu}$, and

after 300 days it would be $+8^{\mu}$
after 400 days it would be $+4^{\mu}$
after 500 days it would be $+2^{\mu}$

This law of decrease may be expressed as follows: *With a constant temperature Z_1 will lose one-half its set in one hundred days.*

This time, 100 days, is probably too great, for during the 100 days of observation the temperature of the comparing-room fell from 77° to 46° F., and this fall of 31° would of itself make a plus set of some amount, as is shown by Z_2, the set of the latter having increased 15^{μ} during the time. It is difficult to say how much effect the fall in temperature had upon Z_1, but whatever its amount the tendency would be to indicate too great a time for losing one-half of the set.

APPENDIX III.

DIFFERENCE OF LONGITUDE BETWEEN DETROIT, MICH., AND CAMBRIDGE, MASS.

REPORT OF A. R. FLINT AND O. B. WHEELER, *Assistant Engineers.*

OFFICE UNITED STATES LAKE SURVEY,
Detroit, Mich., June 30, 1882.

SIR: In compliance with your instructions, we have the honor to make the following report on our recent determination of the difference of longitude between Detroit, Mich., and Cambridge, Mass.

The field-work was done in obedience to the following order:

OFFICE UNITED STATES LAKE SURVEY,
Detroit, Mich., April 22, 1881.

Assistant Engineer A. R. FLINT,
Detroit, Mich.:

SIR: On May 2 the determination of the difference of longitude of Detroit and Cambridge Observatory * * * will be undertaken.

Time-determinations will be made on five nights with yourself at Detroit and Assistant Engineer O. B. Wheeler at Cambridge, using chronographs and instruments of the same construction and size; the observers will then change places, and five more nights be obtained. * * *

Star-places will be taken from 539 Sterne des Fundamental-Catalogs, Berlin, 1881. Eleven wires will be observed on for time-stars, and on slow stars at least seven wires before and after reversal.

Programme for time-determination.

Level readings.
Circumpolar star reversed on.
Level readings.
Five or more well-determined time-stars.
Level readings.
Reversal.
Level readings.
Five or more well-determined time-stars.
Level readings.
Circumpolar star reversed on.
Level readings.

Then will follow the exchange of clock-signals, which will be sent alternately for $1^m 20^s$ from each station till two sets have been sent from each station. The observers will carefully adjust the tensions of their relays and request the repeating stations to do the same before sending clock-signals. After exchange of clock-signals another time-determination according to the above programme will be made. Every care will be taken to secure most accurate work.

Very respectfully,

C. B. COMSTOCK,
Major of Engineers and Brevet Brigadier-General.

The instructions were followed as strictly as the weather and other circumstances would permit.

The two sets of transits, clocks, chronographs, observing-keys, and other electrical apparatus were as nearly alike as possible. The observer used the same transit and observing-key at both stations, in order to eliminate any errors peculiar to their construction.

The transits, made by Würdemann, and numbered 1 and 15, respectively, were of the same size and similarly mounted. They are of about 31 inches focal length, $2\frac{1}{2}$ inches clear aperture of object-glass, are provided with diagonal eye pieces and reticules of nineteen threads, and magnify about sixty-five diameters. Mr. Flint used transit No. 1 and Mr. Wheeler used No. 15. The pivot-corrections and level-values were redetermined for each transit. The values of one division of the levels were tested by means of the level trier, with the tubes in their holders, and were found to be the same as formerly used. The wire-intervals for transit No. 1 were redetermined, but not differing materially from the old values, the latter were used. The wire-intervals for a new set of wires in transit No. 15 were determined by at least twenty observations on slow stars for each wire.

Bond & Sons sidereal clock No. 256 and Bond & Sons spring-governor chronograph No. 216 were used at Detroit. Frodsham sidereal clock No. 1327, belonging to Harvard College Observatory, and Bond & Sons spring-governor chronograph No. 245 were used at Cambridge. This clock is the same as was used by the United States Coast Survey in the transatlantic longitude work of 1872, and was then designated the South Clock. The connection with the clock was through a local circuit with a battery of the same number of cups as was in use on the local circuit of the Observatory. The zinc poles of the two circuits were placed toward each other in order to prevent interference of the circuits with each other. The use of this clock was kindly granted to the Lake Survey by Professor E. C. Pickering, Director of the Observatory, and we may here state that every facility was rendered by the Director and his assistants in the execution of our work at that station.

At Cambridge the observations were made on the south pier of the United States Coast-Survey Observatory used in the transatlantic work of 1872. This station is on the grounds of the Harvard College Observatory, 108 feet, or $0^s.096$ of time, west of the center of the great equatorial dome of the main building (see United States Coast-Survey Report for 1874, p. 164). The two granite piers there described were spanned by a marble slab 3 feet in length and 3 inches in thickness to accommodate our transits. At Detroit the observations were made on the west stone pier of the Lake-Survey Observatory (1871–'82). This pier is 5.1 feet, or in time $0^s.004$, west of the east pier which was used in the Washington longitude work of 1871.

Usually a greater number of stars was observed than was required by the programme, and in selecting for reduction preference was given to those stars observed at both stations on the same night.

The signals were made automatically, the clock breaking the circuit every second, except the sixtieth or zero second. The clock in a local circuit repeated to the main line, and the signals were again repeated to a local clock-and-chronograph circuit at the other station. There were two repeaters on the main line, either at Albany and Buffalo or at New York and Buffalo. The signals were transmitted entirely through closed circuits. Care was always taken that the repeaters were kept in close adjustment during the exchange of signals.

The observations were made on May 13, 23, 24, 26, June 4 and 11, 1881, with Mr. Flint at Detroit and Mr. Wheeler at Cambridge; and on June 21, 22, 23, 24, and 29, 1881, with Mr. Flint at Cambridge and Mr. Wheeler at Detroit. The nights of May 13 and 23, being partially interrupted by clouds, were each considered as of half weight. On May 13, at Detroit, there was no reversal on a slow star, and the stars observed after signals were all with the clamp one way. On May 23, at Cambridge, the stars observed after signals were all with the clamp one way.

The chronographic readings were made in duplicate, and the reductions have been entirely in duplicate, a second person duplicating the work of the first. Collimation was determined from reversals on polar stars, except in two cases, where it was deduced by least squares from the observation-equations. The clock correction at a given epoch, the azimuth, and hourly rate, were determined by least squares from the observed times corrected for errors of level, reduction to middle wire, diurnal aberration, and usually for collimation. The observation-equations from which the most probable values of $\Delta\theta$, a, and ρ were deduced are of the form $Aa+R\rho+\Delta\theta+[t+Bb+C(c+\Delta i+ab'n)]-(\alpha+\text{ assumed }\Delta t)=v$, in which A, B, and C are the azimuth, level, and collimation factors respectively; R, the interval from a star observation to the given epoch, expressed in tenths of an hour; a, b, and c, the azimuth, level, and collimation corrections, respectively; ρ, the rate per hour, + when losing, − when gaining; Δt, the clock correction at the given epoch = assumed $\Delta t+\Delta\theta$; $\Delta\theta$, the correction to the assumed Δt; t, the clock-time of transit, usually the mean of 11 wires; Δi

the reduction to the middle wire; $ab'n$, the correction for diurnal aberration; $\alpha_{,} = A.\ R.$, the right ascension, taken exclusively from the catalogue "Mittlere und scheinbare Oerter für das Jahr 1881 von 539 Sternen"; v, the residual from an observation-equation; t_c is an abbreviation for the term $[t+Bb+C(c+\Delta i+ab'n)]$.

In the reticules were three sets of 5 wires each, the wires being numbered from 1 to 15. Usually the mean of observations on wires 3 to 13 was taken. In some cases the observations were incomplete on these wires, and then observations on other wires were substituted, or a fewer number was used. In such cases the clock-time of transit is from the mean of observations on wires reduced individually to the middle wire, and $\Delta i=0$ in the formula. The size of the correction $C(c+\Delta i+ab'n)$ together with the number of wires observed will sufficiently indicate these exceptions in the reductions.

The pivots of transit No. 1 being irregular in form required a pivot-correction varying with the altitude, so that the factor b generally varies for different stars. This pivot-correction was applied to the observed inclination of the axis as indicated by level-readings, and was taken from a curve plotted from the following data, which were obtained from special observations.

Altitude of star.	B′—B
90°	— 5. 63 divisions of level.
40° North.	— 4. 60 divisions of level.
40° South.	—10. 17 divisions of level.
65° North.	— 8. 67 divisions of level.
65° South.	— 7. 04 divisions of level.

In this table $\left\{ \begin{matrix} B' \\ B \end{matrix} \right\}$ is four times the inclination of the axis for clamp $\left\{ \begin{matrix} \text{east} \\ \text{west} \end{matrix} \right\}$, referring to the special observations, and the value of one division of the level was $0''.918$. Four times the value of pivot-correction, or B′—B, is tabulated for convenience in computation. For transit No. 15 a constant pivot-correction was applied to the level-readings in obtaining the factor b, and in the reduction the value was carried to three decimal places instead of two, as given in the tables following.

All the observations with transit No. 15 have been reduced directly in accordance with the above formula. The collimation from the mean of 19 reversals on slow stars for the six nights at Cambridge was $+0^s.130$ for clamp east, and from 23 reversals at Detroit, for the five nights at Detroit, was $+0^s.085$ for clamp east. (See Table 12.) For the observations with transit No. 1, on May 13 and June 4, c was deduced from the observation-equations. On May 13 the rate was adopted equal to zero, since it was known to be very small, and the data were not good for determining both collimation and rate, since all the stars after signals were with the clamp one way. On June 29 a' was introduced in the observation-equations, since the observations extended over a long space of time and a change of azimuth was suspected. (See the tables following for the individual values for collimation.)

Tables 1 to 11 give the data from which the observation-equations were formed, together with the resulting normal equations and the values of the unknowns, for the observations with transit No. 1. Tables 13 to 23 give similar data for the observations with transit No. 15.

The weight, p, of an observation-equation is derived from the following formulæ. Albrecht gives (see Formeln und Hülfstafeln, &c., Leipzig, 1874, page 7) the probable error of an observed transit over a single wire of a star at any declination

$$\varepsilon^2 = \varepsilon_{\text{eq}}{}^2 + \left(\frac{3.18}{v}\right)^2 \sec^2\delta$$

in which ε_{eq} = probable error of transit over one wire of a star at the equator, and v = the magnifying power of the telescope. We have, from observation, for transits Nos. 1 and 15, $\varepsilon_{\text{eq}} = \pm 0^s.06$ and $v = 65$; therefore

$$\varepsilon = \left\{ (0.06)^2 + (0.049)^2 \sec^2\delta \right\}^{\frac{1}{2}}$$

The United States Coast-Survey Report for 1872, page 224, gives

$$p=\frac{\varepsilon_1^2+\frac{\varepsilon^2}{N}}{\varepsilon_1^2+\frac{\varepsilon^2}{n}}$$

in which ε is as above, ε_1 is the probable error of culmination, reduced to the equator, N is the number of wires to whose mean weight unity is assigned, and n is the number of wires observed. We have used $N=11$ and $\varepsilon_1=\pm 0^s.056$, as given for smaller instruments in the United States Coast-Survey Report referred to above. All stars from 0° to 45° declination have been given weight unity when observed over 11 wires.

Tables 24 and 25 give the means of clock-times of comparison, the clock-corrections, and the resulting differences of time for the several nights. The final results are given in Table 26.

The mean of the results for difference of time between the two observing-stations for the first six nights, giving weight ½ to results for May 13 and 23, is

$$47^m\ 41^s.046$$

and for the last five nights

$$47^m\ 41^s.114$$

One-half the difference of these results, namely, $0^s.034$, which represents the personal and instrumental equation, when added to each of the first six nights' results, and subtracted from each of the last five nights' results, gives the results for difference of longitude for the several nights. Taking a mean of these results, giving weight ½ to results for May 13 and 23, and applying a reduction of $-0^s.004$ to reduce to the east stone-pier of the Lake-Survey Observatory, and a reduction of $+0^s.096$ to reduce to the dome of Harvard College Observatory, there results the final weighted mean difference of longitude. The United States Lake-Survey Observatory, 1871–'82, east pier, west from the dome of Harvard College Observatory, 1872–'82

$$=47^m\ 41^s.172\pm 0^s.031$$

The probable error of this mean is derived from the discrepancies between it and the separate results.

The difference of longitude between Detroit and Washington, given in Chapter XXV, is $24^m\ 00^s.15$. The United States Coast Survey gives for the difference of longitude between Washington and Cambridge, $23^m\ 41^s.042$ (see Report on Telegraphic Longitudes, Appendix No. 6, Coast-Survey Report for 1880). The sum of these gives for a difference of longitude between Detroit and Cambridge $47^m\ 41^s.192$, a value differing less from the result of the present direct determination given above than the probable error of the latter. The value $47^m\ 41^s.172\pm 0^s.031$ by the direct connection will then be adopted.

From the same authority as above quoted we have also the longitude of the dome of Harvard College Observatory west from Greenwich $4^h\ 44^m\ 30^s.994$. Adding the result above given, namely, $47^m\ 41^s.172$, there results for the longitude of the east pier of the Lake Survey Observatory west from Greenwich $5^h\ 32^m\ 12^s.166$.

The value adopted in March, 1874, and upon which all work since reported has been made to depend, was $5^h\ 32^m\ 12^s.24$. All longitudes reported since 1874 in the annual reports and those given in the present volume must, therefore, be diminished by $0^s.074$. Of this correction $0^s.020$ is due to the difference between the direct and indirect determinations of the longitude of Detroit from Cambridge, already mentioned, and $0^s.054$ to a change in the longitude of Cambridge formerly adopted by the United States Coast Survey, this change resulting from an adjustment given in Appendix 6 of Coast-Survey Report for 1880.

In the reductions we have been assisted by Assistant Engineers E. S. Wheeler, T. Russell, and A. Ziwet.

Respectfully submitted.

A. R. Flint,
O. B. Wheeler,
Assistant Engineers.

General C. B. Comstock,
Lieutenant-Colonel of Engineers, and Bvt. Brig.-Gen. U. S. A.

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.

TABLE 1.—TIME DETERMINATION.

[Detroit, Mich , May 13, 1881. A. R. Flint, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit $=t$	Right ascension $=\alpha$	$\alpha-t_c$
			s.	*s.*	*s.*	*s.*	*s.*	*h. m. s.*	*h. m. s.*	*m. s.*
θ Leonis	E.	11	+0.20	+0.19	−0.05	+0.20	0.00	11 09 27.55	11 08 02.30	−1 25.44
ξ Ursæ Maj. med	E.	11	+0.19	+0.22	−0.06	+0.09	0.00	11 13 18.02	11 11 52.66	−1 25.58
σ Leonis	E.	11	+0.23	+0.19	−0.05	+0.26	0.00	11 16 27.79	11 15 02.63	−1 25.35
ι Leonis	E.	11	+0.22	+0.19	−0.05	+0.23	0.00	11 19 11.08	11 17 45.84	−1 25.43
58 Ursæ Maj	E.	11	+0.19	+0.26	−0.07	−0.01	0.00	11 25 32.76	11 24 07.44	−1 25.58
υ Leonis	E.	11	+0.23	+0.17	−0.05	+0.30	0.00	11 32 19.20	11 30 54.05	−1 25.32
γ Cephei, L. C	E.	11	−0.08	+0.17	+0.34	+1.69	0.00	11 35 51.54	23 34 27.8	−1 23.9
χ Ursæ Maj	E.	11	+0.19	+0.28	−0.08	−0.07	0.00	11 41 14.23	11 39 48.78	−1 25.73
β Leonis	E.	11	+0.20	+0.18	−0.05	+0.21	0.00	11 44 27.34	11 43 02.10	−1 25.42
β Virginis	E.	11	+0.23	+0.17	−0.05	+0.28	0.00	11 45 57.72	11 44 32.56	−1 25.33
o Virginis	W.	11	+0.20	+0.17	+0.02	+0.24	0.00	12 00 36.91	11 59 11.67	−1 25.41
33 Bootis	W.	11	+0.08	+0.11	+0.03	−0.03	0.00	14 35 53.26	14 34 27.73	−1 25.64
μ Virginis	W.	11	+0.02	+0.01	+0.02	+0.32	0.00	14 38 15.91	14 36 50.80	−1 25.12
109 Virginis	W.	11	+0.03	+0.02	+0.02	+0.28	0.00	14 41 42.52	14 40 17.31	−1 25.23
47 Hev. Cephei, L. C	W.	8	−0.10	+0.26	−0.24	+1.95	0.00	14 51 42.10	2 50 18.8	−1 23.6
β Bootis	W.	11	+0.07	+0.09	+0.03	+0.01	0.00	14 58 56.22	14 57 31.01	−1 25.30
3 Serpentis	W.	11	+0.04	+0.03	+0.05	+0.26	0.00	15 10 45.02	15 09 19.84	−1 25.21
δ Bootis	W.	11	+0.08	+0.10	+0.02	+0.08	0.00	15 12 10.78	15 10 45.52	−1 25.36
μ Bootis	W.	11	+0.08	+0.10	+0.03	+0.04	0.00	15 21 28.25	15 20 02.83	−1 25.52
β Coronæ Borealis	W.	11	+0.08	+0.09	+0.02	+0.11	0.00	15 24 23.83	15 22 58.57	−1 25.35
ν^1 Bootis	W.	11	+0.08	+0.11	+0.03	+0.01	0.00	15 28 07.96	15 26 42.49	−1 25.58
ν^2 Bootis	W.	11	+0.08	+0.11	+0.03	+0.01	0.00	15 29 00.08	15 27 34.56	−1 25.63

Adopted rate $\rho=0^s.00$.

OBSERVATION-EQUATIONS.

Epoch $13^h.0$ clock time. $\Delta t=-1^m\ 25^s.00+\Delta\theta$.

					Weight.
$+0.46a$	$+1.04c$	$+\Delta\theta$	$+0.45=v=$	$+0.03$	1
+0.21	+1.18	+	+0.59=v=	+0.05	1
+0.59	+1.01	+	+0.36=v=	±0.00	1
+0.53	+1.02	+	+0.44=v=	+0.05	1
−0.03	+1.39	+	+0.59=v=	−0.06	1
+0.68	+1.00	+	+0.33=v=	+0.01	1
+3.87	−4.43	+	−1.1 =v=	+0.3	0.11
−0.16	+1.51	+	+0.75=v=	+0.03	1
+0.47	+1.04	+	+0.43=v=	+0.02	1
+0.64	+1.00	+	+0.34=v=	±0.00	1
+0.55	−1.01	+	+0.37=v=	+0.11	1
$-0.06a$	$-1.41c$	$+\Delta\theta$	$+0.58=v=$	$+0.08$	1
+0.74	−1.00	+	+0.08=v=	−0.10	1
+0.64	−1.00	+	+0.19=v=	−0.03	1
+4.47	+5.21	+	−1.4 =v=	−0.3	0.08
+0.03	−1.32	+	+0.25=v=	−0.22	1
+0.60	−1.01	+	+0.19=v=	−0.05	1
+0.18	−1.20	+	+0.31=v=	−0.10	1
+0.10	−1.26	+	+0.47=v=	+0.03	1
+0.25	−1.15	+	+0.30=v=	−0.08	1
+0.03	−1.33	+	+0.52=v=	+0.05	1
+0.02	−1.33	+	+0.58=v=	+0.11	1

NORMAL EQUATIONS.

$$+6.97a+\ 0.20c+\ 7.26\Delta\theta+1.04=0$$
$$+0.20\quad +31.81\quad -\ 2.90\quad +0.24=0$$
$$+7.26\quad -\ 2.90\quad +20.19\quad +7.92=0$$

RESULTS.

$a=+0^s.437$

$c=-0^s.061$ cl. E.

$\Delta\theta=-0^s.560$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 2.—TIME DETERMINATION.

[Detroit, Mich., May 23, 1881. A. R. Flint, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit $=t$	Right ascension $=a$	$a-t_c$
			s.	s.	s.	s.	s.	h. m. s.	h. m. s.	m. s.
4 Hev. Draconis	E.	8	+0.14	+0.56	+0.00	−0.22	+0.08	12 08 04.78	12 06 40.8	−1 24.5
4 Hev. Draconis	W.	8	+0.02	+0.08	−0.13	−0.22	+0.08	12 08 05.20	12 06 40.8	−1 24.3
η Virginis	W.	11	+0.13	+0.10	−0.05	+0.05	+0.07	12 15 16.51	12 13 51.89	−1 24.67
6 Canum Ven	W.	11	+0.19	+0.25	−0.07	±0.00	+0.07	12 21 26.47	12 20 02.03	−1 24.62
24 Comæ seq	W.	11	+0.16	+0.16	−0.06	+0.03	+0.07	12 30 37.10	12 29 12.49	−1 24.71
γ Virginis med	W.	11	+0.13	+0.09	−0.05	+0.05	+0.06	12 37 05.31	12 35 40.78	−1 24.57
δ Virginis	W.	11	+0.14	+0.11	−0.05	+0.05	+0.05	12 51 04.11	12 49 39.52	−1 24.65
ε Virginis	W.	11	+0.15	+0.13	−0.05	+0.04	+0.05	12 57 42.65	12 56 18.17	−1 24.56
α Ursæ Min., L. C	W.	8	+0.14	−3 97	+1.17	+2.45	+0.04	13 16 10.60	1 14 48.3	−1 19.5
α Ursæ Min., L. C	E.	7	+0.15	−4.25	±0.00	+2.45	+0.04	13 16 13.60	1 14 48.3	−1 21.0
τ Bootis	E.	11	+0.24	+0.23	+0.02	+0.03	+0.03	13 43 04.00	13 41 39.53	−1 24.72
η Ursæ Maj	E.	11	+0.23	+0.35	+0.03	−0.02	+0.03	13 44 18.54	13 42 54.23	−1 24.69
η Bootis	E.	11	+0.23	+0.22	+0.02	+0.03	+0.02	13 50 28.68	13 49 04.23	−1 24.69
τ Virginis	E.	11	+0.27	+0.21	+0.02	+0.05	+0.02	13 57 03.21	13 55 38.68	−1 24.76
d Bootis	E.	11	+0.24	+0.25	+0.02	+0.02	+0.01	14 06 25.86	14 05 01.44	−1 24.69
κ Virginis	E.	11	+0.29	+0.18	+0.02	+0.06	+0.01	14 08 00.88	14 06 36.37	−1 24.71
β Coronæ Bor	E.	11	+0.17	+0.19	+0.03	+0.02	−0.03	15 24 23.17	15 22 58.61	−1 24.78
ν^1 Bootis	E.	11	+0.16	+0.21	+0.03	±0.00	−0.03	15 28 07.04	15 26 42.53	−1 24.75
ν^2 Bootis	E.	11	+0.16	+0.21	+0.03	±0.00	−0.03	15 28 59.08	15 27 34.60	−1 24.72
ι Coronæ Bor	E.	11	+0.17	+0.18	+0.02	+0.02	−0.03	15 31 06.84	15 29 42.24	−1 24.80
φ Bootis	E.	11	+0.16	+0.21	+0.03	±0.00	−0.04	15 35 01.10	15 33 36.44	−1 24.90
ζ Coronæ Bor. seq	E.	11	+0.17	+0.21	+0.03	+0.01	−0.04	15 36 21.65	15 34 56.99	−1 24.90
β Serpentis	W.	11	+0.08	+0.07	−0.05	+0.03	−0.04	15 42 09.73	15 40 45.05	−1 24.70
μ Serpentis	W.	11	+0.05	+0.04	−0.05	+0.05	−0.04	15 44 52.87	15 43 28.17	−1 24.69
ε Serpentis	W.	11	+0.07	+0.06	−0.05	+0.05	−0.04	15 46 21.21	15 44 56.50	−1 24.72
ε Coronæ Bor	W.	11	+0.10	+0.11	−0.06	+0.02	−0.05	15 54 07.55	15 52 42.90	−1 24.70
750 Groombridge, L. C	W.	8	−0.03	+0.22	+0.33	+0.72	−0.05	16 00 59.36	3 59 36.6	−1 23.3
750 Groombridge, L. C	E.	8	±0.00	±0.00	±0.00	+0.72	−0.05	16 01 00.56	3 59 36.6	−1 23.9
δ Ophiuchi	W.	11	+0.08	+0.06	−0.05	+0.05	−0.06	16 09 34.87	16 08 10.17	−1 24.71
ε Ophiuchi	W.	11	+0.08	+0.06	−0.05	+0.05	−0.06	16 13 29.81	16 12 05.12	−1 24.70
η Ursæ Min	W.	8	+0.05	+0.17	−0.11	−0.17	−0.06	16 22 28.86	16 21 03.9	−1 25.0
η Ursæ Min	E.	8	+0.10	+0.34	±0.00	−0.17	−0.06	16 22 28.36	16 21 03.9	−1 24.8

COLLIMATION (Clamp E.)

	s.
1812 Groombridge	+0.014
4 Hev. Draconis	−0.006
α Ursæ Min., L. C	+0.032
750 Groombridge, L. C	+0.041
η Ursæ Maj	+0.040
Mean	+0.024

Adopted collimation $c=+0^s.012$ cl. E. (Mean of May 23 and 24.)

OBSERVATION-EQUATIONS.

Epoch $14^h.5$ clock time. $\Delta t=-1^m\ 24^s.50+\Delta\theta$.

	Weight.
$-\ 2.89a-2.4\rho+\Delta\theta-0.1\ =v=-0.5$	0.104
$+\ 0.67\ \ -2.2\ +\ \ \ +0.17=v=+0.05$	1
$+\ 0.06\ \ -2.1\ +\ \ \ +0.12=v=-0.05$	1
$+\ 0.42\ \ -2.0\ +\ \ \ +0.21=v=+0.07$	1
$+\ 0.68\ \ -1.9\ +\ \ \ +0.07=v=-0.06$	1
$+\ 0.62\ \ -1.6\ +\ \ \ +0.15=v=+0.01$	1
$+\ 0.52\ \ -1.5\ +\ \ \ +0.06=v=-0.09$	1
$+32.62\ \ -1.2\ +\ \ \ -4.2\ =v=-2.0$	0.001
$+\ 0.43\ \ -0.8\ +\ \ \ +0.22=v=+0.04$	1
$-\ 0.20\ \ -0.8\ +\ \ \ +0.19=v=-0.04$	0.640

	Weight.
$+0.42a-0.7\rho+\Delta\theta+0.19=v=\pm0.00$	1
$+0.65\ \ -0.5\ +\ \ \ +0.26=v=+0.09$	1
$+0.32\ \ -0.4\ +\ \ \ +0.18=v=-0.03$	1
$+0.80\ \ -0.4\ +\ \ \ +0.21=v=+0.04$	1
$+0.26a+0.9\rho+\Delta\theta+0.28=v=+0.03$	1
$+0.03\ \ +1.0\ +\ \ \ +0.25=v=-0.02$	1
$+0.03\ \ +1.0\ +\ \ \ +0.22=v=-0.05$	1
$+0.29\ \ +1.0\ +\ \ \ +0.30=v=+0.05$	1
$+0.03\ \ +1.1\ +\ \ \ +0.40=v=+0.12$	1

	Weight.
$+0.12a+1.1\rho+\Delta\theta+0.40=v=+0.13$	1
$+0.46\ \ +1.2\ +\ \ \ +0.20=v=-0.05$	1
$+0.71\ \ +1.2\ +\ \ \ +0.19=v=-0.04$	1
$+0.61\ \ +1.3\ +\ \ \ +0.22=v=-0.01$	1
$+0.29\ \ +1.4\ +\ \ \ +0.20=v=-0.07$	1
$+9.55\ \ +1.5\ +\ \ \ -0.9\ =v=-0.5$	0.018
$+0.72\ \ +1.7\ +\ \ \ +0.21=v=-0.04$	1
$+0.73\ \ +1.7\ +\ \ \ +0.20=v=-0.05$	1
$-2.29\ \ +1.9\ +\ \ \ +0.4\ =v=-0.1$	0.143

NORMAL EQUATIONS.

$$+9.99a-\ 0.67\rho+\ 9.32\Delta\theta+1.47=0$$
$$-0.67\ \ +43.48\ +\ 0.10\ \ +1.56=0$$
$$+9.32\ +\ 0.10\ +23.91\ \ +5.05=0$$

RESULTS.

$a=+0^s.075$

$\rho=-0^s.034$

$\Delta\theta=-0^s.240$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 3.—TIME DETERMINATION.

[Detroit, Mich., May 24, 1881. A. R. Flint, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit $=t$	Right ascension $=\alpha$	$\alpha-t_c$
			s.	*s.*	*s.*	*s.*	*s.*	*h. m. s.*	*h. m. s.*	*m. s.*
4 Hev. Draconis	W.	7	−0. 30	−1. 19	−0. 13	+ 1. 48	0. 00	12 08 04. 84	12 06 40. 8	−1 22. 8
4 Hev Draconis	E.	7	−0. 18	−0. 71	±0. 00	+ 1. 48	0. 00	12 08 04 49	12 06 40. 8	−1 23. 0
η Virginis	E.	11	−0. 09	−0. 07	+0. 02	− 0. 34	0. 00	12 15 17. 16	12 13 51. 88	−1 25. 23
6 Canum Ven	E.	11	−0. 14	−0. 18	+0. 03	− 0. 03	0. 00	12 21 26. 96	12 20 02. 02	−1 24. 79
20 Comæ	E.	11	−0. 12	−0. 12	+0. 02	− 0. 19	0. 00	12 25 12. 59	12 23 47. 40	−1 25. 09
γ Virginis med	E.	11	−0. 09	−0. 07	+0. 02	− 0. 35	0. 00	12 37 06. 09	12 35 40. 77	−1 25. 27
δ Virginis	E.	11	−0. 10	−0. 08	+0. 02	− 0. 32	0. 00	12 51 04. 80	12 49 39. 51	−1 25. 23
ε Virginis	E.	11	−0. 10	−0. 09	+0. 02	− 0. 27	0. 00	12 57 43. 38	12 56 18. 16	−1 25. 15
α Ursæ Min., L. C	E.	8	−0. 17	+4. 83	±0. 00	−16. 67	0. 00	13 16 24. 49	1 14 49. 4	−1 39. 9
α Ursæ Min., L. C	W.	7	−0. 25	+7. 09	+1. 17	−16. 67	0. 00	13 16 22. 00	1 14 49. 4	−1 40. 9
τ Bootis	W.	11	−0. 27	−0. 26	−0. 05	− 0. 22	0. 00	13 43 04. 82	13 41 39. 53	−1 24. 98
η Bootis	W.	11	−0. 26	−0. 25	−0. 06	− 0. 21	0. 00	13 50 29. 54	13 49 04. 23	−1 25. 00
τ Virginis	W.	11	−0. 29	−0. 22	−0. 05	− 0. 33	0. 00	13 57 04. 11	13 55 38. 69	−1 25. 15
α Draconis	W.	11	−0. 29	−0. 65	−0. 12	+ 0. 47	0. 00	14 02 38. 61	14 01 13. 58	−1 24. 26
κ Virginis	W.	11	−0. 31	−0. 20	−0. 05	− 0. 41	0. 00	14 08 01. 83	14 06 36. 36	−1 25. 22
4 Ursæ Min	W.	7	−0. 27	−1. 06	−0. 13	+ 1. 45	0. 00	14 10 49. 20	14 09 24. 6	−1 23. 4
4 Ursæ Min	E.	7	−0. 11	−0. 43	±0. 00	+ 1. 45	0. 00	14 10 48. 77	14 09 24. 6	−1 23. 7
ν^1 Bootis	E.	11	−0. 22	−0. 29	+0. 03	− 0. 02	0. 00	15 28 07. 52	15 26 42. 53	−1 24. 73
ν^2 Bootis	E.	11	−0. 22	−0. 29	+0. 03	− 0. 02	0. 00	15 28 59. 73	15 27 34. 60	−1 24. 87
ε Serpentis	W.	11	−0. 33	−0. 26	−0. 05	− 0. 31	0. 00	15 46 21. 95	15 44 56. 51	−1 25. 13
γ Serpentis	W.	11	−0. 31	−0. 29	−0. 05	− 0. 24	0. 00	15 52 26. 02	15 51 00. 73	−1 24. 95
ε Coronæ Bor	W.	11	−0. 20	−0. 33	−0. 06	− 0. 15	0. 00	15 54 08. 22	15 52 42. 89	−1 24. 94
750 Groombridge, L. C	W.	7	−0. 20	+1. 47	+0. 33	− 4. 88	0. 00	16 01 04. 01	3 59 36. 7	−1 29. 1
750 Groombridge, L. C	E.	7	−0. 26	+1. 91	±0. 00	− 4. 88	0. 00	16 01 04. 09	3 59 36. 7	−1 29. 3
δ Ophiuchi	E.	11	−0. 10	−0. 07	+0. 02	− 0. 37	0. 00	16 09 35. 47	16 08 10. 18	−1 25. 24
ε Ophiuchi	E.	11	−0. 10	−0. 07	+0. 02	− 0. 37	0. 00	16 13 30. 37	16 12 05. 13	−1 25. 19
τ Herculis	E.	11	−0. 16	−0. 23	+0. 03	+ 0. 06	0. 00	16 17 37. 97	16 16 12. 98	−1 24. 79
ω Herculis	E.	11	−0. 14	−0. 13	+0. 02	− 0. 25	0. 00	16 21 24. 06	16 19 58. 77	−1 25. 18
ζ Herculis	W.	11	−0. 26	−0. 30	−0. 06	− 0. 11	0. 00	16 38 16. 51	16 36 51. 24	−1 24. 91
η Herculis	W.	11	−0. 25	−0. 32	−0. 07	− 0. 04	0. 00	16 40 17. 51	16 38 52. 18	−1 24. 94
49 Herculis	W.	11	−0. 28	−0. 26	−0. 05	− 0. 24	0. 00	16 48 08. 61	16 46 43. 14	−1 25. 16
ε Ursæ Min	W.	10	−0. 31	−1. 76	−0. 20	+ 2. 42	0. 00	16 59 42. 56	16 58 17. 6	−1 23. 0
ε Ursæ Min	E.	7	−0. 23	−1. 31	±0. 00	+ 2. 42	0. 00	16 59 41. 80	16 58 17. 6	−1 22. 9

COLLIMATION (Clamp E.)

	s.
4 Hev. Draconis	−0. 013
α Ursæ Min. L. C	+0. 007
4 Ursæ Min	−0. 030
750 Groombridge, L. C	+0. 009
ε Ursae Min	+0. 027
Mean	±0. 000

Adopted collimation $c=+0^s.012$ cl. E. (Mean of May 23 and 24.)

OBSERVATION-EQUATIONS.

Epoch $14^h.7$ clock time. $\Delta t=-1^m\ 24^s.70+\Delta\theta$.

Equation	*Weight.*
$-\ 2.89a-2.6\rho+\Delta\theta-\ 1.8\ =v=-0.4$	0. 101
$+\ 0.67\ \ -2.4\ +\ \ +\ 0.53=v=+0.06$	1
$+\ 0.06\ \ -2.3\ +\ \ +\ 0.09=v=-0.07$	1
$+\ 0.38\ \ -2.3\ +\ \ +\ 0.39=v=+0.07$	1
$+\ 0.68\ \ -2.1\ +\ \ +\ 0.57=v=+0.09$	1
$+\ 0.62\ \ -1.8\ +\ \ +\ 0.53=v=+0.08$	1
$+\ 0.52\ \ -1.7\ +\ \ +\ 0.45=v=+0.05$	1
$+32.62\ \ -1.4\ +\ \ +15.7\ =v=-1.1$	0. 001
$+\ 0.43\ \ -1.0\ +\ \ +\ 0.28=v=-0.07$	1
$+\ 0.42\ \ -0.9\ +\ \ +\ 0.30=v=-0.04$	1

Equation	*Weight.*
$+\ 0.65a-0.7\rho+\Delta\theta+0.45=v=-0.01$	1
$-\ 0.91\ \ -0.7\ +\ \ -0.42=v=-0.08$	0. 354
$+\ 0.80\ \ -0.6\ +\ \ +0.52=v=-0.02$	1
$-\ 2.83\ \ -0.5\ +\ \ -1.1\ =v=+0.2$	0. 104
$+\ 0.03a+0.8\rho+\Delta\theta+0.03=v=-0.12$	1
$+\ 0.03\ \ +0.8\ +\ \ +0.17=v=+0.02$	1
$+\ 0.61\ \ +1.1\ +\ \ +0.43=v=-0.01$	1
$+\ 0.46\ \ +1.2\ +\ \ +0.23=v=-0.14$	1
$+\ 0.29\ \ +1.2\ +\ \ +0.24=v=-0.04$	1

Equation	*Weight.*
$+\ 9.55a+1.3\rho+\Delta\theta+4.5\ =v=-0.5$	0. 018
$+\ 0.72\ \ +1.5\ +\ \ +0.53=v=+0.03$	1
$+\ 0.73\ \ +1.5\ +\ \ +0.48=v=-0.02$	1
$-\ 0.11\ \ +1.6\ +\ \ +0.09=v=+0.02$	0. 690
$+\ 0.49\ \ +1.7\ +\ \ +0.48=v=+0.10$	1
$+\ 0.22\ \ +1.9\ +\ \ +0.21=v=-0.03$	1
$+\ 0.07\ \ +2.0\ +\ \ +0.24=v=+0.07$	1
$+\ 0.47\ \ +2.1\ +\ \ +0.46=v=+0.09$	1
$-\ 4.74\ \ +2.3\ +\ \ -1.7\ =v=+0.6$	0. 047

NORMAL EQUATIONS.

$$+11.08a-\ 0.96\rho+\ 8.34\Delta\theta+6.78=0$$
$$-\ 0.96\ \ +57.24\ +\ 0.68\ \ -0.48=0$$
$$+\ 8.34\ +\ 0.68\ +22.32\ \ +7.25=0$$

RESULTS.

$a=-0^s.511$
$\rho=+0^s.001$
$\Delta\theta=-0^s.134$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 4.—TIME DETERMINATION.

[Detroit, Mich., May 26, 1881. A. R. Flint, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit=t	Right ascension=a	$a-t_c$
			s.	s.	s.	s.	s.	h. m. s	h. m. s.	m. s.
4 Hev. Draconis	E.	8	+0.06	+0.24	+ 2.27	3.18	+0.03	12 08 05.69	12 06 40.0	−1 27.6
4 Hev. Draconis	W.	9	−0.06	−0.24	− 2.42	− 3.18	+0.03	12 08 10.94	12 06 40.6	−1 27.7
η Virginis	W.	11	+0.01	+0.01	− 0.52	− 0.74	+0.03	12 15 16.52	12 13 51.86	−1 24.15
6 Canum Ven	W.	11	+0.06	+0.08	− 0.67	+ 0.07	+0.03	12 21 27.37	12 20 01.90	−1 24.79
20 Comæ	W.	11	+0.04	+0.04	+ 0.56	+ 0.42	+0.03	12 25 12.41	12 23 47.38	−1 24.51
24 Comæ seq	W.	11	+0.04	+0.04	− 0.55	+ 0.46	+0.03	12 30 37.43	12 29 12.46	−1 24.46
γ Virginis med	W.	11	+0.01	+0.01	− 0.52	+ 0.75	+0.03	12 37 05.40	12 35 40.76	−1 24.13
ϵ Virginis	W.	11	+0.02	+0.02	− 0.53	+ 0.57	+0.02	12 57 42.98	12 56 18.15	−1 24.32
α Ursæ Min., L. C	W.	8	−0.07	+1.98	+21.26	+35.81	+0.02	13 15 15.69	1 14 50.9	−0 48.0
α Ursæ Min., L. C	E.	7	−0.01	+0.28	−20.00	+35.81	+0.02	13 16 00.99	1 14 50.9	−0 50.4
τ Bootis	E.	11	+0.09	+0.09	+ 0.51	+ 0.47	+0.01	13 43 03.37	13 41 39.52	−1 24.45
η Ursæ Maj	E.	11	+0.08	+0.12	+ 0.75	− 0.22	+0.01	13 44 18.40	13 42 54.18	−1 25.09
η Bootis	E.	11	+0.09	+0.09	+ 0.52	+ 0.46	+0.01	13 50 28.11	13 49 04.22	−1 24.50
τ Virginis	E.	11	+0.12	+0.09	− 2.17	+ 0.72	+0.01	13 57 04.96	13 55 38.68	−1 24.20
d Bootis	E.	11	+0.08	+0.08	+ 0.54	+ 0.35	+0.01	14 06 25.38	14 05 01.44	−1 24.56
4 Ursæ Min	E.	8	+0.10	+0.40	+ 2.24	− 3.12	+0.01	14 10 50.22	14 09 24.5	−1 28.4
4 Ursæ Min	W.	8	−0.09	−0.35	− 2.39	− 3.12	+0.01	14 10 55.39	14 09 24.5	−1 28.2
μ Bootis	W.	11	−0.02	−0.03	− 0.66	+ 0.11	−0.01	15 21 28.45	15 20 02.86	−1 24.90
β Coronæ Bor	W.	11	−0.03	−0.03	− 0.59	+ 0.29	−0.01	15 24 24.01	15 22 58.62	−1 24.77
ν^1 Bootis	W.	11	−0.02	−0.03	− 0.69	+ 0.03	−0.01	15 28 08.24	15 26 42.52	−1 25.00
α Coronæ Bor	W.	11	−0.03	−0.03	− 2.66	+ 0.32	−0.01	15 31 09.59	15 29 42.24	−1 24.06
ζ Coronæ Bor. seq	W.	11	−0.02	−0.02	− 0.65	+ 0.13	−0.01	15 36 22.41	15 34 56.99	−1 24.75
γ Coronæ Bor	W.	11	−0.03	−0.03	− 0.58	+ 0.33	−0.01	15 39 13.22	15 37 48.00	−1 24.61
750 Groombridge, L. C	W.	5	−0.12	+0.88	+ 5.93	+10.50	−0.02	16 00 44.40	3 59 36.8	−1 14.4
750 Groombridge, L. C	E.	6	−0.07	+0.51	− 5.56	+10.50	−0.02	16 00 55.93	3 59 36.8	−1 14.0
ϵ Ophiuchi	E.	11	+0.04	+0.03	+ 0.49	+ 0.80	−0.02	16 13 28.88	16 12 05.15	−1 24.25
λ Ophiuchi	E.	11	+0.03	+0.02	+ 0.49	+ 0.70	−0.02	16 26 22.00	16 24 58.25	−1 24.26
η Herculis	E.	11	−0.02	−0.03	+ 0.10	+ 0.08	−0.02	16 40 16.95	16 38 52.19	−1 21.83
49 Herculis	E.	11	+0.01	+0.01	− 2.04	+ 0.52	−0.03	16 48 09.59	16 46 43.10	−1 24.40
ϵ Ursæ Min	E.	7	+0.06	+0.34	+ 3.42	− 5.22	−0.03	16 59 44.14	16 58 17.6	−1 30.3
ϵ Ursæ Min	W.	7	−0.18	−1.02	− 3.64	− 5.22	−0.03	16 59 52.47	16 58 17.6	−1 30.2
α Herculis	E.	11	−0.04	−0.04	+ 0.50	+ 0.53	−0.03	17 10 40.58	17 09 16.63	−1 24 41
α Ophiuchi	E.	11	−0.04	−0.04	+ 0.50	+ 0.56	−0.03	17 30 51.81	17 29 27.99	−1 24.28

COLLIMATION (Clamp E.)

	s.
4 Hev. Draconis	+0.485
α Ursæ Min., L. C	+0.510
4 Ursæ Min	+0.457
759 Groombridge, L. C	+0.463
ϵ Ursæ Min	+0.471
Mean	+0.477

Adopted collimation $c=+0^s.477$ cl. E.

OBSERVATION-EQUATIONS.

Epoch $14^h.7$ clock time. $\Delta t=-1^m\ 24^s.50+\Delta\theta$.

	Weight.		Weight.		Weight.
$-2.89a-2.6\rho+\Delta\theta+3.2=v=-0.4$	0.104	$+0.42a-0.9\rho+\Delta\theta\pm0.00=v=+0.04$	1	$+0.30a+1.0\rho+\Delta\theta+0.11=v=\pm0.00$	1
$+0.67\ -2.4\ +\ -0.35=v=-0.01$	1	$+0.65\ -0.7\ +\ -0.30=v=\pm0.00$	1	$+9.55\ +1.3\ +\ -10.3=v=-0.3$	0.017
$+0.06\ -2.3\ +\ +0.20=v=-0.04$	1	$+0.32\ -0.6\ +\ +0.06=v=-0.01$	1	$+0.73\ +1.5\ +\ -0.25=v=+0.10$	1
$+0.38\ -2.3\ +\ +0.01=v=+0.03$	1	$-2.84\ -0.5\ +\ +3.8=v=+0.2$	0.106	$+0.64\ +1.7\ +\ -0.24=v=+0.01$	1
$+0.42\ -2.2\ +\ -0.04=v=+0.02$	1			$+0.07\ +2.0\ +\ +0.33=v=-0.04$	1
$+0.68\ -2.1\ +\ -0.37=v=-0.02$	1	$+0.10a+0.7\rho+\Delta\theta+0.38=v=+0.05$	1	$+0.47\ +2.1\ +\ -0.10=v=-0.04$	1
$+0.52\ -1.7\ +\ -0.18=v=-0.02$	1	$+0.26\ +0.7\ +\ +0.27=v=+0.12$	1	$-4.75\ +2.3\ +\ +5.8=v=+0.1$	0.016
$+32.61\ -1.4\ +\ -35.6=v=-0.2$	0.001	$+0.03\ +0.8\ +\ +0.50=v=+0.09$	1	$+0.48\ +2.5\ +\ -0.09=v=-0.02$	1
$+0.43\ -1.0\ +\ -0.05=v=\pm0.00$	1	$+0.29\ +0.8\ +\ +0.16=v=+0.04$	1	$+0.51\ +2.8\ +\ -0.22=v=-0.12$	1
$-0.20\ -1.0\ +\ +0.59=v=-0.05$	0.625	$+0.12\ +0.9\ +\ +0.25=v=-0.06$	1		

NORMAL EQUATIONS.

$$+9.70a+0.28\rho+7.80\Delta\theta-7.29=0$$
$$+0.28\ +64.46\ +0.49\ \ +0.66=0$$
$$+7.80\ +0.49\ +22.90\ \ +1.30=0$$

RESULTS.

$a=+1^s.698$
$\rho=-0^s.012$
$\Delta\theta=-0^s.431$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

Table 5.—TIME DETERMINATION.

[Detroit, Mich., June 4, 1881. A. R. Flint, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit $=t$.	Right ascension $=\alpha$	$\alpha-t_c$
			s.	s.	s.	s.	s.	h. m. s.	h. m. s.	m. s.
43 Hev. Cephei L. C	W.	8	−0.14	+1.12	−0.03	+0.52	+0.04	12 54 06.70	0 52 42.6	−1 25.3
43 Hev. Cephei L. C	E.	8	−0.09	+0.72	+0.37	+0.52	+0.04	12 54 06.14	0 52 42.6	−1 24.3
43 Comae	E.	11	+0.06	+0.07	±0.00	+0.01	+0.04	13 07 46.41	13 06 22.04	−1 24.44
20 Canum Ven	E.	11	+0.05	+0.07	±0.00	±0.00	+0.04	13 13 39.76	13 12 15.19	−1 24.64
α Ursae Min. L. C	E.	9	−0.04	+1.13	+1.21	+1.64	+0.04	13 16 20.94	1 14 58.2	−1 23.8
17 Hev. Canum Ven	E.	11	+0.06	+0.08	±0.00	+0.01	+0.03	13 30 56.43	13 29 31.85	−1 24.66
τ Bootis	E.	11	+0.08	+0.08	±0.00	+0.02	+0.03	13 43 03.92	13 41 39.46	−1 24.54
η Ursae Maj	E.	11	+0.07	+0.11	−0.01	−0.01	+0.03	13 44 18.49	13 42 54.04	−1 24.56
τ Virginis	E.	11	+0.11	+0.09	±0.00	+0.03	+0.02	13 57 03.08	13 55 38.65	−1 24.52
κ Virginis	W.	11	−0.04	−0.03	−0.03	+0.04	+0.02	14 08 00.91	14 06 36.34	−1 24.54
4 Ursae Min	W.	9	±0.00	±0.00	−0.01	−0.14	+0.02	14 10 49.00	14 09 23.9	−1 25.1
λ Bootis	W.	11	+0.02	+0.03	−0.04	−0.01	+0.02	14 13 19.35	14 11 54.61	−1 24.77
ρ Bootis	W.	11	+0.02	+0.02	−0.03	+0.01	+0.01	14 28 09.80	14 26 45.19	−1 24.63
π Bootis pr	W.	11	±0.00	±0.00	−0.03	+0.02	+0.01	14 36 35.87	14 35 11.24	−1 24.63
μ Virginis	W.	11	−0.03	−0.02	−0.03	+0.04	+0.01	14 38 15.48	14 36 50.84	−1 24.62
109 Virginis	W.	11	−0.02	−0.02	−0.03	+0.03	+0.01	14 41 41.90	14 40 17.36	−1 24.52
γ Ursae Min	W.	11	−0.15	−0.44	−0.09	−0.08	−0.01	15 22 24.20	15 20 59.3	−1 24.4
β Serpentis	W.	11	+0.04	+0.04	−0.03	+0.02	−0.01	15 42 09.68	15 40 45.10	−1 24.62
ε Serpentis	W.	11	+0.02	+0.02	−0.03	+0.03	−0.02	15 46 21.19	15 44 56.57	−1 24.64
ζ Ursae Min	W.	11	+0.04	+0.16	−0.13	−0.14	−0.02	15 49 49.41	15 48 24.2	−1 25.4
γ Serpentis	W.	11	+0.04	+0.04	−0.03	+0.02	−0.02	15 52 25.37	15 51 00.80	−1 24.61
φ Herculis	W.	11	+0.07	+0.09	−0.04	±0.00	−0.02	16 06 29.01	16 05 04.32	−1 24.78
δ Ophiuchi	W.	11	+0.01	+0.01	−0.03	+0.04	−0.03	16 09 34.78	16 08 10.28	−1 24.51
ε Ophiuchi	W.	11	+0.01	+0.01	−0.03	+0.04	−0.03	16 13 29.76	16 12 05.23	−1 24.54
γ Herculis	E.	11	+0.15	+0.15	±0.00	+0.02	−0.03	16 18 08.07	16 16 43.61	−1 24.61
α Herculis	E.	11	+0.15	+0.14	±0.00	+0.02	−0.05	17 10 41.38	17 09 16.74	−1 24.78
π Herculis	E.	11	+0.13	+0.16	±0.00	+0.01	−0.05	17 12 21.87	17 10 57.40	−1 24.63

OBSERVATION-EQUATIONS.

Epoch $15^h.0$ clock time. $\Delta t=-1^m\ 24^s.50+\Delta\theta$.

Equation	Weight.
$+10.31a+13.08c-2.1\rho+\Delta\theta+1.0=v=+1.3$	0.013
$+10.31-13.08-2.1+\quad\pm0.0=v=+0.6$	0.013
$+0.27+1.14-1.9+\quad-0.05=v=-0.12$	1
$+0.03+1.33-1.8+\quad+0.15=v=+0.06$	1
$+32.66-43.20-1.7+\quad\pm0.0=v=+2.1$	0.001
$+0.10+1.26-1.5+\quad+0.17=v=+0.08$	1
$+0.43+1.05-1.3+\quad+0.05=v=-0.02$	1
$-0.20+1.55-1.3+\quad+0.07=v=-0.04$	0.625
$+0.65+1.00-1.0+\quad+0.03=v=-0.04$	1
$+0.80-1.01-0.9+\quad\pm0.00=v=-0.04$	1
$-2.83-4.85-0.8+\quad+0.5=v=+0.3$	0.096
$-0.11-1.46-0.8+\quad+0.22=v=+0.14$	0.700
$+0.23-1.17-0.5+\quad+0.09=v=+0.01$	1
$+0.45-1.05-0.4+\quad+0.09=v=+0.02$	1

Equation	Weight.
$+0.74a-1.00c-0.4\rho+\Delta\theta+0.08=v=+0.03$	1
$+0.65-1.00-0.3+\quad-0.02=v=-0.08$	1
$-1.64a-3.28c+0.4\rho+\Delta\theta-0.2=v=-0.4$	0.206
$+0.46-1.04+0.7+\quad+0.08=v=-0.01$	1
$+0.61-1.00+0.8+\quad+0.10=v=+0.01$	1
$-2.85-4.87+0.8+\quad+0.6=v=+0.4$	0.100
$+0.46-1.04+0.9+\quad+0.07=v=-0.03$	1
$-0.07-1.42+1.1+\quad+0.23=v=+0.12$	0.700
$+0.72-1.00+1.2+\quad-0.03=v=-0.12$	1
$+0.73-1.00-1.2+\quad\pm0.00=v=-0.09$	1
$+0.41+1.06-1.3+\quad+0.12=v=-0.01$	1
$+0.48+1.03+2.2+\quad+0.29=v=+0.14$	1
$+0.12+1.25+2.2+\quad+0.14=v=-0.03$	1

NORMAL EQUATIONS.

$$+10.78a-0.90c+0.78\rho+7.48\Delta\theta+0.30=0$$
$$-0.90+38.70-6.41-3.90-0.02=0$$
$$+0.78-6.41+32.06-0.08+0.55=0$$
$$+7.48-3.90-0.08+20.45+1.86=0$$

Results.

$a=+0^s.050$

$c=-0^s.013$ cl. E.

$\rho=-0^s.021$

$\Delta\theta=-0^s.112$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 6.—TIME DETERMINATION.

[Detroit, Mich., June 11, 1881. A. R. Flint, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit $=t$	Right ascension $=\alpha$	$\alpha-t_c$
			s.	s.	s.	s.	s.	h. m. s.	h. m. s.	m. s.
α Ursæ Min., L. C	W.	4	−0.07	+1.98	−0.47	−3.42	+0.02	13 16 26.10	1 15 04.8	−1 22.8
α Ursæ Min., L. C	E.	5	+0.01	−0.28	+1.82	−3.42	+0.02	13 16 30.28	1 15 04.8	−1 27.0
τ Bootis	E.	11	+0.11	+0.10	−0.02	−0.05	+0.02	13 43 02.98	13 41 39.41	−1 23.65
η Bootis	E.	11	+0.11	+0.11	−0.02	−0.04	+0.02	13 50 27.68	13 49 04.12	−1 23.65
d Bootis	E.	11	+0.10	+0.11	−0.02	−0.03	+0.02	14 06 24.93	14 05 01.33	−1 23.69
λ Bootis	E.	11	+0.09	+0.13	−0.02	+0.01	+0.01	14 13 18.23	14 11 54.51	−1 23.83
π Bootis pr	W.	11	+0.05	+0.05	−0.01	−0.05	+0.01	14 36 35.01	14 35 11.21	−1 23.84
μ Virginis	W.	11	+0.02	+0.01	−0.01	−0.08	+0.01	14 38 14.58	14 36 50.83	−1 23.75
109 Virginis	W.	11	+0.03	+0.02	−0.01	−0.07	+0.01	14 41 41.12	14 40 17.34	−1 23.79
47 Hev. Cephei, L. C	W.	7	−0.10	+0.27	−0.06	−0.47	+0.01	14 51 44.31	2 50 20.8	−1 23.7
47 Hev. Cephei, L. C	E.	6	−0.11	+0.30	+0.22	−0.47	+0.01	14 51 44.72	2 50 20.8	−1 24.4
β Bootis	E.	11	+0.08	+0.11	−0.02	±0.00	+0.01	14 58 54.59	14 57 30.91	−1 23.77
ψ Bootis	E.	11	+0.10	+0.11	−0.02	−0.03	+0.01	15 00 47.69	14 59 23.96	−1 23.[illegible]2
μ Bootis	W.	11	−0.02	−0.03	−0.02	−0.01	±0.90	15 21 26.60	15 20 02.82	−1 23.73
β Coronæ Bor	W.	11	−0.03	−0.03	−0.02	−0.03	±0.00	15 24 22.27	15 22 58.60	−1 23.62
ν^1 Bootis	W.	11	−0.02	−0.03	−0.02	±0.00	±0.00	15 28 06.15	15 26 42.47	−1 23.63
ν^2 Bootis	W.	11	−0.02	−0.03	−0.02	±0.00	±0.00	15 28 58.32	15 27 34.55	−1 23.72
ζ Ursæ Min	W.	7	−0.21	−0.83	+0.05	+0.30	±0.00	15 49 47.54	15 48 23.9	−1 22.8
ζ Ursæ Min	E.	10	−0.05	−0.20	−0.20	+0.30	±0.00	15 49 47.17	15 48 23.9	−1 22.8
750 Groombridge	E.	10	−0.10	+0.73	+0.51	−1.00	±0.00	16 01 02.34	3 59 38.7	−1 24.9
ε Ophiuchi	E.	11	+0.08	+0.06	−0.02	−0.08	±0.00	16 13 29.05	16 12 05.27	−1 23.82
ω Herculis	E.	11	+0.04	+0.04	−0.02	−0.05	±0.00	16 21 22.67	16 19 58.89	−1 23.80
λ Ophiuchi	E.	11	+0.06	+0.05	−0.02	−0.07	±0.00	16 26 22.12	16 24 58.37	−1 23.78
σ Herculis	E.	11	+0.02	+0.03	−0.02	±0.00	±0.00	16 31 42.99	16 30 19.27	−1 23.73
η Herculis	E.	11	+0.01	+0.01	−0.02	−0.01	−0.01	16 40 16.02	16 38 52.29	−1 23.72
ε Ursæ Min	E.	9	+0.04	+0.23	−0.31	+0.50	−0.01	16 59 41.13	16 58 17.3	−1 23.7
ε Ursæ Min	W.	9	−0.12	−0.68	+0.08	+0.50	−0.01	16 59 40.97	16 58 17.3	−1 23.1
α Herculis	W.	11	−0.03	−0.03	−0.01	−0.05	−0.01	17 10 40.56	17 09 16.81	−1 23.71
„ Herculis	W.	11	±0.00	±0.00	−0.02	−0.01	−0.01	17 12 21.20	17 10 57.45	−1 23.73
α Ophiuchi	W.	11	−0.03	−0.03	−0.01	−0.05	−0.01	17 30 51.89	17 29 28.19	−1 23.66
δ Ursæ Min	W.	8	−0.15	−1.81	+0.19	+1.23	−0.02	18 12 12.86	18 10 50.3	−1 20.9
R Lyræ	W.	11	±0.00	±0.00	−0.02	±0.00	−0.02	18 53 09.96	18 51 46.06	−1 23.88
ι Lyræ	W.	11	±0.00	±0.00	−0.02	−0.01	−0.03	19 04 30.33	19 03 06.58	−1 23.73
θ Lyræ	W.	11	±0.00	±0.00	−0.02	−0.01	−0.03	19 13 41.12	19 12 17.44	−1 23.66
κ Cygni	W.	11	−0.01	−0.02	−0.02	+0.03	−0.03	19 15 48.09	19 14 24.32	−1 23.73
2900 Groombridge	W.	7	−0.15	−0.65	+0.06	+0.34	−0.03	19 30 19.66	19 28 55.8	−1 23.3
2900 Groombridge	E.	9	+0.04	+0.17	−0.26	+0.34	−0.03	19 30 19.88	19 28 55.8	−1 24.0

COLLIMATION (Clamp E.)

	s.
α Ursæ Min., L. C	+0.022
47 Hev. Cephei, L. C	+0.042
ζ Ursæ Min	+0.029
ε Ursæ Min	−0.073
2900 Groombridge	−0.096
Mean	−0.027

Adopted collimation $=-0^s.027$ cl. E.

OBSERVATION-EQUATIONS.

Epoch $16^h.0$ clock time. $\Delta t=-1^m\ 23^s.61+\Delta\theta$.

	Weight.
$+32.60a-2.7\rho+\Delta\theta+1.3=v=-2.2$	0.001
$+0.43\ \ -2.3\ +\ \ +0.05=v=-0.08$	1
$+0.42\ \ -2.2\ +\ \ +0.05=v=-0.07$	1
$+0.32\ \ -1.0\ +\ \ +0.09=v=-0.02$	1
$-0.11\ \ -1.8\ +\ \ +0.23=v=+0.15$	0.700
$+0.45\ \ -1.4\ +\ \ +0.24=v=+0.10$	1
$+0.74\ \ -1.4\ +\ \ +0.15=v=-0.02$	1
$+0.65\ \ -1.3\ +\ \ +0.19=v=+0.03$	1
$+4.46\ \ -1.1\ +\ \ +0.4=v=-0.1$	0.090
$+0.03\ \ -1.0\ +\ \ +0.17=v=+0.08$	1
$+0.29\ \ -1.0\ +\ \ +0.22=v=+0.10$	1

	Weight.
$+0.10a-0.6\rho+\Delta\theta+0.13=v=+0.02$	1
$+0.25\ \ -0.6\ +\ \ +0.02=v=-0.11$	1
$+0.03\ \ -0.5\ +\ \ +0.03=v=-0.07$	1
$+0.02\ \ -0.5\ +\ \ +0.12=v=+0.02$	1
$-2.85a-0.2\rho+\Delta\theta-0.8=v=-0.6$	0.105
$+9.55\ \ \pm0.0\ +\ \ +1.2=v=+0.1$	0.017
$+0.73\ \ +0.2\ +\ \ +0.22=v=+0.04$	1
$+0.49\ \ +0.4\ +\ \ +0.20=v=+0.05$	1
$+0.64\ \ +0.4\ +\ \ +0.18=v=+0.01$	1
$-0.01\ \ +0.5\ +\ \ +0.13=v=+0.03$	1

	Weight.
$+0.07a+0.7\rho+\Delta\theta+0.12=v=\pm0.00$	1
$-4.75\ \ +1.0\ +\ \ -0.2=v=+0.2$	0.050
$+0.48\ \ +1.2\ +\ \ +0.11=v=-0.05$	1
$+0.12\ \ +1.2\ +\ \ +0.13=v=+0.01$	1
$+0.51\ \ +1.5\ +\ \ +0.06=v=-0.10$	1
$-11.80\ \ +2.2\ +\ \ -2.7=v=-1.6$	0.008
$-0.03\ \ -2.9\ +\ \ +0.28=v=+0.16$	0.720
$+0.14\ \ +3.1\ +\ \ +0.13=v=-0.01$	1
$+0.10\ \ +3.2\ +\ \ +0.06=v=-0.08$	1
$-0.31\ \ +3.3\ +\ \ +0.13=v=+0.03$	0.570
$-3.26\ \ +3.5\ +\ \ +0.1=v=+0.3$	0.085

NORMAL EQUATIONS.

$$+11.94a-5.09\rho+6.40\Delta\theta+1.88=0$$
$$-5.09\ \ +64.44\ +\ 0.66\ \ +0.06=0$$
$$+6.40\ +\ 0.66\ +24.35\ \ +3.22=0$$

RESULTS.

$a=-0^s.105$

$\rho=-0^s.008$

$\Delta\theta=-0^s.104$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 7.—TIME DETERMINATION.

[Cambridge, Mass., June 21, 1881. A. R. Flint, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	Rp	Clock time of transit $=t$	Right ascension $=a$	$a-t_c$
			s.	s.	s.	s.	s.	h. m. s.	h. m. s.	m. s.
τ Virginis	W.	11	+0.12	+0.09	−0.15	−0.17	+0.04	13 55 59.50	13 55 38.54	−0 20.96
d Bootis	W.	11	+0.16	+0.17	−0.14	−0.08	+0.03	14 05 21.96	14 05 01.23	−0 20.76
4 Ursæ Min	W.	7	+0.02	+0.07	−0.63	+0.74	+0.03	14 09 43.03	14 09 22.6	−0 19.9
4 Ursæ Min	E.	8	+0.14	+0.55	+0.49	+0.74	+0.03	14 09 40.70	14 09 22.6	−0 19.2
φ Virginis	E.	11	+0.25	+0.18	+0.13	−0.18	+0.03	14 22 28.26	14 22 07.57	−0 21.00
ρ Bootis	E.	11	+0.20	+0.23	+0.15	−0.06	+0.02	14 27 05.43	14 26 45.04	−0 20.77
3 Serpentis	E.	11	+0.23	+0.18	+0.13	−0.16	+0.02	15 09 40.53	15 09 19.89	−0 20.95
δ Bootis	E.	11	+0.19	+0.23	+0.15	−0.05	+0.02	15 11 05.77	15 10 45.42	−0 20.73
μ Bootis	E.	11	+0.19	+0.24	+0.16	−0.03	+0.02	15 20 23.14	15 20 02.79	−0 20.82
β Coronæ Bor	E.	11	+0.20	+0.22	+0.14	−0.06	+0.02	15 23 18.99	15 22 58.53	−0 20.82
φ Bootis	W.	11	+0.15	+0.20	−0.20	−0.01	+0.02	15 33 57.09	15 33 36.31	−0 20.78
ζ Coronæ Bor. seq	W.	11	+0.14	+0.18	−0.19	−0.03	+0.01	15 35 17.61	15 34 56.88	−0 20.72
α Serpentis	W.	11	+0.10	+0.08	−0.16	−0.15	+0.01	15 38 48.73	15 38 27.88	−0 20.77
β Serpentis	W.	11	+0.12	+0.11	−0.16	−0.12	+0.01	15 41 05.95	15 40 45.09	−0 20.81
750 Groombridge, L. C	W.	8	−0.01	+0.07	+1.57	−2.47	+0.01	16 00 02.15	3 59 40.4	−0 23.4
750 Groombridge, L. C	E.	8	+0.07	−0.51	−1.23	−2.47	+0.01	16 00 05.66	3 59 40.4	−0 23.5
ω Herculis	E.	11	+0.24	+0.22	+0.13	−0.12	±0.00	16 20 19.48	16 19 58.91	−0 20.92
σ Herculis	E.	11	+0.21	+0.29	+0.17	±0.00	±0.00	16 30 39.58	16 30 19.23	−0 20.81
2373 Groombridge	E.	7	+0.24	+0.92	+0.48	+0.70	±0.00	16 36 09.62	16 35 50.4	−0 20.6
2373 Groombridge	W.	8	+0.14	+0.54	−0.61	+0.70	±0.00	16 36 10.77	16 35 50.4	−0 20.3
ν Ophiuchi	W.	11	+0.13	+0.08	−0.16	−0.21	−0.02	17 52 53.63	17 52 32.55	−0 21.00
72 Ophiuchi	W.	11	+0.16	+0.14	−0.16	−0.14	−0.02	18 02 07.05	18 01 46.16	−0 20.87
o Herculis	W.	11	+0.19	+0.21	−0.18	−0.07	−0.02	18 03 18.30	18 02 57.50	−0 20.83
δ Ursæ Min	W.	7	+0.08	+0.97	−2.20	+3.05	−0.02	18 11 08.90	18 10 50.1	−0 17.6
δ Ursæ Min	E.	6	+0.15	+1.82	+1.72	+3.05	−0.02	18 11 04.38	18 10 50.1	−0 17.8
109 Herculis	E.	11	+0.18	+0.18	+0.14	−0.10	−0.02	18 19 01.66	18 18 41.16	−0 20.82
α Lyræ	E.	11	+0.16	+0.20	+0.16	−0.02	−0.03	18 33 18.26	18 32 57.96	−0 20.66
2655 Groombridge	E.	11	+0.20	+0.75	+0.58	+0.69	−0.03	18 35 52.49	18 35 33.5	−0 20.3
ε Lyræ pr	E.	11	+0.16	+0.21	+0.16	−0.02	−0.03	18 40 47.56	18 40 27.17	−0 20.76
51 Hev. Cephei, L. C	E.	7	+0.17	−2.24	−2.11	−4.12	−0.03	18 44 40.58	6 44 12.5	−0 23.7
λ Aquilæ	W.	11	+0.10	+0.07	−0.16	−0.19	−0.03	19 00 20.96	18 59 59.89	−0 20.98
ι Lyræ	W.	11	+0.16	+0.20	−0.19	−0.04	−0.04	19 03 27.56	19 03 06.72	−0 20.85
θ Lyræ	W.	11	+0.16	+0.20	−0.20	−0.03	−0.04	19 12 38.42	19 12 17.60	−0 20.82

COLLIMATION (Clamp E.)

4 Ursæ Min	+0.190
750 Groombridge, L. C	+0.122
2373 Groombridge	+0.082
δ Ursæ Min	+0.109
Mean	+0.126

Adopted collimation $c=+0^s.116$ cl. E. (Mean of June 21 and 22.)

OBSERVATION-EQUATIONS.

Epoch $16^h.6$ clock time. $\Delta t=-0^m\ 20^s.80+\Delta\theta$.

Equation	*Weight.*
$+0.65a-2.7\rho+\Delta\theta+0.16=v=+0.08$	1
$+0.32\ \ -2.5\ +\ \ -0.04=v=-0.04$	1
$-2.84\ \ -2.4\ +\ \ -1.3\ =v=-0.4$	0.106
$+0.70\ \ -2.2\ +\ \ +0.20=v=+0.10$	1
$+0.23\ \ -2.1\ +\ \ -0.03=v=-0.02$	1
$+0.60\ \ -1.4\ +\ \ +0.15=v=+0.06$	1
$+0.18\ \ -1.4\ +\ \ -0.07=v=-0.05$	1
$+0.10\ \ -1.3\ +\ \ +0.02=v=+0.06$	1
$+0.25\ \ -1.2\ +\ \ +0.02=v=+0.03$	1
$+0.04\ \ -1.0\ +\ \ -0.02=v=+0.04$	1

Equation	*Weight.*
$+0.12a-1.0\rho+\Delta\theta-0.08=v=-0.05$	1
$+0.59\ \ -1.0\ +\ \ -0.03=v=-0.12$	1
$+0.46\ \ -0.9\ +\ \ +0.01=v=-0.05$	1
$+9.55\ \ -0.6\ +\ \ +2.6\ =v=+0.2$	0.018
$+0.48\ \ -0.3\ +\ \ +0.12=v=+0.05$	1
$-0.01\ \ -0.1\ +\ \ +0.01=v=+0.06$	1
$-2.71\ \ \pm0.0\ +\ \ -0.4\ =v=+0.4$	0.110
$+0.80a+1.3\rho+\Delta\theta+0.20=v=+0.02$	1
$+0.55\ \ +1.4\ +\ \ +0.07=v=-0.04$	1
$+0.27\ \ +1.5\ +\ \ +0.03=v=-0.01$	1

Equation	*Weight.*
$-11.80a+1.6\rho+\Delta\theta-3.0\ =v=+0.1$	0.009
$+0.38\ \ +1.7\ +\ \ +0.02=v=-0.05$	1
$+0.08\ \ +2.0\ +\ \ -0.14=v=-0.14$	1
$-2.66\ \ +2.0\ +\ \ -0.5\ =v=+0.2$	0.012
$+0.06\ \ +2.1\ +\ \ -0.04=v=-0.04$	1
$+15.92\ \ +2.1\ +\ \ +2.9\ =v=-1.2$	0.005
$+0.74\ \ +2.4\ +\ \ +0.18=v=+0.01$	1
$+0.14\ \ +2.5\ +\ \ +0.05=v=+0.02$	1
$+0.10\ \ +2.6\ +\ \ -0.02=v=\pm0.00$	1

NORMAL EQUATIONS.

$$+10.67a-3.40\rho+7.08\Delta\theta+2.36=0$$
$$-3.40\ \ +70.82\ \ -1.62\ \ +0.18=0$$
$$+7.08\ \ -1.62\ \ +23.36\ \ +0.64=0$$

RESULTS.

$a=-0^s.259$
$\rho=-0^s.014$
$\Delta\theta=+0^s.050$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 8.—TIME DETERMINATION.

[Cambridge, Mass, June 22, 1881. A. R. Flint, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit=t	Right ascension=a	$a-t_c$
			s.	s.	s.	s.	s.	h. m. s.	h. m. s.	m. s.
α Ursæ Min., L. C	E.	8	+0.06	−1.70	−4.36	−6.75	+0.16	13 15 51.30	1 15 14.8	−0 30.4
α Ursæ Min., L. C	W.	9	+0.03	−0.85	+5.66	−6.75	+0.16	13 15 37.20	1 15 14.8	−0 27.2
d Bootis	W.	11	+0.10	+0.11	−0.17	−0.07	+0.12	14 05 22.79	14 05 01.22	−0 21.51
μ Virginis	W.	11	+0.05	+0.03	−0.16	−0.15	+0.09	14 37 12.38	14 36 50.79	−0 21.46
109 Virginis	W.	11	+0.06	+0.05	−0.15	−0.13	+0.09	14 40 38.96	14 40 17.29	−0 21.57
47 Hev. Cephei, L. C	W.	11	−0.01	+0.03	+0.68	−0.93	+0.08	14 50 43.04	14 50 22.0	−0 21.8
ψ Bootis	W.	11	+0.10	+0.11	−0.18	−0.06	+0.07	14 59 45.44	14 59 23.88	−0 21.49
3 Serpentis	W.	11	+0.07	+0.06	−0.16	−0.12	+0.06	15 09 41.35	15 09 19.89	−0 21.36
β Libræ	W.	11	+0.04	+0.03	−0.16	−0.16	+0.06	15 11 01.52	15 10 39.88	−0 21.51
β Coronæ Bor	E.	11	+0.19	+0.21	+0.15	−0.05	+0.05	15 23 19.63	15 22 58.54	−0 21.45
α Serpentis	E.	11	+0.22	+0.18	+0.13	−0.12	+0.04	15 38 49.22	15 38 27.88	−0 21.65
β Serpentis	E.	11	+0.21	+0.20	+0.13	−0.10	+0.04	15 41 06.26	15 40 45.09	−0 21.50
κ Serpentis	E.	11	+0.20	+0.19	+0.13	−0.09	+0.03	15 43 47.52	15 43 26.36	−0 21.48
ε Serpentis	E.	11	+0.22	+0.18	+0.13	−0.13	+0.03	15 45 17.87	15 44 56.59	−0 21.59
ζ Ursæ Min	E.	7	+0.09	+0.36	+0.50	+0.59	+0.03	15 48 43.23	15 48 23.3	−0 20.8
ζ Ursæ Min	W.	7	−0.02	−0.08	−0.63	+0.59	+0.03	15 48 44.96	15 48 23.3	−0 20.9
750 Groombridge, L. C	W.	7	+0.02	−0.15	+1.57	−1.98	+0.02	16 00 02.90	3 59 40.6	−0 23.7
750 Groombridge, L. C	E.	7	+0.03	−0.22	−1.23	−1.98	+0.02	16 00 06.11	3 59 40.6	−0 24.0
δ Ophiuchi	E.	11	+0.30	+0.21	+0.13	−0.15	+0.02	16 08 31.80	16 08 10.34	−0 21.80
ε Ophiuchi	E.	11	+0.30	+0.21	+0.13	−0.15	+0.01	16 12 26.67	16 12 05.30	−0 21.71
ω Herculis	E.	11	+0.27	+0.25	+0.13	−0.10	+0.01	16 20 20.07	16 19 58.90	−0 21.55
ζ Herculis	E.	11	+0.25	+0.29	+0.15	−0.05	−0.01	16 37 12.49	16 36 51.34	−0 21.59
η Herculis	E.	11	+0.24	+0.31	+0.16	−0.01	−0.02	16 39 13.17	16 38 52.27	−0 21.37
ε Ursæ Min	E.	8	+0.18	+1.02	+0.75	+0.98	−0.03	16 58 35.66	16 58 16.7	−0 20.7
ε Ursæ Min	W.	6	+0.07	+0.40	−0.96	+0.98	−0.03	16 58 37.70	16 58 16.7	−0 20.4
ι Herculis	W.	11	+0.07	+0.10	−0.22	+0.02	−0.06	17 36 31.20	17 36 09.68	−0 21.40
β Ophiuchi	W.	11	+0.03	+0.02	−0.16	−0.13	−0.06	77 38 01.13	17 37 39.40	−0 21.59
μ Herculis	W.	11	+0.06	+0.06	−0.18	−0.06	−0.06	17 42 13.15	17 41 51.54	−0 21.49
ξ Herculis	W.	11	+0.07	+0.08	−0.18	−0.05	−0.07	17 53 33.59	17 53 11.90	−0 21.59
67 Ophiuchi	W.	11	+0.03	+0.02	−0.15	−0.13	−0.07	17 55 06.72	17 54 44.98	−0 21.61
72 Ophiuchi	W.	11	+0.04	+0.03	−0.16	−0.11	−0.08	18 02 07.95	18 01 46.17	−0 21.65
δ Ursæ Min	W.	8	−0.10	−1.21	−2.20	+2.44	−0.09	18 11 10.52	18 10 50.1	−0 17.0
δ Ursæ Min	E.	7	+0.16	+1.94	+1.72	+2.44	−0.09	18 11 06.31	18 10 50.1	−0 19.9
α Lyræ	E.	11	+0.27	+0.35	+0.16	−0.02	−0.11	18 33 19.13	18 32 57.98	−0 21.66
110 Herculis	E.	11	+0.29	+0.29	−0.14	−0.08	−0.12	18 40 57.31	18 40 35.97	−0 21.77

COLLIMATION (Clamp E).

	s.
α Ursæ Min. L. C	+0.153
ζ Ursæ Min	+0.133
750 Groombridge L. C	+0.130
ε Ursæ Min	+0.096
δ Ursæ Min	+0.033
Mean	+0.109

Adopted collimation, $c=+0^s.116$ cl. E. (mean of June 21 and 22).

OBSERVATION-EQUATIONS.

Epoch, $16^h.4$ clock time. $\Delta t=-0^m\,21^s.50+\Delta\theta$.

Equation	Weight.
$+32.60a-3.1\rho+\Delta\theta+7.2=v=+0.6$	0.001
$+0.32\ -2.3\ +\ \ +0.01=v=+0.10$	1
$+0.74\ -1.8\ +\ \ -0.04=v=-0.06$	1
$+0.04\ -1.7\ +\ \ +0.07=v=+0.07$	1
$+4.47\ -1.6\ +\ \ +0.2=v=-0.6$	0.086
$+0.29\ -1.4\ +\ \ -0.01=v=+0.04$	1
$+0.60\ -1.2\ +\ \ -0.14=v=-0.16$	1
$+0.79\ -1.2\ +\ \ +0.01=v=-0.05$	1
$+0.25\ -1.0\ +\ \ -0.05=v=-0.01$	1
$+0.59\ -0.8\ +\ \ +0.15=v=+0.11$	1
$+0.46a-0.7\rho+\Delta\theta\pm0.00=v=-0.04$	1
$+0.42\ -0.7\ +\ \ -0.02=v=-0.04$	1
$+0.61\ -0.6\ +\ \ +0.09=v=+0.03$	1
$-2.85\ -0.6\ +\ \ -0.6=v=+0.0$	0.106
$+9.55\ -0.4\ +\ \ +2.4=v=+0.4$	0.017
$+0.72\ -0.3\ +\ \ +0.30=v=+0.21$	1
$+0.73\ -0.2\ +\ \ +0.21=v=+0.11$	1
$+0.48\ -0.1\ +\ \ +0.05=v=\pm0.00$	1
$+0.22\ +0.2\ +\ \ +0.09=v=+0.07$	1
$+0.07\ +0.3\ +\ \ -0.13=v=-0.12$	1
$-4.74a+0.6\rho+\Delta\theta-0.9=v=+0.1$	0.046
$-0.09a+1.2\rho+\Delta\theta-0.10=v=-0.10$	0.700
$+0.62\ +1.2\ +\ \ +0.09=v=-0.06$	1
$+0.28\ +1.3\ +\ \ -0.01=v=-0.09$	1
$+0.26\ +1.5\ +\ \ +0.09=v=+0.01$	1
$+0.64\ +1.5\ -\ \ +0.11=v=-0.05$	1
$+0.55\ +1.6\ +\ \ +0.15=v=\pm0.00$	1
$-11.80\ +1.8\ +\ \ -3.1=v=-0.7$	0.009
$+0.08\ +2.2\ +\ \ +0.16=v=+0.07$	1
$+0.40\ +2.3\ +\ \ +0.27=v=+0.11$	1

NORMAL EQUATIONS.

$$+13.51a-3.83\rho+10.66\Delta\theta+2.24=0$$
$$-3.83\ +41.16\ -1.22\ +1.31=0$$
$$+10.66\ -1.22\ +23.96\ +1.32=0$$

RESULTS.

$a=-0^s.207$

$\rho=-0^s.050$

$\Delta\theta=+0^s.035$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

Table 9.—TIME DETERMINATION.

[Cambridge, Mass., June 23, 1881. A. R. Flint, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit $=t$	Right ascension $=\alpha$	$\alpha-t_c$
			s.	*s.*	*s.*	*s.*	*s.*	*h. m. s.*	*h. m. s.*	*m. s.*
α Ursæ Min., L. C	W.	7	−0.05	+1.42	−5.44	−9.49	+0.07	13 15 49.62	1 15 15.9	−0 20.7
α Ursæ Min., L. C	E.	8	+0.03	−0.85	+6.74	−9.49	+0.07	13 15 41.60	1 15 15.9	−0 31.6
4 Ursæ Min	E.	7	+0.03	+0.12	−0.76	+0.83	+0.05	14 09 44.71	14 09 22.4	−0 21.7
4 Ursæ Min	W.	6	−0.04	−0.16	+0.61	+0.83	+0.05	14 09 43.09	14 09 22.4	−0 21.2
μ Virginis	W.	11	+0.13	+0.09	+0.10	−0.22	+0.04	14 37 12.97	14 36 50.78	−0 22.38
109 Virginis	W.	11	+0.14	+0.11	+0.10	−0.19	+0.04	14 40 39.51	14 40 17.29	−0 22.43
β Bootis	W.	11	+0.19	+0.25	+0.14	−0.01	+0.03	14 57 52.64	14 57 30.78	−0 22.25
ψ Bootis	W.	11	+0.18	+0.20	+0.11	−0.08	+0.03	14 59 45.89	14 59 23.88	−0 22.32
3 Serpentis	W.	11	+0.15	+0.12	+0.10	−0.17	+0.03	15 09 42.11	15 09 19.89	−0 22.44
β Libræ	W.	11	+0.12	+0.08	+0.10	−0.23	+0.03	15 11 02.09	15 10 39.88	−0 22.39
μ Bootis	W.	11	+0.19	+0.24	+0.14	−0.03	+0.02	15 20 24.61	15 20 02.72	−0 22.27
β Coronæ Bor	W.	11	+0.18	+0.21	+0.13	−0.07	+0.02	15 23 20.45	15 22 58.53	−0 22.26
α Coronæ Bor	W.	11	+0.18	+0.21	+0.13	−0.08	+0.02	15 30 04.04	15 29 42.18	−0 22.20
ζ Coronæ Bor seq	E.	11	+0.26	+0.32	−0.16	−0.03	+0.02	15 35 18.88	15 34 56.89	−0 22.15
α Serpentis	E.	11	+0.30	+0.25	−0.13	−0.17	+0.02	15 38 50.21	15 38 27.89	−0 22.44
β Serpentis	E.	11	+0.28	+0.26	−0.14	−0.13	+0.01	15 41 07.33	15 40 45.09	−0 22.36
κ Serpentis	E.	11	+0.28	+0.27	−0.14	−0.12	+0.01	15 43 48.56	15 43 26.36	−0 22.33
ε Serpentis	E.	11	+0.30	+0.24	−0.13	−0.18	+0.01	15 45 19.00	15 44 56.59	−0 22.52
ε Coronæ Bor	E.	11	+0.27	+0.29	−0.15	−0.08	+0.01	15 53 05.11	15 52 42.92	−0 22.33
δ Ophiuchi	E.	11	+0.31	+0.22	−2.79	−0.21	±0.00	16 08 35.50	16 08 10.34	−0 22.59
ε Ophiuchi	E.	11	+0.32	+0.22	−0.13	−0.21	±0.00	16 12 27.84	16 12 05.30	−0 22.63
ζ Herculis	E.	11	+0.26	+0.30	−0.15	−0.06	−0.01	16 37 13.43	16 36 51.36	−0 22.22
η Herculis	E.	11	+0.25	+0.32	−0.17	−0.02	−0.01	16 39 14.38	16 38 52.29	−0 22.24
ε Ursæ Min	E.	5	+0.10	+0.57	−1.15	+1.38	−0.02	16 58 37.59	16 58 16.7	−0 20.4
ε Ursæ Min	W.	2	+0.11	+0.62	+0.93	+1.38	−0.02	16 58 35.43	16 58 16.7	−0 20.3
θ Herculis	W.	11	+0.22	+0.28	+0.14	−0.03	−0.04	17 52 35.66	17 52 13.68	−0 22.40
67 Ophiuchi	W.	11	+0.17	+0.13	+0.10	−0.19	−0.04	17 55 07.16	17 54 44.98	−0 22.41
ο Herculis	W.	11	+0.21	+0.23	+0.12	−0.08	−0.04	18 03 19.56	18 02 57.52	−0 22.39
δ Ursæ Min	W.	4	+0.18	+2.18	+2.12	+3.43	−0.05	18 11 05.00	18 10 50.0	−0 19.3
δ Ursæ Min	E.	7	+0.30	+3.63	−2.64	+3.43	−0.05	18 11 08.90	18 10 50.0	−0 19.9
α Lyræ	E.	11	+0.21	+0.27	−0.17	−0.02	−0.06	18 33 20.20	18 32 57.98	−0 22.32
51 Hev. Cephei, L. C	E.	6	+0.23	−3.04	+3.22	−4.62	−0.06	18 44 39.46	6 44 12.6	−0 27.1
51 Hev. Cephei, L. C	W.	7	+0.17	−2.24	−2.61	−4.62	−0.06	18 44 42.54	6 44 12.6	−0 25.1
γ Lyræ	E.	10	+0.22	+0.26	−1.76	−0.06	−0.06	18 54 56.83	18 54 32.96	−0 22.37

COLLIMATION (Clamp E.)

	s.	*Weight.*
α Ursæ Min., L. C	−0.119	3.7
4 Ursæ Min	−0.196	3.2
ε Ursæ Min	−0.143	1.4
δ Ursæ Min	−0.159	2.6
51 Hev. Ceph., L. C	−0.094	3.2
Weighted mean	−0.141	

Adopted collimation $c=-0^s.141$ cl. E.

OBSERVATION-EQUATIONS.

Epoch $16^h.3$ clock time. $\Delta t=-0^m\ 22^s.40+\Delta\theta$.

	Weight.		*Weight.*		*Weight.*
$+32.60a-3.0\rho+\Delta\theta+8.28=v=-1.0$	0.001	$+0.29a-0.8\rho+\Delta\theta-0.20=v=-0.11$	1	$+0.07a+0.4\rho+\Delta\theta-0.16=v=-0.04$	1
$-2.84\ \ -2.1\ +\ \ -0.93=v=+0.1$	0.105	$+0.12\ \ -0.7\ +\ \ -0.25=v=-0.11$	1	$-4.74\ \ +0.7\ +\ \ -2.1\ =v=-0.6$	0.040
$+0.74\ \ -1.7\ +\ \ -0.02=v=-0.05$	1	$+0.59\ \ -0.7\ +\ \ +0.04=v=+0.04$	1		
$+0.64\ \ -1.6\ +\ \ +0.03=v=+0.03$	1	$+0.46\ \ -0.6\ +\ \ -0.04=v=-0.01$	1	$+0.11a+1.6\rho+\Delta\theta\pm0.00=v=+0.08$	1
$+0.03\ \ -1.3\ +\ \ -0.15=v=+0.02$	1	$+0.42\ \ -0.6\ +\ \ -0.07=v=-0.03$	1	$+0.64\ \ +1.6\ +\ \ +0.01=v=-0.07$	1
$+0.29\ \ -1.3\ +\ \ -0.08=v=+0.02$	1	$+0.61\ \ -0.5\ +\ \ +0.12=v=+0.10$	1	$+0.27\ \ +1.8\ +\ \ -0.01=v=+0.02$	1
$+0.60\ \ -1.1\ +\ \ +0.04=v=+0.05$	1	$+0.29\ \ -0.4\ +\ \ -0.09=v=-0.01$	1	$-11.80\ \ +1.9\ +\ \ -2.7\ =v=+0.8$	0.009
$+0.79\ \ -1.1\ +\ \ -0.01=v=-0.06$	1	$+0.72\ \ -0.2\ +\ \ +0.19=v=+0.13$	1	$+0.08\ \ +2.3\ +\ \ -0.08=v=-0.01$	1
$+0.10\ \ -1.0\ +\ \ -0.13=v=+0.01$	1	$+0.73\ \ -0.1\ +\ \ +0.23=v=+0.17$	1	$+15.90\ \ +2.4\ +\ \ +3.6\ =v=-0.9$	0.006
$+0.25\ \ -0.9\ +\ \ -0.14=v=-0.04$	1	$+0.22\ \ +0.3\ +\ \ -0.18=v=-0.10$	1	$+0.20\ \ +2.6\ +\ \ -0.03=v=\pm0.00$	1

NORMAL EQUATIONS.

$$+10.60a-3.52\rho+8.81\Delta\theta+1.68=0$$
$$-3.52\ \ +37.07\ \ -4.16\ \ +0.49=0$$
$$+8.81\ \ -4.16\ \ +24.16\ \ -1.15=0$$

Results.

$a=-0^s.291$

$\rho=-0^s.024$

$\Delta\theta=+0^s.150$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 10.—TIME DETERMINATION.

[Cambridge, Mass., June 24, 1881. A. R. Flint, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit $=t$	Right ascension $=\alpha$	$\alpha-t_c$
			s.	s.	s.	s.	s.	h. m. s.	h. m. s.	m. s.
4 Ursæ Min	W.	7	−0.13	−0.51	−0.01	} used for collimation only.				
4 Ursæ Min	E.	9	+0.12	+0.47	±0.00					
47 Hev. Cephei, L. C	E.	11	±0.00	±0.00	±0.00	−1.07	+0.07	14 50 46.78	2 50 22.2	−0 24.5
ψ Bootis	E.	11	+0.18	+0.20	±0.00	−0.07	+0.06	14 59 46.81	14 59 23.87	−0 23.14
3 Serpentis	E.	11	+0.21	+0.16	±0.00	−0.14	+0.06	15 09 43.04	15 09 19.89	−0 23.31
β Libræ	E.	11	+0.23	+0.14	±0.00	−0.19	+0.06	15 11 03.06	15 10 39.88	−0 23.32
μ Bootis	E.	11	+0.16	+0.20	±0.00	−0.02	+0.05	15 20 25.90	15 20 02.71	−0 23.39
β Coronæ Bor	E.	11	+0.17	+0.19	±0.00	−0.06	+0.05	15 23 21.43	15 22 58.53	−0 23.09
ν¹ Bootis	E.	11	+0.16	+0.21	±0.00	−0.01	+0.04	15 27 05.28	15 26 42.36	−0 23.13
ν² Bootis	E.	11	+0.16	+0.21	±0.00	−0.01	+0.04	15 27 57.52	15 27 34.43	−0 23.30
α Coronæ Bor	E.	11	+0.18	+0.19	±0.00	−0.07	+0.04	15 30 05.19	15 29 42.18	−0 23.20
ζ Coronæ Bor. seq	E.	11	+0.16	+0.20	±0.00	−0.03	+0.04	15 35 19.64	15 34 56.88	−0 22.96
α Serpentis	W.	11	+0.02	+0.02	−0.03	−0.14	+0.04	15 38 50.99	15 38 27.89	−0 23.09
β Serpentis	W.	11	+0.04	+0.04	−0.03	−0.11	+0.04	15 41 08.16	15 40 45.08	−0 23.09
κ Serpentis	W.	11	+0.05	+0.05	−0.03	−0.10	+0.04	15 43 49.39	15 43 26.35	−0 23.06
ε Serpentis	W.	11	+0.03	+0.02	−0.03	−0.15	+0.03	15 45 19.66	15 44 56.59	−0 23.06
750 Groombridge, L. C	W.	11	−0.01	+0.07	+0.39	−2.29	+0.03	16 00 05.54	3 59 41.1	−0 24.9
δ Ophiuchi	W.	11	+0.01	+0.01	−0.03	−0.17	+0.02	16 08 33.51	16 08 10.34	−0 23.15
ε Ophiuchi	W.	11	+0.01	+0.01	−0.03	−0.18	+0.02	16 12 28.48	16 12 05.30	−0 23.16
γ Herculis	W.	11	+0.05	+0.05	−0.03	−0.10	+0.01	16 17 06.70	16 16 43.64	−0 23.08
β Herculis	W.	11	+0.05	+0.05	−0.03	−0.09	+0.01	16 25 32.69	16 25 09.65	−0 23.06
η Herculis	W.	11	+0.07	+0.09	−0.04	−0.02	±0.00	16 39 15.41	16 38 52.28	−0 23.18
ε Ursæ Min	W.	8	−0.04	−0.23	−0.06	+1.14	−0.01	16 58 39.35	16 58 16.6	−0 22.5
ε Ursæ Min	E.	8	+0.06	+0.34	−0.17	+1.14	−0.01	16 58 38.02	16 58 16.6	−0 21.6
μ Herculis	E.	11	+0.19	+0.21	±0.00	−0.07	−0.04	17 42 14.52	17 41 51.57	−0 23.16
ξ Herculis	E.	11	+0.19	+0.21	±0.00	−0.06	−0.04	17 53 34.92	17 53 11.93	−0 23.20
67 Ophiuchi	E.	11	+0.23	+0.18	±0.00	−0.15	−0.04	17 55 08.17	17 54 44.99	−0 23.36
72 Ophiuchi	E.	11	+0.22	+0.19	±0.00	−0.13	−0.05	18 02 09.34	18 01 46.19	−0 23.34
δ Ursæ Min	E.	8	+0.09	+1.09	−0.39	+2.83	−0.06	18 11 08.83	18 10 49.9	−0 19.6
δ Ursæ Min	W.	7	−0.06	−0.73	−0.14	+2.83	−0.06	18 11 09.12	18 10 49.9	−0 18.3
α Lyræ	W.	11	+0.18	+0.23	−0.04	−0.02	−0.07	18 33 21.07	18 32 58.00	−0 23.26
51 Hev. Cephei, L. C	W.	6	±0.00	±0.00	+0.17	−3.82	−0.07	18 44 39.67	6 44 12.6	−0 27.2
51 Hev. Cephei, L. C	E.	5	−0.05	+0.66	+0.45	−3.82	−0.07	18 44 39.77	6 44 12.6	−0 28.3
R Lyræ	E.	11	+0.18	+0.25	±0.00	+0.01	−0.08	18 52 09.28	18 51 46.24	−0 23.29
ε Aquilæ	W.	11	+0.15	+0.14	−0.03	−0.12	−0.08	18 54 40.17	18 54 16.95	−0 23.33
ζ Aquilæ	W.	11	+0.14	+0.13	−0.03	−0.12	−0.09	19 00 23.29	19 00 00.08	−0 23.31
ι Lyræ	W.	11	+0.17	+0.21	−0.04	−0.03	−0.09	19 03 29.82	19 03 06.79	−0 23.20

COLLIMATION (Clamp E.)

	s.
4 Ursæ Min	−0.028
ε Ursæ Min	+0.051
δ Ursæ Min	−0.045
51 Hev. Ceph., L. C	+0.018
Weighted mean	−0.007

Adopted collimation $c=-0^s.007$ cl. E.

OBSERVATION-EQUATIONS.

Epoch $16^h.7$ clock time. $\Delta t=-0^m\ 23^s.10+\Delta\theta$.

Equation	Weight.
$+4.45a-1.9\rho+\Delta\theta+1.4=v=+0.4$	0.086
$+0.29\ -1.7\ +\ +0.04=v=+0.02$	1
$+0.60\ -1.5\ +\ +0.21=v=+0.12$	1
$+0.79\ -1.5\ +\ +0.22=v=+0.08$	1
$+0.10\ -1.4\ +\ +0.29=v=+0.31$	1
$+0.25\ -1.3\ +\ -0.01=v=-0.03$	1
$+0.03\ -1.2\ +\ +0.03=v=+0.05$	1
$+0.03\ -1.2\ +\ +0.20=v=+0.22$	1
$+0.30\ -1.2\ +\ +0.10=v=+0.06$	1
$+0.12\ -1.1\ +\ -0.14=v=-0.14$	1
$+0.59\ -1.1\ +\ -0.01=v=-0.12$	1

Equation	Weight.
$+0.46a-1.0\rho+\Delta\theta-0.01=v=-0.09$	1
$+0.43\ -1.0\ +\ -0.04=v=-0.11$	1
$+0.61\ -0.9\ +\ -0.04=v=-0.17$	1
$+9.54\ -0.7\ +\ +1.8=v=-0.5$	0.017
$+0.72\ -0.6\ +\ +0.05=v=-0.11$	1
$+0.73\ -0.5\ +\ +0.06=v=-0.11$	1
$+0.41\ -0.4\ +\ -0.02=v=-0.12$	1
$+0.38\ -0.3\ +\ -0.04=v=-0.13$	1
$+0.07\ \pm0.0\ +\ +0.08=v=+0.05$	1
$-4.74\ +0.3\ +\ -1.0=v=+0.1$	0.047

Equation	Weight.
$+0.28a+1.0\rho+\Delta\theta+0.06=v=-0.06$	1
$+0.26\ +1.2\ +\ +0.10=v=+0.01$	1
$+0.64\ +1.2\ +\ +0.26=v=+0.06$	1
$+0.55\ +1.3\ +\ +0.24=v=+0.05$	1
$-11.80\ +1.5\ +\ -4.1=v=-1.4$	0.009
$+0.08\ +1.9\ +\ +0.16=v=+0.06$	1
$+15.93\ +2.0\ +\ +4.6=v=+0.8$	0.006
$-0.03\ +2.2\ +\ +0.19=v=+0.11$	1
$+0.48\ +2.2\ +\ +0.23=v=+0.02$	1
$+0.49\ +2.3\ +\ +0.21=v=-0.01$	1
$+0.14\ +2.4\ +\ +0.10=v=-0.03$	1

NORMAL EQUATIONS.

$$+12.14a-2.95\rho+10.21\Delta\theta+2.89=0$$
$$-2.95\ +51.72\ -2.34\ +1.18=0$$
$$+10.21\ -2.34\ +27.16\ +2.61=0$$

RESULTS.

$a=-0^s.240$

$\rho=-0^s.037$

$\Delta\theta=-0^s.009$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

Table II.—TIME DETERMINATION.

[Cambridge, Mass., June 29, 1881. A. R. Flint, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit $=t$	Right ascension $=\alpha$	$\alpha-t_c$
			s.	s.	s.	s.	s.	h. m. s.	h. m. s.	m. s.
33 Bootis	W.	11	+0.16	+0.23	− 0.09	−0.04	+0.16	14 34 55.58	14 34 27.24	−0 28.48
109 Virginis	W.	11	+0.11	+0.08	− 0.06	+0.36	+0.16	14 40 45.08	14 40 17.25	−0 27.85
47 Hev. Cephei, L. C	W.	8	−0.18	+0.49	+ 0.20	+2.47	+0.16	14 50 47.86	2 50 22.8	−0 25.8
47 Hev. Cephei, L. C	E.	9	−0.20	+0.54	− 0.04	+2.47	+0.16	14 50 48.01	2 50 22.8	−0 25.7
3 Serpentis	E.	11	+0.07	+0.06	+ 0.03	+0.33	+0.14	15 09 47.03	15 09 19.86	−0 27.86
β Libræ	E.	11	+0.09	+0.06	+ 0.03	+0.44	+0.14	15 11 07.54	15 10 39.86	−0 27.77
β Coronæ Bor	E.	11	+0.03	+0.03	+ 0.04	+0.14	+0.13	15 23 26.41	15 22 58.49	−0 27.99
α Coronæ Bor	E.	11	+0.04	+0.04	+ 0.04	+0.16	+0.13	15 30 10.12	15 29 42.14	−0 28.06
ϕ Bootis	E.	11	+0.02	+0.03	+ 0.04	+0.02	+0.12	15 34 04.30	15 33 36.25	−0 28.12
α Serpentis	E.	11	+0.07	+0.06	+ 0.03	+0.33	+0.12	15 38 55.77	15 38 27.87	−0 27.99
β Serpentis	E.	11	+0.05	+0.05	+ 0.03	+0.25	+0.12	15 41 12.97	15 40 45.06	−0 27.99
ϵ Serpentis	E.	11	+0.07	+0.06	+ 0.03	+0.34	+0.11	15 45 24.37	15 44 56.57	−0 27.89
ζ Ursæ Min	E.	7	−0.06	−0.26	+ 0.04	−1.58	+0.11	15 48 52.54	15 48 22.8	−0 29.5
ζ Ursæ Min	W.	8	−0.26	−1.03	− 0.19	−1.58	+0.11	15 48 53.58	15 48 22.8	−0 29.6
ϕ Herculis	W.	11	±0.00	±0.00	− 0.09	−0.04	+0.10	16 05 32.86	16 05 04.16	−0 28.61
2533 Groombridge	W.	11	−0.02	−0.03	+ 0.03	+0.01	+0.01	18 12 28.51	18 12 00.03	−0 28.48
η Serpentis	W.	11	−0.08	−0.06	+ 0.02	+0.50	+0.01	18 15 41.09	18 15 13.10	−0 27.95
d Sagittarii	W.	11	−0.24	−0.12	+ 0.03	+0.66	−0.03	19 11 12.42	19 10 44.65	−0 27.08
θ Lyræ	W.	11	−0.17	−0.21	+ 0.03	+0.07	−0.03	19 12 46.00	19 12 17.72	−0 28.19
γ Aquilæ	W.	11	−0.21	−0.18	+ 0.03	+0.38	−0.05	19 41 07.81	19 40 39.85	−0 27.81
α Aquilæ	W.	11	−0.21	−0.18	+ 0.02	+0.40	−0.05	19 45 30.41	19 45 02.38	−0 27.87
Ψ Cygni	W.	11	−0.18	−0.29	+11.02	−0.20	−0.06	19 52 54.30	19 52 36.67	−0 28.36
76 Draconis	E.	9	+0.12	+0.67	− 0.58	−3.28	−0.10	20 51 43.61	20 51 12.3	−0 31.4
76 Draconis	W.	8	−0.04	−0.22	+ 0.37	−3.28	−0.10	20 51 43.83	20 51 12.3	−0 31.7
2777 Bradley	W.	7	−0.06	−0.23	+ 0.24	−1.90	−0.11	21 08 26.02	21 07 55.7	−0 30.3
2777 Bradley	E.	6	+0.14	+0.53	− 0.37	−1.90	−0.11	21 08 26.13	21 07 55.7	−0 30.6
1 Pegasi	E.	11	+0.15	+0.14	− 0.06	+0.30	−0.12	21 17 06.64	21 16 38.42	−0 28.30
74 Cygni	E.	11	+0.13	+0.17	− 0.06	+0.04	−0.13	21 32 42.51	21 32 14.17	−0 28.45
ϵ Pegasi	W.	11	+0.04	+0.03	+ 0.02	+0.39	−0.13	21 38 51.96	21 38 23.92	−0 28.09
π^2 Cygni	W.	11	+0.07	+0.11	+ 0.04	−0.12	−0.13	21 42 55.93	21 42 27.28	−0 28.80
16 Pegasi	W.	10	+0.07	+0.07	−11.03	+0.23	−0.14	21 48 21.49	21 47 42.23	−0 28.30
20 Pegasi	E.	11	+0.18	+0.16	− 0.06	+0.36	−0.14	21 55 49.00	21 55 20.90	−0 28.20
α Aquarii	E.	11	+0.20	+0.15	− 0.05	+0.48	−0.15	22 00 11.80	21 59 43.70	−0 28.20
ι Pegasi	E.	11	+0.16	+0.17	− 0.06	+0.23	−0.15	22 01 59.84	22 01 31.62	−0 28.33

COLLIMATION (Clamp E.)

47 Hev. Ceph., L. C	$+0^s.019$
ζ Ursæ Min	$+0^s.027$
76 Draconis	$-0^s.046$
2777 Bradley	$-0^s.093$
Weighted mean	$-0^s.065$

Adopted collimation $c=-0^s.065$ cl. E.

OBSERVATION-EQUATIONS.

Epoch $18^h.5$ clock time. $\Delta t=-0^m\ 28^s.30+\Delta\theta$.

	Weight.
$-0.07a-3.9\rho+\Delta\theta+0.18=v=+0.23$	0.700
$+0.65\quad-3.8\quad+\quad-0.45=v=\pm0.00$	1
$+4.46\quad-3.7\quad+\quad-2.6\ =v=\pm0.0$	0.090
$+0.60\quad-3.3\quad+\quad-0.44=v=-0.04$	1
$+0.79\quad-3.3\quad+\quad-0.53=v=-0.02$	1
$+0.25\quad-3.1\quad+\quad-0.31=v=-0.11$	1
$+0.29\quad-3.0\quad+\quad-0.24=v=-0.02$	1
$+0.03\quad-2.9\quad+\quad-0.18=v=-0.11$	1
$+0.59\quad-2.9\quad+\quad-0.31=v=+0.07$	1
$+0.46\quad-2.8\quad+\quad-0.31=v=-0.02$	1

	Weight.
$+0.61a-2.7\rho+\Delta\theta-0.41=v=-0.03$	1
$-2.85\quad-2.7\quad+\quad+1.2\ =v=-0.3$	0.106
$-0.07\quad-2.4\quad+\quad+0.31=v=+0.30$	0.700
$+0.01a'-0.3\rho+\Delta\theta+0.18=v=+0.13$	1
$+0.71\quad-0.2\quad+\quad-0.35=v=+0.09$	1
$+0.93\quad+0.7\quad+\quad-0.62=v=-0.06$	1
$+0.10\quad+0.7\quad+\quad-0.11=v=-0.14$	1
$+0.54\quad+1.2\quad+\quad-0.49=v=-0.23$	1
$+0.56\quad+1.3\quad+\quad-0.43=v=-0.15$	1
$-0.28\quad+1.4\quad+\quad+0.06=v=-0.27$	0.600

	Weight.
$-4.65a'+2.3\rho+\Delta\theta+3.3\ =v=-0.2$	0.050
$-2.70\quad+2.6\quad+\quad+2.1\ =v=\pm0.0$	0.110
$+0.42\quad+2.8\quad+\quad\pm0.00=v=+0.11$	1
$+0.05\quad+3.0\quad+\quad+0.15=v=-0.01$	1
$+0.55\quad+3.1\quad+\quad-0.21=v=-0.02$	1
$-0.17\quad+3.2\quad+\quad+0.50=v=+0.18$	0.650
$+0.32\quad+3.3\quad+\quad\pm0.00=v=+0.02$	1
$+0.51\quad+3.4\quad+\quad-0.10=v=+0.05$	1
$+0.68\quad+3.5\quad+\quad-0.10=v=+0.16$	1
$+0.33\quad+3.5\quad+\quad+0.03=v=+0.04$	1

NORMAL EQUATIONS.

$$
\begin{array}{rrrrr}
+5.12a & & -13.80\rho & +4.27\Delta\theta & -3.14=0 \\
 & +5.33a' & +9.43 & +4.90 & -3.00=0 \\
-13.80 & +9.43 & +189.92 & -3.50 & +9.12=0 \\
+4.27 & +4.90 & -3.50 & +25.01 & -4.22=0
\end{array}
$$

Results.

$a=+0^s.554$

$a'=+0^s.706$

$\rho=-0^s.042$

$\Delta\theta=-0^s.070$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 12.—COLLIMATION FROM REVERSALS ON SLOW STARS.

[Würdemann transit No. 15. Correction for Clamp East. O. B. Wheeler, Observer.]

Star.	Cambridge, Mass.							
	May 13.	May 23.	May 24.	May 26.	May 31.	June 4.	June 6.	June 11.
4 Hev. Dracon				+0.091				
α Urs. Min. L. C	+0.113	+0.150	+0.170	+0.117	+0.149	+0.162	+0.139	+0.138
47 Hev. Ceph., L. C							+0.155	+0.114
ζ Urs. Min	+0.089							
Gr. 750, L. C				+0.120				
ε Urs. Min			+0.115			+0.154		+0.139
δ Urs. Min			+0.128			+0.116		+0.108
	Mean of 19 results = +0^s.130 Clamp East.							
	Detroit, Mich.							
	June 21.	June 22.	June 23.	June 24.	June 27.	June 28.	June 29.	
α Urs. Min., L. C	+0.107	+0.093	+0.065	+0.098	+0.088	+0.085	+0.075	
47 Hev. Ceph., L. C	+0.101	+0.166	−0.008					
Gr., 750, L C				+0.119	+0.050	+0.078	+0.147	
ε Urs. Min	+0.055	+0.055	+0.122					
δ Urs. Min	+0.122	+0.059	+0.082	+0.074			+0.052	
Gr., 2900							+0.108	
	Mean of 23 results = +0^s.085 Clamp East.							

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

Table 13.—TIME DETERMINATION.

[Cambridge, Mass., May 13, 1881. O. B. Wheeler, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit $=t$	Right ascension $=\alpha$	$\alpha-t_c$
			s.	*s.*	*s.*	*s.*	*s.*	*h. m. s.*	*h. m. s.*	*s.*
υ Leonis	W.	11	−0.04	−0.03	−0.25	+0.01	+0.03	11 30 54.87	11 30 54.05	−0.54
χ Ursæ Maj	W.	11	−0.04	−0.06	−0.38	0.00	+0.03	39 49.83	39 48.79	−0.60
β Leonis	W.	11	−0.04	−0.04	−0.26	0.00	+0.03	43 02.95	43 02.10	−0.55
Gr. 1852	W.	11	−0.04	−0.15	−1.18	−0.02	+0.02	59 18.27	59 16.02	−0.9
Br. 6, L. C	W.	11	−0.04	+0.08	+1.07	+0.03	+0.02	12 09 28.74	0 09 29.23	−0.7
20 Comæ	W.	11	−0.04	−0.04	−0.27	0.00	+0.02	23 48.21	12 23 47.50	−0.40
24 Comæ seq	W.	11	−0.04	−0.04	−0.27	0.00	+0.02	29 13.37	29 12.59	−0.47
γ Virginis med	W.	11	−0.04	−0.03	−0.25	+0.01	+0.02	35 41.53	35 40.84	−0.41
δ Virginis	E.	11	−0.01	−0.00	+0.22	+0.01	+0.02	49 39.86	49 39.58	−0.50
ϵ Virginis	E.	11	−0.01	−0.01	+0.23	0.00	+0.02	56 18.48	56 18.23	−0.47
θ Virginis	E.	11	−0.01	−0.00	+0.22	+0.01	+0.01	13 03 50.75	13 03 50.49	−0.46
α Ursæ Min., L. C	E.	4	+0.01	−0.37	−4.93	+0.29	+0.01	14 48.38	1 14 41.8	−1.3
α Ursæ Min., L. C	W.	7	−0.03	+0.74	+6.32	+0.29	+0.01	14 37.55	1 14 41.8	−2.8
τ Bootis	W.	11	−0.03	−0.03	−0.26	0.00	+0.01	41 40.20	13 41 39.57	−0.34
η Bootis	W.	11	−0.03	−0.03	−0.27	0.00	+0.01	49 04.86	49 04.27	−0.29
ϕ Bootis	W.	11	−0.01	−0.02	−0.33	0.00	−0.01	15 33 37.14	15 33 36.39	−0.40
ζ Coron. Bor. seq	W.	11	−0.01	−0.02	−0.32	0.00	−0.01	34 57.69	34 56.91	−0.42
α Serpentis	W.	11	−0.01	−0.01	−0.15	+0.01	−0.01	38 28.27	38 27.74	−0.37
β Serpentis	W.	11	−0.01	−0.01	−0.26	0.00	−0.01	40 45.72	40 44.97	−0.48
μ Serpentis	W.	11	−0.01	−0.01	−0.25	+0.01	−0.01	43 28.86	43 28.08	−0.52
ζ Ursæ Min	W.	5	−0.01	−0.05	−0.71	−0.03	−0.01	48 25.55	48 24.7	−0.1
ζ Ursæ Min	E.	6	+0.10	+0.41	+0.56	−0.03	−0.01	48 24.22	48 24.7	−0.5
ϕ Herculis	E.	11	+0.09	+0.12	+0.31	0.00	−0.01	16 05 04.64	16 05 04.21	−0.86
δ Ophiuchi	E.	11	+0.09	+0.06	+0.22	+0.01	−0.01	08 10.29	08 10.05	−0.52
ϵ Ophiuchi	E.	11	+0.09	+0.06	+0.22	+0.01	−0.01	12 05.20	12 05.01	−0.47
γ Herculis	E.	11	+0.09	+0.08	+0.23	0.00	−0.02	16 43.62	16 43.41	−0.52
ω Herculis	E.	11	+0.09	+0.08	+0.23	0.00	−0.02	19 58.89	19 58 64	−0.56
λ Ophiuchi	E.	11	+0.09	+0.06	+0.22	+0.01	−0.02	24 58.40	24 58.08	−0.60
σ Herculis	E.	11	+0.07	+0.09	+0.30	0.00	−0.02	30 19.21	30 19.07	−0.53
ζ Herculis	E.	11	+0.07	+0.08	+0.26	0.00	−0.02	36 51.23	36 51.11	−0.46
η Herculis	E.	11	+0.07	+0.08	+0.29	0.00	−0.02	38 52.24	38 52.05	−0.56
κ Ophiuchi	E.	11	+0.07	+0.05	+0.22	0.00	−0.02	52 05.62	52 05.35	−0.54
ϵ Ursæ Min	E.	11	+0.07	+0.37	+1.64	−0.04	−0.02	58 16.00	58 17.3	−0.7

Adopted collimation $=+0^s.130$ Clamp East. (See Table 12.)

OBSERVATION-EQUATIONS.

Epoch $14^h.6$ clock time. $\Delta t=+0^s.00+\Delta\theta$.

	Weight.		*Weight.*
$+0.68a-3.1\rho+\Delta\theta+0.54=v=+0.09$	1	$+0.12a+1.0\rho+\Delta\theta+0.42=v=-0.08$	1
$-0.16\ \ -2.9\ +\ \ +0.60=v=+0.14$	0.65	$+0.59\ \ +1.0\ +\ \ +0.37=v=-0.12$	1
$+0.47\ \ -2.9\ +\ \ +0.55=v=+0.09$	1	$+0.47\ \ +1.1\ +\ \ +0.48=v=-0.02$	1
$-2.67\ \ -2.6\ +\ \ +0.9\ =v=+0.4$	0.11	$+0.71\ \ +1.1\ +\ \ +0.52=v=+0.03$	1
$+3.70\ \ -2.4\ +\ \ +0.7\ =v=+0.2$	0.13	$-2.84\ \ +1.2\ +\ \ +0.3\ =v=-0.2$	0.1
$+0.38\ \ -2.2\ +\ \ +0.40=v=-0.07$	1	$-0.07\ \ +1.5\ +\ \ +0.86=v=+0.36$	0.7
$+0.42\ \ -2.1\ +\ \ +0.47=v=\ \ 0.00$	1	$+0.72\ \ +1.5\ +\ \ +0.52=v=+0.03$	1
$+0.68\ \ -2.0\ +\ \ +0.41=v=-0.05$	1	$+0.73\ \ +1.6\ +\ \ +0.47=v=-0.02$	1
$+0.62\ \ -1.8\ +\ \ +0.50=v=+0.04$	1	$+0.42\ \ +1.7\ +\ \ +0.52=v=+0.01$	1
$+0.52\ \ -1.7\ +\ \ +0.47=v=\ \ 0.00$	1	$+0.48\ \ +1.7\ +\ \ +0.56=v=+0.05$	1
$+0.74\ \ -1.5\ +\ \ +0.46=v=-0.01$	1	$+0.64\ \ +1.8\ +\ \ +0.60=v=+0.10$	1
$+32.60\ \ -1.4\ +\ \ +2.1\ =v=+1.9$	0.001	$-0.01\ \ +1.9\ +\ \ +0.53=v=+0.02$	1
$+0.43\ \ -0.9\ +\ \ +0.34=v=-0.14$	1	$+0.21\ \ +2.0\ +\ \ +0.46=v=-0.05$	1
$+0.42\ \ -0.8\ +\ \ +0.29=v=-0.19$	1	$+0.07\ \ +2.0\ +\ \ +0.56=v=+0.05$	1
		$+0.55\ \ +2.3\ +\ \ +0.54=v=+0.03$	1
$+0.04a+1.0\rho+\Delta\theta+0.40=v=-0.10$	1	$4.74\ \ +2.4\ +\ \ +0.7\ =v=+0.2$	0.04

NORMAL EQUATIONS.

$$+11.66a-2.62\rho+10.69\Delta\theta+5.20=0$$
$$-2.62\ \ +86.16\ \ +1.49\ \ +1.54=0$$
$$+10.69\ \ +1.49\ \ +25.73\ \ +12.62=0$$

Results.

$\Delta\theta=-0^s.493$

$\rho=-0^s.009$

$a=+0^s.009$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 14.—TIME DETERMINATION.

[Cambridge, Mass., May 23, 1881. O. B. Wheeler, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit $=t$	Right ascension $=\alpha$	$\alpha-t_c$
			s.	*s.*	*s.*	*s.*	*s.*	*h. m. s.*	*h. m. s.*	*s.*
χ Ursæ Maj	E.	11	—0.14	—0.22	+0.34	+0.01	+0.21	11 39 56.49	11 39 48.59	—8.02
β Leonis	E.	11	—0.14	—0.13	+0.23	—0.02	+0.21	43 10.13	43 02.00	—8.23
4 Hev. Dracon	E.	12	—0.14	—0.57	+0.56	+0.10	+0.18	12 06 48.96	12 06 40.8	—8.1
Br. 6, L. C	E.	10	—0.14	+0.29	—0.48	—0.14	+0.17	09 39.11	0 09 30.1	—8.8
6 Canum Ven	E.	11	—0.14	—0.18	+0.29	0.00	+0.17	20 10.02	12 20 02.03	—8.10
20 Comæ	E.	11	—0.14	—0.14	+0.24	—0.01	+0.16	23 55.46	23 47.41	—8.15
24 Comæ seq	E.	11	—0.14	—0.14	+0.12	—0.02	+0.15	29 20.63	29 12.49	—8.12
γ Virginis med	E.	11	—0.14	—0.10	+0.22	—0.03	+0.14	35 48.77	35 40.78	—8.11
α Ursæ Min., L. C	W.	5	—0.09	+2.60	+6.31	—1.21	+0.10	13 14 44.96	1 14 48.4	—5.5
α Ursæ Min., L. C	E.	7	—0.20	+5.58	—4.92	—1.21	+0.10	14 54.99	1 14 48.4	—7.3
τ Bootis	E.	11	—0.14	—0.13	+0.23	—0.02	+0.06	41 47.67	13 41 39.53	—8.24
η Ursæ Maj	E.	10	—0.14	—0.21	+0.18	+0.01	+0.06	43 02.58	42 54.23	—8.32
η Bootis	E.	11	—0.14	—0 14	+0.24	—0.02	+0.06	49 12.43	49 04.23	—8.30
τ Virginis	E.	11	—0.14	—0.11	+0.22	—0.02	+0.05	55 46 95	55 38.68	—8.38
d Bootis	W.	11	—0.07	—0.08	—0.28	—0.01	+0.04	14 05 10.00	14 05 01.44	—8.20
κ Virginis	W.	11	—0.07	—0.04	—0.26	—0.03	+0.04	06 44.98	06 36 37	—8.31
4 Ursæ Min	W.	11	—0.07	—0.26	—1.23	+0.10	+0.03	09 34.57	09 24.7	—8.4
λ Bootis	W.	11	—0.07	—0.09	—0.21	0.00	+0.03	12 03.29	11 54.74	—8.25
ρ Bootis	W.	11	—0.07	—0.07	—0.30	—0.01	+0.01	26 53.82	26 45.25	—8.20
π Bootis pr	W.	11	—0.07	—0.06	—0.27	—0.02	+0.00	35 19.90	35 11.26	—8.31
ϕ Herculis	W.	11	+0.02	+0.03	—0.36	0.00	—0.11	16 05 13.22	16 05 04.29	—8.60
δ Ophiuchi	W.	11	+0.02	+0.01	—0.25	—0.03	—0.11	08 18.84	08 10.17	—8.43
ε Ophiuchi	W.	10	+0.02	+0.01	—0.15	—0.03	—0.12	12 13.64	12 05.12	—8.38
γ Herculis	W.	11	+0,02	+0.02	—0.27	—0.02	—0.12	16 52.23	16 43.52	—8.46
η Ursæ Min	W.	10	+0.02	+0.07	—0.60	+0.08	—0.13	21 12.95	21 03.9	—8.5
β Herculis	W.	11	+0.02	+0.02	—0.27	—0 01	—9.13	25 18.18	25 09.52	—8.41
σ Herculis	W.	11	+0.02	+0.02	—0.35	0.00	—0 14	30 27.94	30 19.19	—8.42
ζ Herculis	W.	11	+0.02	+0.02	—0.30	—0.01	—0.14	36 59.91	36 51.23	—8.40
κ Ophiuchi	W.	11	+0.02	+0.02	—0.26	—0.02	—0.17	52 14.32	52 05.50	—8.58

Adopted collimation $=+0^s.130$ Clamp East. (See Table 12.)

OBSERVATION-EQUATIONS.

Epoch $14^h.6$ clock time. $\Delta t=-8^s.00+\Delta\theta$.

	Weight.
$-0.16a-2.9\rho+\Delta\theta+0.02=v=-0.07$	0.65
$+0.47\quad -2.9\quad +\quad +0.23=v=+0.11$	1
$-2.89\quad -2.5\quad +\quad +0.1\ =v=+0.1$	0.1
$+3.70\quad -2.4\quad +\quad +0.8\ =v=+0.5$	0.13
$+0.06\quad -2.3\quad +\quad +0.10=v=-0.04$	1
$+0.38\quad -2.2\quad +\quad +0.15=v=-0.01$	1
$+0.42\quad -2.1\quad +\quad +0.12=v=-0.06$	1
$+0.68\quad -2.0\quad +\quad +0.11=v=-0.09$	1
$+32.59\quad -1.4\quad +\quad -1.6\ =v=-3.0$	0.001
$+0.43\quad -0.9\quad +\quad +0.24=v=-0.03$	1
$-0.21\quad -0.9\quad +\quad +0.32=v=+0.08$	0.63
$+0.42\quad -0.8\quad +\quad +0.30=v=+0.03$	1
$+0.65\quad -0.7\quad +\quad +0.38=v=+0.10$	1
$+0.32\quad -0.5\quad +\quad +0.20=v=-0.08$	1
$+0.80\quad -0.5\quad +\quad +0.31=v=+0.01$	1

	Weight.
$-2.83a-0.4\rho+\Delta\theta+0.4\ =v=+0.2$	0.1
$-0.10\quad -0.4\quad +\quad +0.25=v=-0.03$	0.7
$+0.23\quad -0.2\quad +\quad +0.20=v=-0.11$	1
$+0.45\quad -0.0\quad +\quad +0.31=v=-0.02$	1
$-0.07a+1.5\rho+\Delta\theta+0.60=v=+0.18$	0.7
$+0.72\quad +1.5\quad +\quad +0.43=v=-0.02$	1
$+0.73\quad +1.6\quad +\quad +0.38=v=-0.08$	1
$+0.42\quad +1.7\quad +\quad +0.46=v=+0.01$	1
$-2.29\quad +1.8\quad +\quad +0.5\ =v=+0.2$	0.13
$+0.38\quad +1.8\quad +\quad +0.41=v=-0.04$	1
$-0.01\quad +1.9\quad +\quad +0.42=v=-0.03$	1
$+0.22\quad +2.0\quad +\quad +0.40=v=-0.06$	1
$+0.55\quad +2.3\quad +\quad +0.58=v=+0.08$	1

NORMAL EQUATIONS.

$$+9.70a-1.59\rho+7.62\Delta\theta+2.58=0$$
$$-1.59\quad +62.76\quad -4.36\quad +3.16=0$$
$$+7.62\quad -4.36\quad +22.14\quad +6.76=0$$

RESULTS.

$\Delta\theta=-0^s.307$

$\rho=-0^s.072$

$a=-0^s.037$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 15.—TIME DETERMINATION.

[Cambridge, Mass., May 24, 1881. O. B. Wheeler, Observor.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit $=t$	Right ascension $=\alpha$	$\alpha-t_c$
			s.	s	s.	s	s.	h. m. s.	h. m. s.	s.
ο Virginis	E.	11	−0.13	−0.11	+0.22	−0.02	+0.14	11 59 20.82	11 59 11.58	−9.35
4 Hev. Dracon	E.	11	−0.13	−0.51	+1.09	+0.08	+0.13	12 06 49.61	12 06 40.76	−9.4
Br. 6, L. C	E.	7	−0.13	+0.26	−0.48	−0.10	+0.13	09 40.30	0 09 30.23	−9.8
20 Comæ	E.	11	−0.13	−0.13	+0.24	−0.01	+0.12	23 56.55	12 23 47.40	−9.26
8 Canum Ven	W.	11	−0.08	−0.11	−0.34	0.00	+0.12	28 18.07	28 08.27	−9.35
γ Virginis med	W.	11	−0.08	−0.06	−0.25	−0.02	+0.11	35 50.42	35 40.77	−9.34
δ Virginis	W.	11	−0.08	−0.06	−0.25	−0.02	+0.10	49 49.29	40 39.51	−9.47
θ Virginis	W.	11	−0.08	−0.05	−0.25	−0.02	+0.09	13 04 00.21	13 03 50.43	−9.48
α Ursæ Min., L C	W.	7	−0.12	+3.35	+6.31	−0.91	+0.09	14 51.70	1 14 49.4	−12.0
α Ursæ Min., L. C	E.	7	−0.20	+5.58	−4.92	−0.91	+0.09	15 04.10	1 14 49.4	−15.4
τ Bootis	E.	11	−0.12	−0.12	+0.23	−0.01	+0.07	41 48.82	13 41 39.53	−9.40
η Bootis	E.	11	−0.12	−0.12	+0.24	−0.01	+0.06	49 13.45	49 04.23	−9.34
4 Ursæ Min	E.	11	−0.12	−0.45	+1.08	+0.08	+0.05	14 09 33.74	14 09 24.62	−9.7
φ Virginis	E.	11	−0.12	−0.08	+0.22	−0.02	+0.04	22 17.00	22 07.66	−9.48
ρ Bootis	E.	11	−0.12	−0.14	+0.26	−0.01	+0.04	26 54.62	26 45.25	−9.49
π Bootis, pr	E.	11	−0.12	−0.11	+0.23	−0.01	+0.03	35 20.63	35 11.26	−9.49
47 Hev. Ceph., L. C	E.	11	−0.12	+0.31	−1.16	−0.12	+0.02	50 29.71	2 50 19.4	−9.5
Gr. 750, L. C	E.	5	−0.10	+0.77	−1.37	−0.27	−0.02	15 59 47.86	3 59 36.7	−10.6
ε Ophiuchi	W.	11	−0.01	−0.01	−0.25	−0.02	−0.03	16 12 14.95	16 12 05.13	−9.56
γ Herculis	W.	11	−0.01	−0.01	−0.27	−0.01	−0.04	16 53.39	16 43.54	−9.57
ω Herculis	W.	11	−0.01	−0.01	−0.26	−0.01	−0.04	20 08.64	19 58.77	−9.60
β Herculis	W.	11	−0.01	−0.01	−0.27	−0.01	−0.04	25 19.44	25 09.54	−9.62
ζ Herculis	W.	11	−0.01	−0.01	−0.30	−0.01	−0.05	37 01.08	36 51.24	−9.53
η Herculis	W.	11	−0.01	−0.01	−0.33	0.00	−0.05	39 02.02	38 52.18	−9.50
49 Herculis	W.	11	−0.01	−0.01	−0.26	−0.02	−0.06	46 53.03	46 43.14	−9.62
ε Ursæ Min	W.	9	0.00	0.00	−1.08	+0.13	−0.06	58 28.40	58 17.6	−9.7
ε Ursæ Min	E.	6	+0.01	+0.07	+0.84	+0.13	−0.06	58 26.62	58 17.6	−9.9
κ Herculis	E.	11	−0.02	−0.03	+0.34	0.00	−0.08	17 23 47.49	17 23 38.03	−9.77
β Draconis	E.	11	−0.02	−0.03	+0.18	+0.01	−0.08	27 57.25	27 47.75	−9.65
α Ophiuchi	E.	11	−0.02	−0.02	+0.23	−0.01	−0.08	29 37.29	29 27.96	−9.54
β Ophiuchi	E.	11	−0.02	−0.02	+0.22	−0.02	−0.09	37 48.39	37 39.03	−9.56
μ Herculis	E.	11	−0.02	−0.02	+0.25	−0.01	−0.09	42 00.56	41 51.23	−9.56
θ Herculis	E.	11	−0.02	−0.02	+0.28	0.00	−0.10	52 22.54	52 13.33	−9.47
67 Ophiuchi	E.	11	−0.02	−0.02	+0.22	−0.02	−0.10	54 53.86	54 44.56	−9.50
δ Ursæ Min	E.	8	−0.03	−0.32	+1.92	+0.33	−0.11	18 10 56.97	18 10 49.16	−9.4
δ Ursæ Min	W.	6	−0.07	−0.79	−2.47	+0.33	−0.11	11 01.80	10 49.16	−9.4

Adopted collimation $=+0^s.130$ Clamp East. (See Table 12.)

OBSERVATION-EQUATIONS.

Epoch $15^h.4$ clock time. $\Delta t=-9^s.5+\Delta\theta$.

	Weight.		*Weight.*
$+0.55a-3.4\rho+\Delta\theta-0.15=v=-0.02$	1	$+9.55a+0.6\rho+\Delta\theta+1.1=v=+0.8$	0.01
$-2.89\quad-3.3\;+\quad-0.1=v=+0.1$	0.1	$+0.73\quad+0.8\;+\quad+0.06=v=+0.02$	1
$+3.70\quad-3.2\;+\quad+0.3=v=+0.4$	0.13	$+0.42\quad+0.9\;+\quad+0.07=v=+0.03$	1
$+0.38\quad-3.0\;+\quad-0.24=v=-0.12$	1	$+0.49\quad+0.9\;+\quad+0.10=v=+0.06$	1
$+0.01\quad-2.9\;+\quad-0.15=v=-0.02$	1	$+0.38\quad+1.0\;+\quad+0.12=v=+0.08$	1
$+0.68\quad-2.8\;+\quad-0.16=v=-0.06$	1	$+0.22\quad+1.2\;+\quad+0.03=v=-0.02$	1
$+0.62\quad-2.6\;+\quad-0.03=v=+0.06$	1	$+0.07\quad+1.3\;+\quad0.00=v=-0.04$	1
$+0.74\quad-2.3\;+\quad-0.02=v=+0.06$	1	$+0.47\quad+1.4\;+\quad+0.12=v=+0.05$	1
$+32.59\quad-2.2\;+\quad+4.1=v=+3.3$	0.001	$-4.74\quad+1.6\;+\quad+6.3=v=+0.4$	0.04
$+0.43\quad-1.7\;+\quad-0.10=v=-0.03$	1	$-0.16\quad+2.0\;+\quad+0.27=v=+0.20$	0.65
$+0.42\quad-1.6\;+\quad-0.16=v=-0.10$	1	$-0.28\quad+2.1\;+\quad+0.15=v=+0.09$	0.6
$-2.83\quad-1.2\;+\quad+0.2=v=+0.4$	0.1	$+0.51\quad+2.1\;+\quad+0.04=v=-0.04$	1
$+0.70\quad-1.0\;+\quad-0.02=v=+0.01$	1	$+0.62\quad+2.2\;+\quad+0.06=v=-0.04$	1
$+0.23\quad-1.0\;+\quad-0.01=v=+0.03$	1	$+0.28\quad+2.3\;+\quad+0.06=v=-0.03$	1
$+0.45\quad-0.8\;+\quad-0.01=v=+0.02$	1	$+0.11\quad+2.5\;+\quad-0.03=v=-0.12$	1
$+4.46\quad-0.6\;+\quad0.00=v=-0.1$	0.09	$+0.64\quad+2.5\;+\quad0.00=v=-0.11$	1
		$-11.80\quad+2.8\;+\quad-0.1=v=+0.1$	0.01

NORMAL EQUATIONS.

$$+14.98a-\ 4.89\rho+10.02\Delta\theta+0.12=0$$
$$-\ 4.89\ +100.36\ -\ 2.26\ +3.92=0$$
$$+10.02\ -\ 2.26\ +24.73\ -0.06=0$$

RESULTS.

$\Delta\theta=+0^s.010$
$\rho=-0^s.040$
$a=-0^s.028$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 16.—TIME DETERMINATION.

[Cambridge, Mass., May 26, 1881. O. B. Wheeler, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit$=t$	Right ascension$=a$	$a-t_c$
			s.	s.	s.	s	s	h. m. s.	h. m. s.	s.
o Virginis	W.	11	—0. 13	—0. 11	—0. 26	—0. 04	+0. 19	11 59 23. 56	11 59 11. 55	—11. 64
4 Hev. Dracon	W.	9	—0. 13	—0. 52	—0. 72	+0. 20	+0. 18	12 06 53. 03	12 06 40. 6	—11. 2
4 Hev. Dracon	E.	4	—0. 14	—0. 57	+0. 56	—0. 20	+0. 18	06 52. 19	06 40. 6	—11. 6
η Virginis	E.	11	—0. 13	—0. 10	+0. 22	—0. 05	+0. 17	14 03. 34	13 51. 86	—11. 00
6 Canum Ven	E.	11	—0. 13	—0. 17	+0. 29	0. 00	+0. 17	20 13. 44	20 01. 99	—11. 57
20 Comæ	E.	12	—0. 13	—0. 13	+0. 12	—0. 03	+0. 16	23 59. 04	23 47. 38	—11. 65
24 Comæ seq	E.	11	—0. 13	—0. 13	+0. 24	—0. 03	+0. 16	29 24. 05	29 12. 46	—11. 70
γ Virginis med	E.	11	—0. 13	—0. 09	+0. 22	—0. 05	+0. 15	35 52. 21	35 40. 76	—11. 58
ε Virginis	E.	11	—0. 13	—0. 11	+0. 23	—0. 04	+0. 13	56 29. 59	56 18. 15	—11. 56
α Ursæ Min., L. C	E.	7	—0. 16	+4. 46	—4. 93	—2. 25	+0. 11	13 15 08. 01	1 14 50. 9	—16. 7
α Ursæ Min., L. C	W.	7	—0. 18	+5. 20	+6. 31	—2. 25	+0. 11	14 57. 13	14 50. 9	—17. 8
τ Bootis	W.	11	—0. 13	—0. 12	—0. 27	—0. 03	+0. 08	41 51. 55	13 41 39. 52	—11. 64
η Bootis	W.	11	—0. 07	—0. 06	—0. 27	—0. 03	+0. 07	49 16. 29	49 04. 22	—11. 74
d Bootis	W.	11	—0. 05	—0. 05	—0. 28	—0. 02	+0. 05	14 05 13. 51	14 05 01. 44	—11. 74
4 Ursæ Min	W.	11	—0. 05	—0. 20	—1. 24	+0. 19	+0. 05	09 37. 66	09 24. 5	—11. 7
φ Virginis	W.	11	—0. 05	—0. 03	—0. 25	—0. 05	+0. 04	22 19. 78	22 07. 66	—11. 84
π Bootis, pr	W.	11	—0. 05	—0. 05	—0. 27	—0. 03	+0. 03	35 23. 40	35 11. 26	—11. 82
47 Hev. Ceph., L. C	W.	11	—0. 03	+0. 07	+1. 33	—0. 31	+0. 01	50 29. 71	2 50 19. 5	—11. 6
Gr. 750, L. C	W.	8	—0. 04	+0. 29	+1. 76	—0. 66	—0. 06	15 59 47. 50	3 59 36. 8	—12. 8
Gr. 750, L. C	E.	5	—0. 04	+0. 29	—1. 37	—0. 66	—0. 06	15 59 50. 38	59 36. 8	—12. 5
δ Ophiuchi	E.	11	—0. 08	—0. 06	+0. 22	—0. 05	—0. 07	16 08 21. 85	16 08 10. 20	—11. 81
ε Ophiuchi	E.	11	—0. 08	—0. 06	+0. 22	—0. 05	—0. 07	12 16. 84	12 05. 15	—11. 85
γ Herculis	E.	11	—0. 08	—0. 08	+0. 24	—0. 03	—0. 08	16 55. 26	16 43. 54	—11. 88
ω Herculis	E.	11	—0. 08	—0. 07	+0. 12	—0. 03	—0. 08	20 10. 63	19 58. 79	—11. 89
β Herculis	E.	11	—0. 08	—0. 08	+0. 24	—0. 03	—0. 09	25 21. 27	25 09. 55	—11. 88
σ Herculis	E.	11	—0. 08	—0. 11	+0. 15	0. 00	—0. 09	30 30. 95	30 19. 21	—11. 78
ζ Herculis	W.	11	—0. 03	—0. 03	—0. 30	—0. 02	—0. 10	37 03. 37	36 51. 26	—11. 78
Herculis	W.	11	—0. 03	—0. 04	—0. 33	0. 00	—0. 11	39 04. 52	38 52. 19	—11. 96
κ Ophiuchi	W.	11	—0. 03	—0. 02	—0. 26	—0. 04	—0. 12	52 17. 70	52 05. 54	—11. 88
ε Ursæ Min	W.	11	—0. 03	—0. 14	—1. 08	+0. 33	—0. 12	58 30. 50	58 17. 6	—11. 7
19 Hev. Camelop., L. C	W.	11	—0. 03	+0. 08	+1. 34	—0. 31	—0. 13	17 03 09. 57	5 02 58. 4	—12. 6
α Herculis	W.	11	—0. 03	—0. 02	—0. 26	—0. 03	—0. 14	09 28. 89	17 09 16. 63	—11. 98
π Herculis	W.	12	—0. 03	—0. 04	—0. 18	—0. 01	—0. 14	11 09. 38	10 57. 30	—11. 86

Adopted collimation$=+0^s.130$ Clamp East. (See Table 12.)

OBSERVATION-EQUATIONS.

Epoch $15^h.0$ clock time. $\Delta t=-11^s.7+\Delta\theta$.

	Weight.		Weight.
$+0.55a-3.0\rho+\Delta\theta-0.06=v=+0.04$	1	$+4.46a-0.2\rho+\Delta\theta-0.1=v=-0.5$	0.09
$-2.89\quad -2.9\quad +\quad -0.3=v=\ 0.0$	0.1	$+9.55a+1.0\rho+\Delta\theta+1.0=v=+0.2$	0.01
$+0.67\quad -2.8\quad +\quad -0.10=v=-0.03$	1	$+0.72\quad +1.1\quad +\quad +0.11=v=-0.06$	1
$+0.06\quad -2.7\quad +\quad -0.13=v=-0.01$	1	$+0.73\quad +1.2\quad +\quad +0.15=v=-0.02$	1
$+0.38\quad -2.6\quad +\quad -0.05=v=+0.03$	1	$+0.41\quad +1.3\quad +\quad +0.18=v=+0.02$	1
$+0.42\quad -2.5\quad +\quad 0.00=v=+0.08$	1	$+0.49\quad +1.3\quad +\quad +0.19=v=+0.03$	1
$+0.68\quad -2.4\quad +\quad -0.12=v=-0.07$	1	$+0.39\quad +1.4\quad +\quad +0.18=v=+0.01$	1
$+0.52\quad -2.1\quad +\quad -0.14=v=-0.10$	1	$-0.01\quad +1.5\quad +\quad +0.08=v=-0.06$	1
$+32.58\quad -1.7\quad +\quad +5.5=v=+3.3$	0.001	$+0.22\quad +1.6\quad +\quad +0.08=v=-0.09$	1
$+0.43\quad -1.3\quad +\quad -0.06=v=-0.06$	1	$+0.07\quad +1.7\quad +\quad +0.26=v=+0.10$	1
$+0.42\quad -1.2\quad +\quad +0.04=v=+0.03$	1	$+0.55\quad +1.9\quad +\quad +0.18=v=-0.03$	1
$+0.32\quad -0.9\quad +\quad +0.04=v=+0.02$	1	$-4.74\quad +2.0\quad +\quad +0.0=v=+0.2$	0.04
$-2.83\quad -0.8\quad +\quad 0.0=v=+0.2$	0.1	$+4.51\quad +2.1\quad +\quad +0.9=v=+0.4$	0.09
$+0.70\quad -0.6\quad +\quad +0.14=v=+0.08$	1	$+0.48\quad +2.2\quad +\quad +0.28=v=+0.06$	1
$+0.45\quad -0.4\quad +\quad +0.12=v=+0.07$	1	$+0.12\quad +2.2\quad +\quad +0.16=v=-0.04$	1

NORMAL EQUATIONS.

$$+13.28a-2.70\rho+9.95\Delta\theta+1.27=0$$
$$-2.70\quad +82.22\quad -5.21\quad +4.68=0$$
$$+9.95\quad -5.21\quad +23.43\quad +1.59=0$$

RESULTS.

$\Delta\theta=-0^s.052$

$\rho=-0^s.062$

$a=-0^s.069$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 17.—TIME DETERMINATION.

[Cambridge, Mass., June 4, 1881. O. B. Wheeler, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit$=t$	Right ascension$=\alpha$	$\alpha-t_c$
			s.	s.	s.	s.	s.	h. m. s.	h. m. s.	s.
α Ursæ Min., L C	E.	8	−0.09	+2.60	−4.92	−2.70	+0.08	13 15 20.88	1 14 58.2	−20.4
α Ursæ Min., L. C.	W.	7	−0.03	+0.74	+6.30	−2.70	+0.08	15 08.77	14 58 2	−17.6
ρ Bootis	W.	11	0.00	0.00	−0.30	−0.02	+0.04	14 27 01.50	14 26 45.19	−16.01
π Bootis, pr	W.	11	0.00	0.00	−0.27	−0.04	+0.04	35 27.68	35 11.24	−10.17
μ Virginis	W.	11	0.00	0.00	−0.25	−0.06	+0.04	37 07.14	36 50.84	−16.05
109 Virginis	W.	11	0.00	0.00	−0.25	−0.05	+0.04	40 33.62	40 17.36	−16.01
P. XIV, 221	W.	11	0.00	0.00	−0.26	−0.04	+0.03	50 55.83	50 39.55	−10.02
β Bootis	E.	11	+0.03	+0.03	+0.29	0.00	+0.03	57 46.63	57 30.96	−15.99
ψ Bootis	E.	11	+0.03	+0.03	+0.25	−0.02	+0.03	59 39.71	59 24.00	−15 99
δ Bootis	E.	11	+0.03	+0.03	+0.27	−0.01	+0 02	15 11 01.10	15 10 45.53	−15.87
μ Bootis	E.	11	+0.03	+0.03	+0.28	−0.01	+0.02	20 18.45	20 02.85	−15.91
β Coron. Bor	E.	11	+0.03	+0.03	+0.26	−0.02	+0.02	23 14.26	22 58.62	−15.93
ι Coron. Bor	E.	11	+0.03	+0.03	+0.25	−0.02	+0.01	29 57.91	29 42.26	−15.93
ζ Coron. Bor. seq	E.	11	+0.03	+0.03	+0.28	−0.01	+0.01	35 12.58	34 56.99	−15.90
γ Coron. bor	E.	11	+0.03	+0.03	+0.25	−0.02	+0.01	38 03.63	37 48.02	−15.89
Gr. 750, L. C	E.	9	+0.03	−0.19	−1.37	−0.79	−0.00	15 59 56.60	3 59 37.9	−17.1
γ Herculis	E.	11	+0.03	+0.03	+0.25	−0.03	−0.01	16 16 59.44	16 16 43.61	−16.11
η Ursæ Min	E.	11	+0.03	+0.09	+0.47	+0.19	−0.01	21 19.12	21 03.8	−15.9
ζ Herculis	E.	11	+0.03	+0.03	+0.26	−0.02	−0.02	37 06.95	36 51.31	−15.93
η Herculis	E.	11	+0.03	+0.04	+0.29	−0.01	−0.02	39 07.80	38 52.26	−15.87
49 Herculis	E.	11	+0.03	+0.03	+0.23	−0.04	−0.02	46 59.09	46 43.25	−16.10
κ Ophiuchi	E.	11	+0.03	+0.03	+0.22	−0.05	−0.03	52 21.46	52 05.62	−16.09
ε Ursæ Min	E.	9	+0.08	+0.45	+0.84	+0.39	−0.03	58 32.46	58 17.5	−16.2
ε Ursæ Min	W.	6	+0.08	+0.45	−1.08	+0.39	−0.03	58 34.75	58 17.5	−16.6
α Herculis	W.	11	+0.05	+0.05	−0.26	−0.04	−0.03	17 09 33.08	17 09 16.74	−16.13
δ Herculis	W.	11	+0.05	+0.05	−0.16	−0.03	−0.03	10 28.10	10 11.96	−16.03
α Ophiuchi	W.	11	+0.05	+0.04	−0 26	−0.04	−0.04	29 44.51	29 28.10	−16.19
β Ophiuchi	W.	11	+0.05	+0.04	−0.25	−0.05	−0.05	37 55.49	37 39.19	−16.09
μ Herculis	W.	11	+0.05	+0.06	−0.29	−0.02	−0.05	42 07.65	41 51.38	−16.04
θ Herculis	W.	11	+0.05	+0.06	−0.32	−0.01	−0.06	52 29.75	52 13.49	−16.00
67 Ophiuchi	W.	11	+0.05	+0.04	−0.25	−0.05	−0.06	55 01.07	54 44.73	−16.13
δ Ursæ Min	W.	7	+0.04	+0.48	−2.47	+0.98	−0.06	18 11 07.56	18 10 50.1	−15.5
δ Ursæ Min	E.	8	+0.01	+0.16	+1.92	+0.98	−0.00	11 03.98	10 50.1	−16.0

Adopted collimation$=+0^s.130$ Clamp East. (See Table 12.)

OBSERVATION-EQUATIONS.

Epoch $16^h.0$ clock time. $\Delta t=-16^s.0+\Delta\theta$.

Equation	Weight.	Equation	Weight.
$+32.57a-2.7\rho+\Delta\theta+3.0=v=+0.4$	0.001	$+0.42a+0.3\rho+\Delta\theta+0.11=v=+0.08$	1
$+0.23\ -1.5\ +\ +0.01=v=+0.04$	1	$-2.29\ +0.4\ +\ -0.1=v=+0.1$	0.13
$+0.45\ -1.4\ +\ +0.17=v=+0.18$	1	$+0.22\ +0.6\ +\ -0.07=v=-0.10$	1
$+0.74\ -1.4\ +\ +0.05=v=+0.04$	1	$+0.08\ +0.7\ +\ -0.13=v=-0.15$	1
$+0.64\ -1.3\ +\ +0.01=v=+0.01$	1	$+0.47\ +0.8\ +\ +0.10=v=+0.05$	1
$+0.48\ -1.2\ +\ +0.02=v=+0.02$	1	$+0.55\ +0.9\ +\ +0.09=v=+0.02$	1
$+0.04\ -1.0\ +\ -0.01=v=+0.03$	1	$-4.74\ +1.0\ +\ +0.4=v=+0.8$	0.04
$+0.29\ -1.0\ +\ -0.01=v=+0.01$	1	$+0.48\ +1.2\ +\ +0.13=v=+0.07$	1
$+0.18\ -0.8\ +\ -0.13=v=-0.11$	1	$+0.33\ +1.2\ +\ +0.03=v=-0.02$	1
$+0.10\ -0.7\ +\ -0.09=v=-0.07$	1	$+0.51\ +1.5\ +\ +0.19=v=+0.12$	1
$+0.26\ -0.6\ +\ -0.07=v=-0.06$	1	$+0.62\ +1.6\ +\ +0.09=v=\ 0.00$	1
$+0.30\ -0.5\ +\ -0.07=v=-0.07$	1	$+0.28\ +1.7\ +\ +0.04=v=-0.02$	1
$+0.12\ -0.4\ +\ -0.10=v=-0.09$	1	$+0.11\ +1.9\ +\ 0.00=v=-0.06$	1
$+0.30\ -0.4\ +\ -0.11=v=-0.11$	1	$+0.64\ +1.9\ +\ +0.13=v=+0.03$	1
$+9.55a+0.0\rho+\Delta\theta+1.1=v=+0.3$	0.01	$-11.80\ +2.2\ +\ -0.2=v=+0.7$	0.01

NORMAL EQUATIONS.

$$+9.00a+0.74\rho+8.36\Delta\theta+0.65=0$$
$$+0.74\ +33.67\ +\ 2.21\ \ +1.00=0$$
$$+8.36\ +\ 2.21\ +25.19\ \ +0.40=0$$

RESULTS.

$\Delta\theta=+0^s.014$

$\rho=-0^s.029$

$a=-0^s.083$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 18.—TIME DETERMINATION.

[Cambridge, Mass., June 11, 1881. O. B. Wheeler, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit $=t$	Right ascension $=\alpha$	$\alpha-t_c$
			s.	s.	s.	s.	s.	h. m. s.	h. m. s.	s.
α Ursæ Min., L. C	E.	8	0.00	0.00	—4.92	—4.62	—0.02	13 15 31.79	1 15 04.8	—22.1
α Ursæ Min., L. C	W.	7	—0.02	+0.56	+6.30	—4.62	—0.02	15 19.44	15 05.8	—21.5
τ Bootis	W.	11	—0.05	—0.04	—0.27	—0.06	—0.02	41 55.52	13 41 39.41	—15.80
η Bootis	W.	11	—0.05	—0.04	—0.27	—0.06	—0.02	49 20.18	49 04.12	—15.75
d Bootis	W.	11	—0.05	—0.05	—0.28	—0.05	—0.02	14 05 17.46	14 05 01.33	—15.80
φ Virginis	W.	11	—0.05	—0.04	—0.25	—0.10	—0.01	22 23.70	22 07.63	—15.78
π Bootis pr	W.	11	—0.05	—0.04	—0.27	—0.06	—0.01	35 27.35	35 11.21	—15.83
μ Virginis	W.	11	—0.05	—0.03	—0.25	—0.11	—0.01	37 06.95	36 50.83	—15.84
109 Virginis	W.	11	—0.05	—0.04	—0.25	—0.09	—0.01	40 33.46	40 17.34	—15.83
47 Hev. Ceph., L. C	W.	6	—0.05	+0.14	+0.73	—0.63	—0.01	50 36.17	2 50 20.8	—16.3
47 Hev. Ceph., L. C	E.	6	—0.03	+0.07	—0.59	—0.63	—0.01	50 37.43	50 20.8	—16.1
ψ Bootis	E.	11	—0.03	—0.03	+0.25	—0.04	—0.01	59 39.49	14 59 23.96	—15.75
48 Hev. Ceph., L. C	E.	11	—0.03	+0.07	—1.00	—0.56	—0.01	15 05 34.55	3 05 17.3	—16.3
3 Serpentis	E.	11	—0.03	—0.02	+0.22	—0.09	—0.01	09 35.43	15 09 19.93	—15.70
μ Bootis	E.	11	—0.03	—0.04	+0.28	—0.01	—0.01	20 18.31	20 02.82	—15.73
β Coron. Bor	E.	8	—0.03	—0.03	+0.13	—0.04	—0.00	23 14.11	22 58.60	—15.61
ν^2 Bootis	E.	11	—0.03	—0.04	+0.30	0.00	—0.00	27 49.89	27 34.55	—15.60
ζ Coron. Bor	E.	11	—0.03	—0.04	+0.28	—0.02	—0.00	35 12.42	34 56.97	—15.69
γ Coron. Bor	E.	11	—0.03	—0.03	+0.25	—0.04	—0.00	38 03.43	37 48.02	—15.63
β_1 Serpentis	E	11	—0.03	—0.03	+0.23	—0.07	—0.00	41 00.69	40 45.11	—15.78
κ Serpentis	E.	11	—0.03	—0.03	+0.12	—0.06	—0.00	43 42.03	43 26.38	—15.74
ζ Ursæ Min	E.	11	—0.04	—0.16	+1.08	+0.40	—0.00	48 38.33	48 23.9	—15.3
ε Ursæ Min	E.	9	—0.01	—0.07	+0.84	+0.67	+0.01	16 58 31.73	16 58 17.3	—15.2
ε Ursæ Min	W.	6	—0.03	—0.15	—1.08	+0.67	+0.01	58 33.87	58 17.3	—15.3
ω Herculis	W.	11	—0.01	—0.01	—0.26	—0.07	+0.01	17 09 32.85	17 09 16.81	—15.77
π Herculis	W.	11	—0.01	—0.02	—0.32	—0.02	+0.01	11 13.53	10 57.45	—15.74
β Draconis	W.	11	—0.01	—0.02	—0.42	+0.04	+0.01	28 04.06	27 47.94	—15.68
α Ophiuchi	W.	11	—0.01	—0.01	—0.26	—0.07	+0.01	29 44.36	29 28.19	—15.90
ι Herculis	E.	11	—0.03	—0.04	+0.32	+0.01	+0.01	36 25.01	36 09.66	—15.63
β Ophiuchi	E.	11	—0.03	—0.02	+0.22	—0.09	+0.01	37 54.85	37 39.29	—15.76
μ Herculis	E.	11	—0.03	—0.03	+0.25	—0.04	+0.01	42 06.92	41 51.48	—15.66
θ Herculis	E.	11	—0.03	—0.04	+0.28	—0.02	+0.02	52 29.10	52 13.59	—15.75
67 Ophiuchi	E.	11	—0.03	—0.02	+0.22	—0.09	+0.02	55 00.29	54 44.85	—15.64
δ Ursæ Min	E.	6	—0.03	—0.32	+1.92	+1.68	+0.02	18 11 02.43	18 10 50.3	—13.7
δ Ursæ Min	W.	2	—0.04	—0.48	—2.47	+1.68	+0.02	11 06.35	10 50.3	—13.1

Adopted collimation = + 0s.130 Clamp East. (See Table 12.)

OBSERVATION-EQUATIONS.

Epoch 16h.0 clock time. $\Delta t=-15^s.7+\Delta\theta$.

	Weight.
$+32.57a-2.7\rho+\Delta\theta+6.1=v=+1.5$	0.001
$+0.43-2.3++0.10=v=+0.03$	1
$+0.42-2.2++0.05=v=-0.02$	1
$+0.33-1.9++0.10=v=+0.04$	1
$+0.70-1.6++0.08=v=-0.02$	1
$+0.45-1.4++0.13=v=+0.07$	1
$+0.74-1.4++0.14=v=+0.03$	1
$+0.64-1.3++0.13=v=+0.04$	1
$+4.46-1.2++0.5=v=-0.1$	0.09
$+0.29-1.0++0.05=v=+0.01$	1
$+3.94-0.9++0.6=v=0.0$	0.11
$+0.60-0.8+0.00=v=-0.09$	1
$+0.10-0.7++0.03=v=+0.02$	1
$+0.26-0.6+-0.09=v=-0.12$	0.8
$+0.02-0.5+-0.10=v=-0.09$	1
$+0.12-0.4+-0.01=v=-0.02$	1

	Weight.
$+0.30a-0.4\rho+\Delta\theta-0.07=v=-0.10$	1
$+0.46-0.3++0.08=v=+0.02$	1
$+0.43-0.3++0.04=v=-0.01$	1
$-2.84-0.2+-0.4=v=0.0$	0.1
$-4.74a+1.0\rho+\Delta\theta-0.5=v=+0.2$	0.04
$+0.48+1.2++0.07=v=+0.02$	1
$+0.12+1.2++0.04=v=+0.04$	1
$-0.29+1.5+-0.02=v=+0.04$	0.6
$+0.51+1.5++0.20=v=+0.15$	1
$-0.09+1.6+-0.07=v=-0.04$	0.7
$+0.62+1.6-+0.06=v=-0.01$	1
$+0.28+1.7+-0.04=v=-0.06$	1
$+0.11+1.9++0.05=v=+0.06$	1
$+0.64+1.9+-0.06=v=-0.12$	1
$-11.80+2.2+-2.3=v=-0.6$	0.005

NORMAL EQUATIONS.

$$+11.52a-4.75\rho+9.11\Delta\theta+1.55=0$$
$$-4.75+46.31-4.14-1.01=0$$
$$+9.11-4.14+24.45+1.00=0$$

RESULTS.

$\Delta\theta=+0^s.013$

$\rho=+0^s.008$

$a=-0^s.142$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 19.—TIME DETERMINATION.

[Detroit, Mich., June 21, 1881. O. B. Wheeler, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit=t	Right ascension=α	$\alpha-t_c$
			s.	*s.*	*s.*	*s.*	*s.*	*h. m. s.*	*h. m. s.*	*s.*
α Ursæ Min., L. C.	W.	8	+0.05	—1.49	+4.36	+2.51	—0.01	13 15 08.95	1 15 13.8	+2.0
α Ursæ Min., L. C.	E.	7	+0.03	—0.74	—2.98	+2.51	—0.01	15 17.55	1 15 13.8	0.0
τ Virginis	E.	11	+0.04	+0.03	+0.18	+0.05	—0.01	55 36.55	13 55 38.54	+1.78
d Bootis	E.	11	+0.05	+0.05	+0.20	+0.02	—0.01	14 04 59.14	14 05 01.22	+1.83
4 Ursæ Min	E.	9	+0.06	+0.26	+0.33	—0.22	—0.01	09 20.58	09 22.6	+1.4
φ Virginis	E.	11	+0.07	+0.05	+0.18	+0.05	—0.01	22 05.53	22 07 57	+1.81
ρ Bootis	E.	11	+0.07	+0.08	+0.21	+0.02	—0.01	26 42.88	26 45.04	+1.87
μ Virginis	E.	11	+0.07	+0.04	+0.18	+0.06	—0.01	36 48.73	36 50.79	+1.84
109 Virginis	E.	11	+0.07	+0.05	+0.18	+0.05	—0.01	40 15.19	40 17.30	+1.88
47 Hev. Ceph., L. C	E.	7	+0.08	—0.20	—0.36	+0.34	—0.01	50 20.36	2 50 21.9	+2.1
47 Hev. Ceph., L. C	W.	7	+0.04	—0.10	+0.53	+0.34	—0.01	50 19.14	2 50 21.9	+2.3
48 Hev. Ceph., L. C	W.	11	+0.03	—0.07	+0.95	+0.30	—0.01	15 05 15.61	3 05 18.3	+1.8
3 Serpentis	W.	11	+0.03	+0.02	—0.21	+0.05	—0.01	09 18.20	15 09 19.89	+1.88
δ Bootis	W.	11	+0.01	+0.01	—0.25	+0.01	—0.01	10 43.61	10 45.42	+2.05
β Coron. Bor	W.	11	+0.06	+0.08	—0.23	+0.02	—0.00	22 56.89	22 58.53	+1.81
ζ Coron. Bor. seq	W.	11	+0.06	+0.08	—0.26	+0.01	—0.00	34 55.44	34 56.88	+1.62
γ Coron. Bor	W.	11	+0.06	+0.07	—0.23	+0.02	—0.00	37 46.23	37 47.98	+1.91
β Serpentis	W.	11	+0.06	+0.06	—0.22	+0.04	—0.00	40 43.33	40 45.09	+1.92
ζ Ursæ Min	W.	11	+0.12	+0.47	—1.02	—0.22	—0.00	48 23.03	48 23.4	+0.9
Gr. 2373	W.	11	+0.05	+0.20	—0.98	—0.21	+0.00	16 35 49.72	16 35 50.4	+1.5
49 Herculis	W.	11	+0.05	+0.05	—0.22	+0.04	+0.00	46 41.55	46 43.37	+1.99
ε Ursæ Min	W.	8	+0.05	+0.28	—0.75	—0.37	+0.00	58 15.54	58 16.8	+1.7
ε Ursæ Min	E.	7	+0.06	+0.37	+0.51	—0.37	+0.00	58 14.66	58 16.8	+1.3
α Herculis	E.	11	+0.06	+0.05	+0.18	+0.04	+0.01	17 09 14.87	17 09 16.88	+1.78
π Herculis	E.	11	+0.06	+0.07	+0.22	+0.01	+0.01	10 55.39	10 57.51	+1.83
χ Herculis	E.	11	+0.06	+0.09	+0.27	—0.01	+0.01	23 36.21	23 38.25	+1.68
β Draconis	E.	11	+0.06	+0.10	+0.29	—0.02	+0.01	27 45.75	27 47.97	+1.83
α Ophiuchi	E.	11	+0.05	+0.04	+0.18	+0.04	+0.01	29 26.18	29 28.29	+1.89
β Ophiuchi	W.	11	+0.01	+0.01	—0.21	+0.05	+0.01	37 37.67	37 39.41	+1.94
μ Herculis	W.	11	+0.02	+0.02	—0.24	+0.02	+0.01	41 49.91	41 51.56	+1.87
θ Herculis	W.	11	+0.02	+0.02	—0.26	+0.01	+0.01	52 12.05	52 13.68	+1.87
67 Ophiuchi	W.	11	+0.02	+0.02	—0.21	+0.05	+0.01	54 43.31	54 44.98	+1.86
72 Ophiuchi	W.	11	+0.02	+0.02	—0.21	+0.04	+0.01	18 01 44.63	18 01 46.16	+1.72
δ Ursæ Min	W.	7	+0.03	+0.32	—1.71	—0.91	+0.01	10 49.74	10 50.1	+1.8
δ Ursæ Min	E.	7	0.00	0.00	+1.17	—0.91	+0.01	10 47.10	10 50.1	+1.8

Adopted collimation=+0ˢ.085 Clamp East. (See Table 12.)

OBSERVATION-EQUATIONS.

Epoch 16ʰ.2 clock time. $\Delta t=+1^s.8+\Delta\theta$.

Equation	Weight.
$+32.57a-2.9\rho+\Delta\theta+0.8=v=+3.3$	0.001
$+0.65\ \ -2.3\ +\ \ +0.02=v=+0.07$	1
$+0.32\ \ -2.1\ +\ \ -0.03=v=-0.01$	1
$-2.84\ \ -2.0\ +\ \ +0.4\ =v=+0.2$	0.1
$+0.70\ \ -1.8\ +\ \ -0.01=v=+0.04$	1
$+0.23\ \ -1.8\ +\ \ -0.07=v=-0.05$	1
$+0.74\ \ -1.6\ +\ \ -0.04=v=+0.02$	1
$+0.64\ \ -1.5\ +\ \ -0.08=v=-0.03$	1
$+4.46\ \ -1.4\ +\ \ -0.4\ =v=-0.1$	0.09
$+3.94\ \ -1.1\ +\ \ 0.0\ =v=+0.3$	0.11
$+0.60\ \ -1.0\ +\ \ -0.08=v=-0.03$	1
$+0.18\ \ -1.0\ +\ \ -0.25=v=-0.24$	1
$+0.26\ \ -0.8\ +\ \ -0.01=v=+0.02$	1
$+0.12\ \ -0.6\ +\ \ +0.18=v=+0.20$	1
$+0.30\ \ -0.6\ +\ \ -0.11=v=-0.08$	1
$+0.46\ \ -0.5\ +\ \ -0.12=v=-0.07$	1
$-2.85a-0.4\rho+\Delta\theta+0.9=v=+0.7$	0.1
$-2.71a+0.4\rho+\Delta\theta+0.3=v=+0.1$	0.1
$+0.47\ \ +0.6\ +\ \ -0.19=v=-0.14$	1
$-4.75\ \ +0.8\ +\ \ +0.3\ =v=-0.1$	0.04
$+0.48\ \ +1.0\ +\ \ +0.02=v=+0.08$	1
$+0.12\ \ +1.0\ +\ \ -0.03=v=\ 0.00$	1
$-0.16\ \ +1.2\ +\ \ +0.12=v=+0.13$	0.65
$-0.28\ \ +1.3\ +\ \ +0.03=v=-0.03$	0.6
$+0.51\ \ +1.3\ +\ \ -0.09=v=-0.03$	1
$+0.61\ \ +1.4\ +\ \ -0.14=v=-0.07$	1
$+0.28\ \ +1.5\ +\ \ -0.07=v=-0.03$	1
$+0.11\ \ +1.7\ +\ \ -0.07=v=-0.04$	1
$+0.63\ \ +1.7\ +\ \ +0.06=v=+0.01$	1
$+0.55\ \ +1.8\ +\ \ +0.08=v=+0.14$	1
$-11.81\ \ +2.0\ +\ \ 0.0\ =v=-0.9$	0.01

NORMAL EQUATIONS.

$$+13.94a-3.66\rho+8.41\Delta\theta-1.18=0$$
$$-3.66\ \ +44.66\ \ +2.44\ \ +0.09=0$$
$$+8.41\ \ +2.44\ \ +22.80\ \ -0.96=0$$

RESULTS.

$\Delta\theta=+0^s.014$

$\rho=+0^s.005$

$a=+0^s.077$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 20.—TIME DETERMINATION.

[Detroit, Mich., June 22, 1881. O. B. Wheeler, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit $=t$	Right ascension $=a$	$a-t_c$
			s.	s.	s.	s.	s.	h. m. s.	h. m. s.	s.
α Ursæ Min., L. C.	E	5	+0.12	−3.34	−2.98	−0.13	+0.05	13 15 21.66	1 15 14.8	−0.5
α Ursæ Min., L. C.	W.	7	+0.08	−2.23	+4.36	−0.13	+0.05	15 12.43	1 15 14.8	+0.3
τ Virginis	W.	11	+0.13	+0.10	−0.21	0.00	+0.04	55 34.85	13 55 38.54	+3.80
d Bootis	W.	11	+0.13	+0.14	−0.23	0.00	+0.04	14 04 57.40	14 05 01.22	+3.91
κ Virginis	W.	11	+0.13	+0.08	−0.21	0.00	+0.04	06 32.53	06 36.25	+3.85
4 Ursæ Min	W.	11	+0.13	+0.51	−1.01	+0.01	+0.03	09 19.33	09 22.5	+3.6
φ Virginis	W.	11	+0.13	+0.09	−0.21	0.00	+0.03	22 03.84	22 07.57	+3.85
ρ Bootis	W.	11	+0.13	+0.15	−0.24	0.00	+0.03	26 41.22	26 45.04	+3.91
μ Virginis	W.	11	+0.13	+0.09	−0.21	0.00	+0.03	36 47.05	36 50.79	+3.86
109 Virginis	W.	11	+0.13	+0.10	−0.21	0.00	+0.03	40 13.58	40 17.29	+3.82
47 Hev. Ceph., L. C	W.	7	+0.12	−0.32	+0.53	−0.02	+0.03	50 17.33	2 50 22.0	+4.4
47 Hev. Ceph., L. C	E.	7	+0.07	−0.18	−0.36	−0.02	+0.03	50 18.92	50 22.0	+3.6
ψ Bootis	E.	11	+0.07	+0.07	+0.20	0.00	+0.02	59 19.74	14 59 23.88	+3.87
48 Hev. Ceph., L. C	E.	11	+0.07	−0.16	−0.81	−0.02	+0.02	15 05 15.35	3 05 18.3	+4.0
3 Serpentis	E	11	+0.07	+0.06	+0.18	0.00	+0.02	09 15.74	15 09 19.89	+3.91
β Coron. Bor	E.	11	+0.07	+0.08	+0.20	0.00	+0.02	22 54.32	22 58.54	+3.94
γ Coron. Bor	E.	11	+0.07	+0.08	+0.20	0.00	+0.01	37 43.75	37 47.97	+3.94
α Serpentis	E.	11	+0.07	+0.06	+0.07	0.00	+0.01	38 23.88	38 27.88	+3.87
β Serpentis	E.	11	+0.07	+0.07	+0.18	0.00	+0.01	40 40.90	40 45.09	+3.94
ζ Ursæ Min	E.	11	+0.08	+0.31	+0.86	+0.01	+0.01	48 18.33	48 23.3	+3.8
ε Ursæ Min	E.	8	+0.13	+0.75	+0.51	+0.02	−0.01	16 58 12.00	16 58 16.7	+3.4
ε Ursæ Min	W.	7	+0.12	+0.67	−0.75	+0.02	−0.01	58 12.90	58 16.7	+3.9
θ Herculis	W.	11	+0.11	+0.13	−0.26	0.00	−0.02	17 52 09.99	17 52 13.68	+3.82
67 Ophiuchi	W.	11	+0.08	+0.06	−0.21	0.00	−0.02	54 41.23	54 44.98	+3.90
72 Ophiuchi	W.	11	+0.08	+0.07	−0.21	0.00	−0.02	18 01 42.59	18 01 46.17	+3.72
o Herculis	W.	11	+0.08	+0.09	−0.24	0.00	−0.03	02 53.78	02 57.52	+3.89
δ Ursæ Min	W.	6	+0.05	+0.63	−1.71	+0.05	−0.03	10 46.18	10 50.1	+5.0
δ Ursæ Min	E.	7	+0.05	+0.63	+1.17	+0.05	−0.03	10 44.19	10 50.1	+4.1
109 Herculis	E.	11	+0.05	+0.05	+0.19	0.00	−0.03	18 37.10	18 41.18	+3.84
α Lyræ	E.	11	+0.06	+0.08	+0.23	0.00	−0.03	32 53.93	32 57.98	+3.74
110 Herculis	E.	11	+0.06	+0.06	+0.19	0.00	−0.03	40 32.03	40 35.97	+3.69
β Lyræ	E.	11	+0.06	+0.07	+0.21	0.00	−0.03	45 40.46	45 44.63	+3.89
θ Serpentis	E.	11	+0.06	+0.05	+0.18	0.00	−0.03	50 17.93	50 21.99	+3.83
ε Aquilæ	E.	11	+0.06	+0.06	+0.18	0.00	−0.04	54 12.88	54 16.92	+3.80

Adopted collimation $=+0^{s}.085$ Clamp East. (See Table 12.)

OBSERVATION-EQUATIONS.

Epoch $16^{h}.5$ clock time. $\Delta t=+3^{s}.8+\Delta\theta$.

Equation	Weight.
$+32.57a-3.2\rho+\Delta\theta+3.9\ =v=+3.9$	0.001
$+\ 0.65\ -2.6\ +\quad 0.00=v=+0.09$	1
$+\ 0.32\ -2.4\ +\ -0.11=v=-0.02$	1
$+\ 0.80\ -2.4\ +\ -0.05=v=+0.04$	1
$-\ 2.84\ -2.3\ +\ +0.2\ =v=+0.3$	0.1
$+\ 0.70\ -2.1\ +\ -0.05=v=+0.03$	1
$+\ 0.23\ -2.1\ +\ -0.11=v=-0.03$	1
$+\ 0.74\ -1.9\ +\ -0.06=v=+0.02$	1
$+\ 0.64\ -1.8\ +\ -0.02=v=+0.06$	1
$+\ 4.46\ -1.7\ +\ -0.2\ =v=-0.1$	0.09
$+\ 0.29\ -1.5\ +\ -0.07=v=\ \ 0.00$	1
$+\ 3.94\ -1.4\ +\ -0.2\ =v=-0.2$	0.11
$+\ 0.60\ -1.3\ +\ -0.11=v=-0.04$	1
$+\ 0.26\ -1.1\ +\ -0.14=v=-0.07$	1
$+\ 0.30\ -0.9\ +\ -0.14=v=-0.08$	1

Equation	Weight.
$+\ 0.59a-0.9\rho+\Delta\theta\ -0.07=v=-0.01$	1
$+\ 0.46\ -0.8\ +\ -0.14=v=-0.08$	1
$-\ 2.85\ -0.7\ +\quad 0.0\ =v=+0.1$	0.1
$-\ 4.75a+0.5\rho+\Delta\theta+0.1\ =v=+0.2$	0.04
$+\ 0.11\ +1.4\ +\ -0.02=v=+0.01$	1
$+\ 0.63\ +1.4\ +\ -0.10=v=-0.07$	1
$+\ 0.55\ +1.5\ +\ +0.08=v=+0.11$	1
$+\ 0.28\ +1.5\ +\ -0.09=v=-0.07$	1
$-11.81\ +1.7\ +\ -0.7\ =v=-0.6$	0.01
$+\ 0.39\ +1.8\ +\ -0.04=v=-0.02$	1
$+\ 0.08\ +2.0\ +\ +0.06=v=+0.08$	1
$+\ 0.40\ +2.2\ +\ +0.11=v=+0.13$	1
$+\ 0.19\ +2.3\ +\ -0.09=v=-0.07$	1
$+\ 0.62\ +2.3\ +\ -0.03=v=-0.01$	1
$+\ 0.47\ +2.4\ +\quad 0.00=v=+0.01$	1

NORMAL EQUATIONS.

$$+14.04a-\ 5.38\rho+10.28\Delta\theta-0.56=0$$
$$-\ 5.38\ +79.30\ -\ 3.56\ \ +1.36=0$$
$$+10.28\ -\ 3.56\ +23.45\ \ -1.22=0$$

RESULTS.

$\Delta\theta=+0^{s}.051$

$\rho=-0^{s}.015$

$a=-0^{s}.004$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 21.—TIME DETERMINATION.

[Detroit, Mich., June 23, 1881. O. B. Wheeler, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit $=t$	Right ascension $=\alpha$	$\alpha-t_c$
			s.	s.	s.	s.	s.	h. m. s.	h. m. s.	s.
α Ursæ Min., L. C	E.	9	+0.13	−3.71	−2.98	−1.79	+0.07	13 15 22.40	13 15 15.9	+0.2
α Ursæ Min., L. C	W.	6	+0.07	−1.86	+4.36	−1.79	+0.07	15 14.93	15 15.9	−1.5
d Bootis	W.	11	+0.13	+0.14	−0.23	−0.02	+0.05	14 04 57.56	14 05 01.20	+3.73
χ Virginis	W.	11	+0.13	+0.08	+0.49	−0.04	+0.05	06 31.96	06 36.24	+3.71
φ Virginis	W.	11	+0.13	+0.09	−0.21	−0.04	+0.05	22 03.97	22 07.56	+3.71
ρ Bootis	W.	11	+0.13	+0.15	−0.24	−0.01	+0.05	26 41.33	26 45.01	+3.77
π Bootis pr	W.	11	+0.13	+0.12	−0.22	−0.02	+0.04	35 07.51	35 11.12	+3.71
109 Virginis	W.	11	+0.13	+0.10	−0.21	−0.04	+0.04	40 13.62	40 17.28	+3.77
47 Hev. Ceph., L. C	W.	7	+0.16	−0.43	+0.53	−0.25	+0.04	50 18.61	2 50 22.2	+3.5
47 Hev. Ceph., L. C	E.	6	+0.13	−0.35	−0.36	−0.25	+0.04	50 18.45	2 50 22.2	+4.5
ψ Bootis	E.	11	+0.12	+0.13	+0.20	−0.02	+0.03	59 19.72	14 59 23.87	+3.82
48 Hev. Ceph., L. C	E.	11	+0.12	−0.27	−0.31	−0.22	+0.03	15 05 16.06	3 05 18.45	+3.0
3 Serpentis	E.	11	+0.12	+0.09	+0.18	−0.03	+0.03	09 15.90	15 09 19.88	+3.71
δ Bootis	E.	11	+0.12	+0.14	+0.21	−0.01	+0.03	10 41.23	10 45.39	+3.81
μ Bootis	E.	11	+0.12	+0.15	+0.22	−0.01	+0.02	19 58.66	20 02.71	+3.68
θ Coron. Bor	E.	11	+0.12	+0.14	+0.21	−0.01	+0.02	28 06.87	28 10.95	+3.73
α Coron. Bor	E.	11	+0.12	+0.13	+0.20	−0.02	+0.02	29 38.00	29 42.17	+3.84
ζ Coron. Bor. seq	E.	11	+0.12	+0.15	+0.22	−0.01	+0.02	34 52.79	34 56.87	+3.71
ζ Ursæ Min	E.	11	+0.11	+0.41	+0.86	+0.16	+0.01	48 18.32	48 23.2	+3.6
ε Ursæ Min	E.	7	+0.07	+0.37	+0.51	+0.26	−0.02	16 58 11.47	16 58 16.6	+4.2
ε Ursæ Min	W.	6	+0.11	+0.60	−0.75	+0.26	−0.02	58 13.05	58 16.6	+3.7
α Herculis	W.	11	+0.11	+0.10	−0.22	−0.03	−0.03	17 09 13.23	17 09 16.88	+3.77
„ Herculis	W.	11	+0.11	+0.14	−0.26	−0.01	−0.03	10 53.77	10 57.50	+3.85
β Draconis	W.	11	+0.11	+0.18	−0.34	+0.02	−0.03	27 44 46	27 47.94	+3.64
ι Herculis	W.	11	+0.11	+0.16	−0.30	0.00	−0.04	36 06.06	36 09.68	+3.76
β Ophiuchi	W.	11	+0.11	+0.09	−0.21	−0.03	−0.04	37 35.82	37 39.40	+3.70
μ Herculis	E.	11	+0.12	+0.13	+0.20	−0.02	−0.04	41 47.61	41 51.55	+3.61
θ Herculis	E.	11	+0.12	+0.15	+0.22	−0.01	−0.04	52 09.78	52 13.67	+3.52
ξ Herculis	E.	11	+0.12	+0.13	+0.20	−0.01	−0.04	53 07.94	53 11.90	+3.63
67 Ophiuchi	E.	11	+0.12	+0.09	+0.18	−0.03	−0.94	54 40.91	54 44.97	+3.79
72 Ophiuchi	E.	11	+0.12	+0.10	+0.18	−0.03	−0.05	18 01 42.28	18 01 46.17	+3.61
o Herculis	E.	11	+0.12	+0.13	+0.20	−0.01	−0.05	02 53.55	02 57.51	+3.63
δ Ursæ Min	E.	6	+0.12	+1.43	+1.17	+0.65	−0.05	10 43.78	10 50.0	+3.6
δ Ursæ Min	W.	6	+0.09	+1.10	−1.71	+0.65	−0.05	10 46.90	10 50.0	+3.7

Adopted collimation = + 0s.085 Clamp East. (See Table 12.)

OBSERVATION-EQUATIONS.

Epoch $16^h.2$ clock time. $\Delta t=+3^s.5+\Delta\theta$.

Equation	Weight.
$+32.58a-2.9\rho+\Delta\theta+4.1=v=+2.6$	0.001
$+0.32\ \ -2.1\ +\ \ -0.23=v=+0.04$	1
$+0.80\ \ -2.1\ +\ \ -0.19=v=+0.06$	1
$+0.69\ \ -1.8\ +\ \ -0.21=v=+0.04$	1
$+0.23\ \ -1.8\ +\ \ -0.27=v=+0.01$	1
$+0.45\ \ -1.6\ +\ \ -0.21=v=+0.05$	1
$+0.64\ \ -1.5\ +\ \ -0.27=v=-0.03$	1
$+4.46\ \ -1.4\ +\ \ -0.5=v=-0.5$	0.058
$+0.29\ \ -1.2\ +\ \ -0.32=v+-0.07$	1
$+3.94\ \ -1.1\ +\ \ +0.5=v=+0.6$	0.076
$+0.60\ \ -1.0\ +\ \ -0.21=v=+0.03$	1
$+0.18\ \ -1.0\ +\ \ -0.31=v=-0.05$	1
$+0.10\ \ -0.9\ +\ \ -0.18=v=+0.07$	1
$+0.22\ \ -0.7\ +\ \ -0.23=v=+0.02$	1
$+0.29\ \ -0.7\ +\ \ -0.35=v=-0.11$	1

Equation	Weight.
$+0.12a-0.6\rho+\Delta\theta-0.22=v=+0.03$	1
$-2.85\ \ -0.4\ +\ \ -0.1=v=+0.3$	0.066
$-4.75a+0.8\rho+\Delta\theta-0.5=v=\ 0.0$	0.029
$+0.47\ \ +1.0\ +\ \ -0.27=v=-0.09$	1
$+0.12\ \ +1.0\ +\ \ -0.35=v=-0.15$	1
$-0.28\ \ +1.3\ +\ \ -0.14=v=+0.09$	0.5
$-0.09\ \ +1.4\ +\ \ -0.26=v=-0.06$	1
$+0.61\ \ +1.4\ +\ \ -0.20=v=-0.03$	1
$+0.28\ \ +1.5\ +\ \ -0.11=v=+0.07$	1
$+0.11\ \ +1.7\ +\ \ -0.02=v=+0.17$	1
$+0.26\ \ +1.7\ +\ \ -0.13=v=+0.06$	1
$+0.63\ \ +1.7\ +\ \ -0.29=v=-0.12$	1
$+0.55\ \ +1.8\ +\ \ -0.11=v=+0.05$	1
$+0.27\ \ +1.8\ +\ \ -0.13=v=+0.05$	1
$-11.81\ \ +2.0\ +\ \ -0.2=v=+0.6$	0.006

NORMAL EQUATIONS.

$$+9.48a-3.62\rho+8.15\Delta\theta-1.48=0$$
$$-3.62\ \ +50.13\ \ -1.51\ \ +1.40=0$$
$$+8.15\ \ -1.51\ \ +23.75\ \ -5.16=0$$

RESULTS.

$\Delta\theta=+0^s.235$

$\rho=-0^s.025$

$a=-0^s.055$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 22.—TIME DETERMINATION.

[Detroit, Mich., June 24, 1881. O. B. Wheeler, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit $=t$	Right ascension $=\alpha$	$\alpha-t_c$
			s.	*s.*	*s.*	*s.*	*s.*	*h. m. s.*	*h. m. s.*	*s.*
α Ursæ Min., L. C	W.	8	+0.14	−4.08	+4.36	−2.18	+0.10	13 15 18.40	1 15 17.0	−1.7
α Ursæ Min., L. C	E.	7	+0.11	−2.97	−2.98	−2.18	+0.10	15 25.73	15 17.0	−2.8
d Bootis	E.	10	+0.16	+0.18	−8.65	−0.02	+0.07	14 05 06.33	14 05 01.20	+3.34
χ Virginis	E.	11	+0.16	+0.10	+0.18	−0.05	+0.07	06 32.65	06 36.25	+3.32
φ Virginis	E.	11	+0.16	+0.12	+0.18	−0.05	+0.07	22 04.05	22 07.56	+3.21
ρ Bootis	E.	11	+0.16	+0.19	+0.21	−0.02	+0.07	26 41.18	26 45.02	+3.44
μ Virginis	E.	11	+0.16	+0.11	+0.18	−0.05	+0.06	36 47.20	36 50.78	+3.29
ψ Bootis	E.	11	+0.16	+0.18	+0.20	−0.02	+0.05	59 20.30	59 23.88	+3.20
48 Hev. Ceph., L. C	W.	11	+0.18	−0.42	+0.95	−0.26	+0.04	15 05 14.99	3 05 18.65	+3.1
3 Serpentis	W.	11	+0.18	+0.15	−0.21	−0.04	+0.04	09 16.54	15 09 19.89	+3.41
δ Bootis	W.	11	+0.18	+0.22	−0.25	−0.01	+0.04	10 41.97	10 45.40	+3.46
θ Coron. Bor	W.	11	+0.18	+0.21	−0.25	−0.01	+0.03	28 07.69	28 10.97	+3.32
α Coron. Bor	W.	11	+0.18	+0.20	−0.23	−0.02	+0.03	29 38.90	29 42.19	+3.32
ζ Coron. Bor. seq	W.	11	+0.18	+0.23	−0.26	−0.01	+0.03	34 53.54	34 56.89	+3.38
ε Coron. Bor	W.	11	+0.18	+0.20	−0.23	−0.02	+0.02	52 39.59	52 42.92	+3.36
Gr. 750, L. C	W.	6	+0.18	−1.34	+1.22	−0.64	+0.01	59 38.38	3 59 41.1	+2.8
Gr. 750, L. C	E.	7	+0.11	−0.96	−0.83	−0.64	+0.01	59 40.87	59 41.1	+2.0
β Draconis	E.	11	+0.15	+0.24	+0.29	+0.02	−0.04	17 27 44.26	17 27 47.97	+3.18
α Ophiuchi	E.	11	+0.15	+0.14	+0.18	−0.03	−0.04	29 24.70	29 28.29	+3.27
ι Herculis	E.	11	+0.15	+0.22	+0.25	+0.01	−0.04	36 06.14	36 09.71	+3.10
β Ophiuchi	E.	11	+0.15	+0.12	+0.18	−0.04	−0.04	37 35.84	37 39.42	+3.28
μ Herculis	E.	11	+0.15	+0.17	+0.20	−0.02	−0.04	41 47.88	41 51.57	+3.32
67 Ophiuchi	E.	11	+0.15	+0.12	+0.18	−0.04	−0.05	54 41.44	54 44.90	+3.25
δ Ursæ Min	E.	6	+0.12	+1.43	+1.17	+0.79	−0.06	18 10 43.90	18 10 49.9	+3.4
δ Ursæ Min	W.	7	+0.12	+1.43	−1.71	+0.79	−0.06	10 46.40	10 49.9	+3.8
109 Herculis	W.	11	+0.16	+0.16	−0.23	−0.03	−0.06	18 38.01	18 41.20	+3.26
ε Lyræ pr	W.	11	+0.16	+0.20	−0.27	0.00	−0.07	40 24.18	40 27.21	+3.10
51 Cephei, L. C	W.	5	+0.16	−2.07	+4.32	−1.07	−0.07	44 08.70	6 44 12.7	+1.8
θ Serpentis	W.	11	+0.16	+0.13	−0.21	−0.04	−0.08	50 18.85	18 50 22.02	+3.25
ε Aquilæ	W.	11	+0.16	−0.14	−0.22	−0.03	−0.08	51 13.70	54 16.95	+3.33
ζ Aquilæ	W.	9	+0.16	+0.14	+8.27	−0.03	−0.08	59 48.62	19 00 00.07	+3.04

Adopted collimation $=+0^s.085$ Clamp East. (See Table 12.)

OBSERVATION-EQUATIONS.

Epoch $16^h.4$ clock time. $\Delta t=+3^s.1+\Delta\theta$.

	Weight.		Weight.
$+32.58a-3.1\rho+\Delta\theta+5.4=v=+3.5$	0.001	$-0.28a+1.1\rho+\Delta\theta-0.08=v=+0.11$	0.5
$+0.32\ -2.3\ +\ -0.25=v=+0.01$	1	$+0.51\ +1.1\ +\ -0.17=v=-0.03$	1
$+0.80\ -2.3\ +\ -0.22=v=+0.01$	1	$-0.09\ +1.2\ +\ +0.01=v=+0.19$	1
$+0.69\ -2.0\ +\ -0.11=v=+0.12$	1	$+0.61\ +1.2\ +\ -0.18=v=-0.05$	1
$+0.23\ -2.0\ +\ -0.34=v=-0.08$	1	$+0.28\ +1.3\ +\ -0.24=v=-0.09$	1
$+0.74\ -1.8\ +\ -0.18=v=+0.04$	1	$+0.63\ +1.5\ +\ -0.15=v=-0.03$	1
$+0.29\ -1.4\ +\ -0.10=v=+0.14$	1	$-11.81\ +1.8\ +\ -0.5=v=+0.4$	0.006
$+3.94\ -1.3\ +\ +0.09=v=+0.1$	0.076	$+0.38\ +1.9\ +\ -0.15=v=-0.03$	1
$+0.60\ -1.2\ +\ -0.32=v=-0.11$	1	$+0.06\ +2.3\ +\ -0.01=v=+0.13$	1
$+0.18\ -1.2\ +\ -0.37=v=-0.13$	1	$+15.93\ +2.3\ +\ +1.3=v=+0.4$	0.003
$+0.22\ -0.9\ +\ -0.22=v=+0.01$	1	$+0.62\ +2.4\ +\ -0.16=v=-0.07$	1
$+0.29\ -0.9\ +\ -0.23=v=-0.01$	1	$+0.47\ +2.5\ +\ -0.23=v=-0.13$	1
$+0.12\ -0.8\ +\ -0.29=v=-0.06$	1	$+0.49\ +2.6\ +\ +0.06=v=+0.16$	1
$+0.29\ -0.5\ +\ -0.27=v=-0.06$	1		
$+9.55\ -0.4\ +\ +0.7=v=+0.3$	0.011		

NORMAL EQUATIONS.

$$+9.49a-1.22\rho+9.00\Delta\theta-1.26=0$$
$$-1.22\ +65.66\ +1.17\ +1.81=0$$
$$+9.00\ +1.17\ +22.60\ -4.04=0$$

RESULTS.

$\Delta\theta=+0^s.208$
$\rho=-0^s.032$
$a=-0^s.066$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 23—TIME DETERMINATION.

[Detroit, Mich., June 29, 1881. O. B. Wheeler, Observer.]

Star.	Cl.	No. of wires.	b	Bb	$C(c+\Delta i+ab'n)$	Aa	$R\rho$	Clock time of transit $=t$	Right ascension $=\alpha$	$\alpha-t_c$
				s.	s.	s.	s.	h. m. s.	h. m. s.	s.
α Ursæ Min., L. C	E.	7	+0.12	—3.34	—2.98	+4.79	+0.06	13 15 22.80	1 15 21.6	+5.1
α Ursæ Min., L. C	W.	8	+0.07	—1.86	+4.36	+4.79	+0.06	15 14.94	15 21.6	+4.1
μ Virginis	W.	11	+0.09	+0.06	—0.21	+0.11	+0.04	14 36 50.21	14 36 50.74	+0.68
48 Hev. Ceph., L. C	W.	11	+0.09	—0.20	—5.53	+0.58	+0.03	15 05 24.00	3 05 19.1	+0.8
3 Serpentis	W.	11	+0.09	+0.07	—0.21	+0.09	+0.03	09 19.37	15 09 19.86	+0.63
δ Bootis	W.	11	+0.09	+0.10	—0.25	+0.03	+0.03	10 44.99	10 45.34	+0.50
τ' Serpentis	W.	11	+0.09	+0.08	—0.22	+0.07	+0.03	20 19.10	20 19.50	+0.54
θ Coron. Bor	W.	11	+0.09	+0.10	—0.25	+0.03	+0.03	28 10.66	28 10.91	+0.40
α Coron. Bor	W.	11	+0.09	+0.09	—0.23	+0.04	+0.03	29 41.71	29 42.14	+0.57
ζ Coron. Bor. seq	E.	11	+0.08	+0.10	+0.22	+0.02	+0.02	34 55.87	34 56.83	+0.64
γ Coron. Bor	E.	11	+0.08	+0.08	+0.20	+0.04	+0.02	37 47.03	37 47.93	+0.62
β Serpentis	E.	11	+0.08	+0.07	+0.18	+0.07	+0.02	40 44.17	40 45.06	+0.04
μ Serpentis	E.	11	+0.08	+0.05	+0.18	+0.10	+0.02	43 27.41	43 28.26	+0.62
γ Serpentis	E.	11	+0.08	+0.07	+0.18	+0.07	+0.02	50 59.97	51 00.78	+0.56
ε Coron. Bor	E.	11	+0.08	+0.09	+0.20	+0.04	+0.02	52 42.09	52 42.89	+0.51
Gr. 750, L. C	E.	7	+0.08	—0.58	—0.83	+1.46	+0.02	59 41.90	59 42.2	+1.7
Gr. 750, L. C	W.	7	+0.10	—0.77	+1.22	+1.46	+0.02	59 38.54	59 42.2	+3.2
β Ophiuchi	W.	11	+0.01	+0.01	—0.21	+0.09	—0.01	17 37 39.01	17 37 39.45	+0.64
μ Herculis	W.	11	+0.01	+0.01	—0.24	+0.04	—0.01	41 51.26	41 51.59	+0.56
θ Herculis	W.	11	+0.01	+0.02	—0.26	+0.01	—0.02	52 13.51	52 13.71	+0.44
72 Ophiuchi	W.	11	+0.01	+0.01	—0.21	+0.08	—0.02	18 01 45.87	18 01 46.23	+0.56
ο Herculis	W.	11	+0.01	+0.01	—0.24	+0.04	—0.02	02 57.23	02 57.56	+0.56
δ Ursæ Min	W.	7	+0.04	+0.47	—1.71	—1.74	—0.02	10 52.20	10 49.43	—1.5
δ Ursæ Min	E.	7	+0.07	+0.80	+1.17	—1.74	—0.02	10 50.09	10 49.43	—2.6
109 Herculis	E.	11	+0.06	+0.06	+0.19	+0.06	—0.02	18 40.58	18 41.24	+0.41
α Lyræ	E.	11	+0.06	+0.08	+0.23	+0.01	—0.03	32 57.20	32 58.04	+0.53
110 Herculis	E.	11	+0.06	+0.06	+0.19	+0.06	—0.03	40 35.49	40 36.06	+0.32
51 Cephei, L. C	E.	6	+0.06	—0.79	—1.41	+2.34	—0.03	44 13.12	44 13.0	+2.1
ι Lyræ	E.	11	+0.06	+0.07	+0.23	+0.02	—0.04	19 03 06.02	19 03 06.85	+0.54
θ Lyræ	E.	11	+0.06	+0.08	+0.22	+0.01	—0.04	12 16.88	12 17.72	+0.54
δ Aquilæ	E.	11	+0.06	+0.05	+0.18	+0.09	—0.04	19 32.95	19 33.76	+0.58
Gr. 2900	E.	7	+0.08	+0.34	+0.38	—0.48	—0.04	28 55.89	28 56.4	—0.2
Gr. 2900	W.	7	+0.04	+0.17	—0.55	—0.48	—0.04	28 57.23	28 56.4	—0.5
ψ Cygni	W.	11	+0.06	+0.09	—0.34	—0.04	—0.05	52 36.48	52 36.67	+0.44

Adopted collimation $=+0^s.085$ Clamp East. (See Table 12.)

OBSERVATION-EQUATIONS.

Epoch $17^h.0$ clock time. $\Delta t=+0^s.5+\Delta\theta$.

	Weight.		Weight.
$+32.58a-3.7\rho+\Delta\theta-4.1=v=+0.7$	0.001	$+0.62a+0.6\rho+\Delta\theta-0.14=v=-0.06$	1
$+0.74\ -2.4\ +\ -0.18=v=-0.03$	1	$+0.28\ +0.7\ +\ -0.06=v=-0.03$	1
$+3.94\ -1.9\ +\ -0.3=v=+0.3$	0.076	$+0.11\ +0.9\ +\ +0.05=v=+0.04$	1
$+0.60\ -1.8\ +\ -0.13=v=-0.01$	1	$+0.55\ +1.0\ +\ -0.06-v=\ 0.00$	1
$+0.18\ -1.8\ +\ 0.00=v=+0.06$	1	$+0.27\ +1.0\ +\ -0.06=v=-0.04$	1
$+0.46\ -1.7\ +\ -0.04=v=+0.06$	1	$-11.81\ +1.2\ +\ +2.6=v=+0.8$	0.006
$+0.22\ -1.5\ +\ +0.10=v=+0.16$	1	$+0.38\ +1.3\ +\ +0.09=v=+0.13$	1
$+0.30\ -1.5\ +\ -0.07=v=\ 0.00$	1	$+0.08\ +1.5\ +\ -0.03=v=-0.05$	1
$+0.12\ -1.4\ +\ -0.13=v=-0.09$	1	$+0.40\ +1.7\ +\ +0.18=v=+0.21$	1
$+0.30\ -1.4\ +\ -0.12=v=-0.06$	1	$+15.93\ +1.7\ +\ -1.6=v=+0.7$	0.004
$+0.46\ -1.3\ +\ -0.14=v=-0.05$	1	$+0.14\ +2.1\ +\ -0.04=v=-0.06$	1
$+0.71\ -1.3\ +\ -0.12=v=\ 0.00$	1	$+0.10\ +2.2\ +\ -0.04=v=-0.07$	1
$+0.46\ -1.2\ +\ -0.06=v=+0.03$	1	$+0.64\ +2.3\ +\ -0.08=v=-0.03$	1
$+0.29\ -1.1\ +\ -0.02=v=+0.04$	1	$-3.25\ +2.5\ +\ +0.8=v=+0.3$	0.006
$+9.55\ -1.0\ +\ -2.0=v=-0.5$	0.011	$-0.28\ +2.9\ +\ +0.06=v=-0.03$	1

NORMAL EQUATIONS.

$$+9.78a-5.06\rho+8.35\Delta\theta-1.49=0$$
$$-5.06\ +63.71\ -0.18\ +1.84=0$$
$$+8.35\ -0.18\ +24.16\ -1.12=0$$

Results.

$\Delta\theta=+0^s.005$

$\rho=-0^s.017$

$a=+0^s.147$

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 24.—RESULTS OF CLOCK COMPARISONS.

Signals from—	Date.	Cambridge, O. B. Wheeler, observer.			Detroit, A. R. Flint, observer.			Differences of time.			
		Means of clock times of comparisons.	Clock corrections.	Means of sidereal times of comparisons.	Means of clock times of comparisons.	Clock corrections.	Means of sidereal times of comparisons.	Signals from Cambridge.	Signals from Detroit.	Means.	Mean both ways.
	1881.	*h. m. s.*	*m. s.*	*h. m. s.*	*h. m. s.*	*m. s.*	*h. m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*
Cambridge	May 13	14 48 08.000	−0 00.495	14 48 07.505	14 01 52 000	−1 25.560	14 00 26.440	47 41.065			
		14 55 39.000	−0 00.497	14 55 38.503	14 09 23.000	−1 25.560	14 07 57.440	47 41.063		47 41.064	
Detroit	13	14 42 51.201	−0 00.493	14 42 50.708	13 56 35.000	−1 25.560	13 55 09.440		47 41.268		
		14 51 24.193	−0 00.496	14 51 23.697	14 05 68.000	−1 25.560	14 03 42.440		47 41.257	47 41.262	+47 41.163
Cambridge	23	15 25 25.000	−0 08.367	15 25 16.633	14 39 00.561	−1 24.745	14 37 35.816	47 40.817			
		15 39 52.000	−0 08.385	15 39 43.615	14 53 27.523	−1 24.753	14 54 02.770	47 40.845		47 40.831	
Detroit	23	15 23 16.712	−0 08.365	15 23 08.347	14 36 52.000	−1 24.744	14 35 27.256		47 41.091		
		15 49 41.745	−0 08.397	15 49 33.348	15 03 17.000	−1 24.759	15 01 52.241		47 41.107	47 41.099	47 40.966
Cambridge	24	15 16 15.000	−0 09.485	15 16 05.515	14 29 49.423	−1 24.834	14 28 24.589	47 40.926			
		15 24 34.000	−0 09.490	15 24 24.510	14 38 08.406	−1 24.834	14 36 43.572	47 40.938		47 40.932	
Detroit	24	15 21 39.801	−0 09.488	15 21 30.313	14 35 14.000	−1 24.834	14 33 49.166		47 41.147		
		15 44 17.811	−0 09.504	15 44 08.307	14 57 52.000	−1 24.834	14 56 27.166		47 41.141	47 41.144	47 41.038
Cambridge	26	15 23 52.000	−0 11.777	15 23 40.223	14 37 24.233	−1 24.930	14 35 59.303	47 40.920			
		15 32 09.000	−0 11.785	15 31 47.215	14 45 41.253	−1 24.932	14 44 16.321	47 40.894		47 40.907	
Detroit	26	15 20 41.000	−0 11.773	15 20 29.227	14 34 13.000	−1 24.929	14 32 48.071		47 41.156		
		15 26 14.000	−0 11.779	15 26 02.221	14 39 46.000	−1 24.931	14 38 21.069		47 41.152	47 41.154	47 41.030
Cambridge	June 4	15 47 37.000	−0 15.980	15 47 21.020	15 01 04.689	−1 24.612	14 59 40.077	47 40.943			
		15 52 08.000	−0 15.982	15 51 52.018	15 05 35.679	−1 24.614	15 04 11.065	47 40.953		47 40.948	
Detroit	4	15 45 00.587	−0 15.979	15 44 44.608	14 58 28.000	−1 24.611	14 57 05.389		47 41.219		
		15 50 23.588	−0 15.981	15 50 07.607	15 03 51.000	−1 24.613	15 02 26.387		47 41.220	47 41.220	47 41.084
Cambridge	11	16 32 45.000	−0 15.682	16 32 29.318	15 46 12.180	−1 23.702	15 44 48.478	47 40.840			
		16 40 21.000	−0 15.681	16 40 05.319	15 54 48.167	−1 23.703	15 53 24.464	47 40.855		47 40.848	
Detroit	11	16 29 58.157	−0 15.682	16 29 42.475	15 43 25.000	−1 23.702	15 42 01.298		47 41.177		
		16 35 10.156	−0 15.681	16 34 54.475	15 48 37.000	−1 23.702	15 47 13.298		47 41.177	47 41.177	47 41.012

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 25.—RESULTS OF CLOCK COMPARISONS.

Signals from—	Date.	Cambridge, A. R. Flint, observer.			Detroit, O. B. Wheeler, observer.			Differences of time.			
		Means of clock times of comparisons.	Clock corrections.	Means of sidereal times of comparisons.	Means of clock times of comparisons.	Clock corrections.	Means of sidereal times of comparisons.	Signals from Cambridge.	Signals from Detroit.	Means.	Mean both ways.
	1881.	*h. m. s.*	*s.*	*h. m. s.*	*h. m. s.*	*s.*	*h. m. s.*	*m. s.*	*m. s.*	*m. s.*	*m. s.*
Cambridge	June 21	17 05 23.000	−20.757	17 05 02.243	16 17 19.411	+1.814	16 17 21.225	47 41.018			
		17 11 49.000	−20.758	17 11 28.242	16 23 45.400	+1.815	16 23 47.215	47 41.027	....	47 41.022	
Detroit	21	17 01 46.883	−20.756	17 01 26.127	16 13 43.000	+1.814	16 13 44.814	....	47 41.313		
		17 09 18.833	−20.758	17 08 58.075	16 21 15.000	+1.815	16 21 16.815	....	47 41.260	47 41.286	+47 41.154
Cambridge	22	17 06 30.000	−21.501	17 06 08.499	16 18 23.592	+3.854	16 18 27.446	47 41.053			
		17 12 41.000	−21.505	17 12 19.495	16 24 34.596	+3.852	16 24 38.448	47 41.047	....	47 41.050	
Detroit	22	17 04 14.636	−21.499	17 03 53.137	16 16 08.000	+3.854	16 16 11.854	....	47 41.283		
		17 09 58.657	−21.503	17 09 37.154	16 21 52.000	+3.853	16 21 55.853	....	47 41.301	47 41.292	47 41.171
Cambridge	23	17 14 43.000	−22.273	17 14 20.727	16 26 35.991	+3.728	16 26 39.719	47 41.008			
		17 09 08.000	−22.270	17 08 35.730	16 21 00.988	+3.731	16 21 04.719	47 41.011	....	47 41.010	
Detroit	23	17 05 48.279	−22.269	17 05 26.010	16 17 41.000	+3.733	16 17 44.733	....	47 41.277		
		17 11 18.258	−22.271	17 10 55.987	16 23 11.000	+3.730	16 23 14.730	....	47 41.257	47 41.267	47 41.138
Cambridge	24	17 13 22.000	−23.128	17 12 58.872	16 25 14.592	+3.307	16 25 17.899	47 40.973			
		17 27 38.000	−23.137	17 27 14.863	16 39 30.589	+3.300	16 39 33.889	47 40.974	....	47 40.973	
Detroit	24	17 09 50.677	−23.126	17 09 27.551	16 21 43.000	+3.309	16 21 46.309	....	47 41.242		
		17 24 33.685	−23.135	17 24 10.550	16 36 26.000	+3.301	16 36 29.301	....	47 41.249	47 41.246	47 41.110
Cambridge	29	17 50 29.000	−28.342	17 50 00.658	17 02 19.305	+0.494	17 02 19.799	47 40.859			
		17 57 25.467	−28.347	17 56 57.120	17 09 15.774	+0.492	17 09 16.266	47 40.854	....	47 40.857	
Detroit	29	17 47 05.898	−28.340	17 46 37.558	16 58 55.933	+0.495	16 58 56.428	....	47 41.130		
		17 53 35.442	−28.344	17 53 07.098	17 05 25.467	+0.493	17 05 25.960	....	47 41.138	47 41.134	47 40.995

Difference of Longitude, Detroit, Mich., and Cambridge, Mass.—Continued.

TABLE 26.—FINAL RESULTS.

O. B. WHEELER AT CAMBRIDGE. A. R. FLINT AT DETROIT.

Date.	Difference of time. Signals from Cambridge.	Signals from Detroit.	Mean.	Wt.	Correction.	Corrected results.	Double wave-time, east *minus* west.
1881.	*h. m. s.*	*h. m. s.*	*h. m. s.*		*s.*	*h. m. s.*	*s.*
May 13	0 47 41.064	0 47 41.262	0 47 41.163	0.5	+0.034	0 47 41.197	+0.198
23	40.831	41.099	40.966	0.5	+0.034	41.010	+0.268
24	40.932	41.144	41.038	1	+0.034	41.072	+0.212
26	40.907	41.154	41.030	1	+0.034	41.064	+0.247
June 4	40.948	41.220	41.084	1	+0.034	41.118	+0.272
11	40.848	41.177	41.012	1	+0.034	41.046	+0.329
Weighted mean			0 47 41.046				

A. R. FLINT AT CAMBRIDGE. O. B. WHEELER AT DETROIT.

Date.	Signals from Cambridge.	Signals from Detroit.	Mean.	Wt.	Correction.	Corrected results.	Double wave-time, east *minus* west.
June 21	0 47 41.022	0 47 41.286	0 47 41.154	1	−0.034	0 47 41.120	+0.264
22	41.050	41.292	41.171	1	−0.034	41.137	+0.242
23	41.010	41.267	41.138	1	−0.034	41.104	+0.257
24	40.973	41.246	41.110	1	−0.034	41.076	+0.273
29	40.857	41.134	40.995	1	−0.034	40.961	+0.277
Weighted mean			0 47 41.114				
Difference of weighted means = personal and instrumental equation			= 0.068				
Correction = half of difference			= 0.034				

	h. m. s.
Weighted mean of corrected results	0 47 41.080
Reduction to east pier of Detroit Observatory	− 0.004
East pier of Lake-Survey Observatory, Detroit, west of Coast-Survey post, Cambridge	0 47 41.076
Longitude of Coast-Survey post, Cambridge, Appendix 6, Coast-Survey Report of 1880	4 44 31.090
Longitude of east pier of Lake-Survey Observatory at Detroit west of Greenwich	5 32 12.166

APPENDIX IV.

MAGNETIC WORK OF THE LAKE SURVEY.

By T. RUSSELL, *Assistant Engineer.*

§ **1.** The principal observations to determine the amount and direction of the earth's magnetic force at various places in the region of the Great Lakes were made in the years 1858, 1859, and 1860, while the survey was under the charge of Captain George G. Meade, and in the years 1870 to 1880. For declination and intensity a magnetometer and declinometer, with suspended collimator-magnets, were used. For the determination of dip, one dip-circle by Barrow and two by Troughton & Simms were in use. At most of the places where observations were made in 1858 and 1859 by Lieutenant William Proctor Smith, the same were repeated by Captain A. N. Lee in 1872 and 1873. The instruments were, when it was practicable, set up in the same places as when the observations were first made. The setting of the instruments on the original sites was prevented in some instances by buildings and other improvements. A comparison of the observations of the two periods is interesting, as showing the changes of the magnetic elements from year to year. A great many determinations of magnetic declination have been made in connection with the work of mapping the shore-line of the lakes. These were made with theodolite-compasses by comparing the magnetic bearing of a line with the azimuth determined astronomically. The needles are from four to five inches long and supported on a point. The readings of the ends are made by estimation on a graduated circle a little greater in diameter than the length of the needle.

DESCRIPTIONS OF INSTRUMENTS.

§ **2.** Würdemann magnetometer No. 3, for the measurement of declination and horizontal intensity, has a circle 6 inches in diameter, graduated to 10 minutes, and read by opposite verniers to 30 seconds. The glass tube for the suspending thread of the magnet is 18 inches long. The telescope for viewing the magnet scales is about 8 inches in length. It is mounted on pivots outside the horizontal circle, and swings vertically. A weight on the opposite side of the instrument counterbalances the telescope. Collimator-magnet C_3 is 3 inches long and 0.75 inch in diameter. The scale is 0.05 inch long, and is divided into thirty parts. Magnet C_2 is 3.9 inches long and 0.75 inch in diameter. There are two smaller magnets, both 0.3 inch in diameter, the one 3.6 inches long and the other about 3 inches. These latter magnets were first used in 1876. The instrument is provided with two inertia-rings. The old ring has a weight of 397.44 grains, the external radius is 0.10225 foot, and the internal 0.07975 foot. The new ring, No. 2, for the light magnet, has a weight of 487.07 grains; the external and internal diameters are 2.002 and 1.600 inches.

The Jones declinometer, also used in measuring declination and horizontal intensity, has a circle of 9.5 inches in diameter, graduated to 20 minutes, and read by two verniers to single minutes. The glass tube that surrounds the suspending thread of the magnet is 9 inches long. The telescope is 8 inches long, and is not movable in altitude. Near the eye-piece end of the telescope, and above it, is attached a graduated ivory arc 6 inches long. This scale is used to measure the angle of deflection of the suspended magnet in observations for horizontal intensity by viewing the reflection of the scale in a mirror attached below the suspended magnet. A weight attached to the instrument counterbalances the telescope opposite. Magnet marked "*b*" is a hollow steel cylinder without a lens. It is 3 inches long and 0.25 inch in diameter, and has attached a mirror 0.5 inch by 0.75 inch. Magnet D_5 is 3.6 inches long, and 0.25 of an inch in diameter. A brass cylinder, with a weak magnet inside for adjusting the line of detorsion, is 4.1 inches long and 0.6 inch in diameter. The inertia-ring, No. 3, weighs 830.24 grains; the external and internal diameters are 2.933 and 2.354 inches; the thickness of the ring is 0.16 inch.

The Barrow dip-circle, No. 26, has a vertical circle 6 inches in diameter, graduated to half degrees and read by two verniers to minutes. To the arm carrying the verniers, two microscopes are attached that have threads in the foci extended in the direction of a diameter of the vertical circle. To observe the position of the needle the image of the point is bisected with the thread. The prolongations of the threads pass through the zeroes of the verniers. Two needles have been used with this instrument, each about 3.5 inches long and in form tapering from the center toward the ends. With needles of this shape the moment of inertia is much less for the same weight of needle, without any considerable diminution of its magnetic moment. The pivots of the needles rest on straight agate edges so that in rotating they roll. A framework with two sides of glass incloses the needle, when mounted, to protect it from currents of air. The instrument has a telescope and horizontal circle for bringing the plane of the needle into the magnetic meridian.

The Fox dip-circle, Troughton & Simms No. 1, has two vertical circles on which the needle-ends are read. The plane of one circle is 0.25 inch inside that of the other. The outside circle is 6.5 inches in diameter, and is graduated to 15′; the other is graduated to 30′. The reading of the needle-end is made through the glass side of the box that protects the needle from currents of air The needle-axles rest in jeweled hollows. An attached telescope that moves in altitude, and a horizontal circle serve to bring the plane of the needle into the magnetic meridian. The instrument has three tapering needles, each about 6.5 inches long.

Dip-circle, Troughton & Simms No. 2, has the vertical circle graduated to 15′. The reading of the needle-end is estimated as in Troughton & Simms No. 1. There are three needles, each about 8 inches long. Nos. 3 and 4 are tapering. Needle "A" has the same width throughout, with semi-circular ends. The readings with this needle are made from fine lines on the ends in the direction of its length. The needle-axles rest on agate edges and roll as in the Barrow dip-circle. The instrument has a horizontal circle, but no telescope. There is a compass with a needle about 4 inches long; the box containing the needle fits on two short pins on the horizontal limb.

METHOD OF OBSERVING FOR DIP.

§ **3.** After leveling the instrument and bringing the plane in which the needle rotates into the magnetic meridian, the observations for dip proceed as follows: The ends of the needle, after it comes to rest, are read on the vertical circle. The needle is then reversed on its supports, and the ends read again. The mean of the readings of the ends is free from error arising from the axis of the needle not passing through the center of the vertical circle. Reading with the face of the needle both ways eliminates error due to the magnetic axis of the needle not passing through the needle-points. A similar set of observations is made with the plane of the vertical circle turned 180° in azimuth. The mean of the two sets will be free from error arising from the line joining the zeroes of graduation of the vertical circle not being parallel to the plane of the horizon. After several series of observations have been made as above, the polarity of the needle is reversed and the observations repeated. In reversing the polarity of the needle, care is taken to use the same number of strokes of the bar-magnets from the center of the needle outward on both ends. The polarity is reversed to eliminate error due to the center of gravity of the needle not being in the axis around which it revolves. In determining dip, the mean of the results given by two needles was commonly taken. In the dip-observations of 1858, 1859, and 1860, made by Lieutenant William Proctor Smith, the polarity of needle No. 2 was not reversed, as it was used in determining magnetic intensity by deflections, and for that purpose its magnetic moment had to be kept, so far as possible, constant.

The dip is affected by errors arising from the needle-axles not being truly cylindrical, and also from magnetism of the circle. The method of allowing for this is to determine a constant for the same instrument and needle and apply it to the observed dip. This constant is found by making two complete sets of dip-observations in planes at right-angles to each other, the one 45° east of the magnetic meridian, and the other the same angle west. The correction, c, will then be

$$c = \theta - \theta_1$$

in which

$\theta =$ the true dip,

$\theta_1 =$ the observed dip in magnetic meridian.

$$\cot^2 \theta = \cot^2 \theta' + \cot^2 \theta'',$$

in which θ' and θ'' are the values of the dip found in the planes at right-angles to each other. This correction was always small. For the Barrow dip-circle the correction found in 1858 for needle No. 1 was $+45''$, and for No. 2 $- 4' 37''$. For the same needles, in 1873, Capt. A. N. Lee found the corrections less than the errors of observation.

MAGNETIC DECLINATION.

§ 4. The magnetic declination is the angle made by the vertical plane containing the magnetic axis of a freely suspended needle or magnet with the plane of the true meridian. It varies in the course of a day about $10'$ in summer time and half as much in winter. The end of the needle toward the north has its most easterly position, called the morning eastern elongation, about seven o'clock in the morning. The afternoon western elongation is reached about one o'clock. The values of the declination given in Table II following, beginning with 1876, are the means of these two elongations reduced to the hourly mean for the day. The reduction is made by applying a certain part of the difference of the declinations at the elongations, depending on the time of the year when the observations were made. The part of the difference applied varies from $+0.01$ for the month of June to -0.15 for December. The table used to give the reduction to the mean of hourly observations is in the Coast-Survey Report for 1869, and is derived from a series of magnetic observations made at Girard College Observatory. The declinations given in the table of results for 1858, 1859, and 1860, by Lieutenant William Proctor Smith, are from a single reading of the declinometer taken at any time of the day without regard to the elongation. The declinations of the same years that are marked with a star at Cambridge, Erie, Buffalo, Fort Niagara, Charlotte, and Sacket's Harbor, were derived by finding the magnetic meridian with a dip-circle. In this method the magnetic meridian was found by reading the horizontal limb with the face of the needle toward the north and turned so as to give a dip of exactly 90°, and again with the same face of the needle south and the dip 90°. The mean of the two readings gives the reading corresponding to a line at right angles to the magnetic meridian. The declinations obtained in this way may be in error by a degree or more. In Captain A. N. Lee's work the declinations given are the means of the morning eastern elongation and the elongation at one o'clock in the afternoon, and the same are given in General C. B. Comstock's work in 1870 and 1871.

The declination is determined either with the declinometer or with the magnetometer, the telescope pointing south. The magnet used is a hollow steel cylinder from 3 to 4 inches long, with an external diameter of 0.3 to 0.7 inch. In the end that points to the north is a lens, and in that toward the south a plane glass on which a scale is graduated. The light from the graduations is parallel after leaving the lens so that when viewed with a telescope at a short distance, adjusted to stellar focus, they are plainly visible. The magnet is suspended by one or more fibers of unspun silk. In the older work, with the magnetometer, heavy magnets were used that required six fibers of silk to support them securely. In 1876, beginning with Lieutenant T. N. Bailey's work with the lighter magnets, only one thread was necessary. The thread hangs in a glass cylinder about 1 inch in diameter, and there is a small rectangular wooden box that surrounds the magnet, when suspended, to keep off currents of air. The ends of the box have small windows of glass; through one the magnet-scale is viewed with the telescope, and through the other light from the outside is thrown on the scale by a concave mirror.

Before observations are begun the line of detorsion of the threads suspending the magnet, that is, the direction in which the force of torsion of the threads acts, is brought into the magnetic meridian. This is done approximately by suspending a brass cylinder in place of the magnet, and of about the same weight. After coming to rest the threads are revolved through an angle equal to that which the axis of the cylinder makes with the magnetic meridian. For revolving the threads there is a graduated circle, called the torsion circle, at the top of the glass tube in which the threads hang, and of about the same diameter as the tube. Before making this adjustment the telescope is pointed nearly in the direction of the magnetic meridian, as turning the telescope in azimuth revolves the torsion-circle and changes the plane of detorsion an equal amount. As in declination-observations, the method is to note the scale-reading on the vertical wire and not move the telescope; the torsion of the threads restrains the free motion of the magnet. To allow for this, the declination-magnet is suspended and a reading of the scale is made on the vertical wire; then the torsion-circle is turned 90° and another reading of the scale is made; the torsion-circle is then turned 90° the other side of its first position, and the scale is again read. The mean difference of scale-readings shows the effect on the reading of the scale produced by a twist of the suspending thread of 90°. For the smaller amount of twist of a few minutes or so, such as takes place in changes of declination, the effect is assumed to be in proportion to the angle turned through. For convenience in reduction, the effect of torsion is applied as a correction to the angular value of the scale-division, always increasing that value. The ratio by which the angular value of the scale-interval is increased is usually very small, never being more than $\frac{1}{300}$.

The angle which the magnetic axis of the magnet makes with the meridian is found by noting the reading of the horizontal circle when the vertical wire of the telescope is on a meridian mark or other mark of known azimuth, and again when in coincidence with the division of the scale that marks the magnetic axis of the cylinder. The azimuth of the mark is fixed by observations of the sun or of Polaris. The reading of the scale that corresponds to the magnetic axis is found by noting the reading of the scale when the vertical wire of the telescope is on some division near its middle, then revolving the magnet 180° around its axis and noting the reading of the vertical wire again as soon as the magnet comes to rest. The mean of the two readings gives the reading of the magnetic axis of the cylinder, and is called the scale-zero. The value of a scale-division is between 2′ and 3′. The difference of extreme values in a series of determinations of scale-zero is about 0.2 of a scale-division. As the magnet is nearly always vibrating a little, the mean of the extreme readings of the vertical wire on the scale is taken as the true reading.

The viewing telescope when once fixed with the vertical wire near the zero of the scale in a series of observations is no longer moved, but the changes of declination are shown by the different scale-readings. The angular value of a scale-division must then be known. To find it, the magnet is fixed in the position it has when suspended for observing declination. The vertical wire is brought into coincidence with a division near one end of the scale and a reading of the horizontal circle made; the wire is then brought to the other end of the scale and the circle is read again. The difference of circle-readings, divided by the number of scale-divisions passed over, gives the angular value of a division. The range in a series of values is usually about 0′.06.

Beginning with the work done in 1876, and at only a few places previous to that time, before selecting a place for magnetic observations, tests for local attraction were made. The method of doing this was to lay off two distances of 300 to 600 feet from the proposed position of the instrument and at right angles to each other. The magnetic bearings of these lines were then observed from both ends, and if the differences were found to be considerable, the existence of some local disturbing cause was taken for granted and the site was not occupied. The greatest difference of bearings at the ends of a line for any station where the magnetic elements were finally determined was at Galena, Ill., where it was 3′.

The following were the formulæ used in the reduction for the determination of declination:

$$\frac{H}{F}=\frac{u}{90^\circ-u}=\text{coëfficient of torsion.}$$

$H=$ force of torsion of the thread.

$F=$ horizontal component of the earth's magnetic force.

$u=$ mean difference of scale-readings reduced to angle, caused by turning the suspending thread 90°, first in one direction and then in the other.

$$\delta=\delta'\pm a\left(1+\frac{H}{F}\right)(e-s+fr)$$

$\delta=$ value of declination for the day.

$\delta'=$ angle between the true meridian and the vertical wire of the telescope. It is + when the end of the needle toward the north is west of the meridian.

$a=$ angular value of a scale-division.

$e=$ mean of scale-readings at the elongations. It is + when greater than s.

$s=$ reading of scale corresponding to magnetic axis, or zero of scale, as it is called. It is + when less than e.

$r=$ difference of scale-readings at elongations.

$f=$ factor for reduction to the mean of hourly observations.

The values of f, taken from the United States Coast-Survey Report for 1869, are:

January	−0.089	July	+0.005
February	−0.040	August	−0.023
March	−0.019	September	−0.044
April	−0.068	October	−0.096
May	−0.013	November	−0.096
June	+0.010	December	−0.154

MAGNETIC INTENSITY WITH DIP-CIRCLE.

§ **5.** At the stations where observations were made in 1859 and 1860, except Cambridge and Detroit, the magnetic intensity was determined by deflections with the Barrow dip-circle. Detroit was used as the base-station. The intensity there was determined by deflections and vibrations with the Jones declinometer. The values of total intensity for Detroit that were used in the relative determinations of 1859 and 1860 were 13.710 and 13.785, respectively. With the dip-circle the intensity was determined as follows: Needle No. 2 has four small holes, two that are each 0.5 inch from the needle-end, and the others 0.75 inch. In these holes the weights used in deflecting the needle were inserted. The weights used were small pins. Four angles of deflection as given by different weights were observed at the base-station, and also the angles of deflection produced by the same weights at the other stations where the relative intensity was required. From these the total intensity was derived, and then the horizontal intensity by dividing by the secant of the dip. The range in the four values for each place is on the average about 0.12 in the observations of 1859, and about 0.21 in those of 1860. The largest range was at Minnesota Point, in 1859, the extremes being 13.95 and 14.25.

The formula used in reduction for relative intensity with dip-circle was as follows:

$$R'=R\,\frac{\sin v}{\sin v'}\,\frac{\cos(\Delta'-v')}{\cos(\Delta-v)}$$

$R=$ total intensity at base-station.

$v=$ angle of deflection at base-station.

$\Delta=$ dip at base-station.

$R'=$ total intensity at some other station.

$v'=$ angle of deflection at that station.

$\Delta'=$ dip at that station.

MAGNETIC INTENSITY FROM VIBRATIONS AND DEFLECTIONS.

§ **6.** The method of determining intensity from vibrations and deflections is far more accurate than that which precedes. This was the method followed in 1858 and in the years 1870 to 1880. It consists in determining the angle by which the magnetic axis of a suspended magnet is deflected by another magnet supported at a short distance, and in observing vibrations of the deflecting magnet when suspended. The axes of both magnets in observing deflections must be in the same horizontal plane, and the prolongation of the axis of the deflecting magnet must pass through the center of the suspended magnet. The vibration-observations consist in determining the time in mean solar seconds that it takes the deflecting magnet when suspended to make one vibration. The angle of deflection depends on the ratio of the magnetic moment of the deflecting magnet to the horizontal component of the earth's intensity. The time of vibration depends on the product of these same quantities. Hence the horizontal intensity can be derived, and, by multiplying by the secant of the dip, the total intensity.

The time of vibration of a magnet depends not only on its magnetic moment and the horizontal intensity of the earth, but also on the moment of inertia of the magnet. To obtain this latter, the times of vibration of the magnet are observed when suspended alone and when weighted with a brass ring of which the moment of inertia is known. The ring used for this purpose is made of brass, the diameter about 3 inches, the width of the metal about 0.3 of an inch, and the thickness about half the width. The ring is put on the magnet on its flat side, so that its plane is parallel to the plane in which the magnet vibrates, and with its center in the line of the thread suspending the magnet. As the moment of inertia of a revolving body is the sum of all the products formed by multiplying the mass of each particle by the square of its distance from the center around which it rotates, so in this position of the ring the moment of inertia will be its weight multiplied by one-half the sum of the squares of the internal and external radii. These dimensions of the ring are measured, and the weight is found with a balance. The moment used is in terms of the weight in grains and the radii in parts of a foot. The moment of inertia of the ring will vary slightly with temperature, but the change is so small that it has not been taken into account; neither has the change in the moment of inertia of the magnet been considered, as a difference of 50° F. causes a variation of only about $\frac{1}{10000}$, while the difference of extreme values in a series of ten determinations of this moment is usually $\frac{1}{150}$. The time of vibration is corrected for the change of the magnetic moment when the magnet has a different temperature while vibrating from what it had while being used as a deflector. The rate of the chronometer and the effect of the torsion of the suspending-thread are also allowed for. The arc of vibration was always so small, being about 1°, that no reduction to the time for an infinitely small arc was necessary.

In making vibration experiments the magnet is started to oscillate horizontally by attracting or repelling one of its poles. The oscillation is made at first to surpass the limits of the scale so that it may acquire steadiness of motion before observation is begun. All up-and-down motion is checked. When the motion of the magnet is within the limits of the scale, the extreme readings of the scale on the vertical wire of the telescope are noted. The time that the division of scale half way between these extremes crosses the vertical wire of the telescope is then observed to a fraction of a second. The time of one vibration is derived by dividing the difference of the observed times of two remote crossings of the wire by the number of vibrations the magnet has made in the meantime. An interval of at least 300 vibrations is used in deriving this time. The time of vibration is from five to eight seconds, depending on the magnet. The mean of the times of vibration as given by several sets of observations is used in the reduction. The extreme difference in the values of a vibration in several determinations is usually not more than 0.02 second.

The deflection observations made have been of two kinds, depending on the instrument used. With the magnetometer, the deflecting magnet has its axis perpendicular to the axis of the suspended magnet when deflected. In this case the ratio of the magnetic moment of the magnet to the earth's horizontal intensity is computed with the sine of the angle of deflection. In the case of the declinometer the deflecting magnet remains perpendicular to the magnetic meridian, and the tangent of the angle of deflection is used in the reduction. With the magnetometer, the angle of deflection is measured on the horizontal circle, and with the declinometer, on an ivory scale attached to the telescope. The reflection of the scale is viewed in a mirror fastened to the suspended magnet below its center.

The method of observing with the magnetometer is to bring the vertical wire of the telescope on the scale-zero of the suspended magnet while influenced only by the earth, and then read the horizontal circle. The deflecting magnet is then placed on the deflection-bar at a distance of one foot from the suspended magnet. This distance is measured between the centers of the magnets. The telescope is turned in azimuth until the vertical wire is on the scale-zero, and the circle read again. The difference of the two circle-readings is the angle of deflection. The motion of the telescope also carries the deflecting magnet around. The angle of deflection varies from 10° to 28°, depending on the distances of the magnets apart. The usual distances at which deflections were observed were 1.0 and 1.3 feet. The deflections are observed with the north end of the deflecting magnet toward the suspended magnet, and then with its south end in that direction, and also at the same distances on the opposite side with the north end alternately toward and away from the suspended magnet. Observing under these different circumstances eliminates slight errors arising from inaccuracies in the fixed centers of the magnets and errors due to other instrumental peculiarities.

In Lieutenant William Proctor Smith's observations with the Jones declinometer, in 1858, the distances between centers of magnets were sometimes as great as two feet. With this instrument, the deflecting magnet remaining perpendicular to the magnetic meridian, the observed angles of deflection vary from 1° to 10°. In placing the magnet-centers at a certain distance apart, the errors in the length of the rod used are known and allowed for at the time. The correction for distribution of magnetism in the pair of magnets used, where any was to be made, was derived from observations made specially for that purpose at the two distances 1.0 and 1.32 feet. The lengths of the magnets used were about in the proportion 1 to 1.22, the longer one being used as the deflector. With magnets in this proportion, the correction for distribution of magnetism is a minimum. The change of magnetic moment of the deflecting magnet, caused by temperature, is determined from observed deflections under widely differing temperatures of the deflecting magnet, with the distance from the center of the suspended magnet unchanged. In the early work in 1858 and 1859 the temperature-constant used for the deflecting magnet of the Jones declinometer was determined by Professor Kingston, of the Toronto Magnetic Observatory, and found to be $q=0.00015$ for 1° F. In Captain A. N. Lee's work, in 1872 and 1873, the value used for the deflecting magnet of the Würdemann magnetometer was $q=0.00022$. This value was determined, in 1871, by General C. B. Comstock, from deflections observed when the magnet was at the temperatures 52° and 92° F. in water, both within a quarter of an hour. The coëfficient for the new light magnet first used in 1876 was found to be $q=0.00044$ for 1° F., from observations made when the magnet was packed in melting ice and when in water at temperatures of 90° and 106° F. The observations at the extreme temperatures followed within five minutes of each other.

The effect on the result due to induction is very small, and is neglected. Observations made with the magnetometer require no correction for torsion, as the line of detorsion turns in moving the telescope, and the resistance to the magnet is, therefore, the same when deflected as when hanging freely. In the case of the declinometer, where deflections are measured on the ivory scale by reflection, torsion is taken into account.

The following are the formulæ for magnetic intensity used in reductions:

Vibrations.

$$T^2=T_0^2\left(1+\frac{H}{F}\right)[1-(t'-t)q]$$

T = corrected time of vibration.
T_0 = observed time of vibration.
$\frac{H}{F}$ = coëfficient of torsion, the same as in observations for declination.
t' = temperature of magnet when vibrating.
t = temperature of magnet when deflecting.
q = temperature-constant or change in magnetic moment of deflecting magnet for a temperature-change of 1° F.

The time of vibration is also corrected for rate of chronometer, if considerable.

$$K''=\tfrac{1}{2}(r^2+r'^2)\,w$$

K'' = moment of inertia of ring.
r = outer radius of ring in feet.
r' = inner radius of ring in feet.
w = weight of ring in grains.

$$K = K'' \frac{T^2}{T'^2 - T^2}$$

K = moment of inertia of magnet and stirrup.
T' = corrected time of single vibration of magnet loaded with ring.
T = corrected time of single vibration of unloaded magnet.

$$mX = \frac{\pi^2 K}{T^2}$$

m = magnetic moment of the magnet.
X = horizontal component of the earth's magnetic intensity.
π = 3.14159.

The temperature-constant, q.

$$q = \frac{an \cot u}{t - t_0}$$

q = temperature-constant.
t = higher temperature at which deflection is observed.
t_0 = lower temperature at which deflection is observed.
n = difference of scale-readings corresponding to $(t - t_0)$.
a = angular value of a scale-interval.
u = angle of deflection corresponding to t_0.

Deflections with magnetometer.

$$\frac{m}{X} = \tfrac{1}{2} r^3 \sin u \left(1 - \frac{P}{r^2}\right)$$

m = magnetic moment of deflecting magnet.
X = horizontal component of the earth's magnetic intensity.
r = distance apart of magnet centers in feet.
u = corrected angle of deflection.
P = constant depending on distribution of magnetism in the pair of magnets.

The value of $\frac{m}{X}$ is computed for each distance separately and a mean taken.

$$P = \frac{A - A_1}{\frac{A}{r^2} - \frac{A_1}{r_1^2}} = -\frac{r^2 r_1^5 \sin u' - r_1^2 r^5 \sin u}{r_1^5 \sin u' - r^5 \sin u}$$

A = the value of $\frac{m}{X}$ for the distance in which r is 1 foot.

A_1 = the value of $\frac{m}{X}$ for the distance in which r_1 is 1.32 feet.

The angles of deflection from sets of observations made at different temperatures are reduced to the same temperatures by the following formula:

$$\sin u = \frac{\sin u_0}{1 - (t_0 - t)q}$$

u_0 = observed angle at temperature t_0.
u = angle reduced to standard temperature t.
q = temperature-constant determined as explained above.

Deflections with declinometer.

$$\frac{m}{X} = \tfrac{1}{2} r^3 \tan u \left(1 - \frac{P}{r^2}\right)$$

The notation is the same as in the formula for the magnetometer. The angle u must be corrected for torsion. P is neglected when the least distance of the centers of magnets is not less than four times the magnet-length, and the magnet-lengths are as 1 to 1.22. When the magnets are not of these lengths, P is computed from the formula

$$P = -\frac{r^2 r_1^5 \tan u' - r_1^2 r^5 \tan u}{r_1^5 \tan u' - r^5 \tan u}$$

in which u and u' are angles of deflection observed at two distances, r and r_1.

To find the horizontal intensity of the earth's magnetic force and the magnetic moment of the deflecting magnet,

$$mX = \alpha \text{ from vibrations.}$$

$$\frac{m}{X} = \beta \text{ from deflections.}$$

$$m = \sqrt{\alpha\beta}$$

$$X = \sqrt{\frac{\alpha}{\beta}}$$

The total intensity, φ, is derived from the horizontal intensity and the dip θ.

$$\varphi = X \sec \theta$$

To reduce m to a standard temperature,

$$m_0 = m\ [1 + (t - t_0)q]$$

m_0=value of m at the standard temperature, t_0.
t=temperature at time of deflections.
q=temperature-constant.

CHART OF EQUAL DECLINATIONS FOR THE YEAR 1880.—PLATE XXX.

§ **7.** In compiling this chart all the observations of declination made in the lake region with magnetometer or compass were used. It includes a few values of declination given in the list of Canadian light-houses on the lakes for the year 1880; also recent observations at points on the lakes and in the east made by the United States Coast and Geodetic Survey, given in Appendix No. 9 of the report for 1879. In reducing the declinations observed at different times to the common year 1880, the yearly changes as given in Charles A. Schott's paper in the above-mentioned report were applied. The value of the change for the place nearest the point of observation was the one used. In a table on the chart are given the yearly changes in 1880 for a few places taken from the same report. The method of placing the lines of equal declination on the chart was as follows: The points where observations were made were marked, and the declinations for the year 1880 written after them. Points were then selected by interpolation which were considered to have a declination of a whole number of degrees. The points of equal declination were united by straight lines. Curves were then drawn, with the zig-zag lines as guides, rounding off the angles and making the area between the zig-zag line and the curve the same on both sides. When the observations were numerous in a particular locality a mean was taken for the center of gravity of the group.

§ **8.** TABLE I.—*Relative total intensity for* 1858 *and* 1859 *with weighted dip-circle needles.*

[Observer, Lieutenant William Proctor Smith.]

Station.	Barrow dip-circle.	Dip-circle Troughton & Simms No. 1.
Cambridge, Mass	1.7820	1.7820
Detroit, Mich	1.8157	1.8275
Forestville, Mich	1.8248	....
Thunder Bay Island, Mich	1.8470	....
Sturgeon Point, Mich	1.8267	....
Fort Gratiot, Mich	1.8073	....

Cambridge was used as a base-station in obtaining the relative intensities in Table I.

§ **9.** Table II contains the results of the magnetic work done on the Lake Survey with the magnetometer, declinometer, and dip-circles. The total intensities at most of the stations in 1859 and 1860 are derived from deflections observed with the dip-circle. When not otherwise stated, the intensities are from vibrations and deflections with the declinometer or magnetometer. The dip as given by a single needle is usually the mean of two results with the polarity reversed. In the early work in 1858, 1859, and 1860, the polarity of needle No. 2 was not reversed in observing dip, as its magnetism had to be kept constant, being used in observations for relative total intensity.

TABLE II.

Place.	Latitude.	Longitude.	Date.	Declination.	Needle.	Dip.	Horizontal intensity.	Total intensity.	Instruments; place of observation; observer.
	° ′	° ′		° ′ ″		° ′			
Detroit	42 20	83 03	Apr., 1859	0 42 00 E	1	73 47	3.8379	13.7102	Jones declinometer, Barrow dip-circle, Troughton & Simms dip-circle No. 1. West side of Washington avenue, 200 feet south of Grand River avenue. Lieut. William Proctor Smith.
					2	73 41			
					C	73 35			
						73 41			
Detroit	42 20	83 03	May, 1860	....	....	73 43	3.865	13.785	Jones declinometer, Barrow dip-circle, Troughton & Simms dip-circle No. 1. Same location as above. Lieut. William Proctor Smith.
Detroit	42 20	83 03	May, 1872	0 25 13 E	....	....	3.881	13.720	Würdemann magnetometer, Barrow dip-circle. In park near Lake Survey office, about 400 feet due west of the above location. Capt. A. N. Lee.
			8, 1872	....	2	73 31			
			11, 1872	....	1	73 37			
			29, 1872	....	2	73 36			
						73 35			
Detroit	42 20	83 03	May 5, 1873	....	....	....	3.886	....	Würdemann magnetometer, Barrow dip-circle. In park near Lake Survey office. Capt. A. N. Lee.
			12, 1873	....	1	73 31			
			12, 1873	....	2	73 33			
			14, 1873	0 17 40 E	....	....	3.881		
			14, 1873	....	....	....	3.873		
			16, 1873	....	1	73 37			
			17, 1873	0 16 54 E					
				0 17 17 E		73 34		13.715	
Detroit	42 20	83 03	May 24, 1876	....	1 and 2	73 29.9	....	....	Würdemann magnetometer, Barrow dip-circle. In park near Lake Survey office; coördinates: from lamp-post at southwest corner of Clifford street and West Park Place, S. 13° 06′.3 W.; from northwest corner of Lake-Survey building, S. 26° 01′.0 W. Lieut. T. N. Bailey.
			25, 1876	....	1 and 2	73 38.3			
			June 3, 1876	0 04 49 E	....	....	3.897		
			5, 1876	....	....	....	3.899		
			6, 1876	0 04 36 E	....	....	3.899		
				0 04 42 E		73 34.1		13.781	
Sacket's Harbor	43 57	76 07	June 22, 1859	8 18 W	1	75 47	3.403	13.809	Declination only approximate; observed with dip-circle. Total intensity from deflections with Barrow dip-circle. Place of observation near barracks, 313 feet south of flag-staff. Lieut. William Proctor Smith.
					2	75 41			
						75 44			
Sacket's Harbor	43 57	76 07	June, 1872	8 06 02 W	....	....	3.482	13.847	Würdemann magnetometer, Barrow dip-circle. Barracks, 313 feet south of flag-staff. Capt. A. N. Lee.
			5, 1872	....	1	75 29			
			5, 1872	....	2	75 24			
						75 27			
Sacket's Harbor	43 57	76 07	May 24, 1873	8 17 22 W	....	....	3.472	....	Würdemann magnetometer, Barrow dip-circle. Barracks, 313 feet south of flag-staff. Capt. A. N. Lee.
			26, 1873	....	1	75 25	3.470		
			26, 1873	....	2	75 25			
			27, 1873	8 12 27 W	2	75 23			
				8 15 00 W		75 24		13.770	
Charlotte	43 15	77 37	June 18, 1859	4 31 W	1	75 12	3.561	13.942	Declination only approximate; observed with dip-circle. Total intensity from deflections with Barrow dip-circle. Ninety-one feet northeast of light-house. Lieut. William Proctor Smith.
					2	75 11			
						75 12			
Charlotte	43 15	77 37	June, 1872	3 44 26 W	....	....	3.619	13.920	Würdemann magnetometer, Barrow dip-circle. Latty street, 377 feet southwest from the corner of Broadway. Capt. A. N. Lee.
			7, 1872	....	....	74 57			
			8, 1872	....	....	74 59			
			10, 1872	....	....	74 57			
						74 58			

TABLE II—Continued.

Place.	Latitude.	Longitude.	Date.	Declination.	Needle.	Dip.	Horizontal intensity.	Total intensity.	Instruments; place of observation; observer.
	° ′	° ′		° ′ ″		° ′			
Charlotte	43 15	77 37	May 29, 1873	…………	………	…………	3.634	………	Würdemann magnetometer, Barrow dip-circle. In rear of Methodist Church on Broadway, and about 600 to 800 feet southeast of old station on Latty street. Capt. A. N. Lee.
			30, 1873	3 42 43 W	………	74 51	3.632		
			30, 1873	…………	………	74 49	3.634		
			31, 1873	3 49 36 W	………	74 51			
				3 46 10 W		74 50		13.886	
Buffalo	42 53	78 53	June 11, 1859	2 56 W	1	74 49	3.608	13.743	Declination only approximate; observed with Barrow dip-circle. Total intensity from deflections with Barrow dip-circle. 315 feet south of east end of south pier. Lieut. William Proctor Smith.
			11, 1859	…………	2	74 44			
						74 47			
Buffalo	42 55	78 54	June, 1872	3 52 26 W	………	…………	3.644	13.823	Würdemann magnetometer, Barrow dip-circle. At Fort Porter. Capt. A. N. Lee.
			14, 1872	…………	1	74 43			
			14, 1872	…………	2	74 43			
						74 43			
Buffalo	42 55	78 54	June 3, 1873	…………	………	…………	3.667	………	Würdemann magnetometer, Barrow dip-circle. At Fort Porter. Capt. A. N. Lee.
			4, 1873	3 55 40 W	1	74 26			
			5, 1873	4 01 01 W	………	…………	3.667		
			6, 1873	…………	1	74 30			
			6, 1873	…………	2	74 31			
				3 58 20 W		74 29		13.707	
Cleveland	41 30	81 40	July 4, 1859	…………	1	73 19	3.956	13.794	Total intensity from deflections with Barrow dip-circle. Jones declinometer, Barrow dip-circle. 166 feet south-west of northeast corner of marine hospital. Lieut. William Proctor Smith.
			4, 1859	…………	2	73 21			
			5, 1859	0 46 00 W					
						73 20			
Cleveland	41 30	81 40	June, 1872	0 44 52 W	………	…………	4.017	13.833	Würdemann magnetometer, Barrow dip-circle. Same location as above. Capt. A. N. Lee.
			17, 1872	…………	1	73 07			
			17, 1872	…………	2	73 09			
			18, 1872	…………	1	73 07			
						73 08			
Cleveland	41 30	81 40	June 16, 1873	0 51 17 W	1	73 08	3.998	………	Würdemann magnetometer, Barrow dip-circle. Same location as above. Capt. A. N. Lee.
			17, 1873	0 50 31 W	2	73 08	3.993		
			17, 1873	…………	1	73 09			
				0 50 54 W		73 08		13.773	
Grand Haven	43 05	86 13	Aug. 18, 1859	4 24 15 E	1	74 08	3.815	13.987	Total intensity from deflections with Barrow dip-circle. Jones declinometer, Barrow dip-circle. 100 yards west of Detroit and Milwaukee freight depot. Lieut. William Proctor Smith.
					2	74 12			
						74 10			
Grand Haven	43 05	86 13	Aug. 27, 1873	…………	………	…………	3.854	………	Würdemann magnetometer, Barrow dip-circle. Station in lot in rear of jail in center of lot. Capt. A. N. Lee.
			28, 1873	…………	1	73 55	3.848		
			28, 1873	3 28 43 E	2	74 01	3.848		
			29, 1873	3 27 40 E	1	73 58			
				3 28 12 E		73 58		13.940	
Michigan City	41 43	86 54	Aug. 28, 1859	5 22 39 E	1	73 02	4.013	13.789	Total intensity from deflections with Barrow dip-circle. Jones declinometer, Barrow dip-circle. Northeast corner of light-house inclosure. Lieut. William Proctor Smith.
					2	73 02			
						73 02			

TABLE II—Continued.

Place.	Latitude.	Longitude.	Date.	Declination.	Needle.	Dip.	Horizontal intensity.	Total intensity.	Instruments; place of observation; observer.
	° ′	° ′		° ′ ″		° ′			
Michigan City....	41 43	86 54	Aug. 25, 1873	3 58 23 E	1	72 42	4.097	....	Würdemann magnetometer, Barrow dip-circle. Station on beach 300 feet north of light-house. Capt. A. N. Lee.
			25, 1873	....	2	72 45	4.087		
			26, 1873	3 59 41 E	1	72 43	4.089		
				3 59 02 E		72 43		13.770	
Milwaukee	43 03	87 55	Aug. 20, 1859	6 20 06 E	1	73 56	3.858	13.956	Total intensity from deflections with Barrow dip-circle. Jones declinometer, Barrow dip-circle. In garden southeast corner of Fourth and Poplar streets. Lieut. William Proctor Smith.
			20, 1859	....	2	73 58			
						73 57			
Milwaukee	43 03	87 55	Aug. 22, 1873	6 22 23 E	1	73 43	3.897	....	Würdemann magnetometer, Barrow dip-circle. Station south side of Spring street, between Fourth and Fifth. Capt. A. N. Lee.
			22, 1873	....	2	73 43	3.897		
						73 43		13.900	
Fort Gratiot	43 00	82 25	Oct. 9, 1858	....	1	74 30	3.6890	13.7983	Jones declinometer, Barrow dip-circle. 75 feet southeast of astronomical station. Lieut. William Proctor Smith.
			9, 1858	....	2	74 26			
			13, 1858	1 20 00 E					
						74 28			
Fort Gratiot	43 01	82 25	May 7, 1860	....	....	74 39	3.664	13.840	Total intensity from deflections with Barrow dip-circle. Jones declinometer, Barrow dip-circle. On the shore of Lake Huron, 30 yards due south of geodetic station. Lieut. William Proctor Smith.
			10, 1860	1 23.5 W					
Fort Gratiot	43 01	82 25	July 15, 1873	0 37 32 W	1	74 20	3.716	....	Würdemann magnetometer, Barrow dip circle. Station 200 feet west of south end of old bake-house. Capt. A. N. Lee.
			15, 1873	....	2	74 24	3.713		
			16, 1873	0 36 31 W	1	74 24	3.719		
				0 37 02 W		74 22		13.790	
Forestville........	43 40	82 34	June 19, 1858	....	1	75 02	....	....	Jones declinometer, Barrow dip-circle. 50 yards northeast of astronomical station. Lieut. William Proctor Smith.
			19, 1858	....	2	74 58			
			July, 1858	....	....	....	3.5246	13.6086	
			Aug. 1, 1858	0 18 00 E					
						75 00			
Forestville	43 40	82 34	July 11, 1873	....	1	74 53	3.636	....	Würdemann magnetometer, Barrow dip-circle. Station 100 feet east of astronomical post. Capt. A. N. Lee.
			11, 1873	....	2	74 56	3.631		
			12, 1873	1 32 56 W	2	74 53	3.642		
			13, 1873	1 28 30 W					
				1 30 43 W		74 54		13.957	
Marquette........	46 33	87 23	July 13, 1859	4 01 17 E	1	77 20	3.089	14.034	Total intensity from deflections with Barrow dip-circle. Jones declinometer, Barrow dip-circle. Lake shore, half a mile northwest of light-house. Lieut. William Proctor Smith.
					2	77 14			
						77 17			
Marquette........	46 33	87 23	July 26, 1873	....	......	....	3.441	....	Würdemann magnetometer, Barrow dip-circle. Station on beach 800 feet northwest of light-house. Capt. A. N. Lee.
			28, 1873	4 20 57 E	......	....	3.425		
			29, 1873	4 40 24 E	1	75 50	3.432		
			29, 1873	....	2	75 46			
				4 30 40 E		75 48		13.991	
Copper Harbor....	47 28	87 51	July 14, 1859	....	1	78 06	2.894	14.033	Total intensity from deflections with Barrow dip-circle. Fort Wilkins, 200 yards north of flag-staff. Lieut. William Proctor Smith.
					2	78 05			
						78 06			
Copper Harbor....	47 28	87 51	Aug. 1, 1873	....	....	....	2.923	....	Würdemann magnetometer, Barrow dip-circle. Station Fort Wilkins, 100 feet north of flag-staff. Capt. A. N. Lee.
			2, 1873	4 02 17 E	1	78 04	2.932		
			3, 1873	4 04 23 E	2	78 02	2.928		
			3, 1873	....	1	78 01			
				4 03 20 E		78 02		14.116	

TABLE II—Continued.

Place.	Latitude.	Longitude.	Date.	Declination.	Needle.	Dip.	Horizontal intensity.	Total intensity.	Instruments; place of observation; observer.
	° ′	° ′		° ′ ″		° ′			
Superior City	46 43	92 04	Sept. 20, 1870	10 30 E	1	76 28	3.227	13.790	Würdemann magnetometer, Barrow dip-circle. Station 200 feet north of southwest end of Mr. Robbins' house. Gen. C. B. Comstock.
Minnesota Point	46 47	92 05	July 22, 1859	9 25 15 E	1	76 49	3.222	14.038	Total intensity from deflections with Barrow dip-circle. Jones declinometer, Barrow dip-circle. 75 yards west of light-house. Lieut. William Proctor Smith.
					2	76 39			
						76 44			
Minnesota Point	46 45	92 05	June 16, 1871	....	1	76 21	....	....	Würdemann magnetometer, Barrow dip-circle; declinations mean of elongations morning and afternoon. Station latitude-post, N. 63° 53′ E., and 389 feet from North Base. Gen. C. B. Comstock.
			20, 1871	10 40.7 E					
			25, 1871	....	....	....	3.319	14.065	
			27, 1871	10 39.8 E					
				10 40 E					
Minnesota Point	46 43	92 02	June 23, 1871	....	1	76 26	3.266	13.925	Würdemann magnetometer, Barrow dip-circle. Point 114 feet north of South Base. Gen. C. B. Comstock.
Duluth	46 45	92 05	Aug. 12, 1873	....	....	....	3.391	....	Würdemann magnetometer, Barrow dip-circle. Station on Minnesota Point, 540 feet southeast of Mr. Peck's house. Capt. A. N. Lee.
			13, 1873	....	2	76 18	3.397		
			13, 1873	11 57 33 E	1	76 15			
			14, 1873	....	2	76 19			
			15, 1873	11 47 04 E	....	....	3.397		
				11 52 18 E		76 17		14.318	
Erie	42 09	80 05	June 7, 1859	1 44 W	1 and 2	74 02	3.784	13.661	Declination only approximate; observed with dip-circle Troughton & Simms No. 1; total intensity from deflections with Barrow dip-circle. Station 150 yards northwest of the light-house. Lieut. William Procter Smith.
					C	73 50			
						73 56			
Erie	42 09	80 05	June 11, 1873	....	....	....	3.829	....	Würdemann magnetometer, Barrow dip-circle. Station on peninsula opposite the town, near beacon light-keeper's house. Capt. A. N. Lee.
			12, 1873	....	1	73 46	3.829		
			12, 1873	2 01 20 W	2	73 47			
			13, 1873	2 00 05 W	2	73 45			
				2 00 42 W		73 46		13.697	
Thunder Bay Island.	45 02	83 10	Aug. 20 to 25, 1858.	1 14 00 E	1	76 27	3.2411	13.7916	Jones declinometer, Barrow dip-circle. 75 feet northeast of astronomical station. Lieut. William Proctor Smith.
					2	76 22			
						76 24			
Sand Point, Saginaw Bay.	43 55	83 23	Sept. 13 to 18, 1858.	0 32 00 E	1	75 01	3.5718	13.8348	Jones declinometer, Barrow dip-circle. 15 yards east of astronomical station. Lieut. William Proctor Smith.
					2	75 03			
						75 02			
Sturgeon Point, Mich.	44 43	83 14	Sept. 25 to 29, 1858.	1 02 00 E	1	76 03	3.3388	13.7865	Jones declinometer, Barrow dip-circle. 40 feet east of astronomical station. Lieut. William Proctor Smith.
					2	75 55			
						75 59			
Cambridge, Mass.	42 23	71 08	Mar. 7, 1859	10 48 W	1	74 26	3.5964	13.3180	Declination only approximate; observed with Barrow dip-circle. Jones declinometer, Barrow dip-circle, and dip-circle Troughton & Simms No. 1. West transit room of observatory. Lieut. William Proctor Smith.
					2	74 21			
					C	74 13			
						74 20			

TABLE II—Continued.

Place.	Latitude.	Longitude.	Date.	Declination.	Needle.	Dip.	Horizontal intensity.	Total intensity.	Instruments; place of observation; observer.
	° ′	° ′		° ′ ″		° ′			
Fort Niagara	43 15	79 04	June 15, 1859	2 56 W	1 2	74 54 74 48 74 51	3.584	13.716	Declination only approximate; observed with Barrow dip-circle. Total intensity from deflections with Barrow dip-circle. Station 125 feet west of flag-staff. Lieut. William Proctor Smith.
Toronto, Ont......	43 39	79 23	June 25 to 30, 1859.	2 11 34 W	....	75 24	3.480	13.800	Declination and dip as given by Professor Kingston; total intensity from deflections with Barrow dip-circle. Station Provincial Magnetic Observatory. Lieut. William Proctor Smith.
Ontonagon	46 52	89 20	July 19, 1859	5 03 10 E	1 2	77 30 77 24 77 27	3.064	14.100	Total intensity from deflections with Barrow dip-circle. Jones declinometer, Barrow dip-circle. 50 yards northeast of light-house. Lieut. William Proctor Smith.
Cape Ipperwash, Ont.	43 13	82 00	May 8, 1860	0 03.3 E	....	74 46	3.633	13.940	Total intensity from deflections with Barrow dip-circle. Jones declinometer, Barrow dip-circle. On lake shore, 40 yards south of geodetic station; large amount of iron pyrites in vicinity. Lieut. William Proctor Smith.
Wabley, Mich	43 22	82 32	May 11, 1860	1 05.0 W	....	74 41	3.670	13.893	Total intensity from deflections with Barrow dip-circle. Jones declinometer, Barrow dip-circle. About 50 feet from lake shore, on bluff 30 feet high, 200 yards south of geodetic station. Lieut. William Proctor Smith.
Goderich, Ont.....	43 44	81 43	July 19, 1860	1 42.0 W	....	75 02	3.575	13.841	Total intensity from deflections with Barrow dip-circle. Jones declinometer, Barrow dip-circle. Quarter of a mile south of town hall; 30 yards northeast of astronomical observatory. Lieut. William Proctor Smith.
Mackinac Island ..	45 51	84 36	July 28, 1860	1 42.4 E	....	76 43	3.242	14.108	Total intensity from deflections with Barrow dip-circle. Jones declinometer, Barrow dip-circle. Without the fort, about 100 yards northwest of flag-staff. Lieut. William Proctor Smith.
Cove Island	45 20	81 43	Aug. 28, 1860	*3 58.6 W	....	76 32	3.267	14.063	Total intensity from deflections with Barrow dip-circle. Jones declinometer, Barrow dip-circle. 200 feet southwest of light-house, about 50 feet south of astronomical observatory. Lieut. William Proctor Smith.
Northport	45 08	85 36	Sept. 2, 1860	2 33.7 E	....	76 06	3.340	13.905	Total intensity from deflections with Barrow dip-circle. Jones declinometer, Barrow dip-circle. On shore of Grand Traverse Bay, about 30 yards south of mouth of small stream. Lieut. William Proctor Smith.

* Misprint in report of 1860; 3° 48′.6 should be 3° 58′.6.

TABLE II—Continued.

Place.	Latitude.	Longitude.	Date.	Declination.	Needle.	Dip.	Horizontal intensity.	Total intensity.	Instruments; place of observation; observer.
	° ′	° ′		° ′ ″		° ′			
South Manitou Island.	45 02	86 06	Sept. 11, 1860	3 09.1 E		76 01	3.360	13.937	Total intensity from deflections with Barrow dip-circle. Jones declinometer, Barrow dip-circle. On lake shore, east side of the island, 200 yards southwest of dock. Lieut. William Proctor Smith.
Beaver Island	45 45	85 30	Oct. 2, 1860	2 43.0 E		76 43	3.236	14.084	Total intensity from deflections with Barrow dip-circle. Jones declinometer, Barrow dip-circle. Harbor on lake shore, about 50 yards due west of light-house. Lieut. William Proctor Smith.
Tip-Top, Ont	48 15	86 08	Aug. 26, 1871	0 03 E		78 56	2.708	13.790	Würdemann magnetometer, Barrow dip-circle. Station astronomical post, near shore, on granite rock, vein of magnetic oxide within a mile. General C. B. Comstock.
Sault Ste. Marie, Fort Brady.	46 30	84 20	July 22, 1873		1	77 30	3.041		Würdemann magnetometer, Barrow dip-circle. Station 273 feet northwest of flag-staff. Capt. A. N. Lee.
			22, 1873	0 04 31 W	2	77 32			
			23, 1873	0 05 19 W	2	77 30	3.046		
			23, 1873				3.046		
				0 04 55 W		77 30		14.064	
Saint Paul	44 57	93 05	Aug. 18, 1873				3.646		Würdemann magnetometer, Barrow dip-circle. Station 50 feet west of south front of county court-house. Capt. A. N. Lee.
			19, 1873		1	74 53	3.651		
			19, 1873	10 55 59 E	2	74 56	3.647		
			20, 1873	10 55 30 E	2	74 58			
				10 55 45 E		74 55		14.018	
Wabasha, Minn	44 18	92 07	Aug. 7, 1876			74 19.9			Würdemann magnetometer with light magnets, Barrow dip-circle. Coördinates: from southwest corner of Riverside House 105 feet; from northwest corner of Riverside House 58 feet. Lieut. T. N. Bailey.
			8, 1876	8 04.6 E		74 23.3			
			9, 1876	8 04.0 E					
			11, 1876				3.7208		
			11, 1876				3.7214		
			12, 1876				3.7264		
			12, 1876				3.7268		
				8 04.3 E		74 21.6	3.7238	13.814	
La Crosse, Wis	43 49	91 15	Sept. 18, 1876	8 00.7 E		73 57.2			Würdemann magnetometer with light magnets, Barrow dip-circle. Coördinates from southeast corner of Main street park fence N. 43° 06′.7 W., 97.6 feet. Lieut. C. F. Powell.
			19, 1876			73 57.8			
			20, 1876	8 00.7 E		73 56.6			
			21, 1876				3.8435		
			21, 1876				3.8473		
			22, 1876				3.8447		
			22, 1876				3.8416		
				8 00.7 E		73 57.2	3.8443	13.907	
*Galena, Ill	42 25	90 26	Sept. 28, 1876				3.9953		Würdemann magnetometer with light magnets, Barrow dip-circle. Coördinates from United States Lake-Survey astronomical post, 1873, S. 72° 34′.8 E., 59.2 feet. Lieut. C. F. Powell.
			28, 1876			73 06.3	3.9985		
			29, 1876			73 11.8	3.9899		
			29, 1876	9 08.5 E			3.9865		
			30, 1876	9 08.9 E					
				9 08.7 E		73 09.0	3.9926	13.774	
Rockford, Ill	42 17	89 07	Oct. 5, 1876	5 19.2 E					Würdemann magnetometer with light magnets, Barrow dip-circle. Coördinates from east post at north corner of east park fence S. 12° 24′.4 E., 210.2 feet. Lieut. C. F. Powell.
			6, 1876	5 16.6 E		72 52.2			
			7, 1876				4.0799		
			7, 1876				4.0831		
			9, 1876	5 19.1 E		72 44.7			
			10, 1876				4.0832		
			10, 1876				4.0846		
			11, 1876			72 48.1			
				5 18.3 E		72 48.3	4.0827	13.810	

*Of the results here Lieut. C. F. Powell says: "They may be looked upon with some suspicion, the lead abounding in that region is found imbedded in ferruginous clay, and has associated with it iron ores in small quantities and calamine in larger."

TABLE II—Continued.

Place.	Latitude.	Longitude.	Date.	Declination.	Needle.	Dip.	Horizontal intensity.	Total intensity.	Instruments; place of observation; observer.
	° ′	° ′		° ′		° ′			
Marshall, Mich ...	42 16	84 58	Oct. 15, 1876	1 43.9 E	....	....	....	....	Würdemann magnetometer with light magnets, Barrow dip-circle. Coördinates from United States Lake-Survey astronomical post, 1875, S. 21° 18′.4 W. 95 feet. Lieut. C. F. Powell.
			16, 1876	1 40.8 E	....	73 33.6			
			17, 1876	1 40.6 E	....	73 28.3	3.9231		
			17, 1876	....	....	....	3.9210		
			18, 1876	....	....	....	3.9217		
			18, 1876	....	....	....	3.9273		
				1 41.7 E		73 30.9	3.9233	13.826	
Mount Forest, Ill.	41 45	87 52	Aug. 26, 1876	4 37.3 E	....	....	....	....	Jones declinometer, dip-circle Troughton & Simms No. 2. Coordinates from United States Lake-Survey astronomical post, 1876, north 200 feet. Lieut. D. W. Lockwood.
			28, 1876	4 33.8 E					
			29, 1876	....	....	71 48.6			
			30, 1876	....	....	71 57.1			
			Sept. 1, 1876	....	....	....	3.9730		
			1, 1876	....	....	....	3.9708		
			2, 1876	....	....	....	3.9710		
			2, 1876	....	....	....	3.9705		
				4 35.6 E		71 52.8	3.9713	12.770	
Saginaw, Mich	43 25	83 58	Sept. 16, 1876	....	....	....	3.6724	....	Jones declinometer, dip-circle Troughton & Simms No. 2. Coordinates from United States Lake-Survey astronomical post, 1876, northeast 100 feet. Lieut. D. W. Lockwood.
			16, 1876	....	....	....	3.6709		
			18, 1876	....	....	....	3.6709		
			18, 1876	....	....	....	3.6731		
			20, 1876	0 24.1 E					
			21, 1876	0 22.8 E	....	73 36.4			
			22, 1876	....	....	73 36.0			
				0 23.4 E		73 36.2	3.6718	13.007	
Saint Louis, Mo ...	43 24	84 36	Oct. 12, 1876	1 00.4 E	....	....	....	....	Jones declinometer, dip-circle Troughton & Simms No. 2. Coordinates from United States Lake-Survey astronomical post, 1876, southeast 150 feet. Lieut. D. W. Lockwood.
			13, 1876	0 56.9 E					
			14, 1876	....	....	....	3.6488		
			14, 1876	....	....	73 48.0	3.6513		
			15, 1876	....	....	73 49.8	3.6359		
			15, 1876	....	....	....	3.6369		
				0 58.6 E		73 48.9	3.6432	13.070	
Memphis, Tenn ...	35 09	90 03	July 19, 1877	6 47.1 E	....	....	....	....	Würdemann magnetometer. Magnetic post same as triangulation-station Memphis. Lieut. C. F. Powell.
			20, 1877	6 47.0 E					
				6 47.0 E					
Rock Island, Ill ...	41 31	90 34	Sept. 15, 1878	6 57 52 E	....	....	....	....	Jones declinometer, Barrow dip-circle. Astronomical post to magnetic station, 488 feet, N. 43° 52′ E. Lieut. C. F. Powell.
			16, 1878	6 56 55 E					
			17, 1878	6 58 41 E					
			19, 1878	....	....	....	4.2020		
			19, 1878	....	....	....	4.2059		
			20, 1878	....	....	....	4.2165		
			20, 1878	....	....	....	4.2072		
			Oct. 5, 1878	....	1	72 20.0			
			5, 1878	....	2	72 30.2			
			7, 1878	....	1	71 55.6			
				6 57 49 E		72 15.3	4.2079	13.8059	
Red Wing, Minn ..	44 34	92 32	Oct. 10, 1878	7 48 51 E	....	....	....	....	Jones declinometer, Barrow dip-circle. Astronomical post to magnetic station, 284.2 feet, N. 11° 49′ W. Lieut. C. F. Powell.
			11, 1878	7 50 21 E					
			23, 1878	....	....	....	3.6711		
			23, 1878	....	2	74 20.6	3.6959		
			24, 1878	....	....	....	3.6779		
			24, 1878	....	2	74 16.6	3.6729		
			26, 1878	....	2	74 06.1			
				7 49 36 E		74 14.4	3.6795	13.5474	

§ **10.**—TABLE III.—*Declinations taken from the list of Canadian lights on the Lakes and Saint Lawrence River for* 1880.

[Yearly changes are taken from Appendix No. 9 of Coast-Survey Report for 1879. The plus (+) sign indicates an increasing west declination.]

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.
	° ′	° ′		° ′	′	°
Collingwood	44 31	80 12	1869	2 20 W.	+3.0	2.9 W.
Isle of Coves	45 20	81 43	1870	0 50 W.	+3.3	1.4 W.
Goderich	43 45	81 43	1870	0 50 W.	+3.0	1.3 W.
Long Point	42 33	80 09	1870	1 40 W.	+4.2	2.4 W.
Mohawk Island	42 50	79 37	1870	2 40 W.	+4.2	3.4 W.
Oshaway Port	43 52	78 48	1869	3 30 W.	+4.5	4.3 W.
Darlington	43 52	78 38	1869	3 30 W.	+4.5	4.3 W.
Point Peter	43 51	77 10	1869	6 W.	+4.5	6.8 W.
Between Kingston and Gananoque			1870	7 15 W.	+3.0	7.8 W.
Cornwall Canal	45 00	74 55	1869	9 30 W.	+3.1	10.1 W.

§ **11.**—TABLE IV.—*Declinations derived from Coast-Survey Reports.*

[The yearly change is taken from Appendix No. 9, Coast-Survey Report for 1879, and from the Coast-Survey Report for 1862. The plus (+) sign indicates an increasing west declination.]

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.	Authority.
	° ′	° ′		° ′	′	°	
Silver Lake, Pa	41 57	76 02	Aug. 23, 1841	4 30 W.	+3.0	6.5 W.	Coast-Survey Report for 1862.
Curwinsville, Pa	40 58	78 36	Aug. 1, 1841	1 45 W.	+3.0	3.7 W.	Do.
Ellicotville, N. Y	42 18	78 44	Aug. 14, 1841	2 36 W.	+3.0	4.5 W.	Do.
Bath, N. Y	42 21	77 21	Aug. 11, 1862	4 48 W.	+2.7	5.6 W.	Do.
Williamsport, Pa	41 14	77 02	Aug. 13, 1862	4 26 W.	+2.7	5.3 W.	Do.
Montreal, P. Q.*	45 30.5	73 34.9	Sept. 25, 1879	13 40.5 W.	+3.1	13.8 W.	Coast-Survey Report for 1879.
Burlington, Vt	44 28.2	73 12.3	Oct. 14, 15, 1873	11 19.0 W.	+6.0	11.9 W.	Do.
Albany, N. Y	42 39.2	73 45.8	Oct. 21, 24, 1879	9 51.7 W.	+3.7	9.9 W.	Do.
Oxford, Chenango County, N. Y.	42 26.5	75 40.5	May, June, 1874	6 55.7 W.	+4.3	7.4 W.	Do.
Toronto, Ont	43 39.4	79 23.4	Oct. 18, 1880	3 41.1 W.	+4.5	3.6 W.	Do.
Erie, Pa	42 07.8	80 05.4	Nov., 1877	3 00.0 W.	+4.2	3.0 W.	Do.
Cleveland, Ohio	41 30.3	81 42.0	July 9, 10, 12, 1880	1 38.5 W.	+2.5	1.3 W.	Do.
New York, N. Y	40 42.7	74 00.4	July 17, 18, 1879	7 32.0 W.	+2.5	7.8 W.	Do.
Philadelphia, Pa	39 56.9	75 09.0	Oct. 2, 3, 5, 6, 1877	6 02.2 W.	+4.9	6.3 W.	Do.
Harrisburg, Pa	40 15.9	76 52.9	Sept. 25, 26, 1877	4 53.5 W.	+3.3	5.0 W.	Do.
Sault Ste. Marie, Mich	46 29.9	84 20.1	Aug. 6, 7, 1880	1 04.5 W.	+3.3	1.0 W.	Do.
Grand Haven, Mich	43 05.2	86 12.6	July 20, 21, 1880	2 25.7 E.	+5.1	2.4 E.	Do.

* The declinations for 1880 at this and the following places are computed with periodic formulae. The yearly change given is the rate of change in 1880.

§ **12.** Table V contains magnetic declinations observed on the Lake Survey with theodolite compass-needles. In the years after 1874, on Lakes Erie and Ontario, tests for local attraction were made at most of the sites occupied. It was always found to be very small. The test was made by observing the difference of bearings at the ends of lines radiating from the station. In the later work, 1870 to 1880, the declination is usually the mean of the results given by two and sometimes more instruments on different days. The range in the results given by different instruments at the same place does not often exceed 30′. The plus (+) sign indicates an increasing west declination and a decreasing east declination.

TABLE V.

LAKE SUPERIOR.

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.	Observer.
	° ′	° ′		° ′	′	°	
Agate Harbor	47 28	88 03	1855	5 20 E.	+3.4	3.9 E.	Lt. W. F. Raynolds.
Eagle River	47 25	88 17	1855	6 46 E.	+3.4	5.3 E.	Do.
West of Eagle River	47 23	88 21	1855	6 12 E.	+3.4	4.8 E.	Do.
Ontonagon	46 52	89 20	1855	6 11 E.	+3.4	4.8 E.	Do.
Eagle Harbor	47 28	88 08	1855	2 40 E.	+3.4	1.3 E.	J. U. Mueller.
Grand Island, north end	46 34	86 41	July, 1859	3 40 E.	+3.4	2.5 E.	G. W. Lamson.
Grand Island, south end	46 28	86 40	Sept., 1859	4 13 E.	+3.4	3.0 E.	Do.
Lester River	46 50	92 00	July, 1861	7 39 E.	+4.5	6.2 E.	H. C. Penny.
Amiuicon River, 5½ miles east of.	46 43	91 45	Aug., 1861	10 17 E.	+4.5	8.9 E.	Do.
Knife River	46 57	91 46	Aug., 1861	12 45 E.	+4.5	11.3 E.	Do.
Grand Portage Bay	47 58	89 39	Aug., 1861	6 04 E.	+3.4	5.0 E.	James Carr.
South of Spirit Lake	46 41	92 11	July, 1861	9 42 E.	+4.5	8.3 E.	W. H. Hearding.
Island in Spirit Lake	46 41	92 11	July, 1861	9 50 E.	+4.5	8.4 E.	Do.
Near South Base, Minnesota Point.	46 43	92 02	July 12, 1861	10 12 E.	+4.5	8.8 E.	Do.
Rice's Point	46 45	92 05	July 22, 1861	9 35 E.	+4.5	8.2 E.	Do.
Near North Base, Minnesota Point.	46 45	92 04	July 24, 1861	11 07 E.	+4.5	9.7 E.	Do.
Saint Louis Bay, north of the island.	46 43	92 10	Aug. 9, 1861	11 44 E.	+4.5	10.3 E.	Do.
Fond du Lac	46 39	92 15	Aug. 29, 1861	9 42 E.	+4.5	8.3 E.	Do.
Portage Entry	46 59	88 25	1863	4 37 E.	+3.4	3.6 E.	J. U. Mueller.
Dollar Bay	47 07	88 29	1863	4 03 E.	+3.4	3.1 E.	Do.
Torch Bay	47 05	88 26	1863	3 41 E.	+3.4	2.7 E.	Do.
Portage Lake, north end	47 13	88 36	1863	4 33 E.	+3.4	3.6 E.	Do.
Copper Harbor, Fort Wilkins	47 28	87 51	July 1, 1864	4 44 E.	+3.4	3.8 E.	H. Gillman.
Torch Lake	47 12	88 24	July 23, 1864	5 11 E.	+3.4	4.3 E.	Do.
Keweenaw Bay	46 52	88 28	Sept. 8, 1864	4 45 E.	+3.4	4.0 E.	Do.
Isabelle Point	47 21	87 56	June 1, 1865	4 53 E.	+3.4	4.0 E.	A. Molitor.
Traverse Point, 2 miles north of.	47 11	88 15	Aug. 11, 1865	3 55 E.	+3.4	3.1 E.	Do.
Gratiot River	47 21	88 27	June, 1865	7 37 E.	+3.4	6.7 E.	H. Gillman.
Salmon Trout River	47 09	88 45	July 27, 1865	7 41 E.	+3.4	6.8 E.	Do.
Point above Elm River	47 05	88 55	Aug., 1865	6 41 E.	+3.4	5.8 E.	Do.
Misery River	47 00	88 59	Aug. 25, 1865	7 43 E.	+3.4	6.8 E.	Do.
Ontonagon	46 52	89 19	Sept. 27, 1865	6 31 E.	+3.4	5.7 E.	Do.
Granite Point	46 39	87 27	June, 1866	3 04 E.	+3.4	2.3 E.	A. Molitor.
Little Iron River	46 49	87 35	Aug., 1866	4 21 E.	+3.4	3.6 E.	Do.
Grand Island, near light-house.	46 34	86 40	Aug. 29, 1867	3 06 E.	+3.4	2.4 E.	O. B. Wheeler.
Pine Cliff	46 42	85 53	Sept. 10, 1867	2 01 E.	+3.3	1.3 E.	Do.
Iroquois Point	46 29	84 37	Sept. 26, 1867	0 39 E.	+3.3	0.0	Do.
Whitefish Point	46 46	84 57	Sept., 1867	0 55 E.	+3.3	0.2 E.	A. Molitor.
Laughing Fish River	46 28	86 55	1867	5 00 E.	+3.4	4.3 E.	H. Gillman.
Shot Point	46 31	87 10	Aug., 1867	5 19 E.	+3.4	4.6 E.	Do.
Chocolate River	46 30	87 20	Aug., 1867	5 25 E.	+3.4	4.7 E.	Do.
Scoville Point, Isle Royale	48 10	88 26	1867	6 28 E.	+3.4	5.7 E.	Lieut. B. D. Greene.
Fish Island, Isle Royale	48 09	88 37	1867	5 08 E.	+3.4	4.4 E.	A. C. Lamson.
South Sandy Island, White-fish Bay.	46 48	84 39	1867	0 15 E.	+3.3	0.5 W.	O. N. Chaffee.
Goulais Point	46 41	84 33	July, 1867	0 23 E.	+3.3	0.3 W.	Do.
Iron River	46 50	89 34	May, 1868	6 48 E.	+3.4	6.1 E.	Lieut. J. E. Griffith.
Lone Rock, near Porcupine Mountains	46 48	89 49	June, 1868	11 30 E.	+3.4	10.8 E.	Do.
Wright's Island, Isle Royale.	47 58	88 49	1868	4 16 E.	+3.4	3.6 E.	Lieut. B. D. Greene.
Siskawit Point, Isle Royale.	47 54	88 54	1868	4 30 E.	+3.4	3.8 E.	Do.
South shore of Isle Royale	47 50	89 06	1868	4 56 E.	+3.4	4.3 E.	Do.
Todd's Harbor, Isle Royale	48 05	88 45	June, 1868	6 30 E.	+3.4	5.8 E.	Lieut. J. C. Mallery.
Washington Harbor, Isle Royale.	47 53	89 13	1868	6 36 E.	+3.4	5.9 E.	A. C. Lamson.

TABLE V—Continued.

LAKE SUPERIOR—Continued.

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.	Observer.
	° ′	° ′		° ′	′	°	
Black River	46 40	90 02	July 4, 1868	7 50 E.	+3.4	7.2 E.	H. Gillman.
Little Girl's Point	46 37	90 17	Aug., 1868	8 00 E.	+3.4	7.3 E.	Do.
Oronto River	46 34	90 26	Aug. 8, 1868	6 58 E.	+3.4	6.3 E.	Do.
Terrace Point	47 43	90 26	Aug., 1868	8 12 E.	+3.4	7.5 E.	Do.
Pigeon Point	48 00	89 30	June, 1868	9 30 E.	+3.4	8.8 E.	Lieut. W. E. Rogers.
Grand Portage Island	47 57	89 39	July, 1868	5 50 E.	+3.4	5.2 E.	Do.
Brulé River	47 48	90 03	Aug., 1868	9 30 E.	+3.4	8.8 E.	Do.
Bad River	46 38	90 39	1869	7 30 E.	+3.4	6.9 E.	A. C. Lamson.
Chaquamegon Point	46 41	90 45	1869	7 36 E.	+3.4	7.0 E.	Do.
Bay City, Wis	46 35	90 52	1869	8 10 E.	+3.4	7.6 E.	Do.
Magdalene Island, north side	46 50	90 40	1869	7 38 E.	+3.4	7.0 E.	Do.
Magdalene Island, north end	46 50	90 35	1869	7 08 E.	+3.4	6.5 E.	Do.
Little Island, northeast of Michigan Island.	46 54	90 26	1869	6 21 E.	+3.4	5.7 E.	A. F. Chaffee.
Saint Ignace Harbor observatory post.	48 47	87 49	July 6, 1869	5 08 E.	+3.4	4.5 E.	G. A. Marr
Saint Ignace Harbor observatory post.	48 47	87 49	Sept. 5, 1871	6 26 E.	+3.4	5.9 E.	Do.
South Base, Minnesota Point	46 44	92 03	Oct. 1, 1870	9 46 E.	+4.5	9.0 E.	E. S. Wheeler.

SAINT MARY'S RIVER.

Sugar Island, northeast side	46 30	84 08	1853	0 40 E.	+3.3	0.8 W.	Capt. E. P. Scammon.
Sault Island, north of Drummond Island.	46 02	83 45	1854	1 23 W.	+3.3	2.8 W.	Do.
Sugar Island Rapids	46 29	84 18	1854	0 17 E.	+3.3	1.2 W.	Do.
West Neebish Rapids	46 18	84 12	1854	2 03 W.	+3.3	3.5 W.	Do.
Twin Islands, Mud Lake	46 12	84 06	1854	2 37 W.	+3.3	4.0 W.	Do.
Pointe aux Pins	46 28	84 28	June, 1855	1 24 E.	+3.3	0.0	Do.
Mission Point	46 27	84 36	Aug., 1855	2 09 E.	+3.3	0.8 E.	Do.

STRAITS OF MACKINAC.

Bois Blanc Island, north side	45 49	84 29	July, 1849	1 31 E.	+3.3	0.2 W.	Lieut. J. N. Macomb.
Pointe Saint Ignace	45 51	84 42	July 23, 1849	2 29 E.	+3.3	0.8 E.	Lieut. E. P. Scammon.
Pointe Saint Martin, Station 13	45 59	84 32	July 25, 1849	0 44 E.	+3.3	1.0 W.	Do.
Pointe Saint Martin, Station 12	45 59	84 32	July 25, 1849	1 14 E.	+3.3	0.5 W.	Do.
East of Boiling Spring Point.	46 02	84 35	Aug. 1, 1849	1 32 E.	+3.3	0.2 W.	Do.
Boiling Spring Point	46 02	84 38	Aug. 4, 1849	1 59 E.	+3.3	0.3 E.	Do.
Grosse Pointe	45 58	84 41	Aug. 7, 1849	2 00 E.	+3.3	0.3 E.	Do.
Isle Saint Martin	45 58	84 35	Aug. 21, 1849	0 32 E.	+3.3	1.2 W.	Do.
Rabbit's Back Point	45 52	84 43	Aug., 1849	2 09 E.	+3.3	0.4 E.	Do.
Pointe Brulée, Station D	45 58	84 32	Sept. 3, 1849	1 34 E.	+3.3	0.1 W.	Do.
Pointe Brulée, Station 18	45 58	84 32	Sept. 3, 1849	1 14 E.	+3.3	0.5 W.	Do.
Search Bay, Station A	45 59	84 31	Sept. 6, 1849	0 56 E.	+3.3	0.8 W.	Do.
Pointe Saint Martin, Station 16	45 58	84 31	Sept. 6, 1849	0 39 E.	+3.3	1.1 W.	Do.
Search Bay, Station B	46 00	84 30	Sept. 7, 1849	0 40 E.	+3.3	1.0 W.	Do.
Pointe Brulée, Station E	45 59	84 33	Sept. 10, 1849	1 25 E.	+3.3	0.3 W.	Do.
Pointe Brulée, Station C	45 59	84 34	Sept. 11, 1849	1 04 E.	+3.3	0.6 W.	Do.
Pointe Brulée, Station 17	45 58	84 33	Sept. 17, 1849	1 10 E.	+3.3	0.5 W.	Do.
Pointe Brulée, Station F	46 01	84 33	Sept. 17, 1849	1 11 E.	+3.3	0.5 W.	Do.
Pointe Brulée, Station G	46 00	84 32	Sept. 18, 1849	1 19 E.	+3.3	0.4 W.	Do.
Bois Blanc Island, west side	45 49	84 35	Aug., 1849	1 59 E.	+3.3	0.3 E.	R. W. Burgess.
East of Cheboygan lighthouse.	45 37	84 12	1851	1 54 E.	+3.3	0.3 E.	Lieut. W. F. Raynolds.
Pointe Epoufette	46 03	85 11	1852	3 30 E.	+3.3	2.0 E.	Capt. E. P. Scammon.
McGulpin's Point	45 47	84 47	Oct. 4, 1852	2 19 E.	+3.3	0.8 E.	Lieut. W. F. Raynolds.
Round Island	45 50	84 37	1853	2 05 E.	+3.3	0.6 E.	Do.
Waugoshance Point	45 46	85 00	1853	2 13 E.	+3.3	0.7 E.	Do.

TABLE V—Continued.

LAKE MICHIGAN.

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.	Observer.
	° ′	° ′		° ′	′	°	
North of Little Traverse Bay	45 35	85 06	Aug. 31, 1853	2 38 E.	+3.3	1.1 E.	Lieut. W. F. Raynolds.
Hat Island	45 49	85 18	1853	3 12 E.	+3.3	1.7 E.	Do.
Garden Island	45 47	85 29	1854	2 43 E.	+3.3	1.3 E.	Do.
Whiskey Island	45 49	85 36	1854	3 49 E.	+3.3	2.4 E.	Do.
Point Patterson	45 58	85 39	1854	2 51 E.	+3.3	1.4 E.	G. W. Lamson.
Beaver Island, Station M, east side.	45 39	85 29	1855	3 02 E.	+3.3	1.7 E.	W. H. Hearding.
Beaver Island, Station 76, west side.	45 37	85 37	1855	3 34 E.	+3.3	2.2 E.	Do.
Beaver Island, Station 7, east side.	45 41	85 30	1855	3 22 E.	+3.3	2.0 E.	Do.
Beaver Island, Station 34, south side.	45 35	85 34	1855	4 03 E.	+3.3	2.7 E.	Do.
Beaver Island, Station 40, south side.	45 35	85 35	1855	3 47 E.	+3.3	2.4 E.	Do.
Beaver Island, Station 60, west side.	45 38	85 37	1855	3 36 E.	+3.3	2.2 E.	Do.
Beaver Island, Station R, west side.	45 43	85 34	1855	3 09 E.	+5.3	1.8 E.	Do.
Beaver Island, Station 53, west side.	45 36	85 37	1855	3 25 E.	+3.3	2.0 E.	Do.
Gull Island	45 42	85 50	1855	4 25 E.	+3.3	3.1 E.	G. W. Lamson.
Scott's Point	45 57	85 41	1855	3 06 E.	+3.3	1.7 E.	W. H. Hearding.
Seul Choix Pointe	45 55	85 55	1855	3 56 E.	+3.3	2.6 E.	Do.
River Aux Becs Scies	44 38	86 15	May 1, 1859	4 10 E.	+3.3	3.1 E.	Lieut. O. M. Poe.
Middle Village	45 33	85 08	1860	2 34 E.	+3.3	1.5 E.	W. H. Hearding.
Seven-Mile Point	45 28	85 05	1860	2 41 E.	+3.3	1.6 E.	Do.
Point northwest of Little Traverse Bay.	45 26	85 03	June 9, 1860	2 59 E.	+3.3	1.9 E.	Do.
Point opposite Little Traverse Bay.	45 25	84 59	June 18, 1860	2 25 E.	+3.3	1.3 E.	Do.
Little Traverse Bay	45 24	85 04	June, 1860	2 52 E.	+3.3	1.8 E.	Do.
Do	45 23	84 55	June 15, 1860	3 27 E.	+3.3	2.4 E.	Do.
Do	45 22	85 02	June 27, 1860	3 27 E.	+3.3	2.4 E.	Do.
Do	45 22	85 09	July 2, 1860	2 40 E.	+3.3	1.6 E.	Do.
Do	45 20	85 15	July 12, 1860	2 29 E.	+3.3	1.4 E.	Do.
Near Fisherman's Island	45 17	85 20	July 12, 1860	3 14 E.	+3.3	2.1 E.	Do.
Grand Traverse Bay	45 08	85 22	Aug. 13, 1860	2 11 E.	+3.3	1.1 E.	Do.
Near north end of Torchlight Lake.	45 06	85 22	Aug. 21, 1860	2 20 E.	+3.3	1.2 E.	Do.
Grand Traverse Bay	44 54	85 25	Sept. 3, 1860	1 45 E.	+3.3	0.7 E.	Do.
Do	44 51	85 27	Sept. 4, 1860	1 29 E.	+3.3	0.4 E.	Do.
Do	44 46	85 30	Sept. 17, 1860	1 52 E.	+3.3	0.8 E.	Do.
Do	44 57	85 34	Sept. 20, 1860	2 06 E.	+3.3	1.0 E.	Do.
Do	44 50	85 33	Sept. 22, 1860	2 25 E.	+3.3	1.3 E.	Do.
Grand Traverse Bay, Tucker's Point.	44 53	85 34	1860	2 27 E.	+3.3	1.3 E.	Do.
Grand Traverse Bay, Old Mission Point.	44 58	85 29	1860	2 20 E.	+3.3	1.2 E.	Do.
Grand Traverse Bay, Traverse City.	44 46	85 37	1860	2 24 E.	+3.3	1.3 E.	H. C. Penny.
Grand Traverse Bay, north of Traverse City.	44 51	85 39	1860	2 46 E.	+3.3	1.7 E.	Do.
Grand Traverse Bay, Sutton's Point.	45 00	85 36	Sept. 20, 1860	3 27 E.	+3.3	2.3 E.	Do.
Good Harbor, Leland	44 58	85 47	July 11, 1860	3 31 E.	+3.3	2.4 E.	Do.
Northport	45 08	85 36	Aug. 29, 1860	2 30 E.	+3.3	1.4 E.	Do.
South Manitou Island, east side.	45 02	86 06	1860	3 55 E.	+3.3	2.8 E.	D. F. Henry.
South Fox Island	45 24	85 50	1860	3 50 E.	+3.3	2.7 E.	Do.

Table V—Continued.

LAKE MICHIGAN—Continued.

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.	Observer.
	° ′	° ′		° ′	′	°	
North Manitou Island, west side.	45 06	86 04	1860	3 29 E.	+3.3	2.4 E.	D. F. Henry.
North Manitou Island, east side.	45 07	85 59	1860	4 01 E.	+3.3	2.9 E.	Do.
Glen Arbor	44 54	86 00	June 19, 1860	3 32 E.	+3.3	2.4 E.	H. C. Penny.
North Unity	44 57	85 54	July 19, 1860	3 32 E.	+3.3	2.4 E.	Do.
Manistee	44 15	86 20	1861	4 E.	+3.3	3.0 E.	J. Barney.
South Fox Island	45 23	85 50	Sept. 24, 1862	3 34 E.	+3.3	2.6 E.	J. R. Mayer.
Rawley's Bay	45 12	87 03	Sept., 1863	4 22 E.	+3.3	3.4 E.	Do.
Bayley's Harbor light-house	45 04	87 05	Oct., 1863	4 26 E.	+3.3	3.5 E.	Do.
Pointe Detour	45 40	86 37	1864	3 26 E.	+3.3	2.5 E.	W. T. Casgrain.
South of Portage Bay	45 44	86 32	1864	3 32 E.	+3.3	2.6 E.	Do.
Monistique River	45 57	86 14	1864	3 06 E.	+3.3	2.2 E.	Do.
Pointe aux Barques	45 47	86 20	1864	3 28 E.	+3.3	2.6 E.	Do.
Grand Haven	43 05	86 12	Sept., 1865	4 20 E.	+5.1	3.0 E.	J. de la Camp.
Racine	42 44	87 48	July, 1865	5 10 E.	+4.6	4.0 E.	A. W. Unthank.
Sheboygan, Wis	43 45	87 42	Aug., 1865	5 15 E.	+4.6	4.1 E.	Do
River Aux Bees Scies	44 37	86 15	June 25, 1866	4 17 E.	+3.3	3.6 E.	A. F. Chaffee.
Little Lake Sable	43 59	86 28	Sept. 27, 1866	4 12 E.	+3.3	3.4 E.	Do.
North Bar Lake	44 29	86 15	July 20, 1866	3 16 E.	+3.3	2.5 E.	O. N. Chaffee.
Whitefish Point	44 52	87 12	1866	5 49 E.	+3.3	5.1 E.	Do.
Manistee	44 15	86 20	Sept., 1866	4 E.	+3.3	3.2 E.	W. T. Casgrain.
Whitefish Bay	44 54	87 12	June 27, 1866	5 49 E.	+3.3	5.0 E.	H. Gillman.
One mile north of Station Clay Banks.	44 42	87 21	July 27, 1866	6 18 E.	+3.3	5.5 E.	Do.
Ahnepee	44 36	87 26	Aug. 9, 1866	5 33 E.	+3.3	4.8 E.	Do.
Kewaunee	44 28	87 30	Aug. 29, 1866	6 12 E.	+3.3	5.4 E.	Do.
Rawley's Point	44 11	87 31	Oct. 8, 1866	6 56 E.	+3.3	6.2 E.	Do.
Racine	42 44	87 48	1867	5 12 E.	+4.6	4.2 E.	From a survey un-
Kenosha	42 35	87 49	1870	5 30 E.	+4.6	4.7 E.	der Major Houston.
Benona	43 34	86 30	Aug. 23, 1870	4 56 E.	+5.1	4.1 E.	From a survey un-
Seven miles south of Sheboygan.	43 39	87 44	Oct. 1[illegible], 1870	8 23 E.	+4.6	7.6 E.	der Captain Cuyler. G. A. Marr.
Two miles south of Manitowoc.	44 04	87 39	Aug. 20, 1870	5 03 E.	+4.6	4.3 E.	J. P. Mayer. J. R. Mayer.
Black Lake Harbor	42 46	86 12	1871	2 31 E.	+5.1	1.7 E.	From a survey under Captain Farquhar.
Pentwater	43 47	86 26	1871	4 15 E.	+3.3	3.8 E.	Do.
South Haven	42 24	86 16	1871	3 28 E.	+5.1	2.7 E.	Do.
Saint Joseph	42 06	86 29	1871	2 00 E.	+5.1	1.2 E.	Do.
Pere Marquette	43 57	86 27	1871	4 18 E.	+3.3	3.8 E.	Do.
White River	43 22	86 25	1871	4 14 E.	+3.3	3.7 E.	Do.
Milwaukee	43 02	87 54	1871	6 43 E.	+4.6	6.0 E.	From a survey under Major Houston.
Muskegon	43 13	86 19	July 3, 1871	4 02 E.	+3.3	3.5 E.	L. Foote.
Whitehall	43 22	86 25	June 6, 1871	3 51 E.	+3.3	3.4 E.	Do.
Grand Haven	43 04	86 14	July 31, 1871	3 33 E.	+5.1	2.8 E.	Do.
Black Lake	42 46	86 12	Aug. 19, 1871	2 16 E.	+5.1	1.5 E.	Do.
Michigan City	41 44	86 54	Sept. 11, 1871	4 02 E.	+5.1	3.3 E.	Do.
Grand Calumet River	41 37	87 15	Sept. 30, 1871	4 30 E.	+4.6	3.8 E.	Do.
Saugatuck	42 40	86 12	1871	2 22 E.	+5.1	1.6 E.	H. Custer.
Saint Joseph	42 06	86 30	1871	3 53 E.	+5.1	3.1 E.	Do.
Seven miles north of Saint Joseph.	42 14	86 22	1871	3 57 E.	+5.1	3.2 E.	Do.
South Haven	42 24	86 16	1871	3 33 E.	+5.1	2.8 E.	Do.
Plumberville	42 32	86 14	1871	2 46 E.	+5.1	2.0 E.	Do.
Racine	42 44	87 48	1872	4 29 E.	+4.6	3.9 E.	Do.
New Buffalo	41 48	86 45	1872	4 48 E.	+4.6	4.2 E.	Do.
Kenosha	42 35	87 49	1872	5 00 E.	+4.6	4.4 E.	Do.
Winetka	42 06	87 44	1873	4 58 E.	+4.6	4.4 E.	Do.
Waukegan	42 21	87 50	1873	5 11 E.	+4.6	4.6 E.	Do.

TABLE V—Continued.

GREEN BAY.

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.	Observer.
	° ′	° ′		° ′	′	°	
Four miles north of Big Sturgeon Bay.	44 58	87 22	1843	4 59 E.	+3.3	2.9 E.	Lieut. Jas. H. Simpson.
One mile north of Sugar Creek.	44 48	87 39	1843	6 09 E.	+3.3	4.1 E.	Do.
Two and one-half miles south of Sable Point.	44 32	87 56	1843	6 26 E.	+3.3	4.4 E.	Do.
Detroit Island	45 19	86 55	Oct. 30, 1862	4 14 E.	+3.3	3.3 E.	J. R. Mayer.
Ephraim	45 09	87 10	July 21, 1863	4 42 E.	+3.3	3.8 E.	H. Gillman.
Hedgehog Harbor	45 17	87 02	Aug. 22, 1863	4 38 E.	+3.3	3.7 E.	Do.
Green Island	45 03	87 30	Oct., 1863	4 32 E.	+3.3	3.6 E.	D. F. Henry.
Menomonee River	45 05	87 35	Oct., 1863	6 20 E.	+3.3	5.4 E.	Do.
Pointe Rochereau	45 18	87 26	Oct., 1863	4 43 E.	+3.3	3.8 E.	A. Molitor.
Washington Harbor	45 24	86 56	Sept. 11, 1863	3 38 E.	+3.3	2.8 E.	S. W. Robinson.
Cedar River	45 25	87 21	1863	3 57 E.	+3.3	3.0 E.	H. C. Penny.
Bark River	45 35	87 14	1863	3 37 E.	+3.3	2.7 E.	Do.
Little Bay de Noquette	45 44	87 04	Sept. 8, 1863	1 54 E.	+3.3	1.0 E.	Do.
Little Bay de Noquette, east side.	45 44	86 59	Oct. 21, 1863	3 57 E.	+3.3	3.0 E.	Do.
Chambers' Island	45 10	87 20	Aug., 1864	3 48 E.	+3.3	2.9 E.	A. Molitor.
West of Chippewa Point	45 43	86 54	1864	3 50 E.	+3.3	3.0 E.	A. F. Chaffee.
St. Vital Point	45 48	86 47	1864	3 05 E.	+3.3	2.2 E.	Do.
Peshtigo	44 59	87 38	July, 1865	4 20 E.	+3.3	3.5 E.	Do.
Washington Island	45 25	86 56	1865	3 30 E.	+3.3	2.7 E.	
Egg Harbor	45 03	87 16	Aug. 17, 1865	4 49 E.	+3.3	4.0 E.	H. C. Penny.
Sturgeon Bay, north side	44 54	87 24	July 9, 1865	4 36 E.	+3.3	3.8 E.	A. C. Lamson.
Head of Green Bay	44 33	87 59	Aug. 18, 1865	5 25 E.	+3.3	4.6 E.	O. N. Chaffee.
Little Sturgeon Bay	44 51	87 33	Sept. 13, 1865	6 16 E.	+3.3	5.4 E.	A. C. Lamson.
Near Red River	44 43	87 43	Oct. 2, 1865	6 08 E.	+3.3	5.3 E.	Do.
Oconto	44 53	87 50	July, 1865	5 21 E.	+3.3	4.5 E.	A. F. Chaffee.

LAKE HURON.

Four miles north of Tawas Point.	44 18	83 24	1856	2 05 E.	+3.0	0.9 E.	G. W. Lamson.
Sable River	44 25	83 19	June, 1856	2 12 E.	+3.0	1.0 E.	W. H. Hearding.
Mouth of Saginaw River	43 39	83 50	1856	1 28 E.	+3.0	0.3 E.	Do.
Four miles northwest of Saginaw River.	43 41	83 55	1856	1 28 E.	+3.0	0.3 E.	Do.
Nyahqning Point	43 46	83 56	1856	1 14 E.	+3.0	0.0	Do.
Willow River	44 02	82 50	June 6, 1857	0 12 W.	+3.0	1.3 W.	Do.
Pointe aux Barques	44 04	82 57	July 17, 1857	0 00	+3.0	1.1 W.	Do.
Near Oak Point	43 59	83 12	Sept. 13, 1857	1 05 E.	+3.0	0.0	Do.
Three miles west of Quannakissec River.	43 37	83 43	Sept. 24, 1857	1 32 E.	+3.0	0.4 E.	Do.
Stony Island	43 52	83 26	June 29, 1857	0 24 E.	+3.0	0.8 W.	Lieut. O. M. Poe.
Six miles north of White Stone Point.	44 12	83 33	June 23, 1857	0 34 E.	+3.0	0.6 W.	G. W. Lamson.
Gravelly Point	44 03	83 34	July 25, 1857	1 16 E.	+3.0	0.1 E.	Do.
Partridge River	44 00	83 03	Aug. 5, 1857	0 08 E.	+3.0	1.0 W.	Do.
Pointe aux Gres	43 59	83 40	Aug. 29, 1857	1 30 E.	+3.0	0.4 E.	Do.
Hat Point	44 00	83 06	Sept. 12, 1857	0 40 E.	+3.0	0.5 W.	Do.
Near Gravelly Point	44 03	83 34	Oct. 15, 1857	1 34 E.	+3.0	0.4 E.	Do.
Crane's Point	43 50	82 38	July 13, 1858	0 16 W.	+3.0	1.3 W.	W. H. Hearding.
Sharpe's Bay	43 47	82 36	July 13, 1858	0 18 W.	+3.0	1.4 W.	Do.
White Rock Point	43 43	82 36	Aug. 4, 1858	0 21 W.	+3.0	1.4 W.	Do.
Elk Creek	43 37	82 35	Aug. 11, 1858	0 35 W.	+3.0	1.6 W.	Do.
Cherry Creek	43 30	82 34	Aug. 23, 1858	0 42 W.	+3.0	1.8 W.	Do.
Miller's Creek	43 28	82 33	Sept. 3, 1858	0 43 W.	+3.0	1.8 W.	Do.
Port Sanilac	43 25	82 32	Sept. 17, 1858	0 30 W.	+3.0	1.6 W.	Do.
New London Point	43 23	82 31	Oct. 5, 1858	0 43 W.	+3.0	1.8 W.	Do.

TABLE V—Continued.

LAKE HURON—Continued.

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.	Observer.
	° ′	° ′		° ′	′	°	
Three miles south of New London Point.	43 20	82 31	Oct. 9, 1858	1 14 W.	+3.0	2.3 W.	W. H. Hearding.
Forest Bay	43 53	82 40	1858	0 03 E.	+3.0	1.1 W.	Do.
Stafford	43 57	82 42	1858	0 12 W.	+3.0	1.3 W.	Do.
Two miles north of Stafford.	44 00	82 45	1858	0 08 E.	+3.0	1.0 W.	Do.
Near Pointe aux Barques Light.	44 00	82 46	1858	0 05 E.	+3.0	1.0 W.	Do.
Alpena	45 04	83 25	1858	0 36 W.	+3.3	1.8 W.	G. W. Lamson.
North Point, Thunder Bay	45 02	83 16	1858	1 00 W.	+3.3	2.2 W.	Do.
Presqu' Isle Harbor	45 20	83 27	June 19, 1858	0 48 E.	+3.3	0.4 W.	H. C. Penny.
Middle Island	45 12	83 19	Aug. 5, 1858	1 04 W.	+3.3	2.3 W.	Do.
Four miles south of Harrisville.	44 36	83 19	Aug. 26, 1858	0 29 E.	+3.3	0.7 W.	Do.
The Cove	44 46	83 17	Sept. 11, 1858	0 19 W.	+3.3	1.5 W.	Do.
Adams Point	45 24	83 41	July 6, 1859	0 41 E.	+3.3	0.5 W.	Do.
East of Hammond's Bay	45 29	83 55	July 27, 1859	1 11 W.	+3.3	2.3 W.	Do.
Hammond's Bay	45 31	84 07	Aug., 1859	2 20 E.	+3.3	1.2 E.	Do.
Lake Wawanash, Ontario	43 01	82 19	Sept. 5, 1859	0 40 W.	+3.0	1.7 W.	Do.
Cape Ipperwash, Ontario	43 13	82 00	Oct. 22, 1859	0 22 W.	+3.0	1.4 W.	Do.
Drummond Island, southeast end.	45 55	83 30	July 2, 1859	0 51 W.	+3.3	2.0 W.	W. H. Hearding.
Drummond Island, east side	45 57	83 29	June 24, 1859	0 26 W.	+3.3	1.6 W.	Do.
Drummond Island, near Harbor Island.	45 55	83 34	July 14, 1859	0 50 W.	+3.3	2.0 W.	Do.
Drummond Island, south side.	45 56	83 38	Aug. 4, 1859	0 26 W.	+3.3	1.6 W.	Do.
Drummond Island, southwest point.	45 56	83 42	Aug. 9, 1859	0 13 E.	+3.3	0.9 W.	Do.
One mile south of Lexington	43 15	82 31	Aug. 22, 1859	1 00 W.	+3.0	2.0 W.	Do.
One mile north of Lexington	43 16	82 31	Sept. 1, 1859	1 21 W.	+3.0	2.4 W.	Do.
Two miles south of Lakeport	43 05	82 28	Oct. 1, 1859	0 15 W.	+3.0	1.3 W.	Do.
Four miles south of Lexington.	43 12	82 30	Oct. 3, 1859	0 36 W.	+3.0	1.6 W.	Do.

LAKE AND RIVER SAINT CLAIR.

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.	Observer.
Gratiot	43 00	82 25	Oct. 31, 1859	0 02 E.	+3.0	1.0 W.	W. H. Hearding.
Mouth of Saint Clair River	42 34	82 40	1842	1 25 E.	+3.0	0.5 W.	Lieut. J. N. Macomb.
Middle Pass of Saint Clair River.	42 34	82 41	Oct., 1856	0 48 E.	+3.0	0.4 W.	G. W. Lamson.
Stag Island, Saint Clair River.	42 53	82 27	1866	0 22 E.	+3.0	0.3 W.	F. M. Towar.
Algonac	42 37	82 32	Dec., 1866	0 04 E.	+3.0	0.6 W.	O. N. Chaffee.
River aux Puces	42 18	82 47	Dec. 18, 1868	1 13 E.	+3.0	0.6 E.	Lieut. J. F. Gregory.
Five miles north of Milk River Point.	42 31	82 52	1868	0 42 E.	+3.0	0.1 E.	A. Molitor.
Mouth of Thames River	42 19	82 27	Apr., 1871	0 29 E.	+3.0	0.0	A. C. Lamson.

DETROIT RIVER.

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.	Observer.
Amherstburgh, Ontario	42 07	83 07	June, 1840	1 30 E.	+3.0	0.5 W.	Lieut. J. N. Macomb.
Hamtramck House near Detroit.	42 20	83 00	May, 1842	1 52 E.	+3.0	0.0	Do.
Belle Isle	42 20	83 00	Nov. 7, 1873	0 35 E.	+3.0	0.2 E.	A. C. Lamson.
Bois Blanc Island	42 05	83 07	May 10, 1874	0 32 E.	+3.0	0.2 E.	Do.

LAKE ERIE.

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.	Observer.
Cedar Point, Maumee Bay	41 43	83 31	1844	2 16 E.	+3.0	0.5 E.	Lieut. J. H. Simpson.
Cedar Point, Maumee Bay	41 43	83 21	1844	2 10 E.	+3.0	0.4 E.	Unknown.
Huron Harbor	41 24	82 33	1844 (?)	2 01 E.	+2.5	0.5 E.	J. F. Peter.
Dunkirk	42 30	79 21	1845	1 07 E.	+4.2	1.3 W.	Lieut. J. H. Simpson.

TABLE V—Continued.

LAKE ERIE—Continued.

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.	Observer.
	° ′	° ′		° ′	′	°	
Long Point	42 33	80 05	1845	0 55 W.	+4.2	3.4 W.	Lieut. J. H. Simpson.
Ashtabula	41 54	80 48	1845	0 00	+4.2	2.5 W.	Do.
Pointe aux Pins	42 15	81 52	1845	1 04 E.	+3.0	0.7 W.	Do.
Huron River	41 24	82 33	1845	2 10 E.	+2.5	0.7 E.	Do.
Kelley's Island, southwest side.	41 36	82 43	Oct., 1845	2 20 E.	+2.5	0.9 E.	Do.
Middle Island	41 41	82 41	1845	1 54 E.	+2.5	0.5 E.	Do.
East Sister Island	41 49	82 51	1845	2 18 E.	+3.0	0.6 E.	Do.
Middle Sister Island	41 51	83 00	1845	2 00 E.	+3.0	0.2 E.	Do.
West Sister Island	41 45	83 06	1845	2 20 E.	+3.0	0.6 E.	Do.
Green Island	41 39	82 52	Aug. 23, 1845	2 34 E.	+3.0	0.8 E.	Lieut. J. C. Woodruff.
Port Clinton	41 31	82 56	Oct. 30, 1850	2 05 E.	+3.0	0.6 E.	Lieut. J. N. Macomb.
Near Cedar Point light-house, Sandusky.	41 30	82 40	July, 1862	1 39 E.	+3.0	0.8 E.	W. H. Hearding.
Mouth of Maumee River, east side.	41 42	83 26	Aug. 22, 1862	1 35 E.	+3.0	0.7 E.	Do.
Cleveland, end of east pier	41 30	81 40	May 22, 1865	1 12 E.	+2.5	0.6 E.	W. T. Casgrain.
Ashtabula	41 54	80 48	June, 1865	0 00	+4.2	1.0 W.	Do.
Fairport, Grand River	41 45	81 16	Aug., 1865	0 49 E.	+2.5	0.2 E.	Do.
Black River light-house	41 28	82 11	1865	1 36 E.	+2.5	1.0 E.	Do.
Sandusky	41 27	82 45	June 1, 1872	0 55 E.	+2.5	0.6 E.	A. C. Lamson.
Port Colborne*	42 53	79 14	July 27, Aug. 14, 1875	3 45 W.	+5.0	4.1 W.	J. Eisenmann.
Ridgeway*	42 52	79 04	Aug. 24, 31, 1875	3 33 W.	+5.0	3.9 W.	Do.
Northeast, Pa*	42 15	79 50	Sept. 21, Oct. 1, 1875	2 54 W.	+4.2	3.2 W.	Do.
North Springfield, Pa †	42 00	80 29	Oct. 8, 1875	3 03 W.	+3.0	3.3 W.	Do.
Westfield*	42 20	79 36	Sept. 18, 1875	3 11 W.	+4.2	3.5 W.	F. M. Towar.
Avonia†	42 03	80 18	Oct. 9, 1875	2 02 W.	+4.2	2.3 W.	Do.
Erie*	42 08	80 07	Oct. 9, 1875	2 10 W.	+4.2	2.5 W.	A. C. Lamson.
Long Point	42 34	80 11	Aug. 21, 1876	2 40 W.	+4.2	2.9 W.	F. Terry.
Euclid*	41 34	81 35	Sept., 1876	1 10 W.	+2.5	1.3 W.	Do.
Black River*	41 28	82 10	Oct. 23, 1876	0 17 E.	+2.5	0.1 E.	Do.
North Kingsville, Ohio*	41 56	80 41	June 17, 1876	1 16 W.	+3.0	1.4 W.	F. M. Towar.
Madison	41 50	81 02	June 21, 22, 1876	1 59 W.	+3.0	2.2 W.	Do.
Willoughby*	41 40	81 26	Sept. 13, 1876	1 51 W.	+2.5	2.0 W.	Do.
Rocky River*	41 29	81 52	Oct. 2, 4, 1876	0 11 W.	+2.5	0.3 W.	Do.
Avon Point*	41 31	82 01	Oct. 7, 11, 1876	0 36 W.	+2.5	0.7 W.	Do.
Vermillion*	41 26	82 21	Oct. 21, 1876	0 28 E.	+2.5	0.3 E.	Do.
Ashtabula	41 55	80 48	June 19, 22, 1876	1 46 W.	+3.0	1.9 W.	A. C. Lamson.
Red Creek*	41 53	80 51	Aug. 22, 1876	2 00 W.	+3.0	2.2 W.	Do.
Fairport*	41 45	81 16	Sept. 14, 15, 17, 1876	2 00 W.	+2.5	2.1 W.	Do.
Cleveland	41 30	81 43	Oct. 27, 28, 1876	1 08 W.	+2.5	1.3 W.	Do.
Sand Point, Sandusky*	41 30	82 43	May 30, 1877	0 37 E.	+2.5	0.5 E.	Do.
Kelley's Island*	41 36	82 44	June 16, 22, 1877	0 39 E.	+3.0	0.5 E.	Do.
Catawba Island*	41 35	82 50	July 12, 1877	0 40 E.	+2.5	0.6 E.	Do.
Cedar Point, Maumee Bay*	41 42	83 19	July 22, 1877	0 10 W.	+3.0	0.3 W.	Do.
La Salle, Mich*	41 50	83 25	July 25, 1877	0 13 E.	+3.0	0.1 E.	Do.
Stony Point*	41 56	83 16	Sept. 3, 4, 1877	0 18 E.	+3.0	0.2 E.	Do.
Pigeon Bay	41 59	82 33	Sept. 14, 1877	0 10 W.	+3.0	0.3 W.	Do.
Huron*	41 25	82 35	May 23, 1877	0 29 E.	+2.5	0.4 E.	F. Terry.
Pointe Pelée Island*	41 49	82 41	June 5, 1877	0 15 E.	+3.0	0.1 E.	Do.
Point Clinton*	41 31	82 58	July 18, 1877	0 47 E.	+2.5	0.7 E.	Do.
Toledo*	41 40	83 29	Aug. 23, 1877	0 33 E.	+3.0	0.4 E.	Do.
Kingsville*	42 02	82 45	Sept. 9, 1877	0 30 E.	+3.0	0.4 E.	Do.
North Bass Island*	41 42	82 48	May 25, 1877	1 13 E.	+3.0	1.1 E.	F. M. Towar.
Locust Point	41 36	83 06	July 19, 21, 1877	0 40 E.	+2.5	0.6 E.	Do.
Toledo*	41 42	83 26	Aug. 22, 1877	0 49 E.	+3.0	0.7 E.	Do.
Colchester*	42 00	82 58	Sept. 8, 1877	0 31 E.	+3.0	0.4 E.	Do.
Pointe Pelée*	41 55	82 31	Sept. 18, 1877	0 25 E.	+3.0	0.3 E.	Do.

* Tests for local attraction made. † No tests for local attraction made.

Table V—Continued.

NIAGARA RIVER.

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.	Observer.
	° ′	° ′		° ′	′	°	
Youngstown	43 15	79 03	1864	3 00 W.	+4.5	4.2 W.	J. Barney.
Suspension Bridge	43 07	79 03	July, 1875	2 24 W.	+5.0	2.8 W.	F. M. Towar.
Strawberry Island	42 57	78 55	July, 1875	3 59 W.	+5.0	4.3 W.	F. Terry.
Tonawanda *	43 04	78 56	July 28, 31, 1875	3 50 W.	+5.0	4.2 W.	A. C. Lamson.
Grand Island *	43 00	79 01	Aug. 12, 14, 1875	2 58 W.	+5.0	3.3 W.	Do.

LAKE ONTARIO.

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.	Observer.
Olcott Harbor	43 20	78 43	1870	3 25 W.	+5.0	4.2 W.	Maj. N. Bowen.
Charlotte	43 15	77 36	1870	4 01 W.	+5.0	4.9 W.	Do.
Amherst Island, east end	44 11	76 37	July 24, 1874	7 12 W.	+3.0	7.5 W.	F. M. Towar.
Timber Island	43 57	76 50	Aug. 20, 1874	7 19 W.	+3.0	7.6 W.	Do.
Port Ontario	43 34	76 12	Sept. 18, 1874	8 09 W.	+3.0	8.4 W.	Do.
Sacket's Harbor ‡	43 57	76 07	June 28, 29, 1874	9 46 W.			J. Eisenmann.
Stony Island	43 52	76 20	July, 1874	6 18 W.	+3.0	6.6 W.	Do.
Galloo Island †	43 54	76 25	Aug. 3, 1874	7 28 W.	+3.0	7.7 W.	Do.
Snowshoe Bay †	43 53	76 14	Sept. 6, 1874	7 20 W.	+3.0	7.6 W.	Do.
Stony Creek †	43 49	76 16	Sept. 21, 1874	8 23 W.	+3.0	8.7 W.	Do.
Six miles west of Little Sodus.*	43 18	76 49	Oct. 3, 11, 1874	6 50 W.	+3.0	7.1 W.	Do.
Peninsula Point †	43 58	76 16	Sept. 5, 1874	8 17 W.	+3.0	8.6 W.	J. R. Mayer.
Sandy Creek *	43 42	76 12	Sept. 18, 20, 1874	7 50 W.	+3.0	8.1 W.	F. Terry.
Little Sodus *	43 19	76 43	Oct. 26, 1874	7 03 W.	+3.0	7.3 W.	Do.
Duck Island †	43 56	76 37	Aug. 1, 1874	5 00 W.	+3.0	5.3 W.	C. Donovan.
Lyon's Point	43 16	77 26	June 4, 1875	6 04 W.	+3.0	6.3 W.	F. Terry.
Oak Orchard *	43 22	78 12	July 9, 1875	3 46 W.	+5.0	4.1 W.	Do.
Great Sodus *	43 16	76 58	May 28, 1875	6 50 W.	+3.0	7.1 W.	J. Eisenmann.
East Porter *	43 18	78 55	June, 1875	3 16 W.	+5.0	3.6 W.	Do.
Port Dalhousie *	43 12	79 16	July, 1875	4 22 W.	+5.0	4.7 W.	Do.
Charlotte †	43 16	77 36	June 7, 1875	4 32 W.	+5.0	4.9 W.	F. M. Towar.
Mouth of Niagara River	43 16	79 04	July, 1875	3 41 W.	+5.0	4.1 W.	Do.
Braddock's Point *	43 20	78 43	May 29, 1875	4 48 W.	+5.0	5.2 W.	A. C. Lamson.
Olcott *	43 20	78 43	June 30, 1875	3 40 W.	+5.0	4.0 W.	Do.

SAINT LAWRENCE RIVER.

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.	Observer.
Goose Neck Island	44 55	75 07	June, 1871	9 39 W.	+3.1	10.1 W.	A. C. Lamson.
Barnhart's Island	45 00	74 48	June, 1871	10 22 W.	+3.1	10.8 W.	Do.
Massena Point, Polly's Gut	45 00	74 46	July 1, 1871	10 37 W.	+3.1	11.1 W.	Do.
Four miles southwest of Ogdensburgh.	44 40	75 33	Oct. 7, 1871	9 36 W.	+3.1	10.0 W.	Do.
Two miles northeast of Oak Point.	44 32	75 43	Aug. 19, 1872	11 00 W.	+3.1	11.4 W.	Do.
Alexandria Bay	44 20	75 56	Sept. 16, 1872	7 00 W.	+3.1	7.4 W.	Do.
Wellesley Island	44 17	76 02	Oct., 1872	9 17 W.	+3.1	9.7 W.	Do.
Chippewa Point	44 29	75 46	Aug. 19, 1872	7 42 W.	+3.1	8.1 W.	F. M. Towar.
Picnic Island	44 22	75 52	Oct. 4, 1872	7 56 W.	+3.1	8.3 W.	Do.
Wellesley Island	44 21	76 01	Aug., 1873	8 35 W.	+3.1	8.9 W.	Do.
Halliday's Point §	44 14	76 18	1873	7 30 W.			H. Custer.
Gananoque	44 18	76 12	May 11, 1874	8 33 W.	+3.0	8.8 W.	F. M. Towar.
Wolfe Island, Brown's Point	44 14	76 24	Aug. 31, 1874	6 25 W.	+3.0	6.7 W.	Do.
Wolfe Island, near Garden Island.	44 11	76 29	July 23, 1874	6 45 W.	+3.0	7.0 W.	F. Terry.

* Tests made for local attraction. ‡ Not reliable. † No tests. § Approximate.

TABLE V—Continued.

LAKE CHAMPLAIN.

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.	Observer.
	° ′	° ′		° ′	′	°	
Plattsburgh, N. Y., northeast end of breakwater.	44 45	73 24	Oct. 5, 1870	10 52 W.	+6.0	11.8 W.	J. L. Gillespie.
Burlington, Vt., west of cemetery.	44 30	73 12	Nov. 12, 1870	10 57 W.	+6.0	11.9 W.	G. A. Marr.

MISSISSIPPI RIVER.

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.	Observer.
Cairo*	37 00	89 10	Jan. 5, 1877	5 52 E.			F. M. Towar.
Scanlan's Landing*	35 02	90 16	Jan. 23, 1878	6 46 E.			A. T. Morrow.
Opposite Buck Island*	34 52	90 20	Feb. 16, 1879	6 34 E.			J. Eisenmann.
Triangulation station, Nelms.*	34 58	90 15	Feb. 25, 26, 27, 1879	6 22 E.			J. H. Darling.
Blues Point*	34 50	90 26	Mar. 3, 1879	6 14 E.			J. A. Ockerson.

STATE OF ILLINOIS.

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.	Observer.
Triangulation station, Willow Springs.†	41 44	87 51	July 19, 1879	5 10 E.			J. H. Darling.
Triangulation station, Pilot Grove.†	40 12	87 50	Sept. 4, 1879	4 29 E.			Do.
Triangulation station, Palermo.†	39 53	87 52	Sept. 21, 1879	5 12 E.			Do.
Triangulation station, Belle Air.‡	39 11	87 52	Oct. 13, 1879	5 08 E.			Do.
Triangulation station, East Base.†	38 52	88 02	Oct. 29, 1879	5 00 E.			Do.

* Tests made for local attraction. † No tests; one instrument. ‡ No tests; two instruments.

§ **13.**—Table VI.—*Declinations observed with suspended magnets, given in Table II, reduced to 1880.*

[Yearly change of declination taken from Coast-Survey Report, 1879. The plus sign (+) indicates an increasing west and a decreasing east declination.]

Place.	Latitude.	Longitude.	Date of observation.	Observed declination.	Yearly change.	Declination, 1880.
	° ′	° ′		° ′	′	°
Sand Point, Mich	43 55	83 23	July, 1858	0 32 E.	+3.0	0.6 W.
Thunder Bay, Mich	45 02	83 10	Aug., 1858	1 14 W.	+3.3	2.4 W.
Sturgeon Point, Mich	44 42	83 14	Sept., 1858	1 02 E.	+3.3	0.2 W.
Cape Ipperwash, Ont	43 13	82 00	May, 1860	0 03 W.	+3.0	1.0 W.
Wabley, Mich	43 22	82 32	May, 1860	1 05 W.	+3.0	2.1 W.
Goderich, Ont	43 44	81 43	July, 1860	1 42 W.	+3.0	2.7 W.
Mackinac, Mich	45 51	84 36	July, 1860	1 42 E.	+3.3	0.6 E.
Cove Island, Ont	45 20	81 43	Aug., 1860	3 59 W.	+3.3	5.1 W.
Northport, Mich	45 08	85 36	Sept., 1860	2 34 E.	+3.3	1.5 E.
South Manitou Island, Mich	45 02	86 06	Sept., 1860	3 09 E.	+3.3	2.1 E.
Beaver Island, Mich	45 44	85 30	Oct., 1860	2 43 E.	+3.3	1.6 E.
Superior City, Wis	46 43	92 04	Sept., 1870	10 30 E.	+4.5	9.7 E.
Minnesota Point, North Base	46 45	92 04	June, 1871	10 40 E.	+4.5	10.0 E.
Tip-Top, Ont	48 15	86 08	Aug., 1871	0 03 E.	+3.3	0.4 W.
Sackett's Harbor, N. Y	43 57	76 07	May, 1873	8 15 W.	+4.5	8.8 W.
Charlotte, N. Y	43 15	77 37	May, 1873	3 46 W.	+5.0	4.3 W.
Forestville, Mich	43 40	82 34	July, 1873	1 31 W.	+3.0	1.9 W.
Fort Gratiot, Mich	43 00	82 25	July, 1873	0 37 W.	+3.0	1.0 W.
Marquette, Mich	46 33	87 23	July, 1873	4 31 E.	+3.3	4.1 E.
Copper Harbor, Mich	47 28	87 51	Aug., 1873	4 03 E.	+3.4	3.6 E.
Duluth, Minn	46 45	92 04	Aug., 1873	11 52 E.	+4.5	11.3 E.
Saint Paul, Minn	44 57	93 05	Aug., 1873	10 55 E.	+5.6	10.3 E.
Milwaukee, Wis	43 03	87 55	Aug., 1873	6 22 E.	+4.6	5.8 E.
Michigan City, Ind	41 43	86 54	Aug., 1873	3 59 E.	+4.6	3.4 E.
Detroit, Mich	42 20	83 03	June, 1876	0 05 E.	+3.0	0.1 W.
Wabasha, Minn	44 18	92 07	Aug., 1876	8 04 E.	+3.0	7.9 E.
Mount Forest, Ill	41 45	87 52	Aug., 1876	4 36 E.	+4.6	4.3 E.
La Crosse, Wis	43 49	91 15	Sept., 1876	8 01 E.	+3.0	7.8 E.
Saginaw, Mich	43 25	83 58	Sept., 1876	0 23 E.	+3.0	0.2 E.
Galena, Ill	42 25	90 26	Sept., 1876	9 09 E.	+3.0	9.0 E.
Rockford, Ill	42 17	89 07	Oct., 1876	5 18 E.	+4.6	5.0 E.
Saint Louis, Mich	43 24	84 36	Oct., 1876	0 59 E.	+3.0	0.8 E.
Marshall, Mich	42 16	84 58	Oct., 1876	1 42 E.	+3.0	1.5 E.
Rock Island, Ill	41 31	90 34	Sept., 1878	6 58 E.	+3.0	6.9 E.
Red Wing, Minn	44 34	92 32	Oct., 1878	7 50 E.	+3.0	7.7 E.

APPENDIX V.

VALUE OF METRE R 1876.

1. In the Report upon the Primary Triangulation of the United States Lake Survey, p. 862, it is stated that the standard metre *R* 1876 had been sent to the International Bureau of Weights and Measures at Sèvres, France, for comparison with the standards of that Bureau.

The comparisons have now been made, and I append an extract from Tome III of Travaux et Mémoires du Bureau International des Poids et Mesures giving the results. The Bureau International gives to the metre *R* 1876 the designation U. S. (Repsold).

[Extract from Travaux et Mémoires Bureau International des Poids et Mesures. Tome III, Paris, 1884.]

Règle U. S. (Repsold).

12. Cette règle est un ancien étalon appartenant au Lake Survey, United States, North America. Elle a été construite par Repsold; elle est en acier; elle présente une section rectangulaire découpée en forme de H. Elle est divisée, de décimètre en décimètre, sur des mouches de platine incrustées; dans la longueur du premier décimètre seulement a été incrustée une languette de platine, qui est divisée en millimètres dans toute son étendue; le premier centimètre est sub-divisé en dixièmes de millimètre.

On a fait, en janvier 1883, quelques expériences pour mesurer sa dilatation moyenne entre 0° et 35° environ. Pour ces expériences, on s'est encore servi de la dissolution saturée de borate de soude. La disposition des règles et des thermomètres était toujours la même. Le journal des observations est reproduit ci-après (p. Ç. XLV–XLVI).

La tare des micromètres, fait en commençant la série, a donné pour la valeur d'une division:

	Microscope nord.	Microscope sud.
6 janvier 1883	$0^{\mu}.998485$	$0^{\mu}.999098$

C'est avec ces valeurs qu'ont été réduites toutes les observations. Elles ont conduit, toutes réductions faites, aux résultats ci-dessous.

Observation.	Température de la Règle type II.	Température de la Règle U. S.	Différence U. S.—II.
I	$5^{\circ}.870$	$5^{\circ}.937$	$+\ 29^{\mu}.05$
II	6.037	24.957	+ 228.94
III	6.014	28.121	+ 262.55
IV	5.905	35.223	+ 338.85
V	5.538	5.668	+ 29.07
VI	5.553	0.539	− 23.94

On en déduit, après réduction de la Règle Type II, dans toutes les observations, à la température moyenne de 5° 819, le système d'équations qui suit:

Équation	Obs. — Calc.
$x + 0.539\,y = -\ 26.23$	+ 0.83
$x + 5.668\,y = +\ 26.65$	− 0.47
$x + 5.937\,y = +\ 29.49$	− 0.47
$x + 24.957\,y = +\ 230.82$	− 0.07
$x + 28.121\,y = +\ 264.23$	− 0.08
$x + 35.223\,y = +\ 339.59$	+ 0.25

D'où les équations normales

$$6\,x + 100.445\,y = +\,864.55 \qquad 100.445\,x + 2721.97\,y = +\,25464.36$$

et les valeurs des inconnues

$$x = -\,32.76 \qquad y = +\,10.564$$

dont la substitution dans les équations de condition conduit aux erreurs résiduelles sus-indiquées. On a:

Erreur probable d'une observation . . . . $= r = \pm\,0.36$

et

Erreur probable de x . . . . $= r_x = \pm\,0.24$

Erreur probable de y . . . . $= r_y = \pm\,0.011$

La valeur de x conduit à l'équation

$$\mathrm{US}_{(0)} - \mathrm{II}_{(0)} = +\,17^{\mu}.20$$

et, par suite

$$\mathrm{US}_{(0)} = 1000097^{\mu}.81\,;$$

il en résulte, entre 0° et 36° environ,

$$\alpha(t) = 0.000{,}010563 \pm 0.000{,}000{,}011.$$

2. The value given above for U. S. (Repsold) depends on the values of two metres belonging to the International Bureau, designated respectively I_2 and Type II.

A report on the comparisons of I_2 with the métre des archives may be found in the Procès Verbaux du Comité International des Poids et Mesures, 1882, pp. 69–72. The Comité adopt for I_2 at 0° Centigrade the value

$$I_2 - 6^{\mu} = 1^{\mathrm{m}},$$

and state that when the prototype metre is finally adopted this value can only be changed by some tenths of a micron. (Travaux et Mémoires, T. III, p. C. 16.) Until that prototype is selected the value at 0° Centigrade $I_2 - 6$ is adopted as the unit of length by the Comité. From the report it appears that the residuals of the four equations of conditions resulting from the comparisons of I_2 with the métre des archives varied between $+0^{\mu}.09$ and $-\,0^{\mu}.30$, the probable error of a single series of comparisons being only $0^{\mu}.1$ to $0^{\mu}.2$. (Procès Verbaux, 1882, p. 71.)

The value of I_2 is then known in terms of the mètre des archives, to a high degree of accuracy.

The value of Type II in terms of I_2 is given in Travaux et Mémoires, T. III, p. C. 15, as $II - I_2 = 74^{\mu}.61 \pm 0^{\mu}.02$ at 0°.055 Centigrade. As the coefficients of expansion of I_2 and II are nearly equal, the difference at 0° Centigrade is practically the same.

If the uncertainty of some tenths of a micron yet existing in the value of I_2 referred to the prototype yet to be adopted; the probable error in x above; the probable error in the reduction of x to its value at 0° Centigrade; and the probable error at 0° Centigrade of the value of $II - I_2$; be considered: it will be seen to follow from the statements of the Comité, that the value of U. S. Repsold) given by the Comité, will need no correction in its value at 0° amounting to 1^{μ}, to refer it to the prototype yet to be adopted.

3. On page 849 of the Report on the Primary Triangulation of the Lake Survey, the following value of R 1876 is given by Prof. W. Foerster from the Berlin comparisons:

$R\,1876 = 1^{\mathrm{m}} + 86^{\mu}.18 + 10^{\mu}.651t$ where t is the Centigrade temperature. At 0° this value is $11^{\mu}.63$ less than the value now given by the International Bureau. The difference is very large, and it is of interest to know how it occurred.

Professor Foerster derived the value of R 1876 from comparison with a similar metre belonging to Germany, known as R 1878, which has been compared with metre Type I of the International Bureau. Professor Foerster used for the value of Type I, which was probably the best value then known,

$$\text{Type } I = 1^{\mathrm{m}} + 67^{\mu}.6 + 8^{\mu}.525t + 0^{\mu}.0039t^2,$$

this being based on indirect comparisons with the metre of the archives.

Tome III of Travaux et Mémoires, page C 16, now gives for Type I,

Type $I_0 = 1^{\mathrm{m}} + 76^{\mu}.04$ a value which accounts at once for $8^{\mu}.44$ of the $11^{\mu}.63$ difference at 0° in the two values of R 1876. The remainder may be attributed to the uncertainties in the comparisons and in the coefficients of expansion at the time Professor Foerster wrote.

The change between the value given in Report on Primary Triangulation, p. 849, and the value of R 1876 now given arises then mainly from the adoption by the International Bureau of a more accurate value for the metre, Type I.

Nothing could show more strongly the necessity of such a work, as the International Bureau is now doing, than the fact that in 1880 the value of one of its principal metres supposed known at 0° within 1^μ or 2^μ, has needed a subsequent change of 8^μ (see Procès Verbaux, 1880, p. 106).

4. The new value for U. S. (Repsold) gives a value for the ratio between the metre and the yard which cannot be much in error.

On page 181, Primary Triangulation, is found $R\,1876 = 1^y.09388063$ at $57^\circ.92$ F., the value of the yard being that derived from the Clarke Yard A.

But from the value given for R 1876 by the International Bureau at 0° C., and its mean coefficient of dilatation from 0° to 36° C, there follows

$$R\,1876 = 1^{m}.0002499 \text{ at } 57^\circ.92 \text{ F.}$$

These equations give

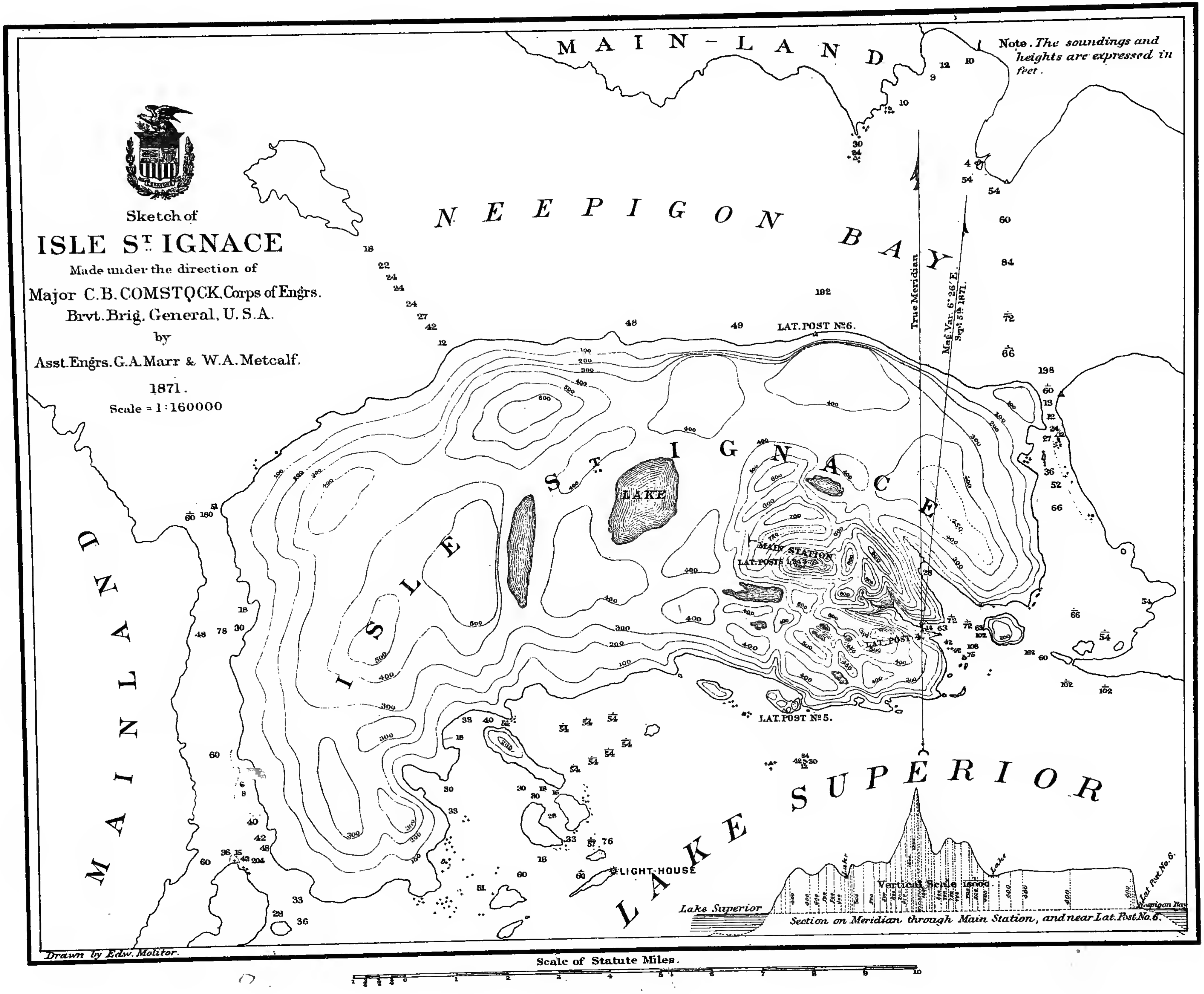
$$\frac{\text{Metre}}{\text{Yard}} = 1.093607,$$

or,

$$\text{Metre} = 39^{in}.36985.$$

C. B. COMSTOCK.

NEW YORK, *February* 28, 1885.


Note. The soundings and heights are expressed in feet.
Sketch of
ISLE ST. IGNACE
Made under the direction of
Major C.B. COMSTOCK, Corps of Engrs.
Brvt. Brig. General, U.S.A.
by
Asst. Engrs. G.A. Marr & W.A. Metcalf.
1871.
Scale = 1:160000
MAIN-LAND
NEEPIGON BAY
ISLE ST. IGNACE
LAKE
MAIN STATION
LAT. POST No. 6.
LAT. POST No. 5.
LAT. POST
True Meridian
Mag. Var. 6° 26' E. Sept. 5th 1871.
MAINLAND
LAKE SUPERIOR
LIGHT-HOUSE
Lake Superior
Neepigon Bay
Vertical Scale 1:6000
Section on Meridian through Main Station, and near Lat. Post No. 6.
Drawn by Edw. Molitor.
Scale of Statute Miles.

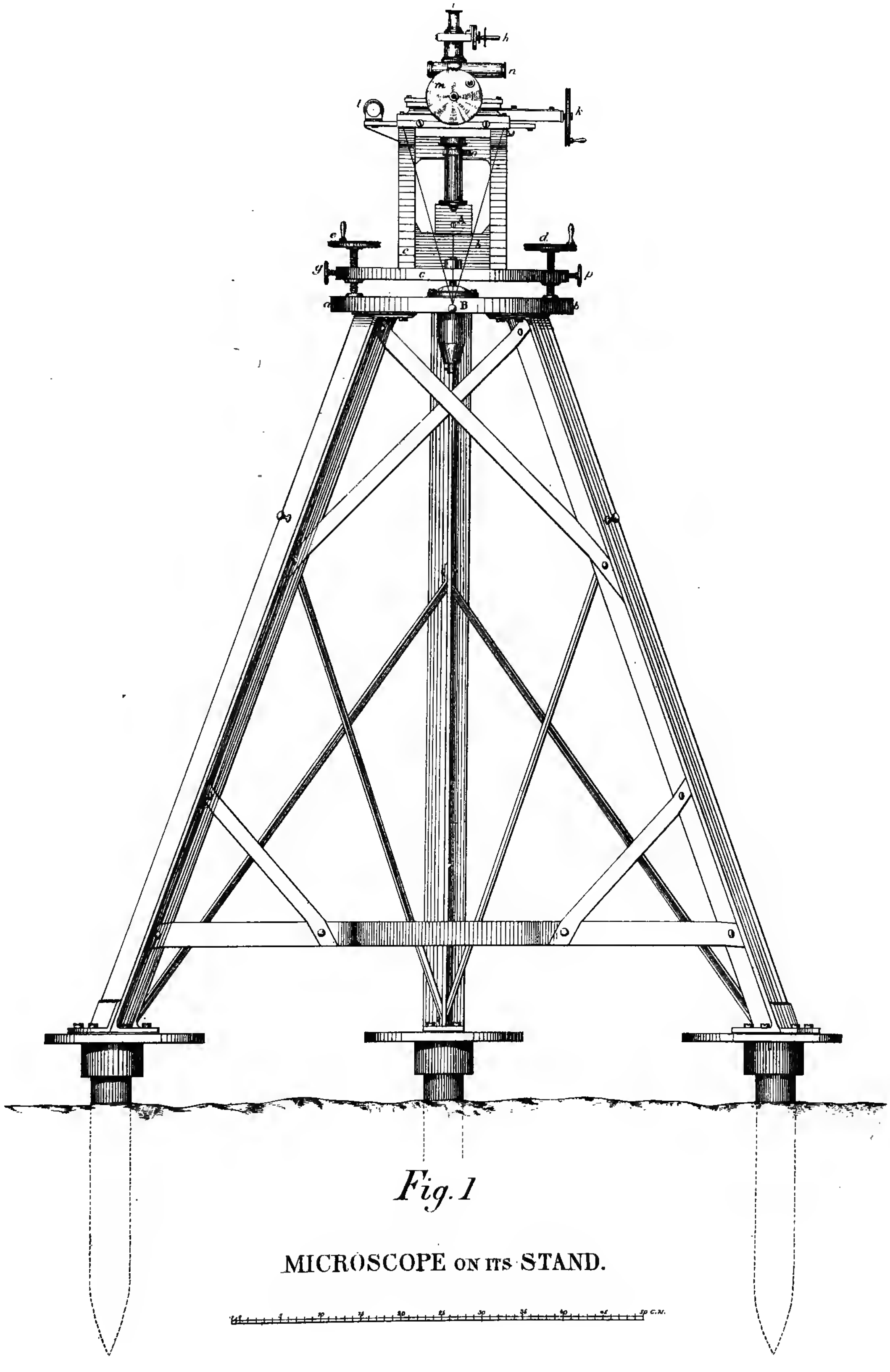


Fig. 1

MICROSCOPE ON ITS STAND.

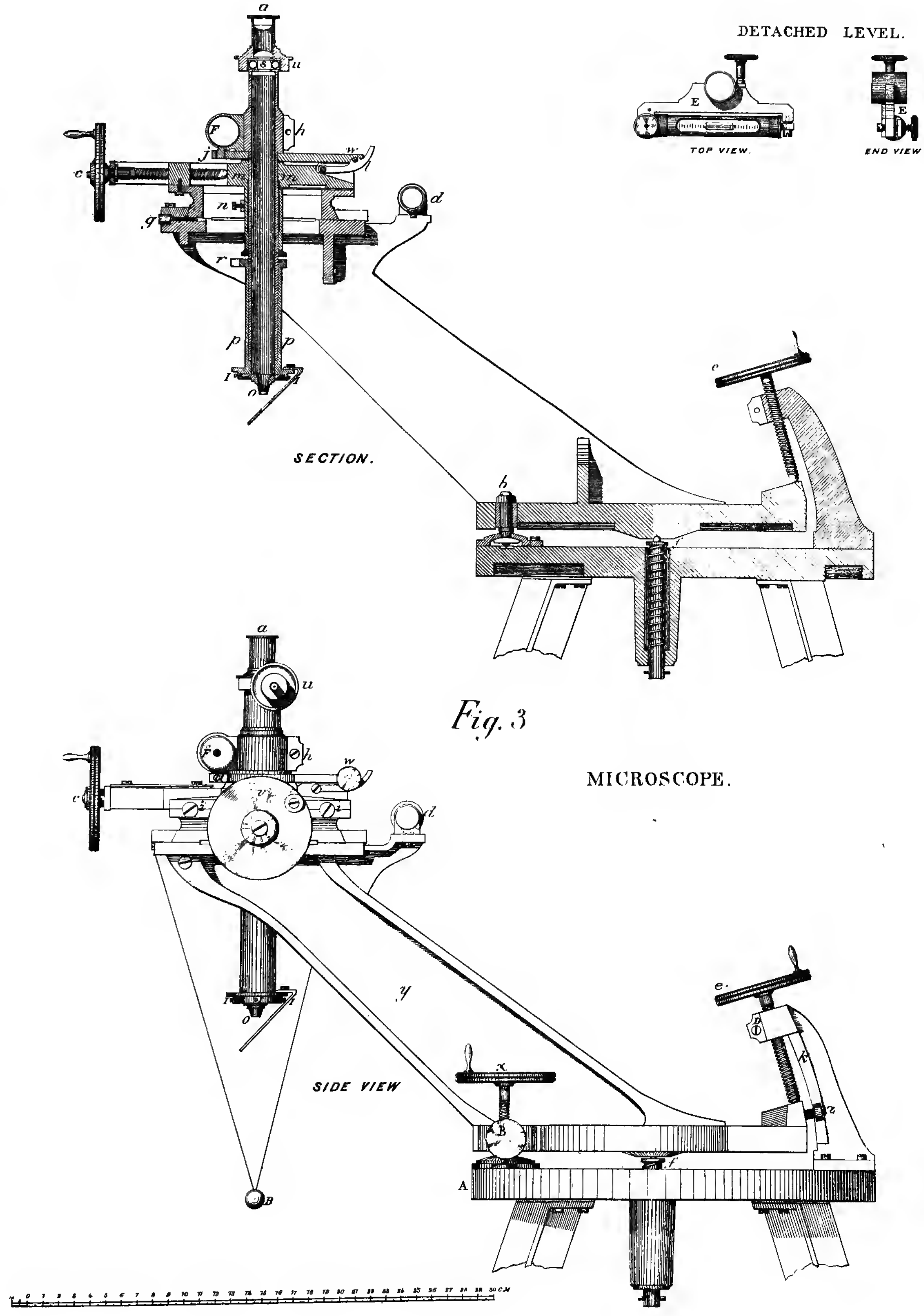
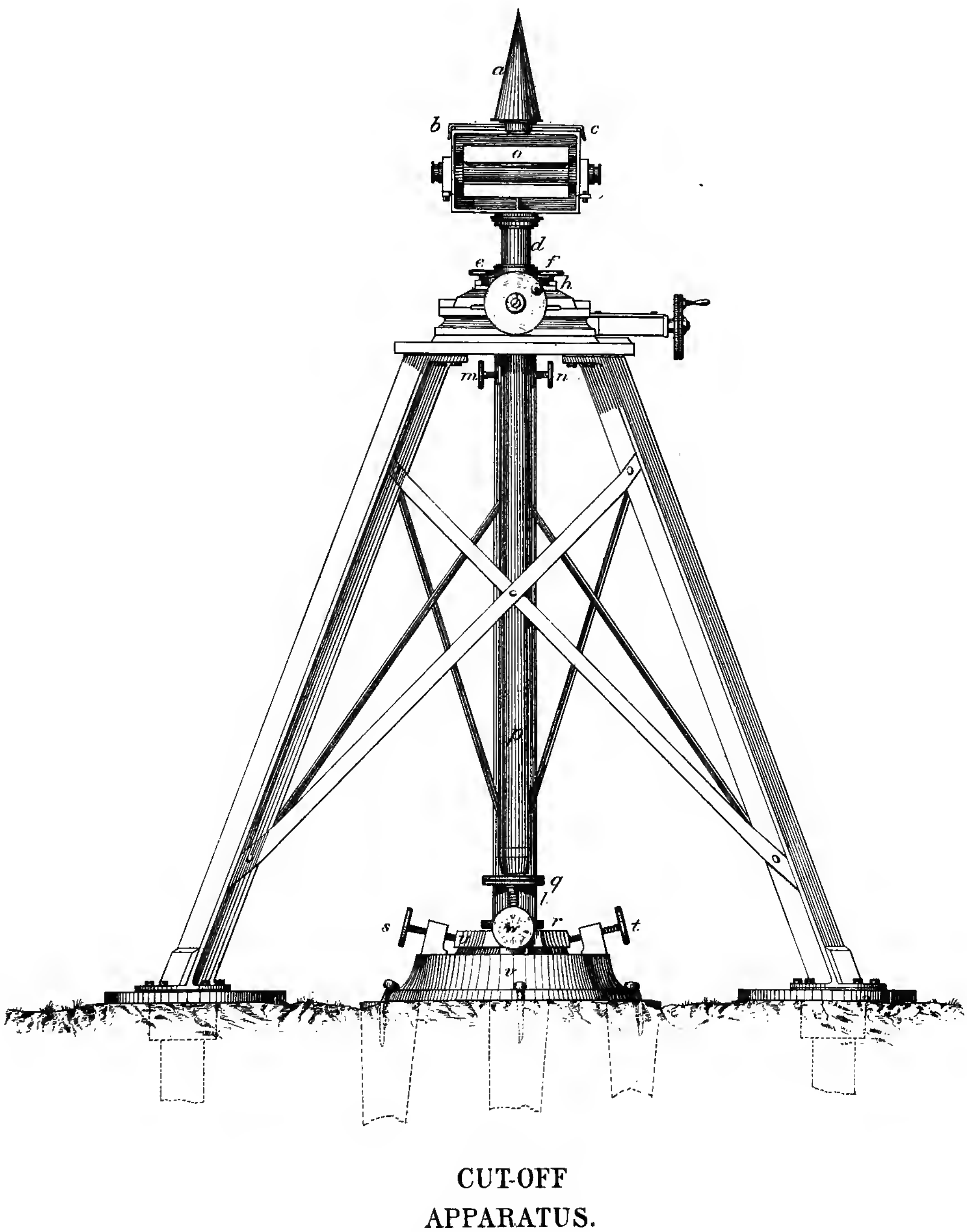

DETACHED LEVEL.
TOP VIEW.
END VIEW.
SECTION.
Fig. 3
MICROSCOPE.
SIDE VIEW
a
u
F
h
j
w
c
n
q
d
r
p
o
e
b
f
x
y
B
A
E
30 C.M.

Fig. 4.



CUT-OFF
APPARATUS.

Fig. 11

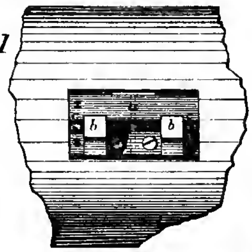
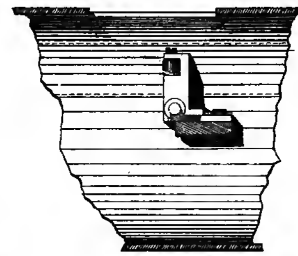


Fig. 12



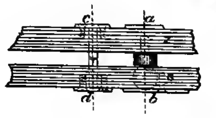
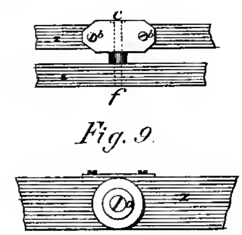
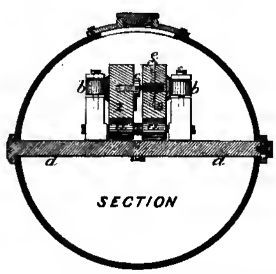


Fig. 10

Fig. 9.





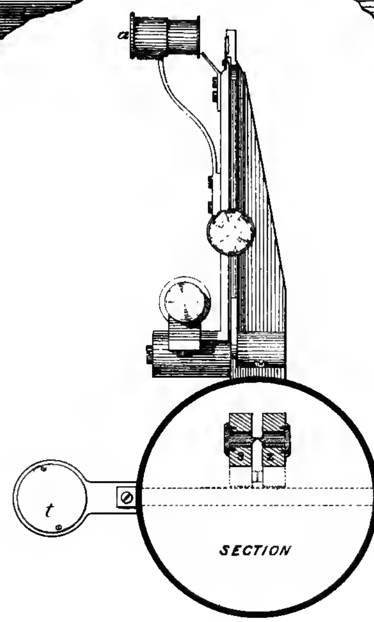


Fig. 6

TUBE near 1m. mark, on line *a b* fig 10.

Fig. 7

TUBE at 1 meter mark, on line *c d* fig. 10.

Fig. 8

TUBE at 3½ meter mark, on line *e f* fig. 9.

HEAD OF TUBE STAND.

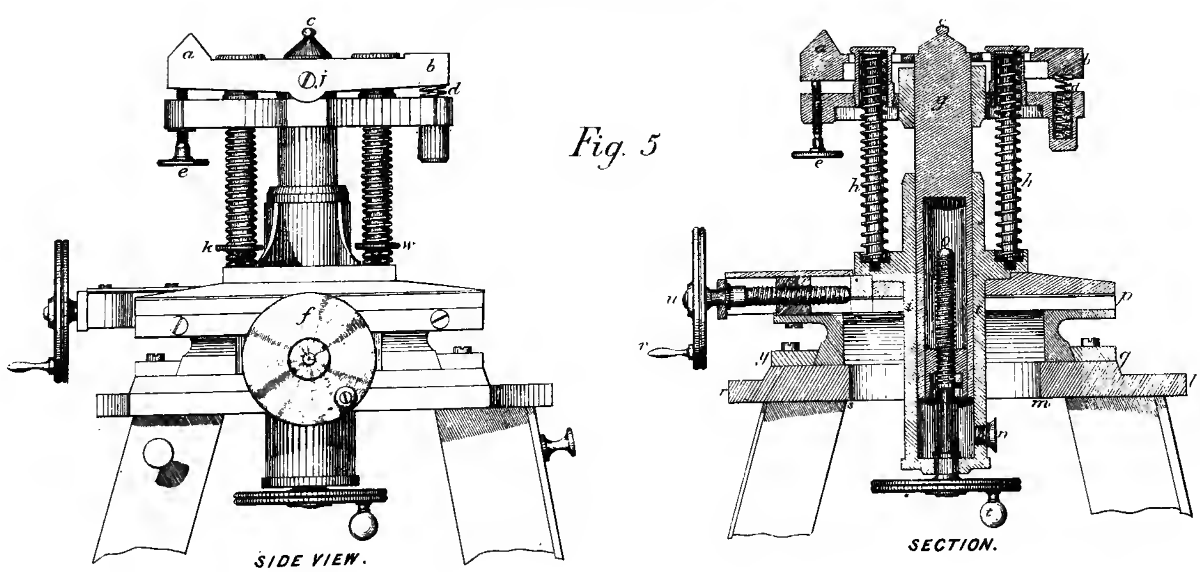
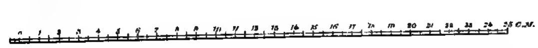


Fig. 5



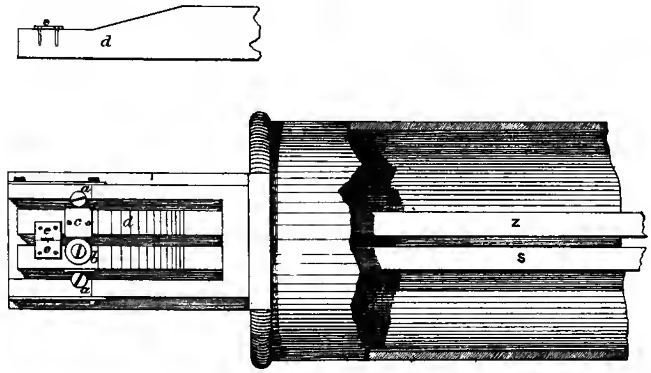
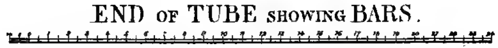


Fig. 15.

END OF TUBE SHOWING BARS.



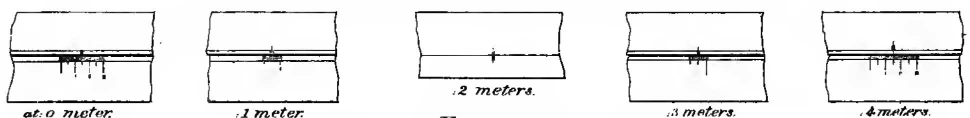


Fig. 14.

ENLARGED VIEWS OF GRADUATION.

END MARK OF BASE LINE.

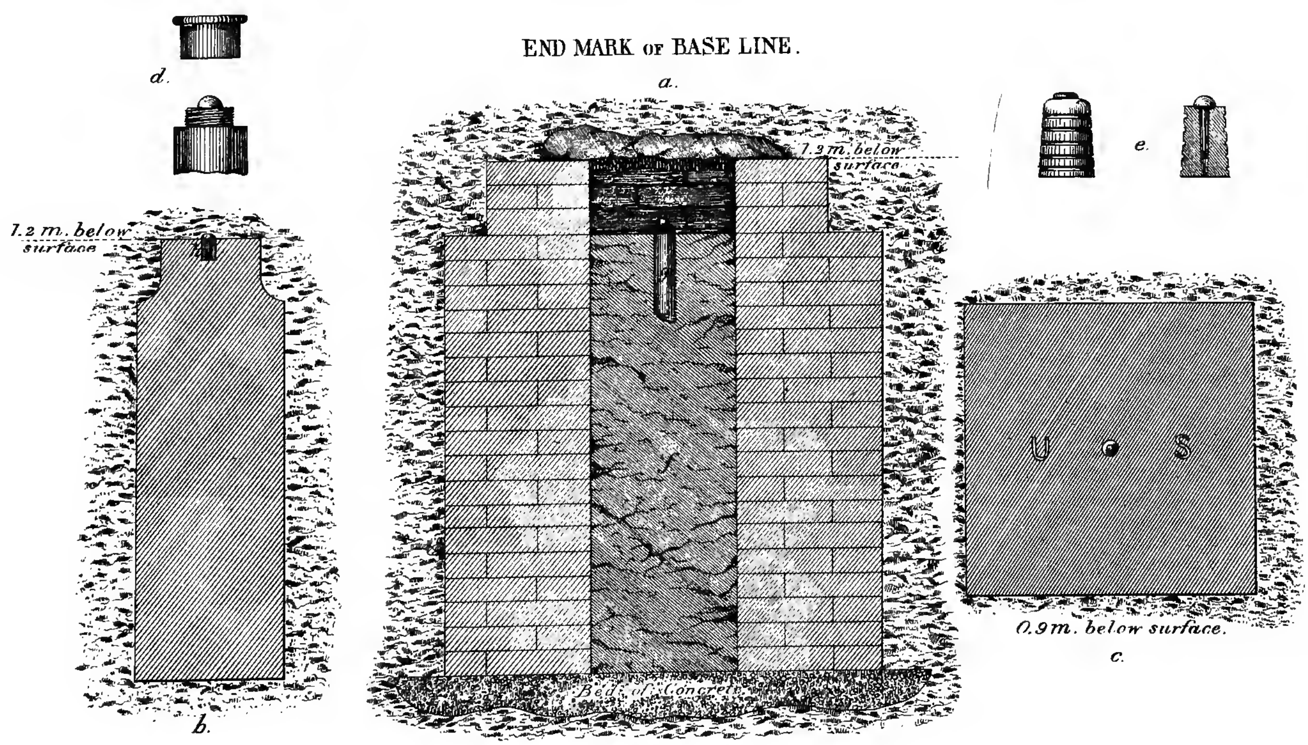
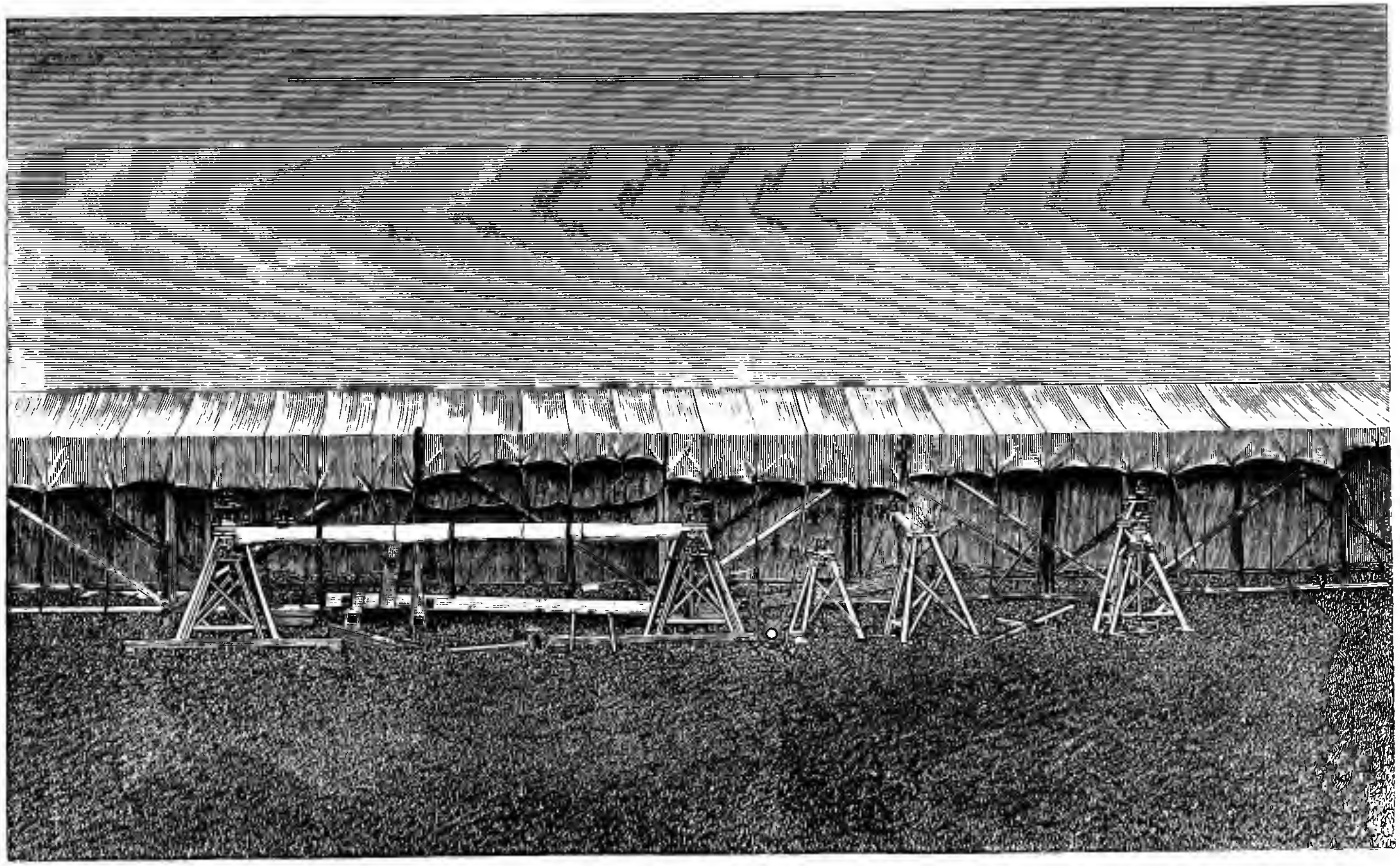
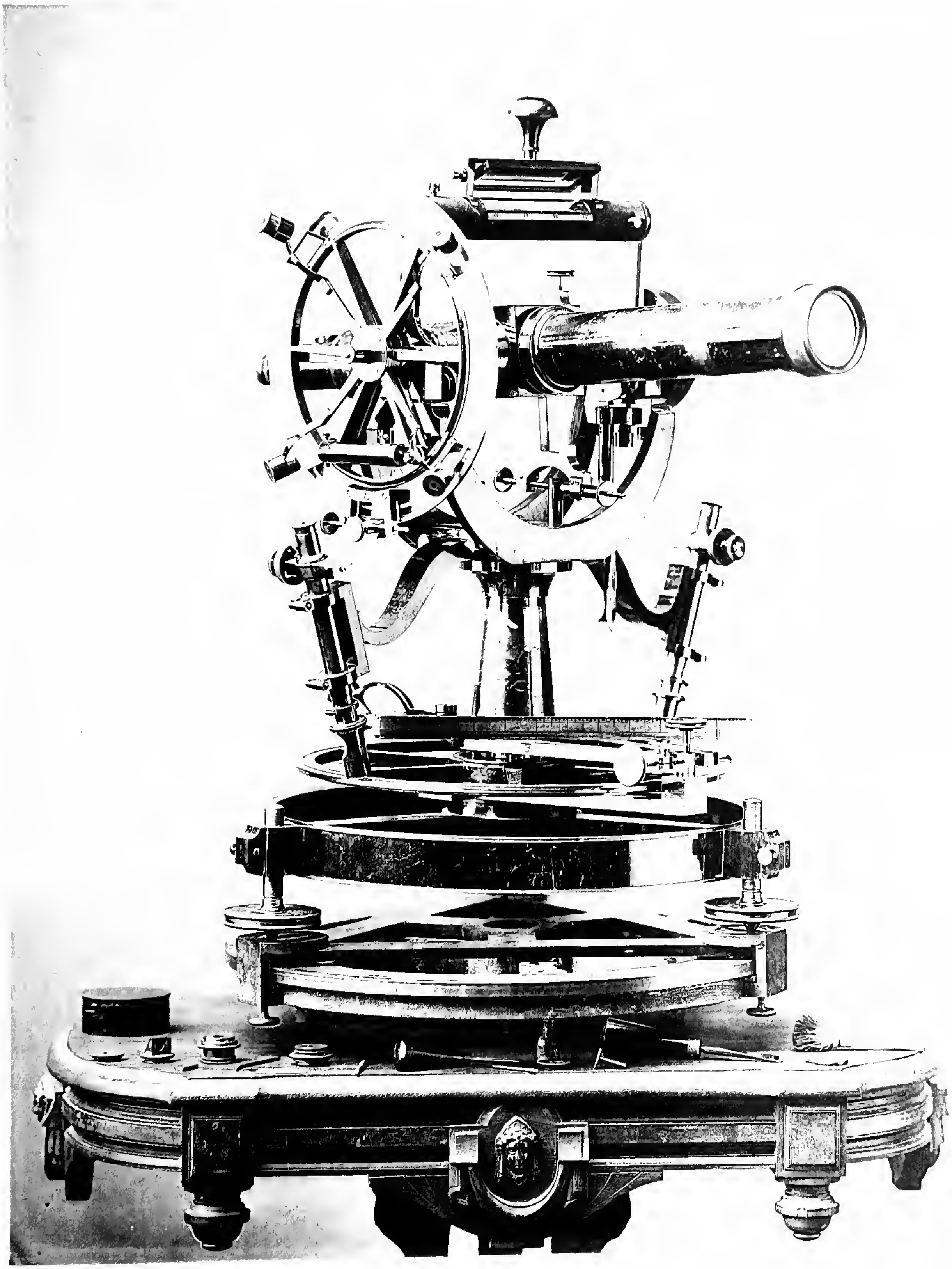


Fig. 13

PLATE XIII.

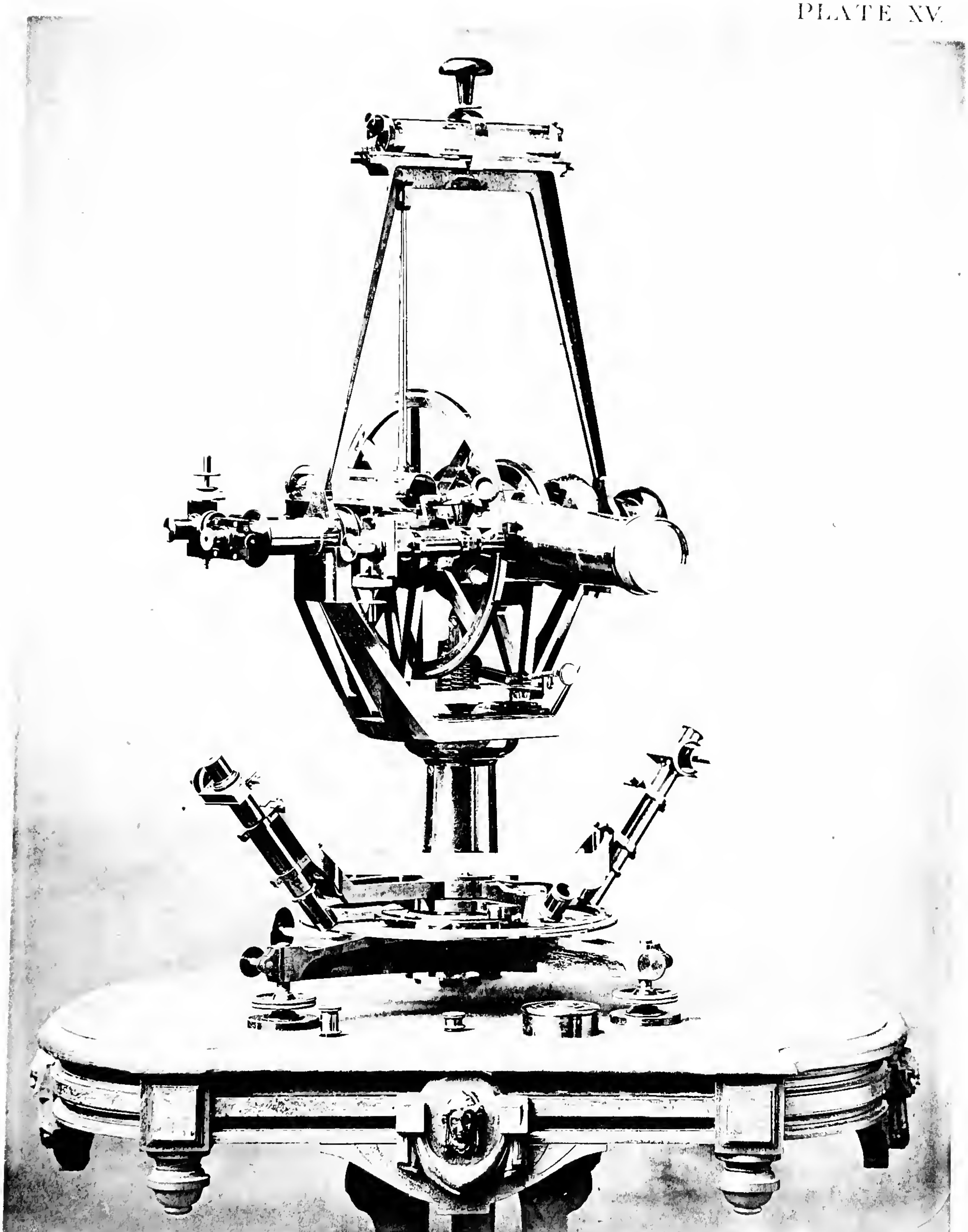


REPSOLD BASE MEASURING APPARATUS, AND AWNINGS.



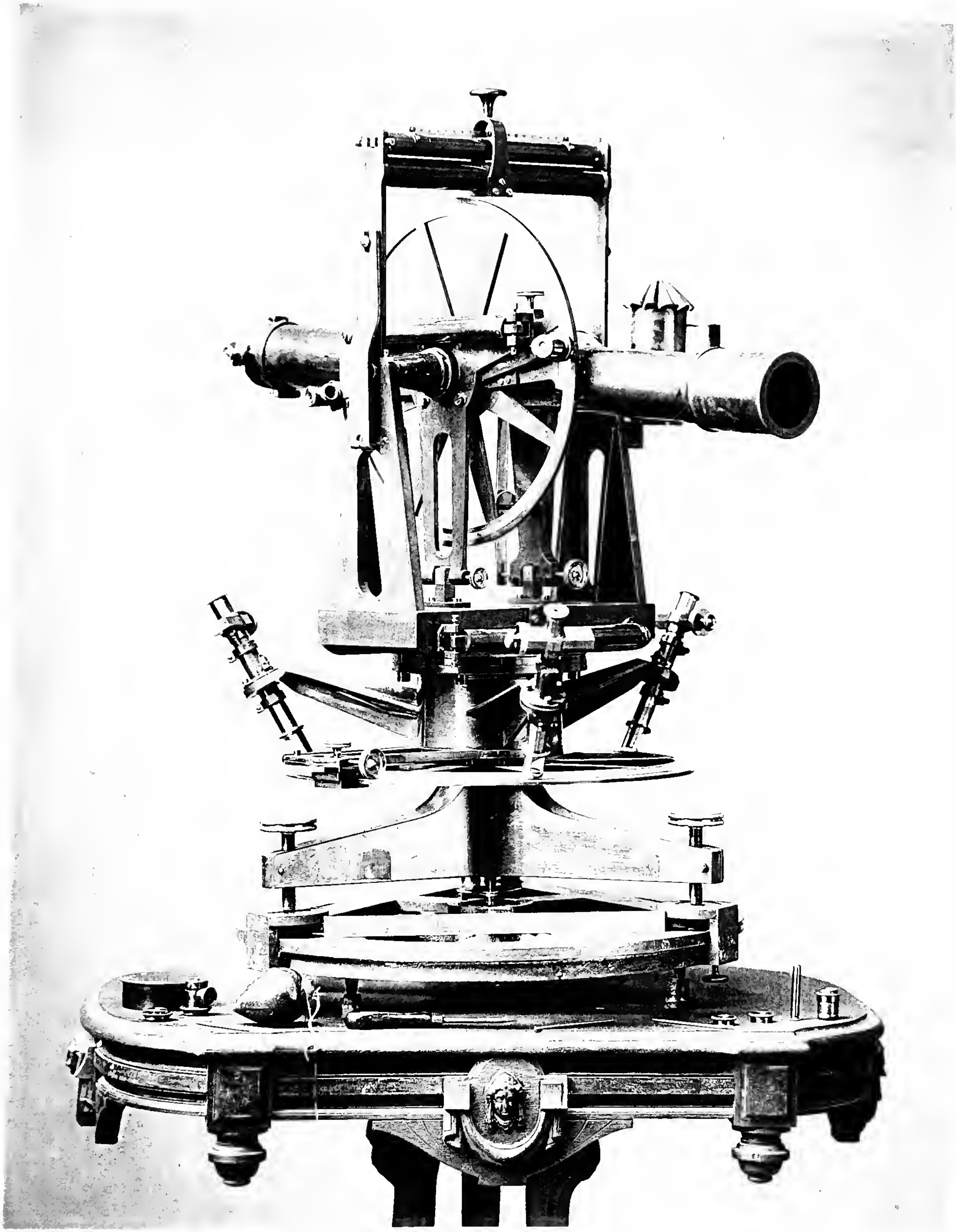
PISTOR & MARTINS

14-inch Theodolite No. 2



REPSOLD

10-inch Universal Instrument No. 1.



TROUGHTON & SIMMS

14-inch Theodolite N° 1

FIG. 16.

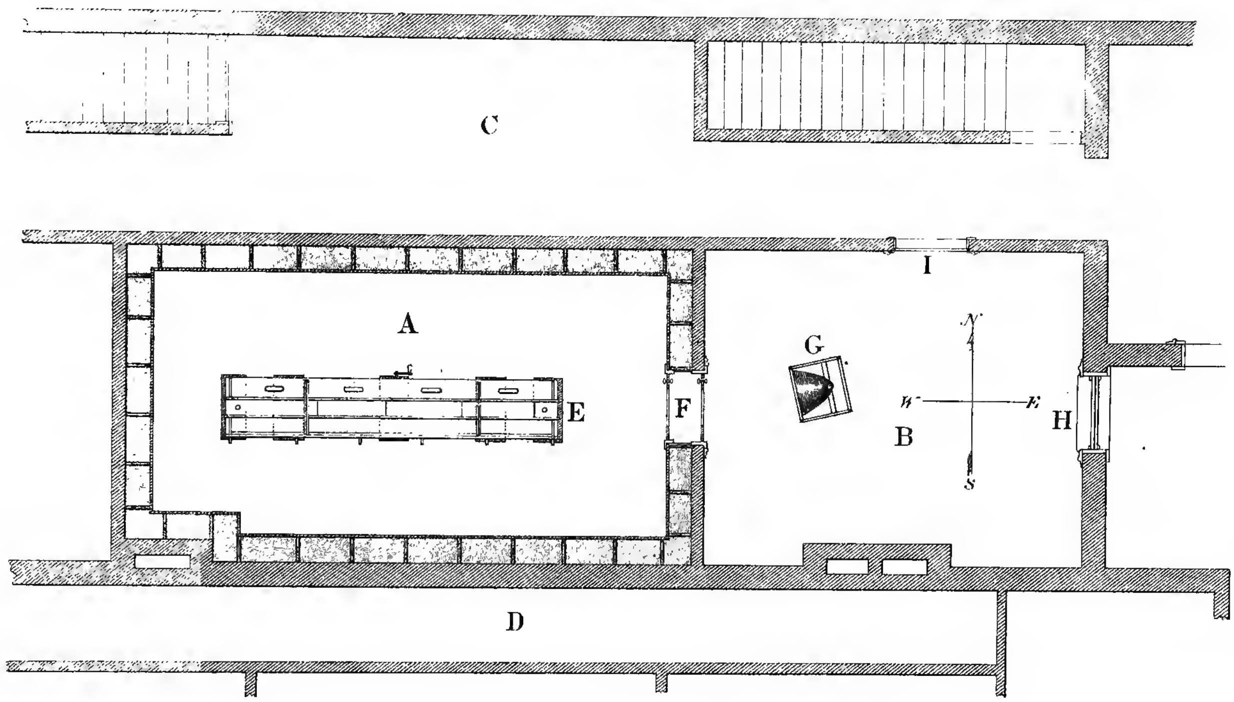
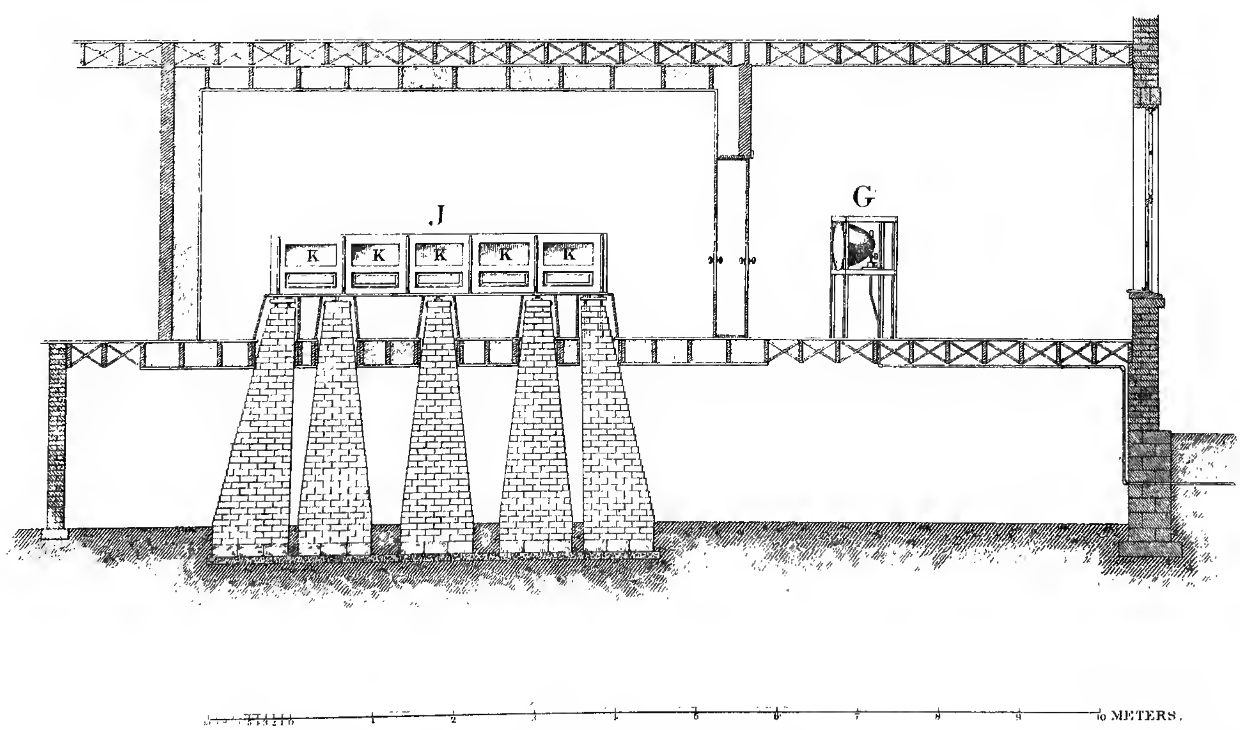


FIG. 17.



drawn by E. Molitor.

PLATE XVIII.

FIG. 18.

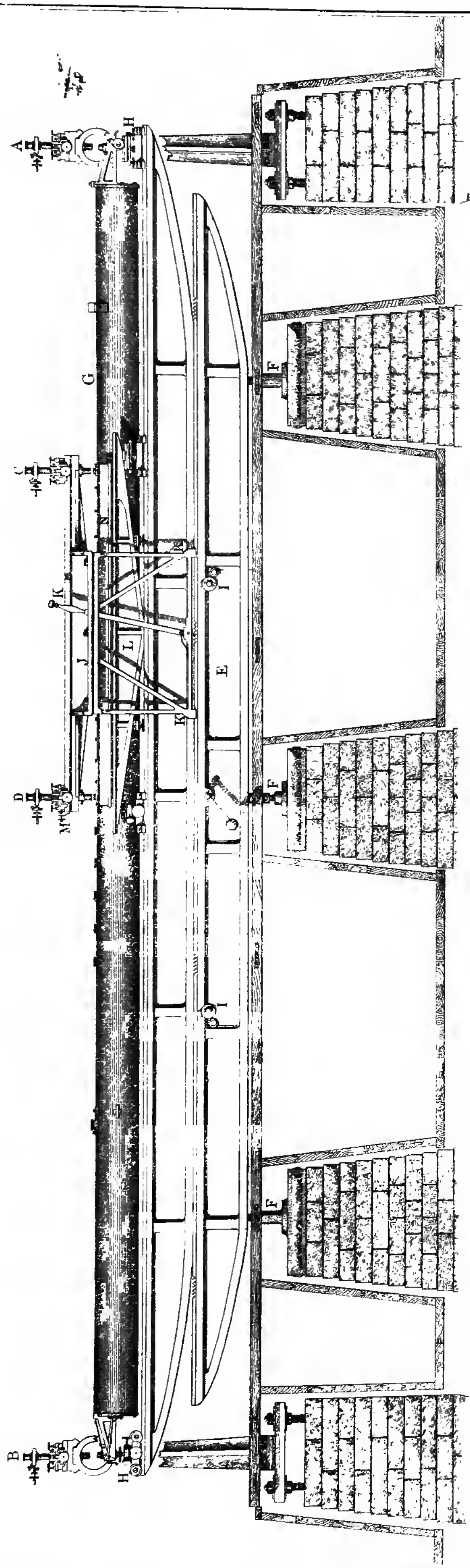
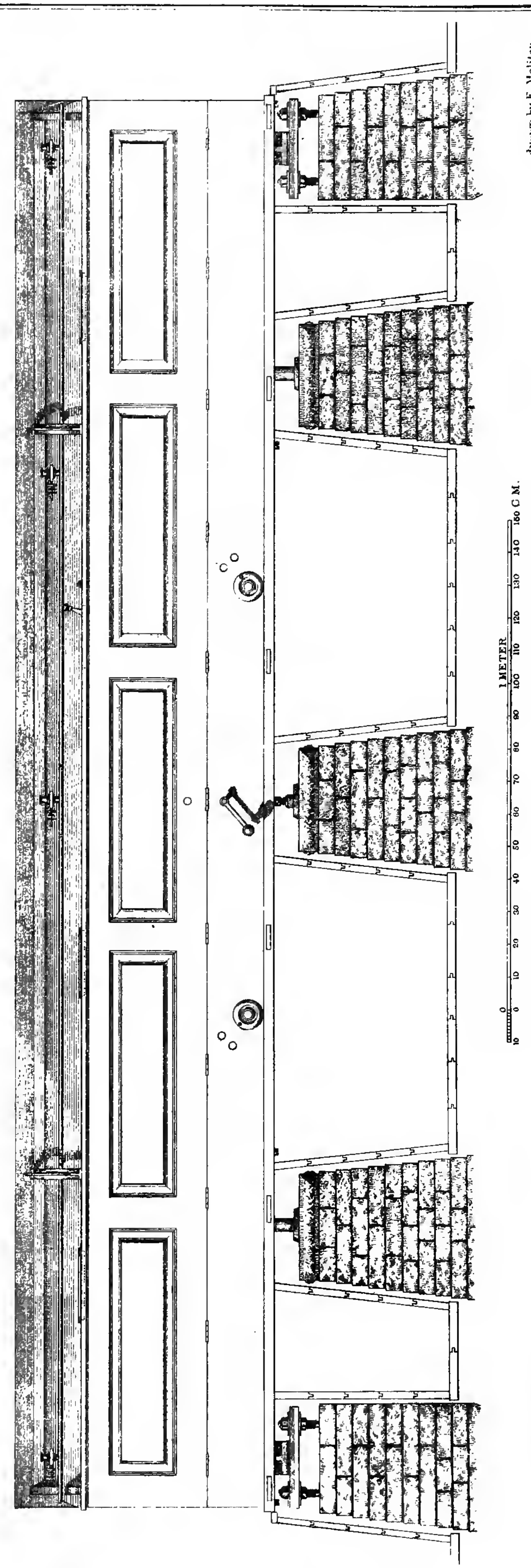


FIG. 19.



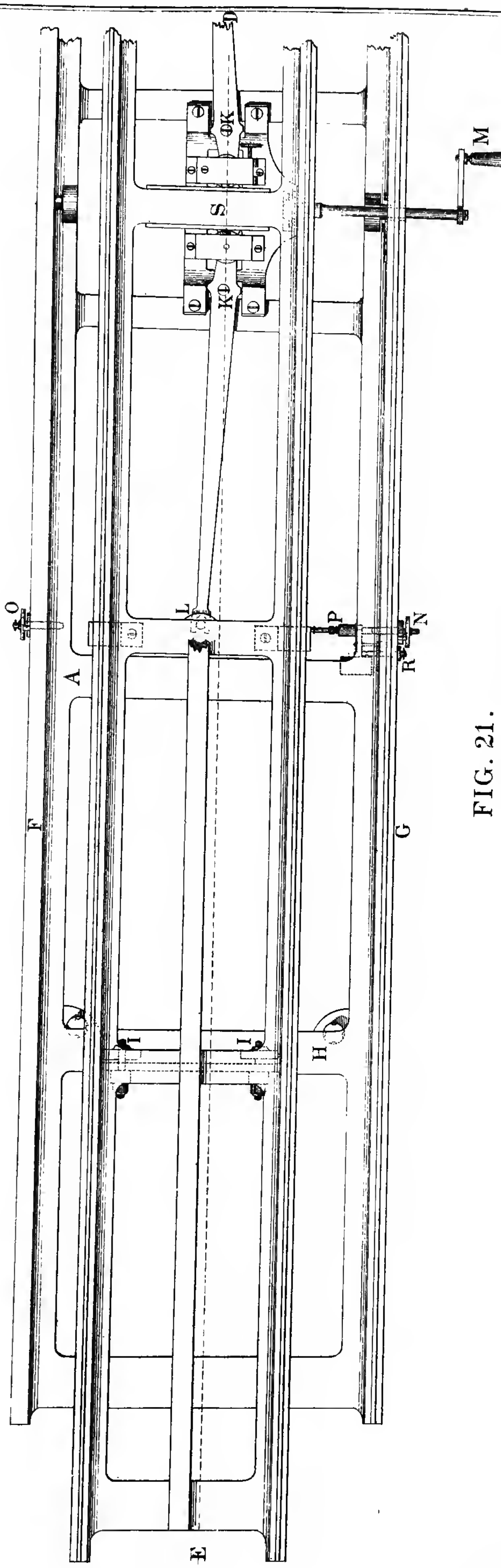


FIG. 20.

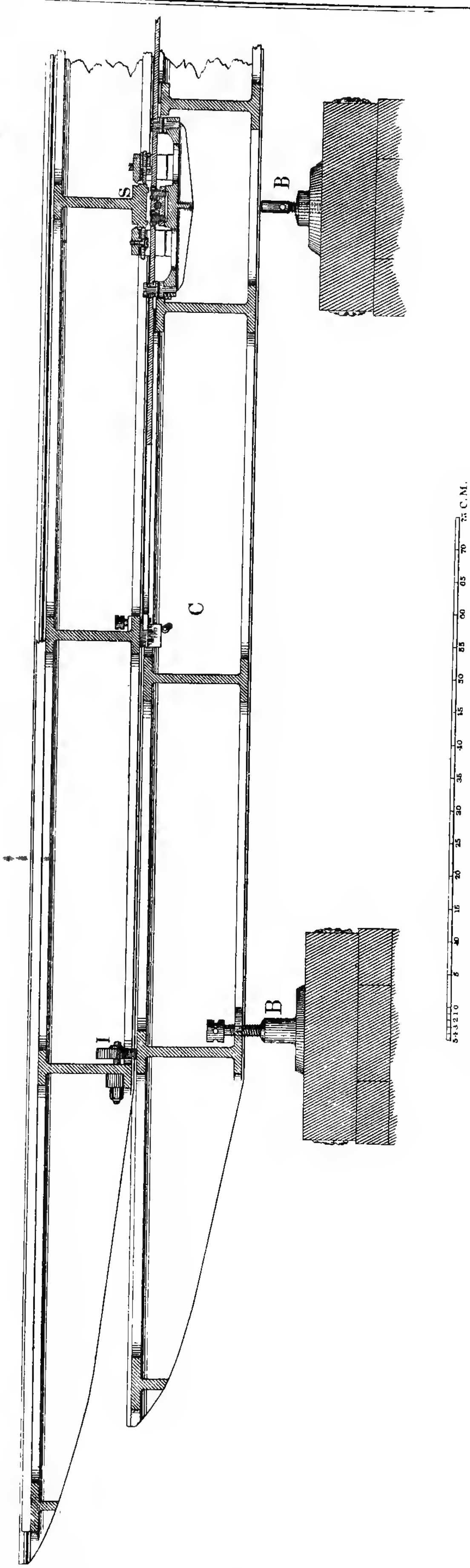


FIG. 21.

FIG. 22.

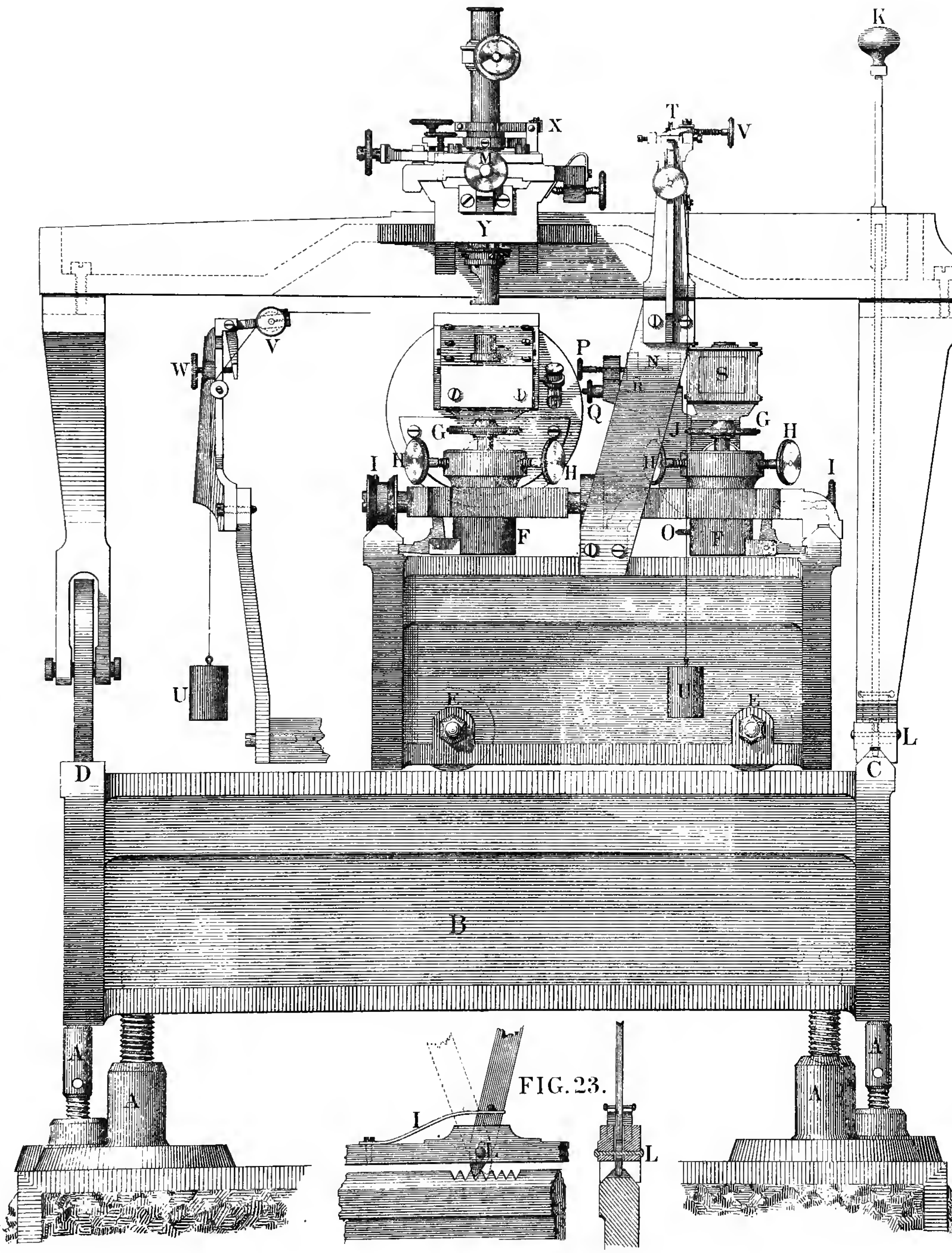


FIG. 23.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 C.M.

drawn by Edw. Molitor

PLATE XXI.

drawn by E. Moltier

FIG. 26.

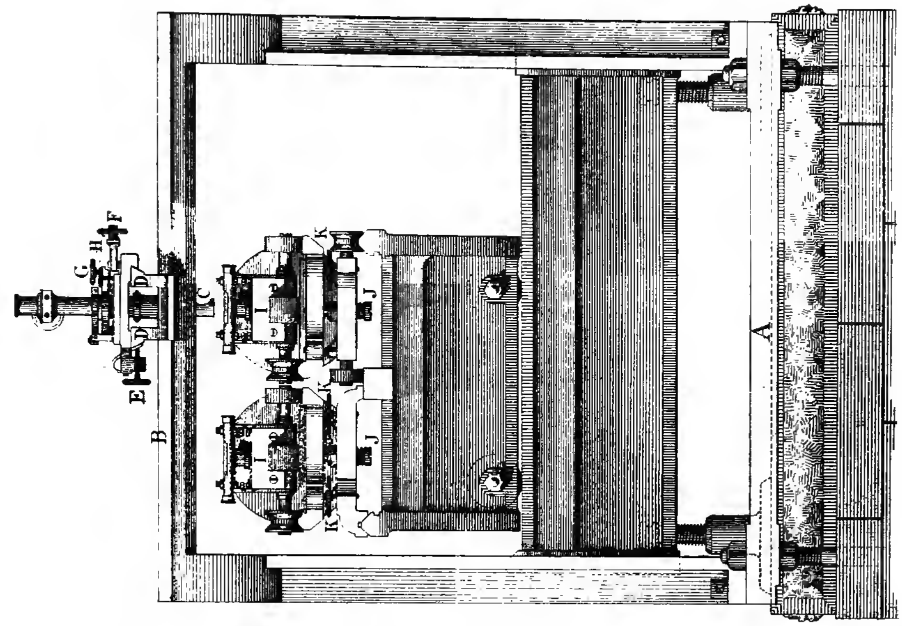


FIG. 25.

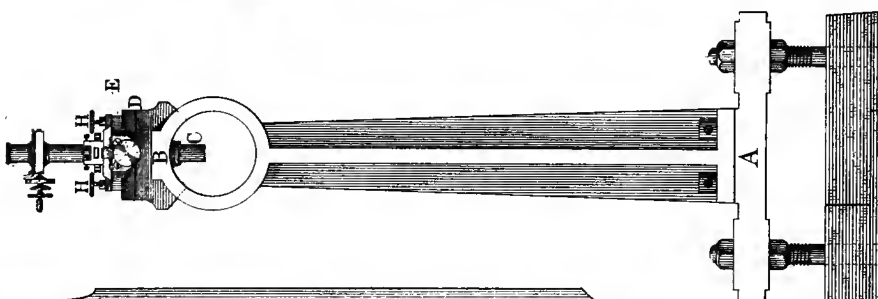


FIG. 24.

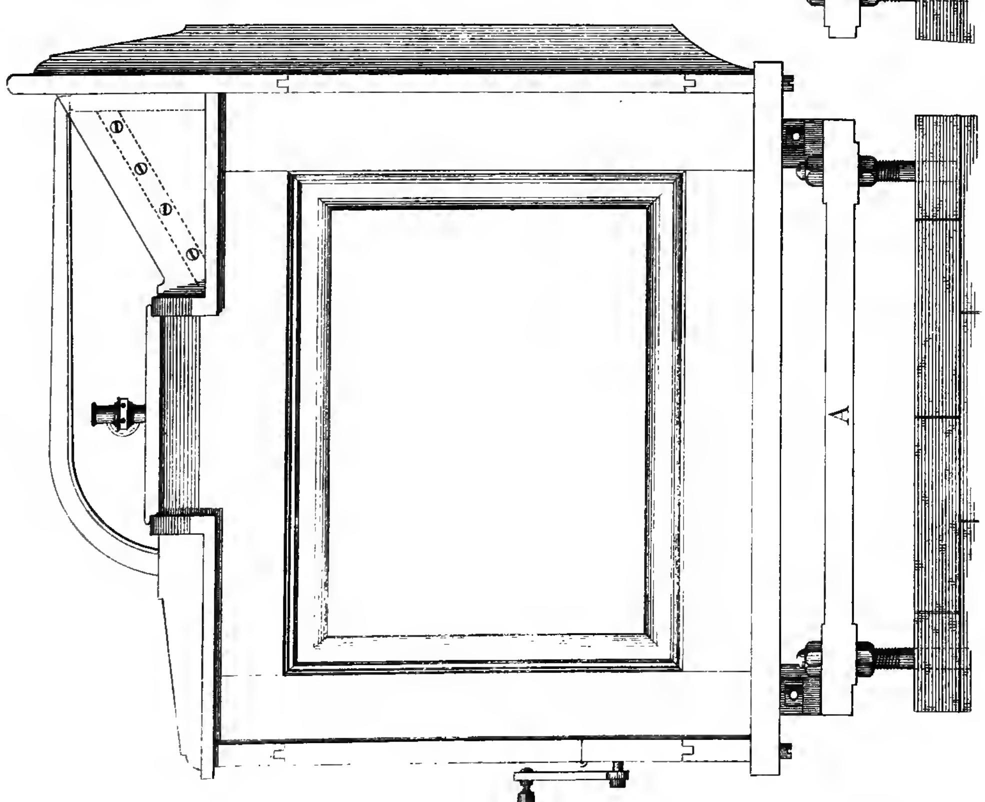


FIG. 27.

PLATE XXII.

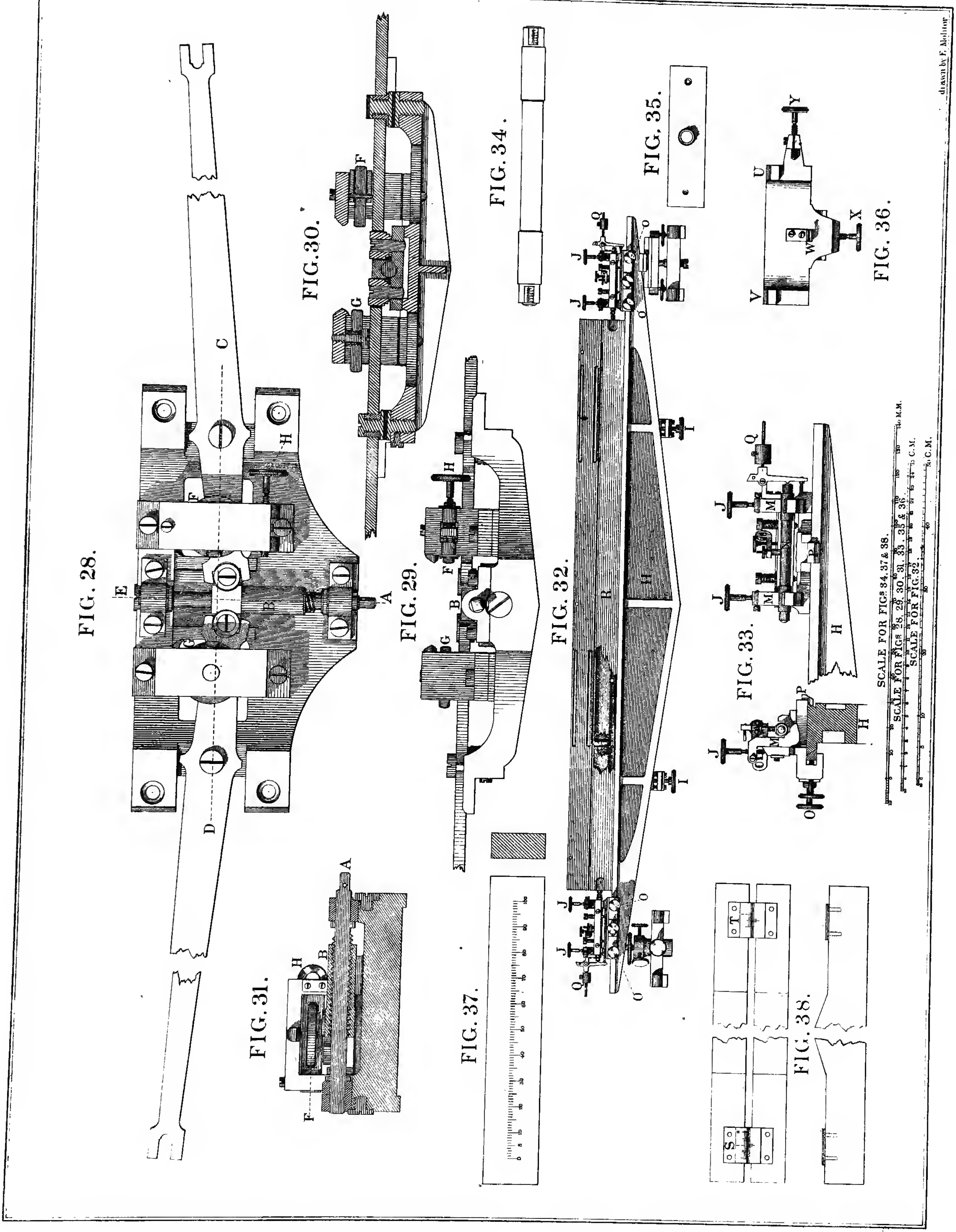
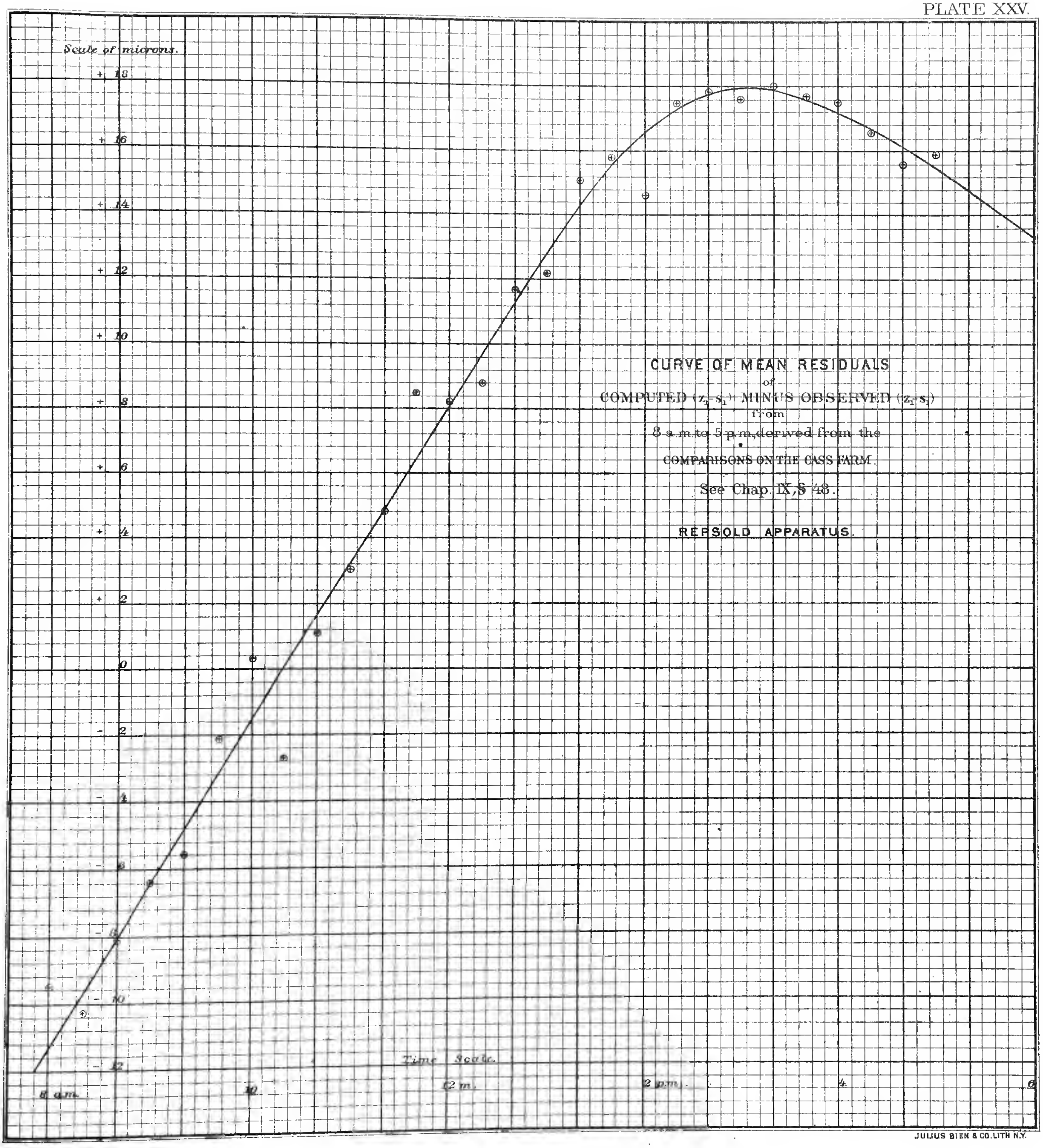


FIG. 28.
FIG. 29.
FIG. 30.
FIG. 31.
FIG. 32.
FIG. 33.
FIG. 34.
FIG. 35.
FIG. 36.
FIG. 37.
FIG. 38.
SCALE FOR FIGs 34, 37 & 38.
SCALE FOR FIGs 28, 29, 30, 31, 33, 35 & 36.
SCALE FOR FIG. 32.
drawn by F. Molitor



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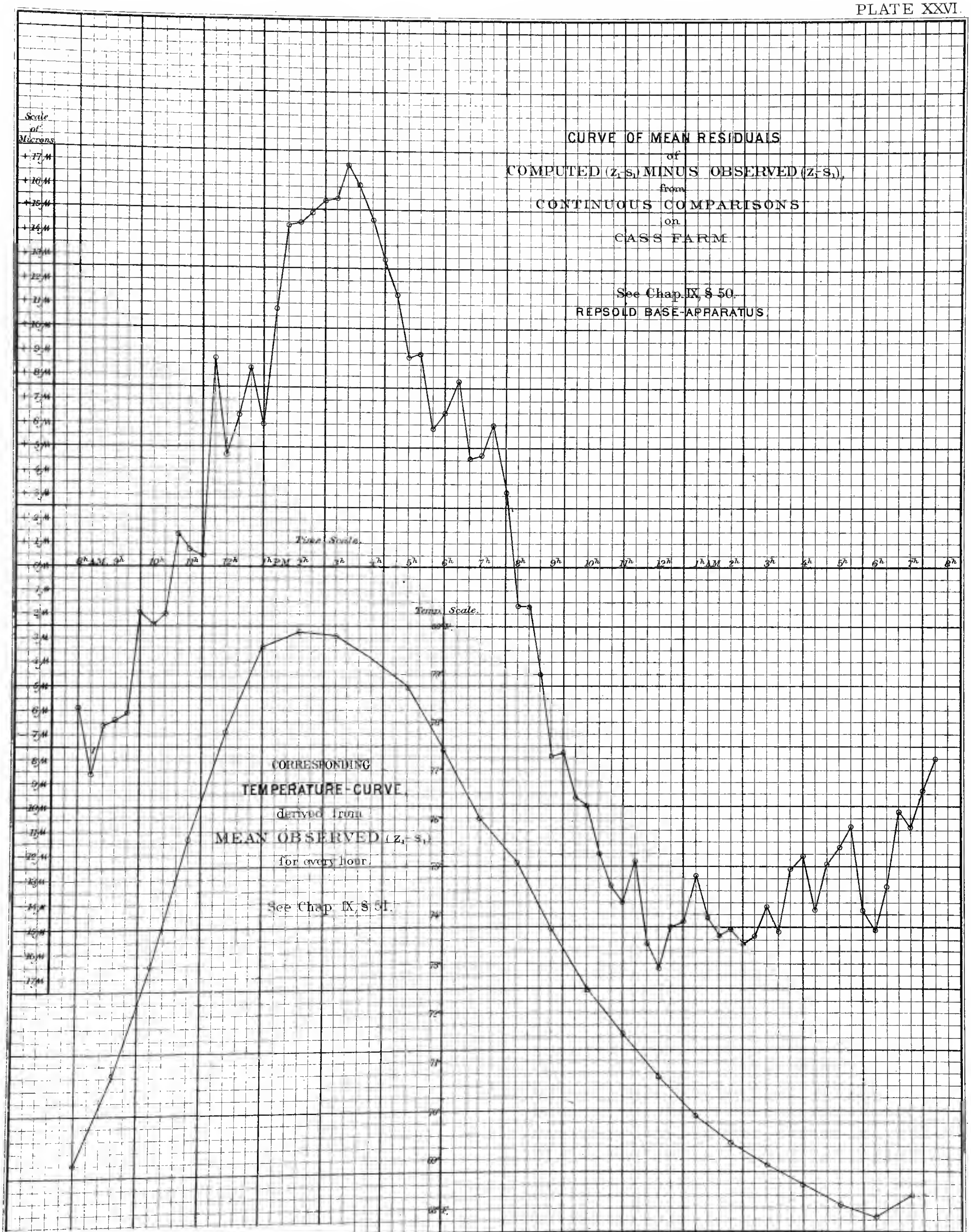
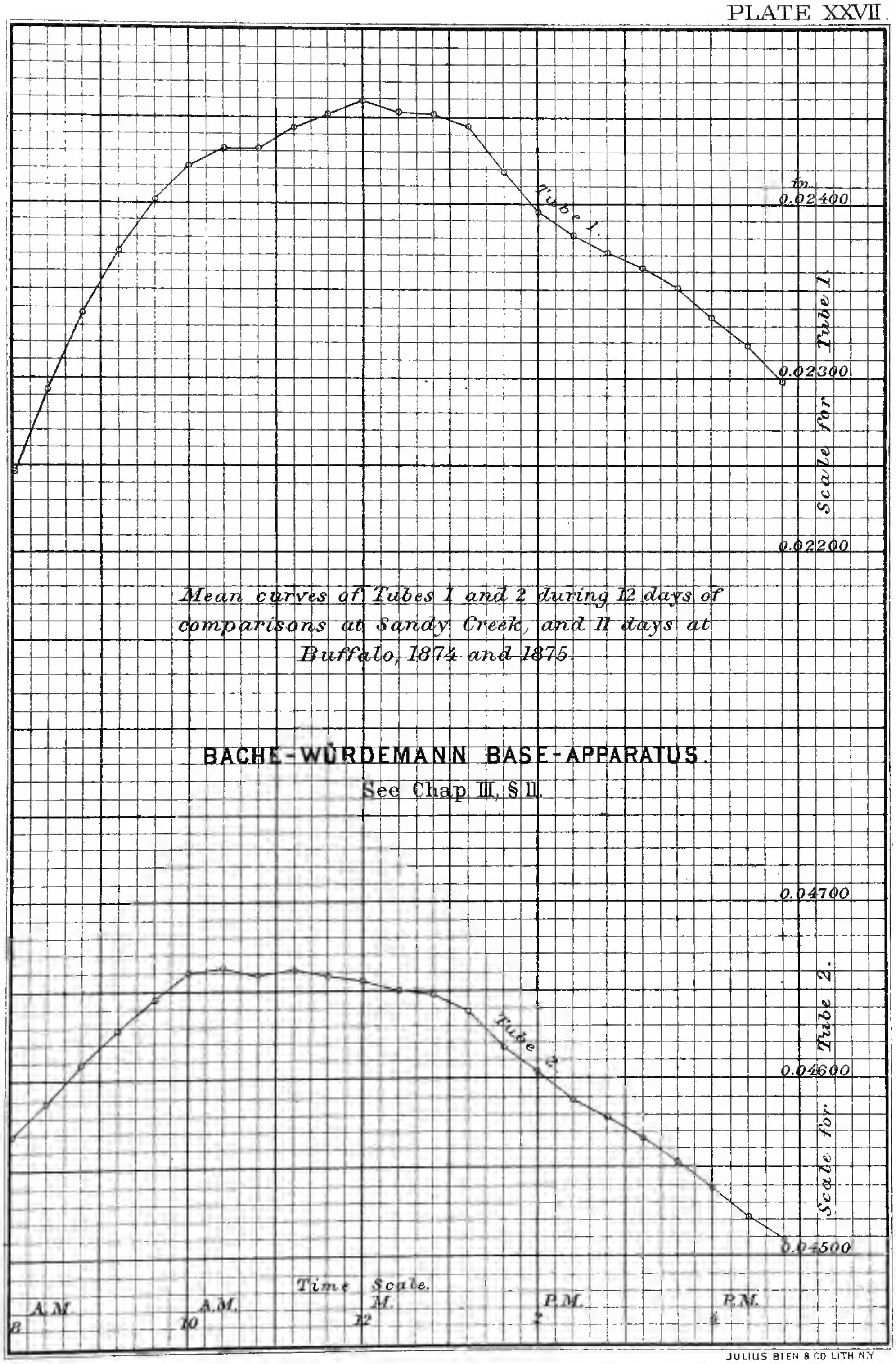


JULIUS BIEN & CO. LITH. N.Y.



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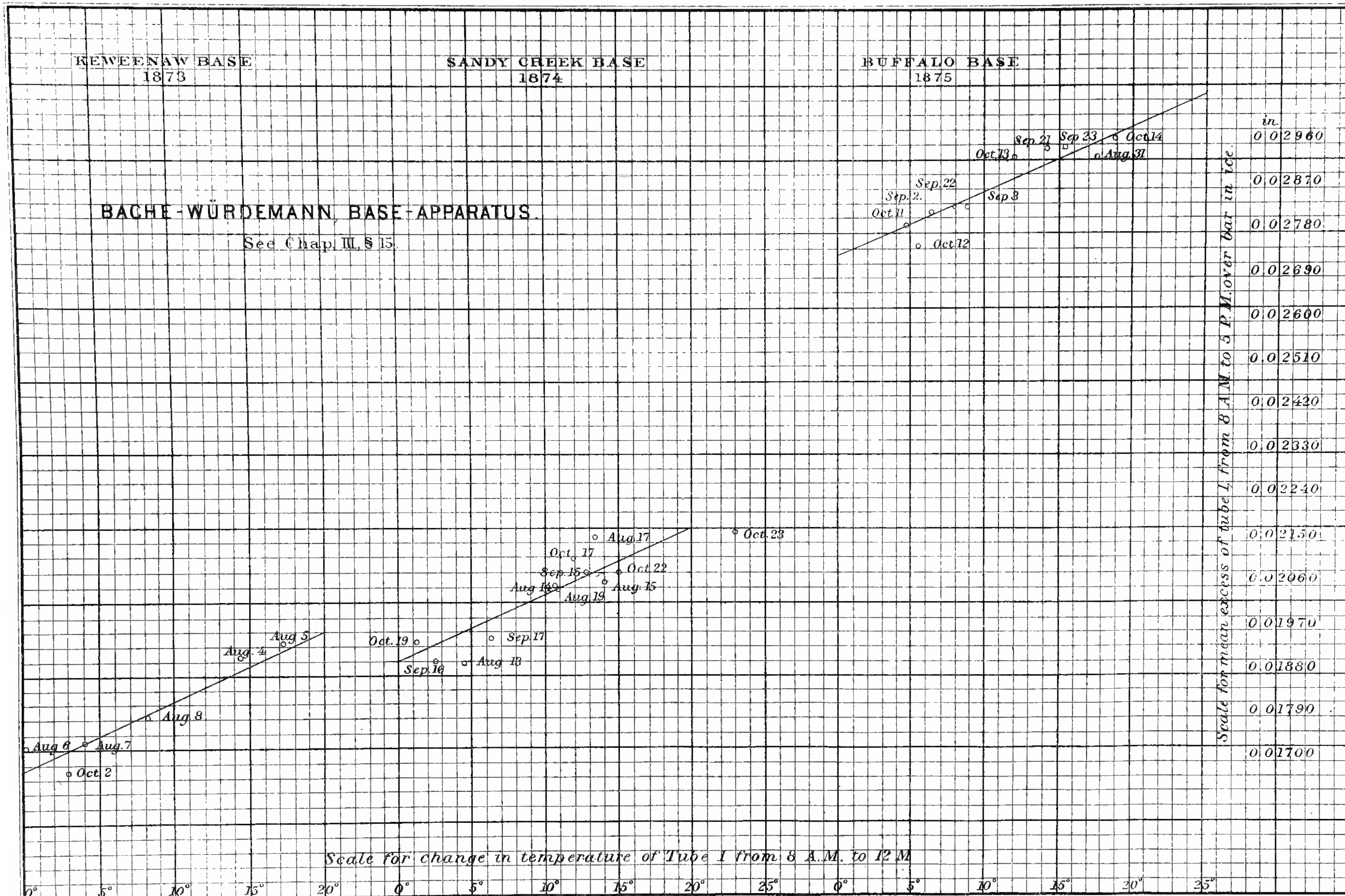

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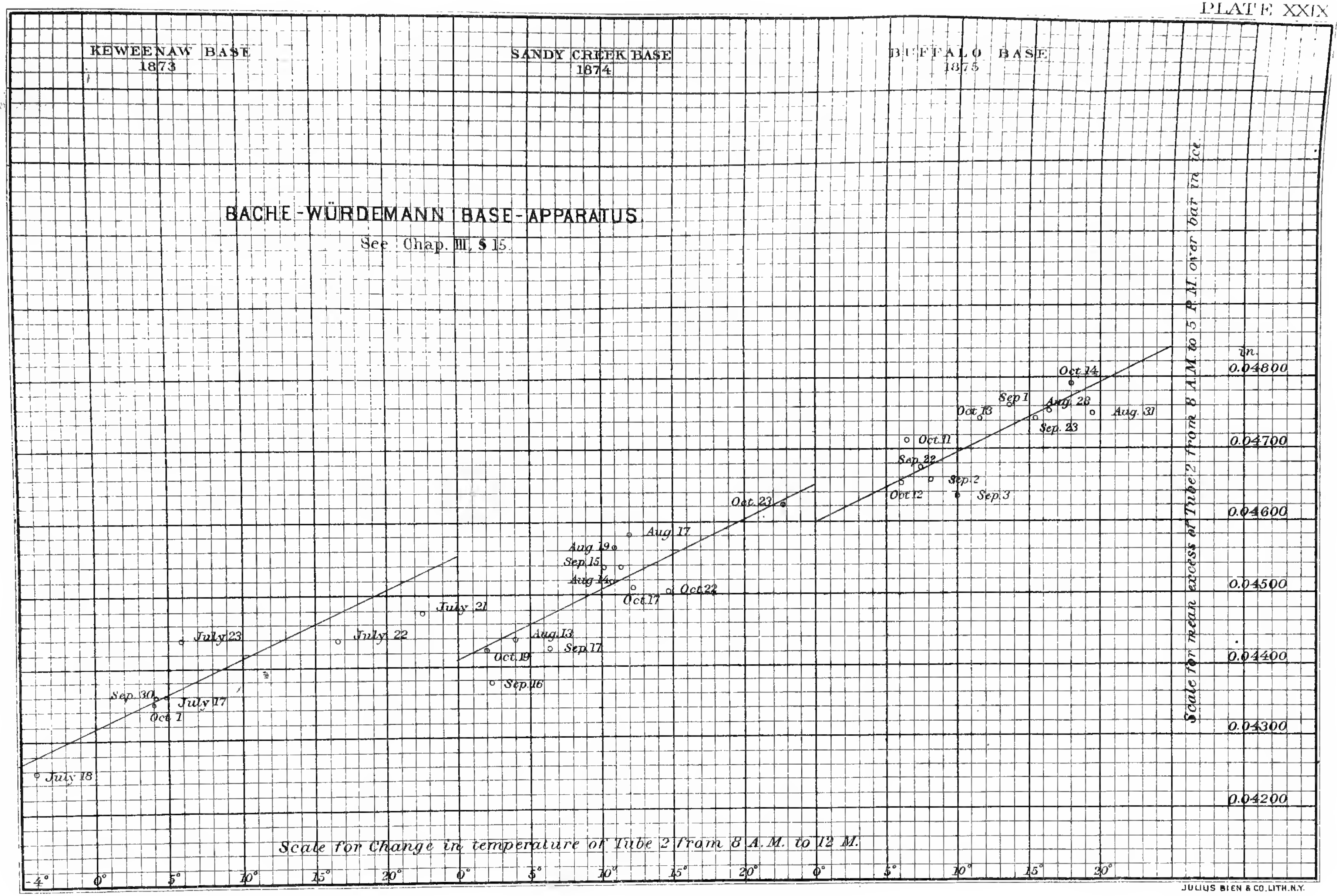

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